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Using Data Envelopment Analysis for the Efficiency and Elasticity Evaluation of Agricultural Farms

by

Kazim Baris Atici

A thesis submitted for the degree of Doctor of Philosophy

Operational Research and Management Science Group

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DECLARATIONS

I declare that I am responsible for the work submitted in this thesis, it is written by me and it has not previously been submitted within a degree programme at this or any other institution.

During the preparation of this thesis a number of publications and conference presentations have taken place stated below:

Publications

Chapter 4 is published as:

Atici, K.B., Podinovski, V.V. (2012) "Mixed partial elasticities in constant returns-to-scale production technologies", ***European Journal of Operational Research***, 220, 262-269.

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Available online at: http://fadntool.sggw.pl/Reviews/WorkPackages

Conferences

Kazim Baris Atici, Victor V. Podinovski "The Integration of Production Trade-offs in Farm Efficiency Evaluation with DEA and into Elasticity Measurement on DEA Efficient Frontiers", ***9th International Conference on Data Envelopment Analysis***, Thessaloniki, Greece, August 2011.

Kazim Baris Atici, Victor V. Podinovski "A Novel Use of DEA with Production Trade-offs for Efficiency and Elasticity Evaluation of Non-homogeneous Agricultural Farms", ***Asia-Pacific Productivity Conference (APPC)***, Bangkok, Thailand, July 2012.

ABSTRACT

Data Envelopment Analysis (DEA) is a well-established relative efficiency measurement technique, which has been widely applied to evaluate the technical efficiency of agricultural units in different countries by focusing on different aspects of agricultural production. This research deals with the evaluation of efficiency through DEA in non-homogeneous agricultural production, where units produce a wide range of different outputs. The objectives are threefold. Firstly, we propose a novel methodological approach of integrating the production trade-offs concept of DEA into non-homogeneous agricultural efficiency evaluation to prevent the overstatement of the efficiency of specialist farms and overcome the issue of insufficient discrimination due to large number of outputs in the models. Secondly, we aim to integrate this methodological perspective to the theory of elasticity measurement on DEA frontiers. The efficient frontiers of DEA are not defined in functional forms as in the classical economic theory, therefore obtaining elasticity measures on them require different considerations. We introduce the production trade-offs to the elasticity measurement and derive the necessary models to calculate the elasticities of response in the presence of production trade-offs. As a third objective, before moving to the introduction of the trade-offs in elasticity measurement, for theoretical completeness, we first consider the elasticity measurement on DEA frontiers of constant returns-to-scale (CRS) technologies. Our proposed methodology and all the developed elasticity theory are illustrated in a real world case of Turkish agricultural sectors. We provide extensive empirical applications covering all the proposed theory and methodology. Among the results of this research, we provide an elasticity measurement framework, which enables us to calculate elasticities of response measures in both VRS and CRS technologies, with or without production trade-offs included. We observe that the integration of production trade-offs provide better discrimination of efficiency scores compared to the models without trade-offs included. We also investigate how changing production trade-offs affect the efficiency and elasticity measures of the evaluated units.

ABBREVIATIONS

BCC: Banker Charnes Cooper

CCR: Charnes Cooper Rhodes

CEEC: Central and East European Countries

CRS: Constant Returns-to-Scale

DEA: Data Envelopment Analysis

DMU: Decision Making Unit

DRS: Decreasing Returns-to-Scale

FADN: Farmer Accountancy Data Network

GAMS: General Algebraic Modeling System

IO: Input-oriented

IPA: Instrument for Pre-Accession Assistance

IRS: Increasing Returns-to-Scale

LHE: Left-hand Elasticity

LP: Linear Programming

MPI: Malmquist Productivity Index

MRS: Marginal Rates of Substitution

NA: Not Applicable

OO: Output-oriented

OR: Operational Research

PPS: Production Possibility Set

RHE: Right-hand Elasticity

RTA: Returns to changing set *A*

RTS: Returns-to-scale

SFA: Stochastic Frontier Analysis

TL: Turkish Lira

TO: Trade-off

UD: Undefined

VRS: Variable Returns-to-Scale

WTO: With Trade-offs

Agricultural Regions in Turkey

AEG: Aegean

EBS: East Black Sea

EM: East Marmara

MA: Middle Anatolia

MEA: Middle East Anatolia

MED: Mediterranean

NEA: North East Anatolia

SEA: South East Anatolia

WA: West Anatolia

WBS: West Black Sea

WM: West Marmara

Chapter 1

Introduction

Performance measurement and benchmarking are important tools for improvement in highly competitive and rapidly changing business environment of our era. Clearly, performance measurement concept is related with the measurement and improvement of efficiency to a great extent. Investigation and further analysis of worst and best performers in a business environment play a key role in deriving useful information to understand the current state of the processes and to identify the opportunities for improvement in efficiency. Several modelling methods can be found in Economics and Operational Research (OR) literature attempting to assess efficiency of different types of business operations, units or processes.

In general, methods of efficiency measurement are based upon the estimation of a production frontier and can be classified into two basic groups as parametric and non-parametric approaches. Parametric frontiers are based on specific functional forms and can be either deterministic or stochastic. On the non-parametric efficiency evaluation side, Data Envelopment Analysis (DEA) is a well-established method aiming to identify relative efficiency of Decision Making Units (DMUs) producing multiple outputs through the use of multiple inputs. It does not require any assumptions about the functional form. The efficiency of a DMU is measured relative to all other DMUs with the simple restriction that all DMUs are members of a production possibility set and they lie on or below an efficient frontier.

DEA has rooted from the study of Farrell (1957) and presented to the OR literature by the seminal paper of Charnes et al. (1978). Generally, a Decision Making Unit (DMU) in DEA is regarded as the entity responsible for converting inputs into outputs and whose performances are to be evaluated. In managerial applications, DMUs may include various private and public sector entities such as banks, department stores, supermarkets, hospitals, schools, public libraries and so forth (Cooper et al., 2006). Since the introduction of the method, various theoretical and methodological improvements have been carried out in order to bring new and advanced approaches to the DEA methodology.

1.1. Motivation and Objectives of The Research

Data Envelopment Analysis (DEA) and related methodologies have been widely applied to evaluate the technical efficiency of agricultural establishments in different countries by focusing on different aspects of agricultural production. In agricultural efficiency evaluations, often, the decision making units considered (although they are located in the same region) can be non-homogeneous in terms of production, in other words, they can be producing more than one type of outputs within the production scope of the same unit. Since, some inseparable resources are devoted to generate all these outputs, it is not possible to ignore the production of some, even though not all units produce them. One interested in efficiency evaluation of non-homogeneous farms through DEA methodology should handle the question of how to measure the agricultural output carefully, since potential complications can arise with different approaches.

One way to deal with outputs in non-homogeneous farms can be integrating physical production amount of agricultural products as separate outputs since DEA method is capable of handling multiple inputs and outputs. In cases where the agricultural production in the evaluated establishments is not widely varied and sample size is large enough, this approach can work without causing an insufficient discrimination problem of efficiency scores. On the other hand, in a very non-homogeneous sample, where a large variety of outputs are produced, inclusion of too many variables (outputs) into DEA models can lead to a insufficient discrimination. Moreover, in several cases, farms producing some specific types of outputs, which are not produced in many of other farms, can gain an advantage and have overestimated efficiency scores, since high level of weights will be attached to such outputs at the unit producing them because other farms would be producing such outputs at the level of zero.

Another way to deal with outputs in evaluating non-homogeneous farms can be the use of aggregated monetary terms (for instance, revenues obtained by the farm through all different products) on the output side as practiced in several previous agricultural efficiency studies[1]. However, under such consideration, a potential drawback can occur related to the prices of the outputs. Price differences between products or price fluctuations depending on other economic forces in the market can considerably affect the monetary value of agricultural production. In this case, models can still lead to an advantage for farms producing specific outputs, which are highly priced relying on the market conditions. Besides, when prices are involved in the evaluation process, the analysis gets apart from assessing the pure production aspect of the units. Nevertheless, using such an approach will not let us to calculate the elasticity of responses for different types of agricultural production since all outputs are aggregated and they are in monetary terms.

We have a real world case in Turkish agricultural farms fitting with above discussions of non-homogeneity in terms of production. The project of establishing a Farm Accountancy Data Network (FADN) database has been started recently in Turkey by the Ministry of Agriculture to accord with the European agricultural policies and initiated by a pilot data collection from several farms all over the country[2]. We begin the research with the objective of evaluating the technical efficiency of the Turkish agricultural farms in this FADN data set and furthermore carry on with the calculation of elasticities of response between different sets of inputs and outputs. Considering above drawbacks of using monetary terms in agricultural evaluations, for our case, we decided that it is more reasonable to use physical production amounts of farms as separate outputs for eliminating the effects of prices and

[1] We provide a review on efficiency measurement in agriculture in Chapter 3.

[2] The specific details about the Turkish FADN data set are given in Chapter 6.

evaluating the pure agricultural production, also to be able to calculate the elasticities of response for different products. However, discrimination problem still remains as an issue to overcome.

In the FADN data set, to avoid the non-homogeneity in terms of environmental factors (such as geography, soil quality, weather or socio-economic differences) and to compare farms with the farms under similar conditions, we rely on the regional classification of the Ministry of Agriculture. In terms of statistical data organizations, the agricultural policy makers in Turkey divided the country into 12 agricultural regions depending on the several factors. In our evaluation, the idea is to evaluate the farms in FADN database in their own regions to avoid the complications that can arise regarding the non-homogeneity of environmental factors. Although non-homogeneity of environmental factors is eliminated through sampling, we still observe a high level of non-homogeneity between the farms of same region, this time, in terms of production. Farms in the data set are mainly crop producers and they produce different types of crops in considerable amounts, the production range is quite widespread and not all farms are producing all crops. Our preliminary analysis revealed the abovementioned discrimination problems, because too many outputs were included in the models. Therefore, it is essential to look for ways of overcoming the problem of insufficient discrimination of efficiency scores that arises in the case of non-homogeneous farms in our data set and can arise in other applications of agricultural efficiency.

Following the above discussions, in this research, first of all, we suggest a novel methodological approach in agricultural efficiency evaluation of non-homogeneous farms through DEA. As mentioned above, we intend to include the production amounts of outputs in each farm as separate output variables and the objective is to avoid the discrimination or overstatement problems of efficiency scores. For this purpose, we aim to establish the relevant production relationships between different types of agricultural production and integrate them into DEA models. It is basically adding more information to the models reflecting the nature of the technology. Such an aim can be achieved through the use of the production trade-offs concept in DEA models, which is introduced by Podinovski (2004a). As discussed more in detail throughout the dissertation, production trade-offs in DEA context can be defined as *'technological judgements representing possible simultaneous changes in the inputs and outputs under the technology considered'* and they can be translated into weight restrictions (Podinovski, 2004a). The production trade-off relations between different types of outputs can be obtained through the expert judgements in the evaluated sectors. These judgements are technological rather than preferences. Unlike in the standard use of weight restrictions in DEA context, which are based on value judgements, in the use of production trade-offs, the technological meaning of the efficiency measures being a radial improvement factor for inputs and outputs is preserved.

The logic behind the proposed methodology is to relate all types of production in an agricultural farm to a base output, which is produced in all units. It can be achieved through the establishment of proper production trade-off relations between outputs and the base output. Such approach will provide restrictions on weights attached to the outputs and therefore provide a better discrimination of efficiency scores, as well as the technological meaning will be preserved.

In recent DEA literature, there is a remarkable interest on measurement of elasticity of response on the DEA frontiers. Actually, since the early literature of DEA, effects of relative changes in outputs compared to the relative changes in inputs has always been an issue to be investigated[3]. However, the investigations were focused merely on the qualitative nature through the identification of Returns-to-Scale (RTS) (Banker et al., 1984; Banker, 1984, Seiford and Zhu, 1999; Banker, 2004). Subsequently, the research interest has shifted towards the quantification of RTS through calculation of scale elasticities (Førsund, 1996;

[3] We provide a comprehensive review of DEA theory and its related issues in Chapter 2.

Førsund and Hjalmarsson, 2004; 2007) and more progressively towards the calculating elasticities of response of any output or input sets to the changing any of inputs and outputs (Podinovski et al., 2009; Podinovski and Førsund, 2010). Following these efforts, the economic notion of elasticity is adapted to the DEA methodology to strengthen a contact with the field of Economics. This tendency towards the elasticity measurement also drew our attention and brought up an additional research direction of matching our proposed methodology with the elasticity measurement on DEA frontiers.

Obviously, the introduction of the trade-off relations between outputs brings new constraints to the DEA models and causes changes in the production possibility set considered and the efficient frontier obtained. Since in DEA, the frontiers are not defined in functional forms as in the classical economic theory, obtaining elasticity measures on DEA frontiers require different considerations, which has been studied in the DEA literature from different perspectives. Introduction of trade-off relations brings up a need for theoretical modifications in the existing models. The necessity of developing the models on how the elasticities of response between different sets of inputs and outputs are calculated when the production trade-offs are present led us to a novel theoretical direction. Therefore, we establish our second objective as deriving a novel theoretical approach to calculate the elasticity scores on DEA frontiers in the presence of production trade-offs and apply this approach to our evaluation of agricultural farms case.

In general, DEA models have two main considerations of returns-to-scale (RTS) as constant (CRS) and variable (VRS). Different RTS assumptions require different modelling of DEA models and the technology dealt is different[4]. Currently, Podinovski and Førsund (2010) have developed the elasticity measures on DEA frontiers of variable returns-to-scale (VRS) technologies. Before moving to the introduction of the trade-offs in elasticity measurement, for theoretical completeness, we first consider the elasticity measurement on DEA frontiers

[4] Discussed in Chapter 2 more in detail.

of constant returns-to-scale (CRS) technologies. It becomes our third main objective. For a proper presentation, we begin our developments with the elasticity models for CRS technologies and following that, we extend our theory to the case of production trade-offs and fulfil our second objective.

1.2. Research Questions

As briefly described above, we can state that this research attempts to achieve three main objectives. Relying on these objectives we aim to answer three main research questions, which lead to several methodological and theoretical outcomes. Research questions and outcomes related to each question are given below.

Research Question 1. *How can non-homogeneous production by agricultural farms be treated in efficiency evaluation with DEA?*

The objective behind asking this question is to propose a novel methodological approach, motivated by the above discussions, which is integrating the production trade-off relations between products of agricultural farms and to incorporate them into efficiency evaluation process with DEA. The outcome is a methodological proposition, which can lead to new uses of production trade-offs concept in agricultural context to overcome discrimination problems.

Research Question 2. *How can elasticities on efficient frontier of DEA models be calculated in the presence of trade-offs?*

The objective of asking this question is to derive a novel theoretical approach for the measurement of elasticity of response on DEA frontiers in the presence of production trade-offs. The outcome is the theoretical work provided in Chapter 5, which can be applied to any real world case, as well as agricultural efficiency evaluation.

Research Question 3. *How can elasticities on efficient frontier be calculated under constant returns-to-scale (CRS) assumption?*

This question evolves out of the preceding research question. The objective of including this question to the scope of the research is the theoretical completeness. The outcome is the theoretical work provided given in Chapter 4, which provide a general framework on elasticity measurement and can be applied to any real world case.

1.3. Contributions of the Research

Following the objectives and the research questions identified, the main theoretical and methodological contributions of this research to DEA can be summarised as follows:

First of all, we propose a novel methodology for agricultural efficiency evaluation with DEA, which can overcome the insufficient discrimination of efficiency scores when the production is highly non-homogeneous. We achieve such a novelty through bringing production trade-offs concept into agricultural efficiency measurement context. We suggest a novel use of production trade-offs, where relationships between different types of crop production are set up based upon a base crop produced by all farms. Also, by employing different trade-off relation ranges, we observe how the changes in these relationships affect the efficiency measures.

Second main contribution is related to the elasticity measurement theory on DEA frontiers. As mentioned as research question 3 above, we begin the theoretical developments with extending the existing partial elasticity measurement theory on variable returns-to-scale

(VRS) technologies (Podinovski and Førsund, 2010) to the constant returns-to-scale (CRS) technologies for the sake of theoretical completeness. We provide a framework and formulate linear programs required for the computation of elasticity measures. We prove an important result, valid in both VRS and CRS technologies, which allow us to identify the reason why the corresponding elasticity measure is undefined at the unit. This enables us to introduce generalizations of the possible solutions obtained from linear programs of elasticity measurement in both technologies. Such a contribution removes the need for a preliminary sorting of the units into those units where the elasticity measure applies and those where it does not. In addition, we identify some special cases that are applicable only in CRS technologies.

The third main contribution of the research is related to answering the question of how elasticity measures can be obtained on DEA frontiers when production trade-offs are present in the given technology. In a way, we extend the theoretical framework we define for the standard VRS and CRS technologies to the cases of production trade-offs, as pointed out as research question 2. We formulate necessary linear programs for elasticity measurement with production trade-offs in both VRS and CRS technologies and provide the generalizations of the possible solutions. We also contribute to DEA theory, by annotating on how the elasticity measures are affected by the changes on the production trade-offs.

We illustrate our proposed methodology and all the developed elasticity theory (together with the existing theory for the VRS technologies) in a real world case of Turkish agricultural sectors. The purpose is to demonstrate how the proposed and developed models can be applied to a real problem, to experiment the propositions and to interpret the results. We provide extensive empirical applications covering all the proposed theory and methodology. We show that elasticity measures can be calculated for any scenario of changing and responding input and outputs sets on DEA frontiers with or without production trade-offs in both VRS and CRS technologies. Among the results in this application, we observe that the integration of production trade-offs provide better discrimination of efficiency scores compared to the models without trade-offs included. The discrimination gets better and better, when trade-off ranges are tightened. The improvement in the discrimination is more extensive from no trade-off model to model with broad trade-offs than broad trade-offs to tighter ones. Such an observation tells us, it is not very crucial to be too accurate in specifying the trade-offs. Even with the broadest range of relations considered, the discriminations of DEA models improve. Also, it is shown both theoretically and empirically that in a production technology, if new production trade-off relations are added or the existing ones are tightened, in other words, when more information about the technology is incorporated, the ranges for one-sided elasticity measures are getting tighter.

The empirical application in the research serves as a first application of partial elasticity measurement on DEA frontiers to a real world problem. We have also a novelty of applying the production trade-offs concept in DEA first time in a real world agriculture problem. In addition, it the first study on the Turkish FADN database. Last but not least, in the scope of this research, we provide a comprehensive review for agricultural efficiency evaluation studies in the literature, which reveals some common practices pursued.

1.4. Dissertation Structure

The dissertation is organized as follows:

Chapter 2 provides a comprehensive review of the DEA theory. It provides the basic DEA concepts together with the formulations and moves forward to more advanced issues of production trade-offs, returns-to-scale investigations and elasticity measurement, which serve as underpinning subjects of this research.

Chapter 3 reviews the previous research on efficiency measurement applying DEA in the agricultural sectors. Main characteristics and methodological considerations of previous research in the literature are discussed. Common properties and practices in terms of general characteristics (countries of application, sources of data and areas of interest) and methodological considerations (methods applied, types of decision making units, selection of variables and return to scale considerations) are reviewed. In addition, a discussion of practices in dealing with non-homogeneous agricultural production is also included in this chapter to provide an insight for further model developments.

Chapter 4 includes theoretical developments on mixed partial elasticity measurement in constant returns-to-scale (CRS) technologies. In this chapter, we progress on the recent work by Podinovski and Førsund (2010) dealing with variable returns-to-scale technologies and we extend their approach to CRS technologies and formulate linear programs required for the computation of elasticity measures. Secondly, we provide a framework on interpretation of elasticity measurement models, which applies both cases of VRS and CRS. Elasticity measures in some special cases that can only arise in the CRS technology are also considered. The chapter aims to answer the Research Question 3 given above. All the proofs of theorems developed in this chapter are given in Appendix A.

Chapter 5 extends the elasticity measurement framework to the case of production trade-offs incorporated into the DEA models. The chapter aims to answer the Research Question 2 given above. The necessary theory is developed both in VRS and CRS technologies in order to incorporate the production trade-offs into elasticity measurement of output and input sets. In addition, we introduce the notion of "*returns to changing set A*" and conceptualise how the changes in production trade-offs affect the elasticity measures. All the proofs of theorems developed in this chapter are given in Appendix B.

Chapter 6 aims to explain the data set used and the design of the empirical analysis in Turkish Agriculture using the proposed methodology of efficiency and elasticity measurement. We provide comprehensive information about the contents of the FADN data set obtained from Turkish Ministry of Agriculture, our sample selection, types of crops in the selected sample and selection of inputs. Information about how the production trade-off relations used throughout the analyses are identified and processed is also given in this chapter. Last but not least, we introduce the brief design of empirical applications performed in Chapters 7 and 8.

Chapter 7 serves as a preliminary exercise for measuring elasticity on DEA frontiers. It includes various illustrative examples of elasticity measurement of DEA frontiers. It aims to demonstrate the applicability of elasticity measures developed in previous chapters under different scenarios of changing and responding input and output sets considering both VRS and CRS technologies with or without trade-offs are incorporated. In this chapter, we use only one region data of the whole data set for our illustrative purposes. We calculate elasticities for either output or input sets under different scenarios of changing and responding sets.

In Chapter 8, we extend our application scope to cover the entire FADN sample we identified in Chapter 6. In this chapter, we introduce different ranges of trade-offs into models in order to observe the effect of changing trade-offs on efficiency and elasticity measures. We pursue two scenarios of elasticity measures for output sets throughout the chapter and interpret the results relying on the methodological aspects. All calculations are performed under both VRS and CRS considerations. The result tables for this chapter are given in Appendix C.

Finally, in Chapter 9, we summarise the key aspects and findings of the entire research and provide the main conclusions derived.

Chapter 2

A Review on Data Envelopment Analysis Theory

This chapter focuses on providing a comprehensive review on fundamental issues and models of DEA together with the discussions of weight restrictions and production trade-offs in DEA models. Further in the chapter, a review on current methodologies dealing with returns-to-scale investigations and calculation of elasticities of response on DEA frontiers are covered in order to provide insight for theoretical improvements undertaken in Chapters 4 and 5.

2.1. Data Envelopment Analysis (DEA)

DEA, being a non-parametric approach, does not require any assumptions about the functional forms. The efficiency of a DMU is measured relative to all other DMUs with the simple restriction that all DMUs lie on or below an efficient frontier (Seiford and Thrall, 1990). A Production Possibility Set (PPS) is constructed, which *'contains all input-output correspondences which are feasible in principle including those observed units being assessed'* (Thanassoulis, 2001). The aim is to determine the efficiently performing units in relative to each other and benchmark the other units relative to the efficient units in the defined PPS. Such an aim is succeeded by determination of units on and below an efficient frontier through the calculation of efficiency scores for the units with linear programming approaches.

2.2. Production Possibility Set (PPS)

In DEA, instead of linking inputs to outputs through functional forms, as a first step, a Production Possibility Set (PPS), which can be defined as the minimum set enveloping the observed units and all the input-output correspondences that are feasible, is constructed.

Figure 2.1 illustrates how PPS is defined and how DEA works in principle with a simple one input and one output example.

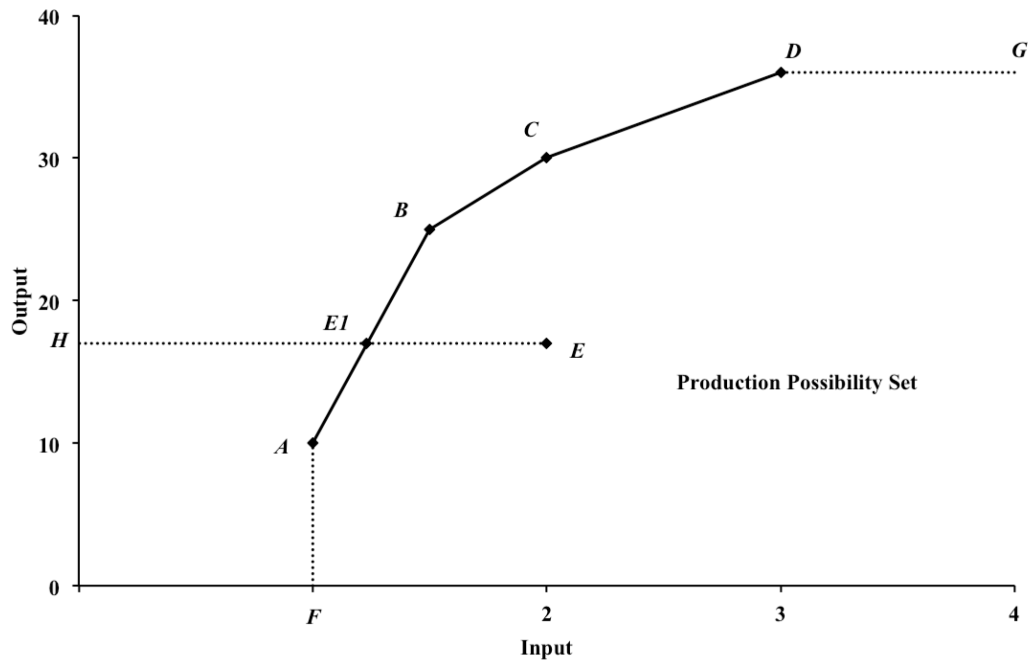

Figure 2.1. Illustration of Production Possibility Set

In Figure 2.1, the observed units *A*, *B*, *C*, *D* and *E* are plotted on the graph. The input-output correspondences lying on linear segments *AB*, *BC* and *CD* are enveloping the data and they are feasible. Since operating at *A* and operating at *B* are both possible, in principle, it is reasonable to deduce through interpolation that to operate at input-output correspondences between those points is also possible (Thanassoulis, 2001). Another assumption that can be made in order to define the PPS is that it is always possible to use more input and produce less output than observed (free disposability principle), in other words, it is possible to operate inefficiently. Therefore, the PPS consists of units on the piece-wise linear boundary *FABCDG* and all units to the right and below of this boundary.

In the given PPS, the piece-wise linear boundary *ABCD* is the efficient frontier since units on this boundary are relatively the best performing units. Units on segments *AF* and *DG* are feasible but not efficient in Pareto sense since units *A* and *D* dominate them, respectively. For the units on *AF*, it is possible to produce more of output with the same input as the unit *A* has achieved. Similarly, for the units on *DG*, it is possible to produce the same output with less input as at unit *G*. Unit *E* lies below the efficient frontier *ABCD*, and thus operating inefficiently relative to the other observed units. It is outperformed by all other observed units, as well as the hypothetical unit *E1*. The unit *E1*, which is an interpolation between units *A* and *B*, is hypothetically producing the same amount of output with the less input than unit *E*. In input terms, the unit *E1* can be a target for unit *E*. In principle, the efficiency of unit *E* is calculated by the ratio *HE1*/*HE*.

To generalize the basic assumptions underlying the PPS in DEA, consider a set of n Decision Making Units (DMUs), $J=\{1,2,..,n\}$. Each unit, DMU_j ($j \in J$) uses m inputs to produce s outputs. The observed units are denoted as pairs (X^j, Y^j), $j \in J$, where vectors $X^j \in R_+^m$ and $Y^j \in R_+^s$. The Production Possibility Set, denoted by *T*, is defined as the set of input and output vectors (X,Y) such that it is possible to produce $Y \geq 0$ from $X \geq 0$.

Conceptually, the Production Possibility Set in DEA is defined as the *minimum technology* that satisfies the following production axioms (Banker et al., 1984; Podinovski, 2004a):

Axiom 2.1. *Feasibility of observed data.* $(X^j, Y^j) \in T$, for any $j \in J$.

Axiom 2.2. *Convexity.* The set *T* is convex.

Axiom 2.3. *Free disposability.* $(X,Y) \in T$, $Y \geq Y' \geq 0$ and $X \leq X'$ implies $(X',Y') \in T$.

DEA models are built under different Returns to Scale (RTS) assumptions. Depending on the RTS assumption, the production technology, and thus the production axioms of the PPS exhibit slight differences. The original model proposed by Charnes et al. (1978) is known as

CCR (Charnes Cooper Rhodes) model. The CCR approach assumes a constant returns-to-scale technology (CRS) and so, proportionality between inputs and outputs. This means that scaled inputs and outputs of DMUs with same proportion are members of the technology. For example, in a two inputs and two outputs case, if we raise both inputs by 10% and expect the outputs to rise by 10%, then CRS assumption is appropriate approach to incorporate for this case. Under CRS assumption, it is assumed that the operating scale of a unit does not have an effect on its efficiency. The CCR approach is modified by Banker et al. (1984) and named as BCC (Banker Charnes Cooper) model, which is assuming variable returns-to-scale (VRS). BCC approach ignores the proportionality assumption.

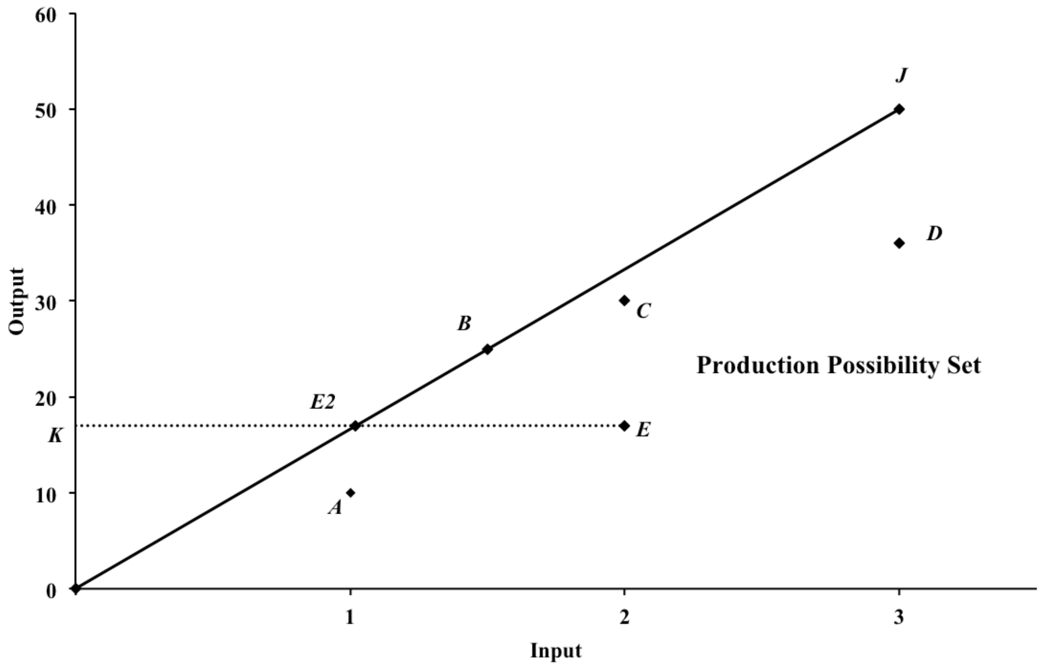

Figure 2.2. Illustration of Production Possibility Set under Constant Returns-to-Scale

The production possibility set for CRS technology is illustrated in Figure 2.2 with the same units as in Figure 2.1. As seen in Figure 2.2, for our one input and one output example, under CRS technology, the efficient boundary has a linear form starting form the origin different than the frontier in the VRS technology as given by Figure 2.1. The PPS is defined as the set of units on or below the ray *OBJ*. Unit *B* is the only unit operating efficiently relative to others. For unit *E*, the efficient target in terms of inputs is the hypothetical unit *E2* and the efficiency of *E* is *KE2*/*KE*.

Production Axioms 2.1, 2.2 and 2.3, stated above define the VRS technology. For the PPS under CRS assumption, proportionality axiom (Axiom 2.4) additional to above 3 axioms is considered, which is referred as “Ray Unboundness” in Banker et al. (1984).

Axiom 2.4. *Proportionality*. $(X,Y) \in T$ and $\alpha \geq 0$ implies $(\alpha X, \alpha Y) \in T$.

One of the main advantages of DEA is that it allows the user to deal with multiple inputs and outputs. More inputs and outputs mean more dimensions for the technology. In the presence of multiple inputs and outputs the PPS has an unbounded polyhedral shape under VRS technology and a conical shape under CRS technology.

2.3. Illustration of Efficiency Measurement with DEA

The efficiency measurement with DEA is achieved through a linear programming (LP) approach. In general, linear programming models of DEA can be thought of two main components. One component is related to how the efficiency is measured. The component sets the nature of objective function of the LP model. The objective of the DEA models can be formulated in two ways as output-oriented (maximisation) or input-oriented (minimisation). In output-orientation, a DMU is not efficient in the given technology if it is possible to augment any output without increasing any input and without decreasing any other output. Similarly, in the input-orientation, a DMU is not efficient in the given technology if it is possible to decrease any input without augmenting any other input and without decreasing any output (Charnes et al., 1981). The second main component of DEA models is related to the properties of the Production Possibility Set (PPS) constructed relying on the production axioms given above and these properties are identified through the constraints of the linear programming model.

2.3.1. Output-oriented DEA Model Example

Following in this section, we explain how the output-oriented and input-oriented LP models for DEA are formulated with two illustrative examples. Let us begin with formulation of output-oriented model. Assume we have four decision making units, (*A*, *B*, *C* and *D*) as given in Table 2.1. Each unit produces 2 outputs through the use of 1 input.

Table 2.1. Data for Output-oriented DEA Illustration

DMU	Input	Output 1	Output 2
A	1	1	4
B	1	3	3
C	1	4	1
D	1	2	1

The units in Table 2.1 are plotted in Figure 2.3. Since we have the same input level for all units, the illustration in Figure 2.3 takes two outputs into consideration. The variable returns-to-scale (VRS) PPS is defined as all the points left and below of the boundary *FABCE* since it is always possible to produce less of outputs. *ABC* is the VRS efficient frontier.

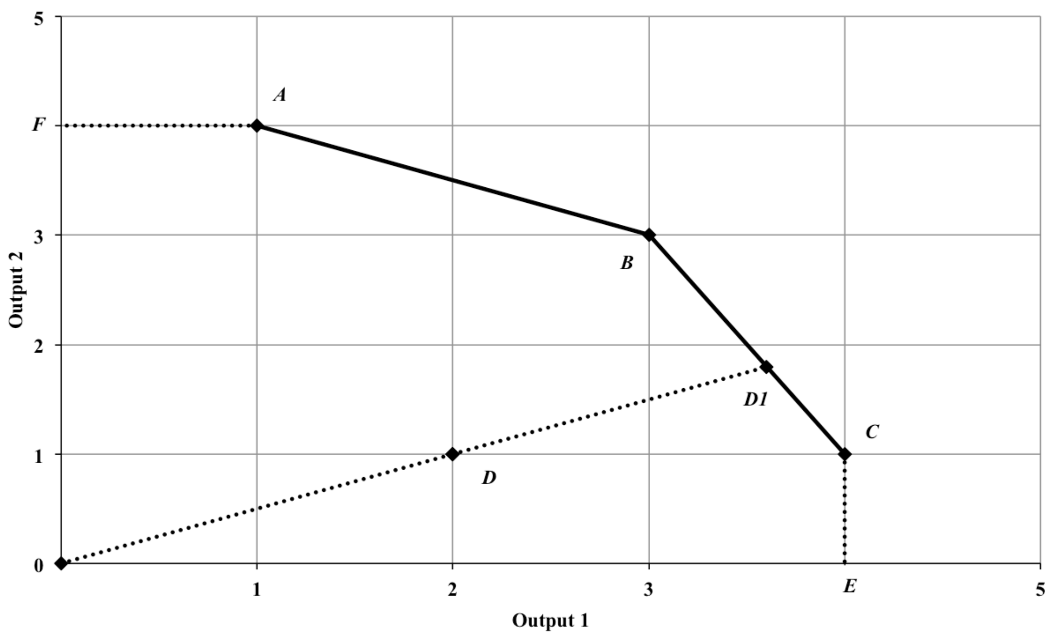

Figure 2.3. Two-Output DEA Example

In modelling DEA, first of all, the production possibility set relying on the axioms given in the previous section is constructed. In principle, the PPS consists of three basic types of units as real (observed), composite and outperformed (dominated) DMUs. Some units can fall into two or three of those categories at the same time.

In our example, units *A*, *B*, *C* and *D* are the real DMUs. We define the PPS as the set consisting of all the pairs *(X, Y)*, which satisfy (2.1). The right-hand sides of in equalities (2.1.1) to (2.1.3) together with (2.1.4) and (2.1.5) represent the composite DMUs that are basically the convex combinations (i.e. weighted averages) of the real DMUs. The λ s represent the weights attached to real units. Outperformed DMUs are the units consume more inputs and/or produce less of outputs than the real or composite DMUs. An outperformed unit is represented as the left-hand sides of the inequalities (2.1.1) to (2.1.3).

$$x_1 \geq 1\lambda_1 + 1\lambda_2 + 1\lambda_3 + 1\lambda_4 \qquad (2.1.1)$$

$$y_1 \leq 1\lambda_1 + 3\lambda_2 + 4\lambda_3 + 2\lambda_4 \qquad (2.1.2)$$

$$y_2 \leq 4\lambda_1 + 3\lambda_2 + 1\lambda_3 + 1\lambda_4 \quad (2.1.3)$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1 \quad (2.1.4)$$

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \geq 0 \quad (2.1.5)$$

The PPS defined by (2.1) satisfies the basic assumptions given for VRS technology as *feasibility of observed data*, *convexity* and *free disposability* in the previous section. The observed units are always feasible because there is always a corresponding λ value that will satisfy (2.1). Any unit on the left and below the boundary operates with less of outputs, therefore, for those units, also (2.1) is always satisfied.

The other important step in formulating DEA is the objective. As mentioned, there are two orientations as input minimisation and output maximisation. Let us formulate the output-oriented VRS DEA model for unit *D*. It is given in (2.2). As seen in Figure 2.3, unit *D* is an inefficient unit and its projection on the frontier is the hypothetical unit *D1* that is an interpolated unit from units *B* and *C*.

$$Max \;\; \phi \quad (2.2.1)$$

Subject to

$$1\lambda_1 + 1\lambda_2 + 1\lambda_3 + 1\lambda_4 \leq 1 \quad (2.2.2)$$

$$1\lambda_1 + 3\lambda_2 + 4\lambda_3 + 2\lambda_4 \geq 2\phi \quad (2.2.3)$$

$$4\lambda_1 + 3\lambda_2 + 1\lambda_3 + 1\lambda_4 \geq 1\phi \quad (2.2.4)$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1 \quad (2.2.5)$$

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \geq 0 \quad (2.2.6)$$

$$\phi \;\; sign\, free \quad (2.2.7)$$

The aim is to consider improving the outputs for the given unit with the maximum proportion possible so that the unit still remains in the technology. We denote the factor by which we improve the outputs of the given unit as ϕ. The unit on the right-hand side of (2.2.1) to (2.2.3) is the improved unit *D* by factor ϕ. We aim to maximise this factor and hit the boundary at *D1* so that we still remain in the technology. It is a sign free variable in our LP model. The optimal ϕ value in output orientation is greater than or equal to 1 since it is an improvement. The optimal ϕ value gives us the value for the radial improvement of unit to the frontier *ABC*. The reciprocal of the optimal ϕ in output-oriented model is known as efficiency score for unit *D*. For the efficient units the efficiency score is obtained as 1 since they are on the frontier, there is no need for an improvement at the output levels.

Optimal solution to (2.2) is $\phi^* = 1.8$, $\lambda_1^* = 0$, $\lambda_2^* = 0.4$, $\lambda_3^* = 0.6$, $\lambda_4^* = 0$. The result indicates that the output radial efficiency for unit *D* is $1/\phi^* = 0.56$. In other words, the unit *D* has 56% technical efficiency. It can improve its outputs by 80% to become efficient. In the given technology, it is possible to produce 80% more of each output with the given input of unit *D*. When output 1 and output 2 values of *D* are both increased by 80%, the unit projects to *D1* (3.6, 1.8), which is on the efficient frontier and an interpolated unit from units *B* and *C* (see Figure 2.3). Units *B* and *C* are labelled as the efficient peers for unit *D* in DEA terminology. 3.6 and 1.8 are the target values for output 1 and output 2 levels of unit *D*, respectively. These target values can also be obtained through the use of optimal λ values when the optimal values above are replaced with the correspondent variables on the left-hand side of (2.2.3) for output 1 and on left-hand side of (2.2.4) for output 2.

2.3.2. Input-oriented DEA Model Example

The input-oriented LP model for DEA can be formulated similarly, this time considering the minimisation of the input factors. Let us consider two inputs-one output case this time. Assume we have four decision making units, (*A*, *B*, *C* and *D*) as given in Table 2.2. Each unit produces 1 output through the use of 2 inputs.

Table 2.2. Data for Input-oriented DEA Illustration

DMU	Input 1	Input 2	Output
A	1	5	2
B	2	2	2
C	4	1	2
D	2	4	2

The units in Table 2.2 are plotted in Figure 2.4. Since we have the same output level for all units, the illustration in Figure 2.4 takes two inputs into consideration. The variable returns-to-scale (VRS) PPS is defined as all the points right and above of the boundary *EABCF* since it is always possible to operate with more inputs. *ABC* is the VRS efficient frontier.

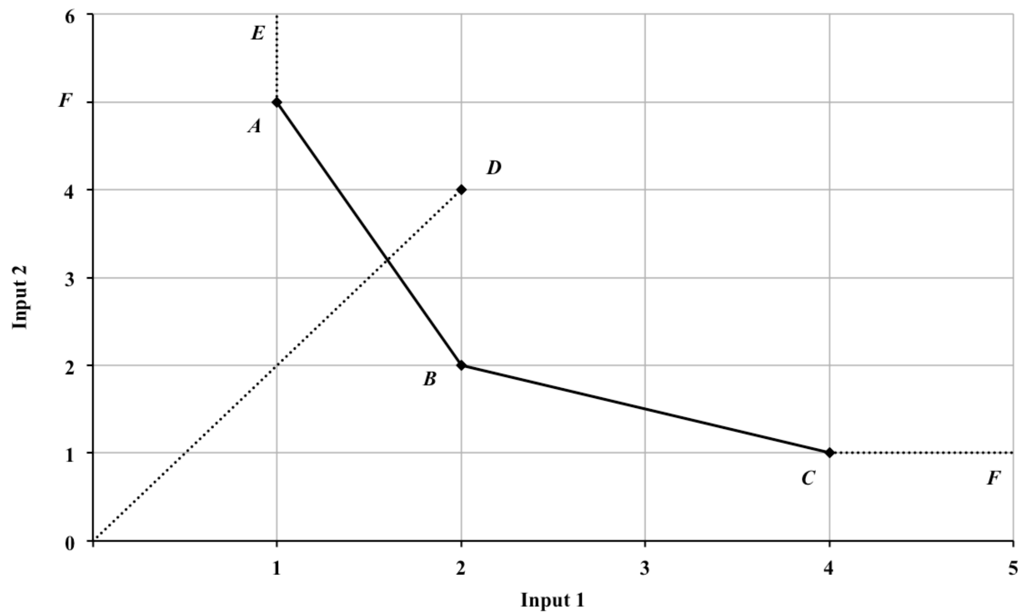

Figure 2.4. Two-Input DEA Example

The LP model for input orientation can be formulated as in (2.3) below. In this case, the aim is to consider changing the inputs for the given unit with the smallest proportion possible so that the unit still remains in the technology. We move radially towards the origin and hit the boundary at *D1*, which is the radial projection for unit *D* (see Figure 2.4). We denote the factor by which we change the inputs of the given unit as θ . The unit on the right-hand side of (2.3.1) to (2.3.3) is the changed unit *D* by factor θ . We aim to minimize this factor so that we reach the frontier and at the same time remain in the technology. The optimal θ values in input orientation are between 0 and 1 ($\theta^* \in]0,1]$). The θ value gives us the value for the radial projection of unit the to the frontier *ABC*, which is known as efficiency score for unit *D*. Similar to the output orientation case, for the efficient units the efficiency score is obtained as 1, since they are on the frontier, there is no need for a change in the input levels.

$$Min \;\; \theta \qquad (2.3.1)$$

Subject to

$$1\lambda_1 + 2\lambda_2 + 4\lambda_3 + 2\lambda_4 \le 2\theta \qquad (2.3.2)$$

$$5\lambda_1 + 2\lambda_2 + 1\lambda_3 + 4\lambda_4 \le 4\theta \qquad (2.3.3)$$

$$2\lambda_1 + 2\lambda_2 + 2\lambda_3 + 2\lambda_4 \ge 2 \qquad (2.3.4)$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1 \qquad (2.3.5)$$

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \ge 0 \qquad (2.3.6)$$

$$\theta \; sign\; free \qquad (2.3.7)$$

Optimal solution to (2.3) is $\theta^* = 0.8$, $\lambda_1^* = 0.4$, $\lambda_2^* = 0.6$, $\lambda_3^* = 0$, $\lambda_4^* = 0$. The result indicates that the input radial efficiency for unit *D* is 0.80. In other words, the unit *D* has 80% technical efficiency. In the given technology, it is possible for unit *D* to reduce the level of its inputs by 80% in order to attain the same output level. When Input 1 and Input 2 values of *D* are both reduced by 80%, the unit projects to *D1* (1.6, 3.2), which is on the efficient frontier and an interpolated unit from units *A* and *B* (see Figure 2.4). Units *A* and *B* are the efficient peers for unit *D*. 1.6 and 3.2 are the target values for Input 1 and Input 2 levels of unit *D*, respectively. These target values can also be obtained through the use of optimal λ values when the optimal values above are replaced with the correspondent variables on the, left-hand side of (2.3.2) for output 1 and on left-hand side of (2.3.3) for output 2.

2.4. Envelopment DEA Models

Following the illustration of DEA models given above, in this section, we provide generalized formulations for the linear programming models of DEA. The original DEA model proposed by Charnes et al. (1978) and known as CCR (Charnes Cooper Rhodes) model builds upon the engineering ratio approach, where the efficiency of a DMU can be expressed by the ratio of its weighted combination of outputs to its weighted combination of inputs. This problem is further transformed into equivalent linear programming models known as Multiplier DEA models (which are given in Section 2.5) and Envelopment DEA models. Models of DEA given in this section are known as the Envelopment forms.

As mentioned in Section 2.2, DEA models also differ in terms of returns-to-scale considerations. The original approach (CCR) assumes a constant returns-to-scale (CRS) technology. This approach is modified by Banker et al. (1984) and named as BCC (Banker Charnes Cooper) model, which is assuming variable returns-to-scale (VRS) technology. The main assumptions of Production Possibility Sets (PPS) of both technologies are discussed in Section 2.2.

2.4.1. Variable Returns-to-Scale Envelopment Models

Following the illustrations of envelopment models above in Section 2.3, we begin with the formulation of the VRS models (also known as BCC models). Consider a set of *n* Decision Making Units (DMUs), $J=\{1,2,..,n\}$. Each unit, DMU_j $j \in J$, uses *m* inputs to produce *s* outputs. The observed units are denoted as pairs (X^j, Y^j), $j \in J$ where vectors $X^j \in R_+^m$ and $Y^j \in R_+^s$. Let $\bar{X}$ and $\bar{Y}$ be the input and output matrices consisting of the input and output vectors X^j and Y^j, respectively.

Table 2.3. Variable Returns-to-Scale Envelopment DEA Models

Output-oriented VRS Envelopment Model		Input-oriented VRS Envelopment Model	
$Max\ \phi$	(2.4.1)	$Min\ \theta$	(2.5.1)
Subject to		Subject to	
$\bar{X}\lambda \le X_0$	(2.4.2)	$\bar{X}\lambda \le \theta X_0$	(2.5.2)
$\bar{Y}\lambda \ge \phi Y_0$	(2.4.3)	$\bar{Y}\lambda \ge Y_0$	(2.5.3)
$e\lambda = 1$	(2.4.4)	$e\lambda = 1$	(2.5.4)
$\lambda \ge 0$	(2.4.5)	$\lambda \ge 0$	(2.5.5)
ϕ *sign free*	(2.4.6)	θ *sign free*	(2.5.6)

Envelopment DEA models for any observed unit DMU_0 under VRS considerations are provided in Table 2.3 with the models (2.4) and (2.5) for the output and input orientations, respectively. X_0 and Y_0 represent the input and output vectors for the unit under evaluation (DMU_0), respectively. In the output orientation, output values of the unit is improved by the maximum possible ϕ, whereas in input orientation, input values are reduced by the minimum possible θ in order to calculate improvement or reduction potentials of the unit in the given technology. In general, the calculations through (2.4) or (2.5) are repeated for each observed unit to identify efficiently and inefficiently performing units. For efficient units, (i.e. for units on the efficient frontier) optimal ϕ and θ values are obtained as 1.

2.4.2. Constant Returns-to-Scale Envelopment Models

The other fundamental type of DEA models regarding the returns-to-scale is the CCR model, which assumes constant returns-to-scale (CRS). Actually, CCR is the original DEA model developed by Charnes et al. (1978) and BCC models given above are proposed as an extension to these original developments. As given in Section 2.2, full proportionality between inputs and outputs is assumed under CRS technology. Scaled units, in addition to real, composite and outperformed units, are also the members of the production possibility set under CRS (see Axiom 2.4 in Section 2.2). However, proportionality is not valid for every real world problem. Especially, when quality of products and services are considered. Also, the discussion of the returns-to-scale specifications is highly related with the size of the operations undertaken by the units. As Coelli et al. (2005) state, '*CRS assumption is appropriate when all firms are operating at an optimal scale. The use of CRS specification when not all firms are operating at the optimal scale, results in measures of technical efficiency that are confounded by scale efficiencies'*. Hence, BCC models, assuming variable returns-to scale, are developed to handle the problems where proportionality is not valid and also to distinguish between technical and scale efficiencies. BCC models eliminate the effect on efficiency created by the scale of the operation and provide a measure for "pure technical efficiency". They permit to identify increasing, decreasing or constant returns to scale at different scale sizes (Charnes et al., 1994). Therefore, VRS frontiers have piece-wise linear shape, which can be observed in Figure 2.1.

For several real world cases, proportionality between inputs and outputs can be valid and so the constant returns-to-scale can be appropriate to assume when constructing DEA models. One common example can be given as the evaluation of university departments through taking the staff as inputs and number of students served and the number of publications as outputs. In principle, it is plausible to expect that by increasing the number of academic staff, the number of students and number of publications increase by the same proportion (Podinovski, 2007). CRS can be an appropriate assumption for such a case.

Table 2.4. Constant Returns-to-Scale Envelopment DEA Models

Output-oriented CRS Envelopment Model		**Input-oriented CRS Envelopment Model**	
$Max\ \phi$	(2.6.1)	$Min\ \theta$	(2.7.1)
Subject to		Subject to	
$\bar{X}\lambda \leq X_0$	(2.6.2)	$\bar{X}\lambda \leq \theta X_0$	(2.7.2)
$\bar{Y}\lambda \geq \phi Y_0$	(2.6.3)	$\bar{Y}\lambda \geq Y_0$	(2.7.3)
$\lambda \geq 0$	(2.6.4)	$\lambda \geq 0$	(2.7.4)
ϕ *sign free*	(2.6.5)	θ *sign free*	(2.7.5)

Output and input oriented CRS Envelopment DEA models are given in Table 2.4 by models (2.6) and (2.7), respectively. Theoretically, the only difference of CRS Envelopment model from the VRS Envelopment model is that in CRS case, we omit the convexity constraint $e\lambda = 1$ given in (2.4.4) and (2.5.4) for output and input oriented VRS models, respectively. In general VRS models envelope the data closer than the CRS models. This can be observed through a comparison between Figures 2.1 and 2.2. Therefore, the discrimination of CRS models are better than VRS since fewer units can be identified as efficient. Also, the distance of a unit to the frontier will be larger under CRS because CRS frontier does not envelope the data as closely as VRS frontier does. Thus, efficiency scores in CRS technology are always less than equal to scores in VRS technology. Moreover, output-oriented and input-oriented efficiency scores of a DMU assessed under CRS are equal ($\theta^* = 1/\phi^*$)

Results obtained through BCC and CCR DEA models provided above can be interpreted as different types of efficiencies. Depending upon the fundamental work by Farrell (1957), *Technical Efficiency* is defined as *'the degree to which a decision making unit produces the maximum feasible output from a given bundle of inputs, or uses the minimum feasible amount of inputs to produce a given level of output'*. These two alternative definitions of technical efficiency leads to two abovementioned measures known as output-oriented and input-oriented efficiency, respectively. *Technical Efficiency* defined above can be further decomposed into two components as *Pure Technical Efficiency* and *Scale Efficiency*. In general, the term *Technical Efficiency* refers to the CCR score. On the other hand, as mentioned BCC results are referred as *Pure Technical Efficiency,* since scale effects are eliminated. If a DMU is fully efficient in both the CCR and BCC models, it is said to be operating at the *Most Productive Scale Size*. If a DMU has full BCC efficiency but a lower CCR score, then it is operating locally efficient but not globally efficient due to the scale size of the DMU. Thus, it is reasonable to characterise the *Scale Efficiency* of a DMU by the ratio of the two scores. (Cooper et al., 2006).

2.5. Mix Inefficiency

One important notion to mention in efficiency measurement through DEA is the mix inefficiencies. In some cases obtaining a DEA efficiency score as 1 for a unit does not always guarantee that the unit is fully efficient (i.e. efficient in Pareto sense). Some units exhibit inefficiency, which cannot be eliminated without changing proportions between its inputs and outputs, however yield to an efficiency score of 1. Suppose we have an observed unit on the *AF* segment of the boundary in Figure 2.3. This unit would produce the same level of output 2, however less of output 1 than unit *A*. Thus, unit *A* would dominate it. Similarly, suppose we have an observed unit on the *CF* segment of the boundary in Figure 2.4. This unit would be using the same level of input 2, but more of input 1 compared to unit *C*. Thus, unit *C* would dominate it. Inefficiency of such units is known as mix inefficiency.

Whether a unit exhibits mix inefficiency or not is tested through a second-stage optimization using the optimal score obtained at the first stage and the slack values in the constraints (2.4.2) and (2.4.3) for output orientation; (2.5.2) and (2.5.3) for input orientation. The input and output slack vectors are represented as s^- and s^+, respectively. The second stage optimization models for testing mix inefficiencies under VRS are given in Table 2.5 by (2.8) and (2.9) for output and input orientations, respectively.

For output-oriented VRS consideration, a unit is fully efficient if both optimal solution to (2.4) is 1 ($\phi^* = 1$) and the optimal solution to (2.8) is 0 ($s^{-*} = 0$ and $s^{+*} = 0$). Similarly, for the input-oriented VRS consideration, a unit is fully efficient if both optimal solution to (2.4) is 1 ($\theta^* = 1$) and the optimal solution to (2.9) is 0 ($s^{-*} = 0$ and $s^{+*} = 0$). This is referred as "Pareto-Koopmans" or "strong" efficiency in DEA terminology (Cooper et al., 2006). If the efficiency is equal to 1 in any DEA model, but the sum of slacks is not equal to 0 in the corresponding second-stage optimization, the unit can be considered to exhibit mix inefficiency. Above statements are also valid for the CRS technology.

Table 2.5. Second-Stage DEA Models for Testing Mix Inefficiency

Output-oriented VRS Second-Stage Model		**Input-oriented VRS Second-Stage Model**	
$Max\ s^- + s^+$	(2.8)	$Max\ s^- + s^+$	(2.9)
Subject to		Subject to	
$\bar{X}\lambda + s^- \leq X_0$		$\bar{X}\lambda + s^- \leq \theta^* X_0$	
$\bar{Y}\lambda - s^+ \geq \phi^* Y_0$		$\bar{Y}\lambda - s^+ \geq Y_0$	
$e\lambda = 1$		$e\lambda = 1$	
$\lambda \geq 0$		$\lambda \geq 0$	

2.6. Multiplier DEA Models

As mentioned earlier, the original model developed by Charnes et al. (1978) has a quite different modelling approach than the envelopment models provided in the preceding sections where the efficiency of a DMU is expressed by the ratio of its weighted combination of outputs to its weighted combination of inputs[5]. This ratio approach is translated into linear programming models known as Multiplier models[6] in the DEA literature. They are the dual models to the envelopment forms. Both approaches lead to the same efficient scores for the units, however the interpretations are different.

The Envelopment DEA models given above measure the efficiency of a unit based on the efficient frontier and as seen in the illustrative examples of Section 2.3, they can provide us the efficiency scores for the units together with the efficient targets and peers for the inefficient ones. They have a technological meaning of efficiency as a possible improvement

[5] Known as virtual outputs and inputs, respectively (Cooper et al., 2006)

[6] Referred as Value-Based DEA models by Thanassoulis (2001)

factor for inputs or outputs. On the other hand, Multiplier DEA models measure the efficiency of a unit relying on the ratio of its outputs to its inputs. They can provide us information about the areas of good and poor performance through the weights attached to the inputs and outputs by the formulated problem. Multiplier forms have more a managerial meaning as the relative standing of the DMU in relation to the other DMUs, assuming the most favourable weights of inputs and outputs (Podinovski, 2007).

2.6.1. Constant Returns-to-Scale Multiplier Models

Since the original ratio form model assumes CRS in Charnes et al. (1978) and multiplier forms have better interpretation under CRS, we begin with the CRS formulation. The mathematical formulation for the Multiplier DEA models under CRS technology is given by (2.10) and (2.11) in Table 2.6 for both output and input orientations. They are dual linear programming models to the models (2.6) and (2.7) in Table 2.4. The vectors v and μ represent the input and output multipliers (i.e. weights), respectively. They are the dual variables (i.e. shadow prices) corresponding to constraints (2.6.2) and (2.6.3) in output orientation and (2.7.2) and (2.7.3) in input orientation.

Table 2.6. Constant Returns-to-Scale Multiplier DEA Models

Output-oriented CRS Multiplier Model		**Input-oriented CRS Multiplier Model**	
$Min\ \ vX_0$	(2.10)	$Max\ \ \mu Y_0$	(2.11)
Subject to		Subject to	
$v\overline{X} - \mu\overline{Y} \geq 0$		$v\overline{X} - \mu\overline{Y} \geq 0$	
$\mu Y_0 = 1$		$vX_0 = 1$	
$v, \mu \geq 0$		$v, \mu \geq 0$	

2.6.2. Variable Returns-to-Scale Multiplier Models

The VRS formulations for the multiplier models are provided by (2.12) and (2.13) in Table 2.7 for output and input orientations, respectively. Model (2.12) is the output-oriented form and is the dual of (2.4), whereas (2.13) is input-oriented and the dual for (2.5) given in Table 2.3. The free variable μ_0 in the VRS Multiplier models is the dual variable corresponding to the constraint $e\lambda = 1$ in the envelopment forms.

Table 2.7 Variable Returns-to-Scale Multiplier DEA Models

Output-oriented VRS Multiplier Model		**Input-oriented VRS Multiplier Model**	
$Min\ \ vX_0 + \mu_0$	(2.12)	$Max\ \ \mu Y_0 + \mu_0$	(2.13)
Subject to		Subject to	
$v\overline{X} - \mu\overline{Y} + e\mu_0 \geq 0$		$v\overline{X} - \mu\overline{Y} + e\mu_0 \geq 0$	
$\mu Y_0 = 1$		$vX_0 = 1$	
$v, \mu \geq 0$		$v, \mu \geq 0$	
μ_0 *sign free*		μ_0 *sign free*	

2.6.3. Illustrative Example on Multiplier Forms

Let us illustrate how the efficiency score for a unit can be calculated with Multiplier DEA approach and how the results can be interpreted. Consider the data set given in Table 2.1 consisting of 4 units as *A*, *B*, *C* and *D*. Suppose we want to calculate the efficiency score of unit *A* using the multiplier model under CRS consideration. Output-oriented CRS Multiplier DEA model for unit *A* will be as in (2.14), where v_1 represent the multiplier (weight) for input; μ_1 and μ_2 represent the multipliers (weights) for Outputs 1 and 2, respectively The efficiency score of the unit *A* is the objective function value of program (2.14).

$$Min \;\; v_1 \tag{2.14}$$

Subject to

$1v_1 - 1\mu_1 - 4\mu_2 \geq 0$

$1v_1 - 3\mu_1 - 3\mu_2 \geq 0$

$1v_1 - 4\mu_1 - 1\mu_2 \geq 0$

$1v_1 - 2\mu_1 - 1\mu_2 \geq 0$

$1\mu_1 + 4\mu_2 = 1$

$v_1, \mu_1, \mu_2 \geq 0$

We have the optimal solution for (2.14) as $v_1^* = 1$, $\mu_1^* \cong 0.11$ and $\mu_2^* \cong 0.22$. The unit *A* is an efficient unit since we have the objective function value as 1. In order to identify the contribution of different inputs and outputs to the efficiency of this unit, virtual inputs and outputs for unit *A* can be calculated through multiplying the actual value of inputs or outputs and the optimal weights attached to them. Virtual input is 100%, since we have only one input and optimal input weight together with input value for unit *A* are both equal to 1. Since our model is output-oriented, the weights attached to outputs are more of interest. For output 1, the virtual output value is 11%, since the optimal weight attached to this output is 0.11 and the output 1 value for unit *A* is 1. For output 2, we have virtual input as 89%. Relying on the virtual outputs, we can conclude that unit *A* is efficient mainly due to the high weight attached to output 2.

2.7. Weight Restrictions and Production Trade-offs in DEA

2.7.1. Weight Restrictions and Value Judgements

In multiplier DEA formulations given in the preceding section, apart from the restriction that weights should be greater or equal to zero ($\nu, \mu \geq 0$), weights on inputs and outputs are only restricted, implicitly through the formulation nature of the model. As a result, it can be concluded that standard multiplier form of DEA models assume a weight flexibility for inputs and outputs. Such weight flexibility in the original formulation leads to some advantages and disadvantages. According to Dyson and Thanassoulis (1988), the weight flexibility allows different weights to be used in computing the relative efficiency of different DMUs, thus eliminating the need to negotiate a common set of weights across the set of DMUs. It can also help to identify aspects in which a DMU could prove an example of good operating practice or where it could improve its performance further (Thanassoulis et al., 1987). On the other hand, the weight flexibility in DEA can lead to a complication of assigning very low weights to some inputs or outputs so that the efficiency score cannot reflect the realistic performance of a DMU as a whole. In other words, some inputs or outputs may be ignored (Dyson and Thanassoulis, 1988).

Thompson et al. (1990) also mention about the free nature of DEA model that it does not require any weights or functional relationships of inputs and outputs; supporting that it can be a strength, but at the same time a weakness. According to Thompson et al. (1990), in order to move from technical efficiency to overall efficiency evaluation, it is necessary to use some restrictions on weights reflecting the realistic assessments on inputs and outputs. Therefore, we can say the use of weight restrictions is often motivated by the observation that the DMU under the assessment may achieve a high efficiency score by using an unrealistic profile of optimal input and output weights in the multiplier model (Podinovski and Thanassoulis, 2007). Moreover, the weight restrictions can also be considered as a tool for increasing discrimination of efficiency scores.

Incorporation of weight restrictions to DEA models can be seen as integrating the value judgements of the decision makers to the efficiency evaluation and evolved through the needs of real world problems. Allen et al. (1997) briefly define the concept of value judgements as *'logical constructs, incorporated within an efficiency assessment study, reflecting the Decision Makers' preferences in the process of assessing efficiency'*. They listed a number of reasons leading to a use of such an approach as below (Allen et al., 1997).

- Incorporating prior views on the value of input and outputs; as well as on efficient and inefficient units,
- Relating the values of certain inputs and outputs,
- Keeping up with the economic notion of input/output substitution,
- Increasing the discrimination of the models.

Different types of weight restriction approaches can be found in DEA literature. Thompson et al. (1986) applied weight restrictions with the development of "Assurance Region (AR)" method through an application for site evaluations to locate a physics laboratory. The AR approach is further improved by Thompson et al. (1990) and Roll and Golany (1993). Another approach on use of weight restrictions, which is known as "Cone-Ratio (CR)" method is developed and improved by Sun (1988), Charnes et al. (1989) and Charnes et al. (1990)[7] through application on commercial banks in U.S.

[7] The reader may refer to Cooper et al. (2006) for further details on formulation of those two methods (AR and CR).

2.7.2. Production Trade-offs in DEA

The major drawback of incorporating weight restrictions based on value judgements is that the resulting efficiency measure can no longer be interpreted as a realistic improvement factor; in other words, the efficiency measures lose their clear technological meaning (Allen et al., 1997; Thanassoulis and Allen, 1998; Podinovski, 2004a). To overcome such drawback and improve the discrimination of DEA models at the same time, Podinovski (2004a) introduces the concept of "Production Trade-offs" which are defined as '*simultaneous changes to the inputs and outputs that are possible in the technology under consideration*'. In trade-offs approach, the possible technological trade-off relations between inputs and outputs are constructed and translated into weight restrictions. Mathematical effect is the same with the weight restrictions, however, the technological meaning of efficiency is preserved and also a more realistic discrimination of DMUs is obtained since the production trade-offs rely on the technological realities rather than managerial value judgements (Podinovski, 2007).

Production trade-offs can be incorporated into both envelopment and multiplier forms of DEA models. In the envelopment form, the composite units are modified and the production possibility set is expanded enabling the technology to reflect the technological judgements incorporated. On the other hand, for incorporating production trade-offs in the multiplier form, they are translated into weight restrictions and added as new constraints to the DEA model. For the practical purposes, multiplier form is more suitable since standard DEA software can handle the weight restrictions.

Consider the data set given in Table 2.1 with four units. Suppose we have two technological judgements that express the relationship between two outputs as below:

Judgement 2.1. *No extra resources can be claimed if the level of output 1 is reduced by 1 and the level of output 2 is increased by 1.*

Judgement 2.2. *No extra resources can be claimed if the level of output 1 is increased by 1 and the level of output 2 is reduced by 1.5.*

Above judgements reveal a range for a two-sided relationship between output 1 and output 2. Relying on these relations, the production possibility set under VRS consideration expands as given in Figure 2.5. As explained in Section 2.2, the original VRS technology is defined as the area bounded by *FABCE*. Incorporation of Judgement 2.1, results in the expansion of the technology over *FB* up to the segment *GB*. This new boundary is obtained from unit *B* by the consecutive replacement of output 1 by the equal number of output 2. Assume we replace 1 unit of output 1 with 1 unit of output 2. In this case, the point *B1* becomes feasible. Consecutively, if we replace 1 more unit of output 1, then point *B2* also becomes feasible. If all 3 units of output 1 are replaced with 3 units of output 2 relying on the judgement 2.1, we reach the point *G*, where it is possible to produce 3 additional units of output 2 through giving up all 3 units of output 1. Then, the new boundary for the technology moves to segment *GB*, where all units in the expanded area under the segment *GB* are also producible because they are obtained from composite units by the application of trade-off, which is technologically realistic. Similarly, Judgement 2.2 expands the technology to the left of *BE* with the new boundary of *BH* because considering unit *B*, it is possible to replace 3 units of output 2 with 2 units of output 1 relying on the Judgement 2.2. The points *G* and *H* and all the hypothetical units under segments *GB* and *BH* become feasible with the incorporation of those relationships and define an expanded production possibility set bounded by *GBH*. In this expanded technology, unit *B* is the only unit that remains on the efficient frontier, which is now defined as piece-wise linear boundary *GBH*.

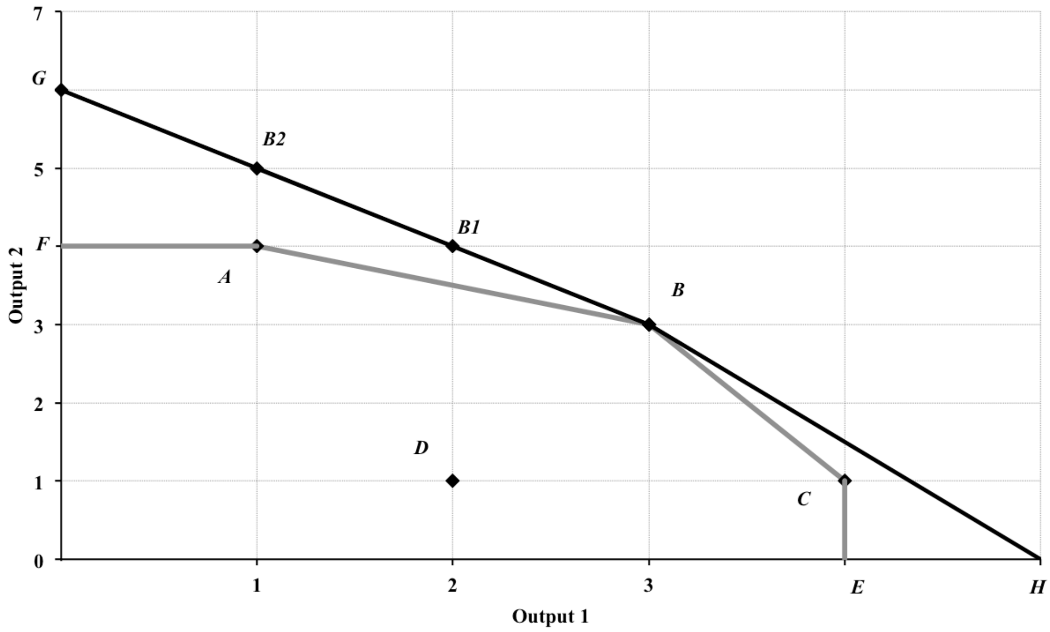

Figure 2.5. Production Possibility Set with Production Trade-offs

The trade-off relations are incorporated to the envelopment form of DEA models as given in (2.15), which measures the output-oriented VRS efficiency of unit *B* in the new technology defined. The non-negative variables π_1 and π_2 modify the composite units in accordance with the judgements given above. They represent the new hypothetical units in the expanded area of the technology obtained through incorporation of judgements. Note that inputs are not modified as seen in (2.15.2), since we do not have any production trade-off judgements related to inputs.

$$Max \;\; \phi \tag{2.15.1}$$

Subject to

$$1\lambda_1 + 1\lambda_2 + 1\lambda_3 + 1\lambda_4 + 0\pi_1 + 0\pi_2 \leq 1 \tag{2.15.2}$$

$$1\lambda_1 + 3\lambda_2 + 4\lambda_3 + 2\lambda_4 - 1\pi_1 + 1\pi_2 \geq 3\phi \tag{2.15.3}$$

$$4\lambda_1 + 3\lambda_2 + 1\lambda_3 + 1\lambda_4 + 1\pi_1 - 1.5\pi_2 \geq 3\phi \tag{2.15.4}$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1 \tag{2.15.5}$$

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \pi_1, \pi_2 \geq 0 \quad (2.15.6)$$

$$\phi \text{ } sign\ free \quad (2.15.7)$$

Before moving to the general formulations of DEA models with trade-offs, let us conceptualise the changes in the production possibility set due to the incorporation of production trade-offs. As mentioned in Section 2.2, for the VRS technology, we have three main axioms defining the PPS given as Axioms 2.1 to 2.3. If we consider CRS technology, then Axiom 2.4 is also considered additional to preceding production axioms. In the case of production trade-offs, we have two more production axioms to consider (Axioms 2.5 and 2.6. below) as given by Podinovski (2004a).

Recall from Section 2.2 that the Production Possibility Set, denoted by *T*, is defined as the set of input and output vectors (X,Y) such that it is possible to produce $Y \geq 0$ from $X \geq 0$. Suppose we have *K* judgements setting up trade-off relationships between inputs and/or outputs. Let the trade-offs are represented as (P^t, Q^t) where $t = 1,2,..,K$. The vectors $P_t \in R^m$ and $Q_t \in R^s$ represent the vectors with trade-off modifications for inputs and outputs, respectively. The vector π represents the weights corresponding to the modification of the composite units.

Axiom 2.5. *Feasibility of trade-offs.* Let $(X,Y) \in T$. Then, for any trade-off *t* in the form of (P^t, Q^t) and any $\pi_t \geq 0$, the unit $(X + \pi_t P_t, Y + \pi_t Q_t) \in T$, provided $X + \pi_t P_t \geq 0$ and $Y + \pi_t Q_t \geq 0$.

Axiom 2.6. *Closedness.* The set *T* is closed

Following the illustrative example and the new PPS considerations given above, the generalized forms of envelopment DEA models under VRS with trade-offs are given for the output and input orientations by (2.16) and (2.17) in Table 2.8.

Table 2.8. Variable Returns-to-Scale Envelopment DEA Models with Trade-offs

Output-oriented VRS Envelopment Model with Trade-offs		Input-oriented VRS Envelopment Model with Trade-offs	
$Max\ \phi$	(2.16)	$Min\ \theta$	(2.17)
Subject to		Subject to	
$\bar{X}\lambda + P\pi \le X_0$		$\bar{X}\lambda + P\pi \le \theta X_0$	
$\bar{Y}\lambda + Q\pi \ge \phi Y_0$		$\bar{Y}\lambda + Q\pi \ge Y_0$	
$e\lambda = 1$		$e\lambda = 1$	
$\lambda, \pi \ge 0$		$\lambda \ge 0$	
ϕ *sign free*		θ *sign free*	

In the envelopment form, the rows of vectors *P* and *Q* represent the outputs or inputs, whereas the columns represent the trade-off judgement. For our example, *P* and *Q* are given as in (2.18). We have two judgements, therefore two columns in both *P* and *Q*. We do not have any coefficients for the inputs in both judgements therefore *P* contains 0s.

$$P = \begin{bmatrix} 0 & 0 \end{bmatrix},\ Q = \begin{bmatrix} -1 & +1 \\ +1 & -1.5 \end{bmatrix} \qquad (2.18)$$

Production trade-offs can be incorporated into multiplier model in the form of weight restrictions. Traditionally, weight restrictions in DEA are based on value judgements. This approach translates the perceived relative importance of input and output factors into the relation between corresponding weights. Podinovski (2004a) provides an alternative approach to use of weight restrictions. It is based on the representation of production trade-offs in terms of weight restrictions. The simple dual relationship means that the incorporation of trade-offs into envelopment model is equivalent to the incorporation of weight restrictions in the multiplier model (Podinovski, 2004a). These restrictions on the weights correspond with hypothetical units added to the technology in the envelopment form. They provide additional constraints to the multiplier DEA linear programs and through these constraints; they reflect the new feasible region (production possibility set). The dual to (2.15) which is given in (2.19) provides us the output-oriented multiplier DEA model under VRS with trade-offs for unit *B*. As seen in (2.19.7) and (2.19.8) the judgements are translated into weight restriction constraints. Note that μ_0 is a sign free variable corresponding to the convexity constraint $e\lambda = 1$ in the envelopment form.

$$Min \;\; v_1 + \mu_0 \tag{2.19.1}$$

Subject to

$$1v_1 - 1\mu_1 - 4\mu_2 + \mu_0 \geq 0 \tag{2.19.2}$$

$$1v_1 - 3\mu_1 - 3\mu_2 + \mu_0 \geq 0 \tag{2.19.3}$$

$$1v_1 - 4\mu_1 - 1\mu_2 + \mu_0 \geq 0 \tag{2.19.4}$$

$$1v_1 - 2\mu_1 - 1\mu_2 + \mu_0 \geq 0 \tag{2.19.5}$$

$$3\mu_1 + 3\mu_2 = 1 \tag{2.19.6}$$

$$0v_1 - (-\mu_1 + \mu_2) \geq 0 \tag{2.19.7}$$

$$0v_1 - (\mu_1 - 1.5\mu_2) \geq 0 \tag{2.19.8}$$

$$v_1, \mu_1, \mu_2 \geq 0 \tag{2.19.9}$$

$$\mu_0 \;\; sign\,free \tag{2.19.10}$$

The generalized forms of multiplier DEA models under VRS and with trade-offs are given for output and input orientations by (2.20) and (2.21) in Table 2.9. The production trade-offs are represented in the form of weight restrictions by the set of constraints (2.20.4) in the output orientation and (2.21.4) in the input orientation. It is worth noting that the trade-offs translate to the same weight restrictions irrespective of the DMU under evaluation and the orientation of the model. Note that the trade-off coefficient matrices *P* and *Q* are transposed. In the multiplier form, the rows of those vectors represent the judgements whereas the columns represent the input and output coefficients, respectively. For our example, *P* and *Q* are given in (2.22). Note that because of the symmetry Q^T is the same with Q, which can be different for other examples.

Table 2.9. Variable Returns-to-Scale Multiplier DEA Models with Trade-offs

Output-oriented VRS Multiplier Model with Trade-offs		**Input-oriented VRS Multiplier Model with Trade-offs**	
$Min\ \ vX_0 + \mu_0$	(2.20.1)	$Max\ \ \mu Y_0 + \mu_0$	(2.21.1)
Subject to		Subject to	
$v\bar{X} - \mu\bar{Y} + e\mu_0 \geq 0$	(2.20.2)	$v\bar{X} - \mu\bar{Y} + e\mu_0 \geq 0$	(2.21.2)
$\mu Y_0 = 1$	(2.20.3)	$vX_0 = 1$	(2.21.3)
$vP^T - \mu Q^T \geq 0$	(2.20.4)	$vP^T - \mu Q^T \geq 0$	(2.21.4)
$v, \mu \geq 0$	(2.20.5)	$v, \mu \geq 0$	(2.21.5)
μ_0 *sign free*	(2.20.6)	μ_0 *sign free*	(2.21.6)

$$P^T = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ Q^T = \begin{bmatrix} -1 & +1 \\ +1 & -1.5 \end{bmatrix} \qquad (2.22)$$

CRS formulations with production trade-offs incorporated for both envelopment and multiplier models are given in Table 2.10. Once again, both orientations are provided. As mentioned, the only difference of CRS models in the envelopment form is convexity constraint $e\lambda = 1$, which is omitted. This corresponds to the absence of free variable μ_0 in the multiplier form.

Table 2.10. Constant Returns-to-Scale DEA Models with Trade-offs

Output-oriented CRS Envelopment Model with Trade-offs	**Input-oriented CRS Envelopment Model with Trade-offs**
$Max\ \phi$ (2.23)	$Min\ \theta$ (2.24)
Subject to	Subject to
$\bar{X}\lambda + P\pi \le X_0$	$\bar{X}\lambda + P\pi \le \theta X_0$
$\bar{Y}\lambda + Q\pi \ge \phi Y_0$	$\bar{Y}\lambda + Q\pi \ge Y_0$
$\lambda, \pi \ge 0$	$\lambda \ge 0$
ϕ *sign free*	θ *sign free*

Output-oriented CRS Multiplier Model with Trade-offs	**Input-oriented CRS Multiplier Model with Trade-offs**
$Min\ vX_0$ (2.25)	$Max\ \mu Y_0$ (2.26)
Subject to	Subject to
$v\bar{X} - \mu\bar{Y} \ge 0$	$v\bar{X} - \mu\bar{Y} \ge 0$
$\mu Y_0 = 1$	$vX_0 = 1$
$vP^T - \mu Q^T \ge 0$	$vP^T - \mu Q^T \ge 0$
$v, \mu \ge 0$	$v, \mu \ge 0$

The production trade-off examples provided by Judgements 2.1 and 2.2 above concern only relationship between two outputs. In principle, the trade-offs can include any set of inputs and outputs. Consider Judgement 2.3 given below revealing a production relationship between the input and output 1.

Judgement 2.3. *It is sufficient to increase the level of input by 3 in order to increase the level of output 1 by 1.*

Judgement 2.3 can be incorporated into envelopment model given in (2.15) through introduction of a new decision variable as π_3 representing this judgement. Constraints (2.15.2) to (2.15.4) will transform to (2.27.1) to (2.27.3), respectively. Since the judgement does not include output 2, the coefficient for π_3 in (2.27.3), which is a constraint corresponding to output 2, is 0.

$$1\lambda_1 + 1\lambda_2 + 1\lambda_3 + 1\lambda_4 + 0\pi_1 + 0\pi_2 + 3\pi_3 \leq 1 \qquad (2.27.1)$$

$$1\lambda_1 + 3\lambda_2 + 4\lambda_3 + 2\lambda_4 - 1\pi_1 + 1\pi_2 + 1\pi_3 \geq 3\phi \qquad (2.27.2)$$

$$4\lambda_1 + 3\lambda_2 + 1\lambda_3 + 1\lambda_4 + 1\pi_1 - 1.5\pi_2 + 0\pi_3 \geq 3\phi \qquad (2.27.3)$$

In the multiplier form, the new judgement expressing the production relationship between the only input and output 1 can be translated into an additional constraint given (2.28) which is imposed to (2.19).

$$3v_1 - \mu_1 \geq 0 \qquad (2.28)$$

One point to mention about construction of DEA models with production trade-offs is the impossibility of infeasible solutions in the multiplier form. It is stated in Podinovski (2004a) that the multiplier DEA models incorporating production trade-offs cannot result in an infeasible model. Infeasibility of the multiplier model is equivalent to the unbounded solution of the envelopment form. In this case, it can be said that at least one of the production trade-offs is expressed incorrectly.

2.7.3. Production Trade-offs vs. Marginal Rates of Substitution

Finally in this section, we aim to point out the difference of Marginal Rate of Substitution (MRS) concept in Economics from the Production Trade-offs concept discussed here, in order to avoid confusion. Some studies can be found in the DEA literature, referring the term "trade-off" implying the rates of substitution or rates of return (e.g. Asmild et al., 2006). The production trade-offs concept in DEA defined by Podinovski (2004a) is completely different. As stated, production trade-offs mentioned in the scope of this study are not calculated values. They are imposed to the technology as in the incorporation of weight restrictions (with a clear difference that trade-offs are not based on managerial judgements) to shape the production possibility set in a more technologically realistic way. They reflect judgements relying on the realities of the production technology and represent on what is possible in the given technology. In the use of production trade-offs, it is not claimed to determine the exact relation between inputs and outputs and moreover, the relationships identified are assumed to be applicable to all the units in the technology. On the other hand, marginal rates of substitution reflect the exact proportions in which the inputs and outputs are changing on the efficient boundary (Podinovski, 2007). It is stated in the seminal work by Charnes et al. (1978), in DEA, ratios of the optimal multipliers provide this information (Asmild et al., 2006). Due to the piecewise linear nature of the DEA frontiers, they can be different for different efficient units (Podinovski, 2007). Therefore, production trade-off concept defined above should not be confused with the trade-off concept used analogous with the marginal rate of substitution in some studies.

2.8. Investigation of Returns to Scale in DEA

The economic notion of returns-to-scale (RTS) has been widely studied within the framework of DEA. Efforts directed to identification of the RTS on DEA frontiers further extended the applicability of DEA and also established a link to standard empirical economic theories of production frontiers. Within the scope of early literature on DEA, the identification largely focused on the qualitative determination of RTS nature, i.e. determination of whether a DMU exhibits decreasing (DRS), increasing (IRS) or constant (CRS) returns-to-scale, rather than to quantify a degree of RTS. Subsequently, the research interest has moved towards the quantification of RTS through calculation of scale elasticities. Such calculations, which require special treatment due to the non-parametric and non-smooth characteristics of DEA frontiers, strengthened the contacts of DEA with the field of Economics. Before moving to the calculation of elasticities, it is essential to provide insight on the notion of RTS investigation, upon which the elasticity measurement builds. Hence, we provide a review on the theory of RTS characterisation in DEA in this section.

The notion of returns-to-scale represents *"the measurement of the increase in output relative to a proportional increase in all inputs, evaluated as marginal changes at a point in input-output space"* (Førsund and Hjalmarsson, 2004). Stated in Banker and Thrall (1992), in economics, RTS is typically defined only for single output situations. RTS are considered to be increasing if a proportionate increase in all the inputs results in a more than proportionate increase in all the single output. Issues of RTS are addressed in the DEA literature with the introduction of VRS efficient frontiers by Banker et al. (1984), which are also known as BCC models and permit to identify increasing, decreasing or constant returns to scale at different scale sizes (Charnes et al., 1994). Two seminal papers by Banker et al. (1984) and Banker (1984) extend the notion of RTS to DEA, which is a multiple output case, referring to the economics literature (Panzar and Willig, 1977) on multi output production. Investigation of RTS in DEA can be seen as identifying the RTS characteristics of DMUs through interpretations of the optimal solutions to the DEA models or to the extensions of them. Although RTS has an unambiguous meaning only if the unit is on the efficient frontier, discussions for the inefficient units are also addressed in the literature, which is also covered further in this section.

2.8.1. Methods of RTS Investigation

Commonly, three basic methods can be recognised in DEA literature regarding the qualitative identification of the RTS nature for a DMU. The first method is referred as *BCC RTS* method by Seiford and Zhu (1999) and proposed in the seminal paper of Banker et al. (1984), where the BCC model is introduced. The method relies on the sign of the optimal value for the free variable μ_0 in the multiplier form of VRS DEA models. The second method employs the use of optimal CCR model results to test the RTS classification of a DMU and discussed in Banker (1984). It relies on the interpretation of sum of optimal λ values in the CCR model and is referred to *CCR RTS* method in Seiford and Zhu (1999). Finally, the last method to consider is developed by Färe et al. (1985; 1994). It is based on the use of optimal radial measures in CCR, BCC and Non-Increasing RTS (described later in the section) models and is referred as *Scale Efficiency Index* method by Seiford and Zhu (1999). Below, we provide the theory on identification of RTS through three different methods regarding the input-oriented DEA models as given in the related papers[8].

Consider the input-oriented CCR model (assuming CRS) given in (2.7) and the input-oriented BCC model (assuming VRS) given in (2.5). Multiplier CCR and BCC models are dual to (2.7) and (2.5) and given by (2.11) and (2.13) in Section 2.6, respectively. *BCC RTS* method given in Banker et al. (1984) requires checking the optimal value of free variable μ_0

[8] Note that input and output orientations may provide different results in terms of RTS findings. Thus, result secured may depend on the orientation used (Banker et al., 2004, pp.347).

in the multiplier model (2.13), which is the dual variable corresponding to the constraint $e\lambda = 1$ in the envelopment form. The interpretation of optimal μ_0 to conclude on the RTS characterisation of the evaluated unit (DMU_0) is given in Theorem 2.1 below.

Theorem 2.1 (Banker et al., 1984). (i) *If* $\mu_0^* = 0$ *in any alternate optima, then constant returns-to-scale (CRS) prevail on* DMU_0. (ii) *If* $\mu_0^* < 0$ *for all alternate optima, then increasing returns-to-scale (IRS) prevail on* DMU_0. (iii) *If* $\mu_0^* > 0$ *for all alternate optima then decreasing returns-to-scale (DRS) prevail on* DMU_0.

On the other hand, as mentioned as *CCR RTS* method and given by Banker (1984), RTS nature of a DMU can also be identified through the employment of CCR model in (2.7). Banker (1984) introduces the notion of *Most Productive Scale Size (MPSS*) and also shows how the CCR models can be employed to estimate the RTS characterisation of the units. The MPSS represents a unit (or multiple units) on the efficiency frontier, at which the outputs produced per unit of inputs is maximised. The unit *B* in Figures 2.1 and 2.2, which is the intersection of VRS and CRS frontier, is an example of a unit performing at MPSS. The sum of optimal λ values is interpreted in order to identify the RTS characterisation for a unit as given in Theorem 2.2.

Theorem 2.2 (Banker, 1984). (i) *If* $e\lambda^* = 1$ *in any alternate optima, then constant returns-to-scale (CRS) prevail on* DMU_0. (ii) *If* $e\lambda^* < 1$ *for all alternate optima, then increasing returns-to-scale (IRS) prevail on* DMU_0. (iii) *If* $e\lambda^* > 1$ *for all alternate optima then decreasing returns-to-scale (DRS) prevail on* DMU_0.

Finally, to conceptualise the RTS investigation through the *Scale Efficiency Index* method developed by Färe et al. (1985), we need the derived Non-Increasing RTS (NIRS) model given as input-oriented in (2.29) below. The NIRS model is basically obtained by imposing the constraint $e\lambda \leq 1$ to the CCR model and the optimal radial efficiency measure for this model can be denoted by θ^*_{NIRS}. Let us also denote the optimal radial efficiency measures in the input-oriented CCR and BCC models given in (2.7) and (2.5) as θ^*_{CRS} and θ^*_{VRS}, respectively. Theorem 2.3 provides the determination of RTS through the optimal radial efficiency scores obtained in CRS, VRS and NIRS models.

$$Min\ \theta_{NIRS} \tag{2.29}$$

Subject to

$$\bar{X}\lambda \leq \theta X_0$$

$$\bar{Y}\lambda \geq Y_0$$

$$e\lambda \leq 1$$

$$\lambda \geq 0$$

$$\theta_{NIRS}\ \textit{sign free}$$

Theorem 2.3 (Färe et al., 1985). (i) $\theta^*_{CRS} = \theta^*_{VRS}$ *if and only if* DMU_0 *exhibits constant returns-to-scale (CRS); otherwise if* $\theta^*_{CRS} < \theta^*_{VRS}$ *or equivalently* $\theta^*_{CRS} \neq \theta^*_{VRS}$ *, then* (ii) $\theta^*_{VRS} > \theta^*_{NIRS}$ *if and only if* DMU_0 *exhibits increasing returns-to-scale (IRS).* (iii) $\theta^*_{VRS} = \theta^*_{NIRS}$ *if and only if* DMU_0 *exhibits decreasing returns-to-scale (DRS).*

2.8.2. Dealing with Multiple Optimal Solutions

Above we provide three fundamental approaches of investigating the RTS nature of a given DMU. One key issue to be addressed is the case of multiple optimal solutions. First two methods given in Theorems 2.1 and 2.2, which rely on the optimal values of some variables, have a possibility of facing a complication due to multiple optimal solutions. Multiple μ_0^* and λ^* yielding different RTS interpretations can be present in some real world problems. This brings out a need for examining the existence of multiple solutions. However, in dealing with real world problems, it may not be always possible or practical to generate all the alternative optimal solutions. Therefore, several further variations or extensions are considered to deal with multiple optimal solutions when RTS is investigated.

Banker and Thrall (1992) identify auxiliary linear programming models in *BCC RTS* method where μ_0^* values are interpreted and determine RTS through the intervals obtained for μ_0^*. Similarly, Banker et al. (1996a) also deal with the multiple optimal solutions in RTS investigation through μ_0^* values and identify a linear programming model, which avoids the examining of all optimal solutions. For the *CCR RTS* method where RTS is evaluated through optimal λ values, Banker et al. (1996b) provide a second-stage linear programming model to check on alternative optimal solutions for λ [9]. On the other hand, the *Scale Efficiency Index* method given by Färe et al. (1985), in which the objective function values of models are interpreted to conclude on RTS classifications, is not affected by the presence of the multiple optimal solutions since it does not require any information related to decision variables.

[9] See Banker et al. (2004) for more details for dealing with multiple optimal solutions in RTS investigation.

In addition to abovementioned treatments of multiple optimal solutions provided by Banker and Thrall (1992) and Banker et al. (1996a: 1996b), an indirect way of identifying the RTS classification of a unit and at the same time omitting the need for examining alternative optimal solutions is put forth by Seiford and Zhu (1999). They set up the relationship between the *Scale Efficiency Index* method and first two approaches and come up with two theorems given as Theorems 2.4 and 2.5 below. These theorems rely on the fact that multiple optimal solutions have nothing to do with RTS identification in the cases of IRS and DRS[10]. Once the units in the CRS classification are identified through the radial efficiency measures in CCR and BCC models regardless of μ_0^* and λ^* values, the units exhibiting IRS and DRS can easily be identified through the interpretation of either μ_0^* or λ^*, since the existence of multiple optimal solutions does not have any effect on them in case of IRS and DRS.

Theorem 2.4 (Seiford and Zhu, 1999). (i) $\theta_{CRS}^* = \theta_{VRS}^*$ *if and only if CRS prevail on* DMU_0*; otherwise if* $\theta_{CRS}^* \neq \theta_{VRS}^*$*, then* (ii) $\mu_0^* > 0$ *if and only if IRS prevail on* DMU_0. (iii) $\mu_0^* < 0$ *if and only if DRS prevail on* DMU_0.

Theorem 2.5 (Seiford and Zhu, 1999). (i) $\theta_{CRS}^* = \theta_{VRS}^*$ *if and only if CRS prevail on* DMU_0*; otherwise if* $\theta_{CRS}^* \neq \theta_{VRS}^*$*, then* (ii) $e\lambda^* < 1$ *if and only if IRS prevail on* DMU_0. (iii) $e\lambda^* > 1$ *if and only if DRS prevail on* DMU_0.

[10] This is given by Corollary 2 and Corollary 3 in Seiford and Zhu (1999) (pp.7).

2.8.3. RTS Investigation for Inefficient Units

Another key issue to be covered is the RTS investigation for units, which are not on the efficient frontier. Methods addressed such as by Banker et al. (1984), Banker (1984) and Banker and Thrall (1992) are only applicable to the units on the frontier. Even though RTS has a clear meaning only for the units on the efficient frontier, some scholars provide methods that can also handle to evaluate RTS for inefficient units. Tone (1996) justifies the need for RTS evaluation for inefficient units with the cases where the majority of the units are inefficient and they remain unanswered in terms of RTS characteristics.

Methods of RTS investigation for inefficient units basically rely on the identification of RTS at the projection of those units on the frontier. Tone (1996) develops a BCC model based method where the RTS of inefficient units are determined automatically from their reference set. Additionally, Golany and Yu (1997) provide a procedure based on linear programming variants of BCC models. They use the optimal values of improvement factors imposed on inputs and outputs to estimate the RTS. Banker et al. (2004) also addresses the issue of RTS investigation for inefficient units through projecting the units onto the BCC efficient frontier and estimating the RTS classification for the projected unit. It is noted in all above papers dealing with RTS investigation of inefficient units that in projecting the units, it can be expected to observe differences between the input and output oriented considerations.

2.8.4. Illustrative Example on RTS Investigation

Let us illustrate the RTS investigation on DEA frontiers using the data set given in Figures 2.1 and 2.2. The data and the efficiency scores together with optimal values of VRS and CRS DEA models for the VRS efficient units are given in Table 2.11 below. As seen in Figure 2.1 the VRS frontier *ABCD* has a piecewise linear shape with different RTS characterisations in different parts. It can be observed that *AB* and *BD* parts exhibit IRS and

DRS, respectively. Unit *B* is on both VRS and CRS frontiers (see Figure 2.2 for the CRS frontier), therefore it is the unit performing at most productive scale size (MPSS). For this unit, $\theta^*_{VRS} = \theta^*_{CRS} = 1$, therefore CRS prevails.

For units *A*, *C* and *D*, the RTS characterisations are identified through both *BCC RTS* (μ_0^* values are interpreted as in Theorem 2.1 or alternatively, Theorem 2.4) and *CCR RTS* (λ^* values are interpreted as in Theorem 2.2 or alternatively, Theorem 2.5) methods in Table 2.11. Unit *A* exhibits IRS since $\mu_0^* > 0$ and $e\lambda^* < 1$. At units *C* and *D*, the VRS frontier exhibits DRS, since $\mu_0^* < 0$ and $e\lambda^* > 1$.

Table 2.11. Illustration of Returns to Scale Investigation

	Input	**Output**	θ^*_{VRS}	θ^*_{CRS}	μ_0^*	λ^*	**RTS**
A	1	10	1	0.75	0.41	0.40	IRS
B	1.5	25	1	1	0	1	CRS
C	2	30	1	0.90	-0.50	1.20	DRS
D	3	36	1	0.68	-1.25	1.44	DRS

As discussed earlier, for inefficient unit *E*, the RTS characterisation can be basically identified through the projection of this unit on the VRS frontier. The projection of *E* is given as *E1* in Figure 2.1 with the input and output values of 1.27 and 17, respectively. For unit *E1*, the optimal values are obtained as $\theta^*_{VRS} = 0.61$, $\mu_0^* = 0.53$, $\theta^*_{CRS} = 0.51$ and $\lambda^* = 0.68$. Therefore, it can be concluded that at the projected unit *E1*, the VRS frontier exhibits IRS since $\mu_0^* > 0$ and $e\lambda^* < 1$.

2.9. Elasticity Measurement on DEA Frontiers

In production theory of empirical economics, it is possible to quantify returns-to-scale (RTS) as scale elasticities, which are measures for the relative change in output compared to the relative change in input. As addressed in the previous section, early DEA literature devotes its focus mostly on identifying the qualitative nature of RTS rather than quantifying it. Subsequently, the research effort has shifted towards the calculation of scale elasticities also on DEA frontiers providing the field of DEA a closer contact with the production theory of Economics. Moreover, the theory of elasticity measures on DEA frontiers have been extended from scale elasticity measurement to more flexible cases of partial elasticities measuring the response of any input or output subset to any mixed input and output bundle at any point of the efficient frontier, in which scale elasticity can be considered as a special case (Podinovski and Førsund, 2010).

In this section, we aim to provide firstly, the basic literature of scale elasticity measurement in DEA literature. Following that, we present the key aspects of mixed partial elasticity measurement theory developed Podinovski and Førsund (2010) upon which our developments build.

2.9.1. Scale Elasticity Literature

Scale elasticity measure can be viewed as a quantitative measure of the strength of the RTS classification (Førsund and Hjalmarsson 2004). In general, elasticity measurement on DEA frontiers is not a straightforward task due to the non-smooth and non-parametric nature of the DEA efficient frontiers. Therefore, classical calculus cannot be applied directly. Different types of difficulties faced in applying the notion of elasticity to DEA frontiers were overcome by several studies, resulting in an inclusive literature on elasticity measurement in DEA (Forsund, 1996; Fukuyama, 2000; Forsund and Hjalmarsson, 2004;

Hadjicostas and Soteriou, 2006; Forsund et al., 2007; Podinovski et al., 2009; Hadjicostas and Soteriou, 2010; Podinovski and Førsund, 2010).

Within the theory of production economics, the calculation of scale elasticity is based on partial derivatives of the transformation function. It requires the smoothness and differentiability on the frontier, which is not the case for piecewise linear DEA frontiers, where differentiability is not valid (i.e. partial derivatives do not exist) at corners or along edges. As stated in Førsund et al. (2007), this is not a limitation of DEA models, but rather is *'a simply feature to be aware of'*. Førsund (1996) states that some adaptation can be made in the piecewise linear technology case, through applying natural rules about right-hand and left-hand derivatives. Subsequently, Førsund and Hjalmarsson (2004), referring to Frisch (1965) and Laitinen (1980), advocate that there exist enough background in economic theory dealing with the case of multiple outputs, which can provide foundation for even the calculation of scale elasticities rather than just qualitatively identify the RTS.

Førsund et al. (2007) review and present two basic ways on calculation of scale elasticity in DEA technologies as indirect and direct approaches. The indirect approach builds upon the work of Førsund and Hjalmarsson (2004), where the scale elasticity formulas for radial projections of inefficient points in the relative interior of facets to the frontier are developed through the efficiency scores and shadow values on the convexity constraint. Førsund et al. (2007) note this indirect approach as not being convenient '*if the purpose of investigating scale properties is to get an overall picture of scale characteristics, not limited to actual observations or their projections'*.

On the other hand, the direct approach provided by Førsund et al. (2007) is developed as a method to calculate scale elasticity at any point on the DEA surface. It is stated as a more general and powerful approach and builds upon the ideas of Krivonozhko et al. (2002; 2004). The direct approach is based on '*cutting through the general multidimensional* *faceted DEA-frontier with a two-dimensional plane in any direction from the origin, and calculating scale elasticity for any point along the intersection of the planes and the frontier. For vertices or points on edges between facets, this method gives scale elasticity based on a right-hand or left-hand derivative in a proportional direction from the origin, corresponding to the basic definition of scale elasticity*'. In Førsund et al. (2007), a real world example is implemented and a high correspondence is observed between indirect and direct methods of calculating scale elasticity.

It is widely accepted (Charnes et al. 1985, Olesen and Petersen 1996, Krivonozhko et al. 2004, Asmild et al. 2006) that, because of the difficulty caused by non-differentiability, most of the previous developments in elasticity measurement on DEA frontiers lacked a rigorous proof at the extreme points of the frontier that represent observed units – exactly where the calculations of elasticity are of the most interest. An exception is the development of scale elasticity computations by Hadjicostas and Soteriou (2006) who offered a complete but technically challenging proof that their results apply to the entire frontier, including its extreme points. In response to the noted analytical difficulties at the extreme points, direct methods mentioned above have been developed in the DEA literature. However, as stated in Podinovski and Førsund (2010), '*they do not result in an analytical expression for the required elasticity measure that is often needed for analysis and interpretation'*.

Podinovski and Førsund (2010) overcome the mentioned difficulties in a different way by extending the earlier results of Podinovski et al. (2009). They proved that a large class of elasticity measures could be expressed as directional derivatives of the optimal values of specially constructed linear programs. Using the known theory of marginal values in linear programming, the calculation of (generally one-sided) elasticities and the proof of corresponding theoretical results became a straightforward task. This approach allowed the introduction of various elasticity measures (such as mentioned above) and substantiation of corresponding computational methods over the entire production frontier, without any simplifying assumptions and in one single development.

2.9.2. Partial Elasticity Measures for Output and Input Sets under VRS Technology

Our proposed theory of elasticity on DEA frontiers in Chapters 4 and 5 build upon the theory of elasticity measurement developed for VRS technologies by Podinovski and Førsund (2010). Therefore, before moving to our developments, it is essential to provide their methodology. In this section, we briefly explain their methodology considering the elasticity measures for output and input sets. In Chapter 4, we extend it to CRS technologies and also improve the implementation, which is applicable to both VRS and CRS technologies. In Chapter 5, production trade-offs are integrated in the measurements.

Consider the VRS technology T_{VRS} with *m* inputs and *s* outputs. The sets *I* and *O* represent the sets of all inputs and all outputs, respectively. Keeping the same notation with our previous discussions, the observed units are denoted as pairs (X^j, Y^j), $j \in J$, where vectors $X^j \in R_+^m$ and $Y^j \in R_+^s$. Recall that $\overline{X}$ and $\overline{Y}$ are the input and output matrices consisting of the input and output vectors X^j and Y^j, respectively.

Throughout the elasticity measurement in both Podinovski and Førsund (2010) and our developments in Chapters 4 and 5, it is assumed that all inputs and outputs can be divided into three disjoint sets as *A*, *B* and *C*. The analyses are concerned with the elasticity of response of the factors in set *B* with respect to the marginal changes of the factors in set *A*, provided the inputs and outputs in set *C* do not change. The set *A* is not empty and may include both inputs and outputs. In elasticity measurement for outputs sets, the set *B* is not empty contains only outputs and $A \cap B = \varnothing$. Then, any unit $(X_0, Y_0) \in T_{VRS}$ can be represented as in (2.30).

$$(X_0, Y_0) = (X_0^A, X_0^C, Y_0^A, Y_0^B, Y_0^C) \tag{2.30}$$

Assuming that sub vector of outputs Y_0^B has at least one strictly positive component and considering the largest amount of β of the output bundle Y_0^B that can be produced in T_{VRS}, given the amount α of the mixed bundle (X_0^A, Y_0^A), under the condition that remaining inputs X_0^C and outputs Y_0^C do not change, the output response function is defined by Podinovski and Førsund (2010) as in (2.31).

$$\bar{\beta}(\alpha) = \max\{\beta \,|\, (\alpha X_0^A, X_0^C, \alpha Y_0^A, \beta Y_0^B, Y_0^C) \in T_{VRS}\} \tag{2.31}$$

Since the elasticity measures of interest are on the efficient frontier, the developments of Podinovski and Førsund (2010) assume that the unit (X_0, Y_0) is efficient, which leads to the condition of $\bar{\beta}(1) = 1$. Assuming that $\bar{\beta}(\alpha)$ is differentiable at $\alpha = 1$, they define the elasticity of response of the output bundle Y_0^B with respect to the mixed bundle (X_0^A, Y_0^A) as in (2.32).

$$\varepsilon_{A,B}(X_0, Y_0) = \bar{\beta}'(1) \tag{2.32}$$

Above definition is an extension of the scale elasticity notion. Note that if $A = I$ and $B = O$ the formula becomes the definition of scale elasticity. Out of this definition, one-sided elasticity measures are defined as follows. In principle, if input and output factors in set A are increased by a factor $\alpha > 1$, the maximum quantity of input or output bundle in set B

possible in the given technology will change by a factor $\varepsilon^{+}_{A,B}(X_0,Y_0)\times(\alpha-1)$, where $\varepsilon^{+}_{A,B}(X_0,Y_0)$ represents the right-hand elasticity of response at the given unit. Inversely, if input and output factors in set A are reduced by a factor $\alpha\in[0,1)$, the maximum quantity of input or output bundle possible in the given technology will change by a factor $\varepsilon^{-}_{A,B}(X_0,Y_0)\times\alpha$, where $\varepsilon^{-}_{A,B}(X_0,Y_0)$ represents the left-hand elasticity of response at the given unit (Podinovski and Førsund, 2010).

Using the theorem of Shapiro (1979) on directional derivatives, right-hand and left-hand elasticities at unit (X_0,Y_0), are defined as the directional derivatives of the output response function $\bar{\beta}(\alpha)$ at $\alpha=1$, which is given as the optimum value of (2.33) under VRS technology in Podinovski and Førsund (2010). The sub matrices $\bar{X}^A$ and $\bar{X}^C$ are defined for inputs and $\bar{Y}^A$, $\bar{Y}^B$ and $\bar{Y}^C$ for outputs representing the inputs and outputs in changing, responding and remaining constant sets.

$$\bar{\beta}(\alpha)=\max \quad \beta \tag{2.33}$$

Subject to

$$\bar{X}^A\lambda\le\alpha X_0^A$$

$$\bar{X}^C\lambda\le X_0^C$$

$$-\bar{Y}^A\lambda\le-\alpha Y_0^A$$

$$-\bar{Y}^B\lambda+\beta Y_0^B\le 0$$

$$-\bar{Y}^C\lambda\le -Y_0^C$$

$$e\lambda=1$$

$\lambda\ge 0$, β sign free

Finally, the calculation of one-sided elasticities derived out of directional derivatives are given and proven by Proposition 1 in Podinovski and Førsund (2010). It provides necessary linear programs for both right and left sides and also guides to explain interpretation of unbounded results in the models with all the proofs. It is given as Theorem 2.6 below.

Theorem 2.6 (Podinovski and Førsund, 2010). (a) *If the function* $\overline{\beta}(\alpha)$ *is defined in some right neighbourhood of* $\alpha = 1$*, then it has a finite right-hand derivative, which can be calculated as in (2.34):*

$$\overline{\beta}'_{+}(1) = \min \quad v^A X_0^A - \mu^A Y_0^A \tag{2.34}$$

Subject to

$$v^A X_0^A + v^C X_0^C - \mu^A Y_0^A - \mu^C Y_0^C + \mu_0 = 1$$

$$v^A \overline{X}^A + v^C \overline{X}^C - \mu^A \overline{Y}^A - \mu^B \overline{Y}^B - \mu^C \overline{Y}^C + \mu_0 \geq 0$$

$$\mu^B Y_0^B = 1$$

$$v^A, v^C, \mu^A, \mu^B, \mu^C \geq 0$$

(b) *If the function* $\overline{\beta}(\alpha)$ *is defined in some left neighbourhood of* $\alpha = 1$*, then it has a finite left-hand derivative, which can be calculated as in (2.35):*

$$\overline{\beta}'_{-}(1) = \max \quad v^A X_0^A - \mu^A Y_0^A \tag{2.35}$$

Subject to

$$v^A X_0^A + v^C X_0^C - \mu^A Y_0^A - \mu^C Y_0^C + \mu_0 = 1$$

$$v^A \overline{X}^A + v^C \overline{X}^C - \mu^A \overline{Y}^A - \mu^B \overline{Y}^B - \mu^C \overline{Y}^C + \mu_0 \geq 0$$

$$\mu^B Y_0^B = 1$$

$$v^A, v^C, \mu^A, \mu^B, \mu^C \geq 0$$

(c) *If the function* $\overline{\beta}(\alpha)$ *is undefined to the right of* $\alpha = 1$, *then the objective function of (2.34) is unbounded. Similarly, if* $\overline{\beta}(\alpha)$ *is undefined to the left of* $\alpha = 1$, *then the objective function of (2.35) is unbounded.*

The similar derivations are made for measuring the elasticity of response of input sets by Podinovski and Førsund (2010). In this case, for the given unit $(X_0, Y_0) \in T_{VRS}$, the elasticity of response of its input bundle X_0^B to mixed bundle (X_0^A, Y_0^A) is considered, provided remaining inputs X_0^C and outputs Y_0^C are kept constant. The sub matrices $\overline{X}^A$, $\overline{X}^B$ and $\overline{X}^C$ are defined for inputs; $\overline{Y}^A$ and $\overline{Y}^C$ for outputs representing the inputs and outputs in changing, responding and remaining constant sets. The output response function in this case is defined as in (2.36) and calculated under VRS technology as in (2.37).

$$\hat{\beta}(\alpha) = \min\left\{\beta \geq 0 \middle| (\alpha X_0^A, \beta X_0^B, X_0^C, \alpha Y_0^A, Y_0^C) \in T_{VRS}\right\}. \tag{2.36}$$

$$\hat{\beta}(\alpha) = \min \quad \beta \tag{2.37}$$

Subject to

$-\overline{X}^A \lambda \geq -\alpha X_0^A$

$-\overline{X}^B \lambda + \beta X_0^B \geq 0$

$-\overline{X}^C \lambda \geq -X_0^C$

$\overline{Y}^A \lambda \geq \alpha Y_0^A$

$\overline{Y}^C \lambda \geq Y_0^C$

$\lambda \geq 0$, β sign free

The elasticity of response of the input bundle X_0^B with respect to the mixed bundle (X_0^A, Y_0^A) is defined as derivative of $\hat{\beta}(\alpha)$ at $\alpha = 1$, given in (2.38).

$$\rho_{A,B}(X_0, Y_0) = \hat{\beta}'(1) \tag{2.38}$$

In a similar manner with the output sets, the elasticity measurement for input sets is explained and proven by Proposition 3 in Podinovski and Førsund (2010) for the VRS technologies. It is given as Theorem 2.7 below.

Theorem 2.7 (Podinovski and Førsund, 2010). (a) *If the function* $\overline{\beta}(\alpha)$ *is defined in some right neighbourhood of* $\alpha = 1$*, then it has a finite right-hand derivative, which can be calculated as in (2.39):*

$$\hat{\beta}'_{+}(1) = \max \quad -v^A X_0^A + \mu^A Y_0^A \tag{2.39}$$

Subject to

$$-v^A X_0^A - v^C X_0^C + \mu^A Y_0^A + \mu^C Y_0^C + \mu_0 = 1$$

$$-v^A \overline{X}^A - v^B \overline{X}^B - v^C \overline{X}^C + \mu^A \overline{Y}^A + \mu^C \overline{Y}^C + \mu_0 \le 0$$

$$v^B X_0^B = 1$$

$$v^A, v^B, v^C, \mu^A, \mu^C \ge 0$$

(b) *If the function* $\overline{\beta}(\alpha)$ *is defined in some left neighbourhood of* $\alpha = 1$*, then it has a finite left-hand derivative, which can be calculated as in (2.40):*

$$\hat{\beta}'_{-}(1) = \min \quad -v^A X_0^A + \mu^A Y_0^A \tag{2.40}$$

Subject to

$$-v^A X_0^A - v^C X_0^C + \mu^A Y_0^A + \mu^C Y_0^C + \mu_0 = 1$$

$$-v^A \bar{X}^A - v^B \bar{X}^B - v^C \bar{X}^C + \mu^A \bar{Y}^A + \mu^C \bar{Y}^C + \mu_0 \leq 0$$

$$v^B X_0^B = 1$$

$$v^A, v^B, v^C, \mu^A, \mu^C \geq 0$$

(c) *If the function* $\bar{\beta}(\alpha)$ *is undefined to the right of* $\alpha = 1$*, then the objective function of (2.39) is unbounded. Similarly, if* $\bar{\beta}(\alpha)$ *is undefined to the left of* $\alpha = 1$*, then the objective function of (2.40) is unbounded.*

2.10. Summary

In this chapter, we provide a comprehensive review on DEA theory. Basic DEA models (envelopment and multiplier forms) with two fundamental returns-to-scale considerations are given with illustrations. Following that, the weight restrictions and the production trade-offs concepts are explained since our proposed methodology involves integration of this notion to agricultural efficiency measurement. Because we also deal with the elasticity measurement on DEA frontiers, a review on this issue beginning from the early approaches of returns-to-scale investigations is also included with the relevant theorems. Then, we move onto the elasticity measurement issues to provide insight for the further developments undertaken in Chapters 4 and 5. We provide the basics of partial elasticity measurement under VRS technologies developed by Podinovski and Førsund (2010). In Chapter 4, their theory is extended to the CRS technologies. In Chapter 5, the notion of trade-offs are integrated into the elasticity measurements for both VRS and CRS technologies. DEA models with production trade-offs and the elasticity measurement models under VRS given in this chapter are used in our empirical applications undertaken in Chapters 7 and 8 together with the developed models in the scope of this research in Chapters 4 and 5.

Chapter 3

A Review on Efficiency Measurement in Agriculture*

Methods of efficiency measurement are widely applied in evaluating agricultural production. A variety of research can be found in Agriculture, Economics and Operational Research literature dealing with measurement and interpretation of efficiency in various agricultural sectors through several parametric or non-parametric approaches at different levels. Since the introduction of the DEA as a non-parametric efficiency evaluation technique by the seminal paper of Charnes et al. (1978), continuous efforts have been put forward in establishing new approaches. Various theoretical and methodological improvements have been carried out. The developed theories and methodologies have been applied to very broad range of areas including agricultural production.

The objective of this chapter is to provide a comprehensive review of previous research on efficiency evaluation applying Data Envelopment Analysis (DEA) in the agricultural sectors. For this purpose, main characteristics and methodological considerations of more than 70 studies in the literature are reviewed. The studies are examined taking different aspects into consideration, which generally can be classified into two main dimensions: general characteristics (*countries of application, sources of data and areas of interest*) and methodological considerations (*methods applied, types of decision making units, selection of variables and return to scale considerations*). Common properties and practices are reviewed and discussed. Together with the key characteristics and methodological considerations of studies, more specifically, a brief discussion of dealing with non-

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homogeneous farms is also included in order to provide an insight for further model developments.

The organization of the chapter is as follows: Section 3.1 reviews the DEA studies in agricultural sectors in terms of their general characteristics such as the countries of applications, sources of data and key areas of interest considered. Section 3.2 provides a review on methodological considerations and specifications undertaken throughout the studies. The section discusses methods applied, type of units and production in the evaluated sectors, selection of inputs and outputs in the models and returns to scale considerations as sub-sections. Section 3.3 aims to provide discussions on dealing with the non-homogeneous farms (farms producing more than one type of product in the same farm) and potential drawbacks that can arise through certain specifications in evaluating those types of farms. Finally, Section 3.4 summarises the observations.

3.1. General Characteristics of the DEA Studies in Agriculture

In this section, a review is provided in terms of general characteristics of the previous research applying DEA in agriculture such as countries of applications, sources of data and the key subjects of interest. DEA studies are conducted in agricultural sectors of several countries through obtaining data from various agriculture-related institutions or databases. Efficiency is measured and interpreted focusing on different subjects through application of several methodological variations of DEA models depending on the context of the study.

3.1.1. Countries of Applications

DEA and related methods are widely applied in agricultural sectors of several countries located in *Europe, America, Asia, Africa* and *Australia*. A number of example studies from agricultural sectors of different countries are presented in this section.

In *Europe*, a considerable amount of studies are dealing with efficiencies of agricultural production in *Central and East European Countries (CEECs)* such as *Bulgaria*, *Czech Republic*, *Estonia*, *Germany*, *Hungary*, *Poland*, *Russia, Slovakia* and *Slovenia*. Examples of studies in *CEE* countries are provided in Table 3.1 with the type of production evaluated.

Table 3.1. Examples of Studies in Central and Eastern European Countries

Country	**Study**	**Type of Production**
Bulgaria & Hungary	Mathijs and Vranken (2000)	Crop & Dairy
Czech Rep. & Slovakia	Mathijs et al. (1999)	Crop & Livestock
Czech Republic	Davidova and Latruffe (2007)	Crop & Livestock
	Latruffe et al. (2008a)	Crop & Livestock
Estonia	Luik et al. (2009)	Crop
Germany	Thiele and Brodersen (1999)	Crop & Livestock
	Cherchye and Van Puyenbroeck (2007)	Crop & Livestock
Poland	Latruffe et al. (2004)	Crop & Livestock
	Balcombe et al. (2008a)	Crop & Livestock
Poland & Russia	Lerman and Schreinemachers (2005)	Crop & Livestock
Russia	Grazhdaninova and Lerman (2005)	Crop & Livestock
Slovenia	Brümmer (2001)	Crop
	Bojnec and Latruffe (2009)	Crop & Livestock

As seen in Table 3.1, in CEE countries, different types of production such as crop, dairy and livestock are assessed through DEA approaches. In some cases, the scope of the study covers units operating in more than one country and more than one type of production.

Apart from *CEECs*, in *Europe*, DEA research on agricultural efficiency is conducted in a number of other European countries such as *Denmark*, *Finland*, *France*, *Greece*, *Netherlands*, *Norway* and *Spain*. Examples are provided in Table 3.2. Farms evaluated in these countries are operating on wide range of production such as crops, dairy, livestock and fishery.

Table 3.2. Examples of Studies in European Countries

Country	Study	Type of Production
Denmark	Andersen and Bogetoft (2007)	Fishery
	Bogetoft et al. (2007)	Crop
	Asmild and Hougaard (2006)	Livestock
Finland	Lansink et al. (2002)	Crop & Livestock
France	Piot-Lepetit et al. (1997)	Crop
	Latruffe et al. (2008b)	Crop & Livestock
Greece	Karkazis and Thanassoulis (1998)	Regional production
	Galanopoulos et al. (2006)	Livestock
Netherlands	Reinhard et al. (2000)	Dairy
	De Koeijer et al. (2002)	Crop
	De Koeijer et al. (2003)	Crop
Norway	Odeck (2009)	Crop
Spain	Millian and Aldaz (1998)	Crop & Livestock
	Aldaz and Millian (2003)	Crop & Livestock
	Iráizoz et al. (2003)	Horticulture
	Reig-Martínez and Picazo-Tadeo (2004)	Crop
	Rodríguez-Díaz et al. (2004)	Irrigation
	Amores and Contreras (2009)	Crop
	André et al. (2010)	Crop
Spain & Germany	Kleinhanß et al. (2007)	Livestock

DEA studies in agricultural sectors are also conducted outside the *Europe*. Table 3.3 provides some examples of research, which take place in agricultural sectors of countries located in *Africa*, *America*, *Asia* and *Australia* with the types of products.

Table 3.3. Examples of Studies outside the Europe

Country	Study	Type of Production
AFRICA		
Botswana	Thirtle et al. (2003)	Crop & Livestock
Ethiophia	Alene et al. (2006)	Crop
South Africa	Townsend et al. (1998)	Crop
Tunisia	Frija et al. (2011)	Crop
AMERICA		
Brazil	Helfland and Levine (2004)	Crop & Livestock
Hawaii / USA	Sharma et al. (1999)	Livestock
Illinois / USA	Färe et al. (1997)	Crop
Kansas / USA	Lilienfeld and Asmild (2007)	Crop
North Carolina / USA	Wossink and Denaux (2006)	Crop
Texas / USA	Haag et al. (1992)	Crop & Livestock
USA	Morrison Paul et al. (2004)	Crop & Livestock
ASIA		
Bangladesh	Coelli et al. (2002)	Crop
	Wadud (2003)	Crop
	Balcombe et al. (2008b)	Crop
China	Zhang (2008)	Crop
	Monchuk et al. (2010)	Crop & Livestock
India	Jha et al. (2000)	Crop
Japan	Sueyoshi (1999)	Cooperatives
Korea	Kim (2001)	Crop
Nepal	Dhungana et al. (2004)	Crop
	Adhikari and Bjorndal (2011)	Crop & Livestock
Turkey	Abay et al. (2004)	Crop
	Binici et al. (2006)	Crop
	Artukoglu et al. (2010)	Crop
Vietnam	Garcia and Shively (2011)	Crop
AUSTRALIA		
Northern Victoria / Australia	Fraser and Cordina (1999)	Dairy
Victoria / Australia	Balcombe et al. (2006)	Dairy

3.1.2. Sources of Data

Data in the previous research on efficiency are obtained from different means of sources such as Farmer Accountancy Data Network (FADN), Ministries of Agriculture, Agricultural

Boards, agriculture related foundations, statistical institutions and surveys conducted by the researchers. Three main sources of data are common. A large number of studies take place in Europe obtain data from Farmer Accountancy Data Network (FADN) databases of the relevant countries. Another common source of data is the surveys conducted by the researchers in the intended region of study. Also, many studies obtain the necessary data through the Ministries of Agriculture and their related institutions in the country of application. Table 3.4 summarises some example studies obtaining data from these different means.

Table 3.4. Examples of Studies according to Source of Data

Source of Data	**Country**	**Example Studies**
FADN	Czech Rep.	Davidova and Latruffe (2007); Latruffe et al. (2008a)
	Estonia	Luik et al. (2009)
	France	Piot-Lepetit et al. (1997); Latruffe et al. (2008b)
	Netherlands	Reinhard et al. (2000); De Koeijer (2003)
	Slovenia	Brümmer (2001); Bojnec and Latruffe (2009)
	Spain	Iráizoz et al. (2003)
	Spain & Germany	Kleinhanß et al. (2007)
Survey Conducted	Australia	Fraser and Cordina (1999); Balcombe et al. (2006)
	Bangladesh	Coelli et al. (2002); Wadud (2003); Balcombe et al. (2008b)
	Bulgaria & Hungary	Mathijs and Vranken (2000)
	USA	Sharma et al. (1999); Wossink and Denaux (2006)
	Ethiophia	Alene et al. (2006)
	Tunisia	Frija et al. (2011)
Ministry of Agriculture	Germany	Thiele and Brodersen (1999)
	Spain	Millian and Aldaz (1998); Aldaz and Millian (2003); Reig-Martínez and Picazo-Tadeo (2004)

3.1.3. Areas of Interest

DEA studies in agricultural sectors deal with efficiency from different points of view and they focus on various subject areas. Some subject areas of interest commonly appear. Most common ones can be listed as *environment*, *irrigation*, *productivity change*, *regional level evaluations* and *subsidies*. Below, we provide a brief reviewing of studies on such common areas of interest. In addition to those, studies with emphasis on subjects as *competitiveness* (Reig-Martínez and Picazo-Tadeo, 2004), *financial management* (Davidova and Latruffe, 2007), *strategic management* (De Koeijer et al., 2003), *sustainability* (Kim, 2001), *decision making* (André et al., 2010) and *quota reallocation* (Andersen and Bogetoft, 2007; Bogetoft et al., 2007) can also be found in the literature.

- ***Environment:*** Environmental performance is one of the key issues taken into consideration in many DEA studies in agriculture. Several research attempts to measure the efficiency of environmental practices of agricultural establishments. Examples include studies by Reinhard et al. (2000), which measure the environmental efficiency of Dutch dairy farms, De Koeijer et al. (2002), which deal with agricultural sustainability with an environmental perspective in sugar beet growing sector of Netherlands and De Koeijer et al. (2003), in which the quality of farmers' environmental performance is assessed through DEA using environmental variables together with the performance indicators in a case study of Dutch arable farms. In addition, Wossink and Denaux (2006) evaluate the pesticide use efficiency in a sample of cotton farms in North Carolina, USA. A regression analysis is performed following DEA in order to analyse the factors affecting efficiency. The study reveals a significant difference in efficiency between different types of cotton produced. Asmild and Hougaard (2006) aim to demonstrate how economic and environmental improvement potentials of Danish pig farms can be estimated using

DEA in the presence of undesirable outputs. The results of the study point out the improvement potentials, especially on the environmental variables.

- ***Irrigation:*** Water use is an inseparable part of agriculture production process. This draws researchers' attention to evaluate the efficiency of water use at both farm and regional level. Therefore, irrigation efficiency is one of the key topics, in agricultural DEA studies. One example can be given from a research conducted in Andalusia region of Spain by Rodríguez-Díaz et al. (2004). The study evaluates the efficiency of irrigation districts in a specific region and depending on the efficiencies, the entire region is divided into three large districts. In the districts, authors identify specific crops, which are cultivated in highly efficient areas. They advocate that this kind of an evaluation can help the policy in terms of considering reductions in labour or water use and substitutions of crops since the results show some relevancy between specific crops and high efficiency.

  In another study, Lilienfeld and Asmild (2007) are looking at the irrigation efficiency, which is conducted in Western Kansas and evaluations are carried out from the perspective of irrigation system type and other factors. DEA models are designed in a water use specific way, which enables the measurement of water use efficiency and excess irrigation water used. A weak relationship between irrigation system types and excess water use is one of the findings of the study. Relationship of excess water use between different factors such as age of farmer, farm size or ground water management is also investigated.

  Also, in a recent study related to irrigation, Frija et al. (2011) investigate farmers' technical efficiency through DEA models considering water use as one of the inputs and additionally, estimate the water demand function using production function approach in Tunisia. The study brings out important implications for the Tunisian water policy.

- ***Productivity change:*** In principle, DEA models are based on relative measurement of efficiency at the same point in time. However, for some problem types, investigation of change in the productivity can be of interest. For this purpose, Malmquist Productivity Index (MPI) approaches (Malmquist, 1953; Caves et al., 1982; Färe et al., 1992) are developed and widely used to observe the changes in productivity during a period of time. Considerable amount of studies in the agricultural efficiency literature deal with the evaluation of the productivity changes overtime. Usually, Malmquist Productivity Index methods and their variations are applied in order to assess the changes. An example is Balcombe et al. (2008a) aiming to evaluate the productivity change in Polish crop and livestock farms between years 1996 and 2000. Productivity indices and farm specific changes in efficiency are calculated through MPI approach. Similarly, Odeck (2009) focuses on the use of a variation to Malmquist Indices and measure the productivity changes. The study designs a procedure for Malmquist Indices with stages and applies this procedure in a sample of grain farms in Norway between years 1987 and 1997.

  Measuring the productivity change has also been studied at regional level as well as the farm level. Millian and Aldaz (1998) and Aldaz and Millian (2003) are examples of regional productivity change studies in Spain. In addition, Thirtle et al. (2003) employ Malmquist Indices and statistical tests in evaluating efficiency change of regional agriculture production in Botswana. Study compares different regions and reveals an interesting finding that the gap between productive and poorer regions is widened. Furthermore, a number of studies on evaluating agricultural production growth at country level through Malmquist Productivity Index approaches are listed by Thirtle et al. (2003). Examples can be given as Thirtle et al. (1995) evaluating

Sub-Saharan Africa countries from 1971 to 1986, Trueblood (1996) and Arnade (1998) working on Worldwide samples of countries from 1961 to 1993, Fulginiti and Perrin (1997) using a sample of Least Developed Countries, from 1961 to 1985 and Suhariyanto et al. (2001) dealing with African and Asian samples of countries, from 1961 to 1991.

- ***Regional level evaluations:*** Although majority of studies in agricultural efficiency studies focus on the evaluations at the farm level, a substantial number of research can be found which is conducted at regional level. Efficiency studies at regional level evaluate the efficiency, changes in efficiency or factors affecting efficiency at a more macro level considering the districts, provinces or regions as decision making units. Examples of regional level DEA studies in agriculture include Zhang (2008) and Monchuk et al. (2010) both conducted in China, where in the former, environmental efficiency of provinces in terms of agricultural production is evaluated and in the latter a derivation of DEA approach is used to determine the technical efficiency at the county level. Some other examples dealing with regions can be given as Millian and Aldaz (1998) and Aldaz and Millian (2003) in Spain, Karkazis and Thanassoulis (1998) in Greece and Thirtle et al. (2003) in Botswana.

- ***Subsidies:*** There are a number of DEA studies dealing with the agricultural efficiency from the perspective of subsidies. These studies focus on the impact of policies related to subsidies or direct payments on the efficiency and investigate the relationship between them. Such studies are usually conducted in European countries and touch the issues of Common Agricultural Policy (CAP) of European Union, which regulates the agricultural subsidies and programmes. One example can be given as Latruffe et al. (2008b), in which relationship with CAP direct payments and managerial efficiency of farms in France is investigated and a negative relationship is identified. Amores and Contreras (2009) propose an allocation system for subsidies through DEA efficiency scores in Spain, considering the aspects of the Common Agricultural Policy in European Union. In a recent study, Zhu and Lansink (2010) evaluate the impact of policy reforms in CAP on the technical efficiency through Stochastic Frontier Analysis (SFA) approach with a large scale study conducted in Germany, Netherlands and Sweden.

3.2. Methodology and Model Specifications of DEA Studies in Agriculture

Depending on the scope of the intended research, DEA studies in agricultural sectors propose different methodological approaches and specify the models in certain ways. This section aims to review methodological considerations and specifications of the studies such as methods applied, types of decision making units in the evaluated sectors, selection of inputs and outputs in the models and returns to scale considerations.

3.2.1. Methods Applied

In agricultural efficiency studies, parametric and non-parametric approaches and their theoretical and methodological variations are widely applied. A large number of studies are implementing more than one method, either as complementary to each other or as alternative approaches to compare. In terms of DEA, different variations and related models are applied such as additive DEA models (Haag et al., 1992), models with allocative input (Färe et al., 1997), sub-vector approach (Piot-Lepetit et al., 1997; Lansink et al., 2002; Asmild and Hougaard, 2006; Lilienfeld and Asmild, 2007), bootstrap DEA approaches (Balcombe et al., 2006; Davidova and Latruffe, 2007; Latruffe et al., 2008a; Balcombe et al., 2008a; 2008b; Odeck, 2009; Monchuk et al., 2010), weight restrictions (Garcia and Shively, 2011) and Malmquist Productivity Indices (Millian and Aldaz, 1998; Balcombe et al., 2008a; Odeck 2009; Thirtle et al., 2003).

Parametric methods such as production function approaches or Stochastic Frontier Analysis (SFA) appear as common techniques applied together with DEA and its related approaches. In addition, regression models have been used in many studies to identify the several factors underlying the inefficiencies. Below, common related approaches of DEA and the methods used together with DEA in the agricultural efficiency evaluation research are reviewed.

- ***Sub-vector Approach:*** Sub-vector approach is one of the variations of DEA models, which is applied in several agricultural efficiency studies. In real world applications of DEA, a distinction of variables can arise being controllable and non-controllable. Sub-vector variation of DEA enables to estimate only the relative input reduction or output expansion potentials in a subset of the inputs or outputs, rather than the reduction or expansion potential in all inputs or outputs simultaneously (Lilienfeld and Asmild, 2007). Sub-vector technical efficiency has been developed by Kopp (1981) and Färe et al. (1983). It is first applied by Banker and Morey (1986).

  The work by Piot-Lepetit et al. (1997) in France is one example of studies, in which sub-vector variation of DEA is employed in agriculture. In this study, sub-vector approach is used to consider land and labour as fixed inputs, whereas other inputs such as equipment, fertilizer, pesticides and seeds are considered to be variable. Lansink et al. (2002) also apply sub-vector approach together with the standard technical and scale efficiency calculations in conventional and organic farming of Finland. Each output is considered separately in different models and capital, land, labour and energy specific models are developed in addition to analysis for standard calculations of technical and scale efficiencies. Furthermore, Asmild and Hougaard (2006) evaluate the efficiency in Danish pig farms using different types of models, two of which are based on sub-vector approach. In one model, the efficiency measured on environmental output variables, where the revenue is kept constant. In another model, efficiency measures are calculated keeping environmental variables fixed and letting only the revenue variable to vary. Another study of sub-vector approach in agriculture is by Lilienfeld and Asmild (2007), evaluating the irrigators in Kansas, USA. In this study, since the irrigation is the main interest, the models are built in water use-specific way and the reduction potential for just this input is investigated.

- ***Malmquist Productivity Index:*** As mentioned in Section 3.1.3, the foregoing use of DEA is based on relative measurement of efficiency at the same point in time. To evaluate the changes in the efficiency overtime, Malmquist Productivity Index (MPI) is introduced by Malmquist (1953) and Caves et al. (1982) and improved further by Färe et al. (1992). Tone (2004) defines Malmquist Productivity Index as *'an index representing Total Factor Productivity (TFP) growth of a Decision Making Unit (DMU), in that it reflects progress or regress in efficiency along with progress or regress of the frontier technology over time under the multiple inputs and multiple outputs framework'*. In other words, Malmquist Productivity Index is a measure of productivity change, which also contains information about the source of this change (Asmild and Tam, 2007). Several studies in agricultural efficiency evaluation apply Malmquist Productivity Index (MPI) approach. Examples can be given as Millian and Aldaz (1998), Balcombe et al. (2008a), Odeck (2009) and Thirtle et al. (2003) as discussed also productivity change studies part in Section 3.1.3.

- ***DEA and Stochastic Approaches:*** Stochastic Frontier Analysis (SFA) is one parametric technique, which is remarkably applied together with DEA in agricultural efficiency studies. It is based on a stochastic frontier production function approach, which is developed by Aigner et al. (1977) and Meeusen and Van den Broeck (1977). The SFA approach requires that a functional form be specified for the frontier production function. An advantage of SFA over DEA is that it takes into account measurement errors and other noise in the data (Latruffe et al., 2004).

In several agricultural studies, SFA technique is used together with DEA and the results are compared with each other. One example can be given as Reinhard et al. (2000), which apply both DEA and SFA approaches to a sample of dairy farms in Netherlands in order to measure the environmental efficiency with the consideration of detrimental inputs. The study compares the results obtained from two methods, together with the discussions of strengths and weaknesses of both approaches in evaluating their case. In Iráizoz et al. (2003), DEA and SFA are applied to horticultural production farms in Spain. Tomato and asparagus production is evaluated separately with both techniques and both of them are found to be highly inefficient. Similarly, Latruffe et al. (2004) aims to measure and compare the technical efficiency through SFA and DEA approaches. A sample of Polish crop and livestock farms are evaluated separately. In this study, SFA findings are generally supported by DEA results. Livestock farms are found to be more efficient. Size-efficiency relationship is found to be positive. Soil quality, degree of integration with downstream markets and education are the variables that are indicated as important determinants of efficiency.

In addition to SFA technique, other similar stochastic approaches are applied together or compared with DEA. As an example, Sharma et al. (1999) apply stochastic efficiency decomposition technique following the Kopp and Diewert (1982) cost decomposition procedure to estimate technical, allocative and economic efficiencies. In the study, the results of both techniques are compared for a sample of swine producers in Hawaii. Results from both models reveal considerable inefficiencies in swine production of Hawaii. Also, DEA is found to be more robust than the parametric approach in overall comparison. Another example of studies comparing results of different approaches can be given as Alene et al. (2006) which also apply a stochastic approach, stochastic frontier production function (SFP), as well as DEA and parametric distance functions (PDF). The study aims to measure the efficiency of different systems in crop production of Ethiopia and compare the performances of three methods. According to the findings of the study, SFP gave the lowest efficiencies. The results reveal that innovative cropping systems contribute to the farmers' efficient use of land and other resources.

- ***DEA and Regression:*** A large number of studies can be found, which evaluate the agricultural efficiency and then investigate the factors underlying the efficiencies or inefficiencies through regression of efficiency scores over sets of various explanatory variables. The relationships between efficiency scores and different variables such as *age of the farmer* (Mathijs and Vranken, 2000; Dhungana et al., 2004,), *education of farmer* (Mathijs and Vranken, 2000; Dhungana et al., 2004; Galanopoulos et al., 2006), *farm size* (Helfland and Levine, 2004; Kleinhanß et al., 2007), *gender* (Mathijs and Vranken, 2000; Dhungana et al., 2004), *land acquisition* (Mathijs and Vranken, 2000; Helfland and Levine, 2004;), *organizational forms* (Mathijs et al., 1999), *product specializations* (Mathijs et al., 1999; Mathijs and Vranken, 2000; Helfland and Levine, 2004), *risk attitude* (Dhungana et al., 2004), *subsidies* (Kleinhanß et al., 2007) and *technology* (Helfland and Levine, 2004) are investigated through regression analyses following DEA. In addition, some studies focus on environmental aspects of the farms and investigate the relationship between efficiencies and *environmental variables* (Wossink and Denaux, 2006) or *environment friendly behavior* of farmers (Mathijs and Vranken, 2000).

- ***Bootstrapping Approaches:*** Simar and Wilson (1998) argue that *'although the literature typically refers to DEA as being deterministic, efficiency is measured relative to an estimate of the true (but unobserved) production frontier. Since* *statistical estimators of the frontier are obtained from finite samples, the corresponding measures of efficiency are sensitive to the sampling variations of the obtained frontier'*. They advocate the bootstrapping introduced by Efron (1979) as a way to analyse the sensitivity of efficiency scores relative to the sampling variations of the estimated frontier. Building upon these discussions, Simar and Wilson (1998; 2000; 2007) propose a bootstrapping methodology allowing the construction of confidence intervals for DEA efficiency scores, which relies on smoothing the empirical distribution (Balcombe et al., 2008a). The approach is also adapted to the case of Malmquist indices in Simar and Wilson (1999). Bootstrapping approaches to DEA introduced by Simar and Wilson are applied widely in DEA literature to estimate and explain technical efficiency.

Bootstrapping approach to DEA is also applied in agricultural efficiency studies. Balcombe et al. (2006) dealing with technical efficiency of Australian dairy farms, Davidova and Latruffe (2007) and Latruffe et al. (2008a) evaluating a sample of crop and livestock farms in Czech Republic, Balcombe et al. (2008b) investigating the technical efficiency and factors behind it in Bangladesh rice farms and Monchuk et al. (2010) measuring the agricultural efficiency of Chinese regions are some examples of studies applying models developed by Simar and Wilson (1998; 2000; 2007). Moreover, the adapted models of bootstrapping to Malmquist Index approach is applied by Odeck (2009) to Norwegian grain farms and Balcombe et al. (2008a) to Polish crop and livestock farms.

3.2.2. Type of DMUs and Production

As mentioned earlier, the agricultural DEA applications in the literature are performed both at farm and regional levels. Majority of the studies apply DEA at a farm level and consider farms as decision making units. Farms considered in the studies operate to produce miscellaneous types of agricultural products. There is research focusing on crop farms (either single or multiple crops), whereas others deal with livestock farms only. A number of studies evaluate farms, which produce both types (crops and livestock). Studies can also be found on the efficiency of farms producing specific types as dairy, fishery, horticulture and organic products. At regional level, the evaluated units are generally agricultural regions, areas or districts.

Crop raising farms evaluated can be grouped in two types: the farms producing a single type of crop or farms with multiple types of crops (we refer this type as non-homogeneous farms). There are several studies in the literature dealing with farms producing single type crops such as *cereals* (Piot-Lepetit et al., 1997), *citrus* (Reig-Martínez and Picazo-Tadeo, 2004), *coffee* (Garcia and Shively, 2011), *corn* (Zhang, 2008), *cotton* (Wossink and Denaux, 2006), *olive* (Amores and Contreras, 2009), *rice* (Kim, 2001; Coelli et al., 2002; Wadud, 2003; Dhungana et al., 2004; Balcombe et al., 2008b), *sugar beet* (De Koeijer et al., 2002; Bogetoft et al., 2007) and *wheat* (Jha et al., 2000). Examples and discussions of research, which attempt to evaluate efficiency in non-homogeneous farms, are covered in Section 3.3.

Only livestock producing farms are also evaluated in several studies. Examples include Sharma et al. (1999) in *swine* producing of Hawaii, Galanapoulos et al. (2006) and Asmild and Hougaard (2006) in *pig* farming of Greece and Denmark and Kleinhanß et al. (2007) in *cattle*, *pig*, *sheep*, *goat* production of Spain and Germany.

In addition to above specifications, it can be noted that a number of studies are conducted in other specific types of agricultural production such as *dairy* (Fraser and Cordina, 1999; Reinhard et al., 2000; Balcombe et al., 2006), *organic farming* (Lansink et al., 2002), *fisheries* (Andersen and Bogetoft, 2007) and *horticulture* (Iráizoz et al., 2003).

It can be observed from the agricultural DEA studies that in order to evaluate farms relative to their analogues, different classifications of samples are undertaken depending on product types or organizational forms. As mentioned, various studies can be found assessing the efficiency in both crop farms and livestock farms. In these type of studies, crop and livestock production are mostly treated separately depending on the farm specialisation (Thirtle et al., 2003, Latruffe et al., 2004; Grazhdaninova and Lerman, 2005; Cherchye and Van Puyenbroeck, 2007; Latruffe et al., 2008a). There are also studies evaluating the multi-product farms producing both crop and livestock in the same farm. As mentioned, discussions of those multi-product farms are discussed in Section 3.3. For some studies, differentiation between organic and conventional production is another consideration to classify the units and evaluate accordingly (Lansink et al., 2002; Artukoglu, 2010).

As well as the differentiation of farms depending on the product type (crop and livestock) in evaluation, researchers also consider to distinguish between the farms according to the organizational forms. In order to assess the relative efficiency of each farm with the similar type in terms of organization form, farms are classified such as being family, cooperative or company farm (Mathijs et al., 1999); individual or corporate farm (Davidova and Latruffe, 2007; Latruffe et al., 2008a); individual, partnership, company or cooperative farms (Thiele and Brodersen, 1999); private, partnership or large-scale successor organization (LSO) farms (Cherchye and Van Puyenbroeck, 2007) and results are interpreted accordingly.

3.2.3. Selection of Inputs and Outputs

One of the major issues to be considered when applying DEA approaches is the selection of appropriate inputs and outputs. As stated in Cooper et al. (2006), the inputs and outputs selected should reflect an interest of an expert so that including these variables into the analysis should make sense in terms of evaluating efficiency. In DEA models, the measurement units of the different inputs and outputs do not need to be analogous. For instance, one input may be measured as number of people whereas another input in the same analysis may be in monetary terms (Cooper et al., 2006).

Outputs: When DEA applications in agricultural sectors are reviewed, it is possible to identify some common inputs and outputs taken into consideration by the majority of the studies. On the output side, most common output used is the *agricultural production* realised by the decision making units. This output is either in the form of monetary values or in the relevant forms of measurement that represent the physical amount produced. In several studies, the output is represented by total monetary value of the production in the form of relevant currency (Piot-Lepetit et al., 1997; Thiele and Brodersen, 1999; Brümmer, 2001; Lansink et al., 2002; Iráizoz et al., 2003; Rodríguez-Díaz et al., 2004; Cherchye an Van Puyenbroeck, 2007; Balcombe et al., 2008a; Adhikari and Bjorndal, 2011) or such as in the studies by Galanopoulos et al. (2006) and Bojnec and Latruffe (2009), revenues realized by the farm.

On the other hand, significant amount of studies consider the production in terms of measures representing the physical production such as kilograms or tonnes (Coelli et al., 2002; Wadud, 2003; Reig-Martínez and Picazo-Tadeo, 2004; Balcombe et al., 2008b; Luik et al., 2009; Odeck, 2009). Additionally, in some studies physical production per a unit of the land are considered as output (De Koeijer et al., 2002; Wossink and Denaux, 2006; Garcia and Shively, 2011). If the evaluated units are dairy farms, total milk production in litres or kilograms are mostly considered as outputs (Fraser and Cordina, 1999; Balcome et al., 2006). A more detailed discussion of considering outputs either in monetary or physical forms from a multi-product farm point of view can be found in Section 3.3.

Inputs: Various inputs have been included in agricultural efficiency evaluation studies with DEA depending on the issues covered and evaluation scope of the research. *Land* and *labour* are the variables that are considered in majority of the studies. *Land* is generally defined as the utilized agricultural area and measured in hectares or homologous measures. *Labour* is measured by different means such as *number of workers* (Grazhdaninova and Lerman, 2005), *labour costs* (i.e. *wages*) (Färe et al., 1997; Kleinhanß et al., 2007; Artukoglu et al., 2010), *annual working units* (Piot-Lepetit et al., 1997; Latruffe et al., 2004; Reig-Martínez and Picazo-Tadeo, 2004; Rodríguez-Díaz et al., 2004; Davidova and Latruffe, 2007; Balcombe et al., 2008a; Latruffe et al., 2008a) or *labour hours* (Fraser and Cordina, 1999; Reinhard et al., 2000; Lansink et al., 2002; Iráizoz et al., 2003; Asmild and Hougaard, 2006; Galanapoulos et al., 2006; Luik et al., 2009).

Naturally, *costs* are among the key input factors. *Costs* are taken into account through different means. On one hand, in many studies, *costs* are integrated into the models as an aggregated variable represented with different labels as '*cultivation costs*' (Iráizoz et al., 2003), '*intermediate consumption*' (Millian and Aldaz, 1998; Latruffe et al., 2004; Davidova and Latruffe, 2007; Balcombe et al., 2008a; Luik et al., 2009), '*materials*' (Alene et al., 2006), '*purchased inputs*' (Helfland and Levine, 2004; Adhikari and Bjorndal, 2011), '*total expenses*' (Amores and Contreras, 2009), '*variable inputs*' (Thiele and Brodersen, 1999; Reinhard et al., 2000; Cherchye and Van Puyenbroeck, 2007; Bojnec and Latruffe, 2009) or '*other expenses*' (Mathijs and Vranken, 2000; Lansink et al., 2002). These aggregated variables can represent the sum of costs on various items in agricultural production process such as energy, fertilizer, feed, fuel, seed, machinery, pesticides, water or farming overheads. The aggregation of *cost* input (i.e. what is included into this item) varies between studies.

On the other hand, some costs or usage of various items in the agricultural production process are considered as separate inputs instead of taking them into account under the aggregated costs. Common examples of such variables are *fertilizers* (Jha et al., 2000; Kim, 2001; Coelli et al. 2002; Wadud, 2003; Lilienfeld and Asmild, 2007; Odeck, 2009; Garcia and Shively, 2011), *fuel* (Grazhdaninova and Lerman, 2005; Andersen and Bogetoft, 2007), *pesticides* (De Koeijer et al., 2002; Reig-Martínez and Picazo-Tadeo, 2004; Wossink and Denaux, 2006), *seed* (Piot-Lepetit et al., 1997; Dhungana et al., 2004; Balcombe et al., 2008b; Odeck, 2009) and *energy consumption* (Morrison Paul et al., 2004; Asmild and Hougaard, 2006; Bogetoft et al., 2007). In significant number of studies, these variables are taken as inputs themselves. Items such as *fertilizers*, *seed* and *pesticides* are represented either with monetary terms or the physical amount purchased.

Another important variable used as an input is the *capital* factor. It has been considered in different forms in several studies. One way undertaken by some studies is to incorporate the *sum of depreciation of fixed assets and the interest payments* as a *capital* factor (Latruffe et al., 2004; Latruffe et al., 2008b; Balcombe et al., 2008a). Another way is to relate *capital* factor to the machinery and other fixed capital such as *hours of used machinery* (Reig-Martínez and Picazo-Tadeo, 2004), *annual costs on capital* (Färe et al., 1997; Lilienfeld and Asmild, 2007; Townsend et al., 1998) or *book value of machinery and inventory* (Lansink et al., 2002; Iráizoz et al., 2003). The use of *total assets* (Brümmer, 2001; Bojnec and Latruffe, 2009), *reported capital in balance sheet* (Mathijs et al., 1999) and *depreciated value of total assets* (Davidova and Latruffe, 2007; Luik et al., 2009) are the other ways considered by the researchers to incorporate capital input into their models.

Inputs identified also vary depending on the product type of the units evaluated. In the research dealing with farms producing livestock or dairy products, it can be observed that animal related inputs are also taken into consideration. Common examples of these variables are *number of animals* (Fraser and Cordina, 1999; Balcombe et al., 2006) and *feed* (Sharma et al., 1999; Fraser and Cordina, 1999; Morrison Paul et al., 2004; Galanopoulos et al., 2006; Balcombe et al., 2006) either in terms of amount or expenditures made for it. *Feed* is usually considered as a separate variable, whereas in some studies it is included in aggregated costs as mentioned above.

Furthermore, the inputs identified for DEA studies in agriculture exhibit slight differences depending on the evaluation context of the study. For instance, in the studies dealing with environmental efficiency, some specific environmentally related inputs such as *nitrogen*, *phosphorus* or *potassium surplus* variables are used. (Reinhard et al., 2000; De Koeijer et al., 2002; 2003; Asmild and Hougaard, 2006). These types of variables are considered as inputs or undesirable outputs and aimed to be minimised, since they are environmentally detrimental factors. Another example can be given the studies dealing with the evaluation of irrigation efficiency. In these types of studies, it is inevitable to consider variables related to water. Rodríguez-Díaz (2004), in which the irrigation districts in Spain are assessed, *water applied in each district* is considered as an input variable. Similarly, Lilienfeld and Asmild (2007) take *water use* and *precipitation* in evaluating the irrigators in Kansas, USA. Recently, in an irrigation efficiency work by Frija et al. (2011), *water use* is considered as an input.

3.2.4. Returns to Scale Considerations

Two fundamental models related to the Returns to Scale (RTS) assumptions are identified in the DEA literature. As thoroughly given in Chapter 2, the original model proposed by Charnes et al. (1978) is known as CCR (Charnes Cooper Rhodes) model and the second model is introduced by Banker et al. (1984), named as BCC (Banker Charnes Cooper) model. CCR model assumes constant returns-to-scale (CRS) and BCC model assumes variable returns-to-scale (VRS) production possibility set.

The tendency of agricultural DEA literature is to consider both fundamental assumptions of returns to scale in application. Numerous studies apply both CRS and VRS methodologies for the same data. CRS and VRS considerations lead to calculation of different types of efficiency as technical, pure technical or scale efficiencies. Relying on application of both approaches, studies measure and interpret different types of efficiency for their sample of units belonging to different types of agricultural production.

It is possible to observe some communality between returns to scale considerations of studies dealing with certain subjects. Studies conducted for the evaluation of efficiency overtime through Malmquist Index approaches at a regional level generally assume constant returns-to-scale (CRS) because of the nature of models applied (Millian and Aldaz, 1998; Aldaz and Millian, 2003; Thirtle et al., 2003). Studies focusing on evaluating environmental performance of units mostly assume variable returns-to-scale since the proportionality between inputs and outputs is not valid for this context (Reig-Martínez and Picazo-Tadeo, 2004; Piot-Lepetit et al., 1997; Reinhard et al., 2000; Asmild and Hougaard, 2006; De Koeijer et al., 2003; Kleinhanß et al., 2007)

As well as being a preliminary assumption to formulate the DEA models, returns to scale (RTS) is an issue that is also investigated as given in Chapter 2. Investigation of returns to scale in agricultural studies is generally performed through parametric methods, specifically Stochastic Frontier Analysis rather than DEA (Tzouvelekas et al., 1997; 2001; Hadley, 2006; Adhikari and Bjorndal, 2011). The research by Townsend et al. (1998) is one example of investigation of returns to scale through DEA models, in which wine production in South Africa is identified as exhibiting constant returns-to-scale.

3.3. Dealing with Non-homogeneous Farms

The aim of this section is to discuss the key considerations of agricultural DEA literature in evaluating the non-homogeneous farms in terms of products, i.e. farms producing more than one type of product in the same farm, in order to provide insight for the further developments. For some farms, since the fixed and variable resources are devoted to production of more than one product in considerable amounts, it is difficult to evaluate efficiency separately for certain types of products. Therefore, it is inevitable to consider the production of all types of products together in the efficiency evaluation process. Several agricultural DEA studies can be found, which deal with non-homogeneous farms under some common considerations. Some examples of these studies are listed in Table 3.5, which summarises the country of application, products and how the outputs are measured. Farms evaluated in given studies produce either multiple crops or multiple products (which include also livestock as well as crops).

As seen in Table 3.5, one of the ways dealing with non-homogenous production is to consider agricultural output in monetary terms. Some studies use an aggregated monetary value for all products as outputs (Brümmer, 2001; Alene et al., 2006; Latruffe et al., 2008b), whereas in some studies monetary value for each product is considered as separate outputs (Iráizoz et al., 2003; Morrison Paul et al., 2004; Adhikari and Bjorndal, 2011). Especially, in the studies dealing with farms producing both crop and livestock products together, it is very common to represent the agricultural production in terms of money (such as revenues, sales or market value). As mentioned earlier in Section 3.2.3, the consideration of agricultural production in monetary values is one of the common practices in efficiency evaluation studies with DEA.

Another way to integrate agricultural outputs into DEA models is to use the physical production amounts, which is also common in various studies as mentioned in Section 3.2.3. This approach is also adapted for the farms producing multiple products, especially in multiple crops case, which can be seen in Table 3.5. There are studies in the literature, which consider the production amount of each crop as a separate output in evaluating the efficiency of non-homogeneous farms. (Färe et al., 1997; Lilienfeld and Asmild, 2007; Luik et al., 2009).

Table 3.5. Examples of Studies Dealing with Non-Homogeneous Farms

Study	Country	Products	Output Measure
Multi-crop			
Färe et al. (1997)	USA	Corn, Soybean, Wheat, Double Crop Soybean	Unit
Iráizoz et al. (2003)	Spain	Tomato and Asparagus	Money
Alene et al. (2006)	Ethiophia	Maize and Coffee	Money
Lilienfeld and Asmild (2007)	USA	Maize, Wheat, Grain Sorghum, Soybeans, Alfalfa Hay, Silage	Unit
Luik et al. (2009)	Estonia	Cereal and Oilseed	Unit
Multi-product			
Brümmer (2001)	Slovenia	Crops and Livestock	Money
Morrison Paul et al. (2004)	USA	Corn, Soybean, Other Crops, Livestock	Money
Grazhdaninova and Lerman (2005)	Russia	Grain, Sunflower, Beef, Milk, Pork	Money
Latruffe et al. (2008b)	France	Cereal, Oilseed, Protein Crops, Beef	Money
Adhikari and Bjorndal (2011)	Nepal	Cereal, Pulse, Cash crops, Other Crops, Livestock	Money

As discussed earlier in the motivations of this research (Chapter 1), potential drawbacks can be brought up for both approaches (using either monetary value or physical production) in consideration of agricultural outputs while evaluating non-homogeneous farms. One potential drawback of using monetary values is that the price differences between products can considerably affect the monetary value of agricultural production. This may yield an advantage for the producers of certain products, which are highly priced even if the production process itself is not efficient for those producers. In addition, dependence of prices on other factors in the market and their fluctuations can affect the efficiency evaluation process so that some farms may gain advantages or possess disadvantages. Moreover, integrating the monetary values may get the evaluation process apart from focusing on the pure production process depending on the context of the study. If a study attempts to evaluate the efficiency of agricultural production process rather than the efficiency of an organization as a whole, considering monetary values may be misleading.

On the other hand, using physical production as outputs may also arise some drawbacks in the evaluation of non-homogeneous farms. Some farms producing specific crops may gain an advantage in the evaluation since other farms may be producing those crops at the level of zero. This may yield to insufficient discrimination of DEA models so that too many units are identified as efficient. Therefore, when farms evaluated are non-homogeneous, modelling the outputs in efficiency measurement is an issue that should be carefully handled, since complications may arise through the both approaches mentioned above. Such complications provide direction for our current research. To overcome the limitations of the both approaches, we propose a novel methodology of integrating production trade-offs approach in DEA literature to the efficiency evaluation in agricultural sectors as stated in the motivations of the research in Chapter 1.

3.4. Summary of the Review

It is possible to find a variety of previous research in Agriculture, Economics and Operational Research literature that are dealing with efficiency measurement in various types of agricultural sectors through application of Data Envelopment Analysis (DEA) and related approaches. In this chapter, we provide a broad review of that previous research. We identify the main characteristics and methodological considerations. Studies are reviewed mainly on two dimensions: 'general characteristics' and 'methodology and model specifications'. In terms of general characteristics, following points can be made:

- DEA studies are conducted in agricultural sectors of several countries located in Europe, America, Asia, Africa and Australia.

- Data in the studies are obtained from different means of sources such as Farmer Accountancy Data Network (FADN), Ministries of Agriculture, Agricultural Boards, agriculture related foundations, statistical institutions and surveys conducted by the researchers. It is very common in studies in Europe to use FADN databases as a source of data.

- Some common subjects of interests can be identified as environment, irrigation, productivity change, regional level evaluations and subsidies. Also, studies with emphasis on subjects as competitiveness, financial management, sustainability, decision-making and quota reallocation can be found in the literature.

Depending on the scope of the intended research, DEA studies in agricultural sectors propose different methodological approaches and specify the models in certain ways. In terms of methodology and model specifications, following points can be made:

- Both parametric and non-parametric approaches and their theoretical and methodological variations are applied. A large number of studies are implementing more than one method either as complementary to each other or as alternative approaches to compare. Stochastic Frontier Analysis (SFA) is one method that is applied together with DEA in a significant amount of studies. Also, many studies investigate the different factors underlying efficiencies through regression type methods.

- Studies are conducted both at farm and regional levels. Majority of the studies apply DEA at a farm level and consider farms as decision making units. Farms evaluated are producing different types of crops and livestock. It can be observed that in order to evaluate farms relative to their analogues, different classifications of samples are undertaken depending on product type (e.g. crop, livestock, dairy etc.) and organizational form (e.g. family farms, corporate farms etc.). Also, several studies can be found in the literature, which perform the evaluations at a more macro level, considering regions, districts or countries as decision making units.

- It is possible to identify some common inputs and outputs taken into consideration by the majority of the studies. Output variable is generally the agricultural production either in monetary or physical units. Land, labour, various types of costs and capital factor are the most common inputs.

- The tendency in terms of returns to scale is to consider both fundamental assumptions of returns to scale in application. Numerous studies apply both CRS and VRS methodologies. Identification of returns to scale is mostly undertaken by applying parametric methods.

An important issue to touch in reviewing the research through DEA in agricultural production is the treatment of non-homogeneous farms, in other words, farms producing a range of products in the same farm. Common practice in dealing with these types of farms is to consider the outputs in monetary terms, especially when the farms are producing both crops and livestock. On the other hand, there are also studies, where the physical production amounts of crops are taken as separate outputs. However, both approaches can acquire some limitations. When more than one type of production is of interest, dealing with the production of these products in efficiency measurement can be an issue to be carefully handled, since some complications arise through the both approaches undertaken in several research up to now.

In conclusion, such limitations drive us to look for ways to use physical production amounts of farms as separate outputs for the sake of evaluating the pure agricultural production, but at the same time it to overcome the problem of poor discrimination of efficiency scores through the employment of production trade-offs approach as stated in the motivations of the research in Chapter 1.

Chapter 4

Mixed Partial Elasticities in Constant Returns-to-Scale Production Technologies*

Recently, Podinovski and Førsund (2010) developed a linear programming approach to the analysis and calculation of a class of mixed partial elasticity measures in variable returns-to scale (VRS) production technologies. In this chapter, we extend their approach to the constant returns-to-scale (CRS) technologies and formulate linear programs required for the computation of elasticity measures. Among other results obtained in this chapter, we prove a new result, valid in both VRS and CRS technologies, which allow us to identify the reason why the corresponding elasticity measure is undefined at the unit. This removes the need for a preliminary sorting of the units into those units where the elasticity measure applies and those where it does not.

4.1. Introduction

In a recent paper, Podinovski and Førsund (2010) introduced a class of elasticity measures for variable returns-to-scale (VRS) production frontiers and methods of their computation. The above study answered the following question: what is the elasticity of response of a subset *B* of outputs (or inputs) with respect to marginal changes of a (generally mixed) subset *A* of inputs and outputs, provided the remaining inputs and outputs in the subset *C* are kept constant? Elasticity measures of this type arise in practical applications of data envelopment analysis (DEA). Examples include problems in which operational inputs and outputs are included in the elasticity calculations while the capacity variables, such as capital equipment or network capacity (Johansen 1972, Salvanes and Tjøtta 1994) or environmental factors (Ruggiero 2000) remain constant.

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The above development was concerned with the technical elasticity measures in that no information on prices or costs was involved. This generalized a number of previous results concerning mostly, but not only, the calculation of scale elasticity and related returns-to-scale (RTS) characteristics. Scale elasticity measure can be viewed as a quantitative measure of the strength of the RTS classification (Førsund and Hjalmarsson 2004). Recent reviews of these results were included in Banker et al. (2004), Førsund et al. (2007) and Hadjicostas and Soteriou (2010).

One of the principal difficulties with the definition and computation of elasticity measures in DEA is the fact that non-parametric efficient frontiers are not smooth and, consequently, classical calculus cannot be applied. It is widely accepted (Charnes et al. 1985, Olesen and Petersen 1996, Krivonozhko et al. 2004, Asmild et al. 2006) that, because of the noted difficulty, most of the previous results lacked a rigorous proof at the extreme points of the frontier that represent observed units – exactly where the calculations of elasticity are of the most interest. An exception is the development of scale elasticity computations by Hadjicostas and Soteriou (2006) who offered a complete but technically challenging proof that their results apply to the entire frontier, including its extreme points.

The above difficulties were overcome in a different way by Podinovski and Førsund (2010) who extended the earlier results of Podinovski et al. (2009). They proved that a large class of elasticity measures could be expressed as directional derivatives of the optimal values of specially constructed linear programs. Using the known theory of marginal values in linear programming, the calculation of (generally one-sided) elasticities and the proof of corresponding theoretical results became a straightforward task. This approach allowed the introduction of various elasticity measures (such as mentioned above) and substantiation of corresponding computational methods over the entire production frontier, without any simplifying assumptions and in one single development.

In this chapter, the development of Podinovski and Førsund (2010) is progressed in a number of ways. First, the class of mentioned elasticity measures from the VRS technology considered in the above paper are extended to the constant returns-to-scale (CRS) technology. Theoretical conditions required for such an extension are obtained and linear programs required for the computation of elasticity measures are formulated.

Second, a new result that complements the study by Podinovski and Førsund (2010) is obtained and it applies equally to the VRS and CRS frontiers. Specifically, it is proven that the case in which an elasticity measure is undefined at a unit makes the corresponding linear program infeasible. This removes the need for a traditionally performed preliminary sorting of the units into those where the elasticity measure applies and those where it does not (the latter would usually include most, but not necessarily all, inefficient units). This result means that we can batch-process the required linear programs for all observed units and the output would either produce the elasticity measure or indicate that the required elasticity measure is undefined and the reason for the latter.

Third, the properties of elasticity measures in some special cases that only arise in the CRS technology are considered. For example, the known fact that the scale elasticity of efficient units in the CRS technology is equal to 1 is generalized. It is also proven that a class of other elasticity measures is also equal to 1 in such a technology. The developments are illustrated by simple numerical examples. All proofs of the chapter are given in Appendix A.

4.2. Elasticity Analysis of Output Sets in CRS Production Technology

Consider a CRS technology T_{CRS} with m inputs and s outputs. The observed units are denoted as pairs (X^j, Y^j), $j = 1,...,n$, where $X^j \in R^m_+$ and $Y^j \in R^s_+$. Vectors X^j and Y^j are not assumed to have positive components, except those specifically required by

Assumptions 4.1 and 4.2. Let $\bar{X}$ and $\bar{Y}$ be the input and output matrices consisting of the input and output vectors X^j and Y^j, respectively.

Following Podinovski and Førsund (2010), it can be assumed that all inputs and outputs can be divided into three disjoint sets: *A*, *B* and *C*. The analysis in this study is concerned with the elasticity of response of the factors in the set *B* with respect to marginal changes of the factors in the set *A*, provided the inputs and outputs in the set *C* do not change. The set *A* is not empty and may include both inputs and outputs. Two scenarios are considered for set *B*: the set *B* contains only outputs or only inputs. This section deals with the elasticity measures for the output scenario. The case of inputs is considered in Section 4.6.

A more general and apparently symmetrical case of the set *B* containing both inputs and outputs can also be considered. However, the resulting notion of elasticity of the factors in *B* with respect to *A* will generally not apply to all efficient units. This is because an efficient unit may not necessarily produce the maximum proportion of its mixed input-output bundle *B* for the given mixed input-output bundle *A*. This makes the exposition and interpretation more technical, and is not pursued in the current study.

Assume that the sets *A* and *B* are not empty, the set *B* contains only outputs, the set *A* may contain either inputs or outputs, or both inputs and outputs. The set *C* contains the remaining inputs and outputs not included in the sets *A* and *B*, and can be empty. Then any unit $(X_0, Y_0) \in T_{CRS}$ can be represented as

$$(X_0, Y_0) = (X_0^A, X_0^C, Y_0^A, Y_0^B, Y_0^C), \tag{4.1}$$

where the superscripts indicate the sub-vectors of X_0 and Y_0 corresponding to the sets A, B and C. If the sets A and C do not contain inputs or outputs, the corresponding sub-vectors are omitted.

For any unit (X_0, Y_0) in the form (4.1), the response of the outputs in the set B to marginal changes of the inputs and/or outputs in the set A is defined only if such a change is feasible in the given technology. This leads to the following two definitions.

Definition 4.1. A *proportional marginal increase* of vectors X_0^A and Y_0^A is feasible in technology T_{CRS} if there exists an $\bar{\alpha} > 1$ such that, for any $\alpha \in [1, \bar{\alpha}]$, there exists a $\beta \geq 0$ (depending on α) for which

$$(\alpha X_0^A, X_0^C, \alpha Y_0^A, \beta Y_0^B, Y_0^C) \in T_{CRS} . \tag{4.2}$$

Definition 4.2. A *proportional marginal reduction* of vectors X_0^A and Y_0^A is feasible in technology T_{CRS} if there exists a $\hat{\alpha} \in [0,1)$ such that, for any $\alpha \in [\hat{\alpha}, 1]$, there exist a $\beta \geq 0$ (depending on α) for which (4.2) holds.

Following Podinovski and Førsund (2010), in order to define the elasticity of response of the output vector Y_0^B to marginal changes of the vectors X_0^A and Y_0^A, first consider the output response function

$$\bar{\beta}(\alpha) = \max\{\beta \,\big|\, (\alpha X_0^A, X_0^C, \alpha Y_0^A, \beta Y_0^B, Y_0^C) \in T_{CRS}\} \tag{4.3}$$

in some neighbourhood of $\alpha = 1$. If a proportional marginal increase or reduction of the vectors X_0^A and Y_0^A is not feasible in T_{CRS} (in the sense of Definitions 4.1 and 4.2), the function $\overline{\beta}(\alpha)$ is undefined in the right or left neighbourhoods of $\alpha = 1$, respectively.

Let $\overline{X}^A$, $\overline{X}^C$, $\overline{Y}^A$, $\overline{Y}^B$ and $\overline{Y}^C$ be the sub-matrices of $\overline{X}$ and $\overline{Y}$ corresponding to the inputs and outputs included in the sets *A*, *B* and *C*. The output response function $\overline{\beta}(\alpha)$ defined in (4.3) is the optimal value in the following linear program, where β is a variable and α is a fixed value:

$$\overline{\beta}(\alpha) = \max \quad \beta \tag{4.4}$$

Subject to

$$\overline{X}^A \lambda \le \alpha X_0^A$$

$$\overline{X}^C \lambda \le X_0^C$$

$$-\overline{Y}^A \lambda \le -\alpha Y_0^A$$

$$-\overline{Y}^B \lambda + \beta Y_0^B \le 0$$

$$-\overline{Y}^C \lambda \le -Y_0^C$$

$\lambda \ge 0$, β sign free

It is common in the DEA literature to define elasticity measures only for efficient units. A unit (X_0, Y_0) is efficient if there exists no other unit (X', Y') in the technology such that, on the component-wise basis, $X_0 \ge X'$, $Y_0 \le Y'$ and $(X_0, Y_0) \ne (X', Y')$. The fact that a unit is technically efficient (that is its radial input or output efficiency is equal to 1), does not guarantee the efficiency of the unit. Testing for efficiency requires the utilization of a two-stage optimisation procedure or an equivalent method (Cooper et al. 2006; Thanassoulis

2001). For practical purposes, a unit is efficient if it coincides with its efficient target, as reported by most DEA programs.

Because the concern is the elasticity of response of a specific subset *B* of outputs, the overall efficiency of the unit is not required, and we only need the unit (X_0, Y_0) to be efficient in the production of its output vector Y_0^B. This is stated below.

Assumption 4.1. *(Selective radial efficiency with respect to the output set B).* The function $\bar{\beta}(\alpha)$ is finite at $\alpha = 1$, and $\bar{\beta}(1) = 1$.

Theorem 4.1. *If the unit $(X_0, Y_0) \in T_{CRS}$ is efficient and the vector Y_0^B has at least one strictly positive component then Assumption 4.1 is satisfied.*

Since zero outputs are allowed in DEA models, the efficiency of the unit itself is not sufficient for the definition of elasticity. For example, if in an efficient unit (X_0, Y_0) output 1 is equal to 1 and output 2 is equal to zero, the elasticity of response of output 2 to output 1 is undefined.

Assumption 4.1 means that the unit (X_0, Y_0), which may be efficient or inefficient, produces the maximum proportion $\beta = 1$ of the output vector Y_0^B possible in the technology for the fixed levels of its inputs and outputs included in the sets *A* and *C*. Any unit (X_0, Y_0) that satisfies Assumption 4.1 is located on the boundary of the technology T_{CRS} but not necessarily on its efficient part (efficient frontier). Even though Assumption 4.1 allows the definition of elasticity measures at some inefficient units, this is different from defining elasticity measures for projections of the inefficient units on the boundary. The units that satisfy Assumption 4.1 are already on the boundary.

Although Assumption 4.1 is needed for the theoretical development of elasticity measures (and can be verified by solving program (4.4) above), in practice no extra effort is required for checking whether this assumption is true. The linear programs developed below in Theorem 4.2 for the calculation of elasticities are self-testing in this respect: according to Theorem 4.3, these become infeasible if Assumption 4.1 is not satisfied.

Following Podinovski and Førsund (2010), if Assumption 4.1 is satisfied and the required derivatives exist, the following definition can be given.

Definition 4.3. The right-hand (left-hand) elasticity of response of the output vector Y_0^B with respect to marginal proportional changes of the vectors X_0^A and Y_0^A is the right (left) derivative of the function $\overline{\beta}(\alpha)$ at $\alpha = 1$:

$$\varepsilon_{A,B}^{+}(X_0, Y_0) = \overline{\beta}'_{+}(1), \tag{4.5}$$

$$\varepsilon_{A,B}^{-}(X_0, Y_0) = \overline{\beta}'_{-}(1). \tag{4.6}$$

The existence of the required one-sided derivatives in (4.5) and (4.6) is established by Theorem 4.2 below. As discussed in Podinovski and Førsund (2010), Definition 4.3 is consistent with conventional definitions of production economics. In particular, this includes the scale elasticity and partial elasticities as special cases.

The following result in Theorem 4.2 extends Proposition 1 of Podinovski and Førsund (2010) to the case of CRS technology. Its proof is given in Appendix A.

Theorem 4.2. *Consider any unit* $(X_0, Y_0) \in T_{CRS}$ *that satisfies Assumption 4.1. (The unit* (X_0, Y_0) *can be either observed or unobserved.)*

(a) *If a proportional marginal increase of vect*ors X_0^A *and* Y_0^A *is feasible in technology* T_{CRS}*, then the right-hand elasticity* $\varepsilon^+_{A,B}(X_0,Y_0)$ *exists, is finite and can be calculated as follows:*

$$\varepsilon^+_{A,B}(X_0,Y_0) = \min \quad v^A X_0^A - \mu^A Y_0^A \tag{4.7.1}$$

Subject to

$$v^A X_0^A + v^C X_0^C - \mu^A Y_0^A - \mu^C Y_0^C = 1 \tag{4.7.2}$$

$$v^A \overline{X}^A + v^C \overline{X}^C - \mu^A \overline{Y}^A - \mu^B \overline{Y}^B - \mu^C \overline{Y}^C \geq 0 \tag{4.7.3}$$

$$\mu^B Y_0^B = 1 \tag{4.7.4}$$

$$v^A, v^C, \mu^A, \mu^B, \mu^C \geq 0 \tag{4.7.5}$$

(b) *If a proportional marginal reduction of vectors* X_0^A *and* Y_0^A *is feasible in technology* T_{CRS}*, then the left-hand elasticity* $\varepsilon^-_{A,B}(X_0,Y_0)$ *exists, is finite and can be calculated by changing the minimisation to maximisation in program (4.7), that is*

$$\varepsilon^-_{A,B}(X_0,Y_0) = \max \quad v^A X_0^A - \mu^A Y_0^A \tag{4.8.1}$$

Subject to

$$v^A X_0^A + v^C X_0^C - \mu^A Y_0^A - \mu^C Y_0^C = 1 \tag{4.8.2}$$

$$v^A \overline{X}^A + v^C \overline{X}^C - \mu^A \overline{Y}^A - \mu^B \overline{Y}^B - \mu^C \overline{Y}^C \geq 0 \tag{4.8.3}$$

$$\mu^B Y_0^B = 1 \tag{4.8.4}$$

$$v^A, v^C, \mu^A, \mu^B, \mu^C \geq 0 \tag{4.8.5}$$

(c) *If a proportional marginal increase (reduction) of vectors* X_0^A *and* Y_0^A *is not feasible in technology* T_{CRS}*, then the objective function in (4.7) (respectively, in (4.8)) is unbounded.*

Comparing (4.7) and (4.8), it can be observed that $\varepsilon^{+}_{A,B}(X_0,Y_0) \le \varepsilon^{-}_{A,B}(X_0,Y_0)$, provided both one-sided elasticities exist. In the case of equality, the output response function $\overline{\beta}(\alpha)$ is differentiable at $\alpha = 1$ and we can define the elasticity $\varepsilon_{A,B}(X_0,Y_0)$ as the derivative $\overline{\beta}'(1)$.

Formally, the use of Theorem 4.2 requires checking Assumption 4.1 first, which can be done by solving model (4.4), where $\alpha = 1$. However, in practical computations this is not necessary. The following result shows that a violation of Assumption 4.1 is equivalent to the infeasibility of linear programs (4.7) and (4.8). Note that programs (4.7) and (4.8) have the same feasible set, and the feasibility of one of them implies the feasibility of the other.

Theorem 4.3. *Assumption 4.1 is true at* (X_0,Y_0) *if and only if both linear programs (4.7) and (4.8) are feasible.*

Theorem 4.3 means that programs (4.7) and (4.8) can in practice be solved for all units, efficient and inefficient. If, for a particular unit (X_0,Y_0), a linear optimizer indicates an infeasible program (4.7) or (4.8), Assumption 4.1 does not hold and the notion of elasticity is not defined at this unit.

It is worth noting that Theorem 4.3 applies to the case of VRS as well and complements the development of Podinovski and Førsund (2010) where the case $\overline{\beta}(1) \ne 1$ is not considered. In the case of VRS, the linear programs (4.7) and (4.8) are modified to incorporate the dual multiplier μ_0 to the convexity constraint, as in Podinovski and Førsund (2010).

4.3. Generalizations of Elasticity Analysis for Output Sets

Traditionally, the calculation of elasticity measures (most often scale elasticity) or analysis of RTS characteristics is performed for efficient units only. This requires the sorting of units into efficient and inefficient units to accompany the analysis of elasticity. In this study, it is argued that elasticity measures of various kinds could be applied to a generally larger set of units on the frontier, including inefficient units, provided they are efficient in the production of outputs from the set *B*, as stated in Assumption 4.1.

Using Theorems 4.2 and 4.3, the analysis and computation of (one-sided) elasticities at any unit (X_0, Y_0) in technology T_{CRS} is straightforward and is based on solving linear programs (4.7) and (4.8) only. The standard diagnostics (optimal solution, unbounded solution or infeasibility) of these two programs provide a complete characterisation of elasticity measures at the unit, including the reasons for a particular elasticity measure to be undefined at the unit.

In practice, programs (4.7) and (4.8) can be solved for all the units (efficient and inefficient) and the results can be interpreted as follows. To be specific, an interpretation of program (4.7) is given.

Case 1. *(Program (4.7) has a finite optimal solution).* According to Theorem 4.3, Assumption 4.1 is satisfied and the notion of elasticity (for the given sets *A*, *B* and *C*) is applicable to the given unit (X_0, Y_0). By Theorem 4.2, a proportional marginal increase of vectors X_0^A and Y_0^A is feasible in technology T_{CRS}. Indeed, if the required marginal increase were infeasible, by part (c) of Theorem 4.2, program (4.7) would have an unbounded objective function. The right-hand elasticity $\varepsilon^{+}_{A,B}(X_0, Y_0)$ is correctly defined and equal to the optimum value of (4.7).

Case 2. *(Program (4.7) has an unbounded optimal solution).* By Theorem 4.3, Assumption 4.1 is satisfied, but part (c) of Theorem 4.2 implies that a proportional marginal increase of vectors X_0^A and Y_0^A is not feasible in the CRS technology. Indeed, if a proportional marginal increase were feasible, by part (a) of Theorem 4.2, the optimum value in (4.7) would be finite, which contradicts the assumption. For this reason, the elasticity $\varepsilon^+_{A,B}(X_0,Y_0)$ is undefined.

Case 3. *(Program (4.7) is infeasible).* If program (4.7) is infeasible (and hence (4.8) is infeasible − as mentioned above, programs (4.7) and (4.8) have the same feasible set, and the feasibility of one of them implies the feasibility of the other.), by Theorem 4.3, Assumption 4.1 is not satisfied. This means that the unit (X_0,Y_0) does not produce the maximum possible amount (proportion) of the output vector Y_0^B and the notion of elasticity is not defined at this unit. Note that this is a different reason for the elasticity of response to be undefined compared to the reason in Case 2. According to Theorem 4.1, this case cannot arise if the unit (X_0,Y_0) is efficient with at least one strictly positive output in the vector Y_0^B. In Section 4.5 the above three cases are illustrated by numerical examples.

4.4. Special Cases of Elasticity Analysis of Output Sets in CRS Production Technology

In this section, special cases corresponding to particular definitions of the sets *A*, *B* and *C* in the CRS technology are considered. The first result generalizes the fact that the scale elasticity in the CRS technology is equal to 1. The second result produces some estimates of elasticity measures.

Theorem 4.4. *Let unit* $(X_0,Y_0)\in T_{CRS}$ *satisfy Assumption 4.1. If the set C is empty, both right-hand and left-hand elasticities exist, and* $\varepsilon^+_{A,B}(X_0,Y_0)=\varepsilon^-_{A,B}(X_0,Y_0)=1$.

The statement that a one-sided elasticity exists implicitly includes the statement that the corresponding proportional marginal increase (for right-hand elasticity) or reduction (for left-hand elasticity) of vectors X_0^A and Y_0^A is feasible in technology T_{CRS}. Theorem 4.4 makes Theorem 4.2 redundant (removing the need to solve programs (4.7) and (4.8)) if the set *C* is empty and Assumption 4.1 is true. However, if Assumption 4.1 has not been verified then solving programs (4.7) and (4.8) has an advantage of being able to test for Assumption 4.1.

One notable special case of Theorem 4.4 is the case of scale elasticity in which the set *A* contains all inputs and the set *B* contains all outputs. In this case, Theorem 4.4 simply restates the fact that the scale elasticity of any unit satisfying Assumption 4.1 in the CRS technology is equal to 1.

Theorem 4.5. *Let unit* $(X_0, Y_0) \in T_{CRS}$ *satisfy Assumption 4.1.*

(a) *If the set B is the set of all outputs, both right-hand and left-hand elasticities exist and satisfy the following inequalities:* $\varepsilon^{+}_{A,B}(X_0, Y_0) \leq 1$ *and* $\varepsilon^{-}_{A,B}(X_0, Y_0) \leq 1$.

(b) *If the set A contains all inputs (and, possibly, some of the outputs), then the right-hand elasticity exists and* $\varepsilon^{+}_{A,B}(X_0, Y_0) \geq 1$.

(c) *If the set A contains only outputs (that is, A does not contain inputs), the left-hand elasticity exists and* $\varepsilon^{-}_{A,B}(X_0, Y_0) \leq 0$.

Note that part (c) of Theorem 4.5 is intuitively clear: if some of the outputs (set A) are marginally increased, this will generally cause the outputs in the set B to decline, provided the inputs and the remaining outputs are kept constant.

4.5. Illustrative Examples for Elasticity Analysis of Output Sets in CRS Technology

In this section, simple numerical examples of computation of elasticity measures in the CRS technology are considered. All examples use the same data set shown in Table 4.1 but use different sets A, B and C to illustrate various possible outcomes.

Table 4.1. The Data Set for Illustrative Example

Unit	Input	Output 1	Output 2	Output radial efficiency in the CRS technology
E	1	2	3	1
F	2	6	5	1
G	1	4	1	1
H	1	1	3	1

The CRS technology induced by the four observed units is shown in Figure 4.1 as the cone spanning the input axis and the rays OK, OE, OF, OG and OM. It is easy to show that all four observed units are technically efficient, that is their output radial efficiency (in the CRS technology, calculating the input radial efficiency would, of course, produce the same result) in the CRS technology is equal to 1. However, only E, F and G are (fully) efficient units. Unit H is outperformed by E and is inefficient.

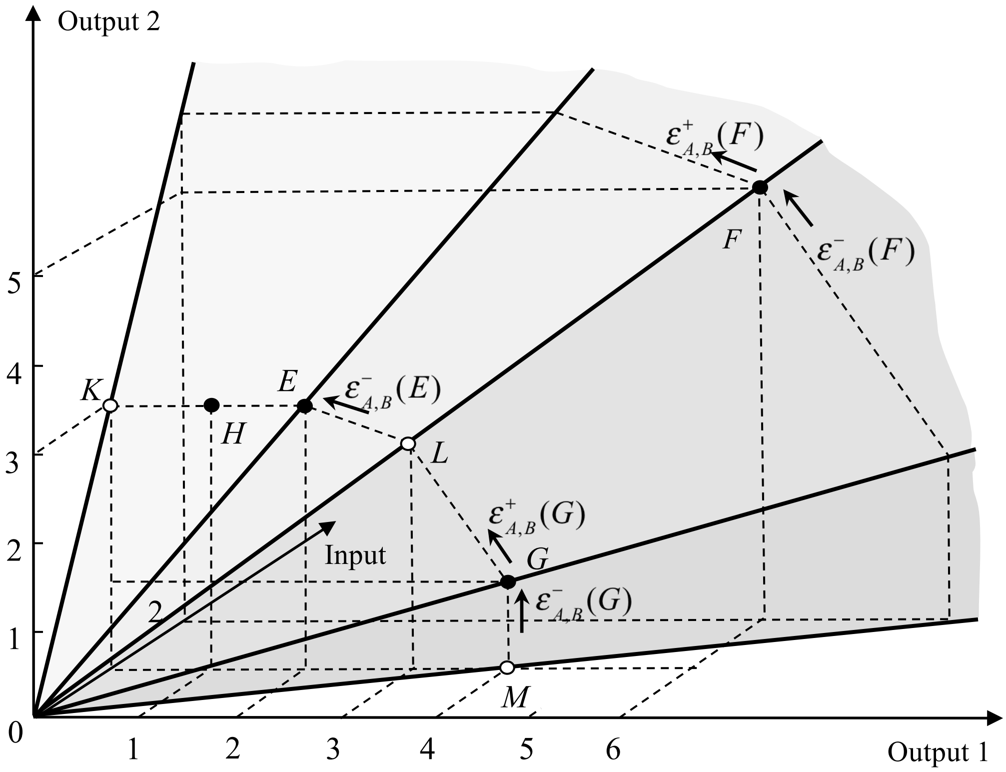

Figure 4.1. The CRS technology and One-sided Elasticities in Scenario 3

Scenario 1. Define *A*={input}, *B*={output 1} and *C*={output 2}. To see if the right-hand and left-hand elasticities exist at the four observed units and, in the case of their existence, calculate their values, programs (4.7) and (4.8) are solved from Theorem 4.2. For example, the right-hand elasticity at unit *E* is found by solving program (4.7):

$\varepsilon^{+}_{A,B}(E) = \min \quad 1v_1$

Subject to

$1v_1 - 3\mu_2 = 1$

$1v_1 - 2\mu_1 - 3\mu_2 \geq 0$

$2v_1 - 6\mu_1 - 5\mu_2 \geq 0$

$1v_1 - 4\mu_1 - 1\mu_2 \geq 0$

$1v_1 - 1\mu_1 - 3\mu_2 \geq 0$

$2\mu_1 = 1$

$v_1, \mu_1, \mu_2 \geq 0$

The results of computations are shown in Table 4.2. For reference, the second column of this table also shows the value of output response function $\overline{\beta}(1)$, which was computed independently by using model (4.4). The computation of $\overline{\beta}(1)$ is not needed in practical situations but it helps us understand some of the results below. Note that $\overline{\beta}(1) = 1$ for units *E*, *F* and *G*, and $\overline{\beta}(1) \neq 1$ for unit *H*. Consequently, the former three units satisfy Assumption 4.1, and unit *H* does not.

Table 4.2. Elasticity Measures for Scenario 1

Unit	$\overline{\beta}(1)$	Optimal value or diagnostics of program (4.7)	Optimal value or diagnostics of program (4.8)	$\varepsilon^{+}_{A,B}$	$\varepsilon^{-}_{A,B}$
E	1	4	Unbounded	4	Undefined
F	1	1.56	2.6	1.56	2.6
G	1	1	1.16	1	1.16
H	2	Infeasible	Infeasible	Undefined	Undefined

As discussed above, the finite optimal values of programs (4.7) and (4.8) mean that the corresponding one-sided elasticities exist and they are equal to those optimal values. These are shown in the last two columns of Table 4.2 and are consistent with part (b) of Theorem 4.5.

For unit *E* program (4.8) is unbounded. This means that a proportional marginal reduction of the input (which constitutes the set *A*) is not possible in the CRS technology under consideration – any such reduction would lead outside the boundaries of the technology. For this reason, the given elasticity measure is not defined at unit *E*.

For unit *H*, both programs (4.7) and (4.8) are infeasible. By Theorem 4.3 this means that unit *H* does not satisfy Assumption 4.1 (because, for this unit, $\overline{\beta}(1) \neq 1$). For this reason, the notion of elasticity (for the given sets *A*, *B* and *C*) is not applicable to unit *H*.

Scenario 2. Define *A*={input, output 1}, *B*={output 2} and *C*=∅. The elasticity measures defined by this scenario can be calculated in two ways. First, programs (4.7) and (4.8) for this scenario may be formulated and solved. For each of the four units, both programs (4.7) and (4.8) have the same optimal value of 1. This means that all four units (including the inefficient unit *H*) satisfy Assumption 4.1 and the elasticity measure for this scenario is equal to 1 at each of them (the reference to one-sided elasticities can be omitted because these are equal).

Second, the same results immediately follow from Theorem 4.4, removing the need to solve programs (4.7) and (4.8). However, Theorem 4.4 is applicable only to units that satisfy Assumption 4.1, which means that the condition $\overline{\beta}(1) = 1$ still needs to be verified. Because units *E*, *F* and *G* are efficient and output 2 is strictly positive, by Theorem 4.1, Assumption 4.1 is true. Unit *H* is inefficient, and the required equality $\overline{\beta}(1) = 1$ can be established by solving model (4.4).

Table 4.3 summarises the results of calculations in Scenario 2. Note that, based on Theorem 4.4, the same results are obtained in the case of standard scale elasticity defined by the following sets: *A*={input}, *B*={output 1, output 2} and *C*=∅.

Table 4.3. Elasticity Measures for Scenario 2

Unit	$\overline{\beta}(1)$	Optimal value or diagnostics of program (4.7)	Optimal value or diagnostics of program (4.8)	$\varepsilon^{+}_{A,B}$	$\varepsilon^{-}_{A,B}$
E	1	1	1	1	1
F	1	1	1	1	1
G	1	1	1	1	1
H	1	1	1	1	1

Scenario 3. Define *A*={output 2}, *B*={output 1} and *C*={input}. By solving programs (4.7) and (4.8) for the given scenario, the results shown in Table 4.4 are obtained. Note that these are consistent with part (c) of Theorem 4.5. The arrows in Figure 4.1 show the directions of marginal movements corresponding to the elasticity calculations in this scenario.

Table 4.4. Elasticity Measures for Scenario 3

Unit	$\overline{\beta}(1)$	Optimal value or diagnostics of program (4.7)	Optimal value or diagnostics of program (4.8)	$\varepsilon^{+}_{A,B}$	$\varepsilon^{-}_{A,B}$
E	1	Unbounded	-3	Undefined	-3
F	1	-1.66	-0.55	-1.66	-0.55
G	1	-0.16	0	-0.16	0
H	2	Infeasible	Infeasible	Undefined	Undefined

For Scenario 3, the right-hand elasticity for unit *G*, left-hand elasticity for unit *E* and both one-sided elasticities for unit *F* are defined with finite negative values given in Table 4.4. These elasticities represent feasible movements along the boundary of the given technology and presented in Figure 4.1 with corresponding arrows. The negative values indicate the inverse relationship since the scenario considers only outputs in both changing and responding sets. A marginal increase of output 2 at the given unit is responded by a reduction in output 1 at that unit and vice versa.

As in Scenario 1, it is observed that the one-sided elasticities may be undefined at particular units for two reasons. Unit *E* has its right-hand elasticity undefined because it is not possible to increase its output 2 (set *A*) while keeping its input (set *C*) constant. Since the plane defined by *OE* and *OK* is the outside boundary of the technology, moving to the right from unit *E* while keeping the input constant results in leaving the boundaries of the technology. This also follows from Theorem 4.2 by noting that the objective function in program (4.7) is unbounded.

At unit *H*, neither one-sided elasticity is defined because, at this unit, $\overline{\beta}(1) \neq 1$. This fact can be established by solving program (4.4) or by using Theorem 4.3 with the fact that both programs (4.7) and (4.8) are infeasible.

At unit *G*, the left-hand elasticity is obtained as 0, which is presented with a vertical arrow in Figure 4.1. While the input is kept constant, a reduction in output 2 is basically a vertical movement along the segment *GM*, which does not cause a marginal change in output 1.

4.6. Elasticity Analysis of Input Sets in CRS Production Technology

In this section, the notion of elasticity of response if the set *B* contains only inputs is briefly outlined. This development is close (and somewhat symmetrical) to the previous case in which the set *B* contained only outputs. The proofs of theorems in this section are also given in Appendix A.

If the set *B* contains only inputs, the input response function is defined as follows:

$$\hat{\beta}(\alpha) = \min\left\{\beta \geq 0 \middle| (\alpha X_0^A, \beta X_0^B, X_0^C, \alpha Y_0^A, Y_0^C) \in T_{CRS}\right\}. \tag{4.9}$$

The input response function $\hat{\beta}(\alpha)$ defined in (4.9) is the optimal value in the following linear program, where β is a variable and α is a fixed value:

$$\hat{\beta}(\alpha) = \min \quad \beta \qquad (4.10)$$

Subject to

$$-\bar{X}^A \lambda \geq -\alpha X_0^A$$

$$-\bar{X}^B \lambda + \beta X_0^B \geq 0$$

$$-\bar{X}^C \lambda \geq -X_0^C$$

$$\bar{Y}^A \lambda \geq \alpha Y_0^A$$

$$\bar{Y}^C \lambda \geq Y_0^C$$

$\lambda \geq 0$, β sign free

If a proportional marginal increase or reduction of vectors X_0^A and Y_0^A is not feasible in T_{CRS}, the function $\hat{\beta}(\alpha)$ is undefined to the right or left of $\alpha = 1$, respectively.

Similar to the case of outputs, the notion of elasticity of response in the case of inputs is applicable to any unit in the CRS technology for which the following assumption holds. Unlike in Assumption 4.1, because $\beta \geq 0$ in (4.8), the requirement that the function $\hat{\beta}$ be finite at $\alpha = 1$ would be redundant.

Assumption 4.2. *(Selective radial efficiency with respect to the input set B).* $\hat{\beta}(1) = 1$.

Theorem 4.6. *If the unit* $(X_0, Y_0) \in T_{CRS}$ *is efficient and the vector* X_0^B *has at least one strictly positive component then Assumption 4.2 is satisfied.*

Definition 4.4. The right-hand (left-hand) elasticity of response of the input vector X_0^B with respect to marginal proportional changes of the vectors X_0^A and Y_0^A is the right (left) derivative of the function $\hat{\beta}(\alpha)$ at $\alpha = 1$:

$$\rho_{A,B}^{+}(X_0, Y_0) = \hat{\beta}'_{+}(1), \tag{4.11}$$

$$\rho_{A,B}^{-}(X_0, Y_0) = \hat{\beta}'_{-}(1). \tag{4.12}$$

The existence of the required one-sided derivatives in (4.11) and (4.12) (and elasticities in Definition 4.4) is established by Theorem 4.7 below.

Theorem 4.7. *Consider any unit* $(X_0, Y_0) \in T_{CRS}$ *that satisfies Assumption 4.2. (The unit* (X_0, Y_0) *can be either observed or unobserved.)*

(a) *If a proportional marginal increase of vecto*rs X_0^A *and* Y_0^A *is feasible in technology* T_{CRS}, *then the right-hand elasticity* $\rho_{A,B}^{+}(X_0, Y_0)$ *exists, is finite and can be calculated as follows:*

$$\rho_{A,B}^{+}(X_0, Y_0) = \max \quad -v^A X_0^A + \mu^A Y_0^A \tag{4.13}$$

Subject to

$$-v^A X_0^A - v^C X_0^C + \mu^A Y_0^A + \mu^C Y_0^C = 1$$

$$-v^A \bar{X}^A - v^B \bar{X}^B - v^C \bar{X}^C + \mu^A \bar{Y}^A + \mu^C \bar{Y}^C \leq 0$$

$$v^B X_0^B = 1$$

$$v^A, v^B, v^C, \mu^A, \mu^C \geq 0$$

(b) *If a proportional marginal reduction of vectors* X_0^A *and* Y_0^A *is feasible in technology* T_{CRS}*, then the left-hand elasticity* $\rho^-_{A,B}(X_0,Y_0)$ *exists, is finite and can be calculated by changing the maximisation to minimisation in program (4.13), that is*

$$\rho^-_{A,B}(X_0,Y_0) = \min \quad -v^A X_0^A + \mu^A Y_0^A \tag{4.14}$$

Subject to

$$-v^A X_0^A - v^C X_0^C + \mu^A Y_0^A + \mu^C Y_0^C = 1$$

$$-v^A \overline{X}^A - v^B \overline{X}^B - v^C \overline{X}^C + \mu^A \overline{Y}^A + \mu^C \overline{Y}^C \le 0$$

$$v^B X_0^B = 1$$

$$v^A, v^B, v^C, \mu^A, \mu^C \ge 0$$

(c) *If a proportional marginal increase (reduction) of vectors* X_0^A *and* Y_0^A *is not feasible in technology* T_{CRS}*, then the objective function in (4.13) (respectively, in (4.14)) is unbounded.*

Similar to the output case, formally, the use of Theorem 4.7 requires checking Assumption 4.2 first. This can be done through solving model (4.10). However, the following result shows that in practice, there is no need to check Assumption 4.2 because solving programs (4.13) and (4.14) validates this requirement automatically.

Theorem 4.8. *Assumption 4.2 is true at* (X_0,Y_0) *if and only if both linear programs (4.13) and (4.14) are feasible.*

Comparing (4.13) and (4.14), we conclude that $\rho^{+}_{A,B}(X_0,Y_0) \geq \rho^{-}_{A,B}(X_0,Y_0)$, provided both one-sided elasticities exist. If these are equal, the function $\hat{\beta}(\alpha)$ is differentiable at $\alpha = 1$ and we can define the elasticity $\rho_{A,B}(X_0,Y_0)$ as the derivative $\hat{\beta}'(1)$.

4.7. Generalizations of Elasticity Analysis for Input Sets

In practice, analysing the elasticity of response of input sets can be performed by solving linear programs (4.13) and (4.14) for all units under consideration. This is similar to the procedure summarized in Section 4.4, with obvious minor modifications. The solution obtained from program (4.13) can be interpreted through cases provided below:

Case 1. *(Program (4.13) has a finite optimal solution).* According to Theorem 4.8, Assumption 4.2 is satisfied and the notion of elasticity (for the given sets *A*, *B* and *C*) is applicable to the given unit (X_0,Y_0). By Theorem 4.7, a proportional marginal increase of vectors X_0^A and Y_0^A is feasible in technology T_{CRS}. Indeed, if the required marginal increase were infeasible, by part (c) of Theorem 4.7, program (4.13) would have an unbounded objective function. The right-hand elasticity $\rho^{+}_{A,B}(X_0,Y_0)$ is correctly defined and equal to the optimum value of (4.13).

Case 2. *(Program (4.13) has an unbounded optimal solution).* By Theorem 4.8, Assumption 4.2 is satisfied, but part (c) of Theorem 4.7 implies that a proportional marginal increase of vectors X_0^A and Y_0^A is not feasible in the CRS technology. Indeed, if a proportional marginal increase were feasible, by part (a) of Theorem 4.7, the optimum value in (4.13) would be finite, which contradicts the assumption. For this reason, the elasticity $\rho^{+}_{A,B}(X_0,Y_0)$ is undefined.

Case 3. *(Program (4.13) is infeasible).* If program (4.13) is infeasible (and hence (4.14) is infeasible − programs (4.13) and (4.14) have the same feasible set, and the feasibility of one of them implies the feasibility of the other, as in the output case.), by Theorem 4.8, Assumption 4.2 is not satisfied. This means that the unit (X_0, Y_0) does not produce the maximum possible amount (proportion) of the input vector X_0^B and the notion of elasticity is not defined at this unit. Note that this is a different reason for the elasticity of response to be undefined compared to the reason in Case 2. According to Theorem 4.6, this case cannot arise if the unit (X_0, Y_0) is efficient with at least one strictly positive input in the vector X_0^B.

As in the elasticity analysis for output sets, a number of special cases can be identified based on particular definitions of sets *A*, *B* and *C*. Below, an analogue of Theorem 4.4 is formulated.

Theorem 4.9. *Let unit* $(X_0, Y_0) \in T_{CRS}$ *satisfy Assumption 4.2. If the set C is empty, both right-hand and left-hand elasticities exist, and* $\rho^+_{A,B}(X_0, Y_0) = \rho^-_{A,B}(X_0, Y_0) = 1$.

4.8. Summary of Elasticity Analysis in CRS Production Technology

The idea of expressing elasticity measures as directional derivatives of the optimal value in linear programs was first suggested and used for the case of scale elasticity in Podinovski et al. (2009). This made it possible to use the theory of marginal values for the definition and calculation of elasticity measures even at the extreme points of the efficient frontier. Podinovski and Førsund (2010) further generalized this to a larger class of elasticity measures in the VRS technology. An important addition compared to the 2009 paper was the non-trivial proof of the fact that, if the elasticity is undefined because a marginal move in the required direction leads outside the technology, the corresponding program becomes unbounded.

In this chapter of the research, abovementioned approach is extended to the CRS technology. An important addition compared to the previous two papers is the proof of the fact that the linear programs used for the calculation of elasticity measures can themselves be used to diagnose if the elasticity measure is correctly defined. The programs are formulated in such a way that if the elasticity measure is undefined at a particular unit, the programs become infeasible.

In conclusion, this chapter does not only extend the earlier approach from the VRS to the CRS technology but it also refines the method (in both VRS and CRS technologies) in that every possible outcome of the linear programs is used for the diagnostic purposes. In particular, a finite optimal value produces the required one-sided elasticity measure. An unbounded solution means that the elasticity of response is undefined because a required marginal change to the inputs or outputs would lead outside the production technology. An infeasible program means that the unit is not efficient in the production of the selected output (or use of the input set), and the required elasticity measure does not apply to it.

Chapter 5

Integration of Production Trade-offs into Mixed Partial Elasticity Measurement

In the scope of our research objectives, we propose to integrate the production trade-offs approach in DEA developed by Podinovski (2004a) to agricultural efficiency and elasticity evaluations. For this purpose, first of all, it is essential to develop the necessary theory, which will enable us to measure elasticities of response on DEA frontiers in the presence of production trade-offs in the given technologies. This is given as Research Question 2 in Chapter 1. In this chapter, we aim to answer Research Question 2 and extend the elasticity measurement approaches developed by us in Chapter 4 for the CRS technology and by Podinovski and Førsund (2010) for VRS technology and to the production technologies with trade-offs. We consider measures for both CRS and VRS technologies, as well as for both output and input sets.

As given in Podinovski (2004a; 2007), production trade-offs represent '*simultaneous changes to the inputs and outputs that are possible in the technology under consideration*'. In this approach, the possible technological trade-off relations between inputs and outputs are constructed and converted into weight restrictions. Section 2.6 of Chapter 2 presents the discussions of production trade-offs in DEA models in detail. Clearly, the introduction of the trade-off relations affects the production possibility set and the efficient frontier. This brings up a new situation to be considered in the analysis of elasticity. We modify the definitions and theorems on elasticity analysis of CRS and VRS DEA technologies in a way that the production trade-off relations between inputs and outputs can be incorporated. In addition, we investigate how the elasticity measures will be affected if the ranges of production trade-offs are changed further in the chapter. The proofs of theorems established in this chapter are given in Appendix B.

The organization of chapter is as follows: Section 5.1 contains the development of elasticity measurement for output sets under CRS technology with trade-offs. In Section 5.2, input sets are considered for the same technology. Section 5.3 provides the elasticity measurement for output sets under VRS technology with trade-offs. Similarly, Section 5.4 considers the input sets for the VRS technology. In Section 5.5, we investigate the effects of changing trade-offs on the elasticity measures. Finally, section 5.6 summarises the findings of this chapter.

5.1. Elasticity Analysis of Output Sets in CRS Technology with Production Trade-offs

In this section, the elasticity measurement of output sets in the CRS technology given in Chapter 4 is modified in order to incorporate the production trade-offs. Definitions 4.1, 4.2 and 4.3 together with Assumption 4.1 given for standard CRS technology (T_{CRS}) in Chapter 4 are also valid for the developments pursued in this section for the expanded technology with trade-offs (T_{CRSTO}). Theorems 4.1, 4.2 and 4.3 are modified into Theorems 5.1, 5.2 and 5.3, respectively. The proofs are given in Appendix B.

Similar to the developments in Chapter 4, the output response function for the expanded CRS technology can be defined as given in (5.1) below.

$$\overline{\beta}(\alpha) = \max\{\beta \mid (\alpha X_0^A, X_0^C, \alpha Y_0^A, \beta Y_0^B, Y_0^C) \in T_{CRSTO}\} \tag{5.1}$$

We begin with the translation of Theorem 4.1 for the expanded CRS technology, which establishes a connection between the efficient unit and the selective radial efficiency given in Assumption 4.1. It is given by Theorem 5.1.

Theorem 5.1. *If the unit $(X_0, Y_0) \in T_{CRSTO}$ is efficient and the vector Y_0^B has at least one strictly positive component then Assumption 4.1 is satisfied.*

Identical with our previous discussions on elasticity measurement, throughout this chapter, we assume that all inputs and outputs can be divided into three disjoint sets as *A*, *B* and *C*. The analyses are concerned with the elasticity of response of the factors in the set *B* with respect to marginal changes of the factors in the set *A*, provided the inputs and outputs in the set *C* do not change. The set *A* is not empty and may include both inputs and outputs. Two scenarios are considered for set *B*: the set *B* contains only outputs or only inputs. In this section, output scenario is considered. Measures for input sets in the CRS technology with production trade-offs are discussed in Section 5.2.

In order to incorporate production trade-offs to the calculation of the output response function in the CRS technology, the trade-off coefficient matrices *P* and *Q* (as given in section 2.7.2) are divided into sub-matrices as P^A, P^C, Q^A, Q^B and Q^C, representing the trade-off coefficients for changing (*A*), responding (*B*) and remaining constant (*C*) sets of inputs and outputs.

In the standard CRS technology, the output response function $\overline{\beta}(\alpha)$ defined by (4.3) can be determined as the solution of (4.4) as given in Section 4.2. The production trade-offs are integrated to the output response model in (4.4) with the same approach given in Section 2.7.2 (see Tables 2.8 and 2.10 in particular) where, trade-offs are integrated to the envelopment DEA models. The output response function $\overline{\beta}(\alpha)$ defined by (5.1) in the expanded CRS technology with trade-offs (T_{CRSTO}) can be determined as the solution of the following linear program given in (5.2), where β is a variable and α is a fixed value.

$$\overline{\beta}(\alpha) = \max \quad \beta \tag{5.2}$$

Subject to

$$\overline{X}^A \lambda + P^A \pi \leq \alpha X_0^A$$

$$\bar{X}^C\lambda + P^C\pi \le X_0^C$$

$$-\bar{Y}^A\lambda - Q^A\pi \le -\alpha Y_0^A$$

$$-\bar{Y}^B\lambda - Q^B\pi + \beta Y_0^B \le 0$$

$$-\bar{Y}^C\lambda - Q^C\pi \le -Y_0^C$$

$\lambda, \pi \ge 0$, β Sign free

Recall from Section 4.2 that the right-hand (left-hand) elasticity of response of the output vector Y_0^B with respect to marginal proportional changes of the vectors X_0^A and Y_0^A is defined as the right (left) derivative of the response function $\bar{\beta}(\alpha)$ at $\alpha = 1$.[11]

Theorem 5.2 below establishes the existence of the required one-sided derivatives in the expanded CRS technology. It is a modification of Theorem 4.2 with the trade-offs incorporated.

Theorem 5.2. *Consider any unit $(X_0, Y_0) \in T_{CRSTO}$ that satisfies Assumption 4.1. (The unit (X_0, Y_0) can be either observed or unobserved.)*

(a) *If a proportional marginal increase of vectors X_0^A and Y_0^A is feasible in technology T_{CRSTO}, then the right-hand elasticity $\varepsilon^+_{A,B}(X_0, Y_0)$ exists, is finite and can be calculated as follows:*

$$\varepsilon^+_{A,B}(X_0, Y_0) = \min \quad v^A X_0^A - \mu^A Y_0^A \tag{5.3.1}$$

Subject to

$$v^A X_0^A + v^C X_0^C - \mu^A Y_0^A - \mu^C Y_0^C = 1 \tag{5.3.2}$$

[11] This is given by (4.5) and (4.6) in Definition 4.3 in Chapter 4.

$$v^A \bar{X}^A + v^C \bar{X}^C - \mu^A \bar{Y}^A - \mu^B \bar{Y}^B - \mu^C \bar{Y}^C \geq 0 \tag{5.3.3}$$

$$v^A P^A + v^C P^C - \mu^A Q^A - \mu^B Q^B - \mu^C Q^C \geq 0 \tag{5.3.4}$$

$$\mu^B Y_0^B = 1 \tag{5.3.5}$$

$$v^A, v^C, \mu^A, \mu^B, \mu^C \geq 0 \tag{5.3.6}$$

(b) *If a proportional marginal reduction of vectors* X_0^A *and* Y_0^A *is feasible in technology* T_{CRSTO}*, then the left-hand elasticity* $\varepsilon_{A,B}^-(X_0, Y_0)$ *exists, is finite and can be calculated by:*

$$\varepsilon_{A,B}^-(X_0, Y_0) = \max \quad v^A X_0^A - \mu^A Y_0^A \tag{5.4.1}$$

Subject to

$$v^A X_0^A + v^C X_0^C - \mu^A Y_0^A - \mu^C Y_0^C = 1 \tag{5.4.2}$$

$$v^A \bar{X}^A + v^C \bar{X}^C - \mu^A \bar{Y}^A - \mu^B \bar{Y}^B - \mu^C \bar{Y}^C \geq 0 \tag{5.4.3}$$

$$v^A P^A + v^C P^C - \mu^A Q^A - \mu^B Q^B - \mu^C Q^C \geq 0 \tag{5.4.4}$$

$$\mu^B Y_0^B = 1 \tag{5.4.5}$$

$$v^A, v^C, \mu^A, \mu^B, \mu^C \geq 0 \tag{5.4.6}$$

(c) *If a proportional marginal increase (reduction) of vectors* X_0^A *and* Y_0^A *is not feasible in technology* T_{CRSTO}*, then the objective function of above models are unbounded.*

Similar to the CRS technology case without trade-offs, the use of Theorem 5.2 initially requires checking Assumption 4.1. In the presence of production trade-offs, this can be done through solving model (5.2). However, in practice, infeasibility of linear programs (5.3) and (5.4) yields to the violation of Assumption 4.1. This is presented in Theorem 5.3, which is simply an analogue of Theorem 4.3.

Theorem 5.3. *Assumption 4.1 is true at* (X_0, Y_0) *if and only if both linear programs (5.3) and (5.4) are feasible.*

The elasticity measurement models in the expanded CRS technology with production trade-offs differ from the standard models for CRS technology in terms of constraint sets (5.3.4) and (5.4.4), which represent the trade-off relations in the form of weight restrictions. This contrast yields to different considerations in the proof of part (c) of Theorem 5.2 than part (c) of Theorem 4.2. On the other hand, the proof of Theorem 5.3 is nearly identical with the proof of Theorem 4.3 with the consideration of the corresponding programs where the trade-offs are present instead of the standard ones.

Using Theorems 5.2 and 5.3, in practice, the analysis and computation of (one-sided) elasticities with production trade-offs for output sets at any unit (X_0, Y_0) in the given technology can be achieved by simply solving programs (5.3) and (5.4) for all the units (efficient and inefficient). The solutions to the programs can be interpreted through three cases identified in Section 5.5.

5.2. Elasticity Analysis of Input Sets in CRS Technology with Production Trade-offs

Following the development of elasticity analysis for input sets in Section 4.6 and the development of elasticity analysis for output sets in CRS technology with the production trade-offs in Section 5.1, the derivation of necessary programs for elasticity analysis of input sets with production trade-offs is a straightforward task. In this case, Definitions 4.1, 4.2 and 4.4 together with Assumption 4.2 given for standard CRS technology (T_{CRS}) in Chapter 4 are also valid for the developments pursued in this section for the expanded technology with trade-offs (T_{CRSTO}). Theorems 4.6, 4.7 and 4.8 are modified into Theorems 5.4, 5.5 and 5.6 such that models incorporate the production trade-offs.

In the expanded CRS technology, the input response function can be defined as below.

$$\hat{\beta}(\alpha) = \min\left\{\beta \geq 0 \middle| (\alpha X_0^A, \beta X_0^B, X_0^C, \alpha Y_0^A, Y_0^C) \in T_{CRSTO}\right\} \tag{5.5}$$

We begin with the translation of Theorem 4.6 for the expanded CRS technology, which establishes a connection between the efficient unit and the selective radial efficiency given in Assumption 4.2. It is given by Theorem 5.4.

Theorem 5.4. *If the unit* $(X_0, Y_0) \in T_{CRSTO}$ *is efficient and the vector* X_0^B *has at least one strictly positive component then Assumption 4.2 is satisfied.*

In the input case, the trade-off coefficient matrices *P* and *Q* are divided into sub-matrices as, P^A, P^B, P^C, Q^A and Q^C representing the trade-off coefficients for changing (*A*), responding (*B*) and remaining constant (*C*) sets of inputs and outputs.

In the standard CRS technology, the input response function $\hat{\beta}(\alpha)$ defined by (4.9) can be determined as the solution of (4.10) as given in Section 4.6. The production trade-offs are integrated to the input response model in (4.10). The input response function $\hat{\beta}(\alpha)$ for the technology T_{CRSTO} defined by (5.5) can be determined as the solution of the following linear program given in (5.6), where β is a variable and α is a fixed value:

.

$$\hat{\beta}(\alpha) = \min \quad \beta \tag{5.6}$$

Subject to

$$-\bar{X}^A \lambda - P^A \pi \geq -\alpha X_0^A$$

$$-\bar{X}^B \lambda - P^B \pi + \beta X_0^B \geq 0$$

$$-\overline{X}^C\lambda - P^C\pi \geq -X_0^C$$

$$\overline{Y}^A\lambda + Q^A\pi \geq \alpha Y_0^A$$

$$\overline{Y}^C\lambda + Q^C\pi \geq Y_0^C$$

$\lambda, \pi \geq 0$, β Sign free

The right-hand (left-hand) elasticity of response of the input vector X_0^B with respect to marginal proportional changes of the vectors X_0^A and Y_0^A is defined as the right (left) derivative of the function $\hat{\beta}(\alpha)$ at $\alpha = 1$[12]. The existence of one sided derivatives and elasticities with trade-offs for input sets is established in Theorem 5.5 below.

Theorem 5.5. *Consider any unit $(X_0, Y_0) \in T_{CRSTO}$ that satisfies Assumption 4.2. (The unit (X_0, Y_0) can be either observed or unobserved.)*

(a) *If a proportional marginal increase of vect*ors *X_0^A and Y_0^A is feasible in technology T_{CRSTO}, then the right-hand elasticity $\rho^+_{A,B}(X_0, Y_0)$ exists, is finite and can be calculated as follows:*

$$\rho^+_{A,B}(X_0, Y_0) = \max \quad -v^A X_0^A + \mu^A Y_0^A \tag{5.7}$$

Subject to

$$-v^A X_0^A - v^C X_0^C + \mu^A Y_0^A + \mu^C Y_0^C = 1$$

$$-v^A \overline{X}^A - v^B \overline{X}^B - v^C \overline{X}^C + \mu^A \overline{Y}^A + \mu^C \overline{Y}^C \leq 0$$

$$-v^A P^A - v^B P^B - v^C P^C + \mu^A Q^A + \mu^C Q^C \leq 0$$

$$v^B X_0^B = 1$$

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[12] This is given by (4.11) and (4.12) in Definition 4.4 in Chapter 4.

$v^A, v^B, v^C, \mu^A, \mu^C \geq 0$

(b) *If a proportional marginal reduction of vectors* X_0^A *and* Y_0^A *is feasible in technology* T_{CRSTO}*, then the left-hand elasticity* $\rho_{A,B}^{-}(X_0, Y_0)$ *exists, is finite and can be calculated by:*

$$\rho_{A,B}^{-}(X_0, Y_0) = \min \quad -v^A X_0^A + \mu^A Y_0^A \tag{5.8}$$

Subject to

$-v^A X_0^A - v^C X_0^C + \mu^A Y_0^A + \mu^C Y_0^C = 1$

$-v^A \overline{X}^A - v^B \overline{X}^B - v^C \overline{X}^C + \mu^A \overline{Y}^A + \mu^C \overline{Y}^C \leq 0$

$-v^A P^A - v^B P^B - v^C P^C + \mu^A Q^A + \mu^C Q^C \leq 0$

$v^B X_0^B = 1$

$v^A, v^B, v^C, \mu^A, \mu^C \geq 0$

(c) *If a proportional marginal increase (reduction) of vectors* X_0^A *and* Y_0^A *is not feasible in technology* T_{CRSTO}*, then the objective function of above models are unbounded.*

Similar to our previous developments, the use of Theorem 5.5 initially requires checking Assumption 4.2. For input sets, in the presence of production trade-offs, this can be done through solving model (5.6). However, in practice, infeasibility of linear programs (5.7) and (5.8) yields to the violation of Assumption 4.2. This is presented in Theorem 5.6, which is simply an analogue of Theorem 4.8 for the models of input sets with production trade-offs.

Theorem 5.6. *Assumption 4.2 is true at* (X_0, Y_0) *if and only if both linear programs (5.7) and (5.8) are feasible.*

Using Theorems 5.5 and 5.6, in practice, the analysis and computation of (one-sided) elasticities with production trade-offs for input sets at any unit (X_0, Y_0) in the given technology can be achieved by simply solving programs (5.7) and (5.8) for all the units (efficient and inefficient). The solutions to the programs can be interpreted through three cases identified in Section 5.5.

5.3. Elasticity Analysis of Output Sets in VRS Technology with Production Trade-offs

The elasticity measures in the VRS production technologies have been introduced by Podinovski and Førsund (2010). Their developments are provided in Section 2.9.2 of Chapter 2. Following these developments and the generalizations for the elasticity analysis in CRS technology introduced in preceding sections, in this and following section, production trade-offs are integrated into the elasticity analysis in the VRS technologies. As in the previous development two scenarios are considered: the changing factor set *B* contains only outputs or only inputs. Output scenario is considered in this section, whereas the input scenario is considered in Section 5.4.

In development, only difference from the CRS technology considerations arises due to the additional convexity constraint of the VRS models that equalizes the sum of λs to 1 in the envelopment form. In VRS technology, the envelopment form of standard output oriented DEA model presented in (2.4) has an additional constraint as $e\lambda = 1$. In the multiplier form, which is dual to the envelopment form, this convexity constraint is represented by free dual variable μ_0 added on the left-hand side of first constraint set as in (2.12).

We begin with the adaptation of basic definitions provided in Chapter 4 for the CRS technologies (T_{CRS}) to the VRS technologies with trade-offs (T_{VRSTO}). Then, elasticity measurement theory for T_{VRSTO} is provided. Once again, assume that the input and output sets A and B are not empty, the set B contains only outputs, the set A may contain either inputs or outputs, or both inputs and outputs. The set C contains the remaining inputs and outputs not included in the sets A and B, and can be empty. Then any unit in the VRS technology with trade-offs $(X_0, Y_0) \in T_{VRSTO}$ can be represented as:

$$(X_0, Y_0) = (X_0^A, X_0^C, Y_0^A, Y_0^B, Y_0^C), \tag{5.9}$$

where the superscripts indicate the sub-vectors of X_0 and Y_0 corresponding to the sets A, B and C.

For any unit (X_0, Y_0) in the form (5.9), the response of the outputs in the set B to marginal changes of the inputs and/or outputs in the set A is defined only if such a change is feasible in the given technology. This leads to the translation of Definitions 4.1 and 4.2 for CRS technology to the Definitions 5.1 and 5.2, which define the proportional marginal increase and reduction of vectors X_0^A and Y_0^A in the VRS technology with trade-offs.

Definition 5.1. A *proportional marginal increase* of vectors X_0^A and Y_0^A is feasible in technology T_{VRSTO} if there exists an $\bar{\alpha} > 1$ such that, for any $\alpha \in [1, \bar{\alpha}]$, there exists a $\beta \geq 0$ (depending on α) for which

$$(\alpha X_0^A, X_0^C, \alpha Y_0^A, \beta Y_0^B, Y_0^C) \in T_{VRSTO}. \tag{5.10}$$

Definition 5.2. A *proportional marginal reduction* of vectors X_0^A and Y_0^A is feasible in technology T_{VRSTO} if there exists a $\hat{\alpha} \in [0,1)$ such that, for any $\alpha \in [\hat{\alpha}, 1]$, there exist a $\beta \geq 0$ (depending on α) for which (5.10) holds.

In order to define the elasticity of response of the output vector Y_0^B to marginal changes of the vectors X_0^A and Y_0^A, consider the output response function in T_{VRSTO}.

$$\bar{\beta}(\alpha) = \max\{\beta \,\big|\, (\alpha X_0^A, X_0^C, \alpha Y_0^A, \beta Y_0^B, Y_0^C) \in T_{VRSTO}\} \tag{5.11}$$

in some neighbourhood of $\alpha = 1$. If a proportional marginal increase or reduction of the vectors X_0^A and Y_0^A is not feasible in T_{VRSTO} (in the sense of Definitions 5.1 and 5.2), the function $\bar{\beta}(\alpha)$ is undefined in the right or left neighbourhoods of $\alpha = 1$, respectively.

The output response function $\bar{\beta}(\alpha)$ defined by (5.11) under T_{VRSTO} can be determined as the solution of the following linear program, where β is a variable and α is a fixed value. As mentioned, (5.12) differs from (5.2) in terms of the additional constraint in (5.12.7).

$$\bar{\beta}(\alpha) = \max \quad \beta \tag{5.12.1}$$

Subject to

$$\bar{X}^A\lambda + P^A\pi \le \alpha X_0^A \tag{5.12.2}$$

$$\bar{X}^C\lambda + P^C\pi \le X_0^C \tag{5.12.3}$$

$$-\bar{Y}^A\lambda - Q^A\pi \le -\alpha Y_0^A \tag{5.12.4}$$

$$-\bar{Y}^B\lambda - Q^B\pi + \beta Y_0^B \le 0 \tag{5.12.5}$$

$$-\bar{Y}^C\lambda - Q^C\pi \le -Y_0^C \tag{5.12.6}$$

$$e\lambda = 1 \tag{5.12.7}$$

$$\lambda, \pi \ge 0,\ \beta \text{ Sign free} \tag{5.12.8}$$

Following Assumption 4.1, which holds for the developments in the VRS technology as well, Theorem 4.1 can be translated into Theorem 5.7 for T_{VRSTO} as below.

Theorem 5.7. *If the unit* $(X_0, Y_0) \in T_{VRSTO}$ *is efficient and the vector* Y_0^B *has at least one strictly positive component then Assumption 4.1 is satisfied.*

Recall from Section 4.2 that the right-hand (left-hand) elasticity of response of the output vector Y_0^B with respect to marginal proportional changes of the vectors X_0^A and Y_0^A is defined as the right (left) derivative of the function $\overline{\beta}(\alpha)$ at $\alpha = 1$ and represented by (4.5) and (4.6) in Definition 4.3.

The existence of the required one-sided derivatives in T_{VRSTO} is established by Theorem 5.8 below, The proof is given in Appendix B.

Theorem 5.8. *Consider any unit* $(X_0, Y_0) \in T_{VRSTO}$ *that satisfies Assumption 4.1. (The unit* (X_0, Y_0) *can be either observed or unobserved.)*

(a) *If a proportional marginal increase of vect*ors X_0^A *and* Y_0^A *is feasible in technology* T_{VRSTO}*, then the right-hand elasticity* $\varepsilon^+_{A,B}(X_0, Y_0)$ *exists, is finite and can be calculated as follows:*

$$\varepsilon^+_{A,B}(X_0, Y_0) = \min \quad v^A X_0^A - \mu^A Y_0^A \tag{5.13}$$

Subject to

$$v^A X_0^A + v^C X_0^C - \mu^A Y_0^A - \mu^C Y_0^C + \mu_0 = 1$$

$$v^A \overline{X}^A + v^C \overline{X}^C - \mu^A \overline{Y}^A - \mu^B \overline{Y}^B - \mu^C \overline{Y}^C + e\mu_0 \geq 0$$

$$\nu^A P^A + \nu^C P^C - \mu^A Q^A - \mu^B Q^B - \mu^C Q^C \geq 0$$

$$\mu^B Y_0^B = 1$$

$$\nu^A, \nu^C, \mu^A, \mu^B, \mu^C \geq 0$$

μ_0 Sign free

(b) *If a proportional marginal reduction of vectors* X_0^A *and* Y_0^A *is feasible in technology* T_{VRSTO}*, then the left-hand elasticity* $\varepsilon_{A,B}^{-}(X_0, Y_0)$ *exists, is finite and can be calculated by changing the minimisation to maximisation in program (5.13), that is*

$$\varepsilon_{A,B}^{-}(X_0, Y_0) = \max \quad \nu^A X_0^A - \mu^A Y_0^A \qquad (5.14)$$

Subject to

$$\nu^A X_0^A + \nu^C X_0^C - \mu^A Y_0^A - \mu^C Y_0^C + \mu_0 = 1$$

$$\nu^A \bar{X}^A + \nu^C \bar{X}^C - \mu^A \bar{Y}^A - \mu^B \bar{Y}^B - \mu^C \bar{Y}^C + e\mu_0 \geq 0$$

$$\nu^A P^A + \nu^C P^C - \mu^A Q^A - \mu^B Q^B - \mu^C Q^C \geq 0$$

$$\mu^B Y_0^B = 1$$

$$\nu^A, \nu^C, \mu^A, \mu^B, \mu^C \geq 0$$

μ_0 Sign free

(c) *If a proportional marginal increase (reduction) of vectors* X_0^A *and* Y_0^A *is not feasible in technology* T_{VRSTO} *, then the objective function in (5.13) (respectively, in (5.14)) is unbounded.*

It is noted in Chapter 4 that Theorem 4.3 applies to the case of VRS as well. Following the modification of Theorem 4.2 and 5.2 to Theorem 5.8 above, Theorem 4.3 and 5.3 can be translated into Theorem 5.9 as below.

Theorem 5.9. *Assumption 4.1 is true at* (X_0, Y_0) *if and only if both linear programs (5.13) and (5.14) are feasible.*

Theorem 5.9 above means that programs (5.13) and (5.14) can in practice be solved for all units, efficient and inefficient. If, for a particular unit (X_0, Y_0), a linear optimizer indicates an infeasible program (5.13) or (5.14), Assumption 4.1 does not hold and the notion of elasticity is not defined at this unit.

Using Theorems 5.8 and 5.9, in practice, the analysis and computation of (one-sided) elasticities with production trade-offs for output sets at any unit (X_0, Y_0) in T_{VRSTO} can be achieved by simply solving programs (5.13) and (5.14) for all the units (efficient and inefficient). The solutions to the programs can be interpreted through three cases identified in Section 5.5

5.4. Elasticity Analysis of Input Sets in VRS Technology with Production Trade-offs

The derivation of necessary programs for elasticity analysis of input sets in VRS technology with production trade-offs is given in this section. Since it is related with input sets, Assumption 4.2 is considered. Definitions, 4.4, 5.1 and 5.2 also hold for the developments in this section. Theorem 4.6 is modified to Theorem 5.10 in order to represent VRS technology. Theorems 4.7 and 4.8 are modified into Theorems 5.11 and 5.12 such that models can handle the production trade-offs. Because of the similarities, the proofs for

Theorems 5.10, 5.11 and 5.12 are not given; related proofs to those theorems are referred in Appendix B.

Theorem 5.10. *If the unit* $(X_0, Y_0) \in T_{VRSTO}$ *is efficient and the vector* X_0^B *has at least one strictly positive component then Assumption 4.2 is satisfied.*

If the set *B* contains only inputs, the input response function is defined in VRS technology with trade-offs (T_{VRSTO}) as follows:

$$\hat{\beta}(\alpha) = \min\left\{\beta \geq 0 \middle| (\alpha X_0^A, \beta X_0^B, X_0^C, \alpha Y_0^A, Y_0^C) \in T_{VRSTO}\right\} \tag{5.15}$$

The input response function $\hat{\beta}(\alpha)$ is defined as in (5.15) and obtained as the optimal value in the following linear program in T_{VRSTO}, where β is a variable and α is a fixed value. In this case, the trade-off coefficient matrices *P* and *Q* are divided into sub-matrices as, P^A, P^B, P^C, Q^A and Q^C representing the trade-off coefficients for changing (*A*), responding (*B*) and remaining constant (*C*) sets of inputs and outputs.

$$\hat{\beta}(\alpha) = \min \quad \beta \tag{5.16}$$

Subject to

$$-\bar{X}^A\lambda - P^A\pi \geq -\alpha X_0^A$$

$$-\bar{X}^B\lambda - P^B\pi + \beta X_0^B \geq 0$$

$$-\bar{X}^C\lambda - P^C\pi \geq -X_0^C$$

$$\bar{Y}^A\lambda + Q^A\pi \geq \alpha Y_0^A$$

$$\bar{Y}^C\lambda + Q^C\pi \geq Y_0^C$$

$$e\lambda = 1$$

$\lambda, \pi \geq 0$, β Sign free

The right-hand (left-hand) elasticity of response of the input vector X_0^B with respect to marginal proportional changes of the vectors X_0^A and Y_0^A is defined as the right (left) derivative of the function $\hat{\beta}(\alpha)$ at $\alpha = 1$ in (4.11) and (4.12), respectively in Definition 4.4. The existence of one sided derivatives and elasticities with trade-offs for input sets is established in Theorem 5.11 below, which is a modification of Theorem 5.4 for the VRS technology.

Theorem 5.11. *Consider any unit $(X_0, Y_0) \in T_{VRSTO}$ that satisfies Assumption 4.2. (The unit (X_0, Y_0) can be either observed or unobserved.)*

(a) *If a proportional marginal increase of vectors X_0^A and Y_0^A is feasible in technology T_{VRSTO}, then the right-hand elasticity $\rho_{A,B}^{+}(X_0, Y_0)$ exists, is finite and can be calculated as follows:*

$$\rho_{A,B}^{+}(X_0, Y_0) = \max \quad -v^A X_0^A + \mu^A Y_0^A \tag{5.17}$$

Subject to

$$-v^A X_0^A - v^C X_0^C + \mu^A Y_0^A + \mu^C Y_0^C + \mu_0 = 1$$

$$-v^A \bar{X}^A - v^B \bar{X}^B - v^C \bar{X}^C + \mu^A \bar{Y}^A + \mu^C \bar{Y}^C + e\mu_0 \leq 0$$

$$-v^A P^A - v^B P^B - v^C P^C + \mu^A Q^A + \mu^C Q^C \leq 0$$

$$v^B X_0^B = 1$$

$$v^A, v^B, v^C, \mu^A, \mu^C \geq 0$$

μ_0 Sign free

(b) *If a proportional marginal reduction of vectors* X_0^A *and* Y_0^A *is feasible in technology* T_{VRSTO}*, then the left-hand elasticity* $\rho_{A,B}^{-}(X_0,Y_0)$ *exists, is finite and can be calculated by:*

$$\rho_{A,B}^{-}(X_0,Y_0) = \min \quad -v^A X_0^A + \mu^A Y_0^A \tag{5.18}$$

Subject to

$$-v^A X_0^A - v^C X_0^C + \mu^A Y_0^A + \mu^C Y_0^C + \mu_0 = 1$$

$$-v^A \bar{X}^A - v^B \bar{X}^B - v^C \bar{X}^C + \mu^A \bar{Y}^A + \mu^C \bar{Y}^C + e\mu_0 \leq 0$$

$$-v^A P^A - v^B P^B - v^C P^C + \mu^A Q^A + \mu^C Q^C \leq 0$$

$$v^B X_0^B = 1$$

$$v^A, v^B, v^C, \mu^A, \mu^C \geq 0$$

μ_0 Sign free

(c) *If a proportional marginal increase (reduction) of vectors* X_0^A *and* Y_0^A *is not feasible in technology* T_{VRSTO}*, then the objective function of above models are unbounded.*

Similar to our previous developments, the use of Theorem 5.11 initially requires checking Assumption 4.2. For input sets, in the presence of production trade-offs, this can be done through solving model (5.16). However, in practice, infeasibility of linear programs (5.17) and (5.18) yields to the violation of Assumption 4.2. This is presented in Theorem 5.12, which is simply an analogue of Theorem 4.8 for the models of input sets under VRS with production trade-offs.

Theorem 5.12. *Assumption 4.2 is true at* (X_0,Y_0) *if and only if both linear programs (5.17) and (5.18) are feasible.*

Using Theorems 5.11 and 5.12, in practice, the analysis and computation of (one-sided) elasticities with production trade-offs for input sets at any unit (X_0, Y_0) in the given technology can be achieved by simply solving programs (5.17) and (5.18) for all the units (efficient and inefficient). The solutions to the programs can be interpreted through three cases identified in Section 5.5.

5.5. Generalizations of Elasticity Analysis with Production Trade-offs

Linear programming (LP) models to measure the elasticity of response at units on DEA frontiers can yield to three types of solutions with different interpretations; optimal solutions, unbounded solutions and infeasible solutions. These three possible cases are summarised in Chapter 4 (Section 4.3 for output sets and Section 4.7 for input sets). This framework is applicable also to both VRS and CRS technologies with production trade-offs. Consider the elasticity measure calculated for any unit (X_0, Y_0) through linear programs given by Theorem 5.2 for the output sets and by Theorem 5.5 for input sets in T_{CRSTO}, or through linear programs given by Theorem 5.8 for output sets and Theorem 5.11 for input sets in T_{VRSTO}. Three cases can be identified regarding to the solutions, briefly given below.[13]

Case 1. *(The program has a finite optimal solution)*. Assuming that the unit satisfies selective radial efficiency assumption (see Assumption 4.1 in Section 4.2 for output case and Assumption 4.2 in Section 4.6 for input case), as proven for any technology with or without trade-offs included, if the LP model has a finite optimal solution, the marginal increase or reduction of the input or output vectors is feasible in the given technology and right-hand or left-hand elasticities are correctly defined as the optimum value of the program.

[13] For more specific discussions of three cases, see Section 4.3 (for output sets) and Section 4.7 (for input sets).

Case 2. *(The program has an unbounded optimal solution)*. Assuming that selective radial efficiency assumption is satisfied, parts (c) in Theorems 5.2, 5.5, 5.8, 5.11 imply that proportional marginal increases or reductions of vectors X_0^A and Y_0^A are not feasible in the given technology. In other words, proportional marginal increases or reductions at the given unit result in leaving the boundaries of the given production possibility set. Indeed, if a proportional marginal increases or reductions were feasible, by parts (a) and (b) of Theorems 5.2, 5.5, 5.8, 5.11, the optimum value of the programs would be finite, which contradict with the case. For this reason, elasticity of response is undefined.

Case 3. *(The program is infeasible)*. Infeasible solutions to the elasticity models indicate that the selective radial efficiency assumption (Assumption 4.1 for output case and Assumption 4.2 for input case) is not satisfied; therefore the elasticity is undefined for that unit.

Note that infeasibility can arise also because there is no strictly positive component in responding set *B*, since we allow zero outputs. Recall the elasticity measures for output sets under CRS explained in Section 4.2. By Theorem 4.1, we know that Assumption 4.1 is satisfied when unit is efficient and has at least one strictly positive component in responding set *B*. On the other hand, by Theorem 4.3, we know that Assumption 4.1 is true if and only if programs (4.7) and (4.8) are feasible. Therefore, the programs (4.7) and (4.8) are always feasible if the unit is efficient and has one strictly positive component in the responding set *B*. However, infeasibility of programs (4.7) and (4.8) can be related to either the inefficiency of the unit (Theorems 4.1 and 4.3) or the absence of at least one strictly positive component in responding output set *B* (constraints (4.7.4) and (4.8.4) are violated in this case). Above notion is also applicable to the any type of elasticity measures (with or without trade-offs included) under any technology.

5.6. Effects of Introducing Production Trade-offs on Elasticity Measures

In this section, we investigate how the introduction of additional trade-off relations in a defined technology can affect the one-sided elasticity measures and the returns to changing set *A* classifications for the units. To begin with the discussion, let us introduce the concept of "*returns to changing set A (RTA)"*. As mentioned in Section 2.9.1, scale elasticity can be viewed as the quantitative measure of the strength of returns-to-scale (RTS) observed at the efficient unit (X_0, Y_0) (Førsund and Hjalmarsson, 2004). Scale elasticity for a unit, being less than, equal to or greater than 1, represents if the efficient frontier exhibits decreasing (DRS), constant (CRS) and increasing (IRS) returns-to-scale at the unit. In the partial elasticity context, scale elasticity is a special case, where set *A* includes all the inputs and set *B* includes all the outputs. In the cases other than scale elasticity, RTS concept can be thought as returns "to changing set *A*" rather than "to scale", since set *A* and *B* may not include all the inputs and outputs. Therefore, a partial elasticity measure can be viewed as the quantitative measure of the strength of the returns to changing set *A* observed at the unit (X_0, Y_0), which satisfies Assumption 4.1 (*selective radial efficiency with respect to set B*).

Depending on whether both one-sided elasticities measured for the unit (X_0, Y_0) under an arbitrary scenario for sets *A* and *B* are less or greater than 1, the efficient frontier exhibits decreasing or increasing returns to changing set *A* at the unit (X_0, Y_0), respectively. For example, if both one-sided elasticities are greater than 1, it can be said that the output bundle Y_0^B exhibits increasing returns with respect to the change of mixed bundle (X_0^A, Y_0^A). In cases where the range defined by one-sided elasticities contains 1, the frontier is thought to be exhibiting constant returns to changing set *A*, at the unit (X_0, Y_0) under the specified scenario.

Let us consider the elasticity measure for output sets with trade-offs, where set *B* includes only outputs, under VRS technology. The same principles can be extended for the elasticity of input sets, as well as the CRS technology. Suppose that unit (X_0, Y_0) satisfies Assumption 4.1 in the technologies T_{VRS} or T_{VRSTO} under some definition of sets *A* and *B*. Right-hand elasticity $\varepsilon^+_{A,B}(X_0, Y_0)$ and left-hand elasticity $\varepsilon^-_{A,B}(X_0, Y_0)$ can be calculated by programs (2.34) and (2.35) for T_{VRS} and by programs (5.13) and (5.14) for T_{VRSTO} (see Theorems 2.6 and 5.8). Let us denote the new technology obtained through the introduction or addition of one or more new production trade-offs as $\hat{T}_{VRSTO}$. One-sided elasticities for the new technology $\hat{T}_{VRSTO}$ can be denoted as $\hat{\varepsilon}^+_{A,B}(X_0, Y_0)$ and $\hat{\varepsilon}^-_{A,B}(X_0, Y_0)$ for right-hand and left-hand, respectively.

Theorem 5.13. *When one or more additional production trade-off relations are added to the technologies T_{VRS} or T_{VRSTO}, if Assumption 4.1 still holds for the unit (X_0, Y_0) in the new technology $\hat{T}_{VRSTO}$ then,*

$$\hat{\varepsilon}^+_{A,B}(X_0, Y_0) \geq \varepsilon^+_{A,B}(X_0, Y_0) \tag{5.19}$$

$$\hat{\varepsilon}^-_{A,B}(X_0, Y_0) \leq \varepsilon^-_{A,B}(X_0, Y_0) \tag{5.20}$$

Basically, adding trade-offs is introduction of new constraints to the defined technology. According to the Theorem 5.13, when one or more trade-offs are added to the existing technology (regardless of already having trade-offs or not), there is a possibility that the unit does not retain the condition of selective radial efficiency ($\overline{\beta}(1) = 1$). In such a case, Assumption 4.1 does not hold for that unit. From Theorem 5.9, programs (5.13) and (5.14) for calculating one-sided elasticities in the presence of production trade-offs will be infeasible, which means that elasticity is not defined for that unit. On the contrary, for the unit, if Assumption 4.1 still holds as in Theorem 5.13, then by adding one or more new trade-offs (which means more constraints are added to the existing linear programs representing the new trade-offs), the objective function values of (5.13) and (5.14) either remain the same or for the right-hand elasticity model, which is a minimisation problem given in (5.13), the optimal solution to the program can be a larger value, whereas for the left-hand elasticity model, which is a maximisation problem given in (5.14), the optimal solution to the program can be a smaller value. Therefore, in such a case, the interval $\left[\varepsilon^{+}_{A,B}(X_0,Y_0),\varepsilon^{-}_{A,B}(X_0,Y_0)\right]$ gets narrower. In addition, tightening the existing trade-offs in the technology is basically analogous to adding new trade-offs, since tighter trade-offs will make the previous ones redundant. Therefore, it can also be concluded that tightening the existing trade-offs can also make the interval $\left[\varepsilon^{+}_{A,B}(X_0,Y_0),\varepsilon^{-}_{A,B}(X_0,Y_0)\right]$ narrower due to restricting the existing constraints.

Furthermore, returns to changing set *A* (RTA) classification for a unit remains the same in the expanded technology $\hat{T}_{VRSTO}$, if output bundle Y_0^B exhibits decreasing or increasing returns with respect to the change of mixed bundle (X_0^A, Y_0^A) in original technology T_{VRS} or T_{VRSTO}. If the frontier exhibits constant RTA in the original technology T_{VRS} or T_{VRSTO}, then it is possible for the RTA characterisation to change to either the decreasing or increasing returns.

5.7. Summary of Elasticity Analysis with Production Trade-offs

In this chapter, elasticity measures on DEA frontier are extended to the production technologies with trade-off relations incorporated. We consider both CRS and VRS technologies and analyses for both output and input sets. Necessary linear programs to calculate elasticity measures are formulated with the proofs provided in Appendix B. The interpretations of the solutions to the linear programs are summarised in Section 5.5 for three possible cases as optimal solutions, unbounded solutions and infeasible solutions.

Also, effects of changing trade-offs on the elasticity measures are investigated. It is proven for the elasticity measures of output sets that by adding one or more new trade-offs, the objective function values for the elasticity models either remain the same or for the right-hand elasticity can get a larger value, whereas for the left-hand elasticity can get a smaller value due to the newly introduced or more restrictive constraints to the linear programs. Furthermore, the notion of *returns to changing set A (RTA)* is introduced, which is analogue of returns-to-scale concept for the context of elasticity measurement. Effects of changing trade-offs on the RTA characterisation of the units are also discussed. If the unit is exhibiting constant RTA in the original technology, then it is possible for the RTA characterisation to change to either the decreasing or increasing returns when trade-offs are added to the technology or the existing trade-offs in the technology are tightened. On the other hand, RTA characterisation is preserved when trade-offs are added or tightened, if the unit exhibits increasing or decreasing RTA in the original technology.

The elasticity measurement models developed throughout the chapter are employed in the empirical applications performed in Chapter 7 and 8. The statements on effects of changing trade-offs are tested and verified with the real world data in Chapter 8.

Chapter 6

The Data Set and the Model Design

This chapter aims to explain the data set used and the design of the empirical analysis in Turkish Agriculture employing the proposed methodology of efficiency and elasticity measurement. We provide comprehensive information about the contents of the FADN data set obtained from Turkish Ministry of Agriculture and our preliminary considerations in this data set for the analyses conducted in the following chapters. We also provide information about how the production trade-off relations used throughout the analyses are identified and processed.

6.1. Farm Accountancy Data Network (FADN)

The data for the applications of proposed methodology is obtained from The Farm Accountancy Data Network (FADN) held by Turkish Ministry of Agriculture. FADN is the agricultural data network project established and defined by the European Union. It is officially defined as *'an instrument for evaluating the income of agricultural holdings and the impacts of the Common Agricultural Policy'*[14]. The FADN concept was first introduced in 1965. Currently, the FADN covers approximately 90% of the total utilized agricultural area and account for about 90% of the total agricultural production of the European Union. The data in the network consist of physical and structural data, as well as economic and financial data, collected through annual surveys carried out by the member states. In general, data is collected from "commercial farms". A commercial farm is officially defined as *'a farm, which is large enough to provide a main activity for the farmer and a level of income sufficient to support his or her family. In practical terms, in order to be classified as commercial, a farm must exceed a minimum economic size'.* [15]

[14] http://ec.europa.eu/agriculture/rica/concept_en.cfm
[15] http://ec.europa.eu/agriculture/rica/methodology1_en.cfm

As a candidate country for European Union, Turkey started a project for establishing a FADN in 2007. For this purpose, a pilot data collection is initiated by the Turkish Ministry of Agriculture with the support of The Instrument for Pre-Accession Assistance (IPA), which aims to *'offer assistance to countries engaged in the accession process to the European Union (EU) for the period 2007-2013'*.[16]

6.2. Sample Selection

The FADN data set provided to us by the Ministry of Agriculture consists of 374 commercial farms. Since 2002, in terms of statistical data collection and organization to shape agricultural policies and practices, the agricultural policy makers in Turkey divided the country into 12 regions depending on several factors. Every region consists of several provinces (Saçlı, 2009). In FADN data collected, 9 regions out of those 12 are covered. In every region, one representative province is selected and the data is collected from the commercial farms located in that specific province. Table 6.1 presents the regions and representative provinces together with the sample sizes and the number of crop types. In order to avoid the non-homogeneity in terms of environmental factors (such as geography, soil quality, weather and socio-economic differences) and to compare farms with the farms under similar conditions, we rely on the regional classification of the Ministry of Agriculture. The samples in each region are evaluated separately.

We aim to illustrate the proposed methodology and developed theoretical models in a real world sample from FADN database of Turkey. In our proposed methodology, the idea is to establish the production trade-off relationships between the several outputs produced by the farms. For the ease of defining such relationships, we focus on the crop production and the livestock production is ignored both on the input and output sides. We observe that in every region, the farms cultivate at least 10 or more types of crops (see the last column of Table

[16] http://europa.eu/legislation_summaries/agriculture/enlargement/e50020_en.htm

6.1). This gives us 10 or more different outputs for each region, resulting in very bad discrimination of the DEA models (the results of evaluations in different regions are provided and discussed in the following chapters). Introduction of production trade-offs is our proposed way to cope with such insufficient discriminations. To make the elicitation of production trade-off relations, a more intuitive process, *wheat*, which is the most common crop cultivated in majority of farms in the data set, is selected as a base crop. Therefore, our sample is narrowed down to wheat producing farms, which consist of 249 commercial farms. Wheat is not only the most common crop also is the primary crop in these farms. The trade-off relationships between production of each crop type and wheat are set up, which is explained later in this chapter.

Table 6.1. Sample Sizes and Number of Crop Types for Each Region in Turkish FADN

	Code	**Regions**	**City**	**# of Farms in FADN**	**# of Wheat Farms**	**Final Sample Sizes**	**# of crop types**
1	TR1	Istanbul	Istanbul	-	-	-	-
2	TR2	West Marmara	Tekirdag	45	44	39	14
3	TR3	Aegean	Izmir	45	19	17	17
4	TR4	East Marmara	Bursa	46	28	27	20
5	TR5	West Anatolia	Konya	43	35	35	17
6	TR6	Mediterranean	Adana	45	35	30	16
7	TR7	Middle Anatolia	Nevsehir	35	31	26	15
8	TR8	West Black Sea	Samsun	-	-	-	-
9	TR9	East Black Sea	Giresun	35	3	-	
10	TRA	North East Anatolia	Erzurum	30	19	14	10
11	TRB	Middle East Anatolia	Malatya	-	-	-	-
12	TRC	South East Anatolia	Sanliurfa	50	35	26	10
			Total	**374**	**249**	**214**	

Table 6.1 presents the number of wheat producing farms in each region. Due to the missing data and incomplete information some farms are excluded from each region sample, ending up with total number of 214 farms. Also, because East Black Sea region includes only 3 farms producing wheat, this region is excluded, resulting in 8 regions in our final sample.

6.3. Crops in Regions

Table 6.2 summarises the crop types in our final sample with the number of farms producing them for each region. As seen in the table, the crop types are quite diversified. In all regions, every farm produces wheat and other additional crops listed. Some additional crop types are cultivated by majority of the farms, whereas some are produced in a very limited number of farms. The production amounts of indicated crops are included in our DEA models as outputs for every region evaluations. As discussed earlier, the aim is to integrate the actual production rather than the aggregated monetary equivalents to assess the production process itself, to avoid the effect of price fluctuations in the evaluation and to be able to calculate the elasticities of response for specific or set of crops.

In a standard DEA application, such number of outputs for every region can be a problem given the sample size. For example, for Aegean region, we have 17 farms and 17 outputs. No matter how many inputs are included in the model, the discrimination of efficiency scores will be very bad (resulting in too many efficient units, even all farms can be found efficient) due to the large number of outputs in the model. The weights attached to the outputs produced in few farms will cause the overestimation of the efficiency score for the farms producing those crops because all others produce them at the level of zero. In order to decrease the number of the outputs, one way could be eliminating the farms producing very specific products from the sample. However, in this case, the sample sizes would get even smaller, which would not help much to increase the discrimination even though we have fewer outputs. Also, our proposed methodology promises to deal with this non-homogeneity in terms of production through applying production trade-offs concept. To observe how our models work even for small samples with many outputs, we prefer to keep all the farms in the region unless they have missing data.

Table 6.2. Crop Types with the Number of Farms in Each Region

West Marmara		#	Aegean		#	East Marmara		#
1	Wheat	39	**1**	Wheat	17	**1**	Wheat	27
2	Sunflower	38	**2**	Fodder Maize	13	**2**	Grain Maize	17
3	Barley	15	**3**	Grain Maize	10	**3**	Fodder Maize	15
4	Oilseed Rape	14	**4**	Vetch	7	**4**	Tomatoes	15
5	Grain Maize	10	**5**	Lucerne	5	**5**	Sugar Beet	10
6	Vetch	10	**6**	Cotton	4	**6**	Lucerne	8
7	Fodder Maize	5	**7**	Barley	4	**7**	Peas	8
8	Watermelon	4	**8**	Olives for Olive oil	3	**8**	Vetch	8
9	Lucerne	3	**9**	Table olives	3	**9**	Sunflower	5
10	Sugar beet	3	**10**	Pepper	3	**10**	Oats	4
11	Onions	3	**11**	Tomatoes	3	**11**	Barley	4
12	Oats	2	**12**	Grapes for wine	2	**12**	Pepper	4
13	Grass	2	**13**	Oats	2	**13**	Table Olives	4
14	Grapes for wine	2	**14**	Tobacco	2	**14**	Onions	3
			15	Aubergine	1	**15**	Beans	2
			16	Potatoes	1	**16**	Watermelon	2
			17	Watermelon	1	**17**	Cherry	1
						18	Melon	1
						19	Pear	1
						20	Potatoes	1

West Anatolia		#	Mediterranean		#	Middle Anatolia		#
1	Wheat	35	**1**	Wheat	30	**1**	Wheat	26
2	Barley	22	**2**	Grain Maize	21	**2**	Barley	19
3	Sugar beet	21	**3**	Sunflower	10	**3**	Table Grapes	12
4	Lucerne	9	**4**	Cotton	7	**4**	Vetch	10
5	Beans	7	**5**	Barley	6	**5**	Potatoes	6
6	Fodder maize	7	**6**	Oranges	4	**6**	Rye	6
7	Sunflower	7	**7**	Fodder Maize	3	**7**	Grain Maize	6
8	Vetch	7	**8**	Watermelon	2	**8**	Lucerne	5
9	Peas	5	**9**	Nuts	2	**9**	Courgette Seed	4
10	Grain Maize	3	**10**	Lemons	1	**10**	Grapes For Wine	4
11	Oats	3	**11**	Table Olives	1	**11**	Peas	4
12	Potatoes	3	**12**	Vetch	1	**12**	Oats	3
13	Apple	2	**13**	Lucerne	1	**13**	Apple	3
14	Cherry	2	**14**	Oilseed Rape	1	**14**	Sugar Beet	2
15	Grass	1	**15**	Tomatoes	1	**15**	Sunflower	1
16	Lentil	1	**16**	Onions	1			
17	Rye	1						

North East Anatolia		#	South East Anatolia		#
1	Wheat	14	**1**	Wheat	26
2	Lucerne	14	**2**	Grain Maize	17
3	Grass	9	**3**	Cotton	13
4	Vetch	8	**4**	Barley	4
5	Barley	8	**5**	Nuts	2
6	Potatoes	4	**6**	Lentil	2
7	Grain Maize	3	**7**	Fodder Maize	2
8	Sugar Beet	3	**8**	Tomatoes	1
9	Rye	1	**9**	Pepper	1
10	Sunflower	1	**10**	Aubergine	1

6.4. Selection of Inputs

FADN data set provided to us consists of more than 2000 items (variables). Of course, not all of those items represent different meaning, there are some aggregation items and also some items intersect with each other. In addition, not all of them are complete. An official document explaining the meaning and content of each item is provided to us together with the data set (Community Committee for FADN, 2009). We rely on these provided descriptions of the data, when identifying the variables for our models.

To determine the inputs for our illustrative DEA models, we also benefit from the results of our review on the agricultural DEA studies in Chapter 3. Availability of data in Turkish FADN for the intended variables was our other concern. When the literature on DEA in agriculture is examined, depending on the context of evaluation or the scope of the study, the selected inputs and outputs vary, but in general it is possible to identify some common types of inputs agreed by the majority of the scholars. For instance, land is used as an input almost in every agricultural efficiency evaluation study. Obviously, land is an inseparable mean of production in agriculture. Efficient use of land is one important indicator of overall performance for farms. Land input in studies is defined as the utilized agricultural area and usually measured in hectares or homologous measures. For our data set, utilized areas for each crop are added to come up with a land measure for each farm. The measurement unit is given as decares (daa), which is 1000 m^2.

Labour is another key indicator employed as input in agricultural efficiency evaluation studies. It is measured by different means such as number of workers, labour costs (i.e. wages), annual working units or labour hours (see Section 3.2.3 in Chapter 3). Regarding the availability of labour related data in Turkish FADN, we use the labour costs (specifically, sum of gross wages and in kind payments paid to the employees). In kind payments represent the payments made to the employees other than the wages in means of rent, farm products, meals and lodging etc. (Community Committee for FADN, 2009). They can be considered as a part of labour costs. The unit of measurement for labour input is Turkish Lira (TL).

Different types of other costs (than labour) are also among the key factors considered in all studies. Costs are integrated into the DEA models regarding two different approaches (see Section 3.2.3 in Chapter 3). First approach is to consider costs as an aggregated variable including various items related to the production of the crops or livestock such as fertilizers, feed, seeds and pesticides etc. In some cases, costs on maintenance of the farm such as energy, fuel, machinery, water or farming overheads is also included to these costs or taken as a separate aggregation as "capital expenditures". Second approach in dealing with costs is the integration of abovementioned items as separate inputs rather than aggregating them together. Such models include several inputs such as fertilizers, fuel, pesticides, seed and energy consumption.

In our models, we use aggregation of several types of costs together; however differentiate between costs spent on crop production and costs spent on the maintenance of the farm (i.e. costs on capital or capital expenditures). The aim with this differentiation is to be able to measure elasticity of response on output or input sets while crop production costs (which are more flexible in the short term) are changed, whereas other types of costs (which are more difficult to adjust in the long term) remain constant.

"Crop production costs" input is obtained by adding up 5 different cost items all measured in Turkish Lira (TL) given as:

- Seeds and seedlings purchased or produced on the farm,
- Purchased fertilizers and soil improvers,
- Crop protection products,

- Other specific crop costs,
- Specific forestry costs.

Another important variable used as an input in the literature is the capital factor. It has been considered regarding different forms in several studies as given in in Section 3.2.3 of Chapter 3. Total assets, reported capital in balance sheet, depreciation, interest payments, annual costs in capital or book value of machinery and inventory are different examples of capital consideration in several studies.

Deciding on the input, which will represent the capital, was a challenging process due to the availability in our data set. Most of the abovementioned measures do not exist or are missing for many farms. Initial intention was using the difference between the opening and closing values of the capital. However, the data is incomplete in terms of those items. Moreover, there is a controversial issue about using such a variable since investments on capital can be considered more like the long-term commitments rather than yielding yearly improvements in the production. Since our evaluations are not over time, a large spending on the improvement of capital by a farm in the certain time period we are dealing will result in a large change between opening and closing values. This can be misleading for such farms invested heavily in measurement year in terms of efficiency measured since capital value for those will be higher than the others. However, return of such investments is a long-term process, which will not create much effect on the measurement year other than underestimation of efficiency.

Among the abovementioned capital items considered by other scholars, "annual costs in capital" is the only item available for majority of the farms in our data set. Such costs can include money spent on the maintaining the farms' capital such as machinery costs, land charges and farming overheads and also considered by several studies as a representative for capital input (Färe et al., 1997; Townsend et al., 1998; Lilienfeld and Asmild, 2007). Also as mentioned above, we already intend to differentiate between these types of costs and specific crop production costs for elasticity measurement reasons. Such differentiation also gives us an opportunity to fulfil the need for an input representing capital and at the same time we separate the costs into two fitting with our aims in elasticity measurement. Therefore, we identify our last input labelled as *"Capital Expenditures"* in Turkish Lira (TL) consisting of three main types of costs and several sub-items given as follows. Those items included in 'capital expenditures' input have a more direct impact on the crop production compared to the difference between opening and closing values of capital.

Capital Expenditures = **Machinery Costs** (*Sum of 'contract works', 'current upkeep of machinery and equipment', 'motor fuels and lubricants' and 'car expenses'*) + **Farming Overheads** (*Sum of 'upkeep of buildings', 'electricity', 'fuels', 'water', 'insurance' and 'other farming overhead'*) + **Land Charges** (*Sum of 'paid rent for land and buildings', 'value of products given to share cropper' and 'tax paid by the farm'*)

The selection of inputs for our illustrative examples fulfils the tendencies in the literature of agricultural efficiency measurement, in which land, labour, costs and capital are the most commonly used inputs. On the output side, we have the physical production amounts of crops as separate outputs, which differ between regions. The inputs and outputs used in our models are summarised in Figure 6.1 below.

Figure 6.1. Inputs and Outputs for the DEA models

6.5. Identification of Production Trade-offs

As stated in section 6.3, the farms of every region in our data set produce a wide range of crop types and this leads to a discrimination problem of efficiency scores due to the large number of outputs in DEA models. To overcome the discrimination problem, we propose to integrate the production trade-offs approach to such agricultural efficiency evaluation problems where the units are non-homogeneous in terms of production. To apply the proposed methodology for our case, we need to identify the production trade-off relations between different types of agricultural outputs. Since we deal with crop production, the necessary information is the relationships between production amounts of different crops. As mentioned in section 6.2, wheat is selected as the base crop and our sample is narrowed down to wheat producers. Therefore, wheat is a crop, which is produced by all the farms in our sample. We design the production trade-offs data collection, in a way that agricultural experts can determine the trade-off relations between each type of crop production and the wheat production.

We collected the trade-offs data from two sources. The main source reflects a practitioner point of view, where the relationships between crops are suggested by agricultural engineers working as consultants in a specific local chamber of agriculture. A group of engineers come up with one trade-off table representing the relationships between crops in three different ranges explained below. As a secondary source to support or to fulfil the gaps of the data provided by the main source, the same questions are also asked of academics working on the agricultural production in a School of Agricultural Engineering of a local university. Fortunately, the data obtained from the main source was satisfying and nearly complete. The missing trade-offs for some crops from the main source are obtained from the second source and finally, the trade-off table presented in Table 6.3 is obtained for nearly all crops produced by the farms in the sample.

The production trade-off relationship questions are designed in a way that the experts comment on the question: "*How much of a certain crop can be produced with the same resources available to produce 1-ton of wheat?*" With the resources available we mean our inputs (land, labour costs, crop production costs and capital expenditures). The experts are asked to identify the production equivalent of crops to the 1-ton of wheat in three different ranges. Answers are designed as stated below:

- As a first guess, it is possible to produce … to … tons of … (crop type) with the same resources available to produce 1 ton of wheat.
- To be more flexible, it can be possible to produce … to … tons of … (crop type) with the same resources available to produce 1 ton of wheat.
- Most likely, it is possible to produce … to … tons of … (crop type) with the same resources available to produce 1 ton of wheat.

Each answer provides us a range of production amounts from tightest to broadest. First answer indicates the tightest range, which represents the very first judgement about the relationship. Second answer extends the range to be on the safer side. Finally, the third answer provides the broadest (robust) range, which represents the plausible range for the relationship. We label these three ranges as *"Tight"*, *"Medium"* and *"Broad"* trade-offs, respectively throughout our analyses. The crop types in our sample are provided to the experts and the question is replied for all types of crops within the answer structure provided above.

The trade-off relations between different crop types in our sample and wheat production are given in Table 6.3. The crops are presented according to the classification considered by the official document given us. We have 5 main classes of crops covering 35 crop types. The data provided in Table 6.3 provides trade-offs in three ranges for all the crop types in our data set except for grass (which is produced by some farms in West Marmara, West Anatolia, and North East Anatolia regions) and nuts (which are produced by some farms in Mediterranean and South East Anatolia regions). For these crops we do not employ any trade-off constraints in our models.

Table 6.3. Production Trade-off Relations between Different Crops and Wheat

	Crops	First Judgement Low	First Judgement Up	Flexible Range Low	Flexible Range Up	Broadest Range Low	Broadest Range Up
	Cereals						
1	Grain Maize	2.4	2.6	2.2	2.8	2	3
2	Barley	0.85	0.9	0.8	0.95	0.75	1
3	Oats	0.3	0.35	0.275	0.375	0.25	0.4
4	Rye	0.6	0.7	0.55	0.75	0.5	0.8
5	Triticale	0.7	0.8	0.65	0.85	0.6	0.9
	Fodder crops						
6	Lucerne	1.5	5.5	1.3	5.9	1.2	6
7	Fodder Maize	14	16	12	18	10	20
8	Grass	-	-	-	-	-	-
	Field crops						
9	Vetch	0.4	0.5	0.35	0.55	0.3	0.6
10	Peas	0.275	0.325	0.25	0.35	0.2	0.4
11	Beans	0.3	0.35	0.275	0.375	0.25	0.4
12	Lentil	0.2	0.25	0.175	0.275	0.15	0.3
13	Sunflower	0.6	0.7	0.5	0.8	0.4	0.9
14	Oilseed Rape	0.7	0.8	0.6	0.9	0.5	1
15	Cotton	1	1.2	0.9	1.3	0.8	1.4
16	Potatoes	5	6	4.5	6.5	4	7
17	Sugar beet	15	18	14	19	13	20
18	Tobacco	0.6	0.7	0.5	0.8	0.4	0.9
	Permanent Crops						
19	Oranges	0.8	1	0.7	1.1	0.6	1.2
20	Lemons	0.5	0.7	0.45	0.75	0.45	0.9
21	Apple	1.5	2	1.25	2.25	1	2.5
22	Cherry	0.6	0.8	0.55	0.85	0.5	1
23	Pear	0.8	1	0.7	1.1	0.6	1.2
24	Nuts	-	-	-	-	-	-
25	Table olives	0.6	0.8	0.5	0.9	0.4	1
26	Olives for olive oil	0.6	0.8	0.5	0.9	0.4	1
27	Table grapes	1.7	2.2	1.6	2.4	1.5	2.5
28	Grapes for wine	1.7	2.2	1.6	2.4	1.5	2.5
	Vegetables & Non-perennial fruits						
29	Watermelon	5	6	4.75	6.25	4.5	6.5
30	Melon	4	4.5	3.75	4.75	3.5	5
31	Tomatoes	8	10	6	12	4	14
32	Pepper	4	5	3.5	5.5	3	6
33	Courgette Seed	0.2	0.25	0.175	0.275	0.175	0.3
34	Aubergine	3	3.5	2.75	3.75	2.5	4
35	Onion	1	1.2	0.9	1.3	0.8	1.5

Let us explain how we interpret the information given in Table 6.3 with a specific example of grain maize. The experts identify the first judgement for grain maize in relation to wheat as 2.4 to 2.6. This means that as a first judgement, it is possible to produce 2.4 to 2.6 tons of grain maize with the devoted resources (land, labour costs, crop production costs and capital expenditures) to produce 1-ton of wheat. This range extends to 2.2 to 2.8 as the expert is asked to be more flexible. Finally, expert states the broadest range of 2 to 3, where it can be translated as 1-ton of wheat production is most likely equivalent with 2 to 3 tons of grain maize production.

Trade-off relations identified represent the technological judgements of the experts in the area. Of course, it is not possible to obtain them exactly; therefore they are questioned in terms of ranges. However, since they rely on the expert opinions, they reflect a realistic point of view. Unlike the weight restrictions based on value judgements, they reflect a technological meaning as stated earlier in Section 2.7.2. Despite the mathematical representation is the same (they are translated into weight restrictions), the technological meaning is preserved. Moreover, to measure how different ranges of trade-off relations affect the discrimination of our models, we obtain them in three ranges (tightest to broadest). Such an approach enables us to conclude also on the sensitivity of efficiency score discriminations and elasticities to the changing trade-offs.

6.6. Interpretation of Production Trade-offs

Production trade-off relations provided in Table 6.3 establish two-sided relationships between the production of crops listed and the wheat production. They can be seen as the judgements as in the production trade-off methodology developed by Podinovski (2004a). Consider the broadest range of relationship between wheat and grain maize, which is given as 2 to 3. This range provides us appropriate information to come up with two judgements and establish a two-sided relationship between wheat output and grain maize output.

Judgement 1. *No extra resources can be claimed if the production of wheat is reduced by 1 ton and the grain maize production is increased by 2 tons.*

Judgement 2. *No extra resources can be claimed if the production of wheat is increased by 1 ton and the grain maize production is reduced by 3 tons.*

Because the DEA models with trade-off in this work are solved in the multiplier form, above judgements need to be translated into weight restrictions and included in DEA models as additional constraints as described in Podinovski (2004a) and Section 2.7.2 of Chapter 2.

6.7. Design of Empirical Applications

In the following two chapters, we illustrate the proposed methodology and theory on efficiency and elasticity using the Turkish FADN data set identified above. The chapters are designed to reflect different implications of our methodology. We aim to provide an insight on the use of production trade-offs in agricultural efficiency evaluation with DEA and on the elasticity measures under different considerations of returns-to-scale, elasticity scenarios[17] and ranges of trade-offs. Our objective is to illustrate and observe whether the method we propose, the statements we provide and theory we develop throughout the research can be verified in a real world case. The design of empirical applications are summarised in Figure 6.2.

Throughout the empirical applications, all the calculations are performed using *General Algebraic Modeling System (GAMS)*[18]. Linear programming (LP) models of DEA and elasticity measures with or without trade-offs under both VRS and CRS are coded and

[17] Elasticity scenarios refer to the choice of changing, responding and remaining constant sets of inputs and outputs

[18] http://www.gams.com/

solved. Relying on the preliminary experiments done, as an LP solver, MOSEK solver[19] embedded in GAMS is used. With the default LP solver of GAMS, some problems arise in solving the elasticity models related to handling of the infeasible and unbounded solutions, therefore after several experimenting, MOSEK solver is found more reliable and used in calculations.

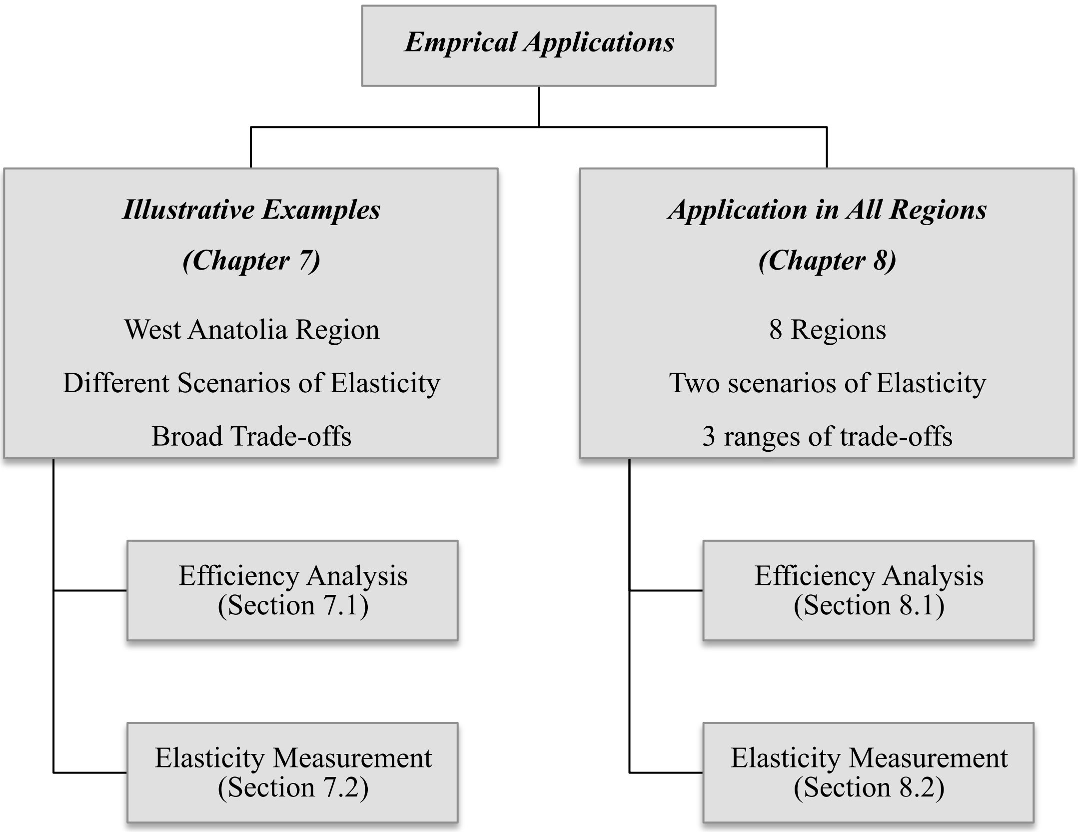

Figure 6.2. Summary of Empirical Applications in Turkish FADN

Chapter 7 includes illustrative examples of elasticity measures. It aims to demonstrate the applicability of elasticity measures under different scenarios of changing and responding sets of inputs and outputs considering both VRS and CRS assumptions with or without trade-offs are incorporated. We use the West Anatolia sample of the data set and the broad range of trade-offs identified for our illustrative purposes. We calculate elasticities for either

[19] http://www.gams.com/solvers/solvers.htm#MOSEK

output or input sets under different scenarios of changing and responding sets. The chapter serves as a preliminary exercise for measuring elasticity on DEA frontiers.

In Chapter 8, we extend our application scope to all over the 8 regions' data. In this chapter, we also introduce different ranges of trade-offs into models in order to observe the effect of changing trade-offs on efficiency and elasticity measures. We pursue two scenarios of elasticity measures for output sets throughout the chapter and interpret the results relying on the methodological aspects. In both scenarios, changing set consists of *"Crop Production Costs"* and *"Labour"* inputs. Responding set contains *"Cereals"* in one scenario and *"Field Crops"* in the other. All calculations are also performed under both VRS and CRS considerations.

Chapter 7

Illustrative Examples of Efficiency and Elasticity Measures in Turkish Agriculture

In this chapter, we cover a series of illustrative examples on the proposed methodology. The aim is to demonstrate how efficiency scores with and without trade-offs differ in a sample of farms and more importantly, how elasticity measures (both existing in the literature or developed in this research) can be applied to a real world sample considering different scenarios of changing and responding sets. We design a bundle of examples addressing different scenarios. We calculate efficiency and elasticity measures using DEA methodology. Both variable returns-to-scale (VRS) and constant returns-to-scale (CRS) technologies with and without trade-offs included in the models are considered. We also verify the discussions on special cases of CRS models provided in Section 4.4 of Chapter 4.

For the illustrative examples in this chapter, we use the West Anatolia region sample in our data set consisting of 35 farms producing 17 types of crops. The crop types produced in this region are given with their classifications and the number of farms producing them in Table 7.1 below. DEA models include 17 outputs representing the production amount of each crop in tons. The inputs selected are land (in daa), labour (as labour costs in TL), crop production costs (TL) and capital expenditures (TL). The selection of inputs is discussed thoroughly in the Chapter 6.

Throughout the examples, as production trade-off relations, the broadest ranges provided to us by the experts are used in order to show how the models work even with a broadest range of trade-offs. A detailed discussion of how the changes in trade-off ranges affect the efficiency scores and the elasticity measures is covered in extensive applications given in Chapter 8. The broadest trade-off relations for the crops produced by the farms in West Anatolia region are also provided in Table 7.1. We have the up and low limits of production amounts for each crop in relation to the wheat production. Only, the trade-off relation of grass is missing (which is not provided by the experts); therefore, we do not include any trade-off constraints for grass production. The trade-offs are translated into judgements and then to weight restrictions as explained in previous discussions (see Section 6.6 in Chapter 6 and Section 2.7.2 in Chapter 2).

Table 7.1. Crops in West Anatolia Region

	Crops	Class	# of Farms	TO related to 1 ton of Wheat	
				Low	Up
1	Wheat	Cereals	35	-	-
2	Barley	Cereals	22	0.75	1
3	Sugar beet	Field crops	21	13	20
4	Lucerne	Fodder crops	9	1.2	6
5	Sunflower	Field crops	7	0.4	0.9
6	Vetch	Field crops	7	0.3	0.6
7	Fodder maize	Fodder crops	7	10	20
8	Beans	Field crops	7	0.25	0.4
9	Peas	Field crops	5	0.2	0.4
10	Potatoes	Field crops	3	4	7
11	Oats	Cereals	3	0.25	0.4
12	Grain Maze	Cereals	3	2	3
13	Apple	Permanent crops	2	1	2.5
14	Cherry	Permanent crops	2	0.5	1
15	Grass	Fodder crops	1	-	-
16	Rye	Cereals	1	0.5	0.8
17	Lentil	Field crops	1	0.15	0.3

Two types of examples regarding the orientation of the models are considered: output-oriented and input-oriented. First of all, the efficiency scores with and without trade-offs are calculated for both orientations in order to discuss how the discrimination of efficiency scores is affected in the presence of production trade-offs. After the efficiency analysis, elasticities of responses are evaluated under different scenarios of changing and responding sets. In the elasticity analysis of output sets, the responding set consists of only outputs (i.e. a set of outputs are responding to the changes in input and/or output sets), whereas in the analysis of input sets, changing set consists of only inputs (i.e. a set of inputs are responding to the changes in inputs and/or output sets). All the calculations are performed for both variable returns-to-scale (VRS) and constant returns-to-scale (CRS) technologies.

7.1. Efficiency Analysis

Output-oriented (OO) and input-oriented (IO) VRS and CRS DEA efficiency scores for the farms in West Anatolia region, with (WTO) and without trade-off relations included in the models, are provided in Table 7.2. In calculations, multiplier forms are considered. Standard DEA models in multiplier forms are given in Table 2.7 for VRS and Table 2.6 for CRS in Chapter 2. The multiplier models with trade-offs are given in Table 2.9 for VRS and Table 2.10 for CRS in Chapter 2. 35 farms are included in the analysis and the farm codes given in the second column of Table 7.2 indicate the label of the farm in the original FADN data set.

It can be observed in Table 7.2 that in both VRS and CRS cases and in both orientations, the discrimination of efficiency scores gets better when the trade-off relations are integrated to the models. In VRS models, nearly all the farms are obtained as efficient due to the large number of outputs in the model. When the trade-off relations are considered, the number of efficient units dropped to 19 and the average efficiency is reduced to 86% in output orientation and 87% in input orientation. In the CRS cases, (efficiency scores are the same for both orientations as discussed in Section 2.4.2), the discrimination gets even better where in the presence of trade-offs only 7 farms are efficient and the average efficiency score drops from 94% to 72% with the inclusion of production trade-offs to DEA models. Above results reveal that even with a small sample of units, the integration of production trade-offs (even when broadest trade-offs are considered) leads to a considerable improvement in the efficiency score discrimination.

Table 7.2. Efficiency Scores of Farms in West Anatolia

	Farm Code	OO VRS	OO VRS (WTO)	OO CRS	OO CRS (WTO)	IO VRS	IO VRS (WTO)	IO CRS	IO CRS (WTO)
1	**288**	1	1	1	0.95	1	1	1	0.95
2	**289**	1	1	0.87	0.61	1	1	0.87	0.61
3	**290**	1	1	0.58	0.36	1	1	0.58	0.36
4	**291**	0.99	0.81	0.87	0.62	0.97	0.70	0.87	0.62
5	**293**	1	0.89	1	0.64	1	0.74	1	0.64
6	**295**	1	0.54	0.99	0.53	1	0.61	0.99	0.53
7	**296**	1	1	1	1	1	1	1	1
8	**297**	1	0.94	1	0.65	1	0.92	1	0.65
9	**298**	1	0.82	1	0.78	1	0.78	1	0.78
10	**300**	0.22	0.14	0.19	0.13	0.47	0.47	0.19	0.13
11	**301**	1	0.93	1	0.88	1	0.92	1	0.88
12	**302**	1	0.66	1	0.59	1	0.62	1	0.59
13	**303**	1	0.28	1	0.26	1	0.91	1	0.26
14	**304**	1	1	1	0.86	1	1	1	0.86
15	**306**	1	0.69	1	0.67	1	0.69	1	0.67
16	**307**	1	1	1	1	1	1	1	1
17	**309**	1	0.45	1	0.37	1	0.38	1	0.37
18	**310**	1	1	1	0.69	1	1	1	0.69
19	**311**	1	1	0.97	0.84	1	1	0.97	0.84
20	**312**	1	0.65	0.80	0.42	1	0.52	0.80	0.42
21	**313**	1	0.72	0.90	0.55	1	0.57	0.90	0.55
22	**314**	1	1	1	0.81	1	1	1	0.81
23	**315**	1	1	1	0.94	1	1	1	0.94
24	**316**	1	0.67	0.93	0.67	1	0.88	0.93	0.67
25	**317**	1	1	1	0.94	1	1	1	0.94
26	**318**	1	1	1	1	1	1	1	1
27	**319**	1	1	1	1	1	1	1	1
28	**320**	1	1	1	1	1	1	1	1
29	**321**	1	0.99	1	0.62	1	0.99	1	0.62
30	**322**	1	0.85	1	0.62	1	0.79	1	0.62
31	**323**	1	1	1	0.94	1	1	1	0.94
32	**325**	1	1	1	0.79	1	1	1	0.79
33	**326**	1	1	1	1	1	1	1	1
34	**327**	1	1	1	1	1	1	1	1
35	**328**	1	1	0.91	0.38	1	1	0.91	0.38
Average		0.98	0.86	0.94	0.72	0.98	0.87	0.94	0.72
# of Efficient		33	19	25	7	33	19	25	7
# of Inefficient		2	16	10	28	2	16	10	28

7.2. Elasticity Analysis

In this section, we present the elasticity measures for different output and input sets under different considerations of changing and responding sets of inputs and outputs. The scenarios considered are for illustrative purposes; we do not claim to come up with any policy implications regarding the agriculture. The main objective is to demonstrate that it is possible to calculate mixed partial elasticities on DEA frontiers considering any subset of inputs and/or outputs. In elasticity measurement on DEA frontiers context, set *A* represents the changing set of inputs and/or outputs, set *B* is the responding set consisting of either inputs or outputs (outputs considered in section 7.2.1 and inputs considered in section 7.2.2) and set *C* is the set consist of inputs and/or outputs which remain constant. The analysis is concerned with the elasticity of response of the factors in the set *B* with respect to marginal changes of the factors in the set *A*, provided the inputs and outputs in the set *C* do not change. We obtain two-sided elasticities where right-hand elasticity (RHE) at a given unit represents response of the outputs (inputs) in set *B* to the proportional marginal *increase* of the input and/or outputs in set *A* under the given technology (VRS or CRS). On the other hand, left-hand elasticity (LHE) at a given unit stands for the response of the outputs (inputs) in set *B* to the proportional marginal *reduction* of the input and/or outputs in set *A* under the given technology (VRS or CRS).

In principle, if input and output factors in set *A* are increased by a factor $\alpha > 1$, the maximum quantity of input or output bundle in set *B* possible in the given technology will change by a factor $\varepsilon^{+}_{A,B}(X_0,Y_0)\times(\alpha-1)$, where $\varepsilon^{+}_{A,B}(X_0,Y_0)$ represents the right-hand elasticity of response at the given unit. Inversely, if input and output factors in set *A* are reduced by a factor $\alpha \in [0,1)$, the maximum quantity of input or output bundle possible in the given technology will change by a factor $\varepsilon^{-}_{A,B}(X_0,Y_0)\times\alpha$, where $\varepsilon^{-}_{A,B}(X_0,Y_0)$ represents the left-hand elasticity of response at the given unit (for more details, see Section 2.9.2).

In designing the scenarios of changing and responding sets of outputs, we benefit from the classifications of crops provided in Table 7.1. Each class of outputs existing in West Anatolia sample (Cereals, Field Crops, Fodder Crops and Permanent Crops) represents different outputs. We use these classifications just to give us a direction on how to group the output types for designing experiments and easier representation of the scenarios. In principle, we can use every output combination as responding or changing sets.

In calculations of elasticities of response under VRS technology, we rely on the models for output sets and input sets developed by Podinovski and Førsund (2010) provided by Theorems 2.6 and 2.7 in section 2.9.2 of Chapter 2. For the CRS technology, we employ elasticity measures of output sets and input sets developed in the scope of this research and provided in Chapter 4 (see Theorem 4.2 in Section 4.2 for output sets and Theorem 4.7 in Section 4.6 for input sets). For the models with trade-offs, we implement the theory developed in Chapter 5 (see Theorem 5.2 in Section 5.1 for output sets and Theorem 5.5 in Section 5.2 for input sets under CRS; see Theorem 5.8 in Section 5.3 for output sets and Theorem 5.11 in Section 5.4 for input sets under VRS).

As explained in Chapters 4 and 5, linear programming (LP) models to measure the elasticity of response at units on DEA frontiers can yield to three types of solutions with different interpretations; optimal solutions, unbounded solutions and infeasible solutions. These three possible cases are summarised in Chapters 4 and 5 (see Section 4.3 for output sets, Section 4.7 for input sets or Section 5.5 for trade-offs case). This framework is applicable to both VRS and CRS technologies with or without trade-offs. Assuming that the unit satisfies selective radial efficiency assumption (see Assumption 4.1 in Section 4.2 for output case and Assumption 4.2 in Section 4.6 for input case), as proven for any technology with or without trade-offs included, if the LP model has a finite optimal solution, the marginal increase or reduction of the input or output vectors is feasible in the given technology and RHE or LHE is correctly defined as the optimum value of the program. Again assuming that selective radial efficiency assumption is satisfied, unbounded solutions indicate that proportional marginal increases or reductions of vectors X_0^A and Y_0^A are not feasible in the given technology. Therefore, elasticity of response is undefined. Finally, infeasible solutions to the elasticity models indicate that the selective radial efficiency assumption (Assumption 4.1 for output case and Assumption 4.2 for input case) is not satisfied; therefore the elasticity is undefined for that unit.

Note that infeasibility may arise also because there is no strictly positive component in responding set *B*, since we allow zero outputs. This is explained in Section 5.5 of Chapter 5. In presenting the results in this chapter and the following chapter, we differentiate between the infeasibility related to the violation of selective radial efficiency and the infeasibility due to the lack of strictly positive component in responding set *B*. For the units yielding infeasible solution (so the elasticity is not defined because of violation of selective radial efficiency assumption), the corresponding cells are left blank. For the examples in this section, we do not have the latter case of infeasibility due to the lack of strictly positive component in responding set *B*. In all scenarios undertaken in this section, the units have at least one positive output in set *B*. Therefore; infeasibility in examples of this chapter is only caused by the violation of selective radial efficiency assumption. For this reason, in the following examples, a blank cell corresponding to a unit indicates that the unit is inefficient in the given technology. For the unbounded solutions, in the result tables, the elasticity is given as “UD” (Undefined), which indicates that selective radial efficiency assumption is satisfied; however it is not possible to marginally increase or reduce the factors in set *A* at the given unit and remain in the given technology. Note that in some cases, we have extremely large values of elasticities, the interpretation of such very large values of elasticity is provided with an illustrative example in Section 8.2.1 of Chapter 8.

7.2.1. Elasticity Analysis of Output Sets

Following the efficiency analysis, in this section, we apply elasticity measures with different changing and responding set scenarios, where responding set contains only outputs. The models for the calculations are given in Theorem 2.6 in Section 2.9.2 for the VRS technology, in Theorem 4.2 (in Section 4.2) for the CRS technology and in Theorems 5.2 (Section 5.1) and 5.8 (Section 5.3) for the CRS and VRS technologies with trade-off relations included, respectively.

7.2.1.1. Scale elasticity of Output Sets

Suppose we have sets as A={All inputs}, B={All outputs} and C=∅. This scenario is known as scale elasticity, where all outputs are responding to the changes in all inputs. The scale elasticity of the output sets for the farms in West Anatolia is given in Table 7.3. As stated in Theorem 4.4 (see Chapter 4., Section 4.4), the scale elasticity for CRS technology is equal to 1 for all units, since full proportionality between inputs and outputs are assumed. For the VRS technology, we do not have proportionality assumption; therefore it is possible to obtain elasticity values different than 1 and even unbounded solution as seen in Table 7.3.

Blank cells in Table 7.3 in both VRS and CRS cases, as explained, indicates the unit is not fulfilling the selective radial efficiency assumption (i.e. is not efficient in the given technology for this specific example since we have at least one strictly positive output in set B for all units); therefore the elasticity is not defined. This can be verified through observing the output-oriented (OO) efficiency scores for those units in Table 7.2. For example, farms 4 and 10, which do not correspond to any elasticity value in Table 7.3, are not efficient neither in VRS and CRS technologies with efficiency scores of 0.99 and 0.22 in VRS, 0.87 and 0.19 in CRS respectively. Therefore, they are omitted in Table 7.3. On the other hand, some farms are efficient in VRS technology but not in CRS. For example, farm 19 has defined

RHE and LHE under VRS consideration; however, CRS RHE and LHE do not exist for this farm since it is not on the efficient frontier under CRS.

As discussed in Section 2.9 and Section 5.6, scale elasticity can be viewed as the quantitative measure of the strength of returns-to-scale (RTS) observed at the efficient unit. (Førsund and Hjalmarsson, 2004). At a unit, if both one-sided scale elasticities are, respectively, less than or greater than 1, the frontier exhibits decreasing returns-to-scale (DRS) or increasing returns-to-scale (IRS), respectively. It can be observed in VRS technology elasticity results given in Table 7.3 that farms 2, 6, 19, 20 and 21 are examples of units at which the efficient frontier exhibits DRS. On the other hand, at units 3, 24 and 35, the frontier exhibit IRS. If the range defined by one-sided elasticities contains 1, the frontier exhibits constant returns-to-scale (CRS) at that unit (Podinovski and Førsund 2010). Majority of the farms in West Anatolia sample are examples of units where the frontier is exhibiting CRS.

Table 7.3. Scale Elasticities for Farms in West Anatolia

Farm	VRS LHE	VRS RHE	CRS LHE	CRS RHE
1	1.04	0	1	1
2	0.99	0		
3	UD	1.86		
5	1.44	0.04	1	1
6	0.99	0.56		
7	2.81	0	1	1
8	1.18	0	1	1
9	1.83	0	1	1
11	1.77	0.10	1	1
12	1.16	0.84	1	1
13	11.15	0.21	1	1
14	1.42	0	1	1
15	1.56	0	1	1
16	2.96	0	1	1
17	1.07	0.61	1	1
18	1.04	0	1	1
19	0.99	0.29		
20	0.83	0.25		
21	0.88	0.29		
22	1.23	0	1	1
23	1.20	0.01	1	1
24	2.70	1.30		
25	UD	0.59	1	1
26	2.00	0.08	1	1
27	1.10	0	1	1
28	9.44	0.13	1	1
29	1.11	0	1	1
30	1.24	0.05	1	1
31	1.20	0.24	1	1
32	1.10	0.20	1	1
33	1.12	0	1	1
34	UD	0.01	1	1
35	UD	1.15		

7.2.1.2. Special Cases of CRS Technology

In this section, we cover three different scenarios defined as special cases of CRS technology, in Theorem 4.5 (see Chapter 4, Section 4.4). The results are presented in Table 7.4. We deal with the same technology in all scenarios (CRS); therefore the efficient and inefficient units are the same for all three experiments. Since the elasticities are not defined for the inefficient units, they (10 farms; see OO CRS column in Table 7.2) are omitted in Table 7.4.

In the first scenario, the set *B* includes all outputs, part (a) of Theorem 4.5 states that, if set *B* is the set of all outputs, both RHE and LHE exist and less than equal to 1, no matter which inputs are included in set *A*. We experiment this bit of the theorem with a scenario where the changing set includes Land, Cost and Labour inputs. The responding set *B* is all outputs. As seen in Table 7.4, under scenario 1, the results verify the above statement and the elasticity values are all less than or equal to 1 on both side. Again, we want to remind that the blank cells indicate that the corresponding unit does not satisfy the selective radial efficiency assumption. Farms that do not satisfy this assumption in any type of technology are omitted in Table 7.4.

The part (b) of Theorem 4.5 states that under CRS technology, if the set *A* contains all inputs, then RHE exists and is greater than or equal to 1. We test this statement with a scenario where a set of outputs (Cereals, which include 5 outputs as Wheat, Barley, Oats, Grain Maize and Rye productions in West Anatolia sample; see Table 7.1) is responding to the proportional marginal increase in set of all inputs (Land, Labour, Crop Production Costs and Capital Expenditures). The results for this scenario presented in Table 7.4 (Scenario 2) reveals that in this case all the defined RHE are greater than or equal to 1 as stated in the Theorem 4.5.

Finally, the part (c) of Theorem 4.5, which states that if the changing set (set *A*) contains only outputs under CRS technology, the LHE exists and less than or equal to 0, is tested. In other words, response of an output set to a marginal reduction in a set of outputs is always equal or less than 0 under CRS. This is verified through experimenting the scenario where set *A* consists of Field Crops (6 outputs as Sugar beet, Vetch, Beans, Peas, Potatoes and Lentil productions; see Table 7.1) and the responding set *B* contains Cereals (5 outputs as Wheat, Barley, Oats, Grain maize and Rye productions). As seen in the last column of Table 7.4, elasticities under this scenario is less than or equal to 0 for all farms in the West Anatolia sample.

Table 7.4. Special Cases of CRS Technology

	Scenario 1		Scenario 2	Scenario 3
Farm	**LHE**	**RHE**	**RHE**	**LHE**
1	1	0	1.00	0
5	0.71	0.11	2.02	-1.02
7	1	0	1.13	0
8	1	0	1.51	-0.51
9	1	0.05	1	0
11	1	0	1.03	0
12	0.90	0.43	14.17	-13.17
13	1	0.20	2.40	-1.40
14	1	0	2.34	-1.34
15	1	0	6.34	0
16	1	0	1	0
17	1	0.96	14.40	-1.45
18	1	0	2.77	-1.77
22	1	0	1.76	-0.75
23	1	0	4.02	-1.42
25	1	0.39	1.00	0
26	1	0	2.29	0
27	1	0	4.02	0
28	1	0	1	0
29	1	0	1	0
30	1	0	1	0
31	1	0.45	1.53	0
32	1	0.47	1.63	-0.63
33	1	0	4.17	-1.62
34	1	0	1	0

7.2.1.3. Elasticity Analysis of Outputs to Changing Outputs

Suppose we apply the third scenario for the CRS technology above, where sets A and B contain only outputs (A={Field Crops}, B={Cereals}) (set C contains all inputs and the remaining outputs) to both VRS and CRS technologies with or without trade-offs. The results are provided in Table 7.5. Farms 4 and 10 are omitted, since these units do not satisfy the selective radial efficiency assumption in any technology of VRS and CRS (with or without trade-offs). Note that CRS LHE results are those given in Table 7.4 (Scenario 3) as a special case of CRS technology.

Table 7.5. Elasticity Measures for Cereals in response to Changing Field Crops

Farm	VRS LHE	VRS RHE	VRS LHE (WTO)	VRS RHE (WTO)	CRS LHE	CRS RHE	CRS LHE (WTO)	CRS RHE (WTO)
1	0	UD	-0.38	-0.62	0	UD		
2	0	UD	-0.21	-0.35				
3	0	0	0	0				
5	-0.23	UD			-1.02	UD		
6	0	0						
7	0	UD	-0.50	-1.18	0	UD	-0.50	-1.18
8	-0.07	UD			-0.51	UD		
9	0	-10.49			0	-4.04		
11	0	UD			0	UD		
12	-11.08	UD			-13.17	UD		
13	-1.30	UD			-1.40	UD		
14	-0.80	UD	-2.20	-2.50	-1.34	UD		
15	0	0			0	0		
16	0	UD	-0.13	-0.32	0	UD	-0.23	-0.31
17	-0.44	UD			-1.45	-4.17		
18	0	UD	-0.63	-1.08	-1.77	UD		
19	0	-4.03	-0.40	-0.63				
20	-0.24	-1.16						
21	0	0						
22	0	UD	-1.77	-2.66	-0.75	UD		
23	0	UD	-3.06	-4.99	-1.42	UD		
24	-0.23	-1.12						
25	0	UD	-0.19	-0.29	0	UD		
26	0	UD	-1.28	-2.20	0	UD	-1.28	-2.92
27	0	UD	-1.47	-2.42	0	UD	-1.47	-2.42
28	0	UD	-0.54	-0.87	0	UD	-0.54	-0.87
29	0	UD			0	UD		
30	0	-0.43			0	-0.42		
31	0	UD	-0.22	-0.33	0	UD		
32	0	UD	-0.56	-0.57	-0.63	UD		
33	0	UD	-4.30	-7.62	-1.62	UD	-4.30	-7.62
34	0	0	0	0	0	0	0	0
35	0	UD	-0.32	-0.58				

It can be observed in Table 7.5 that at several units, we have negative right-hand and left-hand elasticities revealing an inverse relationship between selected sets of outputs, where at those units, proportional increases (reductions) in the production level of Field Crops are responded with a reduction (increase) in the production level of Cereals.

Another interesting result here is that the integration of production trade-offs provides us more of finite elasticities in both VRS and CRS technologies. We have several undefined elasticities (i.e. unbounded results) without the trade-offs, whereas even the consideration of broadest trade-offs cause tighter ranges of elasticities and in our specific example here, we do not have any unbounded results with the trade-offs included.

7.2.1.4. Elasticity Analysis of Outputs to Changing Inputs and Outputs

Suppose we have sets as *A*={Cost, Capital Expenditures, Fodder Crops}, *B*={Cereals} and the remaining inputs and outputs are kept constant in set *C*. The difference of such scenario is that in the changing set *A*, we have both inputs and outputs. In West Anatolia sample, Fodder crops represent two outputs as Lucerne and Fodder Maize production, whereas Cereals represent a set of 5 outputs as Wheat, Barley, Oats, Grain Maize and Rye productions. The elasticity measures for this scenario are presented in Table 7.6 for both VRS and CRS technologies and either with or without trade-offs included. Once again, farms 4 and 10 are omitted, since these units do not satisfy the selective radial efficiency assumption in any technology of VRS and CRS (with or without trade-offs).

Without the trade-offs, we have several farms at which RHE and LHE are undefined for both VRS and CRS technologies; integration of trade-offs provides finite ranges for some of such farms (e.g. farm 26 and farm 33). Also, the number of unbounded results is less in the models with trade-offs compared to models without trade-offs, since the frontier becomes more gradual with the expansion of the technology through the inclusion of new constraints representing production trade-offs.

Table 7.6. Elasticity Measures for Cereals in response to Cost, Capital and Fodder Crops

Farm	VRS LHE	VRS RHE	VRS LHE (WTO)	VRS RHE (WTO)	CRS LHE	CRS RHE	CRS LHE (WTO)	CRS RHE (WTO)
1	UD	0	1.59	0	UD	0		
2	UD	UD	1.19	-1.19				
3	UD	0	UD	0				
5	UD	0			UD	0.15		
6	0.99	0.45						
7	UD	0	1.16	0	UD	0	1.15	0
8	UD	0			UD	0		
9	7.45	-1.05			3.99	0.53		
11	UD	0			UD	0		
12	UD	2.37			UD	3.15		
13	UD	0			UD	0		
14	UD	0	2.28	0	UD	0		
15	UD	0			UD	0		
16	UD	0	3.29	1.15	UD	0	1.31	1.21
17	UD	0			0.87	0		
18	UD	0	1.78	0	UD	0		
19	1.57	0	0.69	0.04				
20	0.85	0						
21	0.88	0.29						
22	UD	UD	2.72	0.36	UD	-0.03		
23	UD	UD	3.32	-0.49	UD	UD		
24	4.02	0.95						
25	UD	0	UD	0	UD	0		
26	UD	UD	5.27	-3.08	UD	UD	3.63	-3.08
27	UD	UD	UD	UD	UD	UD	3.42	UD
28	UD	0	UD	0	UD	0	1.83	0
29	UD	UD			UD	UD		
30	1.45	0			1.40	0		
31	UD	UD	-0.10	-1.47	UD	UD		
32	UD	0	0.27	0.13	UD	0		
33	UD	UD	6.02	-3.18	UD	UD	2.94	-3.18
34	UD	UD	UD	-1.65	1.00	UD	1.00	-1.65
35	UD	0	UD	0				

7.2.1.5. Elasticity Analysis of Outputs to Changing Inputs

Another possible scenario to consider for the elasticity analysis of output sets is in which we have only some inputs in the changing set and only some outputs in the responding set. This scenario is covered in an extensive application in the following chapter (Chapter 8) for all regions in our data set as well as the West Anatolia region. The application in Chapter 8 is designed with two different sets of responding outputs (such as Cereals and Field Crops) and changing set contains Labour and Crop Production Cost inputs throughout all application. Since the calculations under this scenario are covered, not to repeat, we leave the discussion and presentations of this scenario for Chapter 8.

7.2.2. Elasticity Analysis of Input Sets

For completeness of discussions, in this section, we apply elasticity measures with different changing and responding set scenarios, where responding set contains this time only inputs. Theorem 2.7 in Section 2.9.2 provides the models for the calculations for the VRS technology, and Theorem 4.7 in Section 4.6 provides the models for the CRS technology. Theorems 5.5 (Section 5.2) and 5.11 (Section 5.4) give the elasticity measures of for the CRS and VRS technologies with trade-off relations included, respectively. Similar to the previous section, farms 4 and 10 omitted from the result tables since those units do not satisfy the selective radial efficiency assumption in any input-oriented model.

7.2.2.1. Scale elasticity of Input Sets

Suppose we have A={All outputs}, B={All inputs} and C=∅. This scenario represents the reverse of the scale elasticity, where all inputs are responding to changes in all outputs. The results are provided in Table 7.7. Naturally, for the VRS technology, the results in this scenario are the reciprocals of those in Section 7.2.1.1, which are given in Table 7.3. Basically, sets A and B are switched. Returns-to-scale considerations for the units exhibiting DRS and IRS are the inverses of those in Section 7.2.1.1 under this scenario. In other words, units exhibiting DRS in the preceding exhibits IRS in this scenario and vice versa. As stated in Theorem 4.9 in Section 4.7, under CRS consideration, the scale elasticities for all units are equal to 1, since full proportionality between inputs and outputs are assumed.

Table 7.7. Scale Elasticities of Input Sets for Farms in West Anatolia

Farm	VRS LHE	VRS RHE	CRS LHE	CRS RHE
1	0.96	UD	1	1
2	1.02	UD		
3	0	0.54		
5	0.70	28.56	1	1
6	1.01	1.78		
7	0.36	UD	1	1
8	0.85	UD	1	1
9	0.55	UD	1	1
11	0.57	10.25	1	1
12	0.86	1.20	1	1
13	0.09	4.68	1	1
14	0.71	UD	1	1
15	0.64	UD	1	1
16	0.34	UD	1	1
17	0.93	1.64	1	1
18	0.96	UD	1	1
19	1.01	3.44		
20	1.20	3.95		
21	1.14	3.46		
22	0.82	UD	1	1
23	0.83	196.60	1	1
24	0.37	0.77		
25	0	1.69	1	1
26	0.50	12.97	1	1
27	0.91	UD	1	1
28	0	7.51	1	1
29	0.90	UD	1	1
30	0.81	18.43	1	1
31	0.84	4.11	1	1
32	0.91	5.05	1	1
33	0.90	UD	1	1
34	0	220.35	1	1
35	0	0.87		

7.2.2.2. Elasticity Analysis of Inputs to Changing Inputs

In this scenario, we experiment the responses of inputs to changing inputs. Suppose we have A={Land, Capital Expenditures}, B={Cost}. Set C contains all outputs and the one remaining input (Labour). We consider both technologies (VRS and CRS) with and without trade-offs included. The results obtained under this scenario are given in Table 7.8.

Table 7.8. Elasticity Measures for Cost in response to Changing Land and Capital

Farm	VRS LHE	VRS RHE	VRS LHE (WTO)	VRS RHE (WTO)	CRS LHE	CRS RHE	CRS LHE (WTO)	CRS RHE (WTO)
1	UD	0	UD	0	UD	-0.78		
2	UD	0	UD	0				
3	UD	0	UD	0				
5	UD	0			UD	0		
6	UD	-0.03						
7	UD	0	UD	-5.92	UD	0	UD	-7.08
8	UD	0			UD	0		
9	UD	0			UD	0		
11	UD	0			UD	0		
12	UD	-2.58			UD	-2.83		
13	UD	0			UD	0		
14	UD	0	UD	0	UD	0		
15	UD	0			UD	0		
16	UD	0	-0.39	0	UD	0	-0.07	0
17	UD	0			UD	0		
18	UD	0	UD	0	UD	0		
19	UD	0	-47.00	-0.91				
20	UD	0						
21	-0.26	0						
22	UD	0	-2.87	0	UD	0		
23	UD	0	UD	-0.09	UD	0		
24	UD	-5.80						
25	UD	0	UD	0	UD	0		
26	UD	0	UD	0	UD	0	UD	-0.06
27	UD	0	UD	0	UD	0	UD	0
28	UD	0	UD	0	UD	0	UD	-0.15
29	UD	0			UD	0		
30	UD	0			UD	0		
31	UD	0	UD	-2.07	UD	-0.63		
32	UD	0	-6.79	-2.83	UD	0		
33	UD	0	UD	0	UD	0	UD	-1.88
34	UD	0	UD	0	UD	0	UD	0
35	UD	-0.12	UD	-0.14				

As seen in Table 7.8, we have most LHE measures as undefined and RHE measures as 0. For the units that the elasticities are defined and different than 0, the values are all negative indicating an inverse relationship between inputs similar to output-to-output case considered in Section 7.2.1.3 above. For some units, the integration of trade-offs result in a narrower range of LHE and RHE (e.g. farms 19 and 32 in VRS; farm 16 in CRS)

7.2.2.3. Elasticity Analysis of Inputs to Changing Inputs and Outputs

Suppose we have *A*={Land, Labour, Cereals}, *B*={Cost, Capital Expenditures} and the remaining outputs are kept constant in set *C*. In this scenario, set *A* includes both inputs (Land and Labour) and a set of outputs (Cereals, which represent a set of 5 outputs as Wheat, Barley, Oats, Grain Maize and Rye). The results of elasticity analysis under this scenario are given in Table 7.9. We have majority of left-hand elasticities as undefined. Also in the VRS technology, majority of right-hand elasticity are undefined as well. The inclusion of trade-offs in the models causes a transition to a narrower range of elasticities (for this case, farms 19, 22, 23 and 32 are examples in VRS technology and farm 16 is an example in CRS technology).

7.2.2.4. Elasticity Analysis of Inputs to Changing Outputs

Final illustrative example we consider in elasticity analysis of input sets is the output-to-input scenario. Suppose we have *A*={Field Crops}, *B*={Cost, Labour}, where set C contains all the remaining inputs and outputs as the ones kept constant. Field Crops set represent 6 outputs as Sugar beet, Vetch, Beans, Peas, Potatoes and Lentil productions (see Table 7.1) in West Anatolia. Cost and Labour inputs are responding to the changes in given multiple outputs. The results of this scenario are given in Table 7.10. As in all of the previous cases, the integration of trade-off relations provides an improvement to a more number of defined elasticities.

Table 7.9. Elasticity Measures for Cost and Capital in response to Land, Labour, Cereals

Farm	VRS LHE	VRS RHE	VRS LHE (WTO)	VRS RHE (WTO)	CRS LHE	CRS RHE	CRS LHE (WTO)	CRS RHE (WTO)
1	UD	UD	UD	UD	UD	1.00		
2	UD	UD	UD	UD				
3	UD	0.16	UD	0.09				
5	UD	UD			-83.97	0.50		
6	1.01	1.94						
7	UD	UD	UD	-0.38	UD	0.32	UD	-0.39
8	UD	UD			UD	0.66		
9	UD	UD			UD	1.00		
11	UD	UD			UD	0.97		
12	-11.76	-0.34			-8.43	-0.43		
13	UD	UD			UD	0.32		
14	UD	UD	UD	UD	UD	0.43		
15	UD	UD			UD	0.09		
16	UD	UD	0.23	0.87	UD	1.00	0.76	0.81
17	UD	-6.90			UD	-17.58		
18	UD	UD	UD	UD	UD	0.33		
19	UD	UD	-7.45	1.39				
20	UD	UD						
21	1.14	3.46						
22	UD	UD	-1.72	0.91	UD	0.40		
23	UD	UD	-5.11	0.29	UD	0.12		
24	-0.03	0.51						
25	UD	UD	UD	0.27	UD	1.00		
26	UD	UD	UD	0.38	UD	0.28	UD	0.14
27	UD	UD	UD	UD	UD	0.12	UD	0.06
28	UD	UD	UD	UD	UD	1.00	UD	0.53
29	UD	UD			UD	1.00		
30	UD	UD			UD	1.00		
31	UD	UD	UD	UD	UD	-0.45		
32	UD	UD	0.29	0.91	UD	0.61		
33	UD	UD	UD	0.16	UD	0.07	UD	-1.14
34	UD	UD	UD	UD	UD	1.00	UD	0.76
35	UD	0.25	UD	0.15				

Table 7.10. Elasticity Measures for Cost and Labour in response to Changing Field Crops

Farm	VRS LHE	VRS RHE	VRS LHE (WTO)	VRS RHE (WTO)	CRS LHE	CRS RHE	CRS LHE (WTO)	CRS RHE (WTO)
1	0	UD	0.33	UD	0	UD		
2	0	UD	0.10	UD				
3	0	0	0	0				
5	0.50	UD			0.51	6.73		
6	0	0						
7	0	UD	1.33	UD	0	UD	1.43	UD
8	0.13	UD			0.70	UD		
9	0	UD			0	UD		
11	0	UD			0	UD		
12	3.40	UD			3.83	UD		
13	0.09	4.82		0	0.58	3.53		
14	0.64	UD	1.67		0.70	UD		
15	0	0		0	0	0		
16	0	UD	0.05		0	UD	0.19	0.25
17	0.07	0.22			0.08	0.11		
18	0	UD	0.54	6.38	0.80	UD		
19	0	UD	0.92	19.77				
20	0.62	UD						
21	0	0						
22	0	UD	0.89	3.78	0.44	UD		
23	0	UD	1.17	UD	0.43	UD		
24	0.80	UD						
25	0	UD	0	682.17	0	UD		
26	0	UD	0.43	UD	0	UD	0.58	UD
27	0	UD	0	UD	0	UD	0	UD
28	0	UD	0	UD	0	UD	0.53	UD
29	0	UD			0	UD		
30	0	UD			0	UD		
31	0	UD	0.81	5.03	0	UD		
32	0	UD	1.82	2.51	0.39	UD		
33	0	UD	0.97	UD	0.84	UD	2.13	UD
34	0	0	0	0	0	0	0	0
35	0	UD	0	UD				

7.3. Summary of Illustrative Example Results

In this chapter, we illustrate different elasticity scenarios and demonstrate that the mixed partial elasticities can be calculated for any subset of inputs and outputs. The examples in this chapter serve as a practice for elasticity measurement under different scenarios. We consider both output and input orientation and show that our models work for any orientation. The examples considered are summarised in Table 7.11, with the corresponding table numbers providing the results for each example. Scenarios are given as changing set to responding set depending on the elements included in such sets (e.g. inputs to outputs or inputs and outputs to inputs).

Table 7.11. Summary of Illustrative Elasticity Analysis in West Anatolia

Examples	Tables	Remark
Measures for output sets		
Scale elasticity of output sets	Table 7.3	For CRS, always equal to 1
Special cases of CRS	Table 7.4	Theorem 4.5 is verified
Outputs to Outputs	Table 7.5	Negative elasticities indicating inverse relationship
Inputs and Outputs to Outputs	Table 7.6	Trade-offs provide more finite results
Inputs to Outputs	Table 7.7	Covered in Chapter 8
Measures for input sets		
Scale elasticity of input sets	Table 7.8	Reciprocal of output sets. For CRS, equal to 1
Inputs to Inputs	Table 7.9	Negative elasticities indicating inverse relationship
Inputs and Outputs to Inputs	Table 7.10	Trade-offs provide more finite results
Outputs to Inputs	Table 7.11	Trade-offs provide more finite results

We show that elasticity measures can be calculated for outputs or inputs under any scenario of changing sets under any technology, with or without trade-offs integrated. It can be observed that integration of production trade-offs, even with a broadest manner, provide a better discrimination of efficiency and more finite elasticity measures. We provide scale elasticity measures and in addition, verify the statements for the CRS consideration given in Chapter 4.

Another important result out of all the experiments performed is that efficiency and elasticity analyses exhibit 100% consistency with each other. If a unit is not efficient, the elasticity measure is infeasible; otherwise, we have either finite or unbounded results. This supports our generalization in Chapter 4 that eliminates the necessity of preliminary check for the efficiency since elasticity measures already provide information about the efficiency of the units as well.

In the following chapter, we extend our application scope to all regions and integrate different ranges of trade-offs in order to evaluate the effect of changing trade-offs on efficiency and elasticity measures.

Chapter 8

Empirical Application in Turkish FADN Data set

In Chapter 7, we illustrate the proposed methodology on efficiency and elasticity in a sample of farms from a specific region (West Anatolia). We show that the partial elasticity measures can be calculated for any subset of input and outputs. The examples provide a practice on how the elasticity measures developed are applicable to different scenarios of elasticity. They also serve as a preliminary illustration of incorporating production trade-offs in efficiency measurement and the results reveal that even with a broad range of production trade-offs, the discrimination of efficiency scores is affected positively. In this chapter, we extend our application to the whole sample of Turkish FADN data set consisting of farms from 8 different regions. The general characteristics of our data set, the inputs and outputs used in the models, crop types in each region with their classifications and identification of production trade-off relations are described in Chapter 6.

The first part of analysis (presented in Section 8.1) deals with the efficiency of farms in each region with and without trade-offs incorporated. In efficiency analysis, we consider all three ranges of trade-offs together with the models without any trade-offs and aim to observe how the discrimination of efficiency responds to integration of trade-offs and the changes in trade-off ranges. In the second part (presented in Section 8.2), we deal with the elasticity measures in each region, once again with and without trade-off relations included. In the elasticity analysis, to be consistent, the same scenarios of elasticity are applied to all regions. We also incorporate three different ranges of trade-offs in every region to observe how the changing trade-off ranges affect the elasticity measures. Analyses in this chapter are performed for both returns-to-scale (RTS) considerations (variable and constant). For a better presentation, the result tables of the analysis in this chapter are given in Appendix C.

8.1. Efficiency Analysis in All Regions

First of all, we measure the efficiency of farms in our data set extracted from Turkish FADN database and described in Chapter 6. We consider 4 different models for each returns-to-scale consideration (variable and constant): models with no trade-offs and models with 3 different ranges of trade-offs (Broad, Medium and Tight). Because we consider the response of the output sets to a set of changing inputs for the elasticity analysis in the following section, we use the DEA models for the efficiency evaluation in this section as output-oriented (OO). The output-oriented DEA efficiency scores for each region are presented in Tables C.1 to C.8 in Appendix C. Using the information from efficiency result tables in Appendix C, the average efficiency scores and number of efficient and inefficient units for each region are summarised in Table 8.1 below.

As seen in Table 8.1, without trade-offs incorporated, the DEA models do not discriminate well, which is an expected result due to the large number of outputs included in the models. For instance, in East Marmara region, in which we have 20 different types of crops (therefore, 20 outputs) produced by 27 farms, all units are obtained as efficient in both VRS and CRS considerations. As expected, CRS models discriminate better than VRS models, but still the discrimination is quite insufficient without any trade-offs introduced.

When the production trade-off relations are introduced into the DEA models in the form of weight restrictions, the discriminations of both VRS and CRS models improve for all regions. In particular, average efficiency scores for Aegean and Mediterranean regions change dramatically. For regions like West Marmara and South East Anatolia, even though the average efficiency level remains high, the number of efficient and inefficient farms changes to a large extent. For instance, in West Marmara region, without trade-offs, the majority of the farms are efficient, but with the tightest trade-offs incorporated, it can be observed that only 8 farms remain efficient.

Table 8.1. Summary of Efficiency Analysis

Regions	Sample sizes		OO VRS				OO CRS			
			No TO	**Broad TO**	**Medium TO**	**Tight TO**	**No TO**	**Broad TO**	**Medium TO**	**Tight TO**
1.West Marmara (WM)	39	**Average**	0.98	0.96	0.95	0.92	0.97	0.92	0.88	0.84
		# of Efficient.	35	24	20	15	30	12	9	8
		# of Inefficient	4	15	19	24	9	27	30	31
2.Aegean (AEG)	17	**Average**	1.00	0.23	0.22	0.21	0.99	0.10	0.09	0.09
		# of Efficient.	17	3	3	3	16	1	1	1
		# of Inefficient	0	14	14	14	1	16	16	16
3.East Marmara (EM)	27	**Average**	1.00	0.74	0.69	0.63	1.00	0.70	0.63	0.56
		# of Efficient.	27	10	10	6	27	8	6	5
		# of Inefficient	0	17	17	21	0	19	21	22
4.West Anatolia (WA)	35	**Average**	0.98	0.86	0.84	0.81	0.94	0.72	0.69	0.65
		# of Efficient.	33	19	17	14	25	7	5	5
		# of Inefficient	2	16	18	21	10	28	30	30
5.Mediterranean (MED)	30	**Average**	1.00	0.51	0.47	0.44	0.99	0.24	0.22	0.21
		# of Efficient.	28	8	8	6	26	3	3	3
		# of Inefficient	2	22	22	24	4	27	27	27
6.Middle Anatolia (MA)	26	**Average**	0.97	0.81	0.79	0.76	0.95	0.68	0.67	0.64
		# of Efficient.	22	13	13	12	22	8	8	8
		# of Inefficient	4	13	13	14	4	18	18	18
7.North East Anatolia (NEA)	14	**Average**	1.00	0.97	0.96	0.95	0.97	0.89	0.89	0.87
		# of Efficient.	13	11	11	11	12	9	9	8
		# of Inefficient	1	3	3	3	2	5	5	6
8.South East Anatolia (SEA)	26	**Average**	0.99	0.97	0.96	0.96	0.97	0.92	0.91	0.89
		# of Efficient.	24	20	20	18	19	12	10	10
		# of Inefficient	2	6	6	8	7	14	16	16

One interesting result about the efficiency discriminations is that the improvement in the discrimination is more extensive from no trade-off model to model with broad trade-offs than broad trade-offs to tighter ones. Such an observation tells us, it is not very crucial to be too accurate in specifying the trade-offs. Even with the broadest range of relations considered, the discrimination improves.

8.2. Elasticity Analysis in All Regions

The second part of the analysis focuses on the elasticity measurement for farms in our data set considering different ranges of trade-off relations. In this section, we aim to interpret the results of analysis obtained for 8 regions under two different elasticity scenarios for each region. In both scenarios, the changing set is the same and consists of two inputs as "Crop Production Costs" and "Labour". We keep "Land" and "Capital Expenditures" constant since these factors are less flexible to change in short-term. Therefore, we are interested in the effect of the changes in costs and labour inputs on the production of crops in the responding set.

When identifying the outputs in the responding set, we use the classification of crop types given in Table 6.3 and design two scenarios. In the first scenario, the responding set consists of "Cereals", which represent a set of outputs such as Wheat, Barley, Grain Maize, Oats and Rye productions and we measure the response of Cereals to the changing crop production and labour costs. In the second scenario, we measure the response of "Field Crop" production, which include a set of outputs such as Vetch, Beans, Peas, Lentil, Sunflower, Oilseed Rape, Cotton, Potatoes, Sugar Beet and Tobacco productions to the changes in the cost and labour inputs. Not all crop types are produced in every region, therefore the elements of responding set varies between regions (The crop types produced by the farms in each region are given in Table 6.2).

The reason behind identifying Cereals and Field Crops as responding sets is that the crop types under these classes are the ones, for which most of the crop production is concentrated in every region. In principle, we can also calculate the elasticity of response for other classes of crops such as Fodder Crops, Permanent Crops and Vegetables, however, for majority of farms elasticity measures are not applicable in all regions, since not many of the farms are producing these types of crops. Therefore, we illustrate our case for two classes of crop types, on which most of the crop production is focused.

The results of elasticity calculations are presented in Tables C.9 to C.24 of Appendix C. For each region we have two tables representing two different scenarios of elasticity measurement (Cereals responding and Field Crops responding). In calculations of elasticities of response under VRS technology, we rely on the models for output sets and input sets developed by Podinovski and Førsund (2010) provided by Theorems 2.6 and 2.7, respectively, in Section 2.9.2 of Chapter 2. For the CRS technology, we employ elasticity measures of output sets developed in the scope of this research and provided in Chapter 4 (see Theorem 4.2 in Section 4.2). For the models with trade-offs, we implement the theory developed for output sets in Chapter 5 (see Theorem 5.2 in Section 5.1 under CRS; see Theorem 5.8 in Section 5.3 under VRS).

Linear programming (LP) models to measure the elasticity of response at any unit of DEA technology can yield to three types of solutions with different interpretations; optimal, unbounded and infeasible solutions. Given three possible cases are summarised in Chapters 4 and 5 (see Section 4.3 for output sets, Section 4.7 for input sets or Section 5.5 for trade-offs case). The framework is applicable to both VRS and CRS technologies with or without trade-offs. If the unit is efficient in the given technology, optimal solutions to models give us the elasticity measure, whereas unbounded solutions indicate that it is not possible to marginally increase or reduce the factors in set A at the given unit and remain in the given technology. Unbounded solutions are represented as "*Undefined* (*UD)*" in the result tables.

As explained in Section 5.5, in principle, we can have infeasibility either when the unit is not efficient or does not have any strictly positive output in set *B*. These two cases of infeasibility are differentiated in our result tables. (In the examples of Chapter 7, we do not have the latter case because in those examples, all farms are producing at least one of the crops in the responding set). As in the illustrative examples in Chapter 7, when presenting elasticity results, for the units yielding infeasible solution because they violate the selective radial efficiency assumption, (i.e. they are inefficient) the corresponding cells are left blank. As stated, infeasibility can also occur when the unit does not have any strictly positive output in the responding set; for such units, the elasticity is also not applicable. We represent such units with "*Not Applicable (NA)*" in the result tables. Examples can be seen for many result tables in which Field Crops are the responding set (such as Table C.12, Table C.14, Table C.16 and Table C.18). The term "NA" for a farm reveals that the farm does not produce any of the outputs in the responding set (it may or may not be efficient); therefore elasticity measures are not applicable.

In Tables C.9 to C.24 in Appendix C, we have many undefined and 0 elasticities for both VRS and CRS considerations without the production trade-offs incorporated. When the trade-offs are considered, the several farms are becoming inefficient as mentioned in the preceding section, resulting in inapplicability of elasticity measures for those units. For the ones efficient, we can clearly observe that the proportion of finitely defined elasticities to undefined ones is increasing for all regions.

In regions that exhibit a significant change in the discrimination of efficiency, obviously, the number of units for which elasticity measures are applicable, is quite low. Especially, in Aegean region (see Tables C.11 and C.12), due to the considerably changing efficiency discrimination, the number of efficient farms in the models with trade-offs is just 3 for VRS and 1 for CRS case. Also, for the scenario in which the Field Crops are responding set, for several farms, elasticity is not applicable because those farms are not producing any of Field Crops (see Table C.12).

Following in this chapter, we intend to draw general interpretations through the elasticity measures obtained for all the regions. First of all, in Section 8.2.1, we comment on the very large elasticity values that can be observed in results of many regions. Secondly, in Section 8.2.2, we evaluate the effects of changing trade-offs on the elasticity measures and observe whether the propositions in Chapter 5 are verified by the results. In Section 8.2.2, we also touch the notion of "*Returns to changing set A (RTA)"*, which is introduced in Chapter 5.

8.2.1. Interpretation of Large Elasticities

When the elasticity measures in Tables C.9 to C.24 are examined, it can be observed that for some farms the elasticity of response values obtained are very large values. (For examples of such a situation, see farm 29 in Table C.9; farm 38 in Table C.10; farm 16 in Table C.11; farm 27 in Table C.14; farm 23 in Table C.17; farm 23 in Table C.19; farm 1 in Table C.22). In some cases elasticities are extremely large. Such large elasticity values are obtained due to the steepness of the frontier at the given unit in the given scenario. Let us explain such a case with a simple one input-one output example. Suppose we have a production possibility set defined by 4 units given in Table 8.2 together with the output-oriented (OO) variable returns-to-scale (VRS) DEA scores and two-sided elasticity measures for the units. We consider the scenario where input is changing and output is responding (scale elasticity). The units are plotted in Figure 8.1.

Table 8.2. Illustrative Example For Large Elasticities

Units	Input	Output	OO VRS	LHE	RHE
A	0.8	10	1	UD	12.8
B	0.85	18	1	7.6	0.28
C	2.2	26	1	0.5	0
D	2	17	0.69		

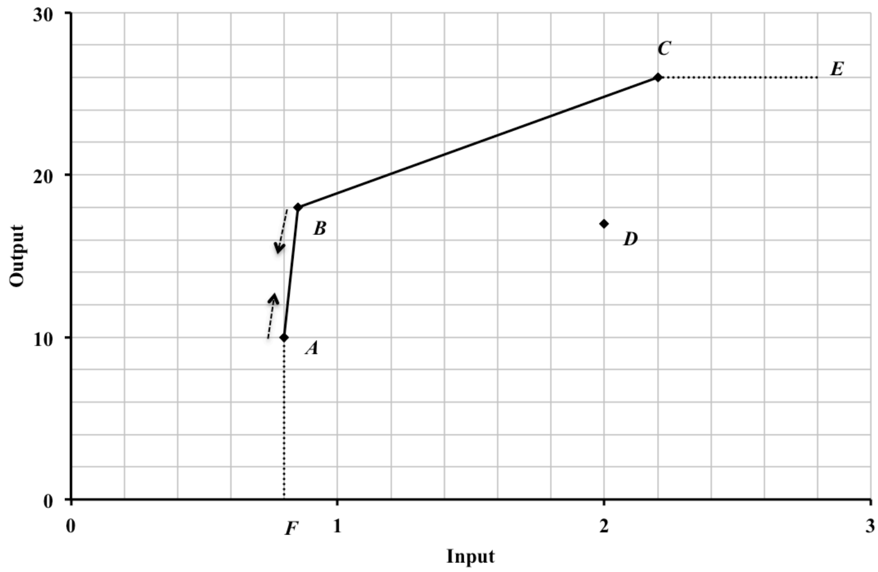

Figure 8.1. One input-One Output Illustrative Example for Elasticity

In VRS technology, 3 units (*A*, *B* and *C*) define the efficient frontier. Unit *D* is inefficient; therefore the elasticity measures are infeasible for that unit. Right-hand elasticity for unit *A* (12.8) and left-hand elasticity for unit *B* (7.6) are relatively large numbers indicating that between *A* and *B* the frontier is steep resulting in large values of elasticity of responses for units *A* and *B* towards the direction of the steep bit of the frontier. This can be observed in Figure 8.1. The ray *AB* on the frontier is extremely steep. The upward dashed arrow from unit *A* represents the right-hand movement for this unit. Similarly, the downward dashed arrow represents the left-hand movement from unit *B*. These movements towards ray *AB* of the frontier cause the corresponding elasticities to be large since the slope is large on that part of the frontier.

Another point to make through the given example above is the illustration of undefined and 0 elasticities for the units. As in Figure 8.1 right-hand movement from unit *C* is a horizontal movement towards point *E*. The right-hand elasticity for unit *C* is 0, since the movement is horizontal. Increasing the input does not change the output level in the given technology. On the other hand, left-hand movement from unit *A* causes a move out of the given technology since the vertical ray *AF* defines the boundary. It results in an unbounded objective function for left-hand elasticity measure. Such a result can be translated that the left-hand elasticity of unit *A* is undefined in the given technology.

8.2.2. Effects of Changing Trade-off Ranges on Elasticity Measures

In Section 5.6 of Chapter 5, we assert that addition of new trade-off relations to an existing technology (regardless of already having trade-offs or not) cause the interval $\left[\varepsilon^{+}_{A,B}(X_0,Y_0),\varepsilon^{-}_{A,B}(X_0,Y_0)\right]$ either remaining the same or getting narrower provided that the unit is still holding the selective radial efficiency assumption. Theorem 5.13 states and proves this notion and it is applicable also for the case of tightening the trade-off ranges, since tightening is analogous to adding new trade-offs, which make the previous ones redundant.

Abovementioned statement is supported by the elasticity measurement results presented in Tables C.9 to C.24 in Appendix C. It can be observed in all elasticity result tables given by Appendix C that the left-hand elasticities are either remaining same or getting smaller, whereas the right-hand elasticities are either remaining the same or getting bigger, resulting in a same or narrower interval of $\left[\varepsilon^{+}_{A,B}(X_0,Y_0),\varepsilon^{-}_{A,B}(X_0,Y_0)\right]$ provided that the farm remains efficient (i.e. still holds the selective radial efficiency assumption).

We can give farm 4 in South East Anatolia region as a specific example in which all the elasticity measures are finite for all models when Cereals are in the responding set (see Table C.23). Under VRS, the left-hand and right-hand elasticities are 1.17 and 0.03, respectively. When the trade-offs incorporated, the range narrows down. With the tightest trade-offs, we have 1.12 and 0.24 for left and right hand, respectively. Under CRS, left-hand elasticity remains the same as 1 for any model with or without trade-offs, whereas with the inclusion and tightening of trade-offs the right-hand elasticity is changing from 0.20 to 0.51. Note that for this farm, the two-sided elasticity measures are not applicable under the scenario of Field Crops responding to changes in the inputs in set *A*, (see Table C.24) indicating that farm 4 in South East Anatolia is not involved in production of any crop types classified as Field Crops.

Particularly, in many farms there is a transition from an undefined elasticity score to a finite elasticity score when the trade-offs are incorporated. For instance, see farm 18 in West Anatolia region in Tables C.15 and C.16. When cereals are responding, under VRS consideration, without trade-offs are included, farm 18 has undefined left-hand and zero right-hand elasticity (see Table C.15). When the broad trade-offs are included, the left-hand elasticity is measured as 1.74, where as right-hand elasticity is 0.14. The tighter the trade-offs, left and right-hand elasticity range is getting narrower. When the tightest trade-offs are incorporated, left-hand elasticity is 1.46 and right-hand is 0.21. A similar case can be observed for this unit in Table C.16, where Field Crops are in the responding set. Note that under CRS consideration, this farm is efficient only when trade-offs are not included. With trade-offs under CRS, two-sided elasticities are not defined because unit is not efficient in any model with trade-offs.

One interesting point to mention is that usually, the change in transition from broadest to tightest trade-offs is not that deep as in the transition from no-trade-offs to broadest trade-offs. This is similar to the observation for the efficiency discrimination. Observe the left-hand elasticities of farm 35 in West Marmara (Table C.9), farm 16 in Aegean region (Table C.12), farms 13 and 21 in East Marmara (Table C.14) or farm 8 in Middle Anatolia (Table C.19). In all of these farms, the left-hand elasticities change to finite values from undefined values. However, the change in left-hand elasticities is quite small between the models with different trade-off ranges. On the contrary, in a very few cases, tighter trade-offs than the existing ones can create big difference and help the elasticities become more reasonable. Such a case can be observed in farm 4 of East Marmara region (Table C.13). Left-hand elasticity values are quite large when broad trade-offs are used (16.56 and 14.88 for VRS and CRS considerations, respectively). In this specific farm, incorporating tighter trade-offs yield more reasonable left-hand elasticities (0.45 and 1.08 for VRS and CRS considerations, respectively).

In section 5.6, we also introduce the notion of "*returns to changing set A*" (RTA) in partial elasticity measurement analogous to returns-to-scale concept. A partial elasticity measure can be viewed as the quantitative measure of the strength of the "returns to changing set *A*" observed at the unit (X_0, Y_0), which satisfies the selective radial efficiency with respect to set *B*. Depending on whether both one-sided elasticities measured for the unit (X_0, Y_0) under an arbitrary scenario for sets *A* and *B* are less or greater than 1, the efficient frontier exhibits decreasing or increasing returns to changing set *A* at the unit (X_0, Y_0), respectively. When the range defined by one-sided elasticities contains 1, the frontier is thought to be exhibiting constant returns to changing set *A*, at the unit (X_0, Y_0) under the specified scenario.

It is stated in Section 5.6 of Chapter 5 that when new trade-offs are added to a technology (or the existing trade-offs are tightened), returns to changing set *A* classification for a unit remains the same in the expanded technology, if output bundle Y_0^B exhibits decreasing or increasing returns with respect to the change of mixed bundle (X_0^A, Y_0^A) in original technology. If at a unit, the frontier exhibits constant returns to changing set *A* in the original technology, then it is possible for the returns to changing set *A* characterisation to change to either the decreasing or increasing returns. Of course above statements are valid if the unit still holds the selective radial efficiency assumption in the expanded technology.

Results reveal that a majority of farms in all regions exhibit constant returns to changing set *A* (RTA). In our case, set *A* consists of cost and labour inputs. When production trade-offs incorporated into the models, usually the constant RTA characterisation is preserved. However, there are a number of farms from different regions changing RTA from constant to decreasing with addition of the trade-offs to the elasticity measurement models. Examples of such farms are provided in Table 8.3 below.

Table 8.3. Farms Changing RTA from Constant to Decreasing when Trade-offs Added

Region-Farm	**Responding Set**	**No TO**		**Broad TO**	
		LHE	**RHE**	**LHE**	**RHE**
WM-27	Cereals	2.45	0	0.23	0.01
WM-19	Field Crops	1.57	0	0.09	0
EM-22	Cereals	2.12	0	0.74	0
WA-19	Cereals	1.33	0	0.68	0.03
WA-14	Field Crops	1.57	0	0.60	0.10
MED-7	Cereals	1.01	0	0.46	0
SEA-1	Cereals	3.20	0	0.96	0.07
SEA-26	Cereals	1.04	0	0.67	0

Table 8.3 is basically extracted from VRS results parts of the elasticity tables presented in Appendix C. The responding set information is also provided to identify the elasticity scenario under which those results are obtained. All the farms Table 8.3 exhibit constant RTA in the elasticity measurement without trade-offs and change to decreasing RTA when the broadest trade-offs are incorporated into models.

Changes from constant RTA to decreasing RTA, are also observed in transition from broad trade-offs to tighter trade-offs. Some examples are provided in Table 8.4, which is also extracted from result tables in Appendix C. Farms presented in Table 8.4 exhibit constant RTA according to the results of the VRS elasticity model with broad trade-offs. They change RTA characterisation from constant to decreasing when the medium trade-offs are incorporated.

Table 8.4. Farms Changing RTA from Constant to Decreasing when Trade-offs Tightened

Region-Farm	Responding Set	Broad TO		Medium TO	
		LHE	RHE	LHE	RHE
WM-32	Cereals	1.41	0	0.94	0
WM-16	Field Crops	1.06	0	0.46	0
EM-10	Field Crops	5.98	0	0.91	0
WA-22	Field Crops	1.12	0.27	0.96	0.44
WA-31	Field Crops	1.24	0.20	0.89	0.33
WA-33	Field Crops	1.03	0	0.95	0
MED-7	Cereals	2.75	0	0.84	0
MA-1	Cereals	1.33	0.10	0.99	0.65

Above results verify the statement that if the unit is exhibiting constant RTA, then it is possible for the RTA characterisation to change to either the decreasing or increasing returns when trade-offs are added to elasticity measurement or the existing trade-offs are tightened. In our case to change is always to decreasing RTA. It is also stated that the RTA characterisation is preserved when trade-offs are added or tightened, if the unit exhibits increasing or decreasing RTA in the original technology. It is also verified by our elasticity results. Not many units exhibit increasing or decreasing RTA in the original models without trade-offs, but for the ones that exhibit, the RTA characterisation remains the same, when trade-offs are incorporated or tightened provided that the unit still holds the selective radial efficiency assumption. Examples can be given as farms 14 and 19 in West Marmara (Table C.10) and farm 7 in South East Anatolia (Table C.23) for the decreasing RTA or farm 16 in West Anatolia (Table C.15) for increasing RTA.

Chapter 9

Summary and Conclusions

9.1. Summary of the Objectives and the Scope of the Research

This research aims to contribute to the field of Operational Research, specifically, to the methodology and theory of Data Envelopment Analysis (DEA). As a well-established linear programming based relative efficiency evaluation technique, DEA is widely applied in the public and private sectors of business environment all over the world. Since the introduction of the technique, the performance measurement in agricultural production is one of the key issues of DEA scholars have been dealing with. This research is motivated based upon the non-homogeneity of the production in the evaluated units (mostly farms within the agriculture context). In evaluating agricultural establishments, non-homogeneous production is one of the common cases, which is faced in many real world applications. Even though the units evaluated are located in the same agricultural regions, which induce the non-homogeneity in terms of environmental factors to minimum, type of production can still exhibit a high level of variety since the production of different crop types is possible in a single agricultural area. DEA is an appropriate technique to handle when there are multiple inputs and outputs in the evaluation context. However, it is a well-known fact that the incorporation of too many variables can lead the DEA models to discriminate the efficiency scores insufficiently. Therefore, it is essential to be cautious when the variables are identified when the production is non-homogeneous.

In the case of non-homogeneous agricultural production, selection of outputs is challenging since it is not possible to ignore some of the outputs produced by the minority of the farms. The resources are devoted to the production of all given multiple outputs and excluding the ones, which are not produced by many of others, will result in an unrealistic and incomplete evaluations. One way to deal with non-homogeneity is to define the output side in terms of aggregated monetary equivalents of the agricultural production. Under such a consideration all the agricultural production realised by a farm can be captured in terms of revenues. However, the major drawback, which can occur with this approach, is due to the prices of different agricultural products. Because the prices fluctuate and they may depend on other economic and political factors, producers of some specific types of crops, which are highly priced in the market, will experience high output levels even though their production may not be relatively efficient.

Relying on the drawback of using monetary equivalents, a prominent way to evaluate the production efficiency of agricultural establishments can be the identification of each type of production as separate outputs in terms of physical production amounts. Yet, the possibility of insufficient discrimination of efficiency scores still remains as an issue to overcome. If the samples dealt are not large enough, the inclusion of too many outputs into the DEA models will result in a very insufficient discrimination where nearly all farms will be obtained as efficient. Nevertheless, due to the weight flexibility of DEA models, very large weights will be attached to the producers of very rare crops. This can lead to overestimation of efficiency for many of the farms.

Considering above discussions, in this research, we aim to establish a novel methodology to agricultural efficiency evaluation, which enables us to use every kind of agricultural production realized by the farms as separate outputs, but at the same time, avoid the potential insufficient discrimination. The motivation to overcome the discrimination problem lies behind incorporating more information to the DEA models reflecting the nature of the technology better. This can be achieved through the integration of production trade-offs concept introduced by Podinovski (2004a) to the agricultural context. Production trade-offs can be briefly defined as *'technological judgements representing possible simultaneous changes in the inputs and outputs under the technology considered'*. They can be represented as weight restrictions in DEA models, but they are different from weight restrictions based on value judgements in a sense that technological meaning of the efficiency measures is preserved.

The first of the main objectives of the research is to bring the use of production trade-offs to agricultural efficiency evaluation context. We propose a novel use of production trade-offs, where the relationships between different types of agricultural production is set up based on a crop which is produced by all the farms in the data set. This will provide us restrictions on the weights attached to the outputs and therefore avoid the overestimation of efficiency scores since additional constraints will be present in the linear programming models of DEA. Because the relations defined are technologically achievable, the production possibility set will be reshaped relying on the new constraints added.

One of the remarkable issues in the recent DEA literature is the elasticity measurement on DEA frontiers. Since the DEA frontiers are not defined in functional forms as in the classical economic theory, obtaining elasticity measures on DEA frontiers require different considerations. In the early literature, the investigations were mostly on the identification of qualitative nature through returns-to-scale (RTS) studies. However, recently, the interest has shifted towards quantifying the effects of relative changes in outputs compared to the relative changes in inputs or vice versa. The economic notion of elasticity is adapted to the DEA methodology and it strengthened the contact with the field of Economics. We consider this contemporary issue of elasticity measurement on DEA frontier and bring out the question of how elasticities can be calculated when production trade-offs are present in the technology as in our proposed methodology of efficiency measurement.

Since the introduction of the trade-off relations between outputs causes changes in the production possibility set considered and the efficient frontier obtained, theoretical modifications in the existing elasticity measurement models are required. We devoted the second main objective of the research to the elasticity measurement on DEA frontiers when production trade-offs are present. However, before moving into that, there exists a gap to be fulfilled in the elasticity measurement literature of DEA. Recently, partial elasticity measurement models which enable us to calculate elasticities of response for input or output sets to the changes in any subset of input and outputs have been developed just for variable returns-to-scale (VRS) technologies by Podinovski and Førsund (2010). For the theoretical completeness of discussions for partial elasticity measurement with production trade-offs, we first handle the issue of elasticity measurement under constant returns-to-scale (CRS) technologies. This establishes our third main objective in this research.

We answer three main research questions listed in the introduction, which are in accordance with the three main objectives summarised above. We illustrate the proposed methodology for efficiency measurement and the developed theory on elasticity measurement through empirical applications to a sample of farms Turkish Farmer Accountancy Data Network (FADN) that is obtained from Turkish Ministry of Agriculture.

The scope of the research is structured as follows: Before moving to theoretical developments on elasticity measurement and the empirical work following them, we provide two comprehensive reviews on the existing DEA literature. Chapter 2 reviews the theoretical foundation and the key considerations of DEA literature, which are closely related to the issues dealt by this research. In Chapter 3, we look at the efficiency evaluation literature in agriculture and identify the main characteristics and methodological considerations pursued in the previous research.

Following two extensive reviews, we begin the theoretical developments with the elasticity measurement in CRS technologies, which is listed as our third main objective. Necessary theoretical developments of elasticity measurement under CRS technologies are pursued in Chapter 4. The proofs for theorems established in this chapter are given in Appendix A. In the scope of this chapter, progressing upon previous discussions on VRS technologies by

Podinovski and Førsund (2010) and the measures developed for CRS technologies, we introduce a generalized framework of calculating mixed partial elasticities on DEA frontiers and interpreting the results, which is applicable to both VRS and CRS technologies. Then, we extend this framework to the cases where production trade-off relations between inputs and outputs in Chapter 5. The proofs for theorems in Chapter 5 are provided in Appendix B.

Finally, Chapters 6, 7 and 8 present the empirical applications conducted in a real world evaluation case of Turkish commercial farms.

In Chapter 6, we provide comprehensive information about our data set and our model design. We explain the key characteristics of the Turkish FADN dataset and our sample extracted from it. Also, detailed information about input selection, identification of production trade-offs and design of our empirical applications in Chapters 7 and 8 are provided by Chapter 6.

Chapter 7 serves as a real world exercise for measuring elasticity of responses on DEA frontiers under different scenarios of changing and responding input and output sets. It includes various illustrative examples pursued in one region data of the whole data set. It aims to demonstrate the applicability of different elasticity measures developed in previous chapters under both VRS and CRS technologies with or without trade-offs are incorporated.

In Chapter 8, we extend our application scope to cover the entire FADN sample we identified in Chapter 6. We introduce different ranges of trade-offs into models in order to observe the effect of changing trade-offs on efficiency and elasticity measures. We pursue two scenarios of elasticity measures for output sets throughout the chapter and interpret the results. All calculations are performed under both VRS and CRS considerations. The result tables for this chapter are given in Appendix C.

9.2. Conclusions of the Research

Throughout this research, we establish several methodological and theoretical contributions to the fields of DEA and agricultural efficiency evaluation. The key contributions and conclusions can be summarised as follows:

- A comprehensive review on agricultural efficiency evaluation studies applying Data envelopment Analysis (DEA) reveals that common practice in dealing with non-homogeneous production of farms is to consider the outputs in monetary terms such as revenues obtained by the farms, especially when the farms are producing both crops and livestock. On the other hand, there are also studies, where the physical production amounts of crops are taken as separate outputs. However, in such cases, the production ranges are not very diversified. When the agricultural production is non-homogeneous to a great extent, insufficient discrimination can be observed.

- We propose a novel methodology for agricultural efficiency evaluation with DEA, which can overcome the insufficient discrimination of efficiency scores when the production is highly non-homogeneous. We achieve such a novelty through bringing production trade-offs concept into agricultural efficiency measurement context, which will avoid the assignment of unrealistic weights to some outputs, which are not produced by majority of other farms while the technological meaning of efficiency is still preserved. We also consider the elasticity measurement and extend our proposed methodology to calculations of elasticities of response on DEA frontiers.

- We suggest a novel use of production trade-offs, where relationships between different types of crop production are set up based upon a base crop produced by all farms. These relationships are then translated into weight restrictions and integrated into multiplier DEA models. We identify the production trade-offs through expert opinions in the agricultural sectors in three ranges more robust to more flexible way, which enables us to observe the effects of changing trade-offs on our results of efficiency and elasticity.

- We illustrate our proposed methodology in a real world case of Turkish agricultural sectors using the Turkish Farmer Accountancy Data Network (FADN) data set provided by Turkish Ministry of Agriculture. The research is the first academic study on the farm efficiency of FADN farms in Turkey. We provide extensive empirical applications covering all the proposed methodology and theory.

- We contribute to the elasticity of response measurement on DEA frontiers. We extend the earlier approach of Podinovski and Førsund (2010), which deals with variable returns-to-scale (VRS) technologies to the case of constant returns-to-scale (CRS) production technologies. We formulate the linear programs required for the computation of one-sided elasticity measures in Chapter 4 of the research. It is a theoretical contribution that is applicable to any real world problem, where constant returns-to-scale can be assumed.

- We prove an important result in elasticity measurement on DEA frontiers, valid in both VRS and CRS technologies, that the linear programs used for the calculation of elasticity measures can themselves be used to diagnose if the elasticity measure is correctly defined. The programs are formulated in such a way that if the elasticity measure is undefined at a particular unit, the programs become infeasible. This enables us to introduce generalizations of the possible solutions obtained from linear programs of elasticity measurement in both technologies. Such a contribution removes the need for a preliminary sorting of the units into those units where the elasticity measure applies and those where it does not.

- We identify some special cases in elasticity measurement that are applicable only in CRS technologies. These cases are verified through an empirical application in Chapter 7.

- We extend the theoretical framework for the standard VRS and CRS technologies to the cases of production trade-offs in Chapter 5. We derive the necessary linear programs for one-sided elasticity measurement when production trade-offs are present in both VRS and CRS technologies. This provides a theoretical contribution that is applicable to any real world problem, where production trade-offs can be used.

- We provide several illustrative examples of elasticity measurement in Chapter 7 of the research. It is shown that elasticity measures can be calculated for any scenario of changing and responding input and outputs sets on DEA frontiers with or without production trade-offs in both VRS and CRS technologies.

- We observe in the empirical applications that integration of production trade-offs, even with a broadest manner; provide a better discrimination of efficiency and more finite elasticity measures.

- We conceptualise the effects of introducing or changing the production trade-offs in the existing production technology. It is shown both theoretically and empirically that in a production technology, when the new production trade-off relations are added or the existing ones are tightened, in other words, when more information about the technology is incorporated, the ranges for one-sided elasticity measures are getting tighter.

- We introduce the notion of *returns to changing set A (RTA)* that is the analogue of returns-to-scale (RTS) concept for the context of partial elasticity measurement. In the partial elasticity context, scale elasticity is a special case, where set *A* includes all the inputs and set *B* includes all the outputs. In the cases other than scale elasticity, RTS concept can be thought as returns "to changing set *A*" rather than "to scale", since set *A* and *B* may not include all the inputs and outputs. Therefore, a partial elasticity measure can be viewed as the quantitative measure of the strength of the returns to changing set *A* observed at the unit, which satisfies selective radial efficiency with respect to set *B*.

- We discuss the effects of changing trade-offs on the RTA characterisation of the units. We show and empirically verify that if the unit is exhibiting constant RTA in the original technology, then it is possible for the RTA characterisation to change to either the decreasing or increasing returns when trade-offs are added to the technology or the existing trade-offs in the technology are tightened. On the other hand, RTA characterisation is preserved when trade-offs are added or tightened, if the unit exhibits increasing or decreasing RTA in the original technology.

- Empirical application results reveal that majority of farms in all regions of our Turkish FADN sample exhibit constant returns to changing set *A* (RTA), where set *A* consists of cost and labour inputs and set *B* consists of cereals or field crops. When production trade-offs incorporated into the models, usually the constant RTA characterisation is preserved. However, there are a number of farms from different regions changing RTA from constant to decreasing with the incorporation of production trade-offs.

- One interesting result about the efficiency discriminations is that the improvement in the discrimination is more extensive from no trade-off model to model with broad trade-offs than broad trade-offs to tighter ones. Such an observation tells us, it is not very crucial to be too accurate in specifying the trade-offs. Even with the broadest range of relations considered, the discriminations of DEA models improve. A similar observation can be made for elasticity analysis with and without trade-offs. The changes observed in transition from broadest to tightest trade-offs are not that deep as in the transition from no-trade-offs to broadest trade-offs.

- The empirical application conducted in the research serves as a first application of partial elasticity measurement on DEA frontiers to a real world problem. We have also a novelty of applying the production trade-offs concept in DEA first time in a real world agriculture problem.

9.3. Further Research Directions

We can mention two possible further research directions related to this research. Firstly, as stated in Chapter 6, Turkish Ministry of Agriculture collected the FADN data set used in this research as a pilot study for initializing the development of a FADN in accordance with the European Union regulations. Therefore, it is a relatively small sample when the overall agricultural production in Turkey is considered. It serves perfectly for our illustration purposes, since our aim is not to draw very specific policy conclusions about the Turkish agriculture and we aim just to empirically test and illustrate our proposed methodology or theory. One further research direction can be extending the scope of application to a more extensive data set when the data collection efforts of the Ministry are widened to the whole country. Through collaboration with the authorities, a wider application can be conducted which can provide more generalizable conclusions about the efficiency of the agriculture in Turkey. This research can serve as a guide to larger scale applications, since the necessary theory is developed and an initial application is performed. In addition, elasticity measurement can be a tool for guiding the crop decisions since it enables us to calculate the responses for any inputs or outputs to the changes in any inputs and outputs. With a more large-scale and a more representative data set, the proposed methodology can help us the draw more policy conclusions.

The second possible direction is on the theory side. A research can be conducted to extend the elasticity measurement to hybrid returns-to-scale (HRS) technologies, where selective proportionality between inputs and outputs considered. In some cases of efficiency evaluation, only some of the inputs and outputs can be proportional to each other while the remaining ones are not part of this proportionality. As full proportionality assumption is not valid for that kind of cases the only option is to use VRS approach. However, by using VRS approach, the proportionality between some inputs and outputs would be ignored; so the model will not reflect the true scope of the feasible technology. To overcome this problem, Podinovski (2004b) develops a Hybrid Returns to Scale (HRS) approach where the DEA model is reformulated to handle the situations that include proportionality between some inputs and/or outputs and no proportionality between remaining. In this case CRS and VRS model become special cases of HRS models (Podinovski, 2004b). Elasticity measurement in such type of technologies can be an issue to investigate as a further theoretical research direction.

Appendix A

Proofs for Theorems in Chapter 4

Proof of Theorem 4.1. Because $(X_0, Y_0) \in T_{CRS}$, $\beta = 1$ is feasible in (4.3) with $\alpha = 1$. Suppose there is a feasible $\beta^* > 1$. Because at least one component of vector Y_0^B is strictly positive, the unit $(X_0^A, X_0^C, Y_0^A, \beta^* Y_0^B, Y_0^C) \in T_{CRS}$ dominates (X_0, Y_0), which contradicts the efficiency of the latter unit. □

Proof of Theorem 4.2. The proof of parts (a) and (b) follows closely the proof of part (a) of Proposition 1 in Podinovski and Førsund (2010). The function $\overline{\beta}(\alpha)$ defined in (4.3) is the optimal value in the linear program in (4.4).

The function $\overline{\beta}(\alpha)$ can be viewed as the function $\Phi(Z)$ of the vector $Z = (\alpha X_0^A, X_0^C, -\alpha Y_0^A, 0, -Y_0^C) \in R^{m+s}$ on the right-hand side of (4.4). As proved in Podinovski and Førsund (2010), $\beta'_+(1)$ is equal to the directional derivative of the function $\Phi(Z)$ taken at $\hat{Z} = (X_0^A, X_0^C, -Y_0^A, 0, -Y_0^C)$ in the direction $\hat{d} = (X_0^A, 0, -Y_0^A, 0, 0) \in R^{m+s}$ that is $\overline{\beta}'_+(1) = \Phi'(\hat{Z}; \hat{d})$. Similarly, we have $\overline{\beta}'_-(1) = -\Phi'(\hat{Z}; -\hat{d})$.

The proof is completed by the application of the theorem of marginal values in linear programming (see Theorem 2.2 in Shapiro 1979). Consider the dual to (4.4) with $\alpha = 1$:

$$\overline{\beta}(1) = \min \quad v^A X_0^A + v^C X_0^C - \mu^A Y_0^A - \mu^C Y_0^C \tag{A.1}$$

Subject to

$$v^A \overline{X}^A + v^C \overline{X}^C - \mu^A \overline{Y}^A - \mu^B \overline{Y}^B - \mu^C \overline{Y}^C \geq 0$$

$$\mu^B Y_0^B = 1$$

$v^A, v^C, \mu^A, \mu^B, \mu^C \geq 0$

Let Ω be the set of optimal solutions to (A.1). By the assumption of parts (a) and (b) of Theorem 4.2, (4.4) is feasible to the right (left) of $\alpha = 1$. Then, by the theorem of marginal values, the directional derivatives of Φ at $\hat{Z}$ exist, $\Phi'(\hat{Z};\hat{d}) = \min\{\omega\hat{d} \mid \omega \in \Omega\}$ and $\Phi'(\hat{Z};-\hat{d}) = -\max\{\omega\hat{d} \mid \omega \in \Omega\}$. This leads to programs (4.7) and (4.8) which include the constraints of (A.1) and the condition (4.7.2) that equates the objective function in (A.1) to its optimal value of 1. □

Part (c) of Theorem 4.2 is proved in Lemmas A.1 and A.2 below.

Lemma A.1. *If a proportional marginal increase of vectors X_0^A and Y_0^A is not feasible in T_{CRS}, the objective function in (4.7) is unbounded.*

Proof of Lemma A.1. Under the conditions of Lemma A.1, in other words, to have the proportional marginal increase of vectors X_0^A and Y_0^A as infeasible, the set *C* must include at least one non-zero input ($X_0^C \neq 0$). Because if not, it means that all the inputs are in set *A* and this contradicts with part (b) of Theorem 4.5 which states that when all inputs are in set *A*, right hand elasticity exists therefore marginal increase is feasible.

Because the optimal value of (4.4) and its dual (A.1) is $\overline{\beta}(1) = 1$, the program (4.7) is feasible. Suppose that, contrary to the statement of part (c) of Theorem 4.2, program (4.7) has a finite optimal solution. Then the dual to (4.7) is feasible:

Max $\beta + \delta$ (A.2.1)

Subject to

$$\bar{X}^A\lambda + \delta X_0^A \le X_0^A \tag{A.2.2}$$

$$\bar{X}^C\lambda + \delta X_0^C \le 0 \tag{A.2.3}$$

$$\bar{Y}^A\lambda + \delta Y_0^A \ge Y_0^A \tag{A.2.4}$$

$$\bar{Y}^B\lambda - \beta Y_0^B \ge 0 \tag{A.2.5}$$

$$\bar{Y}^C\lambda + \delta Y_0^C \ge 0 \tag{A.2.6}$$

$$\lambda \ge 0;\ \beta,\ \delta \text{ sign free} \tag{A.2.7}$$

Because $X_0^C \ne 0$, (A.2.3) implies that $\delta \le 0$.

Suppose that $\delta < 0$. By dividing the constraints of (A.2) by $-\delta > 0$ and rearranging the terms, (A.3) below is obtained:

$$\bar{X}^A\tilde{\lambda} \le \tilde{\alpha} X_0^A \tag{A.3}$$

$$\bar{X}^C\tilde{\lambda} \le X_0^C$$

$$\bar{Y}^A\tilde{\lambda} \ge \tilde{\alpha} Y_0^A$$

$$\bar{Y}^B\tilde{\lambda} \ge \tilde{\beta} Y_0^B$$

$$\bar{Y}^C\tilde{\lambda} \ge Y_0^C$$

where $\tilde{\lambda} = -\lambda/\delta \ge 0$, $\tilde{\alpha} = 1 - 1/\delta > 1$ and $\tilde{\beta} = -\beta/\delta$. If $\tilde{\beta} < 0$, we redefine $\tilde{\beta}$ by changing it to zero. This still satisfies the second last inequality in (A.3), because $\bar{Y}^B\tilde{\lambda} \ge 0$.

Suppose that $\delta = 0$. Then we have $\bar{X}^A\lambda \le X_0^A$, $\bar{X}^C\lambda \le 0$, $\bar{Y}^A\lambda \ge Y_0^A$, $\bar{Y}^B\lambda \ge \beta Y_0^B$, $\bar{Y}^C\lambda \ge 0$. Because $(X_0, Y_0) \in T_{CRS}$, there exists a vector $\lambda^* \ge 0$ such that $\bar{X}^A\lambda^* \le X_0^A$, $\bar{X}^C\lambda^* \le X_0^C$, $\bar{Y}^A\lambda^* \ge Y_0^A$, $\bar{Y}^B\lambda^* \ge Y_0^B$, $\bar{Y}^C\lambda^* \ge Y_0^C$. Add the corresponding constraints together and denote

$\tilde{\lambda} = \lambda + \lambda^*$. Then the vector $\tilde{\lambda} \geq 0$ and scalar β satisfy the following inequalities: $\bar{X}^A \tilde{\lambda} \leq 2X_0^A$, $\bar{X}^C \tilde{\lambda} \leq X_0^C$, $\bar{Y}^A \tilde{\lambda} \geq 2Y_0^A$, $\bar{Y}^B \tilde{\lambda} \geq (1+\beta)Y_0^B$, $\bar{Y}^C \tilde{\lambda} \geq Y_0^C$. Define $\tilde{\alpha} = 2$. Also, if $1+\beta \geq 0$ define $\tilde{\beta} = 1+\beta$, otherwise define $\tilde{\beta} = 0$. Then the vector $\tilde{\lambda} \geq 0$ and scalars $\tilde{\alpha} > 1$ and $\tilde{\beta} \geq 0$ satisfy (A.3).

Inequalities (A.3) mean that in both cases, $\delta < 0$ and $\delta = 0$, condition (4.2) holds for some $\tilde{\alpha} > 1$ and $\tilde{\beta} \geq 0$. Because T_{CRS} is convex, for every $\alpha \in [1, \tilde{\alpha}]$ there exists a $\beta \geq 0$ such that (4.2) is true. This contradicts the assumption of Lemma A.1 and completes the proof □

Lemma A.2. *If a proportional marginal reduction of vectors X_0^A and Y_0^A is not feasible in T_{CRS}, the objective function in (4.8) is unbounded.*

Proof of Lemma A.2. In this case, to have the proportional marginal reduction of vectors X_0^A and Y_0^A as infeasible, the set *A* must include at least one non-zero input ($X_0^A \neq 0$). Because if not, all the inputs will be in set *C*. Suppose we have all the inputs in set *C*, in other words, all the inputs are kept constant. In this case, sets *A* and *B* will include only outputs. Because left-hand elasticity considers the left of $\alpha = 1$ (reduction of inputs or outputs), due to the fact that CRS technology satisfies free disposability assumption of inputs and outputs (this means that if the unit $(X, Y) \in T$, and we have $Y \geq \tilde{Y} \geq 0$ and $X \leq \tilde{X}$, then the unit $(\tilde{X}, \tilde{Y}) \in T$), it will be always possible to reduce the outputs and remain feasible.

Because $\bar{\beta}(1) = 1$, the program (4.8) is feasible. Suppose that contrary to the statement of part (c) of Theorem 4.2, (4.8) has a finite optimal solution. Then its dual is feasible:

Min $\beta + \delta$ (A.4.1)

Subject to

$\bar{X}^A \lambda - \delta X_0^A \leq -X_0^A$ (A.4.2)

$\bar{X}^C \lambda - \delta X_0^C \leq 0$ (A.4.3)

$\bar{Y}^A \lambda - \delta Y_0^A \geq -Y_0^A$ (A.4.4)

$\bar{Y}^B \lambda + \beta Y_0^B \geq 0$ (A.4.5)

$\bar{Y}^C \lambda - \delta Y_0^C \geq 0$ (A.4.6)

$\lambda \geq 0$; β, δ sign free (A.4.7)

Because $X_0^A \neq 0$, from (A.4.2) we have $\delta > 0$. Divide the constraints of program (A.4) by δ and define $\tilde{\lambda} = \lambda / \delta \geq 0$, $\tilde{\alpha} = 1 - 1/\delta < 1$. Because $\bar{X}^A \tilde{\lambda} \geq 0$ and $X_0^A \neq 0$, (A.4.2) implies $\alpha \geq 0$. Finally, if $-\beta / \delta \geq 0$, define $\tilde{\beta} = -\beta / \delta$, otherwise let $\tilde{\beta} = 0$. In either case all the inequalities in (A.3) are satisfied. This means that condition (4.2) holds for some $\tilde{\alpha} \in [0,1)$ and $\tilde{\beta} \geq 0$. Because T_{CRS} is convex, for every $\alpha \in [\tilde{\alpha}, 1]$ there exists a $\beta \geq 0$ such that (4.2) is true. This contradicts the assumption of Lemma A.2 and completes the proof. □

Proof of Theorem 4.3. Assume that Assumption 4.1 is not true. If $\bar{\beta}(\alpha)$ is unbounded at $\alpha = 1$, (4.4) has an unbounded solution, and its dual (A.1) is infeasible. Then programs (4.7) and (4.8) that have an additional constraint (4.7.2) are also infeasible. If $\bar{\beta}(\alpha)$ is finite at $\alpha = 1$ but $\bar{\beta}(1) \neq 1$, then $\bar{\beta}(1) > 1$. The optimum value in the dual (A.1) is also equal to $\bar{\beta}(1) > 1$. Therefore, (4.7.2) is inconsistent with the other constraints of (4.7) and (4.8), and programs (4.7) and (4.8) are infeasible. Conversely, let Assumption 4.1 be true. Then the conditions of Theorem 4.2 are satisfied and the infeasibility of programs (4.7) and (4.8) is impossible. □

Proof of Theorem 4.4. If $C = \varnothing$, the terms X_0^C and Y_0^C in (4.2) are omitted. According to the assumption of CRS, condition (4.2) is true for any $\alpha \geq 0$ and $\beta = \alpha$. This means that both proportional marginal increase and reduction of the vectors X_0^A and Y_0^A are feasible. By Theorem 4.2, the corresponding one-sided elasticities exist and can be calculated by solving programs (4.7) and (4.8), respectively. Because $C = \varnothing$, the equality (4.7.2) coincides with their objective functions. Therefore, the optimal value of both programs is equal to 1. □

Proof of Theorem 4.5. In the proof of this theorem we use the fact that the CRS technology T satisfies the assumption of free (strong) disposability of input and outputs. This means that, if the unit $(X,Y) \in T$, and we have $Y \geq \tilde{Y} \geq 0$ and $X \leq \tilde{X}$, then the unit $(\tilde{X},\tilde{Y}) \in T$.

To prove part (a) of Theorem 4.5, note that condition (4.2) becomes $(\alpha X_0^A, X_0^C, \beta Y_0^B) \in T_{CRS}$. By the free disposability of inputs, for $\alpha > 1$ this condition is satisfied with $\beta = 1$, and for $0 \leq \alpha < 1$ this is satisfied by $\beta = \alpha$ (the resulting unit $(\alpha X_0^A, X_0^C, \alpha Y_0^B)$ is dominated by the unit $(\alpha X_0^A, \alpha X_0^C, \alpha Y_0^B) \in T_{CRS}$ and is therefore feasible). This means that both proportional marginal increase and reduction of the vectors X_0^A and Y_0^A are feasible. By Theorem 4.2, both one-sided elasticities (4.5) and (4.6) exist. Equality (4.7.2) becomes $v^A X_0^A + v^C X_0^C = 1$. Because $v^C X_0^C \geq 0$, in both programs (4.7) and (4.8) the objective function $v^A X_0^A = 1 - v^C X_0^C \leq 1$.

To prove part (b), note that condition (4.2) becomes $(\alpha X_0^A, \alpha Y_0^A, \beta Y_0^B, Y_0^C) \in T_{CRS}$. By free disposability of output, for any $\alpha > 1$ this is satisfied with $\beta = \alpha$. This means that a proportional marginal increase of the vectors X_0^A and Y_0^A is feasible. By Theorem 4.2, the right-hand elasticity exists and can be calculated by program (4.7). Noting that the term $v^C X_0^C$ is omitted from the program, the objective function (4.7.1) can be evaluated using (4.7.2): $v^A X_0^A = 1 + \mu^A Y_0^A + \mu^C Y_0^C \geq 1$.

To prove part (c), note that condition (4.2) becomes $(X_0^C, \alpha Y_0^A, \beta Y_0^B, Y_0^C) \in T_{CRS}$. By the free disposability of outputs, for any $0 \leq \alpha < 1$ this is satisfied by $\beta = 1$. This means that a proportional marginal reduction of the vector Y_0^A is feasible. By part (b) of Theorem 4.2, the left-hand elasticity exists and can be calculated by program (4.8) in which the terms $v^A X_0^A$ are omitted. The objective function (4.8) becomes $-\mu^A Y_0^A \leq 0$. □

Proof of Theorem 4.6. Theorem 4.6 can be proven with a very similar way to Theorem 4.1. $\beta = 1$ is feasible in (4.9) with $\alpha = 1$ since $(X_0, Y_0) \in T_{CRS}$. Suppose that there is a feasible $\beta^* < 1$. In this case, the unit $(X_0^A, \beta^* X_0^B, X_0^C, Y_0^A, Y_0^C) \in T_{CRS}$ dominates (X_0, Y_0) since at least one component of vector X_0^B is strictly positive. It contradicts the efficiency of the unit (X_0, Y_0). □

Proof of Theorem 4.7. The proof of parts (a) and (b) follows closely the proof of part (a) of Proposition 1 in Podinovski and Førsund (2010) and the proof of Theorem 4.2. In this case, the function $\overline{\beta}(\alpha)$ defined in (4.9) is the optimal value in the linear program in (4.10).

Similar to the output case, the function $\hat{\beta}(\alpha)$ can be viewed as the function $\Phi(Z)$ of the vector $Z = (-\alpha X_0^A, 0, -X_0^C, \alpha Y_0^A, Y_0^C) \in R^{m+s}$ on the right-hand side of (4.10). As proved in Podinovski and Førsund (2010), $\beta'_+(1)$ is equal to the directional derivative of the function

$\Phi(Z)$ taken at $\hat{Z} = (-X_0^A, 0, -X_0^C, Y_0^A, Y_0^C)$ in the direction $\hat{d} = (-X_0^A, 0, Y_0^A, 0, 0) \in R^{m+s}$, that is $\hat{\beta}'_+(1) = \Phi'(\hat{Z}; \hat{d})$. Similarly, we have $\hat{\beta}'_-(1) = -\Phi'(\hat{Z}; -\hat{d})$.

The proof is again completed by the application of the theorem of marginal values in linear programming (see Theorem 2.2 in Shapiro 1979). Consider the dual to (4.10) with $\alpha = 1$:

$$\hat{\beta}(1) = \max \quad -v^A X_0^A - v^C X_0^C + \mu^A Y_0^A + \mu^C Y_0^C \tag{A.5}$$

Subject to

$$-v^A \overline{X}^A - v^C \overline{X}^C + \mu^A \overline{Y}^A + \mu^B \overline{Y}^B + \mu^C \overline{Y}^C \leq 0$$

$$v^B X_0^B = 1$$

$$v^A, v^B, v^C, \mu^A, \mu^C \geq 0$$

Let Δ be the set of optimal solutions to (A.5). By the assumption of parts (a) and (b) of Theorem 4.7, (4.10) is feasible to the right (left) of $\alpha = 1$. Then, by the theorem of marginal values, the directional derivatives of Φ at $\hat{Z}$ exist, $\Phi'(\hat{Z}; \hat{d}) = \min\{\omega \hat{d} \mid \omega \in \Delta\}$ and $\Phi'(\hat{Z}; -\hat{d}) = -\max\{\omega \hat{d} \mid \omega \in \Delta\}$. This leads to programs (4.13) and (4.14) which include the constraints of (A.5) and the condition (4.13.2) that equates the objective function in (A.5) to its optimal value of 1. □

Part (c) of Theorem 4.7 is proved in Lemmas A.3 and A.4 below.

Lemma A.3. *If a proportional marginal increase of vectors X_0^A and Y_0^A is not feasible in T_{CRS}, the objective function in (4.13) is unbounded.*

Proof of Lemma A.3. Under the conditions of Lemma A.3, the set *C* must include at least one non-zero input ($X_0^C \neq 0$). Because if not, due to the free disposability and proportionality assumption of CRS models, marginal increase of vectors X_0^A and Y_0^A will always be feasible in T_{CRS}.

Because the optimal value of (4.10) and its dual (A.5) is $\hat{\beta}(1) = 1$, the program (4.13) is feasible. Suppose that, contrary to the statement of part (c) of Theorem 4.7, program (4.13) has a finite optimal solution. Then the dual to (4.13) is feasible:

Min $\beta + \delta$ (A.6.1)

Subject to

$\bar{X}^A \lambda + \delta X_0^A \leq X_0^A$ (A.6.2)

$\bar{X}^B \lambda - \beta X_0^B \leq 0$ (A.6.3)

$\bar{X}^C \lambda + \delta X_0^C \leq 0$ (A.6.4)

$\bar{Y}^A \lambda + \delta Y_0^A \geq Y_0^A$ (A.6.5)

$\bar{Y}^C \lambda + \delta Y_0^C \geq 0$ (A.6.6)

$\lambda \geq 0$; β, δ sign free (A.6.7)

Because $X_0^C \neq 0$, (A.6.3) implies that $\delta \leq 0$.

Suppose that $\delta < 0$. By dividing the constraints of (A.2) by $-\delta > 0$ and rearranging the terms, the constraint set in (A.7) is obtained:

$\bar{X}^A \tilde{\lambda} \leq \tilde{\alpha} X_0^A$ (A.7)

$\bar{X}^B \tilde{\lambda} \leq \tilde{\beta} X_0^B$

$\bar{X}^C\tilde{\lambda} \le X_0^C$

$\bar{Y}^A\tilde{\lambda} \ge \alpha Y_0^A$

$\bar{Y}^C\tilde{\lambda} \ge Y_0^C$

where we denote $\tilde{\lambda} = -\lambda/\delta \ge 0$, $\alpha = 1 - 1/\delta > 1$ and $\tilde{\beta} = \beta/\delta$.

Suppose that $\delta = 0$. Then, we have $\bar{X}^A\lambda \le X_0^A$, $\bar{X}^B\lambda \le \beta X_0^B$, $\bar{X}^C\lambda \le 0$, $\bar{Y}^A\lambda \ge Y_0^A$, $\bar{Y}^C\lambda \ge 0$ Because $(X_0, Y_0) \in T_{CRS}$, there exists a vector $\lambda^* \ge 0$ such that $\bar{X}^A\lambda^* \le X_0^A$, $\bar{X}^B\lambda^* \le X_0^B$, $\bar{X}^C\lambda^* \le X_0^C$ $\bar{Y}^A\lambda^* \ge Y_0^A$, $\bar{Y}^C\lambda^* \ge Y_0^C$. Add the corresponding constraints together and denote $\tilde{\lambda} = \lambda + \lambda^*$. Then the vector $\tilde{\lambda} \ge 0$ and scalar β satisfy the following inequalities: $\bar{X}^A\tilde{\lambda} \le 2X_0^A$, $\bar{X}^B\tilde{\lambda} \le (1+\beta)X_0^B$, $\bar{X}^C\tilde{\lambda} \le X_0^C$, $\bar{Y}^A\tilde{\lambda} \ge 2Y_0^A$, $\bar{Y}^C\tilde{\lambda} \ge Y_0^C$. Define $\tilde{\alpha} = 2$. Also, as $1+\beta \ge 0$ define $\tilde{\beta} = 1+\beta$. Then the vector $\tilde{\lambda} \ge 0$ and scalars $\tilde{\alpha} > 1$ and $\tilde{\beta} \ge 0$ satisfy (A.7).

Inequalities (A.7) mean that in both cases, $\delta < 0$ and $\delta = 0$, condition (4.2) holds for some $\tilde{\alpha} > 1$ and $\tilde{\beta} \ge 0$. Because T_{CRS} is convex, for every $\alpha \in [1, \tilde{\alpha}]$ there exists a $\beta \ge 0$ such that (4.2) is true. This contradicts the assumption of Lemma A.3 and completes the proof. □

Lemma A.4. *If a proportional marginal reduction of vectors X_0^A and Y_0^A is not feasible in T_{CRS}, the objective function in (4.14) is unbounded.*

Proof of Lemma A.4. In this case, to have marginal reduction as infeasible, the set *A* must include at least one non-zero input ($X_0^A \ne 0$). Because if not, when the set *A* does not include any input, since inputs and outputs are assumed to be proportional to each other in CRS technology, proportional marginal reduction of vector Y_0^A will always remain feasible.

Because $\hat{\beta}(1)=1$, the program (4.14) is feasible. Suppose that (4.14) has a finite optimal solution. Then its dual is feasible:

$$\text{Max } \beta+\delta \tag{A.8.1}$$

Subject to

$$\bar{X}^A\lambda-\delta X_0^A \le -X_0^A \tag{A.8.2}$$

$$\bar{X}^B\lambda+\beta Y_0^B \le 0 \tag{A.8.3}$$

$$\bar{X}^C\lambda-\delta X_0^C \le 0 \tag{A.8.4}$$

$$\bar{Y}^A\lambda-\delta Y_0^A \ge -Y_0^A \tag{A.8.5}$$

$$\bar{Y}^C\lambda-\delta Y_0^C \ge 0 \tag{A.8.6}$$

$$\lambda \ge 0;\ \beta,\ \delta \text{ sign free} \tag{A.8.7}$$

Because $X_0^A \ne 0$, from (A.8.2) we have $\delta > 0$. Divide the constraints of program (A.8) by δ and define $\tilde{\lambda}=\lambda/\delta \ge 0$, $\tilde{\alpha}=1-1/\delta<1$. Because $\bar{X}^A\tilde{\lambda} \ge 0$ and $X_0^A \ne 0$, (A.8.2) implies $\alpha \ge 0$. Finally, if $-\beta/\delta \ge 0$, define $\tilde{\beta}=-\beta/\delta$, otherwise let $\tilde{\beta}=0$. In either case all the inequalities in (A.7) are satisfied. This means that condition (4.2) holds for some $\tilde{\alpha}\in[0,1)$ and $\tilde{\beta}\ge 0$. Because T_{CRS} is convex, for every $\alpha\in[\tilde{\alpha},1]$ there exists a $\beta \ge 0$ such that (4.2) is true. This contradicts the assumption of Lemma A.4 and completes the proof. □

Proof of Theorem 4.8. Theorem 4.8 can be proven with a similar way to Theorem 4.3. Assume that Assumption 4.2 is not true. If $\hat{\beta}(\alpha)$ is unbounded at $\alpha=1$, (4.10) has an unbounded solution, and its dual (A.5) is infeasible. Then programs (4.13) and (4.14) that have an additional constraint (4.13.2) are also infeasible. If $\hat{\beta}(\alpha)$ is finite at $\alpha = 1$ but $\hat{\beta}(1) \neq 1$, then $\hat{\beta}(1) < 1$. The optimum value in the dual (A.5) is also equal to $\hat{\beta}(1) < 1$. Therefore, (4.13.2) is inconsistent with the other constraints of (4.13) and (4.14), and programs (4.13) and (4.14) are infeasible. Conversely, let Assumption 4.2 be true. Then the conditions of Theorem 4.7 are satisfied and the infeasibility of programs (4.13) and (4.14) is impossible. □

Proof of Theorem 4.9. Theorem 4.9 is the analogue of Theorem 4.4 and proven in a same way. If $C = \varnothing$, the terms X_0^C and Y_0^C in (4.2) are omitted. According to the assumption of CRS, condition (4.2) is true for any $\alpha \geq 0$ and $\beta = \alpha$. This means that both proportional marginal increase and reduction of the vectors X_0^A and Y_0^A are feasible. By Theorem 4.7, the corresponding one-sided elasticities exist and can be calculated by solving programs (4.13) and (4.14), respectively. Because $C = \varnothing$, the equality (4.13.2) coincides with their objective functions. Therefore, the optimal value of both programs is equal to 1. □

Appendix B

Proofs for Theorems in Chapter 5

Proof of Theorem 5.1. Because $(X_0, Y_0) \in T_{CRS}$, $\beta = 1$ is feasible in (5.1) with $\alpha = 1$. Suppose there is a feasible $\beta^* > 1$. Because at least one component of vector Y_0^B is strictly positive, the unit $(X_0^A, X_0^C, Y_0^A, \beta^* Y_0^B, Y_0^C) \in T_{CRSTO}$ dominates (X_0, Y_0), which contradicts the efficiency of the latter unit. □

Proof of Theorem 5.2. The proof of parts (a) and (b) of Theorem 5.2 is almost identical with the Theorem 4.2. In the application of theorem of marginal values of linear programming, the dual to (5.2) given below is considered.

$$\bar{\beta}(1) = \min \quad v^A X_0^A + v^C X_0^C - \mu^A Y_0^A - \mu^C Y_0^C \qquad \text{(B.1)}$$

Subject to

$$v^A \bar{X}^A + v^C \bar{X}^C - \mu^A \bar{Y}^A - \mu^B \bar{Y}^B - \mu^C \bar{Y}^C \geq 0$$

$$v^A P^A + v^C P^C - \mu^A Q^A - \mu^B Q^B - \mu^C Q^C \geq 0$$

$$\mu^B Y_0^B = 1$$

$$v^A, v^C, \mu^A, \mu^B, \mu^C \geq 0$$

The only difference of (5.3) and (5.4) than (4.7) and (4.8) is the constraint sets (5.3.4) and (5.4.4), which represent the production trade-offs. As the objective function $\bar{\beta}(\alpha)$ is the same, the directional derivatives of this function can be obtained as in the proof of Theorem 4.2. □

Part (c) of above Theorem 5.2 can be proved in Lemmas B.1 and B.2 below:

Lemma B.1. *If a proportional marginal increase of vectors* X_0^A *and* Y_0^A *is not feasible in* T_{CRSTO}*, the objective function in (5.3) is unbounded.*

Because the optimal value of (5.2) and its dual (B.1) is $\bar{\beta}(1)=1$, the program (5.3) is feasible. Suppose that, contrary to the statement of part (c) of Theorem 5.2, program (5.3) has a finite optimal solution. Then the dual to (5.3) is feasible:

$$\text{Max } \beta+\delta \tag{B.2}$$

Subject to

$$\bar{X}^A\lambda+\delta X_0^A+P^A\pi \leq X_0^A$$

$$\bar{X}^C\lambda+\delta X_0^C+P^C\pi \leq 0$$

$$\bar{Y}^A\lambda+\delta Y_0^A+Q^A\pi \geq Y_0^A$$

$$\bar{Y}^B\lambda-\beta Y_0^B+Q^B\pi \geq 0$$

$$\bar{Y}^C\lambda+\delta Y_0^C+Q^C\pi \geq 0$$

$\lambda,\pi \geq 0$; $\beta,\ \delta$ Sign free

Because $(X_0,Y_0)\in T_{CRSTO}$, there exists vectors $\lambda^*\geq 0$ and $\pi^*\geq 0$ such that

$$\bar{X}^A\lambda^*+P^A\pi^* \leq X_0^A \tag{B.3}$$

$$\bar{X}^C\lambda^*+P^C\pi^* \leq X_0^C$$

$$\bar{Y}^A\lambda^*+Q^A\pi^* \geq Y_0^A$$

$$\bar{Y}^B\lambda^*+Q^B\pi^* \geq Y_0^B$$

$$\bar{Y}^C\lambda^*+Q^C\pi^* \geq Y_0^C$$

If the constraints in (B.2) are multiplied by a very small positive number ($\gamma > 0$) and the corresponding constraints in (B.3) are added to them, the constraint set in (B.4) is obtained.

$$\bar{X}^A(\lambda^* + \gamma\lambda) + P^A(\pi^* + \gamma\pi) \le \gamma X_0^A + (1-\gamma\delta)X_0^A \tag{B.4}$$

$$\bar{X}^C(\lambda^* + \gamma\lambda) + P^C(\pi^* + \gamma\pi) \le (1-\gamma\delta)X_0^C$$

$$\bar{Y}^A(\lambda^* + \gamma\lambda) + Q^A(\pi^* + \gamma\pi) \ge \gamma Y_0^A + (1-\gamma\delta)Y_0^A$$

$$\bar{Y}^B(\lambda^* + \gamma\lambda) + Q^B(\pi^* + \gamma\pi) \ge (1+\gamma\beta)Y_0^B$$

$$\bar{Y}^C(\lambda^* + \gamma\lambda) + Q^C(\pi^* + \gamma\pi) \ge (1-\gamma\delta)Y_0^C$$

If the constraints in (B.4) are divided by $(1-\gamma\delta)$, the set of constraints in (B.5) are obtained.

$$\bar{X}^A\tilde{\lambda} + P^A\tilde{\pi} \le \tilde{\alpha}X_0^A \tag{B.5}$$

$$\bar{X}^C\tilde{\lambda} + P^C\tilde{\pi} \le X_0^C$$

$$\bar{Y}^A\tilde{\lambda} + Q^A\tilde{\pi} \ge \tilde{\alpha}Y_0^A$$

$$\bar{Y}^B\tilde{\lambda} + Q^B\tilde{\pi} \ge \tilde{\beta}Y_0^B$$

$$\bar{Y}^C\tilde{\lambda} + Q^C\tilde{\pi} \ge Y_0^C$$

where we donate $\tilde{\alpha} = 1+\gamma > 1$, $\tilde{\lambda} = (\lambda^* + \gamma\lambda)/(1-\gamma\delta) \ge 0$, $\tilde{\pi} = (\pi^* + \gamma\pi)/(1-\gamma\delta) \ge 0$ and $\tilde{\beta} = (1+\gamma\beta)/(1-\gamma\delta) \ge 0$.

Inequalities (B.5) mean that condition (4.2) holds for some $\tilde{\alpha} > 1$ and $\tilde{\beta} \ge 0$. Because T_{CRSTO} is convex, for every $\alpha \in [1, \tilde{\alpha}]$ there exists a $\beta \ge 0$ such that (4.2) is true. This contradicts the assumption of Lemma B.1 and completes the proof. □

Lemma B.2. *If a proportional marginal increase of vectors* X_0^A *and* Y_0^A *is not feasible in* T_{CRSTO}*, the objective function in (5.4) is unbounded.*

Because the optimal value of (5.2) and its dual (B.1) is $\bar{\beta}(1)=1$, the program (5.4) is feasible. Suppose that, contrary to the statement of part (c) of Theorem 5.2, program (5.4) has a finite optimal solution. Then the dual to (5.4) is feasible:

Min $\beta+\delta$ (B.6)

Subject to

$\bar{X}^A\lambda-\delta X_0^A+P^A\pi\leq -X_0^A$

$\bar{X}^C\lambda-\delta X_0^C+P^C\pi\leq 0$

$\bar{Y}^A\lambda-\delta Y_0^A+Q^A\pi\geq -Y_0^A$

$\bar{Y}^B\lambda+\beta Y_0^B+Q^B\pi\geq 0$

$\bar{Y}^C\lambda-\delta Y_0^C+Q^C\pi\geq 0$

$\lambda,\pi\geq 0$; β, δ sign free

Because $(X_0,Y_0)\in T_{CRS}$, there exist vectors $\lambda^*\geq 0$ and $\pi^*\geq 0$ as in (B.3). If constraints in (B.6) are multiplied by a very small positive number ($\gamma>0$) and the corresponding constraints in (B.3) are added to them, the constraint set in (B.7) is obtained.

$\bar{X}^A(\lambda^*+\gamma\lambda)+P^A(\pi^*+\gamma\pi)\leq -\gamma X_0^A+(1+\gamma\delta)X_0^A$ (B.7)

$\bar{X}^C(\lambda^*+\gamma\lambda)+P^C(\pi^*+\gamma\pi)\leq (1+\gamma\delta)X_0^C$

$\bar{Y}^A(\lambda^*+\gamma\lambda)+Q^A(\pi^*+\gamma\pi)\geq -\gamma Y_0^A+(1+\gamma\delta)Y_0^A$

$\bar{Y}^B(\lambda^*+\gamma\lambda)+Q^B(\pi^*+\gamma\pi)\geq (1-\gamma\beta)Y_0^B$

$\bar{Y}^C(\lambda^*+\gamma\lambda)+Q^C(\pi^*+\gamma\pi)\geq (1+\gamma\delta)Y_0^C$

If the constraints in (B.7) are divided by $(1+\gamma\delta)$, the set of constraints in (B.5) is obtained, where we denote $\tilde{\alpha} = 1-\gamma < 1$, $\tilde{\lambda} = (\lambda^* + \gamma\lambda)/(1+\gamma\delta) \geq 0$, $\tilde{\pi} = (\pi^* + \gamma\pi)/(1+\gamma\delta) \geq 0$ and $\tilde{\beta} = (1-\gamma\beta)/(1+\gamma\delta) \geq 0$.

Inequalities (B.5) mean that condition (4.2) holds for some $\tilde{\alpha} < 1$ and $\tilde{\beta} \geq 0$. Because T_{CRSTO} is convex, for every $\alpha \in [\tilde{\alpha}, 1]$ there exists a $\beta \geq 0$ such that (4.2) is true. This contradicts the assumption of Lemma B.2 and completes the proof. □

Proof of Theorem 5.3. The proof of Theorem 5.3 is identical with the proof of Theorem 4.3. In this case programs (5.2), (5.3), (5.4) and (B.1) are considered instead of (4.4), (4.7), (4.8) and (A.1), respectively. □

Proof of Theorem 5.4. $\beta = 1$ is feasible in (5.5) with $\alpha = 1$ since $(X_0, Y_0) \in T_{CRSTO}$. Suppose that there is a feasible $\beta^* < 1$. In this case, the unit $(X_0^A, \beta^* X_0^B, X_0^C, Y_0^A, Y_0^C) \in T_{CRSTO}$ dominates (X_0, Y_0) since at least one component of vector X_0^B is strictly positive. It contradicts the efficiency of the unit (X_0, Y_0). □

Proof of Theorem 5.5. The proof of parts (a) and (b) of Theorem 5.5 is almost identical with the Theorem 4.7. In this case, when applying the theorem of marginal values of linear programming, the dual to (5.8) is considered. In the proof of part (c), the theorem is proven with a similar approach to proof of part (c) in Theorem 5.2. In this case, duals to (5.7) and (5.8) are considered for increase and reduction scenarios, respectively. □

Proof of Theorem 5.6. See Theorem 4.8 and its proof in Appendix A. □

Proof of Theorem 5.7. The proof of Theorem 5.7 is very similar to the Theorem 5.1. In this case VRS technology is considered instead of CRS technology. Since $(X_0, Y_0) \in T_{VRSTO}$, $\beta = 1$ is feasible in (5.11) with $\alpha = 1$. Suppose that there is a feasible $\beta^* > 1$. Because at least one component of vector Y_0^B is strictly positive, the unit $(X_0^A, X_0^C, Y_0^A, \beta^* Y_0^B, Y_0^C) \in T_{VRSTO}$ dominates (X_0, Y_0), which contradicts the efficiency of (X_0, Y_0). □

Proof of Theorem 5.8. The proof of parts (a) and (b) of Theorem 5.8 is almost identical with the Theorem 4.2 and the proof of part (a) in Proposition 1 in Podinovski and Førsund (2010). In the application of theorem of marginal values of linear programming, the dual to (5.12) given below is considered.

$$\overline{\beta}(1) = \min \quad v^A X_0^A + v^C X_0^C - \mu^A Y_0^A - \mu^C Y_0^C + \mu_0 \tag{B.8}$$

Subject to

$$v^A \overline{X}^A + v^C \overline{X}^C - \mu^A \overline{Y}^A - \mu^B \overline{Y}^B - \mu^C \overline{Y}^C + e\mu_0 \geq 0$$

$$v^A P^A + v^C P^C - \mu^A Q^A - \mu^B Q^B - \mu^C Q^C \geq 0$$

$$\mu^B Y_0^B = 1$$

$$v^A, v^C, \mu^A, \mu^B, \mu^C \geq 0$$

μ_0 Sign free

Part (c) of above Theorem 5.8 can be proved in Lemmas B.3 and B.4 below:

Lemma B.3. *If a proportional marginal increase of vectors X_0^A and Y_0^A is not feasible in T_{VRSTO}, the objective function in (5.13) is unbounded.*

Because the optimal value of (5.12) and its dual (B.8) is $\overline{\beta}(1)=1$, the program (5.13) is feasible. Suppose that, contrary to the statement of part (c) of Theorem 5.8, program (5.13) has a finite optimal solution. Then the dual to (5.13) is feasible:

$$\text{Max } \beta+\delta \tag{B.9}$$

Subject to

$$\overline{X}^{A}\lambda+\delta X_0^A+P^A\pi \le X_0^A$$

$$\overline{X}^{C}\lambda+\delta X_0^C+P^C\pi \le 0$$

$$\overline{Y}^{A}\lambda+\delta Y_0^A+Q^A\pi \ge Y_0^A$$

$$\overline{Y}^{B}\lambda-\beta Y_0^B+Q^B\pi \ge 0$$

$$\overline{Y}^{C}\lambda+\delta Y_0^C+Q^C\pi \ge 0$$

$$e\lambda+\delta=0$$

$\lambda,\pi \ge 0$; β, δ Sign free

Because $(X_0,Y_0)\in T_{VRS}$, there exists vectors $\lambda^* \ge 0$ and $\pi^* \ge 0$ such that

$$\overline{X}^{A}\lambda^*+P^A\pi^* \le X_0^A \tag{B.10}$$

$$\overline{X}^{C}\lambda^*+P^C\pi^* \le X_0^C$$

$$\overline{Y}^{A}\lambda^*+Q^A\pi^* \ge Y_0^A$$

$$\overline{Y}^{B}\lambda^*+Q^B\pi^* \ge Y_0^B$$

$$\overline{Y}^{C}\lambda^*+Q^C\pi^* \ge Y_0^C$$

$$e\lambda^*=1$$

If the constraints in (B.9) are multiplied by a very small positive number $\gamma > 0$ and the corresponding constraints in (B.10) are added to them, the constraint set in (B.11) is obtained.

$$\bar{X}^A(\lambda^* + \gamma\lambda) + P^A(\pi^* + \gamma\pi) \leq \gamma X_0^A + (1-\gamma\delta)X_0^A \quad \text{(B.11)}$$

$$\bar{X}^C(\lambda^* + \gamma\lambda) + P^C(\pi^* + \gamma\pi) \leq (1-\gamma\delta)X_0^C$$

$$\bar{Y}^A(\lambda^* + \gamma\lambda) + Q^A(\pi^* + \gamma\pi) \geq \gamma Y_0^A + (1-\gamma\delta)Y_0^A$$

$$\bar{Y}^B(\lambda^* + \gamma\lambda) + Q^B(\pi^* + \gamma\pi) \geq (1+\gamma\beta)Y_0^B$$

$$\bar{Y}^C(\lambda^* + \gamma\lambda) + Q^C(\pi^* + \gamma\pi) \geq (1-\gamma\delta)Y_0^C$$

$$e(\lambda^* + \gamma\lambda) + \gamma\delta = 1$$

If we divide the constraints in (B.11) by $(1-\gamma\delta)$, the set of constraints in (B.12) are obtained.

$$\bar{X}^A\tilde{\lambda} + P^A\tilde{\pi} \leq \tilde{\alpha}X_0^A \quad \text{(B.12)}$$

$$\bar{X}^C\tilde{\lambda} + P^C\tilde{\pi} \leq X_0^C$$

$$\bar{Y}^A\tilde{\lambda} + Q^A\tilde{\pi} \geq \tilde{\alpha}Y_0^A$$

$$\bar{Y}^B\tilde{\lambda} + Q^B\tilde{\pi} \geq \tilde{\beta}Y_0^B$$

$$\bar{Y}^C\tilde{\lambda} + Q^C\tilde{\pi} \geq Y_0^C$$

$$e\tilde{\lambda} = 1$$

where we donate $\tilde{\alpha} = 1 + \gamma > 1$, $\tilde{\lambda} = (\lambda^* + \gamma\lambda)/(1-\gamma\delta) \geq 0$, $\tilde{\pi} = (\pi^* + \gamma\pi)/(1-\gamma\delta) \geq 0$ and $\tilde{\beta} = (1+\gamma\beta)/(1-\gamma\delta) \geq 0$.

Inequalities (B.12) mean that condition (5.8) holds for some $\tilde{\alpha} > 1$ and $\tilde{\beta} \geq 0$. Because T_{VRSTO} is convex, for every $\alpha \in [1, \tilde{\alpha}]$ there exists a $\beta \geq 0$ such that (5.12) is true. This contradicts the assumption of Lemma B.3 and completes the proof. □

Lemma B.4. *If a proportional marginal increase of vectors X_0^A and Y_0^A is not feasible in T_{VRSTO}, the objective function in (5.14) is unbounded.*

Because the optimal value of (5.12) and its dual (B.8) is $\bar{\beta}(1) = 1$, the program (5.14) is feasible. Suppose that, contrary to the statement of part (c) of Theorem 5.8, program (5.14) has a finite optimal solution. Then the dual to (5.14) is feasible:

Min $\beta + \delta$ (B.13)

Subject to

$\bar{X}^A \lambda - \delta X_0^A + P^A \pi \leq -X_0^A$

$\bar{X}^C \lambda - \delta X_0^C + P^C \pi \leq 0$

$\bar{Y}^A \lambda - \delta Y_0^A + Q^A \pi \geq -Y_0^A$

$\bar{Y}^B \lambda + \beta Y_0^B + Q^B \pi \geq 0$

$\bar{Y}^C \lambda - \delta Y_0^C + Q^C \pi \geq 0$

$\lambda - \delta = 0$

$\lambda, \pi \geq 0$; β, δ sign free

Because $(X_0, Y_0) \in T_{VRS}$, there exist vectors $\lambda^* \geq 0$ and $\pi^* \geq 0$ as in (B.10). If constraints in (B.13) are multiplied by a very small positive number ($\gamma > 0$) and the corresponding constraints in (B.10) are added to them, the constraint set in (B.14) is obtained.

$$\bar{X}^A(\lambda^*+\gamma\lambda)+P^A(\pi^*+\gamma\pi)\le -\gamma X_0^A+(1+\gamma\delta)X_0^A \tag{B.14}$$

$$\bar{X}^C(\lambda^*+\gamma\lambda)+P^C(\pi^*+\gamma\pi)\le (1+\gamma\delta)X_0^C$$

$$\bar{Y}^A(\lambda^*+\gamma\lambda)+Q^A(\pi^*+\gamma\pi)\ge -\gamma Y_0^A+(1+\gamma\delta)Y_0^A$$

$$\bar{Y}^B(\lambda^*+\gamma\lambda)+Q^B(\pi^*+\gamma\pi)\ge (1-\gamma\beta)Y_0^B$$

$$\bar{Y}^C(\lambda^*+\gamma\lambda)+Q^C(\pi^*+\gamma\pi)\ge (1+\gamma\delta)Y_0^C$$

$$e(\lambda^*+\gamma\lambda)-\gamma\delta=1$$

If we divide the constraints in (B.14) by $(1+\gamma\delta)$, the set of constraints in (B.12) is obtained, where we donate $\tilde{\alpha}=1-\gamma<1$, $\tilde{\lambda}=(\lambda^*+\gamma\lambda)/(1+\gamma\delta)\ge 0$, $\tilde{\pi}=(\pi^*+\gamma\pi)/(1+\gamma\delta)\ge 0$ and $\tilde{\beta}=(1-\gamma\beta)/(1+\gamma\delta)\ge 0$.

Inequalities (B.12) mean that condition (5.8) holds for some $\tilde{\alpha}<1$ and $\tilde{\beta}\ge 0$. Because T_{VRSTO} is convex, for every $\alpha\in[\tilde{\alpha},1]$ there exists a $\beta\ge 0$ such that (5.10) is true. This contradicts the assumption of Lemma B.4 and completes the proof. □

Proof of Theorem 5.9. The proof of Theorem 5.9 is identical with the proof of Theorem 4.3. In this case programs (5.12), (5.13), (5.14) and (B.8) are considered instead of (4.4), (4.7), (4.8) and (A.1), respectively. □

Proof of Theorem 5.10. See Theorem 4.6 and its proof in Appendix A. □

Proof of Theorem 5.11. The proof of parts (a) and (b) of Theorem 5.11 is almost identical with the Theorem 4.7. In this case, when applying the theorem of marginal values of linear programming, the dual to (5.16) is considered. In the proof of part (c), the theorem is proven with a similar approach to proof of part (c) in Theorem 5.8. In this case, duals to (5.17) and (5.18) are considered for increase and reduction scenarios, respectively. □

Proof of Theorem 5.12. See Theorem 4.8 and its proof in Appendix A. □

Proof of Theorem 5.13. Let Ω be the set of optimal solutions to the original technology (B.8), which is formulated for T_{VRSTO}. Let $\hat{\Omega}$ be the set of optimal solutions to (B.8), which, in this case, is formulated for the expanded technology $\hat{T}_{VRSTO}$ with the introduction of additional trade-offs. Since additional trade-offs bring new constraints to the original technology, the set of optimal solutions to the extended model is the subset of the optimal solutions to the original model ($\hat{\Omega} \subseteq \Omega$). The proof follows from (5.13) and (5.14). □

Appendix C

Result Tables for Analysis in Chapter 8

Appendix C presents the result tables for the empirical analyses conducted for Chapter 8. Tables C.1 to C.8 provides the output-oriented (OO) efficiency scores for the farms in each region under VRS and CRS considerations with and without the production trade-offs included. "Farm Code" column in each table represent the labels for the farms in the original FADN data set. Efficiency score tables also summarise the average efficiency scores and number of efficient and in efficient units for each type of model. In the analyses with trade-offs, we consider three types of trade-off relations for each crop (provided in Table 6.3) as Broad, Medium and Tight Trade-offs (TO). The identification of trade-offs is explained in section 6.5 thoroughly.

Tables C.9 to C.24 provide the elasticity analysis results for each region. Right-hand Elasticities (RHE) and Left-Hand Elasticities (LHE) are calculated and presented for each farm without trade-offs and as well as with three different ranges of trade-offs (as broad, medium and tight) under both variable returns-to-scale (VRS) and constant returns-to-scale (CRS) considerations.

For each of 8 regions, we have two elasticity result tables. One is giving the results for the scenario, where the responding output set consists of Cereals (crop types as Wheat, Barley, Grain Maize, Oats and Rye) and the other is providing the results for the scenario, where Field Crops (crop types as Vetch, Beans, Peas, Lentil, Sunflower, Oilseed Rape, Cotton, Potatoes, Sugar Beet and Tobacco) are in the responding output set. The details of elasticity scenarios considered are given in Chapter 8. The elements of the responding sets vary between the regions depending on the product range of the region specifically. The crop types produced by the farms in each region are given in Table 6.2 and the classifications of crop types are given in Table 6.3.

The elasticity measures for the farms that are not efficient in any model (VRS or CRS, with or without trade-offs) are omitted in the relevant tables. The omitted farms are Farms 13, 24, 28 and 30 in West Marmara; 10 and 20 in Mediterranean; 5, 18, 19 and 21 in Middle Anatolia; 6 in North East Anatolia and 22 and 23 in South East Anatolia. It can be observed from the relevant efficiency score tables (C.1 to C.8) that these farms are not on the frontier of any model.

The term “NA” in elasticity result tables stands for the non-applicability of elasticity measurement at the corresponding unit because the farm is not producing any type of outputs in the responding set, which is also explained more in detail by Chapter 8.

Table C.1. DEA Efficiency Scores for Farms in West Marmara

	Farm Code	OO VRS	OO VRS WTO			OO CRS	OO CRS WTO		
			Broad TO	Medium TO	Tight TO		Broad TO	Medium TO	Broad TO
1	**142**	1	1	1	1	1	1	1	1
2	**143**	1	1	0.97	0.94	1	0.99	0.96	0.93
3	**144**	1	1	1	0.91	1	1	0.97	0.87
4	**145**	1	1	1	1	1	1	1	1
5	**146**	1	0.96	0.92	0.87	1	0.90	0.85	0.81
6	**147**	1	0.93	0.92	0.89	0.93	0.80	0.74	0.68
7	**148**	1	1	1	1	1	0.86	0.79	0.73
8	**149**	1	1	0.98	0.95	1	0.99	0.97	0.94
9	**150**	1	1	1	1	1	1	1	1
10	**152**	1	1	1	1	1	0.95	0.83	0.74
11	**153**	1	0.91	0.90	0.89	1	0.90	0.89	0.88
12	**154**	1	1	1	0.96	1	0.87	0.77	0.69
13	**155**	0.71	0.67	0.64	0.60	0.69	0.65	0.62	0.58
14	**157**	1	1	0.98	0.90	1	0.95	0.92	0.84
15	**158**	1	0.87	0.84	0.81	1	0.87	0.83	0.80
16	**159**	1	1	1	0.96	1	1	0.93	0.84
17	**160**	1	1	1	1	1	1	0.96	0.88
18	**161**	1	1	1	0.96	0.98	0.94	0.92	0.87
19	**162**	1	1	0.99	0.93	1	0.91	0.86	0.79
20	**163**	1	0.97	0.95	0.94	1	0.90	0.88	0.85
21	**164**	1	0.95	0.90	0.85	0.95	0.88	0.83	0.77
22	**165**	1	1	1	1	1	1	1	1
23	**166**	1	0.93	0.91	0.89	1	0.92	0.90	0.87
24	**167**	0.97	0.93	0.92	0.90	0.96	0.89	0.85	0.82
25	**168**	1	1	1	1	1	1	1	1
26	**169**	1	1	1	1	1	0.97	0.91	0.85
27	**171**	1	1	0.95	0.88	1	0.97	0.92	0.85
28	**172**	0.98	0.91	0.85	0.78	0.91	0.84	0.73	0.65
29	**173**	1	1	1	1	1	1	1	1
30	**175**	0.74	0.70	0.66	0.60	0.74	0.67	0.64	0.58
31	**176**	1	1	0.98	0.91	1	0.91	0.83	0.75
32	**177**	1	1	1	1	1	0.95	0.89	0.84
33	**178**	1	1	1	1	0.81	0.78	0.74	0.69
34	**180**	1	0.88	0.85	0.82	1	0.87	0.85	0.82
35	**181**	1	1	1	1	1	1	1	1
36	**182**	1	1	1	1	1	1	1	1
37	**183**	1	0.86	0.84	0.83	0.84	0.75	0.71	0.66
38	**184**	1	1	1	1	1	1	1	1
39	**185**	1	1	1	0.97	1	0.95	0.92	0.88
Average		0.98	0.96	0.95	0.92	0.97	0.92	0.88	0.84
# of Efficient		35	24	20	15	30	12	9	8
# of Inefficient		4	15	19	24	9	27	30	31

Table C.2. DEA Efficiency Scores for Farms in Aegean

	Farm Code	OO VRS	OO VRS WTO			OO CRS	OO CRS WTO		
			Broad TO	Medium TO	Tight TO		Broad TO	Medium TO	Tight TO
1	**2**	1	1	1	1	1	0.02	0.01	0.01
2	**3**	1	0.11	0.09	0.08	1	0.10	0.09	0.07
3	**4**	1	0.03	0.03	0.02	0.99	0.01	0.01	0.01
4	**5**	1	0.03	0.03	0.02	1	0.02	0.02	0.01
5	**8**	1	0.03	0.03	0.03	1	0.01	0.01	0.01
6	**9**	1	0.12	0.10	0.08	1	0.02	0.02	0.02
7	**14**	1	0.07	0.06	0.05	1	0.01	0.01	0.01
8	**15**	1	0.31	0.29	0.26	1	0.22	0.20	0.18
9	**19**	1	0.01	0.01	0.01	1	0.01	0.01	0.01
10	**22**	1	0.01	0.01	0.01	1	0.01	0.01	0.01
11	**23**	1	0.04	0.03	0.03	1	0.01	0.01	0.01
12	**26**	1	0.02	0.02	0.02	1	0.02	0.02	0.01
13	**29**	1	1	1	1	1	0.17	0.14	0.12
14	**34**	1	0.04	0.04	0.04	1	0.01	0.01	0.01
15	**38**	1	0.01	0.01	0.01	1	0.01	0.01	0.01
16	**41**	1	1	1	1	1	1	1	1
17	**45**	1	0.02	0.02	0.01	1	0.01	0.01	0.01
Average		1	0.23	0.22	0.21	1	0.10	0.09	0.09
# of Efficient		17	3	3	3	16	1	1	1
# of Inefficient		0	14	14	14	1	16	16	16

Table C.3. DEA Efficiency Scores for Farms in East Marmara

	Farm Code	OO VRS	OO VRS WTO			OO CRS	OO CRS WTO		
			Broad TO	Medium TO	Tight TO		Broad TO	Medium TO	Broad TO
1	**329**	1	0.37	0.33	0.30	1	0.34	0.30	0.27
2	**332**	1	0.77	0.68	0.58	1	0.69	0.55	0.45
3	**333**	1	0.48	0.42	0.36	1	0.47	0.41	0.36
4	**336**	1	1	1	1	1	1	1	0.83
5	**339**	1	0.58	0.47	0.38	1	0.53	0.44	0.35
6	**340**	1	0.79	0.68	0.56	1	0.78	0.61	0.47
7	**341**	1	0.36	0.31	0.26	1	0.35	0.29	0.25
8	**342**	1	1	1	1	1	1	1	1
9	**345**	1	1	1	0.75	1	0.93	0.78	0.56
10	**348**	1	1	1	0.84	1	1	0.97	0.77
11	**349**	1	0.82	0.73	0.65	1	0.79	0.70	0.63
12	**351**	1	0.48	0.43	0.38	1	0.47	0.43	0.38
13	**353**	1	1	1	1	1	1	1	1
14	**355**	1	0.69	0.57	0.47	1	0.51	0.42	0.35
15	**356**	1	0.80	0.75	0.69	1	0.76	0.70	0.59
16	**357**	1	1	1	1	1	1	1	1
17	**358**	1	0.51	0.44	0.37	1	0.51	0.43	0.35
18	**359**	1	0.41	0.34	0.29	1	0.39	0.32	0.27
19	**360**	1	0.61	0.42	0.32	1	0.52	0.38	0.29
20	**361**	1	0.85	0.76	0.70	1	0.75	0.70	0.65
21	**362**	1	1	1	1	1	1	1	1
22	**364**	1	1	1	0.89	1	1	0.86	0.74
23	**365**	1	1	1	0.91	1	0.81	0.69	0.59
24	**369**	1	0.47	0.45	0.41	1	0.47	0.44	0.41
25	**371**	1	0.38	0.33	0.29	1	0.34	0.30	0.26
26	**373**	1	0.61	0.55	0.50	1	0.49	0.43	0.38
27	**374**	1	1	1	1	1	1	1	1
Average		1	0.74	0.69	0.63	1	0.70	0.63	0.56
# of Efficient		27	10	10	6	27	8	6	5
# of Inefficient		0	17	17	21	0	19	21	22

Table C.4. DEA Efficiency Scores for Farms in West Anatolia

	Farm Code	OO VRS	OO VRS WTO			OO CRS	OO CRS WTO		
			Broad TO	Medium TO	Tight TO		Broad TO	Medium TO	Broad TO
1	**288**	1	1	1	1	1	0.95	0.91	0.87
2	**289**	1	1	1	1	0.87	0.61	0.60	0.58
3	**290**	1	1	1	1	0.58	0.36	0.33	0.31
4	**291**	0.99	0.81	0.74	0.68	0.87	0.62	0.56	0.51
5	**293**	1	0.89	0.82	0.76	1	0.64	0.58	0.54
6	**295**	1	0.54	0.52	0.50	0.99	0.53	0.51	0.49
7	**296**	1	1	1	0.94	1	1	0.99	0.91
8	**297**	1	0.94	0.85	0.80	1	0.65	0.58	0.55
9	**298**	1	0.82	0.76	0.70	1	0.78	0.73	0.68
10	**300**	0.22	0.14	0.13	0.13	0.19	0.13	0.13	0.12
11	**301**	1	0.93	0.88	0.82	1	0.88	0.83	0.78
12	**302**	1	0.66	0.61	0.57	1	0.59	0.54	0.50
13	**303**	1	0.28	0.27	0.26	1	0.26	0.25	0.24
14	**304**	1	1	0.91	0.79	1	0.86	0.72	0.62
15	**306**	1	0.69	0.61	0.55	1	0.67	0.60	0.54
16	**307**	1	1	1	1	1	1	0.98	0.92
17	**309**	1	0.45	0.44	0.42	1	0.37	0.35	0.33
18	**310**	1	1	1	1	1	0.69	0.67	0.65
19	**311**	1	1	1	1	0.97	0.84	0.81	0.79
20	**312**	1	0.65	0.62	0.60	0.80	0.42	0.39	0.38
21	**313**	1	0.72	0.71	0.69	0.90	0.55	0.55	0.53
22	**314**	1	1	1	0.96	1	0.81	0.74	0.68
23	**315**	1	1	1	1	1	0.94	0.90	0.86
24	**316**	1	0.67	0.64	0.62	0.93	0.67	0.64	0.62
25	**317**	1	1	1	1	1	0.94	0.91	0.87
26	**318**	1	1	1	1	1	1	1	1
27	**319**	1	1	1	1	1	1	1	1
28	**320**	1	1	1	1	1	1	1	1
29	**321**	1	1	0.95	0.91	1	0.62	0.59	0.55
30	**322**	1	0.85	0.81	0.77	1	0.62	0.59	0.56
31	**323**	1	1	1	1	1	0.94	0.91	0.89
32	**325**	1	1	0.96	0.91	1	0.79	0.75	0.72
33	**326**	1	1	1	1	1	1	1	1
34	**327**	1	1	1	1	1	1	1	1
35	**328**	1	1	1	1	0.91	0.38	0.36	0.34
Average		0.98	0.86	0.84	0.81	0.94	0.72	0.69	0.65
# of Efficient		33	19	17	14	25	7	5	5
# of Inefficient		2	16	18	21	10	28	30	30

Table C.5. DEA Efficiency Scores for Farms in Mediterranean

	Farm Code	OO VRS	OO VRS WTO			OO CRS	OO CRS WTO		
			Broad TO	Medium TO	Tight TO		Broad TO	Medium TO	Broad TO
1	**96**	1	0.51	0.44	0.37	1	0.10	0.08	0.07
2	**97**	1	0.58	0.46	0.37	1	0.56	0.44	0.35
3	**99**	1	1	1	0.88	1	0.30	0.24	0.19
4	**100**	1	0.12	0.11	0.10	1	0.11	0.10	0.09
5	**102**	1	1	1	1	1	1	1	1
6	**103**	1	1	1	1	1	0.52	0.43	0.36
7	**104**	1	1	1	1	1	0.06	0.05	0.04
8	**105**	1	1	1	1	1	0.17	0.14	0.11
9	**107**	1	0.31	0.27	0.23	1	0.31	0.26	0.22
10	**108**	0.96	0.27	0.23	0.19	0.95	0.06	0.05	0.34
11	**109**	1	0.54	0.44	0.36	1	0.47	0.38	0.31
12	**110**	1	1	1	1	1	1	1	1
13	**111**	1	0.41	0.37	0.32	1	0.09	0.07	0.06
14	**112**	1	0.28	0.24	0.20	1	0.11	0.09	0.08
15	**113**	1	1	1	1	1	1	1	1
16	**115**	1	0.32	0.27	0.23	1	0.02	0.02	0.02
17	**117**	1	1	1	0.86	1	0.12	0.10	0.08
18	**118**	1	0.24	0.20	0.17	1	0.06	0.05	0.04
19	**119**	1	0.37	0.32	0.28	1	0.04	0.03	0.03
20	**120**	0.91	0.24	0.20	0.17	0.89	0.07	0.06	0.05
21	**121**	1	0.19	0.16	0.14	0.99	0.09	0.08	0.07
22	**123**	1	0.31	0.28	0.24	0.88	0.15	0.13	0.12
23	**126**	1	0.20	0.17	0.15	1	0.10	0.09	0.08
24	**127**	1	0.47	0.43	0.38	1	0.08	0.07	0.07
25	**132**	1	0.39	0.34	0.29	1	0.06	0.05	0.04
26	**134**	1	0.41	0.36	0.31	1	0.37	0.32	0.28
27	**136**	1	0.38	0.34	0.29	1	0.07	0.06	0.05
28	**137**	1	0.34	0.28	0.24	1	0.04	0.03	0.03
29	**138**	1	0.32	0.27	0.23	1	0.06	0.05	0.04
30	**140**	1	0.10	0.09	0.08	1	0.07	0.06	0.05
Average		1	0.51	0.47	0.44	0.99	0.24	0.22	0.21
# of Efficient		28	8	8	6	26	3	3	3
# of Inefficient		2	22	22	24	4	27	27	27

Table C.6. DEA Efficiency Scores for Farms in Middle Anatolia

	Farm Code	OO VRS	OO VRS WTO			OO CRS	OO CRS WTO		
			Broad TO	Medium TO	Tight TO		Broad TO	Medium TO	Tight TO
1	**251**	1	1	1	0.95	1	0.92	0.90	0.88
2	**252**	1	0.69	0.65	0.61	1	0.32	0.28	0.25
3	**253**	1	0.85	0.73	0.63	1	0.84	0.73	0.62
4	**254**	1	1	1	1	1	1	1	1
5	**255**	0.73	0.55	0.54	0.54	0.56	0.46	0.46	0.45
6	**256**	1	1	1	1	1	0.27	0.46	0.42
7	**257**	1	0.99	0.87	0.76	1	0.99	0.86	0.73
8	**259**	1	1	1	1	1	1	1	1
9	**260**	1	1	1	1	1	1	1	1
10	**261**	1	1	1	1	1	1	1	1
11	**262**	1	0.97	0.93	0.89	1	0.95	0.91	0.87
12	**263**	1	0.56	0.49	0.43	1	0.46	0.40	0.35
13	**264**	1	1	1	1	1	0.31	0.28	0.24
14	**265**	1	0.55	0.51	0.44	1	0.55	0.51	0.43
15	**266**	1	0.68	0.61	0.55	1	0.64	0.58	0.53
16	**267**	1	1	1	1	1	1	1	1
17	**269**	1	1	1	1	1	1	1	1
18	**270**	0.54	0.26	0.25	0.24	0.47	0.26	0.25	0.23
19	**271**	0.87	0.24	0.23	0.21	0.68	0.24	0.23	0.21
20	**274**	1	1	1	1	1	0.72	0.69	0.66
21	**277**	1	0.50	0.45	0.40	0.99	0.49	0.45	0.40
22	**279**	1	0.67	0.69	0.67	1	0.56	0.58	0.56
23	**280**	1	1	1	1	1	1	1	1
24	**281**	1	1	1	1	1	0.32	0.42	0.38
25	**284**	1	1	1	1	1	1	1	1
26	**285**	1	0.51	0.49	0.47	1	0.50	0.48	0.46
Average		0.97	0.81	0.79	0.76	0.95	0.68	0.67	0.64
# of Efficient		22	13	13	12	22	8	8	8
# of Inefficient		4	13	13	14	4	18	18	18

Table C.7. DEA Efficiency Scores for Farms in North East Anatolia

	Farm Code	OO VRS	OO VRS WTO			OO CRS	OO CRS WTO		
			Broad TO	Medium TO	Tight TO		Broad TO	Medium TO	Tight TO
1	**221**	1	1	1	1	1	0.71	0.67	0.61
2	**223**	1	0.89	0.85	0.80	1	0.84	0.83	0.79
3	**224**	1	1	1	1	1	1	1	1
4	**227**	1	1	1	1	1	1	1	1
5	**229**	1	0.89	0.85	0.82	1	0.72	0.69	0.65
6	**230**	0.95	0.77	0.76	0.75	0.75	0.52	0.50	0.47
7	**233**	1	1	1	1	1	1	1	1
8	**234**	1	1	1	1	1	1	1	1
9	**236**	1	1	1	1	1	1	1	1
10	**238**	1	1	1	1	0.78	0.72	0.72	0.72
11	**244**	1	1	1	1	1	1	1	1
12	**247**	1	1	1	1	1	1	1	1
13	**248**	1	1	1	1	1	1	1	1
14	**250**	1	1	1	1	1	1	1	1
Average		1	0.97	0.96	0.95	0.97	0.89	0.89	0.87
# of Efficient		13	11	11	11	12	9	9	8
# of Inefficient		1	3	3	3	2	5	5	6

Table C.8. DEA Efficiency Scores for Farms in South East Anatolia

	Farm Code	OO VRS	OO VRS WTO			OO CRS	OO CRS WTO		
			Broad TO	Medium TO	Tight TO		Broad TO	Medium TO	Tight TO
1	**49**	1	1	1	1	1	0.98	0.97	0.95
2	**50**	1	1	1	1	1	1	0.98	0.88
3	**53**	1	1	1	0.98	1	0.93	0.87	0.82
4	**54**	1	1	1	1	1	1	1	1
5	**55**	1	1	1	1	1	1	1	1
6	**59**	1	1	1	1	0.94	0.84	0.83	0.82
7	**60**	1	1	1	1	1	1	1	0.98
8	**61**	1	1	1	1	1	0.92	0.88	0.83
9	**62**	1	0.67	0.64	0.60	0.98	0.63	0.57	0.52
10	**64**	1	1	1	1	1	0.80	0.78	0.75
11	**69**	1	1	1	1	1	1	1	1
12	**70**	1	1	1	1	1	1	1	1
13	**71**	1	0.94	0.88	0.83	1	0.92	0.86	0.82
14	**72**	1	1	1	1	1	1	1	1
15	**74**	1	1	1	1	1	0.99	0.99	0.97
16	**75**	1	1	1	1	1	1	1	1
17	**78**	1	1	1	1	1	1	1	1
18	**83**	1	1	1	1	0.94	0.86	0.84	0.83
19	**87**	1	1	1	1	1	0.91	0.85	0.80
20	**88**	1	0.88	0.86	0.85	0.75	0.72	0.72	0.72
21	**90**	1	0.99	0.97	0.95	1	0.90	0.86	0.82
22	**91**	0.99	0.90	0.88	0.85	0.99	0.84	0.81	0.77
23	**92**	0.81	0.80	0.79	0.79	0.75	0.74	0.74	0.74
24	**93**	1	1	1	1	1	1	1	1
25	**94**	1	1	1	1	1	1	1	1
26	**95**	1	1	1	1	1	1	1	1
Average		0.99	0.97	0.96	0.96	0.97	0.92	0.91	0.89
# of Efficient		24	20	20	18	19	12	10	10
# of Inefficient		2	6	6	8	7	14	16	16

Table C.9. West Marmara Region Elasticity Measures (Cereals are responding)

	VRS								CRS							
	No TO		Broad TO		Medium TO		Tight TO		No TO		Broad TO		Medium TO		Tight TO	
Farms	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**
1	UD	0	1.49	0	1.21	0	0.98	0	UD	0	1.33	0	1.05	0.04	0.73	0.12
2	2.73	0							2.70	0						
3	UD	0	4.75	0	1.56	0.66			UD	0	1.57	0				
4	UD	0	UD	0.03	UD	0.05	UD	0.07	2.13	0	1.45	0.03	1.36	0.05	0.91	0.08
5	UD	0							UD	0						
6	0.77	0														
7	UD	0	UD	0	UD	0	UD	0	UD	0						
8	0.86	0							0.76	0						
9	UD	0	1.62	0	1.50	0	1.42	0	1.03	0	0.84	0	0.80	0.01	0.76	0.03
10	UD	0	UD	0	UD	0	UD	0	UD	0						
11	1.14	0							1.12	0.73						
12	UD	0	1.32	0	0.75	0			3.00	0						
14	UD	0	0.33	0					0.20	0						
15	2.84	0							2.51	0						
16	2.82	0	0.63	0	0.27	0			2.76	0	0.61	0				
17	UD	0	1.79	0	1.17	0	0.69	0	UD	0	0.51	0				
18	0.40	0	0.24	0	0	0										
19	UD	0	0.08	0					UD	0						
20	UD	0							UD	0.11						
21	UD	0.14														
22	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0
23	2.33	0							1.11	0.08						
25	UD	0	UD	0	UD	0	UD	0	0.36	0	0.19	0	0.16	0.08		
26	UD	0	1.91	0	1.68	0	1.51	0	UD	0						
27	2.45	0	0.23	0.01					2.24	0						
29	UD	0	16.91	0	13.87	0	11.57	0	UD	0	10.85	0	9.12	0	7.76	0
31	1.06	0	0.19	0					1.03	0						
32	UD	0	1.41	0	0.94	0	0.55	0	8.35	0						
33	UD	0	1.62	0	1.44	0	1.32	0								
34	UD	0							UD	0						
35	UD	0	2.13	0	1.95	0	1.83	0	UD	0	1.74	0	1.60	0	1.51	0
36	UD	0	UD	0	UD	0.01	UD	0.02	UD	0	2.54	0	2.23	0.01	2.02	0.02
37	UD	0														
38	UD	0	UD	0.01	UD	0.03	UD	0.06	UD	0	2.31	0.02	2.15	0.03	1.99	0.08
39	UD	0	0.77	0.01	0.69	0.34			0.41	0						

Table C.10. West Marmara Region Elasticity Measures (Field Crops are responding)

	VRS								CRS							
	No TO		Broad TO		Medium TO		Tight TO		No TO		Broad TO		Medium TO		Tight TO	
	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE
1	UD	0	2.30	0	2.07	0	1.85	0	UD	0	1.97	0	1.78	0.07	1.39	0.23
2	UD	0							UD	0						
3	1.58	0	1.14	0	0.47	0.20			1.11	0	0.38	0				
4	UD	0	UD	0.03	UD	0.06	UD	0.09	13.35	0	2.18	0.03	1.85	0.06	1.15	0.09
5	UD	0							UD	0						
6	UD	0														
7	UD	0	UD	0	UD	0	UD	0	UD	0						
8	UD	0							UD	0						
9	UD	0	3.33	0	3.04	0	2.75	0	UD	0	2.12	0	1.82	0.02	1.52	0.05
10	UD	0	UD	0	UD	0	UD	0	UD	0						
11	UD	7.22							65.83	8.32						
12	UD	0	1.14	0	0.79	0			7.53	0						
14	0.85	0	0.25	0					0.20	0						
15	UD	0							UD	0						
16	UD	0	1.06	0	0.46	0			UD	0	0.96	0				
17	UD	0	0.68	0	0.54	0	0.37	0	1.47	0	0.19	0				
18	0.36	0	0.24	0	0.01	0										
19	1.57	0	0.09	0					1.53	0						
20	UD	0							UD	0.19						
21	1.02	0.14														
22	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0
23	UD	0							UD	0.22						
25	UD	0	UD	0	UD	0	UD	0	UD	0	0.52	0.02	0.40	0.12		
26	UD	0	1.67	0	1.58	0	1.49	0	1.42	0						
27	UD	0	0.31	0.02					UD	0						
29	UD	0	2.07	0	1.98	0	1.90	0	UD	0	1.32	0	1.30	0	1.27	0
31	UD	0	0.29	0					4.49	0						
32	UD	0	0.90	0	0.66	0	0.44	0	UD	0						
33	UD	0	2.06	0	1.93	0	1.80	0								
34	UD	0							UD	0						
35	UD	0	4.77	0	4.37	0	3.98	0	UD	0	3.90	0	3.59	0	3.28	0
36	UD	0	UD	0	UD	0	UD	0.02	UD	0	2.35	0	2.22	0.01	2.08	0.02
37	2.02	0														
38	UD	0	UD	0.47	UD	1.32	UD	2.35	UD	0	128.89	0.68	113.79	1.34	100.04	3.26
39	UD	0	1.07	0.01	0.96	0.44			0.67	0						

Table C.11. Aegean Region Elasticity Measures (Cereals are responding)

	VRS								CRS							
	No TO		Broad TO		Medium TO		Tight TO		No TO		Broad TO		Medium TO		Tight TO	
	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE
1	UD	0	UD	0	UD	0	UD	0	UD	0						
2	UD	0							UD	0						
3	1.74	0														
4	UD	0							UD	0						
5	UD	0							UD	0						
6	UD	0							UD	0						
7	UD	0							UD	0						
8	1.39	0							1.00	0						
9	UD	0							UD	0						
10	1.66	0							1.65	0						
11	UD	0							UD	0						
12	UD	0							UD	0						
13	UD	0	UD	10.05	UD	10.97	UD	12.08	UD	0						
14	UD	0							UD	0						
15	UD	0							6.12	0						
16	UD	0	436.28	0	366.49	0	313.46	0	UD	0	331.43	0	278.42	0	238.14	0
17	UD	0							UD	0						

Table C.12. Aegean Region Elasticity Measures (Field Crops are responding)

	VRS								CRS							
	No TO		**Broad TO**		**Medium TO**		**Tight TO**		**No TO**		**Broad TO**		**Medium TO**		**Tight TO**	
	LHE	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**
1	UD	0	UD	0	UD	0	UD	0	UD	0						
2	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
3	UD	0														
4	UD	0							UD	0						
5	UD	0							UD	0						
6	UD	0							UD	0						
7	UD	0							UD	0						
8	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
9	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
10	UD	0							UD	0						
11	UD	0							2.19	0						
12	UD	0							UD	0						
13	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
14	UD	0							UD	0						
15	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
16	UD	0	1.33	0	1.33	0	1.32	0	UD	0	1.01	0	1.01	0	1.01	0
17	UD	0							UD	0						

Table C.13. East Marmara Region Elasticity Measures (Cereals are responding)

	VRS								**CRS**							
	No TO		**Broad TO**		**Medium TO**		**Tight TO**		**No TO**		**Broad TO**		**Medium TO**		**Tight TO**	
	LHE	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**
1	UD	0							170.98	0.17						
2	2.85	0							2.84	0						
3	UD	0							UD	0						
4	UD	0	16.56	0	6.84	0	0.45	0	UD	0	14.88	0	1.08	0		
5	UD	0							UD	0						
6	UD	0							UD	0						
7	UD	0							UD	0						
8	UD	0	UD	0	UD	0	UD	0	UD	0	1.91	0.10	1.72	0.15	1.58	0.20
9	UD	0	30.44	0	1.70	0			UD	0						
10	UD	0	29.66	0	5.90	0			UD	0	26.89	0				
11	UD	0							UD	0						
12	UD	0							UD	0						
13	UD	0	26.29	0	22.18	0	18.90	0	UD	0	11.32	0	9.83	0	8.66	0
14	UD	0							UD	0						
15	UD	0							UD	0						
16	UD	0	1.10	0	1.08	0	1.06	0	1.60	0	1.09	0	1.07	0	1.06	0
17	UD	0							UD	0						
18	UD	0							UD	0						
19	UD	0							UD	0.39						
20	UD	0							UD	0						
21	UD	0	7.28	0	5.92	0	4.29	0	UD	0	6.42	0	4.85	0	3.84	0
22	UD	0	4.39	0	0.63	0			UD	0	1.89	0				
23	UD	0	13.99	0	3.62	0			UD	0						
24	UD	0							UD	0						
25	UD	0							UD	0						
26	UD	0							UD	0						
27	UD	0	1.67	0	1.57	0	1.50	0	UD	0	1.27	0.08	1.21	0.11	1.17	0.15

Table C.14. East Marmara Region Elasticity Measures (Field Crops are responding)

	VRS								CRS							
	No TO		**Broad TO**		**Medium TO**		**Tight TO**		**No TO**		**Broad TO**		**Medium TO**		**Tight TO**	
	LHE	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**
1	56.83	0							55.35	0.41						
2	UD	0							UD	0						
3	UD	0							3.95	0						
4	UD	0	1.85	0	0.99	0	0.08	0	UD	0	1.66	0	0.19	0		
5	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
6	UD	0							UD	0						
7	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
8	UD	0	UD	0	UD	0	UD	0	UD	0	18.72	0.85	15.50	1.27	12.96	1.68
9	UD	0	36.52	0	1.43	0			UD	0						
10	UD	0	5.98	0	0.91	0			UD	0	5.01	0				
11	UD	0							UD	0						
12	UD	0							0.51	0						
13	UD	0	2.87	0	2.79	0	2.70	0	UD	0	1.39	0	1.33	0	1.28	0
14	UD	0							UD	0						
15	UD	0							UD	0						
16	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
17	UD	0							UD	0						
18	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
19	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
20	UD	0							UD	0						
21	UD	0	1.40	0	1.26	0	1.01	0	UD	0	1.19	0	1.02	0	0.91	0
22	2.12	0	0.74	0	0.14	0			2.07	0	0.37	0				
23	UD	0	3.23	0	0.77	0			UD	0						
24	UD	0							UD	0						
25	2.98	0							2.51	0						
26	UD	0							UD	0						
27	UD	0	42.07	0	34.28	0	27.59	0	UD	0	31.60	0.95	26.01	1.60	21.33	2.43

Table C.15. West Anatolia Region Elasticity Measures (Cereals are responding)

	VRS								CRS							
	No TO		Broad TO		Medium TO		Tight TO		No TO		Broad TO		Medium TO		Tight TO	
	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE
1	UD	0	1.28	0	1.24	0	1.21	0	UD	0						
2	UD	0	2.28	0	2.15	0	1.99	0								
3	UD	0	UD	0	UD	0	UD	0								
4																
5	UD	0							UD	0.39						
6	0.76	0														
7	UD	0	0.73	0	0.21	0			UD	0	0.62	0				
8	UD	0							UD	0						
9	8.36	0							4.64	0						
10																
11	UD	0							UD	0						
12	UD	0							UD	0						
13	UD	0.47							UD	0.47						
14	UD	0	1.50	0.24					UD	0						
15	UD	0							UD	0						
16	UD	0	3.36	1.15	3.18	1.33	3.07	1.50	UD	0	1.31	1.21				
17	UD	2.04							40.94	13.64						
18	UD	0	1.74	0.14	1.59	0.17	1.46	0.21	UD	0						
19	1.33	0	0.68	0.03	0.55	0.05	0.40	0.12								
20	0.85	0														
21	0.88	0.29														
22	UD	0	2.82	0.57	2.08	0.91			UD	0						
23	UD	0	4.18	0	2.92	0.05	1.46	1.03	UD	0						
24	0.62	0														
25	UD	0	UD	0	UD	0.01	UD	0.02	UD	0						
26	UD	0	6.73	0	4.57	0	2.93	0	UD	0	5.05	0	3.62	0	2.52	0
27	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0
28	UD	0	UD	0	UD	0	UD	0	UD	0	1.62	0	1.33	0	1.01	0
29	UD	0							UD	0						
30	1.35	0							1.26	0						
31	UD	0	0.41	0.07	0.27	0.10			UD	0						
32	UD	0	0.31	0.23					UD	0						
33	UD	0	7.84	0	6.45	0	5.31	0	UD	0	2.97	0	2.17	0	1.44	0
34	UD	0	UD	0	UD	0	UD	0	UD	0	2.65	0	2.47	0	2.24	0
35	UD	0	UD	0	UD	0	UD	0								

Table C.16. West Anatolia Region Elasticity Measures (Field Crops are responding)

	VRS								CRS							
	No TO		Broad TO		Medium TO		Tight TO		No TO		Broad TO		Medium TO		Tight TO	
	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE
1	UD	0	3.02	0	2.86	0	2.67	0	UD	0						
2	UD	0	9.92	0	9.06	0	8.06	0								
3	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
4																
5	2.01	0							1.98	0.15						
6	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
7	UD	0	0.75	0	0.24	0			UD	0	0.70	0				
8	7.59	0							1.43	0						
9	UD	0							UD	0						
10	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
11	UD	0							UD	0						
12	0.29	0							0.26	0						
13	11.15	0							1.72	0.28						
14	1.57	0	0.60	0.10					1.44	0						
15	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
16	UD	0	21.86	3.90	19.19	5.35	17.09	6.94	UD	0	5.33	4.09				
17	14.79	0							12.31	9.42						
18	UD	0	1.86	0.16	1.80	0.20	1.73	0.25	1.25	0						
19	UD	0	1.09	0.05	0.96	0.09	0.75	0.23								
20	1.62	0	0													
21	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
22	UD	0	1.12	0.27	0.96	0.44			2.26	0						
23	UD	0	0.86	0	0.69	0.01	0.40	0.29	2.36	0						
24	1.25	0														
25	UD	0	UD	0	UD	0.03	0	0.06	UD	0						
26	UD	0	2.31	0	1.87	0	1.40	0	UD	0	1.73	0	1.48	0	1.21	0
27	UD	0	UD	0	UD	0	0	0	UD	0	UD	0	UD	0	UD	0
28	UD	0	UD	0	UD	0	2.03	0	UD	0	1.88	0	1.69	0	1.41	0
29	UD	0							UD	0						
30	UD	0							UD	0						
31	UD	0	1.24	0.20	0.89	0.33			UD	0						
32	UD	0	0.55	0.40					2.59	0						
33	UD	0	1.03	0	0.95	0	0.87	0	1.19	0	0.47	0	0.37	0	0.26	0
34	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
35	UD	0	UD	0	UD	0	0	0								

Table C.17. Mediterranean Region Elasticity Measures (Cereals are responding)

	VRS								CRS							
	No TO		Broad TO		Medium TO		Tight TO		No TO		Broad TO		Medium TO		Tight TO	
	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE
1	6.53	0.08							2.57	0.08						
2	UD	0							UD	0						
3	UD	0	2.75	0	0.84	0			UD	0						
4	0.95	0							0.91	0						
5	UD	0	UD	2.29	UD	2.71	UD	3.21	UD	0	UD	3.63	UD	4.20	UD	4.89
6	UD	0	UD	3.42	UD	3.83	UD	4.33	3.33	0.20						
7	1.01	0	0.46	0	0.31	0	0.15	0	0.67	0						
8	UD	0	19.45	0	14.27	0	8.55	0	UD	0						
9	1.13	0							1.00	0						
11	UD	0							UD	0						
12	UD	0	UD	0	UD	0	UD	0	UD	0	132.25	0	102.23	0	82.06	0
13	1.05	0							1.02	0						
14	UD	0							UD	0						
15	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0
16	UD	0							UD	0						
17	UD	0	2.64	0	1.12	0.24			UD	0						
18	UD	0							UD	0						
19	UD	0							UD	0						
21	0.18	0														
22	0.49	0.01														
23	137.08	0							4.18	0						
24	0.80	0							0.80	0						
25	1.13	0							1.09	0						
26	1.33	0.03							1.00	0.09						
27	1.16	0							1.10	0						
28	UD	0							UD	0						
29	UD	0							UD	0						
30	0.60	0							0.45	0.39						

Table C.18. Mediterranean Region Elasticity Measures (Field Crops are responding)

	VRS								**CRS**							
	No TO		**Broad TO**		**Medium TO**		**Tight TO**		**No TO**		**Broad TO**		**Medium TO**		**Tight TO**	
	LHE	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	UD	0							UD	0						
3	UD	0	13.30	0	3.78	0			UD	0						
4	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
5	UD	0	UD	10.90	UD	15.34	UD	20.96	UD	0	UD	17.27	UD	23.78	UD	31.91
6	UD	0	UD	4.79	UD	6.70	UD	9.09	UD	1.43						
7	UD	0	7.86	0	4.62	0	1.86	0	UD	0						
8	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
9	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
11	UD	0							UD	0						
12	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
13	UD	0							UD	0						
14	UD	0							UD	0						
15	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
16	1.30	0							1.28	0						
17	UD	0	9.10	0	3.48	0.57			UD	0						
18	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
19	UD	0							UD	0						
21	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
22	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
23	UD	0							UD	0						
24	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
25	UD	0							UD	0						
26	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
27	UD	0							UD	0						
28	UD	0							9.86	0						
29	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
30	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

Table C.19. Middle Anatolia Region Elasticity Measures (Cereals are responding)

	VRS								CRS							
	No TO		**Broad TO**		**Medium TO**		**Tight TO**		**No TO**		**Broad TO**		**Medium TO**		**Tight TO**	
	LHE	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**
1	UD	0	1.33	0.10	0.99	0.65			3.74	0						
2	UD	0							1.43	0						
3	UD	0							UD	0						
4	UD	0	UD	0	UD	0	UD	0	UD	0	1.58	0	1.46	0	1.41	0.02
6	UD	0	UD	0	UD	0	UD	0	UD	0						
7	UD	0							UD	0						
8	UD	0	1.24	0	1.20	0	1.16	0	2.17	0	1.24	0	1.20	0	1.13	0
9	UD	0	UD	0.07	UD	0.10	UD	0.14	UD	0	1.60	0.25	1.50	0.30	1.43	0.35
10	1.43	0	1.10	0	1.08	0.04	0.98	0.33	1.42	0	1.08	0	1.04	0.04	0.80	0.34
11	0.90	0							0.85	0						
12	UD	0							UD	0						
13	UD	0	UD	6.92	UD	19.38	UD	24.06	UD	0						
14	UD	0							UD	0						
15	UD	0							UD	0						
16	1.54	0	1.29	0.11	1.27	0.13	1.24	0.15	1.00	0	1.00	0.12	1.00	0.14	1.00	0.15
17	UD	0	1.29	0	1.21	0	1.14	0	UD	0	0.77	0	0.48	0	0.12	0
20	4.09	0	2.69	0.39	2.49	0.55	2.35	0.69	1.00	0						
22	3.08	0							2.84	0.54						
23	UD	0	19.13	0	14.95	0	12.09	0	UD	0	13.28	0	10.61	0	8.68	0
24	UD	0	UD	0	UD	0	UD	0	UD	0						
25	UD	0	3.06	0	2.56	0	2.14	0	UD	0	2.34	0	2.12	0	1.93	0
26	UD	0							UD	0						

Table C.20. Middle Anatolia Region Elasticity Measures (Field Crops are responding)

	VRS								**CRS**							
	No TO		**Broad TO**		**Medium TO**		**Tight TO**		**No TO**		**Broad TO**		**Medium TO**		**Tight TO**	
	LHE	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**
1	UD	0	4.00	0.27	3.26	2.15			2.85	0						
2	UD	0							UD	0						
3	1.30	0							1.05	0						
4	UD	0	UD	0	UD	0	UD	0	UD	0	5.18	0	4.55	0	4.21	0.06
6	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
7	UD	0							UD	0						
8	UD	0	10.34	0	9.08	0	7.97	0	UD	0	10.23	0	8.86	0	7.62	0
9	UD	0	UD	0.16	UD	0.27	UD	0.39	UD	0	6.00	0.56	5.36	0.76	4.79	0.99
10	UD	0	18.06	0	15.18	0.38	10.60	3.57	UD	0	17.97	0	14.82	0.43	8.70	3.63
11	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
12	1.08	0							1.06	0						
13	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
14	UD	0							UD	0						
15	0.85	0							0.78	0						
16	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
17	UD	0	8.75	0	7.40	0	6.22	0	UD	0	4.82	0	2.51	0	0.59	0
20	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
22	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
23	UD	0	2.08	0	1.95	0	1.83	0	UD	0	1.48	0	1.41	0	1.33	0
24	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
25	UD	0	5.44	0	4.51	0	3.66	0	UD	0	4.40	0	3.91	0	3.44	0
26	13.39	0							3.91	0						

Table C.21. North East Anatolia Region Elasticity Measures (Cereals are responding)

	VRS								**CRS**							
	No TO		**Broad TO**		**Medium TO**		**Tight TO**		**No TO**		**Broad TO**		**Medium TO**		**Tight TO**	
	LHE	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**
1	UD	0	10.75	0	9.07	0	6.59	0	UD	0						
2	UD	0							0.69	0						
3	UD	0	1.65	0	1.59	0	1.52	0		0	1.37	0	1.29	0	1.20	0
4	UD	0	1.93	0	1.71	0	1.54	0		0	1.86	0	1.65	0	1.49	0
5	UD	0							1.51	0						
7	UD	0	UD	0	UD	0	UD	0	1.91	0	0.90	0	0.73	0	0.55	0
8	UD	0.15	21.64	1.13	18.80	1.23	16.43	1.37	2.96	0.19	2.09	1.44	2.06	1.57	2.02	1.74
9	UD	0	UD	0	UD	0	UD	0	1.01	0	0.26	0.07	0.23	0.08		
10	UD	4.26	UD	49.49	UD	52.47	UD	56.57								
11	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0
12	UD	0	6.65	0	5.82	0	5.07	0	UD	0	5.29	0	4.68	0	4.12	0.03
13	UD	0	1.31	0	1.19	0	1.09	0	UD	0	1.28	0	1.17	0	1.07	0
14	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0

Table C.22. North East Anatolia Region Elasticity Measures (Field Crops are responding)

	VRS								CRS							
	No TO		Broad TO		Medium TO		Tight TO		No TO		Broad TO		Medium TO		Tight TO	
	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE	LHE	RHE
1	UD	0	676.06	0	530.24	0	355.44	0	UD	0						
2	UD	0							UD	0						
3	UD	0	5.22	0	4.81	0	4.40	0	UD	0	4.30	0	3.88	0	0	3.44
4	UD	0	1.31	0	1.22	0	1.13	0	UD	0	1.29	0	1.20	0	0	1.10
5	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
7	UD	0	UD	0	UD	0	UD	0	UD	0	8.61	0	6.08	0	0	3.93
8	UD	0.90	24.58	1.50	22.25	1.61	20.03	1.75	UD	1.34	3.45	2.20	3.11	0	2.33	2.78
9	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
10	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
11	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
12	UD	0	3.62	0	3.32	0	2.99	0	UD	0	3.07	0	2.78	0	0.02	2.48
13	UD	0	1.87	0	1.70	0	1.53	0	UD	0	1.86	0	1.68	0	0	1.50
14	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	0	UD

Table C.23. South East Anatolia Region Elasticity Measures (Cereals are responding)

	VRS								CRS							
	No TO		**Broad TO**		**Medium TO**		**Tight TO**		**No TO**		**Broad TO**		**Medium TO**		**Tight TO**	
	LHE	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**
1	3.20	0	0.96	0.07	0.75	0.11	0.43	0.16	0.20	0.05						
2	UD	0	2.95	0	2.38	0	1.94	0	UD	0	2.33	0.25				
3	UD	0	1.88	0	1.56	0.27			UD	0						
4	1.17	0.03	1.13	0.18	1.12	0.21	1.12	0.24	1.00	0.20	1.00	0.46	1.00	0.48	1.00	0.51
5	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0	UD	0
6	1.07	0	1.04	0	1.02	0	1.00	0								
7	0.68	0	0.26	0	0.18	0			0.06	0	0.01	0				
8	UD	0	1.56	0	1.39	0	1.25	0.31	27.89	0						
9	UD	0														
10	UD	0	2.16	0	2.02	0	1.91	0	3.40	2.07						
11	UD	0	UD	0	UD	0	UD	0	1.00	0	1.00	0	1.00	0	1.00	0
12	1.30	0.06	1.29	0.07	1.29	0.08	1.29	0.08	1.00	0.19	1.00	0.20	1.00	0.20	1.00	0.21
13	UD	0							UD	0						
14	1.43	0.08	1.28	0.30	1.22	0.39	1.16	0.49	1.00	0.10	1.00	0.31	1.00	0.41	1.00	0.52
15	1.00	0.15	1.00	0.29	0.99	0.30	0.96	0.32								
16	UD	0	1.69	0	1.42	0	1.10	0	UD	0	1.68	0	1.37	0	0.97	0
17	UD	0	UD	0	UD	0	UD	0	0.80	0.14	0.79	0.17	0.79	0.17	0.79	0.17
18	2.92	0	1.61	0	1.52	0.53	1.44	0.58								
19	UD	0	2.83	0.48	2.49	0.09	2.19	0.62	UD	0.04						
20	0.62	0														
21	1.92	0							1.92	1.72						
24	UD	0	1.64	0	1.53	0	1.44	0	UD	0	1.51	0	1.44	0	1.38	0
25	1.09	0	0.99	0	0.99	0	0.98	0	1.00	0	0.96	0	0.94	0	0.92	0
26	1.04	0	0.67	0	0.57	0	0.44	0	1.00	0	0.59	0	0.45	0	0.29	0

Table C.24. South East Anatolia Region Elasticity Measures (Field Crops are responding)

	VRS								CRS							
	No TO		**Broad TO**		**Medium TO**		**Tight TO**		**No TO**		**Broad TO**		**Medium TO**		**Tight TO**	
	LHE	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**	**LHE**	**RHE**
1	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	UD	0	3.29	0	2.72	0	2.21	0	UD	0	2.03	0.21				
3	2.26	0	1.74	0	1.63	0.28			2.14	0						
4	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
5	UD	0	2.19	0	1.92	0	1.75	0	UD	0	1.75	0	1.65	0	1.57	0
6	UD	0	14.96	0	13.73	0	12.58	0								
7	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
8	UD	0	2.10	0	1.99	0	1.90	0.44	1.38	0						
9	0.17	0														
10	UD	0	2.79	0	2.64	0	2.49	0	1.93	1.49						
11	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
12	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
13	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
14	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
15	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
16	1.59	0	1.30	0	1.23	0	1.05	0	1.54	0	1.29	0	1.18	0	0.93	0
17	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
18	UD	0	3.53	0.95	3.30	1.06	3.10	1.18								
19	UD	0	1.86	0	1.70	0.06	1.57	0.44	UD	0.02						
20	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
21	3.65	0							2.23	2.03						
24	UD	0	5.07	0	4.68	0	4.31	0	4.79	0	4.79	0	4.66	0	4.25	0
25	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
26	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

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