# Minimum Cost Decision for the Tradeoff between Finished Goods Inventory and Production Capacity 

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M.S., Mechanical Engineering

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#### Abstract

The goal of the production planning system is to specify when and how much product to build while satisfying demand requirements and minimizing cost. In the face of nonuniform, uncertain demand, this often requires a tradeoff between finished goods inventory and production capacity

This problem is examined in the context of operations at the Billerica Manufacturing Site of Bay Networks for a subset of their in-house production. The historical planning process, which relies heavily on the experience and knowledge of schedulers, planners, and production managers, is analyzed and evaluated from a cost performance standpoint. Comparisons are made with production plans generated from deterministic demand; singleperiod, stochastic demand; and multiple-period, stochastic demand models.

The historical production planning process performs well from a total cost standpoint relative to a minimum cost benchmark. There is, however, some room for improvement. The extent of this potential is sensitive to the treatment of penalty costs related to shortages and end-ofquarter finished goods inventory levels. The multiple-period model, which is uniquely formulated to allow for the parameterization of these costs, is used as the basis for the development of a production planning tool. This tool provides a useful minimum cost input to the production planning process. In addition, it allows modeling of the effect of management policies for the compromise between customer service levels and inventory turnover.

Thesis Supervisors: Donald B. Rosenfield, Senior Lecturer Stanley B. Gershwin, Senior Research Scientist


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## NOTATION



## 1 INTRODUCTION

This thesis examines the production planning process at the Billerica Manufacturing Site of Bay Networks. ${ }^{1}$ More specifically, the process used to make the tradeoff between production capacity and finished goods inventory (FGI) is analyzed to assess its cost effectiveness. Additionally, a number of inventory models are applied, the most advanced of which forms the basis for the development of a production planning tool intended to provide a minimum cost input to the planning process.

As a means of introduction, what first follows is a non-technical problem statement that describes the context, objectives, and economic justification for this work. Next, a brief literature review provides a survey of applicable modeling techniques. Finally, the structure of the remainder of the thesis is presented.

### 1.1 Non-Technical Problem Statement

Production planning, in a limited definition, is the process of taking available information and making decisions about how much of a product to build and when to build it. The goal is to maximize profits by fulfilling demand with minimum cost. Achieving this goal requires the optimization of the tradeoff between production capacity and finished goods inventory.

Throughout the quarter, the capacity planning team at Bay Networks repeatedly evaluates the revenue position in terms of shipments relative to forecast. Typically, because of a nonuniform demand profile, demand uncertainty, and production capacity constraints, at some point within the quarter they modify a fundamental build-to-order strategy by accumulating some finished goods inventory. Planning team members decide exactly when and how much to inventory to store based on their knowledge and experience. The focus of this thesis is to provide a more quantitative, minimum-cost based approach to making this decision. Specifically, this thesis has two objectives:

- a cost performance analysis of the current production planning process
- a production planning tool to support weekly production planning decisions

[^0]
### 1.2 Literature Review

The popularity of operations research has created a wealth of literature on inventory modeling techniques that extends well beyond journal articles to edited compilations of articles, handbooks, and textbooks. Nahmias provides a recent introduction to commonly understood inventory models as part of his text on production and operations analysis (Production Analysis 211-330). Multiple-period, stochastic models are included with other advanced models in a text by Hadley and Whitin and also in a survey of techniques by Scarf (345-49; 194-205). These classic works, though both published in the early 1960's, are still relevant. However, the same information can be found in sections of more recent, comprehensive handbooks such as those by Lee and Nahmias or Nahmias (27-28; Inventory Models 463-65). Also, texts on the subject of dynamic programming frequently use the multiple-period, stochastic inventory model as an example (Cooper and Cooper 197-201; Bertsekas 1-5, 65-72). These provide useful information on the numerical solution implementation rather than equation derivations.

While there is ample literature on the topic of inventory modeling, they tend to be overly theoretical. In addition, the complexity of these models and their solutions quickly escalates beyond what operations managers might practically apply in many situations. This research did not reveal any industry examples of the application of the multiple-period, stochastic models used in this thesis. This is not to say that this type of model or, likely, more comprehensive ones have not been applied in industry. It is a reasonable supposition that some of the advanced, commercial supply chain planning packages use these techniques. However, if they do, it seems that any specific results have escaped publication.

### 1.3 Thesis Overview

The next section of this thesis contains the problem description. It provides general information on the Billerica Manufacturing Site products and operations along with more detailed information on the production planning process. Following this, Section 3 presents the solution technique. It describes the approach, formulation, and inputs used to measure and model the planning process. This is done in three subsections, one each for the historical performance evaluation, single-period model, and multiple-period model. Using this same
structure, Section 4 provides the results and their interpretation. Finally, Section 5 completes this work with a report of conclusions.

## 2 PROBLEM DESCRIPTION

This section describes the production planning problem that is the main subject of this thesis. It does so by providing an overview of the relevant, current state of affairs at the Billerica Manufacturing Site. First, background information on the site's products and operations provides some context. Subsequently, a more focused look at the objectives and details of the current planning process serves to define the problems that motivate this project.

### 2.1 Background

The Billerica Manufacturing Site is a producer of high-technology electronic equipment. As such, its in-house production capabilities are mainly of the light assembly variety typical of the industry. Following is a description of the specific products and manufacturing operations that the production planning processes studied in this thesis affect.

### 2.1.1 Products

The Billerica Manufacturing Site produces networking hardware such as hubs, routers, and switches. These network boxes range from standardized products selling for less than one thousand dollars to highly configurable products selling for tens of thousands of dollars. They generate about $80 \%$ of the site's annual revenues with the remaining $20 \%$, stemming from other peripheral networking hardware and services. Total revenues are projected to be around $\$ 1$ billion for fiscal year 1999.

At the start of this project, approximately ten product families had an in-house manufacturing component, and these realized roughly sixty percent of annual revenues. Of the ten, three families became the focus of the project and will be referred to herein as Family 1, Family 2, and Family 3 . ${ }^{2}$

Together, these three families represent nineteen percent of annual revenues and account for a majority of the site's direct labor use. In addition, they represent the range of product configurability. At one extreme, Family 2 is sold in 14 basic configurations with a multitude of possible options. At the other extreme, Family 3 is available in four basic configurations

[^1]with only memory options. Family 1 occupies the middle ground with eight basic configurations and a handful of available options.

### 2.1.2 Operations

The three product families are manufactured with a manual light assembly operation. As is typical in this type of operation, capital equipment requirements are fairly minor, and the challenge is in managing the direct labor force.

## Manufacturing Process

Figure 1 schematically depicts the manufacturing process. Subassemblies and materials such as sheet metal boxes, power supplies, motherboards, and option cards are supplied on pallets or carts next to the assembly line. Workers stand at workstations and snap or screw components into chassis that are placed on trays and slid down the assembly line on rollers. Following the two to three assembly stations is a test station where the product is checked for proper operation. If called for, software is also loaded at this point. From the test station the product moves to a packout station where power cords and literature are added along with packing materials into a cardboard shipping box. An automated roller line transports the packaged product to the shipping area. There, handlers either palletize the product and load it onto a truck for delivery or place it in finished goods inventory.


Figure 1. Schematic of assembly operations.

## Production Equipment

The production equipment used in the manufacturing process is simple and relatively inexpensive. Its generic nature allows for ample flexibility between lines. Aside from providing the appropriate material and components to the line-side, changeover to accommodate another product family primarily requires only different test cable assemblies and software. This flexibility allows for the existence of a "universal" line that can accommodate any of the product families when there is a capacity shortfall.

The test function is typically the bottleneck process restricting line capacity. Production lines are generally set up with a maximum capacity that can accommodate on the order of twice the expected quarterly demand.

## Direct Labor

The direct labor force at the site is fairly flexible, both in terms of the ability to work on multiple product lines and in size.

The flexibility between product lines is especially apparent for the three product families that are the focus of this project. Workers must learn the various product configurations and options that are unique to each product line, but the basic assembly operations are very similar.

The flexibility in the size of the workforce is possible due to the relatively simple nature of the assembly tasks and, therefore, the limited amount of training necessary to bring new workers up to expected productivity levels. This allows the site to use a direct labor force that includes a $20 \%$ temporary worker component.

In spite of this apparent flexibility, production managers still have a significant incentive to stabilize the workforce. In addition to the issues of maintaining quality, productivity, and morale, there is always a concern about the availability of temporary workers. Factors such as season (e.g., summer and the availability of students), economic conditions, and competing employment opportunities can affect the quality of available workers as well hiring lead times.

### 2.2 Production Planning

The production planning process specifies how much product to build and when to build it, driving materials, capacity, and labor requirements. The planning process, therefore,
significantly impacts financial performance, and in fact, a production plan's relative merit is based on this impact.

Below, the planning process is broken into two major components, forecasting and scheduling. For each, the current process, objectives, and problem areas are described.

### 2.2.1 Forecasting

As described by Rosa in his research at Bay Networks, the planning process includes a judgmental forecasting technique (18). Business planners assess a wide array of information including product positioning, revenue goals, promotions, discontinuations, price, competitive response, and industry trends to establish quarterly, line-item forecasts. They then make adjustments to ensure that the aggregated line-item forecasts mesh with corporate revenue goals.

While the aggregate revenue projections are generally acknowledged to be fairly accurate, the line-item forecasts are usually considered to be poor. Since no formal attempts at tracking forecast accuracy are made, this qualitative evaluation is based on hearsay. In addition, planners do not forecast demand variability. Instead, schedulers address variability by increasing forecasted volumes with a "flex" percentage. The flex percentage can vary by product, but is typically in the ten to twenty percent range.

Normally, planners formally update forecasts twice during the course of a quarter. In addition to these formal updates, production schedulers make minor adjustments throughout the quarter, especially near its end, based on assessments of the order pipeline. News of pending orders and the chance they will become firm influence the production plan. In addition, orders that are booked for subsequent quarters can be pulled in, materials shipments can be expedited or delayed, and finished goods inventory can be expanded or reduced, in an attempt to match revenue projections while minimizing inventory levels.

Although the formal forecasting procedure provides only quarterly numbers, it is generally accepted that demand follows what is referred to as a "hockey-stick" profile over the course of a quarter. The hockey-stick profile describes a demand pattern in which as much as $25 \%$ of a quarter's total demand materializes in its last two weeks. While internal sales and marketing incentives to meet quarterly revenue targets are at least partially responsible, external factors such as common capital budgeting policies and industry-wide
discounting practices are thought to drive this hockey-stick effect. Accommodating this demand profile, along with the associated uncertainty, is what complicates the production planning process.

### 2.2.2 Scheduling

Master schedulers use a build-to-order strategy in developing a production schedule.
However, because a forecast is never perfect and production capacity is constrained, they must often modify this fundamental approach by building some amount of finished goods inventory.

Figure 2 depicts a typical example. The circle indicates the forecasted volume.
Maximum capacity, marked by the long-dashed line, well exceeds that required for a levelload plan, marked by the solid line (a level-load plan is one in which production is uniformly distributed over the quarter). Demand, marked by the solid squares, falls below the levelload plan as expected with a typical hockey-stick demand profile. Build-to-order production then, follows this demand curve. On a weekly basis, the capacity planning team evaluates the current position and the resources required to make the forecast. In the example, currently at week eight, an informal two-week projection (marked by the unfilled squares) and resulting plan (short-dashed line) to reach the quarterly target is shown. If the capacity


Figure 2. Typical production schedule.
requirement of the resulting plan exceeds current capacity, the team considers options such as hiring workers, installing equipment, or building inventory.

The income resulting from the execution of a schedule determines a schedule's relative merit. When using income as a criterion, items with direct impact on revenues or expenses, such as satisfying demand come immediately to mind. However, the effect of less direct objectives such as customer service levels or cost of raising capital should also be included. Three key objectives, demand fulfillment, customer service, and inventory levels, are discussed below.

## Demand Fulfillment

A primary objective of the manufacturing operation is to meet revenue projections by fulfilling all available demand. Operations managers take great strides in order to meet demand. Actions taken include expediting component supply, using overtime, and purchasing new equipment. Perhaps the following statement, as heard around the master scheduling area, best describes the importance place on satisfying demand. "You can get your 'rear' chewed for having too much inventory, but you can lose your job for missing revenue."

## Customer Service

There is no doubt that customer satisfaction is important. However, exactly what customer service level is acceptable is subject to debate. Common practice generally accepts that if an order ships within two weeks of order entry, customers are satisfied, i.e., orders shipped the same day or two weeks after the order is taken results in the same customer satisfaction.

Customers receive lead-time quotes based on an Available To Promise (ATP) method that takes into account variables such as materials availability and production capacity. Unusually high demand levels or production problems can cause lead times to exceed the two-week limit. In these cases, addressing capacity constraints to reduce lead times to an acceptable level has top priority.

However, when lead times do extend beyond the two-week limit, nobody tracks lost orders or other measures of dissatisfaction. In fact, orders that are booked beyond the twoweek limit are not considered late if they ship when promised. Thus, there is a bit of a selffulfilling prophecy with this system since orders that cannot be shipped on time may never be booked.

Other factors impacting customer satisfaction are also not measured. Whether a product is new and based on leading-edge technology or near the end of its life cycle and fairly well commoditized should affect what lead-times customers will accept without dissatisfaction. Also, it seems likely that customers who receive their order within a few days are likely to be left with a more positive impression than those that wait for two weeks. As the network box business becomes more commoditized, service level is one means of providing some differentiation, and its strategic effects should also be considered.

## Inventory

There are two components to inventory costs that warrant attention. The first is the holding cost. Holding cost here is defined comprehensively to reflect the cost of capital, re-work, administration, taxes, insurance, labor, shrinkage, and obsolescence. The second component is related to the inventory numbers reported to the financial community. There is high emphasis placed on the importance of keeping inventory at a minimum on the last day of each quarter so that reported numbers are low and inventory turns are high. Meeting target inventory on these days is a key performance measure for the Materials group. There are even contractual stipulations with suppliers that allow materials to be held at the supplier at the end of each quarter as a means of achieving inventory numbers.

The reason for the emphasis on these numbers is unclear. Reporting of poor numbers has an effect on the stock price and therefore, plausibly, the cost of capital to the company. However, using a single-day measure is perhaps not the best number to reflect the actual asset utilization. There is no doubt that it is important to individuals in the company since it is tied to the compensation and reward systems.

## 3 SOLUTION TECHNIQUE

As an objective of this thesis, the need to deliver a useful production-planning tool elevates the importance of selecting methodologies that are easy to use, intuitive, and comfortable for potential users. This being the case, the approach used here is to begin with as simple an analysis as possible. Complexity is then only added if results fall short of that required to fulfill the objectives of the thesis. This approach yields a progression of analysis that is broken into three discrete steps: historical performance; single-period, stochastic model; and multiple-period, stochastic model.

The starting point is to quantify historical production costs resulting from the existing planning process and measure it against what might have been done had demand been known in advance with certainty, i.e., with deterministic demand. While this helps to provide a feel for how the existing process is performing and perhaps might suggest what sorts of approaches will yield lower cost solutions, it doesn't provide a means of planning for in the future where demand is uncertain, i.e., stochastic.

To address this shortcoming, the next logical step is to account for the demand uncertainty. Treating a fiscal quarter as a single period and assuming sort of distribution for demand allows an optimum production volume to be determined using the classic "newsboy" approach in which expected overage and underage costs are minimized. While this provide a production plan that accounts for demand uncertainty, treating the entire quarter as a single period does not account for how demand arrives over the course of the quarter. In other words, while it provides an overall production volume target for the quarter, it doesn't address when in the quarter the production should be carried out.

Therefore, the next logical step is to go to a multiple-period, stochastic demand model. While this is a significant jump in the level of complexity, it is required to perhaps provide an input to the planning process that formalizes implicit knowledge of the demand profile and dynamic adjustment as information arrives during the quarter. It is this model on which the production-planning tool is based.

With this introduction each of the following sub-sections describes the three stages of the analysis described above. Each details the assumptions, mathematical formulation, and specific solution methods.

### 3.1 Historical Performance

Historical performance of the production planning system is analyzed by quantifying historical costs for a particular period and then comparing these to pure strategies that could have been implemented had demand been known in advance.

### 3.1.1 Historical Costs

Cost baselines for three product families are established by first examining the historical demand for a particular fiscal quarter. The costs of the production plans responding to this demand is then quantified by considering three primary cost categories: labor, materials, and inventory.

## Demand

Order bookings is the proxy used for demand. Ideally, one could simply pull the required booking values from a data warehouse. Not surprisingly though, using this approach reveals some amount of inconsistency with other reported values. For example, bookings as calculated from shipment and backlog level data do not match with database booking values.

The cause of such inconsistencies can largely be attributed to the fact that not all data is rigorously tracked and available. The accuracy of informally tracked data values and reporting frequency can make it difficult to reconstruct a precise history. Consider again the example of demand calculation from shipment and backlog level change data. While backlog levels are reported as part of a daily report, they are not formally archived, and compiling historical levels depends upon tracking down reports stored in personal files. The result is that reporting dates between shipments and backlogs for a particular week might differ by as much as four days.

In order to provide the most consistent basis for measuring performance, order bookings are calculated from shipment and backlog level data. While database values of bookings can be accurately associated with a booking date, the effects of delayed customer request dates, large channel orders that are smoothed out over a number of weeks, and non-revenue sales are difficult to sort out. Therefore, backing out bookings provides a more accurate picture of what demand the production planning process is responding to.

## Labor Cost

Weekly labor cost is calculated by multiplying the number of workers staffing a production line by a labor rate. The number of people working full shifts on each production line is reconstructed through informal capacity planning and performance meetings. A standard eight-hour shift and five-day workweek is used with an hourly labor rate based on a production manager's estimate of the average temporary worker's wage.

Because temporary workers are used to adjust labor level, no additional hiring and firing costs are incurred. The temporary worker staffing level is managed through an on-site temporary service representative, and the labor rate actually reflects that paid to the temporary service.

Overtime costs are a small fraction of labor costs and an even smaller fraction of total costs, which are largely driven by material costs. They are not accounted for in the historical baseline.

## Material Cost

Material costs are calculated as the product of production volume and unit material cost for each product family. Production volumes are obtained by reviewing certain material movements for the final assembly part numbers in the ERP system. Product family unit material cost is calculated as the volume weighted average of all the unit material costs of product family members shipped in the quarter being analyzed.

## Inventory \& Backlog Costs

Inventory levels are calculated as cumulative production less shipments. Weekly shipment volumes are available in a data warehouse and production volumes are obtained as described above. Costs are calculated by applying a comprehensive holding rate against the level. The holding rate, provided by the manufacturing finance group, includes administrative costs, insurance, handling, shrinkage, damage, and obsolescence in addition to the cost of capital.

Backlog costs are more difficult to establish. There are no established rules within the company to quantify demand and customer good-will losses. However, it's generally agreed that if orders are shipped within two weeks there is no penalty cost. Looking at backlog numbers pulled from informal records of daily production reports does not resolve whether lead times exceed the two-week limit, since some backlog is the result of delayed customer
request dates. Given this complexity, and since the high priority placed on shipping within two weeks generally results in strong on-time performance, backlog costs are not assessed.

### 3.1.2 Pure Strategy Comparison

Quantifying historical cost figures does little good without having some relative measure to indicate whether performance is good or bad. Two pure, aggregate planning strategies, as described by Nahmias, provide the means of comparison (Production Analysis 130-136). These are the chase (zero-inventory) and constant workforce strategies. Both of these strategies represent extremes and, in most cases, it is most likely that a compromise between the two would result in a minimum cost plan. However, it is because they are pure extremes that they serve as good benchmarks.

The chase strategy assumes complete labor flexibility in order to produce exactly to demand thereby eliminating inventory. The required weekly labor is calculated as bookings (demand) divided by weekly labor productivity. Here, because the quotient is rounded up to an integer value and maintained at a constant level for each week, the resulting production levels do not match demand exactly and a minimal amount inventory and associated holding costs result.

The constant workforce strategy, as the name implies, assumes complete labor inflexibility. For each week, the required labor is calculated as cumulative bookings divided by cumulative labor productivity. The maximum of the resulting values provides the labor requirement under a no backlog constraint. The value calculated for the final period provides the labor requirement without any constraint on backlog levels.

### 3.2 Single-Period, Stochastic Demand Model

As its name indicates, with a single-period model an entire fiscal quarter is treated as a single period. As described in almost any text dealing with inventory control, the simplest model that addresses stochastic demand is the Newsboy Model (e.g., Nahmias, Production Analysis $272-80$ ). The derivation is not repeated here since it is commonly available, and it is essentially shown as part of the derivation in following section on the multiple-period model. However a brief description of the formulation and inputs is provided.

### 3.2.1 Formulation

The approach is to treat demand as a continuous, non-negative random variable, $D$, with probability distribution function $f(d)$ and cumulative distribution function $F(d)$. The decision variable is the target inventory level, $y$. In the case of the actual Newsboy scenario, $y$ represents the number of papers to buy at the beginning of a sales period. Here, the $y$ is equal to the initial inventory (or backlog) position plus production planned for the period at its beginning, i.e., before any actual demand occurs. An expression for total expected production costs is written as a function of $D$ and $y$ and minimized to produce $y^{*}$, the optimum target inventory level. The costs considered are the variable production cost, $c$, inventory carrying cost, $h$, and the shortage cost, which in this case is taken to be equal to the selling price, $s$. The solution is expressed as:

$$
\begin{equation*}
F\left(y^{*}\right)=\frac{s-c}{s+h} \tag{Eq.1}
\end{equation*}
$$

This equation assumes finished goods inventories are perishable, and therefore, overage costs include the loss of variable costs associated with the inventory. If instead finished goods inventories are treated as durable, the denominator of the above equation is reduced by the variable costs producing the following.

$$
\begin{equation*}
F\left(y^{*}\right)=\frac{s-c}{s+h-c} \tag{Eq.2}
\end{equation*}
$$

Both equations produce target inventories for the entire quarter. In order to translate to a weekly production plan, some assumption about how production is distributed within the quarter needs to be made. In this case, a "level loaded" plan in which production is uniformly distributed is assumed.

### 3.2.2 Inputs

The solution to the single-period model requires inputs for the demand distribution, variable cost, holding cost, and selling price.

## Demand

As is typically done, the demand distribution is taken to be normal. While this technically violates the assumption of non-negativity, as long as the standard error (the standard
deviation divided by the mean) is less than about 0.3 , the error in the solution is not significant.

Quarterly demand parameters for each of the product families are estimated based on historical booking data for six recent, consecutive quarters.

## Variable Cost

Variable costs are the sum of labor and material costs. Unit labor costs are calculated based on labor productivity and wage rates. Unit material cost is calculated as the volumeweighted average of the unit material costs for each product family member shipped in the quarter being analyzed.

## Holding Cost

Unit holding costs are based on a holding rate applied to material cost of inventory items. As described previously, the holding rate, provided by the manufacturing finance group, captures G \& A costs, insurance, handling, shrinkage, damage, and obsolescence in addition to the cost of capital.

## Selling Price

Unit selling price, like unit material cost, is calculated as the volume weighted average of the selling price for each of the product family members shipped in the quarter being analyzed.

### 3.3 Multiple-Period, Stochastic Demand Model

The technique used to solve the multiple-period, stochastic demand model is dynamic programming. This general approach breaks down the cost equation into current costs and expected future costs to provide a time-iterative algorithm.

### 3.3.1 Formulation

The general dynamic programming algorithm for the multiple-period, stochastic demand model is commonly available in references on inventory control such as those by Scarf or Nahmias (197-200; Inventory Models 463-64). The formulation described simply follows these standard derivations. However, this formulation is somewhat unique in that it allows for the parameterization of shortage cost parameters. This formulation allows for some flexibility in setting costs that are typically difficult to establish. Also, it enables shortage costs and holding costs at the end of the time horizon to be different than those occurring
within the time horizon as a means of modeling incentives to minimize end of quarter inventory (to maximize reported inventory turns).

Demands for each period, $i$, are treated as continuous, non-negative, independent random variables, $D_{i}$, with probability distribution functions $f_{i}(d)$ and cumulative distribution function $F_{i}(d)$. The decision variables are the target inventory levels, $y_{i}$, for each period. A target inventory level is equal to the initial inventory (or backlog) position, $x_{i}$, plus production planned for the period at its beginning, i.e., before any actual demand occurs. The costs considered are the variable production cost, $c$, inventory carrying cost, $h$, and the shortage cost, $s$ (equal to the sales price).

There are two loss parameters. The end-of-horizon inventory loss parameter, $\alpha$, indicates what fraction of inventory held at the end of the time horizon is lost. At one extreme, $\alpha=0$ indicates that no inventory is lost and "full credit" for the material cost associated with the overage is given, i.e., the inventory will be sold in future periods. At the other extreme, when $\alpha=1$ the full variable cost of end-of-horizon overage is incurred, i.e., this inventory is written off.

The revenue loss parameter, $\beta$, indicates the fraction of revenue lost for backlog level in excess of a target backlog constraint, $\tau$, specified in terms of time (e.g., a two-week backlog constraint). In other words, a penalty cost of $\beta s$ is applied to backlog in excess of $\tau \mu_{i}$, where $\mu_{i}$ is the mean demand in period $i$. Note that only the penalty cost is applied and the backlog itself is not perishable, i.e., it is assumed to carry over from period to period.

The general cost function is

$$
c\left(y_{i}-x_{i}\right)+[h-c(1-\alpha)] \max \left(y_{i}-D_{i}, 0\right)+\beta s \max \left(D_{i}-y_{i}, 0\right)
$$

where

$$
0 \leq \alpha \leq 1 \text { and } 0 \leq \beta \leq 1 \text { except } \beta=0 \text { when } D_{i}-y_{i} \leq \tau \mu_{i}
$$

There are two things that require clarification. The end-of-horizon inventory loss term, $c(1-\alpha)$, is only included in the final period and is otherwise equal to zero. The term is included here in the general cost function to avoid repetition of the formula. Where it should and shouldn't be included is made clear below. Also, because the backlog constraint,
specified in units of time, must be converted to a unit volume in order to assess penalty costs, it is an approximation.

This is the Newsboy formulation with the addition of parameterized penalty and end-ofhorizon inventory variable costs. It is extended to a multiple period formulation using the dynamic programming approach of separating cost into current and future terms. Allowing $C_{i}\left(x_{i}\right)$ to represent minimum expected costs with $i$ periods remaining produces the following equation

$$
\begin{gather*}
C_{i}\left(x_{i}\right)=\min _{y_{i}}\left\{c\left(y_{i}-x_{i}\right)+[h-c(1-\alpha)] \int_{0}^{y_{i}}\left(y_{i}-\xi\right) f_{i}(\xi) d \xi+\beta s \int_{y_{i}}^{\infty}\left(\xi-y_{i}\right) f_{i}(\xi) d \xi\right.  \tag{Eq.3}\\
\left.+\int_{0}^{\infty} C_{i-1}\left(y_{i}-\xi\right) f_{i}(\xi) d \xi\right\}
\end{gather*}
$$

It is worth emphasizing that this formulation follows standard notation with the index $i$ representing the number of periods remaining. Therefore, the index decreases chronologically from the first period in which, e.g., $i=13$, toward the final period in which $i=1$. This notation makes sense since the solution technique begins with the final period and steps in reverse chronological order toward the initial period.

For the last period, $i=1$, expected future costs are taken to be zero $\left[C_{0}\left(x_{0}\right)=0\right]$. Also, a unique end-of-quarter revenue loss factor, $\beta_{1}$, is used to reflect that cost of shortages at the end of quarter is usually considered more significant than are intra-quarter shortages. Thus, Equation 3 simplifies to
$C_{1}\left(x_{1}\right)=\min _{y_{1}}\left\{c\left(y_{1}-x_{1}\right)+[h-c(1-\alpha)] \int_{0}^{y_{1}}\left(y_{1}-\xi\right) f_{1}(\xi) d \xi+\beta_{1} s \int_{y_{1}}^{\infty}\left(\xi-y_{1}\right) f_{1}(\xi) d \xi\right\}$
Separating out the initial inventory term and allowing

$$
G_{1}\left(y_{1}\right)=c\left(y_{1}\right)+[h-c(1-\alpha)] \int_{0}^{y_{1}}\left(y_{1}-\xi\right) f_{1}(\xi) d \xi+\beta_{1} s \int_{y_{1}}^{\infty}\left(\xi-y_{1}\right) f_{1}(\xi) d \xi
$$

produces

$$
C_{1}\left(x_{1}\right)=\min _{y_{1}}\left[G_{1}\left(y_{1}\right)\right]-c x_{1}
$$

Since $G_{1}\left(y_{1}\right)$ is convex, taking its derivative and setting it equal to zero yields the following equation defining the minimum cost target inventory level, $y_{1}^{*}$.

$$
\begin{equation*}
F_{1}\left(y_{1}^{*}\right)=\frac{\beta_{1} s-c}{h+\beta_{1} s-c(1-\alpha)} \tag{Eq.4}
\end{equation*}
$$

Thus, for the final period, the solution reduces to that of the Newsboy Model and can be solved analytically.

For all other periods, the end-of-horizon inventory loss term, $c(1-\alpha)$, goes away leaving

$$
\begin{gather*}
C_{i}\left(x_{i}\right)=\min _{y_{i}}\left\{c y_{i}+h \int_{0}^{y_{i}}\left(y_{i}-\xi\right) f_{i}(\xi) d \xi+\beta s \max \left[\int_{y_{i}}^{\infty}\left(\xi-y_{i}\right) f_{i}(\xi) d \xi-\tau \mu_{i}, 0\right]\right.  \tag{Eq.5}\\
\left.+\int_{0}^{\infty} C_{i-1}\left(y_{i}-\xi\right) f_{i}(\xi) d \xi\right\}-c x_{i}
\end{gather*}
$$

The solution is given by choosing the critical $y_{i}^{*}$ values that satisfy this equation subject to the following conditions and constraints:
$\tau \quad$ The desired time backlog constraint, i.e., customers should receive their orders within this time.
$0 \leq \beta \leq 1 \quad$ When expected backlog (underage) exceeds $\tau \mu_{i}$ (volume backlog constraint), the penalty cost will be equal to $\beta s$ for each unit exceeding $\tau \mu_{i}$. When the expected underage does not exceed $\tau \mu_{i}$ there is no penalty cost.
$y_{i}^{*} \geq x_{i} \quad$ The target level cannot be less than the initial inventory position (i.e., production cannot be negative).
$y_{i}^{*}-x_{i} \leq K$ The amount ordered (produced) cannot exceed some hard capacity constraint, $K$.

### 3.3.2 Inputs

The multiple-period model requires inputs for initial conditions, weekly demand distributions, capacity constraints, loss factors, and backlog constraints in addition to the same cost and price inputs required for the single-period model described in Section 3.2.2. Following is a description of the inputs that are unique to the multiple-period model.

## Initial Conditions

In solving the multiple-period problem it is necessary to specify two initial conditions. These are the initial time and inventory position.

The initial time is specified in terms of the total number of periods to be analyzed. Because the implementation of the multiple period model was targeted to look at thirteenweek fiscal quarters, this number ranges from one to thirteen weeks and can be described as the number weeks left until the end of the quarter.

The initial inventory position is simply the number of units in inventory at the start of the analysis period. Initial backlog positions are specified as a negative inventory position.

## Demand

Gamma distributions are used to describe the weekly demand variables necessary for input to the multiple-period model. As described in Section 3.2.2, the quarterly demand for the single-period model can be treated as normally distributed since the violation of the nonnegativity requirement is not significant. However, the variance of the weekly demand distributions used in the multiple period models is much greater. Standard error values are typically greater than 0.5 and were on occasion greater than 0.9 . Therefore, the use of a normal distribution by ignoring its negative component proves infeasible. A gamma distribution, which does not have a negative component, is a good compromise.

Weekly booking data for six recent, consecutive quarters is used to determine the distribution parameters for each of the three product families.

## Capacity Constraint

Equipment production capacity is treated as fixed over the time-horizon of the analysis. Values of this hard capacity constraint for each product family are obtained through the constraint analyses and working experience of the manufacturing engineering group responsible for production equipment.

## Loss Factors \& Backlog Constraint

The loss factors are parameterized precisely because they are difficult to quantify. Manipulating the soft backlog constraint and loss factors as input to various "what-if" scenarios provides a means of investigating management policy effects (this is described in greater detail in Section 4.2.4). "Standard" values for the parameters used in the multiple-
period model are a backlog constraint of two weeks ( $\tau=2$ ), revenue loss factor of ten percent within the quarter ( $\beta=0.1$ ) and $100 \%$ at the end ( $\beta=1$ ), and an end-of-quarter finished goods loss factor of $100 \%(\alpha=1)$. The loss parameter values, though chosen somewhat arbitrarily, are scaled relative to each other in manner that fits within the bounds of current production planning objectives. Similarly, the production plans resulting from running the model with these parameters make intuitive sense.

### 3.3.3 Numerical Implementation

The approach used to solving the dynamic programming formulation of the multiple-period problem is to begin by first solving the final period problem for a range of initial inventory values. As shown above, this problem can be solved analytically since the lack of future expected costs results in Equation 4.

Given the cost function developed for the final period, the next-to-final period problem can be solved. In Equation 5, the terms for current, in this case the next-to-final period, expected overage and underage costs could be solved analytically. However, the expected future cost term requires a numerical integration of the final period cost function.
Furthermore a numerical approach to finding the critical target inventory level, i.e., the inventory level that minimizes total expected costs, is necessary.

The procedure used to solve the next-to-final period problem is then applied repeatedly to solve preceding period problems until the initial period is solved. At this point, the critical target inventory levels and total cost function for an array of initial inventory position exists, and the specific solution for a given initial inventory position is interpolated.

A brief description of the key numerical techniques follows. Appendix A contains the Microsoft Visual Basic for Application code for some of the key routines.

## Inventory Range and Resolution

For computational reasons, it is important to choose an appropriate range and resolution of initial inventory positions over which the problem will be solved. The appropriate range of inventory positions is determined by the need to cover the range of integration in calculating expected future cost term taken from Equation 5.

$$
\begin{equation*}
\int_{0}^{\infty} C_{i-1}\left(y_{i}-\xi\right) f_{i}(\xi) d \xi \tag{Eq.6}
\end{equation*}
$$

In this equation, the cost function is evaluated at $y_{i}-\xi$. The maximum value occurs at the lower limit of integration (zero). Thus $\max \left(y_{i}\right)$ drives the maximum initial inventory level. The minimum value occurs at the upper limit of integration (infinity). Choosing the mean plus six standard deviations as an acceptable substitute for infinity produces $\min \left(y_{i}\right)+\mu+6 \sigma$ as the driver of the minimum initial inventory level.

The maximum value of $y_{i}$ is chosen based on the solution to the infinite horizon problem with stationary demand. As described by Nahmias, for this problem the series of critical values continually increases as the period considered moves away from the final one towards the initial and approaches the critical value as shown below (464).

$$
y_{1}^{*} \leq y_{2}^{*} \leq \cdots \leq y_{\infty}^{*}=F^{-1}\left(\frac{s}{h+s}\right)
$$

Thus, taking the maximum of the values resulting from the application of the above equation to each of the weekly demand distributions in the quarter provides a conservative maximum initial inventory value that should not be exceeded during computation of the solution

The minimum value of $y_{i}$ is defined heuristically by making a conservative estimate of excess capacity. In other words, $y_{i}$ is chosen to be the backlog position that would result from maximum production rates over the time horizon. The minimum initial inventory position is then based on this negative inventory value less the demand distribution mean and six standard deviations.

A typical initial inventory position range in this project is $-15,000$ to 5000 units. This range is discretized using a step of 100 units. This resolution was chosen because it produced solutions of reasonable accuracy without too much sacrifice in run time.

## Minimization

A numerical minimization function is needed to find the critical target inventory level, $y_{i}^{*}$, from Equation 5. Press et. al. describes an effective approach to one-dimensional numerical minimization that begins by bracketing a minimum point between two others (277-282).

A fairly brute-force, bisection type approach to bracketing a minimum is used. In essence, the cost function is evaluated at the endpoints and midpoint of the possible target inventory range. If the value at the midpoint is less than that of either endpoint, the
minimum is bracketed. If not, the midpoint becomes a new endpoint, the new range is bisected, and the process is repeated until either the minimum is bracketed or an endpoint is effectively reached (in which case the endpoint is the minimum).

Once a minimum is bracketed, any number of standard routines can be used to isolate it to the required degree. In this case a routine based on the Golden Section Search as described by Press et. al. is used (282). It too is a simple, brute-force routine that does not require any derivatives of the cost function to be calculated.

## Integration

Calculation of expected future cost requires numerical integration of Equation 6. A routine based on the basic trapezoidal rule is used.

The lower limit of integration is taken to be the greater of zero and the mean less six standard deviations. The upper limit is taken to be approximately six standard deviations above the mean. This integration range is discretized based on a division equal to a standard deviation divided by thirty. These values are chosen on the results of experimentation that sought to strike a balance between acceptable accuracy and run time.

## 4 RESULTS \& INTERPRETATION

Results of historical, single-period model, and multiple-period model production plans are presented in this section. Though the sample is limited, historical performance shows that the current production planning approach yields generally favorable results in terms of total costs. However, there appears to be some room for improvement. Results for the singleperiod model confirm this is the case. Specifically, the trade-off between customer service levels and low end-of-quarter finished goods inventories may be skewed in favor of the latter. The multiple-period model, on which the final production-planning tool is based, identifies viable production plans that may provide cost improvements relative to the current planning process. However, the magnitude of these cost savings is sensitive to how penalties against shortages and end-of-quarter finished goods inventories are levied.

Results are described generally without reference to absolute dollar and volume values in the interest of protecting proprietary information. Also to this end, plots of production plans are shown without ordinate values and the data may be manipulated.

### 4.1 Historical Performance

The historical performance for each product family is shown in Figure 3 through Figure 5 located in the following three subsections. In each figure, the "Demand" curve represents actual demand based on order bookings. The "Production Plan" curve shows the actual, historical production plan for the quarter.

There are also two "Constant Workforce" curves. As described in Section 3.1.2, these represent production plans based on the assumption that labor levels cannot be adjusted in the quarter. The difference between the two constant workforce strategies is that one allows the accumulation of backlog and the other does not.

A separate curve for a production plan based on the zero-inventory or "Chase" strategy is not shown since, as described in Section 3.1.2, it is based on adjusting labor to meet demand (i.e., the chase strategy and demand curve are essentially identical). With the minimum variable costs necessary to fulfill demand and virtually no inventory holding costs, the chase strategy is considered to be a minimum cost benchmark.

### 4.1.1 Product Family No. 1

The actual production plan for product Family No. 1 is shown in Figure 3. It is driven by a forecast for total quarter volume that proved to be within five percent of the actual level. Initially, production is primarily build-to-order and thus trails demand by a week or so. This lasts through the seventh week at which time some finished goods inventory is accumulated in anticipation of the end-of-quarter demand surge. Nearly all this inventory is shipped by the end of week thirteen and very little finished goods inventory is carried into the subsequent quarter.


Figure 3. Historical performance for Product Family No. 1.

Though specific numbers are not shown, the total cost of the production plan is within a few percent of that yielded by the chase strategy. The total cost of the CWF with backlog strategy also comes close to that of the chase strategy. However, because total costs are dominated by material costs, using it as a metric masks performance differences based on inventory holding costs.

Even though holding cost performance differences are a small percentage of total costs, they may still be significant on an absolute dollar basis. For this product family, for example, inventory holding costs resulting from the production plan are on the order of $\$ 10,000$ more than that resulting from a chase strategy. On this measure, historical
production planning methods are clearly superior to the CWF with backlog strategy, which has $70 \%$ greater inventory holding costs. The CWF without backlog strategy is the poorest performer. A demand spike early in the quarter drives up the workforce requirement and resulting inventory.

### 4.1.2 Product Family No. 2

The actual production plan for Product Family No. 2 is shown in Figure 4. As is the case with Product Family No. 1, this plan is based on a forecasted quarterly demand volume that proved to be accurate within five percent. However, in contrast to the plan for Product Family No. 1, very little finished goods inventory is accumulated at any time during the quarter. This may be attributed to two factors. First, this product family is highly configurable, making pre-building of finished goods more difficult. Second, demand for this product family is relatively strong compared to previous quarters possibly leading to a more reactive approach.


Figure 4. Historical performance for Product Family No. 2.

Total cost performance is within a few percent of that yielded by a chase strategy. Given the lack of finished goods inventory in this plan, their costs are minimal. As would be expected in this situation, the inventory holding costs associated with a constant workforce
approach are relatively large. Note that there is no difference between the constant workforce plans because no demand spikes within the quarter exist.

### 4.1.3 Product Family No. 3

The actual production plan for Product Family No. 3, depicted in Figure 5, is at the opposite end of the spectrum from the plan for the second product family. As is shown, production slightly exceeds that of the constant workforce plans. Perhaps contributing to this outcome is that this product family is the least configurable of the three. The forecasted quarterly volume, though less than ten percent in error, is the most inaccurate and the only one that over-predicted demand.


Figure 5. Historical performance for Product Family No. 3.

In this case, total cost performance is more than five percent greater than that of the chase strategy. In addition, the inventory holding costs exceed those of the constant workforce strategies (as with the second product family, there is no distinction between the two constant workforce strategies due to the relatively smooth demand profile).

### 4.1.4 Interpretation of Results

Results for Product Family No. 1 are consistent with the planning approach described in Section 2.2.2 - an initial build-to-order strategy is modified mid-way through the quarter to
build finished goods inventories in anticipation of the end-of-quarter demand surge. However, results for Product Families 2 and 3 show that historical plans can also look much more like pure build-to-order and build-to-stock approaches. While this might be due to variation in the planning process, it is also consistent with the planned consideration of product family configurability characteristics.

In spite of the variation in production plans, each yields a cost performance that matches favorably with a minimum cost benchmark based on the chase strategy. However, in each case production plans benefit from accurate quarterly forecasts. Given a broader base of comparison, it is likely that situations in which either higher levels of finished goods inventories or shortages would arise.

In addition, it is possible that the self-fulfilling prophecy with respect to demand forecast (see Section 2.2.2) does not allow for the unmasking of true demand. For example, with Product Family No. 2, capacity constraints that produce long lead-time quotes in the Available To Promise system may be drive down demand (and potential shortages). Similarly for Product Family No. 3, finished goods inventory levels may be reduced by pulling in orders originally booked for the following quarter.

While this historical performance assessment yielded generally positive cost performance results, there is likely still room for improvement.

### 4.2 Single-Period, Stochastic Demand Model

Comparisons of production plans resulting from the single-period, stochastic demand model (i.e., Newsboy Model) with actual, historical production plans and demand are shown for each product family in Figure 6 through Figure 8. In each of the figures, the demand and production plan curves are repeated exactly as in Section 4.1. The other two curves depict the output of the single-period model. The first model plan, with a lower total cumulative production volume, represents the standard Newsboy formulation in which finished good inventories are assumed to be perishable (see Equation 1). The other model plan assumes finished goods inventories are durable and that no value is lost by carrying it over to the next quarter (see Equation 2). Recall from Section 3.2.1 that a level-loaded production plan is used to reach the quarterly target level.

### 4.2.1 Product Family No. 1

The single-period model production plans for the first product family are shown in Figure 6. The forecast, a key input for both the current production planning process and the model, is accurate, under-predicting demand by less than five percent.

For the case in which FGI is lost at the end of the quarter, an inventory target equal to the 60th percentile of the demand distribution results. While both the model and actual production planning process both produce plans in which very little FGI exists at quarter-end, the model plan clearly carries a greater amount of inventory throughout the quarter. This results in $30 \%$ higher inventory holding costs.

For the case in which FGI is carried over at the end of the quarter, the inventory target increases to the 97 th percentile of the demand distribution. This results in inventory holding costs more than three times that of the actual production plan. Not quantified, however, are the offsetting benefits of improved customer service and level production that higher inventories enable.


Figure 6. Single-period model results for Product Family No. 1.

### 4.2.2 Product Family No. 2

The single-period model production plan for the second product family is shown in Figure 7. The input forecast is very accurate, under-predicting demand by only two percent.

The margins for the second product family are such that the application of the model in which quarter-end FGI is lost results in a target inventory level that is equal to the 45 th percentile of the demand distribution. In other words, the expectation is that there will be a shortage at the end of the quarter. Thus, in spite of inventory costs that more than double those of the actual production plan, demand is left unfulfilled.

For the case in which FGI is carried over at the end of the quarter, the inventory target increases to the 94th percentile of the demand distribution. This results in inventory holding costs that are 19 times greater than for the actual production plan.


Figure 7. Single-period model results for Product Family No. 2.

### 4.2.3 Product Family No. 3

The single-period model production plans for the second product family are shown in Figure
8. In this case the input forecast over-predicts demand by about five percent.


Figure 8. Single-period model results for Product Family No. 3.

As with the second product family, the margins for the third product family are such that under the assumption of lost end-of-quarter FGI, the model target inventory level is less than the mean demand (in this case, it is equal to the 40th percentile level) and produces a shortage expectation. The higher inventory levels of the actual production play yields holding costs that are eight times greater than for the model plan. However, the offsetting penalty cost of the shortage is not assessed.

For the case in which FGI is carried over at the end of the quarter, the inventory target increases to the 94th percentile of the demand distribution. This results in inventory holding costs that are close to four times greater than for the actual production plan.

### 4.2.4 Interpretation of Results

For each product family, the single-period model brackets a range of possible, level-loaded production plans by using two extreme assumptions for the treatment of quarter-end FGI. It is interesting that in each case, the actual, historical production plan and demand falls much closer to the lower end of this bracket at roughly the 50 percentile of the demand distribution.

This calls attention to the following points. First, the forecasted demand for these three examples proved to be fairly accurate. Second, the existing planning process yields
production plans in which it appears that FGI is treated as though it is perishable. While these plan may make some sense in light of the desire to keep asset utilization high (i.e., high inventory turns), they contradict the reality that the products are more durable than perishable. The low levels of FGI also conflict with the strong emphasis placed on customer service.

A single-period model formulation could be adjusted to treat the trade-off between overage and underage costs in a manner the better reflects management objectives. While quantifying these costs would be a challenge, doing so might yield an effective means of adjusting quarterly point forecasts to account for demand variability, margin, and holding costs. This could result in a quantitative replacement for the current experientially based "flex" adjustments made to current forecasts.

The single-period model results also suffer because of the assumption of a uniform production rate over the course of the quarter. Conceivably, performance might be improved by assuming some sort of standard production rate schedule, e.g., one that results in production of 20 percent of the forecast in the first month of the quarter, 30 percent in the second month, and 50 percent in the third.

Overall, the single-period model results reinforce the interpretation of the historical performance results (Section 4.1.4) by showing that in spite of the strong performance of the existing production planning process on the limited sample considered here, there is some room for improvement. It appears that perhaps the protection of customer service levels in light of the high historical demand variability is sacrificed in favor of minimizing end of quarter FGI levels. While the single-period model has some shortcomings, the concepts provide some insight into the development of a multiple-period approach.

### 4.3 Multiple Period, Stochastic Demand Model

The results in this section are obtained using the production-planning tool developed on the basis of the multiple-period, stochastic model. The tool has an intuitive user interface executed in Microsoft Excel and Visual Basic for Applications. Appendix B shows the tool's main screens, including an instruction sheet, and a sample of its output. The production plan for each of the product families are generated by running the planning tool thirteen times in succession, i.e., once for each week in the quarter, using the standard parameters described in

Section 3.3.2. Before each such run, actual demand and resulting inventory positions are entered, but no adjustment to forecast for remaining weeks is made.

The first three of the following five subsections compare the production plans from the multiple-period model with historical plans and demand profiles for the three product families. The fourth subsection contains results that demonstrate some of the effects that adjusting the model parameters have on the output. Finally, the fifth subsection contains an interpretation of the results.

### 4.3.1 Product Family No. 1

The multiple-period model production plan for Product Family No. 1 is shown in Figure 9. The key input variables, the weekly demand forecasts, proved to be fairly accurate. After the first three weeks, all actual demand levels fall within a single standard deviation of the weakly means. Generally, the model production plan has a profile that is similar to the actual production plan. Production lags behind bookings until the latter half of the quarter when some inventory is produced.

However, there are some differences between the two plans. The multiple-period model produces a plan that lags behind demand by amount closer to the acceptable two-week


Figure 9. Multiple-period model results for Product Family No. 1.
backlog constraint than does the historical plan. The model plan also delays by a week the time at which inventory production begins. In addition, the model plan maintains higher inventory levels over the last few weeks in the quarter.

The total cost of the model plan is less costly than the historical plan by only a few percent, though it is still significant in absolute dollar terms. However, the dollar difference is sensitive to policies on how penalties for shortages and end-of-quarter inventories are assessed. Inventory holding cost for the model plan is within twenty percent of that of the historical plan.

### 4.3.2 Product Family No. 2

The multiple-period model results for Product Family No. 2, shown in Figure 10, again show the basic pattern of production first trailing demand in build-to-order fashion, and then at some point moving ahead of demand in anticipation of an end-of-quarter demand surge. As is the case with the first product family, the forecasted weekly demand distribution inputs are good in the sense that the actual demand levels all fall within a standard deviation of their means. However, relative to Product Family No. 1, the standard deviations are significantly higher. This explains the greater inventories and earlier build-up depicted for the model plan in Figure 10 relative to that of Figure 9.


Figure 10. Multiple-period model results for Product Family No. 2.

In spite of the clear difference in inventory build-up shown for the actual and model production plans, the total cost difference between the two plans is relatively small. This is the case since holding costs are such a small percentage of the material driven total costs. Differences between the holding costs for the two plans, which can still be significant in absolute dollar terms, is highly sensitive to how shortage penalty cost are assessed and how quarter-end FGI is treated.

### 4.3.3 Product Family No. 3

The multiple-period model results for the third product family, shown in Figure 11, shows the familiar, basic profile associated with delayed inventory build-up. Here however, compared with the results for the first two product families, the inventory build-up is very slight and delayed until late in the quarter.

This may be partially explained by the weekly demand distribution forecasts used as input to the model. The actual demand levels for the three final weeks are more that a standard deviation from the mean. In addition, the resulting overall forecast for the quarter is the worst, over-predicting demand by more than five percent.


Figure 11. Multiple-period model results for Product Family No. 3.

In this case the total cost resulting from the model production plan exceeds actual costs by approximately ten percent. Given the build-to-stock nature of the actual production plan for this product family, it is not surprising that inventory costs exceed those of the model plan more than 20 times. However, as before, the absolute dollar difference between the two plans is sensitive to the treatment of penalty costs for shortages and end-of-quarter inventories.

### 4.3.4 Parameter Effects

As described in the preceding subsections, the costs associated with a production plan are sensitive to the parameters that establish how various penalties for shortage and end-ofquarter inventories are assessed. This section allows isolation of these parameter effects from confounding, demand fluctuation effects.

Figure 12 illustrates the effect of the backlog constraint and associated penalty costs have on a plan. The "Demand" curve is based on forecast, i.e., there is no forecast error. The "Baseline" curve is the production plan from the multiple-period model using the "standard" input parameters described in section 3.3.2. Among these parameters are a backlog constraint of two weeks and a backlog revenue loss factor of ten percent. The "Backlog Penalty" curve decreases the backlog constraint to zero weeks. In other words, any backlog


Figure 12. Effect of backlog constraint on multiple-period model results.
will be subject to a penalty cost equal to ten percent of the sales price. As shown, this plan produces in advance of demand to eliminate the penalty cost. At the end of the quarter, the FGI penalty costs dominate, driving inventory levels down. These sorts of penalty costs are not accounted for in the historical performance evaluation (Section 4.1), since no penalty costs are associated with backlog within the two-week constraint.

Figure 13 illustrates the effect of end-of-quarter FGI penalties. The forecasted demand curve and baseline curve based on "standard" inputs are the same as in Figure 12. The parameters for the baseline curve includes an end-of-quarter FGI loss factor equal to one, i.e., a penalty equal to the total variable costs of all finished goods inventory at the end of the period is assessed.


Figure 13. Effect of FGI penalty on multiple-period model results.

The "No FGI Penalty" curve shows the production plan resulting from the multipleperiod model with all end-of-quarter FGI penalties eliminated. Initial inventory levels are adjusted to equal end-of-quarter inventories to reflect the steady-state condition that would be achieved for multiple quarters with the same demand profile. As shown, without the penalty on quarter-end inventories, FGI levels are much higher to protect revenue in the face of demand uncertainty. This in some sense is how the model should be run since end-of-quarter inventory is not perishable. However, management policy dictates that inventory levels be
lowered at the end of the quarter for financial reporting purposes. These reports, based on single-day inventory levels, are not a good indication of operational performance but do perhaps affect investor reaction. In this light, there may be a real cost in terms of the company's ability to raise capital and its cost. The end-of-quarter FGI loss parameter provides a means for management to adjust the production plan to account for this cost effect.

### 4.3.5 Interpretation of Results

Results for the multiple-period model using standard input parameters consistently yields production plans with the same general profile. Initially, production trails demand in build-to-order fashion. However, at some point within the quarter, inventories are built in anticipation of the end-of-quarter demand surge.

The difference between model production plans for specific product families includes when within the quarter and how much FGI is built. The model provides a quantitative approach to production planning that accounts for such product line differences as capacity constraints, backlog (customer service) constraints, and demand uncertainty. In addition, the dynamic nature of the solution technique, in which the model is updated with actual production and demand information each week, provides much better performance than a single-period modeling approach does. As can be seen by comparing the single and multiple period model results for the third product family (Figure 8 and Figure 11), this is especially true when forecast accuracy is poor. The shortages that occur with the single-period model plan are virtually eliminated with the multiple-period approach.

However, the shape of production plan and its associated costs are sensitive to the model's input parameters. In particular, the parameters that specify the treatment of penalties for backlog constraint violation and end-of-quarter finished goods inventories are important in generating production plans that are in alignment with the business and product strategy. The model allows management to better assess tradeoffs between factors such as capacity, inventory levels, and customer service.

## 5 CONCLUSIONS

The primary objectives of this thesis are a cost performance analysis of the current production planning process and a production-planning tool to support weekly production planning decisions. The conclusion and recommendations that follow are discussed along these two lines.

### 5.1 Current Planning Process Assessment

The current production planning process yields plans that match favorably with minimum cost benchmarks. These benchmarks, based on what might be achieved with deterministic demand, suggest that there is room for total cost improvements of at most a few percent. This amount, though, may still be significant from an absolute dollar standpoint. Across the three product families, quarterly results for the multiple-period model range from a potential saving of approximately $\$ 900,000$ to a loss of $\$ 15,000$ depending on how shortage cost and inventory penalties are evaluated.

Furthermore, arguably, this is a conservative evaluation. While the historical performance for the three product families considered here is fairly strong from a total cost perspective, the sample size is clearly limited. In each case forecasts proved to be fairly accurate. It is reasonable to assume a broader sample would likely yield more extreme cases where either large shortages or high FGI levels occur. Also, as discussed in Section 4.1.4, there are some organizational incentives that lead toward the self-fulfillment of demand forecasts. This may also contribute towards masking actual shortage and FGI levels.

However, it remains unclear as to how much of the potential savings can actually be captured. The actual production plans considered in this paper include ones that resemble ones based on pure build-to-stock and build-to-order strategies. This may represent an opportunity for cost savings, or it may be a result of the current planning process accounting for product family configurability characteristics. The multiple-period planning model does not do this. Nor does it account for limitations in labor flexibility such as hiring lead-time or
a minimum permanent labor force. ${ }^{3}$ A production-planning model may reduce the overhead dedicated to the planning process, but it may require more for generating and updating weekly demand forecasts.

Regardless, there is some room for production planning improvement. Since material costs dominate total production costs, it might appear that this would be a better area to expend management resources. However, material cost reductions are hard to come by, especially in the short-term. On the other hand, production planning is an area in which operations managers may be able to exercise their abilities to achieve more immediate cost savings or better align their manufacturing and business strategies.

### 5.2 Production Planning Tool

The production-planning tool (Appendix B), developed on the basis of the multiple-period model, can be a useful input to the capacity planning process. It provides a quantitative means of deciding when and how much finished goods inventory should be built that takes into account demand variability, margins, service levels, and inventory holding costs. The tool may allow the capture of some cost savings, but perhaps more interesting is that it allows the effects of management policies on inventory turns and service levels to be evaluated. Running the model requires specification of parameters that force recognition of some of the tradeoffs involved.

In spite of these possible benefits, there are a number of challenges that need to be addressed before the tool can be implemented as a regular part of the planning process. In addition to user training and general acceptance of the tool, these include an increase in forecasting requirements, formalization of management policies on service level and inventory, and a determination of how the tool can be used in the changing business environment.

The planning tool requires input of weekly demand distributions for each product family analyzed. The current forecasting provides only quarterly point estimates of demand with no indication of variation. Early, practical feedback from potential planning tool users suggest that any requirement to do additional work such as rigorous demand analysis is likely to slow

[^2]adoption. To address this, the planning tool contains standard, historical demand distribution profiles that automatically scale to the input of a single-period demand forecast (see Appendix B). Of course, these profiles need to be updated periodically. An automated process that queries a database for historical demand data and extracts distribution parameters on some rolling-time basis is possible, but no work has been done in this regard.

Complicating the forecasting issue is that historical data may be of little value under rapidly changing business conditions. The validity of the tool's outputs is dependent on the accuracy of its inputs. Bad input data may result in the output of operational decisions that are worse than those based on intuition and experience-based rules of thumb. Developing techniques for adjusting demand to accommodate factors such as economic changes, market information, and product life cycle may be at this stage more of an art than a science. However, given the sensitivity of the production planning process to forecast, perhaps they are worth investing in.

The output of the planning tool is also sensitive to specification of the parameters that quantify how service levels and end-of-quarter finished goods inventories are treated. Currently, among the qualitative goals of the planning process are exceptional customer service and efficient asset utilization. These are measured in terms of customer delivery lead-time and inventory turnover. What does not seem to be resolved though, is that there is a direct conflict between these requirements that necessitates a policy decision to establish how the compromise should be made. This type of policy decision is necessary to provide the proper inputs to the planning tool. The planning tool could actually be used to assist in making such a tradeoff by allowing various policies to be modeled.

A final challenge to implementation of the model is the trend toward the outsourcing of manufacturing. During the course of this internship project, the three product families studied in this thesis were contracted out to suppliers. This is the result of a strategic decision to focus internal resources on new technology products for which time-to-market is critical. Commodity-like products further along in their life cycles are contracted out to bring down costs. The result is that forecasting of demand distributions for in-house products will become more uncertain, bringing into question the value of applying mathematical techniques like the one the planning tool is based on.

In this light, some thought has been given to using the planning tool as a low-effort means of assessing contract manufacturer performance for the more mature, outsourced products. Supply managers could use the tool to ascertain whether contract manufacturer inventory levels and production capacities are adequate for anticipated, end-of-quarter demand surges. Alternatively, the contractors themselves could adapt the tool for use in their own production environment.

While challenges to the implementation of the model as an effective, full-scale production-planning tool remain, the tool in its current form provides a simple means of considering the effects of alternative planning strategies and can serve as useful input to the current production planning process.

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## APPENDIX A

## SELECTED CODE LISTING

```
Public Sub MinCost()
This routine is the main solution algorithm. It initializes the InitInvarray to
a range of initial inventory positions that accommodates the solution of the multiple-
period problem. It then finds the minimum cost solution which is stored in costarray
and TargetArray.
11-23-98, W. Dutcher
'get parameters from the input sheet
TimeLeft = Worksheets("Inputs").Range("time_left").Value
InitPosition = Worksheets("Inputs").Range("init_pos").Value
UnitVarCost = Worksheets("Inputs").Range("variable_cost").Value
UnitHoldCost = Worksheets("Inputs").Range("holding_cost").Value
UnitHBPenaltyCost = Worksheets("Inputs").Range("hb_penalty_cost").Value
UnitEOQPenaltyCost = Worksheets("Inputs").Range("eoq_penalty_cost").Value
EOQFGILossFactor = Worksheets("Inputs").Range("eoq_fgilf").Value
BacklogLimit = Worksheets("Inputs").Range("backlog_constraint").Value
HardCap = Worksheets("Inputs").Range("hard_capw").Value
'range of initial inventory positions from hidden cells on the input sheet
'MaxLevel based on infinite horizon solution
'MinLevel based on "excess" capacity
MaxLevel = Worksheets("Inputs").Range("max_level").Value
MinLevel = Worksheets("Inputs").Range("min_level").Value
'Imax is the number of initial inventory positions in the range
IMax = (MaxLevel - MinLevel) / INCR + 1
'begin last period solution
'last period demand distribution parameters from hidden cells on input sheet
Alpha = Worksheets("Inputs").Range("alpha_last").Value
Beta = Worksheets("Inputs").Range("beta_last").Value
Mean = Alpha * Beta
'the closed form solution to the last period from a hidden cell on the input sheet
LastWkTarget = Worksheets("Inputs").Range("last_wk_target").Value
SolutionArray(1) = LastWkTarget 'store target in SolutionArray
For iLvl = 1 To IMax
    InitInvArray(iLvl) = MinLevel + (iLvl - l) * INCR 'initialize inventory positon
    'array
    'target must be greater than or
    'equal to the initial inventory
    'position
    Else
        Production = LastwkTarget - InitInvArray(iLvl)
        If Production <= HardCap Then 'target must be less than or
            TargetArray(1, iLvl) = LastWkTarget 'equal to the initial inventory
        Else
    'position plus the hard capacity
    'constraint
                TargetArray(1, iLvl) = InitInvArray(iLvl) + HardCap
        End If
    End If
    Target = TargetArray(1, iLvl)
    Production = Target - InitInvArray(iLvl)
    'calculate expected overage and underage for target level
    Overage = Target * GammaDist2(Target, Alpha, Beta, True) _
        - GammaExpVal(Target, Alpha, Beta)
    Underage = Mean - GammaExpVal{Target, Alpha, Beta)
        - Target * (1 - GammaDist2(Target, Alpha, Beta, True))
```

```
    'calculate costs and store in costArray
    VarCost = UnitVarcost * Production
    HoldCost = (UnitHoldCost - UnitVarCost * (1 - EOQFGILossFactor)) * Overage
    PenaltyCost = UnitEOQPenaltyCost * Underage
    CostArray(1, iLvl) = VarCost + HoldCost + PenaltyCost
    'calculate solution progress parameters and update run status form
    CurTime = Time
    SolFirstElapTime = CurTime - SolStartTime
    Progress = iLvl / IMax
    Call UpdateRunStatus(Progress, False)
Next
'solve remaining periods in reverse chronological order (from the second-to-last
'period to the first period)
For TIndx = 2 To TimeLeft
    'calculate solution progress parameters and update run status form
Progress = TIndx - 1 + 1 / IMax
Call UpdateRunStatus(Progress, False)
'get demand distribution data for current period from hidden cells on input sheet
Alpha = Worksheets("Inputs").Range("alpha_last").Offset(-TIndx + 1, 0).Value
Beta = Worksheets("Inputs").Range("beta_last").Offset(-TIndx + I, 0).Value
Mean = Alpha * Beta
'set endpoints of inventory range to provide to solution bracketing routine
AX = MinLevel
CX = MaxLevel
'routine to find an interval which defines a minimum
Call BracketMin(AX, BX, CX, 0, TIndx - 1, Alpha, Beta, UnitVarCost,
    UnitHoldCost, UnitHBPenaltyCost, BacklogLimit, FlagEndPt, Progresss)
If FlagEndPt = False Then
    'if the minimum does not occur at an endpoint, call a routine to find the target
    'that yields the minimum cost
    Target = GetTarget(AX, BX, CX, 0, TIndx - 1, Alpha, Beta, UnitVarCost, _
            UnitHoldCost, UnitHBPenaltyCost, BacklogLimit, Progress)
Else
    'the target occurs at an endpoint and was returned as BX from BracketMin
    Target = BX
End If
SolutionArray(TIndx) = Target 'store target in solution array
For iLvl = 1 To IMax
    If InitInvArray(iLv1) >= Target Then 'target must be greater than or equal to
                            'the initial inventory position
        TargetArray(TIndx, iLvl) = InitInvArray(iLvl)
    Else
        Production = Target - InitInvArray(iLvl)
        f Production <= HardCap Then 'target must be less than or equal to
            TargetArray(TIndx, iLvl) = Target 'the initial inventory position plus
        Else 'the hard capacity constraint
            TargetArray(TIndx, iLvl) =
                    InitInvArray(iLvL) + HarrdCap
        End If
    End If
    'calculate costs and store in CostArray
    CostArray(TIndx, iLvl) = TotalCost(TargetArray(TIndx, iLvl),
        InitInvArray(iLvl), TIndx - 1, Alpha, Beta, UnitVarCost, -
        UnitHoldCost, UnitHBPenaltyCost, BacklogLimit)
```

```
        'calculate solution progress parameters and update run status form
        Progress = TIndx - 1 + iLvl / IMax
        Call UpdateRunStatus(Progress, True)
        Next
    Next
    Unload FormRunStatus 'unload the run status form since numerical solution is complete
End Sub
Public Sub BracketMin(AX, BX, CX, InitInvLevel, ValCol, Alpha, Beta,
    UnitVCost, UnitHCost, UnitPCost, BacklogLimit, FlagEndPt, Progress)
1 This routine returns a minimum bracketed by AX and CX for input to the
' the isolation routine.
1 11-23-98, W. Dutcher
    FlagEndPt = False
    FA = TotalCost(AX, InitInvLevel, ValCol, Alpha, Beta, -
        UnitVCost, UnitHCost, UnitPCost, BacklogLimit)
    FC = TotalCost(CX, InitInvLevel, ValCol, Alpha, Beta, _
        UnitVCost, UnitHCost, UnitPCost, BacklogLimit)
    Do
        BX = WorksheetFunction.RoundUp((AX + CX) / 2, INCR_EXP)
        If BX = AX Or BX = CX Then
            If FA < FC Then
                    BX = AX
            Else
                    BX = CX
            End If
            FlagEndPt = True
            Exit Do
        End If
        FB = TotalCost(BX, InitInvLevel, ValCol, Alpha, Beta, _
            UnitvCost, UnitHCost, UnitPCost, BacklogLimit)
        If FB<EA Then
            If FB<EC Then
                    Exit Do
            Else
                    AX = BX
                    FA = FB
                    GoT0 Line2
            End If
        Else
            CX = BX
            FC=FB
        End If
Line2:
    Call UpdateRunStatus(Progress, False)
    Loop
End Sub
Public Function TotalCost(Target, InitInvLevel, ValCol, Alpha, Beta, _
    UnitVCost, UnitHCost, UnitPCost, BacklogLimit)
    This function returns the total production cost.
    11-23-98, W. Dutcher
    Mean = Alpha * Beta
    VolBacklogLimit = BacklogLimit * Mean
    Production = Target - InitInvLevel
    Overage = Target * GammaDist2(Target, Alpha, Beta, True) _
        - GammaExpVal(Target, Alpha, Beta)
    Underage = Mean - GarmaExpVal(Target, Alpha, Beta)
        - Target * (1 - GammaDist2(Target, Alpha, Beta, True))
```

```
    VarCost = UnitVCost * Production
    HoldCost = UnitHCost * Overage
    If Underage > VolBacklogLimit Then
        PenaltyCost = UnitPCost * (Underage - VolBacklogLimit)
    Else
    PenaltyCost = 0
End If
FutureCost = ExpFutureCostGamma(Target, ValCol, Alpha, Beta)
TotalCost = VarCost + HoldCost + PenaltyCost + FutureCost
End Function
```

```
Public Function ExpFutureCostGamma(Target, ValCol, Alpha, Beta)
P
    This function returns expected future costs by performing a numerical
    integration using the trapezoidal rule.
    11-23-98, W. Dutcher
    Mu = Alpha * Beta
    Sigma = (Alpha * Beta ^ 2) ^ (0.5)
    LoLimit = WorksheetFunction.Max(0, -DEV INFINTY * Sigma + Mu}
    If LoLimit = 0 Then
        IntRange = WorksheetFunction.Round((Mu / Sigma + DEV_INEINTY) * STD_DEV_DIV, 0)
    Else
        IntRange = 2 * DEV_INFINTY * STD_DEV_DIV
    End If
    Delta = Sigma / STD_DEV_DIV
    ExpFutureCostGamma = 0
    ValPrior = GetFuncVal((Target - LoLimit), InitInvArray, CostArray, ValCol)
    fx = GammaDist2(LoLimit, Alpha, Beta, False)
    ValPrior = ValPrior * fx
    For i = 1 To IntRange
        XVal = LoLimit + Delta * i
        ValCur = GetFuncVal((Target - XVal), InitInvArray, CostArray, ValCol)
        fx = GammaDist2(XVal, Alpha, Beta, False)
        ValCur = ValCur * fx
        ExpFutureCostGamma = ExpFutureCostGamma + ((ValCur + ValPrior) / 2) * Delta
        ValPrior = ValCur
    Next
End Function
```

Public Function GetFuncVal(InpVal, XArray, YArray, ValCol)
This function returns the value of a function defined by XArray and YArray
for an input value of InpVal. YArray can be a multi-dimensional array defining
a series of functions. The function returned is determined by ValCol. XArray
is a one-dimensional array containing the abscissa values.
11-23-98, W. Dutcher
LoVal $=$ XArray (1)
HiVal = XArray(IMax)
If InpVal < LoVal Then
$m=($ YArray (ValCol, 2) - YArray (ValCol, 1)) / (XArray (2) - XArray(1))
GetFuncVal $=m$ * (InpVal - XArray (1)) + YArray(ValCol, 1)
ElseIf InpVal $>$ HiVal Then
MsgBox ("Input Value to GetFuncVal Above Range: " \& InpVal \& " >" \&
HiVal)
Stop
Else

```
        IVal1 = LoVal
        IStop = False
        indx = 2
        Do While IStop = False
            If indx > IMax Then MsgBox ("This shouldn't happen!")
            IVal2 = XArray(indx)
            If InpVal <= IVal2 Then
                m = (YArray(ValCol, indx) - YArray(ValCol, indx - I)) / (IVal2 - IVal1)
                GetFuncVal = m * (InpVal - IVal1) + YArray(ValCol, indx - 1)
                IStop = True
            Else
                indx = indx + 1
                IVal1 = IVal2
        End If
            Loop
    End If
End Function
```


## APPENDIX B

## PRODUCTION PLANNING TOOL



Figure B-1. Main Input Screen.

STANDARD DEMAND PROFILES

| Edit | Add | Delete |
| :---: | :---: | :---: |
| Family No. 1 |  |  |
| Week | Mean | Std Error |
| 1. | 3.000 | 0.599 |
| 2 | 6. 125 | 0.577 |
| 3 | 6.125 | 0.555 |
| 4 | 6. 125 | 0.534 |
| 5 | 6.125 | 0.512 |
| 6 | 6.125 | 0.491 |
| 7 | 6.125 | 0.469 |
| 8 | 6.125 | 0.447 |
| 9 | 6.125 | 0.426 |
| 10 | 9.000 | 0.404 |
| 11 | 9.000 | 0.383 |
| 12 | 13.000 | 0.361 |
| 13 | 17.000 | 0.339 |
| Total: | 100.000 |  |


| Family |  |  |
| ---: | :---: | ---: |
| No. 2 |  |  |
| Week | Mean | Std Error |
|  | 3.000 | 0.936 |
| 2 | 6.125 | 0.918 |
| 3 | 6.125 | 0.900 |
| 4 | 6.125 | 0.882 |
| 5 | 6.125 | 0.864 |
| 6 | 6.125 | 0.846 |
| 7 | 6.125 | 0.828 |
| 8 | 6.125 | 0.810 |
| 9 | 6.125 | 0.791 |
| 10 | 9.000 | 0.773 |
| 11 | 9.000 | 0.755 |
| 12 | 13.000 | 0.737 |
| 13 | 17.000 | 0.719 |
| Total: | 100.000 |  |






Figure B-2. Standard Demand Distribution Profile Screen.

## INSTRUCTIONS

This model is intended to provide an input to the capacity planning process. It is designed to be run on a weekly basis to incorporate the latest information on demand, production rates, and inventory levels. Instructions on how to operate the model are provided below.

## INPUT PARAMETERS

The input sheet is divided into six groups. To change the input values for a particular group, press the appropriate edit button. Following are specific instructions and parameter definitions related to each group.

## Time Horizon/Demand Forecast

Enter a value for the number of weeks remaining in the quarter using the spin-button control and press the OK/Next button to display the Demand form. There are two means of entering a demand profile:
(1) Select each week in the quarter using the spin-button control and enter non-zero values for the mean and standard deviation. Note that existing values will be displayed if available. When complete, click the OK button.
(2) Press the Load Standard button to display the Standard Demand Profile form. Select an available profile name using the spin-button control. Enter a value for the demand remaining in the quarter and click the OK button. Note - Standard demand distributions are discussed below.

Demand values are rounded and stored to the nearest tenth but displayed to the nearest integer.

## Inventory/Backlog

Enter the values as defined here and click the OK button.
Backlog constraint - If the number of units in backlog is greater than the expected demand in this period, a penalty cost is applied (the penalty cost is discussed further under "Financial"
Initial inventory position - The amount of finished goods inventory at the beginning of the time period being analyzed.

## Labor

Enter the four labor-related parameters and click the OK button when complete. Note that the value for work hours should reflect hours in a single shift.

## Production Equipment

Enter the value for hard capacity and click the OK button. This equipment related capacity constraint is expressed in terms of units/day and should reflect total capacity for all work shifts combined (e.g., if the equipment allows 200 units/shift and there are two shifts/day, the hard capacity should be set to 400 units/day).

## Product

Enter the values for the material cost and selling price of the product and click the OK button.

Figure B-3. Instruction Screen No. 1.

## Finance

Enter the values as defined here and click the OK button.
Holding rate - The cost of holding finished goods inventory expressed as an annual percentage rate. Reflects cost of capital, administration, insurance, storage, damage, obsolescence, etc.

Revenue loss factor - Penalty cost due to shortages within the quarter expressed as a fraction of the selling price (limited to the range from 0 to 1 ). So, for example, if this value is set to 0.5 , then a cost of $50 \%$ of the selling price will be applied to each unit of unsatisfied demand. Note that unsatisfied demand is still backlogged and not lost.
End of quarter revenue loss factor-Similar to the above revenue loss factor except this factor is applied to the shortage existing at the end of the time horizon. It is expected that this factor will generally be the larger of the two, reflecting a more severe penalty for letting potential revenue slide into the next quarter. In addition, the range of acceptable values for this factor is more restricted. While the upper limit remains 1 , the lower limit is dependent on the selling price and variable costs. An automatic check for acceptability of the value entered is performed when the solution is initiated.

End of quarter finished goods inventory loss factor - Fraction of the lost variable cost of the finished goods inventory at quarter end (limited to the range from 0 to 1). For example, if this parameter is set to 1, then all the variable cost associated with the end of quarter finished goods inventory is applied (i.e., as if the FGI is written off). At the other extreme, if this parameter is 0 , these costs are not included (i.e., as if the FGl is carried over and sold in the following quarter).

## SOLUTION

Once the input parameters are specified, click on the Press to Solve button. As mentioned in the description for the end of quarter revenue loss factor, some automatic checks are performed to ensure a solution. If prompted, adjust the necessary parameter and click the Press to Solve button again. Once started, a form will display the status of the solution algorithm.

Upon completion, a new workbook will be opened. This workbook contains a sheet labeled "Output" and another labeled "Graph." The Output sheet contains the following information:

## This week's actions

Contains the build plan for the current week.

## Remainder of quarter expectations

Contains expected cost of the plan for the remainder of the quarter. Also shown are the expected inventory/backlog levels, labor requirements, and capacity shortfall (should the optimum solution be constrained by hard capacity limitations). The demand mean and standard deviation shown are those inputted by the user and are repeated here for clarity.

## Input parameters

Lists all the input parameter less the demand distribution (which is shown as part of the "Remainder of quarter expectations").

## Solution times

Lists the date, start time, finish time, and elapsed time of the solution.

Figure B-4. Instruction Screen No. 2.

## STANDARD DEMAND DISTRIBUTIONS

As described above, standard demand distributions can be used. These user-defined distributions are stored on the sheet labeled "Demand Distributions." For each distribution, weekly values of the mean expressed as percentage of the total quarter demand and the standard error (standard deviation/mean) are listed. Plots of these values are also provided. Standard distributions can be edited, added, or deleted using by clicking on the appropriate button.

## Edit

Choose the profile name to be edited using the spin-button control and click the OK button. Modify the name, mean, or standard error values and click OK. Note that values are rounded to the nearest thousandth and the weekly mean values must total to 100 (remember the mean values are entered as a percentage of the total quarter demand).

## Add

Enter the name, mean, and standard error values and click OK. Note that values are rounded to the nearest thousandth and the weekly mean values must total to 100 (remember the mean values are entered as a percentage of the total quarter demand).

## Delete

Choose the profile name to be deleted using the spin-button control and click the OK button.

Figure B-5. Instruction Screen No. 3.

## Output

This week's actions
Build Plan 0.00

| Solution Times |  |
| ---: | ---: |
| Date: | $03 / 27 / 99$ |
| Start: | $16: 31: 19$ |
| Finished: | $16: 42: 34$ |
| Elapsed: | $0: 11: 15$ |

Remainder of quarter expectations

| Cost | \$5,433,120 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Week | Demand Forecast [units] | Standard Deviation [units] | Target [units] | TnitialFGI <br> (Backlog) [units] | Build [units] | FinalFGI (Backlog) [units] | $\begin{gathered} \text { Labor } \\ \text { Required } \\ \text { [workers] } \end{gathered}$ | Capacity Shortfall [units] |
| 1 | 381.6 | 228.6 | 125.0 | 125.0 | 0.0 | -256.6 | 0.0 |  |
| 2 | 779.2 | 449.6 | -252.4 | -256.6 | 4.2 | -1031.6 | 0.1 |  |
| 3 | 779.2 | 432.5 | -359.9 | -1031.6 | 671.7 | -1139.1 | 9.0 |  |
| 4 | 779.2 | 416.1 | -439.9 | -1139.1 | 699.3 | -1219.1 | 9.3 |  |
| 5 | 779.2 | 399.0 | -500.1 | -1219.1 | 718.9 | -1279.3 | 9.6 |  |
| 6 | 779.2 | 382.6 | -583.5 | -1279.3 | 695.8 | -1362.7 | 9.3 |  |
| 7 | 779.2 | 365.4 | -654.4 | -1362.7 | 708.3 | -1433.6 | 9.4 |  |
| 8 | 779.2 | 348.3 | -709.5 | -1433.6 | 724.1 | -1488.7 | 9.7 |  |
| 9 | 779.2 | 331.9 | 192.7 | -1488.7 | 1681.3 | -586.5 | 22.4 |  |
| 10 | 1144.9 | 462.5 | 1119.5 | -586.5 | 1706.1 | -25.4 | 22.7 |  |
| 11 | 1144.9 | 438.5 | 2004.8 | -25.4 | 2030.2 | 859.9 | 27.1 |  |
| 12 | 1653.7 | 597.0 | 2943.3 | 859.9 | 2083.3 | 1289.6 | 27.8 |  |
| 13 | 2162.6 | 733.1 | 2249.0 | 1289.6 | 959.4 | 86.4 | 12.8 |  |

Input parameters for this solution (with demand forecast shown above)

| Time Horizon |  |
| :---: | :---: |
| Time Left in Quarter | 13 [weeks] |
| Inventory/Backlog |  |
| Backlog Constraint | 2.00 [weeks] |
| Initial Inventory Position | 125.00 [units] |
| Labor |  |
| Labor Rate | 16.00 [\$/hr] |
| Work Hours | 8.00 [hr/day] |
| Work Days | 5.00 [day/wk] |
| Productivity | 15.00 [units/day] |
| Labor Cost | 8.53 [\$/unit] |
| Production Equipment |  |
| Hard Capacity | 500.00 [units/day] |
| Weekly Hard Capacity | 2500.00 [units] |
| Product |  |
| Material Cost | 400.00 [\$/unit] |
| Selling Price | 1000.00 [\$/unit] |
| Financial |  |
| Holding Rate | 18.00 [\%/year] |
| High Backlog Rev Loss Factor | 0.10 [fraction] |
| End of Qtr Revenue Loss Factor | 1.00 [fraction] |
| End of Qtr FGl Loss Factor | 1.00 [fraction] |
| Variable Cost | 408.53 [\$/unit] |
| Weekly Holding Cost | 1.38 [\$/unit] |
| High Backlog Penalty Cost | 100.00 [\$/unit] |
| End of Qtr Penalty Cost | 1000.00 [\$/unit] |

Figure B-6. Solution Screen.

Remainder of Quarter Expectations

-Production = - - Demand

| Week in <br> Quarter | Weekly <br> Demand | Cumulative <br> Demand | Weekly <br> Production | Cumulative <br> Production |
| ---: | ---: | ---: | ---: | ---: |
| 0 | 0 | 0 | 0 | 125 |
| 1 | 382 | 382 | 0 | 125 |
| 2 | 779 | 1161 | 4 | 129 |
| 3 | 779 | 1940 | 672 | 801 |
| 4 | 779 | 2719 | 699 | 1500 |
| 5 | 779 | 3498 | 719 | 2219 |
| 6 | 779 | 4278 | 696 | 2915 |
| 7 | 779 | 5057 | 708 | 3623 |
| 8 | 779 | 5836 | 724 | 4347 |
| 9 | 779 | 6615 | 1881 | 6029 |
| 10 | 1145 | 7760 | 1706 | 7735 |
| 11 | 1145 | 8905 | 2030 | 9765 |
| 12 | 1654 | 10559 | 2083 | 11848 |
| 13 | 2163 | 12721 | 959 | 12808 |

Figure B-7. Solution Plot Screen.


[^0]:    ${ }^{1}$ Bay Networks and Northern Telecom officially merged at the end of August 1998. Soon after, Northern Telecom officially changed its brand name to Nortel Networks. Bay Networks is now a Nortel Networks business unit.

[^1]:    ${ }^{2}$ During the course of the project all three families were outsourced as a result of a manufacturing strategy shift which focused internal capabilities on newer technology products.

[^2]:    ${ }^{3}$ However, labor costs are such a small fraction of total costs that even treating them as fixed at the maximum level required in the quarter does not significantly affect the resulting production plan.

