Power Allocation of Multi-Rate Transmissions over a Jammed Broadcast Channel

by

John L. Benko

Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of

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Abstract

This thesis deals with the Power Allocation of Multi-Rate Multi-Priority Transmissions in a hostile environment. Previous efforts have examined systems with data-streams of different bit-rates but none, to the authors knowledge, have explored multi-rate data-streams with different priorities. A definition of a prioritizing scheme is given and analyzed in two jammer environments. First, the jammer is modeled as full-band additive white Gaussian noise. Second, the jammer is modeled using optimal partial band jamming. Spread Spectrum is used with non-coherent modulation/demodulation in order to hide the transmitted data-streams as well as improve overall system performance.

Thesis Supervisor: Vincent W. S. Chan
Title: Joan and Irwin Jacobs Professor of EECS and Aero/Astro

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I would like to thank my family for all their support, especially my mother and father for making it possible for me to attend this institution.

This thesis would not exist without the guidance and helpful suggestions of my advisor, Vincent Chan.

I dedicate this thesis to my father, Laszlo Benko (February 24, 1931 - August 16, 1999), who gives me the example and motivation to pursue my dreams and accomplish unbounded goals. Thanks Dad.

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Chapter 1

Background

1.1 Introduction

Reliable communications in the presence of jamming signals is critical for the successful operation of many military, as well as commercial, operations in today's technologically advanced society. A specific problem that presents itself is the robust one-way communication of multiple messages of varying importance in the presence of a jammer. A real world example of this might arise in secure military communications from a base command center or communications satellite to a unit of soldiers in a tactical theater. In this case, the top priority message might be mission instructions from the Pentagon as to where to attack next. Information of secondary importance might include maps of the surrounding area, to facilitate finding certain strategic locations. The lowest priority message might be a video signal of the President addressing the nation.

This thesis will analyze efficient and effective power allocation and modulation/ demodulation strategies used to transmit multi-priority multi-rate data streams over a jammed broadcast channel. The word data-stream will be used here on instead of message, since message implies finite length data, where data-networking issues are relevant. Data-networking will not be discussed here.

In secure communications over a hostile channel, the best way to mitigate the effects of a jamming signal is to avoid it completely. This might be accomplished

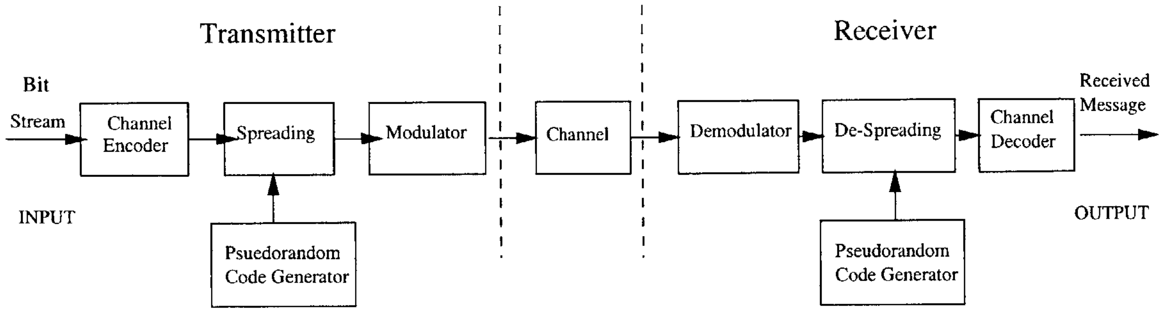

Figure 1-1: Block Diagram of a Spread Spectrum Communications System.

by hiding or disguising the transmitted signal in the background noise, so that the jammer does not know in which frequency to transmit. A popular and effective means of accomplishing this is through spreading the signal energy over a large bandwidth. This spreading decreases the magnitude of the power spectral density of the signal in the occupied bandwidth, so that it resembles noise. With the signal resembling noise, the jammer's receiver will have a low probability of intercepting the signal(LPI). LPI systems, as they relate to anti-jamming scenarios, are discussed in detail in [3]. Numerous methods have been developed to accomplish this task, and combined they carry the name of spread spectrum. Figure 1-1 shows a block diagram of a general spread spectrum communications system.

1.2 Spread Spectrum

Spread spectrum also has the added bonus of a low probability of detection or demodulation (LPD.) Once a receiver detects that a signal is present, the next step is demodulation. LPD is achieved by spreading the signal with a pseudo-random binary code, usually produced by a feedback shift register. Extensive research and theory has been developed in designing shift registers and there are many books and papers that proficiently cover shift register design and implementation. An example of a linear shift register is shown in Figure 1-2. The multipliers $a_0, a_1, \ldots, a_{m-1}$ take on values of either (0,1). The designing of a linear feedback shift register requires choosing values for the multipliers and the initial values in the registers $R_0, R_1, \ldots, R_m$. In order to correctly de-spread the signal, the receiver must know the topology of the

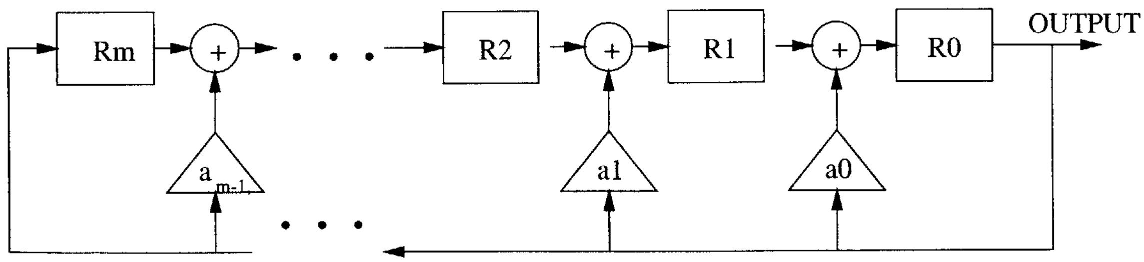

Figure 1-2: Linear Feedback m-state Shift Register

particular shift register used to spread the signal as well as the initial values loaded into the register. As long as the unintended receiver doesn't have these two vector variables, it remains virtually impossible for the jammer to de-spread the signal. It is virtually impossible to successfully demodulate a spread signal, if it is properly designed. However, it is relatively easy for a jammer to determine the variables from a section of the output sequence of a linear shift register. For this reason, non-linear feedback shift registers are more commonly used, since it is virtually impossible to determine the topology of a non-linear feedback register from the output sequence. For a good introduction to shift registers and the generation of pseudo-random codes see [7, 4].

There are three major spreading techniques: Direct Sequence, Frequency Hopped and Time-Hopped. In Direct-Sequence Spread Spectrum (DS-SS) a waveform is produced from a pseudo-random binary code (values (+1,-1)) of length N. The period of each bit in the waveform (called a chip) is $T_c = T_b/N$, where T_b is the period of the unencoded bit stream. This waveform is then multiplied to the original bit stream waveform, to produce the spread sequence (see Figure 1-3.) Note that since the new bit (chip) period T_c is N times smaller than the original bit period T_b, the coded (or spread) spectrum has a bandwidth equal to N times the original bandwidth. This SS system is said to have a processing gain of N since a jammer, transmitting band-limited Gaussian noise, must increase the signal power by N in order to maintain the same bit error rate (BER). DS-SS is used mainly with coherent detection. The focus of this thesis will be on systems with non-coherent detectors (for reasons to be explained later), so DS-SS will not be considered any further.

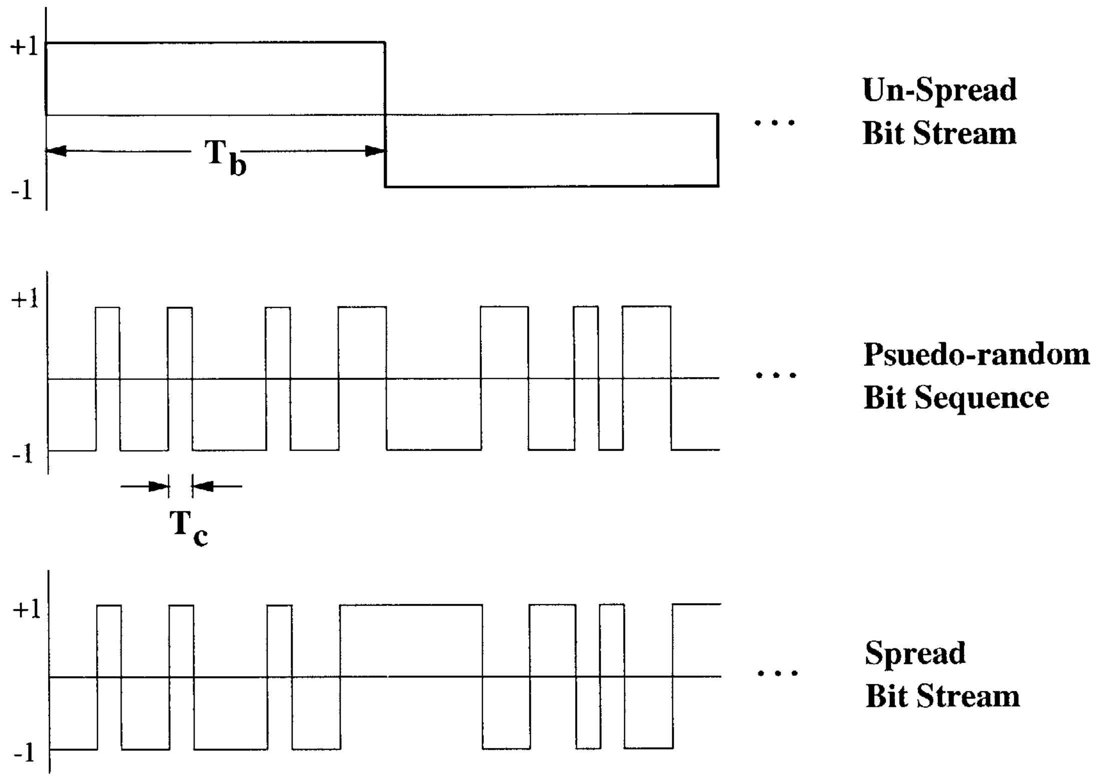

Figure 1-3: Direct-Sequence Spreading

The most commonly used spreading technique with non-coherent detection is Frequency Hopped Spread Spectrum (FH-SS). Here, bandwidth spreading is achieved by pseudo-randomly hopping the carrier frequency so that the signal appears to have a larger bandwidth. Again, the pseudo-random hopping patterns are produced by a feedback shift register. An FH-SS system with processing gain of N, has N possible frequency slots in which to transmit. The frequency slots do not necessarily have to be adjacent to one another, as they can exist in different frequency bands. Figure 1-4 shows an example of FH transmissions.

FH-SS is sub divided into two types. The difference between the two hopping schemes lies solely on the frequency hop rate, $R_h = \frac{1}{T_h}$. When the FH rate is faster than the symbol rate (or $T_s > T_h$), it is called Fast Frequency Hopped Spread Spectrum (FFH-SS). Here the frequency carrier, controlled by a digitally controlled variable controlled oscillator (VCO), hops over multiple frequency slots during the period of one symbol transmission. When the hop rate is slower than or equal to the symbol

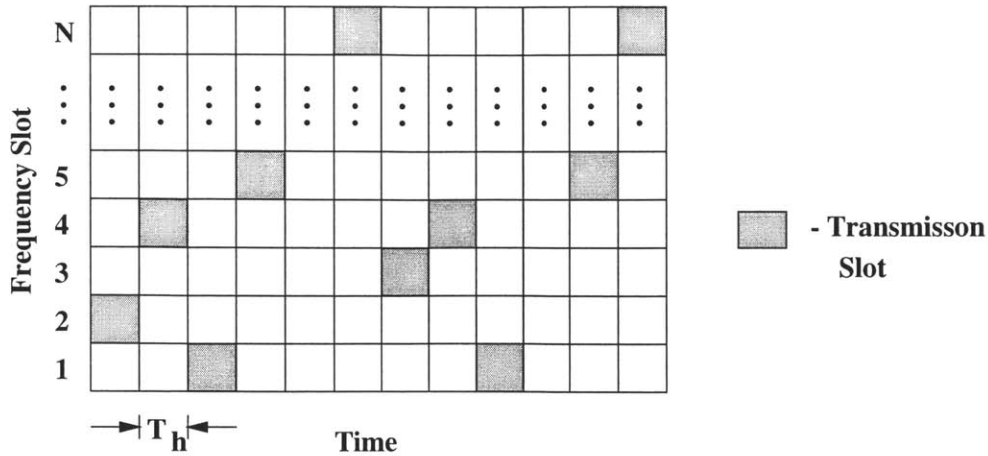

Figure 1-4: FH transmissions with adjacent frequency

rate (or $T_s < T_h$), it is called Slow-Frequency Hopped Spread Spectrum (SFH-SS). In this scheme, there is one or more symbols transmitted in each frequency slot. Caution must be used when choosing R_h, for if the hop rate is too slow, it is conceivable for an intelligent intercept system to follow the hopping, and effective spreading does not occur. The differences in performance between the FFH-SS and SFH-SS systems are seen primarily in complex channel models, that include fading or multi-path propagation effects. Channel fading will not be discussed and so FFH-SS and SFH-SS will have identical performances here.

The last type of spreading technique discussed here will be Time Hopped Spread Spectrum (TH-SS). This is the time dual of FH-SS. Instead of hopping about frequency slots, the time axis is sub-divided in N time-slots into which one slot is randomly selected for transmission. In order to maintain the same data rate as the original message, the symbol period must be reduced by N. This symbol period reduction by N increases the signal bandwidth by N. The pseudo-random output sequence of a shift register is used to decide into which time-slot to transmit. Since the period of the pulse gets smaller, synchronization effects become more of a factor here. In order to detect a shorter pulse length the synchronization of the demodulator has to be much more precise. Slight offsets in the time synchronization may cause the demodulator to sample the signal when the modulator is not transmitting, resulting in errors. In this respect, FH-SS systems are easier to implement and a favorite among SS system designers.

Hybrid systems can also be devised that have both TH-SS and FH-SS. In these systems there are N frequency slots and M time slots. This increases the signal bandwidth NM times. Depending on the nature of the jammer's signal, hybrid systems might prove to give good anti-jam protection. For the simple channel models discussed here, hybrid systems have no advantage over the conventional SS systems.

1.3 Modulation/Demodulation

Non-coherent demodulation will be used in the systems here. In a hostile communications environment jamming signals can easily disrupt the operation of an intended receivers phase-tracking device, such as a phase-locked-loop (PLL), forcing it into an unlocked state. In coherent demodulators, when the phase of the carrier signal cannot be determined (which is the result of an unlocked PLL), the required matched filtering operation does not get successfully performed, therefore rendering the detection sub-optimal. In fact, most coherent demodulators will not perform the maximum likelihood detection until the PLL is locked. Non-coherent demodulators do not utilize phase information because envelope or square-law detectors are used. Envelope detection takes the square root of the sum of the squares of the orthogonal (quadrature) components of the signal; that is, the magnitude of the signal. This manifests itself in completely different decoding statistics, from that of coherent detectors (which are based on the Q-function.) Jamming signals with Gaussian statistics when passed through an envelope detector yield Rayleigh statistics. The Rayleigh probability density function (pdf) is:

$$f_r(r) = \frac{r \cdot e^{-\frac{r^2}{2\sigma^2}}}{\sigma^2} \tag{1.1}$$

When a jamming signal of band-limited Gaussian noise is added to a signal and passed through envelope detection, Ricean statistics are obtained. The Ricean pdf is:

$$f_r(r) = \frac{r \cdot e^{-\frac{r^2+A^2}{2\sigma^2}}}{\sigma^2} I_0(\frac{rA}{\sigma^2}) \tag{1.2}$$

Where I_0 is a zero-order modified Bessel Function of the first kind. The derivation of these pdfs are given in Appendix A.

Two types of modems (modulators/demodulators) will be analyzed. The first is Frequency Shift Keying (FSK), of which the most basic form is Binary Frequency Shift Keying (BFSK). In BFSK, transmission occurs with two distinct frequency carriers. It operates by transmitting a pulse in frequency slot 1 if the bit has value '0' or a pulse in frequency slot 2 if the bit value is '1'. The associated matched filter demodulator is a bank of four orthogonal filters followed by a sampler, an envelope detector, a summer, and a comparator. Figure 1-5 shows a correlation demodulator for BFSK. From Appendix A, the BER for BFSK is:

$$P_b = \frac{1}{2} e^{-\frac{E_b}{2N_T}} = \frac{1}{2} e^{-\frac{p}{2R(N_T)}} \tag{1.3}$$

Where E_b is the energy per bit, N_T is the total noise power spectral density, R is the rate, and p is the power transmitted, where $p = E_b R$.

This can be extended from two frequency slots to M frequency slots (MFSK.) The symbol error rate for MFSK from [1] is:

$$P_s = \sum_{k=1}^{M-1} \frac{(-1)^{k+1}}{k+1} \begin{pmatrix} M \\ k \end{pmatrix} e^{-\frac{k}{k+1}(\frac{E_b}{N_T})} \tag{1.4}$$

By a combinatorial argument $P_b = \frac{M}{2(M-1)} P_s$. With M slots, each pulse transmission yields $\log_2 M$ bits of information. This implies that $E_b = \frac{E_s}{\log_2 M}$, where E_s is the energy transmitted per symbol.

In the upcoming analysis, it will be required to solve explicitly for the power p, from the BER rate equations. This is not possible for MFSK, since the BER involves a sum of exponentials that cannot be factored. For this reason, only BFSK will be

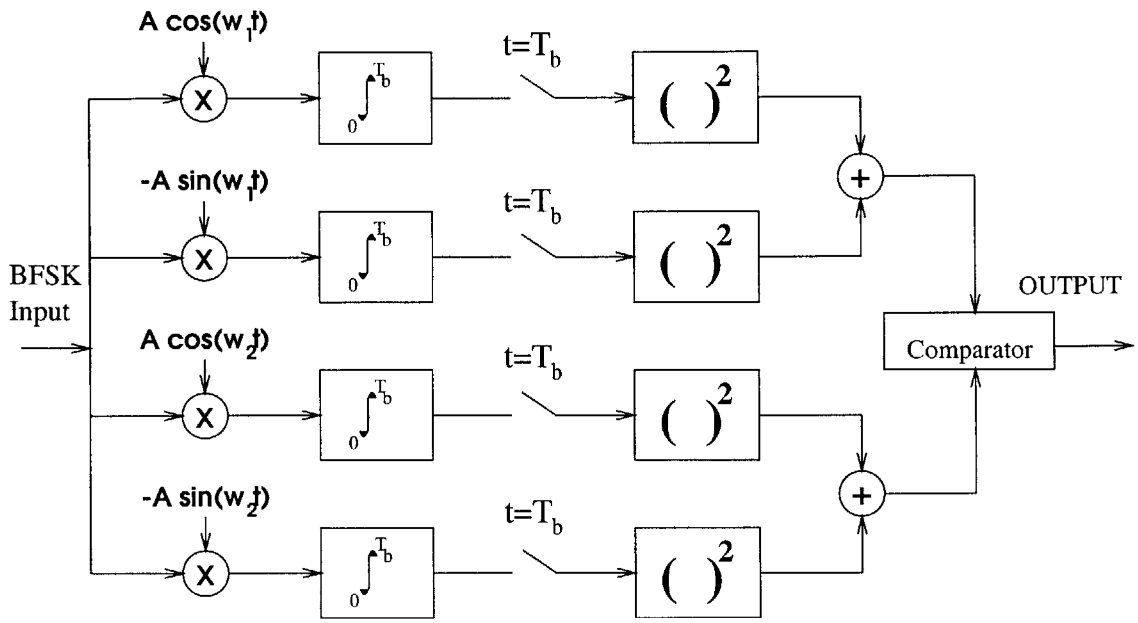

Figure 1-5: Correlation Demodulator for BFSK

considered in the analysis of chapter 2.

Pulse Position Modulation (PPM) is the time dual of MFSK. The time axis is divided into M slots and a time pulse is transmitted in one of them, so that $T_S = MT_p$. In Binary-PPM there are 2 adjacent time slots, $T_b = 2T_p$. If the incoming message bit has a value of '0' then a pulse is transmitted in the first slot, otherwise a pulse is transmitted in second. The associated demodulator is a correlation receiver that integrates, samples twice, and then compares the sampled values. If the first sampled value is larger, a '0' is declared. A diagram of a M-ary PPM demodulator is shown in Figure 1-6. Since the noise is modeled as white Gaussian, the register values in Figure 1-6 from different time samples are uncorrelated and orthogonal, just as in FSK. Because of this, the BER equations are exactly the same as in FSK(see Appendix B).

Just as in MFSK, each pulse transmission yields $\log_2 M$ bits, resulting in a modulation rate of $\frac{\log_2 M}{M}$. The rate is the ratio of number of input bits used, over the number of output pulse-periods used to transmit the information (provided that the pulse period remains constant.) See Figure 1-7 for an example transmission pattern for 8-PPM. In this example the rate is $\frac{3}{8}$, that is, for every 3 input message bits, 8

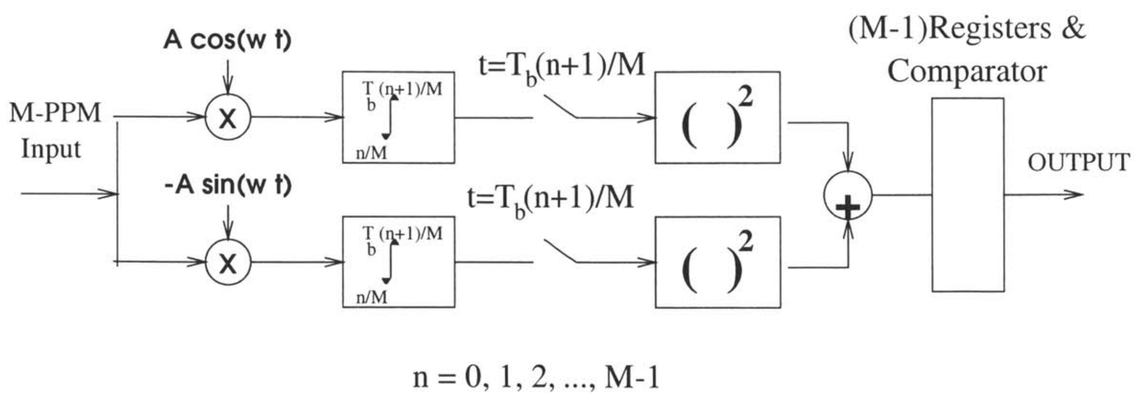

Figure 1-6: Correlation Demodulator for M-ary PPM

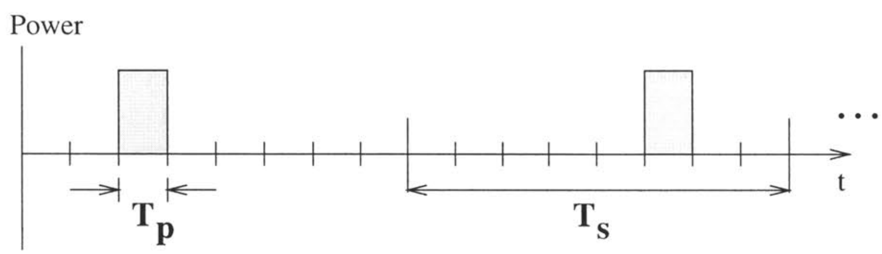

Figure 1-7: Example of transmission progression of 8-PPM

output bits are produced.

1.4 Characteristics of the Jammer

To simplify analysis, only a few types of jamming signals will be considered. The signals will be derived from Additive White Gaussian Noise (AWGN.) Full band continuous jamming along with pulsed jamming and partial band jamming will be considered. The benefits of using the AWGN model are; one, a simplified analysis and, two, possible application to multi-user communications.

In the design of anti-jam (AJ) systems, the jammer is assumed to have a complete characterization of the communications system (i.e knowledge of the type of modulation, spreading, and frequency location). It has all the information that the intended receiver has, except for pseudo-random spreading code generator. It is the randomness of the spreading code, that gives the AJ advantage of SS.

Given a specific jammer model, the optimal signal power of each message will be determined in order to obtain the best overall system performance. Since the receiver is in a hostile environment, transmitting will not be allow, and therefore feedback or channel equalization cannot be utilized. Channel coding will also not be considered; however, it is well known that gains of around 3dB can be achieved with appropriate channel coding.

Chapter 2

Power Allocation

2.1 Formulation

The objective of this thesis is to find power allocation methods that give good BERs for different data-streams of varying importance. The first question to address is how to quantitatively assign importance values to the data-streams. This depends mostly on the specific application.

A certain application might require minimum BERs for the given data-streams. If this is the case, and the data-streams are assigned such that the most important objective is that data-stream 1 maintains a minimum BER, followed by data-stream 2 and so on, a simple solution exists. After a characterization of the jammer noise is made, power is allocated to data-stream 1 such that the minimum BER is barely ensured. After the minimum BER for data-stream 1 is satisfied the process is repeated for data-stream 2. This process is continued until either all the available power is used, or the minimum BERs for all data-streams are satisfied. If the latter is satisfied, the remaining power can be divided up in a number of ways; however, this is not relevant, since the original problem presented has already been solved.

The solution to the specific problem above is quite obvious and subsequently not very interesting. A new formulation of the power allocation problem is now posed that will be used here on. It is based on assigning costs to each data-stream, and then minimizing the sum of the BERs, weighted by the costs. With these assignments, we can reduce the power allocation problem to a non-linear programming optimization problem.

M independent data-streams of rates $R_1, R_2, \ldots, R_M$ with corresponding importance values $I_1, I_2, \ldots, I_M$, are broadcasted through a corrupted channel. The rates will be indexed such that $I_1 \geq I_2 \geq \ldots \geq I_M$(i.e. data-stream 1 will be the most important, followed by 2, etc.) A rate normalized cost will be defined as the importance multiplied by the rate, or $C_i = I_i R_i$. Importance will be defined with the least important message assigned a one ($I_M = 1$), making the cost of the lowest priority message equal to its rate ($C_M = R_M$). As an example, if the second data-stream of a two data-stream system is 10 times as important as the first data-stream then $I_1 = 10$ and $I_2 = 1$. The reasoning behind this assigning protocol will be explained later in this chapter. What is left is to minimize the sum of the BERs for the M data-streams, weighted by the rate-normalized cost numbers.

2.2 BFSK with FH-SS

The form of a transmitted bit from the i^{th} data-stream is [5]:

$$x_i(t) = \sqrt{\frac{p_i}{2}} \cos(\omega_c + \omega_i + \omega_m)t \qquad , \ (0 \leq t \leq T_i) \tag{2.1}$$

Where p_i corresponds to the transmitted power of the i-th message, and is constant through out the bit period T_i (i.e. no pulse shaping). The bit period is equal to the inverse of the data-rate, $T_i = \frac{1}{R_i}, for\ i = 1 \ldots M$. The carrier frequency is ω_c, while ω_i is the hopping frequency, and ω_m is the modulation frequency. All M data-streams will be transmitted simultaneously over the hostile channel. To avoid inter-symbol interference (ISI), the bandwidth must be at least twice the inverse of the symbol period. However, since the BFSK system uses two frequency bands for modulation, twice the minimum bandwidth is required, or $B_i = 4R_i = \frac{4}{T_i}$. The total bandwidth without spreading is then:

$$B = \sum_{i=1}^{M} B_i \;=\; 4\sum_{i=1}^{M} R_i \;=\; 4R, \qquad\qquad R = \sum_{i=1}^{M} R_i.$$

After the data-stream is spread via frequency hopping, the resultant total spread bandwidth is B_{SS}. Since the jammer is assumed to have full knowledge of the spreading scheme (excluding the pseudorandom hopping pattern), it is most beneficial to utilize the maximum bandwidth allowable for every data-stream. Therefore, each data-stream will be spread along the entire bandwidth, giving different processing gains for streams with different data rates, $N_i = \frac{B_{SS}}{B_i}$. The pseudorandom hopping scheme will be carefully chosen such that no two data-streams are occupying the same frequencies at any given time. This is called orthogonal signaling. Note that if the N_i are too small, it might not be possible for transmission of all data-streams to occur on non-overlapping (orthogonal) channels. It will be assumed that the N_i's are sufficiently large. The total transmit power is defined as the sum of the individual powers, $P = \sum_{i=1}^{M} p_i$.

2.2.1 Full-Band Gaussian Noise Jammer

Here the transmitted signal gets corrupted by thermal noise plus a jammer transmitting Gaussian noise of average power J. The thermal noise can be modeled as additive white Gaussian noise (AWGN) with double sided power spectral density (PSD) $\frac{N_o}{2}$. The jammer signal can also be modeled as AWGN, with PSD $\frac{J}{2B_{SS}}$ (the jammer has to distribute his power along the total spread bandwidth, B_{SS}.) Since the two interference signals are independent Gaussian processes, the sum is also Gaussian with PSD equal to the sum of the individual PSDs, that is $\frac{N_T}{2} = \frac{N_o}{2} + \frac{J}{2B_{SS}}$. Now, this system can be modeled as the transmitted signal going through an AWGN channel with PSD $\frac{N_T}{2}$.

The following cost function to be minimized is now defined:

$$\sum_{i=1}^{M} C_i \cdot P_b(i) \tag{2.2}$$

where $P_b(i)$ is the probability of bit error, or the bit error rate (BER). Lagrange Multipliers are used to find values of p_i that minimize Equation 2.2. From Appendix A, the BER for BFSK with the current model is:

$$P_{b_i} = \frac{1}{2}e^{-\frac{E_b}{2N_T}} = \frac{1}{2}e^{-\frac{p_i}{2R_i(N_o+\frac{J}{B_{SS}})}} \tag{2.3}$$

The optimization is then to:

$$\min\left(\sum_{i=1}^{M}\frac{1}{2}C_i e^{-\frac{p_i}{2R_i(N_o+\frac{J}{B_{SS}})}}\right) \tag{2.4}$$

with the constraints that $P = \sum_{i=1}^{M} p_i$ and $p_i > 0$. So :

$$\phi(p_1,\ldots,p_M) = \sum_{i=1}^{M}\frac{1}{2}C_i e^{-\frac{p_i}{2R_i(N_o+\frac{J}{B_{SS}})}} + \lambda\left(\sum_{i=1}^{M}p_i - P\right) \tag{2.5}$$

$$\frac{\partial\phi}{\partial p_i} = -\frac{C_i}{4R_i(N_o+\frac{J}{B_{SS}})}e^{-\frac{p_i}{2R_i(N_o+\frac{J}{B_{SS}})}} + \lambda = 0 \tag{2.6}$$

$$p_i = -2R_i(N_o+\frac{J}{B_{SS}})\ln\left(\frac{4\lambda R_i}{C_i}(N_o+\frac{J}{B_{SS}})\right) \tag{2.7}$$

$$\sum_{i=1}^{M}p_i = P = -2(N_o+\frac{J}{B_{SS}})\left[\ln\left(4\lambda(N_o+\frac{J}{B_{SS}})\right)\sum_{i=1}^{M}R_i + \sum_{i=1}^{M}R_i\ln(\frac{R_i}{C_i})\right]$$

Now, the reason for the particular definition for C_i is made clear. In order to get the correct cancellation, C_i must have the same units as R_i. Eliminating λ, substituting $I_i = C_i/R_i$ and some algebra results in:

$$p_i = \frac{R_i}{R}\left[P + 2(N_o+\frac{J}{B_{SS}})\sum_{j=1}^{M}R_j\ln(\frac{I_i}{I_j})\right] \tag{2.8}$$

From Equation 2.8, it is important to note that the energy per bit is a function of the importance value, and can be written in the form:

$$\frac{p_i}{R_i} = f(I_i) = \alpha + \beta\ln(I_i)$$

Using the condition that $p_i > 0$, in Equation 2.8, results in the minimum total power condition of:

$$P > 2(N_o + \frac{J}{B_{SS}})\sum_{j=1}^{M} R_j \ln(\frac{I_j}{I_i}) \quad , \quad i = 1, 2, \ldots, M \tag{2.9}$$

yielding in a minimum signal to noise power ratio (SNR) of:

$$SNR = \frac{P}{(N_o B_{SS} + J)} > \frac{2}{B_{SS}} \sum_{j=1}^{M} R_j \ln(\frac{I_j}{I_i}), \qquad i = 1, 2, \ldots, M. \tag{2.10}$$

Substituting the optimized value for p_i into Equation 2.3 yields a BER of:

$$P_{b_i} = \frac{1}{2} \exp\left(-\frac{P}{2R(N_o + \frac{J}{B_{SS}})} - \sum_{j=1}^{M} \frac{R_j}{R} \ln(\frac{I_i}{I_j})\right) = \frac{1}{2I_i} \exp\left(-\frac{P}{2R(N_o + \frac{J}{B_{SS}})}\right) \prod_{j=1}^{M} I_j^{\frac{R_j}{R}} \tag{2.11}$$

giving a minimized objective function of:

$$\sum_{i=1}^{M} C_i P_{b_i} = \sum_{i=1}^{M} \frac{R_i}{2} e^{-\frac{P}{2R(N_o + \frac{J}{B_{SS}})}} \prod_{j=1}^{M} I_j^{\frac{R_j}{R}} \tag{2.12}$$

To verify that the above is a local minimum, the second partials of $\phi(p_1, \ldots, p_M)$ must be examined. From [6] conditions for a local minimum are given as $\phi_i'(p_1, \ldots, p_M) = 0$, and $D_i > 0$,for $i = 1, 2, \ldots, M$, where D_i are determinants of the Hessian matrix of ϕ, and have the form:

$$D_i = \begin{vmatrix} \phi''_{p_1p_1} & \phi''_{p_1p_2} & \cdots & \phi''_{p_1p_i} \\ \phi''_{p_2p_1} & \phi''_{p_2p_2} & \cdots & \phi''_{p_2p_i} \\ \vdots & & & \vdots \\ \phi''_{p_ip_1} & \cdots & \cdots & \phi''_{p_ip_i} \end{vmatrix} \tag{2.13}$$

The first condition above was already used to find the values of p_i, so the constraint $D_i > 0$ must now be verified. In evaluating the determinant, it is clear that all the elements outside the main diagonal go to zero, or $\phi''_{p_ip_j} = 0$ when $i \neq j$. Thus $D_i = \prod_{j=1}^{i} \phi''_{p_jp_j}$. Differentiating Equation 2.7 results in:

$$\frac{\partial^2 \phi}{\partial p_i^2} = \frac{C_i}{2(2R_i(N_o + \frac{J}{B_{SS}}))^2} \, e^{-\frac{p_i}{2R_i(N_o+\frac{J}{B_{SS}})}} \tag{2.14}$$

which is always positive since the exponential e^x is a strictly positive (non-zero) function for real finite values of x. Now since the product of positive numbers is also positive, and $D_i > 0$ (for $i = 1, 2, \ldots M$), the value obtained for p_i in Equation 2.10 yields a minimum of Equation 2.2.

Interpretation of Results

The simplest form of this problem is achieved under the constraint that $C_i = R_i$, or $I_i = 1$, for $i = 1, 2, \ldots, M$ (i.e. all the bits are equally valuable). In this case, the logarithms from Equation 2.8 go to zero resulting in $p_i = \frac{R_i}{R}P$ and $E_b = \frac{P}{R}$. This gives $P_b(i) = \frac{1}{2}e^{-\frac{P}{2R(N_o+\frac{J}{B_{SS}})}}$, and a objective function of $\sum_{i=1}^{M} C_i P_{b_i} = P_{b_i} \sum_{i=1}^{M} C_i = RP_{b_i}$. Because all streams are of equal importance, it makes sense that the power allocation that minimizes the combined weighted probability of bit error, assigns the same E_b for every data-stream. This results in each data-stream having the same BER, as expected.

The following are a few simple examples.

Example 1: Equal Importance Values, Equal Rates:

$R_1 = \cdots = R_M = C_1 = \cdots = C_M$.

Thus, we get $p_i = \frac{P}{M}$, and $P_{b_i} = \frac{1}{2}e^{-\frac{P}{2R_1 M(N_o+\frac{J}{B_{SS}})}}$.

We indeed expect the total power P to be divided equally among the transmitted data-streams, since all the streams are assigned the same importance. As M decreases (we have fewer data-streams), we allow more power in each data-stream and, the probability of error decreases.

Example 2: Equal Importance Values, Unequal Rates:

$R_2 = \alpha R_1$ and $I_2 = I_1 = 1$.

Here, $p_1 = \frac{1}{1+\alpha}P$ and $p_2 = \frac{\alpha}{1+\alpha}P$, or $p_2 = \alpha p_1$. $P_{b_1} = P_{b_2} = \frac{1}{2}e^{-\frac{P}{2R(N_o+\frac{J}{B_{SS}})}}$.

This is the two rate version of what was discussed in the beginning of this section.

Example 3: Equal Rates, Unequal Importance Values:

$R_1 = R_2$, and $I_1 = \alpha$, $I_2 = 1$, $\qquad \alpha > 1$.

Here, $p_1 = \frac{P}{2} + \ln(\alpha) R_1 (N_o + \frac{J}{B_{SS}})$ and $p_2 = \frac{P}{2} - \ln(\alpha) R_1 (N_o + \frac{J}{B_{SS}})$.

$P_{b_1} = \frac{1}{2\sqrt{\alpha}} e^{-\frac{P}{2R(N_o + \frac{J}{B_{SS}})}}$, and $P_{b_2} = \frac{\sqrt{\alpha}}{2} e^{-\frac{P}{2R(N_o + \frac{J}{B_{SS}})}}$.

This yields $P_{b_2} = \alpha P_{b_1}$.

Example 4: Unequal Rates, Unequal Importance Values:

$R_2 = \alpha R_1$ and $I_1 = \beta$, $I_2 = 1$. $\qquad \beta > 1$.

Here, $p_1 = \frac{1}{1+\alpha} P + 2(\frac{1}{1+\alpha}) \ln(\beta)(N_o + \frac{J}{B_{SS}}) R_2$, and

$p_2 = \frac{\alpha}{1+\alpha} P - 2(\frac{\alpha}{1+\alpha}) \ln(\beta)(N_o + \frac{J}{B_{SS}}) R_1$.

$P_{b_1} = \frac{1}{2\beta^{\frac{\alpha}{1+\alpha}}} e^{-\frac{P}{2R(N_o + \frac{J}{B_{SS}})}}$, and $P_{b_2} = \frac{\beta^{\frac{1}{1+\alpha}}}{2} e^{-\frac{P}{2R(N_o + \frac{J}{B_{SS}})}}$. We get $P_{b_2} = \beta P_{b_1}$.

As expected, the probability of error follows the importance values.

In the next example, specific values are given for the system in Example 4.

Example 5:

$R_1 = 20 kbs/sec$, $R_2 = 100 kbs/sec$, and $I_1 = e^3 \approx 20$, $I_2 = 1$.

Let $P = 2\ W$, $J = 10\ W$, $B_{SS} = 1\ MHz$, $N_o = 10^{-7}\ WHz^{-1}$. Plugging the values here into the results from Example 4 gives, $p_1 = 1.34\ W$, and $p_2 = 0.66\ W$. This yields $P_{b_1} = 9.9 \cdot 10^{-6}$, and $P_{b_2} = 2 \cdot 10^{-4}$.

The above system has the jammer transmitting 5 times as much power as the transmitter. With out spread spectrum, this would lead to extremely poor BERs. Spreading the signal over a larger spectrum results in the good BERs seen above. Data-stream 1 has a processing gain of 50, while data-stream 2 has a processing gain of 10. Note that the thermal noise has a negligible effect on the above system. Over the spread bandwidth, it contributes merely $0.1W$ of power compared to the $10W$ from the jammer. Further improvement of the BER can be achieved by either increasing the transmitter power or increasing the bandwidth.

In all of the above examples, it was assumed that Equation 2.10 was satisfied. If this is not the case and P is not large enough for a given set of constraints, then the minimization gives a negative value for some p_i. This is clearly not correct, so the end-points must be examined for a minimum. The next example will demonstrate this case.

Example 6: Degenerate Case

$R_1 = R_2 = 50kbs/sec$, and $I_1 = e^2 \approx 7$, $I_2 = 1$.

Specifically define: $P = 1W,\ J = 10W,\ B_{SS} = 1MHz,\ N_o = 0$.

From Example 3, $p_1 = \frac{P}{2} + \ln(\alpha)R_1(N_o + \frac{J}{B_{SS}})$, and $p_2 = \frac{P}{2} - \ln(\alpha)R_1(N_o + \frac{J}{B_{SS}})$. This accordingly gives $p_2 = -\frac{1}{2}$, which is not possible. Examining the end-points gives us a minimum of the objective function at $p_1 = 1$, and $p_2 = 0$, resulting in $P_{b_1} = \frac{1}{2}e^{-1}$, and $P_{b_2} = \frac{1}{2}$.

In general, the transmitter should operate in a region sufficiently above the required SNR.

2.2.2 Partial Band Jamming

Here, the jamming (additive Gaussian noise) is limited to a fraction α, $(0 < \alpha \leq 1)$ of the total spread spectrum bandwidth B_{SS}. Since a smaller bandwidth is jammed, the power spectral density of the jamming signal can increase ($N_J = \frac{J}{\alpha B_{SS}}$) while maintaining the same average power J. This benefit, however, does not come without a price. If the jamming is over a bandwidth of αB_{SS}, then the probability that the signal is in the jammed band is α. In order to facilitate this analysis an important assumption must be made. Namely, if the signal hops into the jammed band, it will be assume that the entire unspread signal spectrum is jammed. Cases when only a fraction of the unspread signal spectrum is jammed, will not be considered. The exact spectral location of the partial-band jamming is irrelevant if the above assumption holds. With this in mind the BER is as follows:

$$P_{b_i} = \frac{\alpha}{2}\, e^{-\frac{p_i}{2R_i(N_o + \frac{J}{\alpha B_{SS}})}} + \frac{1-\alpha}{2}\, e^{-\frac{p_i}{2R_i(N_o)}} \tag{2.15}$$

For a specified α, the same optimization as in the previous section is performed to find optimal values of p_i. To be optimized is:

$$\min \sum_{i=1}^{M} \left(\frac{\alpha C_i}{2} e^{-\frac{p_i}{2R_i(N_o + \frac{J}{\alpha B_{SS}})}} + \frac{(1-\alpha)C_i}{2} e^{-\frac{p_i}{2R_i N_o}} \right) \tag{2.16}$$

with the constraints that $P = \sum_{i=1}^{M} p_i$ and $p_i > 0$. Define:

$$\phi(p_1, \ldots, p_M) = \sum_{i=1}^{M} \left(\frac{\alpha C_i}{2} e^{-\frac{p_i}{2R_i(N_o + \frac{J}{\alpha B_{SS}})}} + \frac{(1-\alpha)C_i}{2} e^{-\frac{p_i}{2R_i N_o}} \right) + \lambda \left(\sum_{i=1}^{M} p_i - P \right) \tag{2.17}$$

$$\frac{\partial \phi}{\partial p_i} = -\frac{\alpha C_i}{4R_i(N_o + \frac{J}{\alpha B_{SS}})} e^{-\frac{p_i}{2R_i(N_o + \frac{J}{\alpha B_{SS}})}} - \frac{(1-\alpha)C_i}{4R_i N_o} e^{-\frac{p_i}{2R_i N_o}} + \lambda = 0 \tag{2.18}$$

Unfortunately the above equation cannot be solved explicitly for p_i. A number of assumptions can be made to make this problem more manageable. Namely, if it is assumed that the second term above is negligible compared to the first the following results:

$$p_i = \frac{R_i}{R} \left[P + 2(N_o + \frac{J}{\alpha B_{SS}}) \sum_{j=1}^{M} R_j \ln(\frac{I_i}{I_j}) \right] \tag{2.19}$$

This form is very similar to what was seen in the first section. To see when this assumption is accurate, a few two-stream systems will be examined. In each of these examples, p_i is calculated using the above approximation and is then compared to p_i found by using the Nelder-Mead simplex (direct search) method from MATLAB. We plot the values of p_i, and the associated BERs, as α varies from 0 to 1.

In Figure 2-1, $N_J = 10^{-5}$, which implies that $J = B_{SS} \cdot 10^{-5}$. Some system parameters that satisfy this expression would be $J = 10$ W, $B_{SS} = 1$ MHz or $J = 100$ W, $B_{SS} = 10$ MHz. It is apparent that $p_2 > p_1$, for all α. Even though the power in data-stream 2 is larger, the BER is smaller, since the energy per bit in data-stream 1 is greater than the energy per bit in data-stream 2. Note that as α decreases from 1 to about 0.24, the system performance deteriorates (the cost function gets larger.) The estimation used in Equation 2.19 is accurate for values of α as low a 0.06. At $\alpha = 0.06$ the cost function, calculated from the estimated values of p_1 and p_2, increases rapidly.

Figure 2-2 has the same system parameters as Figure 2-1, except for an increase in the magnitude of the jammer PSD. Here, $N_J = 10^{-4}$, which is 100 times as large as N_o. Values such as $J = 5$ W, $B_{SS} = 500$ kHz or $J = 50$ Watts, $B_{SS} = 5$ MHz, are possible system values. Since the jammer has a larger PSD here, the performance clearly deteriorates. The estimation is valid on a small interval. The estimated values of p_i start straying from the optimal p_i at $\alpha = 0.56$, although a significant increase in the cost function does not occur until about $\alpha = 0.48$.

Figure 2-3 differs from Figure 2-1 in that data-stream 1 has an importance value 100 times greater than data-stream 2. With full-band jamming ($\alpha = 1$), the power allocation for p_1 is increased only about a half a watt from Figure 2-1. This is due to p_i's logarithmic dependence on I_i, in Equation 2.8. The estimation fails for $\alpha < 0.1$.

In Figure 2-4 the magnitude of the jammer PSD is increased ten times from that in Figure 2-3. Here, with $N_J = 10^{-4}$, the estimation fails for all values of $\alpha \neq 1$. Because of the large difference in importance values and relatively small SNR, at $\alpha = 1$, the BER for data-stream 2 is very poor ($P_{b_2} \approx 0.4$.)

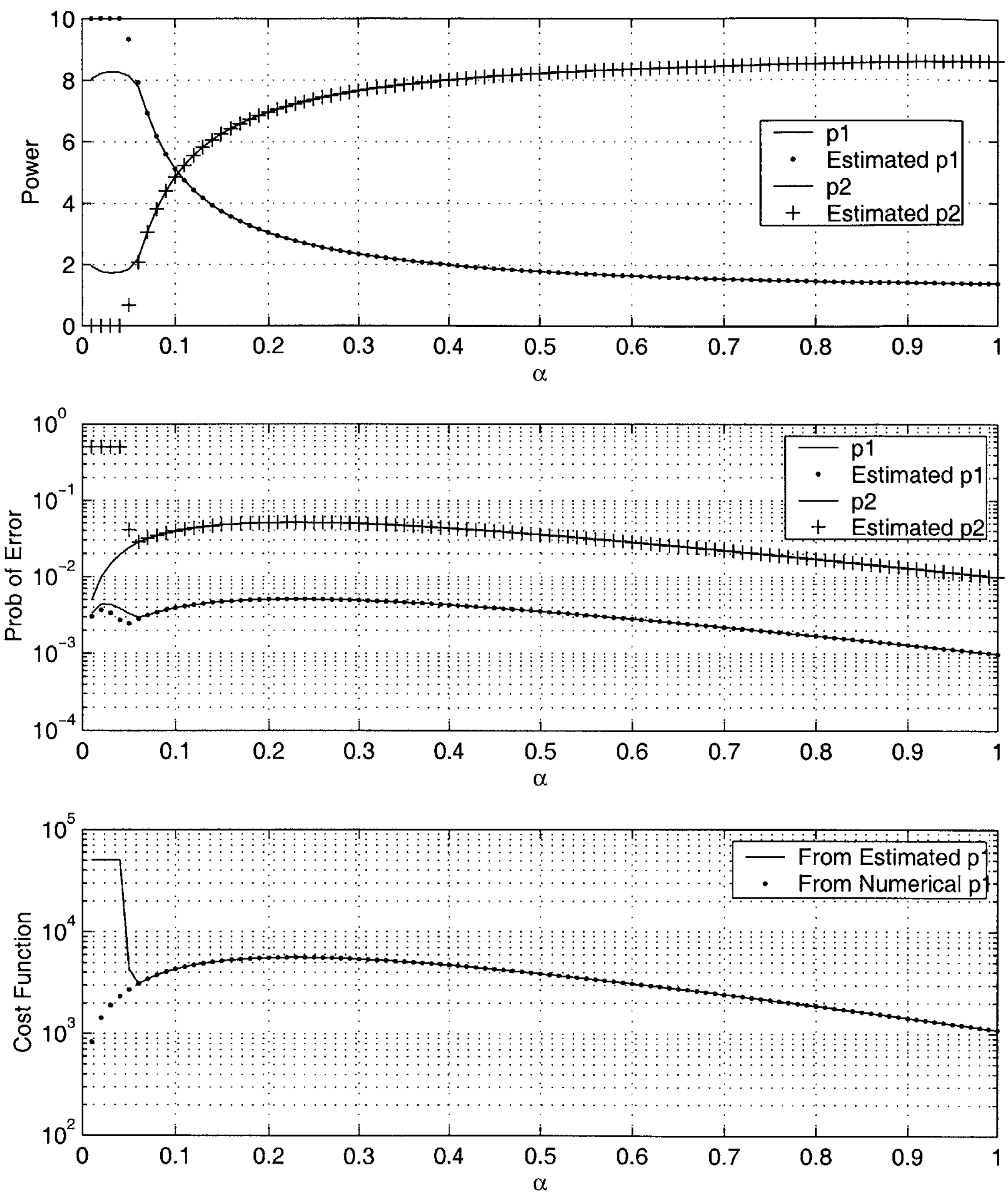

Figure 2-1: $P = 10\ W$, $I_1 = 10$, $I_2 = 1$, $R_1 = 10\ kb/s$, $R_2 = 100\ kb/s$, $N_o = 10^{-6}\ WHz^{-1}$, $N_J = 10^{-5}\ WHz^{-1}$.

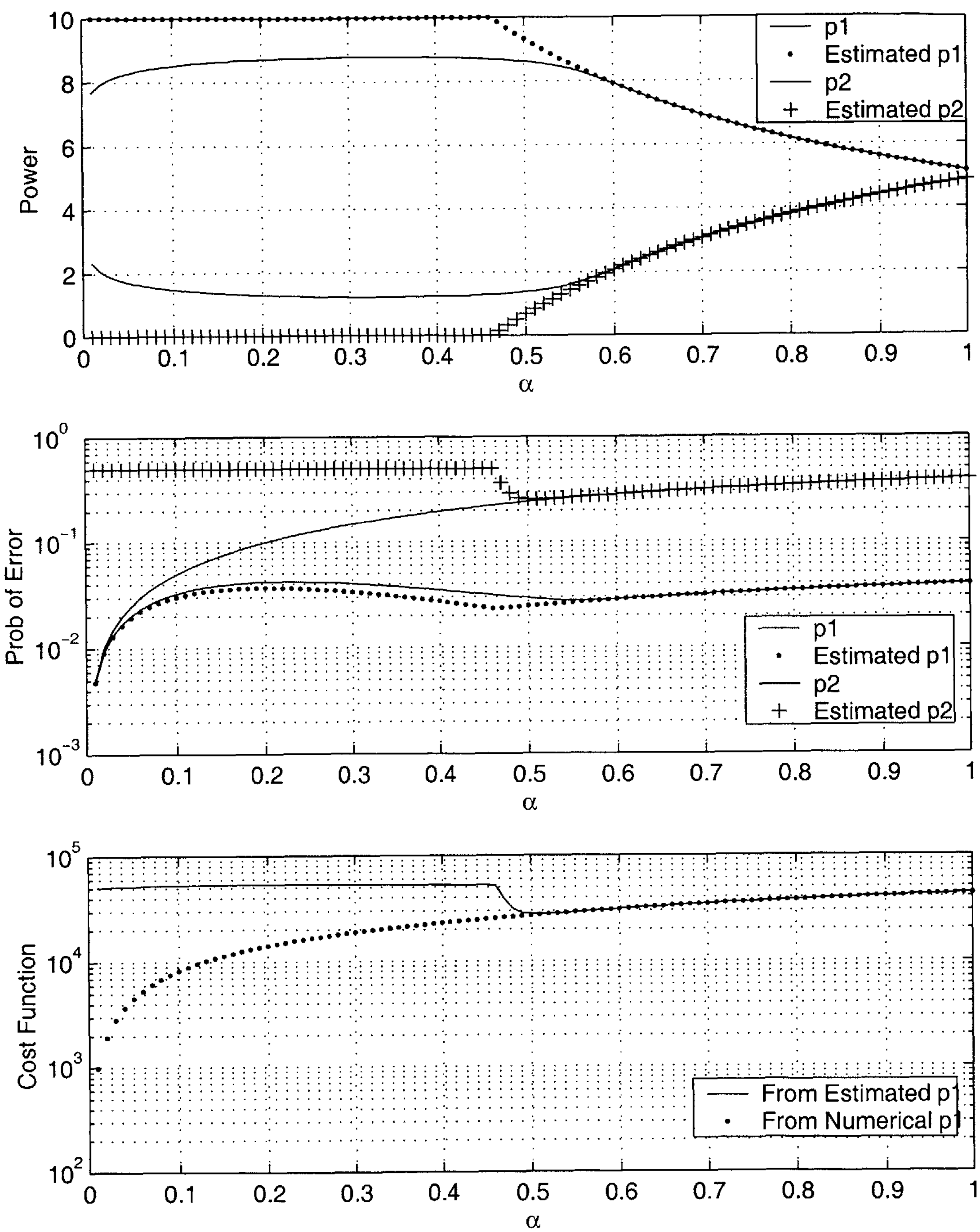

Figure 2-2: $P = 10\ W$, $I_1 = 10$, $I_2 = 1$, $R_1 = 10\ kb/s$, $R_2 = 100\ kb/s$, $N_o = 10^{-6}\ WHz^{-1}$, $N_J = 10^{-4}\ WHz^{-1}$.

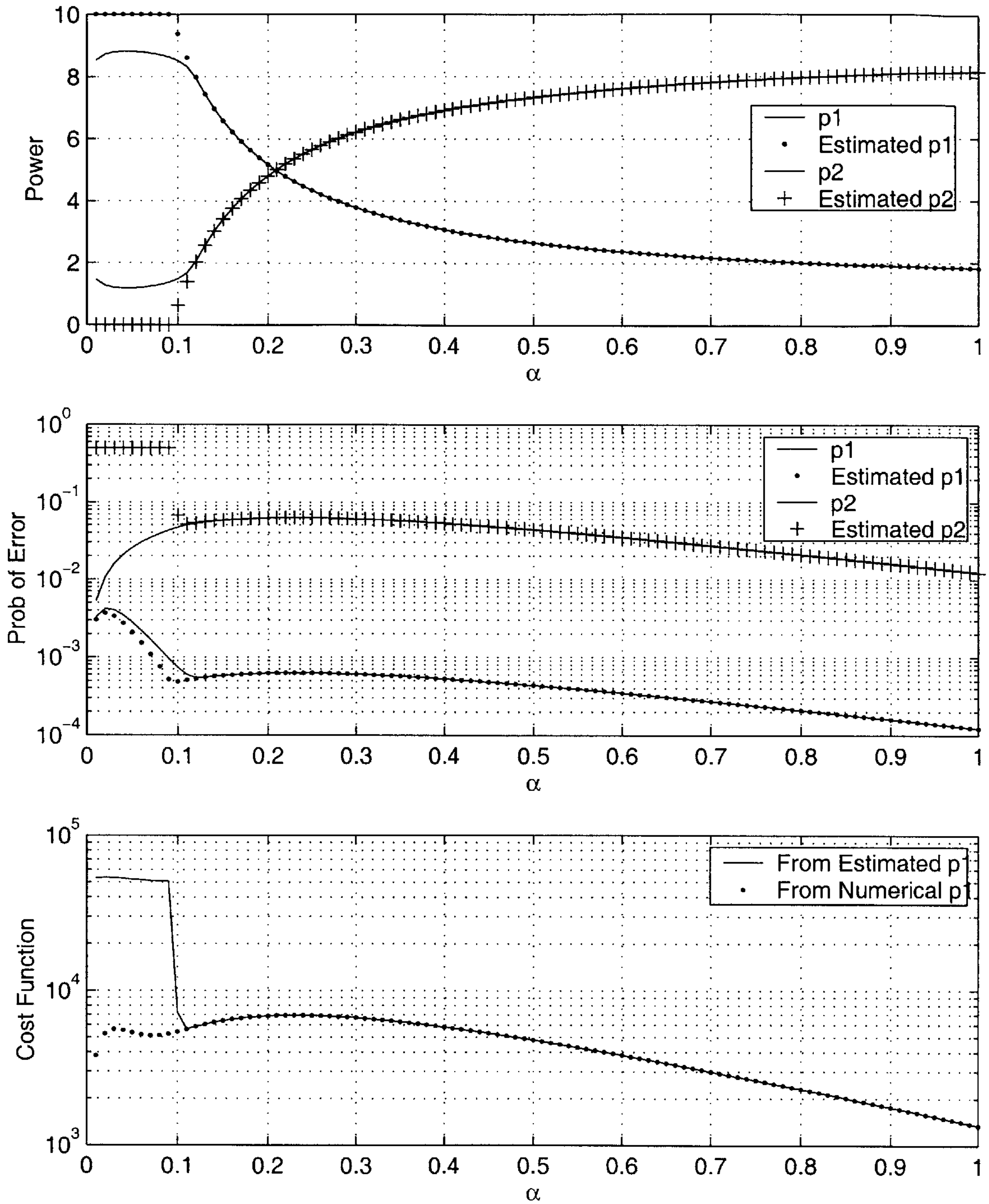

Figure 2-3: $P = 10\ W$, $I_1 = 100$, $I_2 = 1$, $R_1 = 10\ kb/s$, $R_2 = 100\ kb/s$, $N_o = 10^{-6}\ WHz^{-1}$, $N_J = 10^{-5}\ WHz^{-1}$.

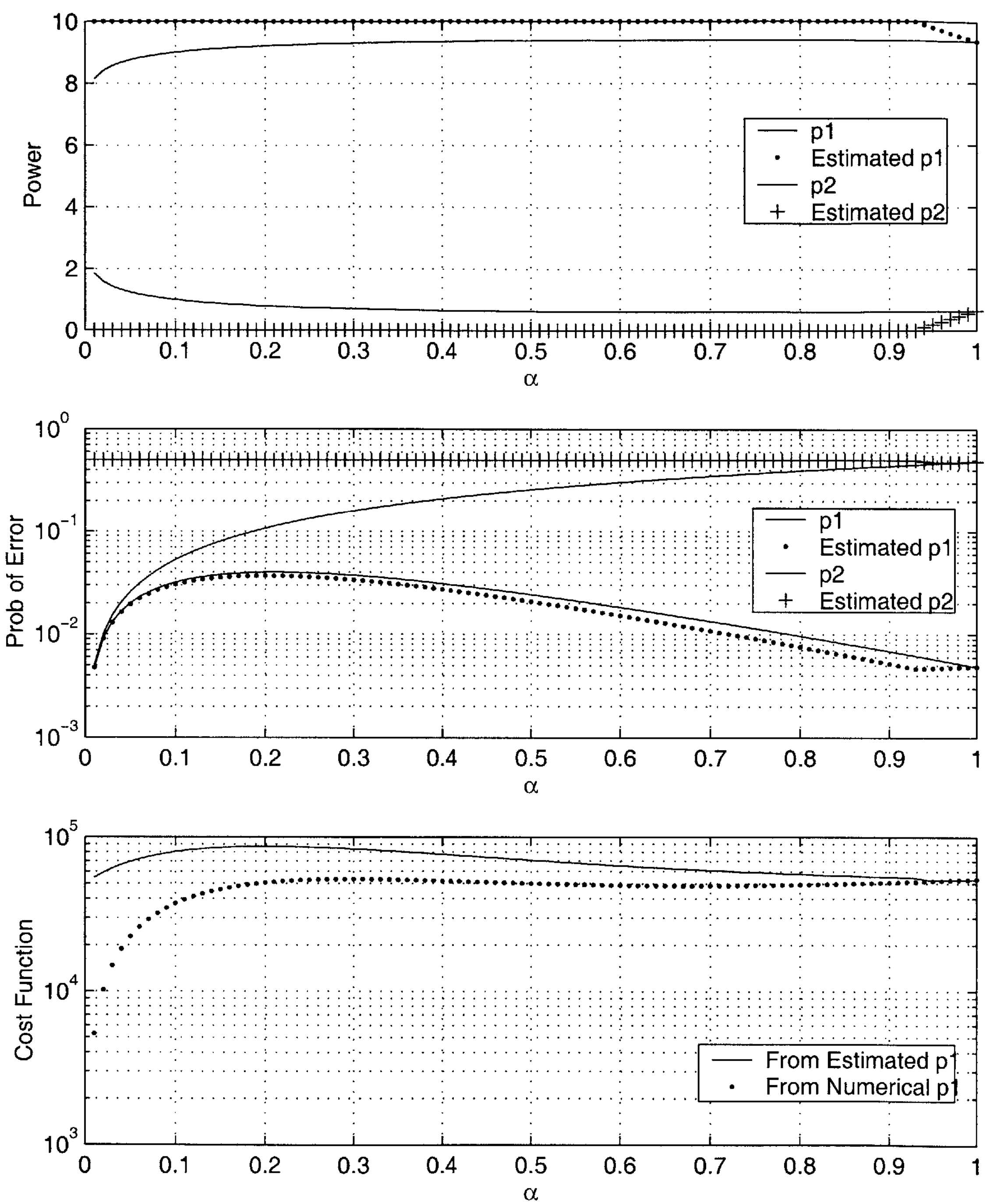

Figure 2-4: $P = 10\ W$, $I_1 = 100$, $I_2 = 1$, $R_1 = 10\ kb/s$, $R_2 = 100\ kb/s$, $N_o = 10^{-6}\ WHz^{-1}$, $N_J = 10^{-4}\ WHz^{-1}$.

Optimum Partial-Band Jamming

Which value of α is most beneficial for the jammer, or what α will maximize the probability of error? This is what is called a mini-max problem. The goal of the jammer is to:

$$\max_{\alpha}(\min_{p_i}\sum_{i=1}^{M} C_i \cdot P_b). \quad ,0 < \alpha \leq 1. \quad 0 \leq p_i \leq 1, \forall i, \quad \sum_{i=1}^{M} p_i = P. \tag{2.20}$$

On the opposing side, the goal of the transmitter is to:

$$\min_{p_i}(\max_{\alpha}\sum_{i=1}^{M} C_i \cdot P_b). \quad ,0 < \alpha \leq 1. \quad 0 \leq p_i \leq 1, \forall i, \quad \sum_{i=1}^{M} p_i = P. \tag{2.21}$$

It is well known from mini-max theory that Equation 2.20 and Equation 2.21 are equal. To simplify the optimization it will be assumed that the PSD of the jammer is much larger than the thermal noise ($N_J >> N_o$) and that the thermal noise is very small ($N_o << 1$). The optimization is performed by differentiating the combined probability of errors and setting the result equal to zero. The simplified weighted objective function to maximize in Equation 2.20 is:

$$\frac{\alpha}{2}\sum_{i=1}^{M} C_i e^{-\frac{\alpha p_i B_{SS}}{2R_i J}} \tag{2.22}$$

Differentiating the BER by α, and setting the result equal to zero:

$$\frac{\partial P_b}{\partial \alpha} \approx \sum_{i=1}^{M}\left(1 - \frac{\alpha p_i B_{SS}}{2R_i J}\right)\frac{C_i}{2}e^{-\frac{\alpha p_i B_{SS}}{2R_i J}} = 0 \tag{2.23}$$

By setting the term in the parenthesis equal to zero for all i, the above equation equals zero. This can only be done if the rates are equal and all have the same importance values. If this is not the case, an analytic solution for α can not be found. Equal rates and importance values gives the constraint that:

$$\alpha = \frac{2JR_i}{p_i B_{SS}} = \frac{2}{E_{b_i}/N_J}, \quad i = 1, 2, \ldots, M. \tag{2.24}$$

Since $0 < \alpha \leq 1$, the above only holds when $\frac{E_{b_i}}{N_J} \geq 2$. When $\frac{E_{b_i}}{N_J} < 2$, we set $\alpha = 1$, which implies that full-band jamming gives the best results. Assuming that the transmitter performs that optimization of the cost function, knowing that optimal partial band jamming is used, the worst case BER is:

$$P_{b_i} \approx \frac{JR_i}{p_i B_{SS}} e^{-1} = \frac{1}{E_{b_i}/N_J} e^{-1}. \tag{2.25}$$

To verify that the above is a maxima, the second derivative evaluated at the extrema must be negative.

$$\frac{\partial^2 P_b}{\partial \alpha^2} \approx \sum_{i=1}^{M} \left[(1 - \frac{\alpha p_i B_{SS}}{2R_i J}) \frac{-p_i B_{SS} C_i}{4R_i J} e^{-\frac{\alpha p_i B_{SS}}{2R_i J}} - \frac{p_i B_{SS} C_i}{2R_i J} e^{-\frac{\alpha p_i B_{SS}}{2R_i J}} \right]. \tag{2.26}$$

The first term is zero and the second term is always negative, giving a maximum. This method does not produce a very useful maximum, since the focus here is on multi-rate multi-priority systems. However, even if the system is single rate and has equal importance values, the problem arises that α is function of p_i and p_i is a function of α. Since there is no analytic solution to this problem, numerical methods must be utilized.

Seeing that numerical methods will be used, the approximation in Equation 2.22 need not be used. The mini-max problem is solved with MATLAB. Table 2.1 shows a comparison of the achievable BER in the previously used examples. The full-band and (the numerical results of) the optimal partial band jammer models are compared.

2.3 Binary PPM with Time Hopping

The analysis of this type of system is facilitated by noticing the duality relationship it has with BFSK FH-SS. While BFSK transmits in one of two frequencies (with non-overlapping spectrums), Binary PPM transmits in one of two non-overlapping time slots. This time-frequency duality seen between BFSK and Binary PPM can be also be seen between TH and FH Spread Spectrum. Because of this, the two are

Figure	Optimal α	Data Stream	BER ($\alpha = 1$)	BER (Optimal α)	Cost ($\alpha = 1$)	Cost (Optimal α)
2-1	0.23	1	$9.9 \cdot 10^{-4}$	0.0051	$1.1 \cdot 10^{3}$	$5.6 \cdot 10^{3}$
		2	$9.9 \cdot 10^{-3}$	0.051		
2-2	1	1	0.039	0.039	$4.3 \cdot 10^{4}$	$4.3 \cdot 10^{4}$
		2	0.39	0.39		
2-3	0.49	1	$1.2 \cdot 10^{-4}$	$6.3 \cdot 10^{-7}$	$1.3 \cdot 10^{3}$	$8.6 \cdot 10^{3}$
		2	$1.2 \cdot 10^{-2}$	0.086		
2-4	1	1	0.0048	0.0048	$5.3 \cdot 10^{4}$	$5.3 \cdot 10^{4}$
		2	0.48	0.48		

Table 2.1: Performance of Full-Band Jamming and Optimal Partial Band Jamming.

mathematically isomorphic.

2.4 Conclusion

It was found that for a multi-rate multi-priority system as described in Section 2.1, the optimal power allocation is related to the logarithm of its importance value. Furthermore, for data-streams with large signal to noise ratios, optimal partial-band jamming was found to degrade system performance significantly, in comparison to full-band jamming.

Appendix A

Derivation of the BER for BFSK, from [1].

The following is a derivation for the BER for BFSK signaling in white Gaussian noise. A correlation demodulator with a square-law detector is used, as shown in Figure A-1.

With BFSK, the two possible signals that can be sent to represent a bit are:

$$\begin{aligned} s_m(t) &= A\cos(\omega_m t + \phi_m) \\ &= A\cos\phi_m \cos\omega_m t - A\sin\phi_m \sin\omega_m, \qquad 0 \le t \le T_b, \quad m = 0,1 \end{aligned} \tag{A.1}$$

where m= (0,1) correspond to the input stream bit-values. A is the amplitude, T_b is the bit period, and ϕ_m is a random phase value with uniform distribution (See Figure A-2). After going through the channel and getting corrupted by additive white Gaussian noise $n(t)$ with two-sided PSD, $\frac{N_J}{2}$, the demodulator receives the signal, $r_m(t) = s_m(t) + n(t)$. The probability of bit error (BER) is:

$$\mathrm{Pr_b} = \mathrm{Pr}(0\ \mathrm{detected}|s_1)\cdot \mathrm{Pr}(s_1) + \mathrm{Pr}(1\ \mathrm{detected}|s_0)\cdot \mathrm{Pr}(s_0). \tag{A.2}$$

The *a priori* probability that s_0 or s_1 is sent is 1/2, and by symmetry, the Pr(0

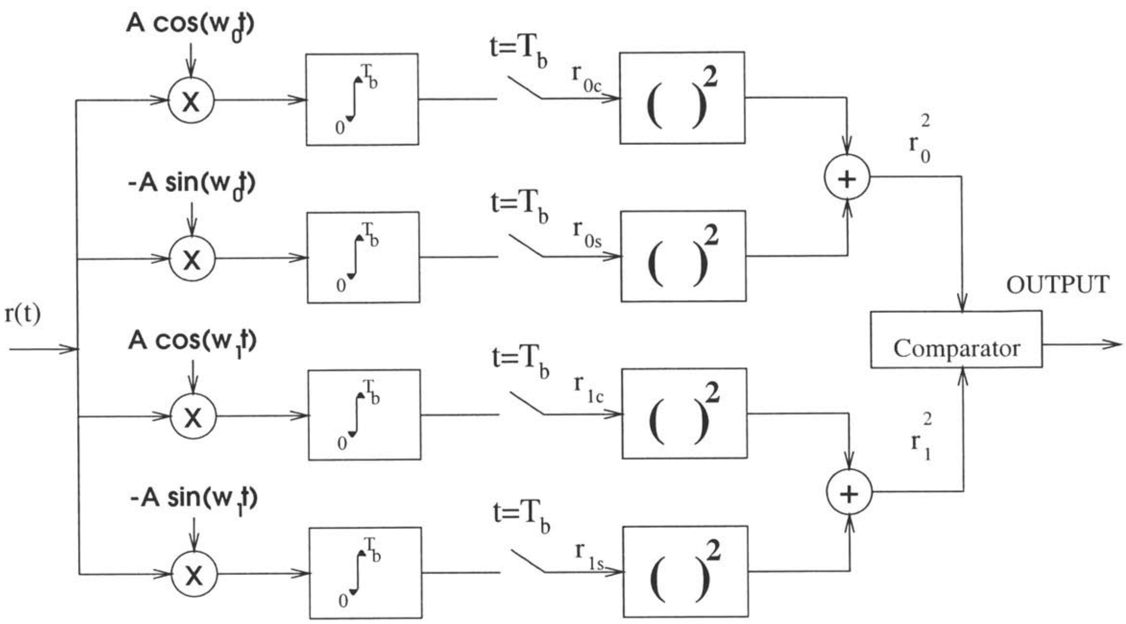

Figure A-1: Correlation Demodulator for BFSK

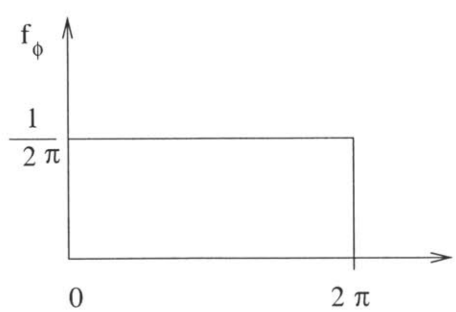

Figure A-2: Probability Density Function for ϕ.

detected$|s_1$) = Pr(1 detected$|s_0$). The probability of error then reduces to:

$$\mathrm{Pr_b} = \mathrm{Pr(0\ detected|s_1) = Pr(r_0 > r_1|s_1)}, \tag{A.3}$$

since the comparison of $r_0^2 > r_1^2$, which takes place in the comparator of Figure A-1, is equivalent to $r_0 > r_1$. Given that s_1 is sent, the received signals are:

$$\begin{aligned} r_{1c} &= E_b \cos\phi_1 + n_{1c}, \qquad r_{1s} = E_b \sin\phi_1 + n_{1s} \\ r_{0c} &= n_{0c}, \qquad\qquad\qquad\quad r_{0s} = n_{0s}. \end{aligned} \tag{A.4}$$

with the following definitions

$$E_b = \int_0^{T_b} s_m^2(t) = \frac{A^2 T_b}{2}, \quad n_{mc} = \int_0^{T_b} n(t){\cdot}A\cos\omega_m t\,dt, \quad n_{ms} = -\int_0^{T_b} n(t){\cdot}A\sin\omega_m t\,dt.$$

Where n_{mc} and n_{ms} are both uncorrelated Gaussian random variables with variance, $\sigma^2 = \frac{E_b N_J}{2}$. Now, the pdf of $\mathbf{r}$, given that s_1 was sent, for some ϕ_1 is:

$$f(\mathbf{r}|s_1, \phi_1) = f(r_{1c}, r_{1s}|s_1, \phi_1) f(r_{0c}, r_{0s}|s_1). \tag{A.5}$$

where,

$$f(r_{1c}, r_{1s}|s_1, \phi_1) = \frac{1}{\pi E_b N_J} e^{-\frac{(r_{1c} - E_b\cos\phi_1)^2 + (r_{1s} - E_b \sin\phi_1)^2}{E_b N_J}} \tag{A.6}$$

$$f(r_{0c}, r_{0s}|s_0) = \frac{1}{\pi E_b N_J} e^{-\frac{r_{0c}^2 + r_{0s}^2}{E_b N_J}} \tag{A.7}$$

Since the pairs (r_{1c}, r_{1s}) and (r_{0c}, r_{0s}) are uncorrelated and Gaussian, they are independent and their marginal pdf's can be multiplied together in Equation A.5. Note that the last pdf in Equation A.5 is not dependent on ϕ_1. Since r_{mc} and r_{ms} are themselves uncorrelated, the cross-correlation term of the bi-variate Gaussian distribution is zero, resulting in the above equations. Making a change of variables

$$r_{mc} = r_m \cos\gamma, \qquad r_{ms} = r_m \sin\gamma,$$

and integrating ϕ_1 out of Equation A.6 yields:

$$\begin{aligned} f(r_{1c}, r_{1s}|s_1) &= \frac{1}{\pi E_b N_J} e^{-\frac{r_1^2+E_b^2}{E_b N_J}} \cdot \frac{1}{2\pi} \int_0^{2\pi} e^{\frac{2}{N_J}(r_1 \cos\gamma \cos\phi_1 + r_1 \sin\gamma \sin\phi_1)}\, d\phi_1 \\ &= \frac{1}{\pi E_b N_J} e^{-\frac{r_1^2+E_b^2}{E_b N_J}} \cdot I_0\left(\frac{2}{N_J} r_1\right). \end{aligned} \tag{A.8}$$

Since the angle formula $[r_1 \cos\gamma \cos\phi_1 + r_1 \sin\gamma \sin\phi_1 = r_1 \cos(\phi_1 - \gamma)]$ reduces the integral to a zero-order Modified Bessel Function of the 1^{st} kind:

$$I_0\left(\frac{2}{N_J} r_1\right) = \frac{1}{2\pi} \int_0^{2\pi} e^{\frac{2}{N_J} r_1 \cos\phi_1}\, d\phi_1. \tag{A.9}$$

Equation A.6 now yields: $f(r_{0c}, r_{0s}|s_1) = \frac{1}{\pi E_b N_J} e^{-\frac{r_0^2}{E_b N_J}}$, which implies that,

$$f(\mathbf{r}|s_1) = \frac{1}{(\pi E_b N_J)^2} e^{-\frac{r_1^1+r_0^2+E_b^2}{E_b N_J}}\ I_0\left(\frac{2}{N_J} r_1\right). \tag{A.10}$$

Since the decision rule is based on the random variables r_0 and r_1, a change of variable in the pdf must also occur. So,

$$f(r_m|s_1) = \int_{2\pi} f(r_{ms}, r_{mc}, \gamma|s_1)\, d\gamma\, (r_m\, dr_m) = 2\pi r_m\ f(r_{mc}, r_{ms}|s_1). \tag{A.11}$$

And finally,

$$\begin{aligned} \mathrm{Pr_b} = \Pr(r_0 > r_1|s_1) &= \int_0^\infty f(r_1|s_1) \int_{r_1}^\infty f(r_0|s_1)\, dr_0\, dr_1 \\ &= \int_0^\infty f(r_1|s_1) \int_{r_1}^\infty \frac{2r_0}{E_b N_J} e^{-\frac{r_0^2}{E_b N_J}}\, dr_0\, dr_1 \\ &= \frac{1}{2} e^{-\frac{E_b}{2N_J}} \underbrace{\int_0^\infty \frac{2r_1}{\frac{E_b N_J}{2}} e^{-\frac{r_1^2+\frac{E_b^2}{4}}{\frac{E_b N_J}{2}}}\ I_0\left(\frac{2}{N_J} r_1\right) dr_1}_{1} \\ &= \frac{1}{2} e^{-\frac{E_b}{2N_J}}. \end{aligned} \tag{A.12}$$

where the third line of Equation A.12 is the integral of the Ricean pdf over all possible values of r_1. The final result, after some tedious computation, is a simple exponential.

Appendix B

Derivation of BER for Binary PPM

The detector for Binary PPM is shown in Figure B-1. The derivation of the BER for Binary PPM is identical to BFSK as shown in Appendix B1, with the following substitution. Equation A.1 is now:

$$\begin{aligned} s_m(t) &= A\sqrt{2}\cos(\omega t + \phi_m) \\ &= A\sqrt{2}(\cos\phi_m \cos\omega t - \sin\phi_m \sin\omega t), \quad \frac{mT_b}{2} \leq t \leq \frac{(m+1)T_b}{2}, \quad m = 0, 1 \end{aligned}$$

Since the transmitter is only transmitting half of the time, the amplitude of the carrier is increased by $\sqrt{2}$ to keep the average transmitted power constant. Equation A.4 is now:

$$\begin{aligned} r_{1c} &= E_b \cos\phi_1 + n_{1c}, \qquad r_{1s} = E_b \sin\phi_1 + n_{1s} \\ r_{0c} &= n_{0c}, \qquad r_{0s} = n_{0s}. \end{aligned}$$

with the following definition:

$$E_b = \int_{\frac{T_b}{2}}^{T_b} s_1^2(t) = \frac{(A\sqrt{2})^2 T_b}{4} = \frac{A^2 T_b}{2}.$$

The final result for the BER is the same as BFSK.

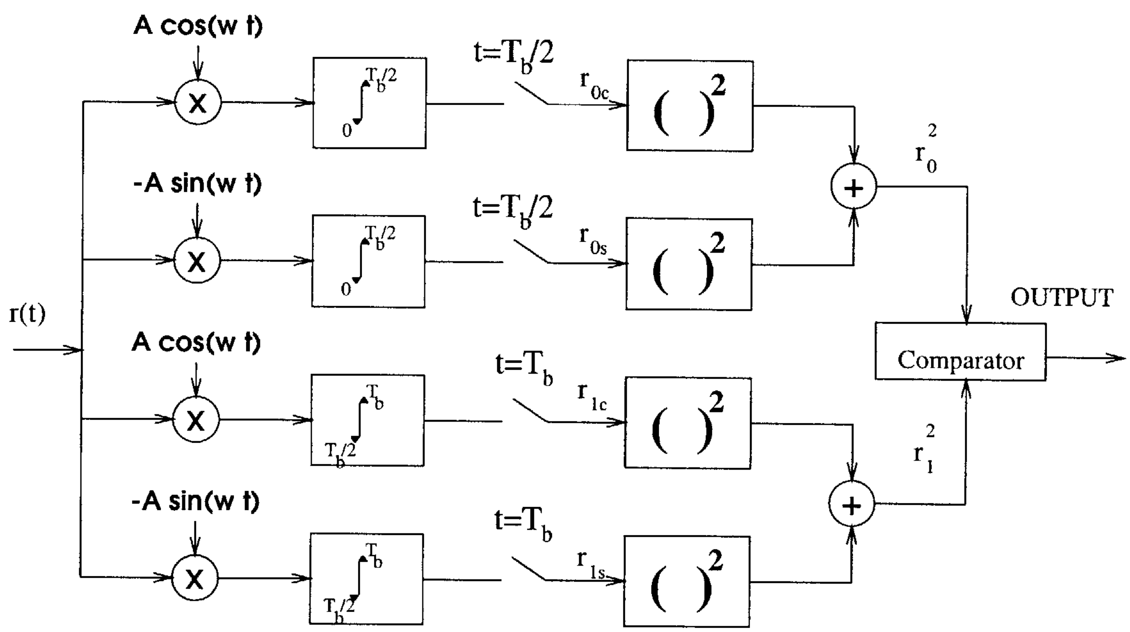

Figure B-1: Correlation Demodulator for Binary PPM

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