Formulating Effective Performance Measures for Systems with Multiple Quality Characteristics

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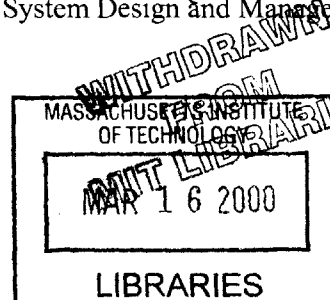
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Abstract

One of the main objectives in engineering – whether design or manufacturing -- is to minimize variations in quality characteristics of a system. The system can either be a product or a process. Minimizing variation requires defining a quantitative measure of variation; in other words a *performance measure*. Several performance measures have been defined and continue to be utilized primarily to design systems with a single dominant quality characteristic. However, almost every system has more than one quality characteristics considered important by the designer of the process or the consumer of the product. Designing robust systems with multiple – often competing – quality characteristics is difficult for a designer because of the uncertain correlation among the design objectives. It is the purpose of this paper to suggest a method for improving the quality of a system with multiple quality characteristics. The desired properties a performance measure should possess are outlined. Measures such as quality loss, signal-to-noise ratio, information content, and rolled throughput yield are examined and their use extended for systems with multiple quality characteristics. These are also looked at in the context of the desired properties such measures should possess. The concept of differential entropy, as defined in information theory, is presented as a candidate performance measure for both single and multiple quality characteristics systems. The suitability of differential entropy as a measure of variation is then compared to existing measures. A case study is presented which demonstrates the use of performance measures in engineering design.

Thesis Advisor: Daniel Frey
  Assistant Professor

بسم الله الرحمن الرحيم

اقرا باسم ربك الذى خلق   خلق الانسان من علق   اقرا و ربك الاكرم
الذى علم بالقلم   علم الانسان ما لم يعلم

In the name of God, the Most Gracious, the Most Merciful

Read in the name of thy Lord, who has created –
created the humn being out of a germ-cell!
Read – for thy Lord is the Most Bountiful One
who has taught (humankind) the use of the pen –
taught humankind what it did not know!

Quran 96, v. 1-5

To My Parents

To My Wife

&

To My Children

Acknowledgements

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1. Introduction

1.1 Motivation

In engineering design, one of the famous heuristics has been the Occam's Razor. William of Occam said "Entities shall not be multiplied beyond necessity," or to paraphrase it, "The simplest explanation is the best," and "It is vain to do with more what can be done with less." This almost universally accepted heuristic demands that engineering design be made as simple as possible. In other words, the complexity associated with a design must be minimized. However, it is not clear what the metric for complexity should be in engineering design. Borrowing from computer science, complexity is the minimal description length. Description length can also be described in terms of the information required to reproduce a given data string. The more random the string of data, the more complex it is and the more information is needed to reproduce it. We will draw on this relationship between complexity and information to propose a new performance measure. It will become clear that in engineering design as well, the concepts of complexity, information and variation are inextricably intertwined. Complexity of an engineering system is very closely related to the degree of deviation of the system's quality characteristics from their target values. Thus, minimizing complexity in a system can be said to be equivalent to minimizing variation in the quality characteristics of a system.

The goal of my thesis is to provide a performance measure which can capture the complexity of a design by measuring the variation in quality characteristics of a system. By minimizing this measure one hopes to end up with a robust product which is insensitive to noise in product or process parameters and thus minimizes economic loss to society.

Choosing the proper measure of variation is perhaps the primary activity of engineering design in general and probabilistic engineering design in particular. Principles that provide guidance in this choice of measure are among the most important tools of a design. This thesis discusses the criteria that have been used for many years but have neither been explicitly recognized nor applied with much conviction. However, the emergence of emphasis on cost sensitive quality design and manufacturing has sharpened the focus on such criteria by making more apparent the situation and the reasons why they apply. This thesis articulates the argument explicitly so as to examine the nature of the desired properties and see how general the existing measures really are. It is hoped that these criteria will provide a rationale in defining more general performance measures.

The methodology I will follow is to first describe the state of the art in measures of variation for systems with a single quality characteristic. Such measures of variation have been utilized mainly to design products/ processes with a single dominant quality characteristic. Most of the research work in robust design has also focused on finding strategies for determining control factors to optimize a single quality characteristic. However, almost every product or process has more than one quality characteristics considered important by the designer of the process or the consumer of the product. Designing robust products with multiple – often competing – quality characteristics is difficult for a designer because of the uncertain correlation among the design objectives. Changes in the control factors, which reduce mean square error in one quality characteristic can adversely affect the variance or the mean in another quality characteristic. Consequently, control factors that may affect multiple quality characteristics cannot and must not be adjusted solely on the basis of the dominant quality characteristic. Therefore, the effect of changes in control factors on all the quality characteristics must be considered jointly.

In the second part of my thesis, I will extend the single-quality-characteristic-system performance measures to systems with multiple quality characteristics. The objective of defining a joint performance measure is to find an optimum solution such that each quality characteristic attains a compromise optimal value. Similar to single-response systems, I will synthesize fragments of existing theory and knowledge into a new framework for understanding robust design of multi-response systems. In formulating this framework, I will draw from several somewhat disparate research areas: design theory, information theory, statistics, and robust design. I will then use this framework to illuminate, with examples, how the robust design of the product relates to general multi-response systems.

1.2 Thesis Organization

This thesis is organized in six chapters. The present chapter describes the motivation behind the thesis and gives a brief overview of past research in the relevant areas. Chapter 2 describes the case study that will be used throughout the rest of the thesis to quantify various performance measures. The main part of the thesis starts with Chapter 3 where the desired properties of a performance measure for systems with multiple quality characteristics are laid out. Chapters 4 and 5 make the largest contribution towards enhancing insight into multi-response system optimization. Chapter 4 describes the existing performance measures for systems with a single quality characteristic. The use of transformations to improve statistical behavior of response variables is also briefly touched upon. The use of entropy as a candidate performance measure is proposed and illustrated through an example. This chapter sets the stage for Chapter 5 where the formulation of existing single-response systems is extended to systems with multiple responses or quality characteristics. The case study presented in Chapter 3 is used to illustrate the use of

multiple quality characteristic performance measures. Chapter 6 summarizes the contributions made in this thesis and suggests future research directions.

While Chapters 1-6 form the main part of the thesis, I have added two appendices that, in my mind, raise interesting philosophical and engineering questions. Appendix A examines Axiomatic Design with more emphasis on minimizing complexity than on de-coupling the design. Several of the theorems and corollaries are examined in this context. Appendix B tries to broaden the concept of robust design into robust architecture. The premise is that by pushing the robustness issue even earlier in the design cycle, significant economic and societal advantages can be obtained.

1.3 Literature Review

There are three main areas of research I will draw from to formulate my ideas. They are 1) robust design using Taguchi methods, 2) axiomatic design, and 3) information theory. A brief survey of these three areas as it relates to topics in this thesis is given.

1.3.1 Taguchi Methods in Robust Design

In designing robust products/ processes, Taguchi methods have been applied successfully in many industries over the years and have resulted in improved product quality and manufacturing processes. The purpose of these methods is to make the product/ process insensitive to uncertain and uncontrollable operating environments. A historical perspective on Taguchi methods has been provided by Phadke [1989] and Creveling [1995]. Over the last ten to fifteen years, Taguchi methods have received a lot of attention in the U.S. and Europe. Most of the research reported in books and technical journals has been in applying the theory of robust design to single-

response systems. However, some strategies have been studied which extend Taguchi methods to multi-response systems.

The case of polysilicon deposition presented in Phadke [1989] is an example of products with multiple characteristics such as surface defects and thickness. An informal argument based on engineering insight and practicality is used to arrive at a compromise solution. Logothetis and Haigh [1988] utilize data transformations to achieve better statistical behavior for each of the quality characteristics and use linear programming techniques to arrive at an optimum. Chen [1997] transforms the individual signal-to-noise ratios for each quality characteristic into the degrees of satisfaction based on their relative importance. Control factors for each quality characteristic are determined and a corresponding regression model is fitted. Then, a mathematical programming problem is formulated and solved to give the optimum settings which maximize the designer's overall satisfaction. The approach is tedious because of its formulation and use of mathematical programming. It does not have the elegance and simplicity of Taguchi methods which is its main attraction. Pignatiello [1993] presents a quadratic loss function for multi-response quality engineering problems. He shows that the quadratic loss function is a generalization of the single-response quadratic loss function used by Taguchi. However, in determining the function suggested by Pignatiello, the loss incurred by each response should be estimated carefully. The determination of the cost parameters of the function will be difficult when there are more than two responses. Pignatiello discusses some strategies that minimize the expected loss function. However, some disadvantages are found with this approach. For example, the costs of experimentation may be expensive, or special prior knowledge may be required for the partitioned strategy. In addition, Pigantiello's approaches are mainly useful for the nominal-the-best type problems. Although he suggests the priority based strategy for mixed design objectives, it is not a generalized approach for producing optimum settings. Similar to

Pignatiello's approach, Elsayed and Chen [1993] address a multi-response model based on the quadratic loss function. Instead of signal-to-noise ratio, they suggest the use of PerMIA (Performance Measure Independent of Adjustment) and tabulate it for different types of problems. The method is outlined for a system with a single control factor and a possible way of extending the method to multiple control factors is presented. Lai and Chang [1994] use Tanaka and Ishibuchi's [1991] quadratic possibility distributions to optimize control factors for a multi-response system. The method is based on a fuzzy multi-response optimization procedure to search an appropriate combination of process parameter settings based on multiple quality characteristics or responses. A die casting example is used to illustrate the approach and appropriate machine settings (actually only the furnace temperature) are derived which simultaneously optimize both casting quality and die life. This approach also uses linear programming to arrive at the final optimal solution. The references cited above also discuss some previous work in the area of multi-response system optimization.

In summary, the research to date on multi-response systems has mostly been on minimizing quadratic loss functions. Several approaches have been discussed but all involve some sort of programming and iterations to find an optimal solution. Pignatiello [1993] refers to his personal communication with Taguchi where Taguchi recommends handling the multiple response case by adding the signal-to-noise ratios. While this would work if the quality characteristics are independent, the procedure would result in sub-optimal results for the case of correlation among the quality characteristics.

1.3.2   Axiomatic Design

Suh [1997, 1995, 1993, and 1990] gives representation techniques of the design process and discusses the axiomatic design principles in the context of different design applications. Albano and Suh [1993] examine the information axiom and its implications on design process. They offer a homogeneous approach to multi-objective design problems and show that the Information Axiom can be used without the need for weighting factors or relative preference. There have also been several graduate level theses from MIT dealing with different aspects of Axiomatic Design and applying the principles to practical cases. Ashby [1992] provides additional insight into information content and complexity of systems with dimensional tolerances. He defines an easier-to-use information content based on geometrical mean dimension and geometrical mean precision of a product and presents a complexity-size chart for different manufacturing processes.

Filippone [1988] and Suh [1990] draw parallels between Axiomatic Design and Robust Design and show, through an example, that the two approaches lead to the same result. Bras and Mistree [1995] use compromise decision support problems to model engineering decisions involving multiple trade-offs. They focus on fulfilling the objectives of axiomatic design and robust design simultaneously. In their paper, axiomatic design is assumed to mean picking the least coupled design and robust design involves maximizing the signal-to-noise ratio. The signal-to-noise ratio is treated as an information measure. In other words, the objective of the decision support problem is to simultaneously satisfy the two axioms.

1.3.3   Information Theory

Shannon [1948] laid the foundation of the mathematical theory of communication that led to further work in information theory. Cover and Thomas [1991] give a very good introduction

and a summary of recent advances in information theory. The concept of entropy has been related to numerous physical sciences but so far has not been studied extensively in the context of engineering design. Suh [1990] does talk about similarities between entropy and information content but the topic is left for further research. Tribus [1969] relates maximum entropy formalisms to engineering design but the work is decision-theory based rather than using entropy as a criterion for comparing engineering designs.

2. Case Study

In order to define a performance measure meaningfully, a system must have a mathematical model that relates its ouptuts to its inputs. This mathematical representation is discussed in the next section followed by a description of the case study to be used in the rest of the thesis.

2.1 System Model

Robust design uses a P diagram to illustrate the relationship of system's outputs called response variables to its various inputs.
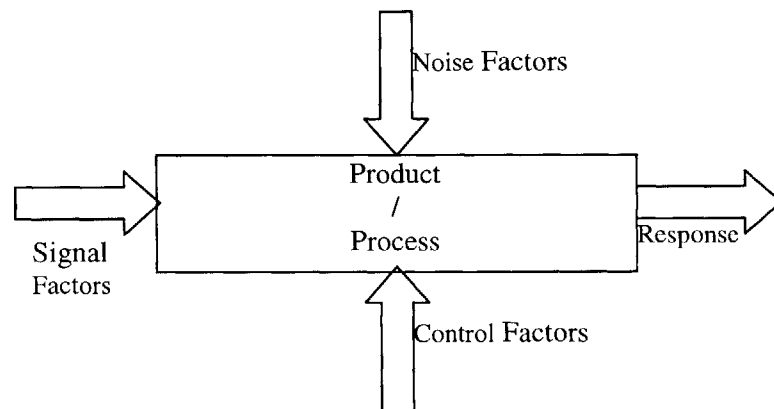
Figure 2.1 Block diagram of a product/process: P Diagram

Fowlkes and Creveling [1995] use the term quality characteristics to define the measured response of a design. Suh [1990] defines design as the mapping process between the functional requirements in the functional domain and design parameters in the physical domain. In other words, an engineering system takes design parameters as the inputs and produces functional requirements as the outputs. In this thesis, I use the term quality characteristic for functional requirements, response variables, or outputs. Since a system transforms a set of given inputs into a set of desired outputs or quality characteristics, the transformation can be represented

mathematically. The set of quality characteristics can be represented by a vector Y with n components, where n is the number of quality characteristics. Similarly, the inputs to a system may be represented by a vector X with m components, where m is the number of inputs which may include control factors, noise factors, and signal factors. The design process involves choosing the right set of inputs to satisfy the desired quality characteristics, which may be expressed as

$$Y = f(X) \tag{2.1}$$

where the function f transforms the inputs into outputs. If the relationship between inputs and outputs is linear, Equation 2.1 can be rewritten as

$$Y = AX \tag{2.2}$$

where A is the design matrix. Suh [1990] calls Equation 2.2 as the Design Equation. In the subsequent case study and the following chapters, I will use this equation to model the system and compute various performance measures accordingly.

2.2 Passive filter design problem

This case study is an adaptation of Example 4.2 from Suh [1990] concerning the design of an electrical passive filter. The two proposed circuit designs are given in Figure 1 as Network a and Network b.

Displacement transducer and demodulator | C | R=R$_2$ | Galvanometer type 348

Network   Oscillograph

Displacement transducer and demodulator | C | R=R$_3$ | Galvanometer type 348

Network b   Oscillograph
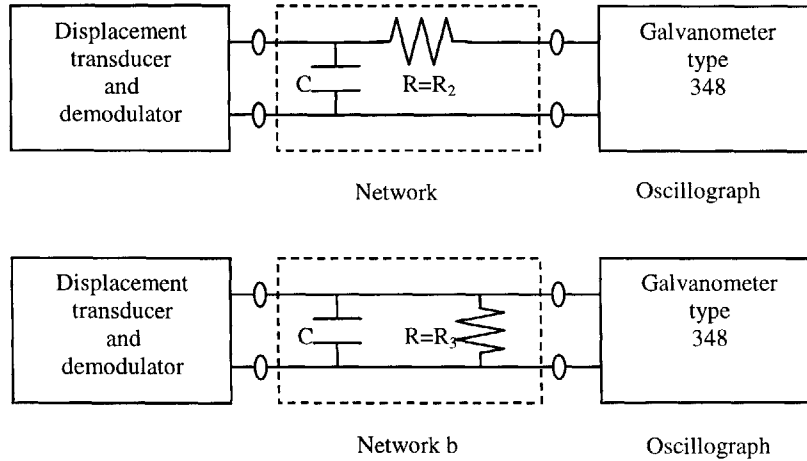
Figure 2.2: Passive Filter Design Problem [Suh, 1990]

Furthermore, two types of displacement transducers are to be evaluated: a Pickering precision LVDT model DTM-5 and a displacement transducer based on a four-active-arm strain gage bridge. Therefore, we have four design options based on possible combinations of two network options and two displacement transducer options. We denote these four design options as LVDT(a), SG(a), LVDT(b) and SG(b).

The quality characteristics of the system have been specified as

ω_c = Design a low-pass filter with a filter pole at 6.84 Hz.

D = Obtain D.C. gain such that the full-scale deflection results in ± 3 in. light beam deflection.

The two design variables are capacitance C and resistance R (i.e., R_2 for Network a and R_3 for Network b). The expressions for D and ω_c can be obtained by using Kirchhoff current law and are given in the following table.

	Network a	Network b
ω_c	$\dfrac{2\pi(R_s + R_g + R_2)}{CR_s(R_g + R_2)}$	$\dfrac{2\pi[(R_g + R_3)R_s + R_3 R_g]}{CR_3 R_g R_s}$
D	$\dfrac{R_g V_{in}}{G_{sen}(R_s + R_g + R_2)}$	$\dfrac{R_3 R_g V_{in}}{G_{sen}[(R_g + R_3)R_s + R_3 R_g]}$

Table 2.1 Equations for Networks a and b

We next define the quality characteristic vector $\mathbf{Y} = [\Delta\omega_c \quad \Delta D]^T$ where $\Delta\omega_c$ and ΔD represent deviation from the ideal values of ω_c and D, respectively. Similarly, the design variable vector is defined as $\mathbf{X} = [\Delta C \quad \Delta R]^T$ where ΔC and ΔR represent deviation from the nominal values of C and R, respectively. The design of the passive filter network can then be represented in the form of Equation 2.2 ($\mathbf{Y} = \mathbf{AX}$) where the matrix $\mathbf{A}$ for the two networks is given as

	Network a	Network b
$\mathbf{A}$	$\begin{bmatrix} -\dfrac{\omega_c}{C} & -\dfrac{\omega_c R_s}{(R_s + R_g + R_2)(R_2 + R_g)} \\ 0 & -\dfrac{D}{(R_s + R_g + R_2)} \end{bmatrix}$	$\begin{bmatrix} -\dfrac{\omega_c}{C} & -\dfrac{\omega_c R_g R_s}{R_3(R_3 R_s + R_g R_s + R_3 R_g)} \\ 0 & \dfrac{DR_g R_s}{R_3(R_3 R_s + R_g R_s + R_3 R_g)} \end{bmatrix}$

Table 2.2 The Matrix $\mathbf{A}$ for Networks a and b

In order to solve for the matrix $\mathbf{A}$ and the Design variables, we assume the following nominal values:

Characteristic	Nominal Value
$R_s\,(\Omega)$	120 (SG), 1860 (LVDT)
$V_{in}\,(V)$	0.015 (SG), 4.5 (LVDT)
$R_g\,(\Omega)$	98
$G_{sen}\,(\mu V\,/\,in.)$	657.58

Table 2.3 Passive Filter Design Characteristic Values

The following table summarizes the results obtained for the matrix **A** and design variables for the four design options.

	SG(a)	SG(b)	LVDT(a)	LVDT(b)
A	-7.4964E+02 -1.7620E-03 0.0000E+00 -4.0260E-03	-1.1751E+02 -2.8390E-02 0.0000E+00 -1.2484E-02	-1.3735E+04 -2.5672E-07 0.0000E+00 -1.3420E-05	-6.0716E+00 -6.5723E-02 0.0000E+00 -1.5315E-03
C	9.1243E-03	5.8205E-02	4.9801E-04	1.1266E+00
R	5.2716E+02	2.2308E+01	2.2159E+05	8.2260E-01

Table 2.4 Values for the design matrix and design parameters

We next assume a 1% specification on C and R, and represent it as $\Delta C = 0.01C$ and $\Delta R = 0.01R$. We also assume that the values of C and R are normally distributed and that there is no correlation between the values of the design variables. This assumption of statistical independence is reasonable if the components come from different vendors or different manufacturing processes. Given a tolerance ΔC as a percentage of capacitance C, we will calculate its variance to be $(\Delta C/3)^2$. Similarly, the variance of resistance R will be calculated as $(\Delta R/3)^2$ if ΔR is given as its percentage tolerance. We let σ_C^2 and σ_R^2 denote the variances

associated with the capacitance C and the resistance R, respectively. The covariance matrix for the design variables, $\mathbf{K}_x$, can then be represented as

$$\mathbf{K}_x = \begin{bmatrix} \sigma_C^2 & 0 \\ 0 & \sigma_R^2 \end{bmatrix}$$

(2.3)

The following table summarizes the values for σ_R and σ_C for the four design options.

	SG(A)	SG(B)	LVDT(A)	LVDT(B)
σ_C	3.0414E-05	1.9402E-04	1.6600E-06	3.7552E-03
σ_R	1.7572E+00	7.4361E-02	7.3863E+02	2.7420E-03

Table 2.5 Values for the standard deviation of design variables

For normally distributed random variables and given $\mathbf{Y} = \mathbf{AX}$, the covariance matrix for the quality characteristics, $\mathbf{K}$, is therefore

$$\mathbf{K} = \mathbf{AK}_x\mathbf{A}^T$$

(2.4)

The following table summarizes the values for the covariance matrix of quality characteristics for the four design options.

	SG(a)		SG(b)		LVDT(a)		LVDT(b)	
K	5.2943E-04	2.1903E-05	5.2430E-04	1.9598E-06	5.1988E-04	1.8796E-06	5.1987E-04	7.5680E-10
	2.1903E-05	5.0048E-05	1.9598E-06	8.6178E-07	1.8796E-06	9.8256E-05	7.5680E-10	1.7636E-11

Table 2.6 Values for the covariance matrix

The values in Tables 3-6 will be used to calculate the quantitative measures of variations given in the following chapters. Also, it will be assumed that the sample mean of quality

characteristics is equal to the target values, i.e., the design variables are chosen such that there is no bias in the system.

In order to evaluate some of the performance measures, a desired tolerance band on individual quality characteristics indicating acceptability of the design would be required. Tolerance limits of 1% and 0.1% are specified on quality characteristics ω and D, respectively as acceptable. For systems with a single quality characteristic to be discussed in Chapter 4, separate expressions for the two quality characteristics would be evaluated for each of the four design options. Similarly, for systems with multiple quality characteristics, performance measures would be evaluated for each of the four design options. Our objective is to find the best design option according to each of the performance measures.

3. Desired Characteristics of a Performance Measure

Numerous measures of variation have been used in engineering design in the past without explicitly stating the assumptions or the criteria that they must satisfy. With the increasing emphasis on quality in design and reducing variation in a product, choosing the "right" measure of variation has become exceedingly important. This paper articulates the desired properties of such performance measures so that any measure of variation can be examined against these properties and its suitability for a specific application judged accordingly. It is also hoped that these criteria will provide a rationale in defining more general performance measures.

The following characteristics are presented as desired properties of a measure of variation:

1. It is an indicator of economic loss resulting from deviation of quality characteristics from the desired specifications.

2. It is easily computable from the information regarding variations in input variables or alternatively from observed statistical properties of the quality characteristics.

3. It makes a minimum of assumptions about the problem structure (probability distribution, heteroscadicity, etc.).

4. It accounts for correlation among quality characteristics.

5. Preference for certain quality characteristics can be incorporated in it.

In Chapter 4 the performance measures for systems with a single quality characteristic will be measured against the desired properties set forth above with the exception of properties 4 and 5 since they apply to systems with multiple quality characteristics only. I will discuss the relevance of these criteria for multiple quality characteristics performance measures in Chapter 5 of the thesis. For both a single quality characteristic system and a multiple quality characteristic system, differential entropy is presented as a performance measure and its relative merits against other more common measures discussed. It is hoped that an insight will be gained into the proper choice of a measure of variation through this process.

3.1 Indicator of Economic Loss

What is the objective of an engineering design or a manufacturing process? Is it to deliver products that are within some specified tolerance band (good enough) or is it to deliver products that are as close to the target specifications as possible (perfection)? Traditionally, part acceptance after a design or manufacturing process has been binary; if the part is within some tolerance limits it is acceptable, otherwise it is defective. The main contribution made by Taguchi was to point out the pitfalls inherent in this "goal post" approach. He proposed the concept of economic loss caused to society by deviation of a product from its desired quality characteristics. Designers and manufacturers must strive to deliver products that are as close to the quality characteristics as possible. Consistently adequate products are not going to win new customers and bring profits to the industry. Instead continuously improving products that aim to meet the customers requirements are going to succeed in the market. Performance measures should be designed such that they measure deviation from the ideal quality characteristics and not just the probability of staying within some tolerance limits. In other words, the criterion for deciding the

"goodness" of a design should not be the probability of remaining within some tolerance band but the potential of coming closest to the desired quality characteristics.

3.2 Easily Computable

The major reason for the success of the Taguchi methods in the engineering community has been its ease of use. The signal-to-noise ratio is easy to compute and is a good measure of deviation from the target characteristics. Although statistics provides better measures of variation and of deviation from the mean, they can be difficult to use. In the case of systems with multiple quality characteristics, the situation is even more complicated since reliance is often made on linear programming or other response surface methods to obtain the optimum control factors. Even when there is a joint performance measures proposed, it is often difficult to compute without the use of numerical integration.

3.3 Assumptions about the Problem Structure

The performance measures used in design must make a minimum of assumptions about the problem structure. This must be balanced against the property of easy computability just discussed. One of the criticisms on Taguchi's signal-to-noise ration is that it assumes a linear relationship between the variance and the mean of the quality characteristic. Although that may be true in most of the cases, care must be used to apply Taguchi methods in cases when it is not. A performance measure should ideally be independent of the problem structure whether it relates to mean-variance relationship or to probability distributions of input or output variables.

3.4 Correlation Among Quality Characteristics

As discussed in the introduction to this thesis, correlation among quality characteristics is a rule rather than an exception. Changes in control factors, which reduce variation in one quality characteristic can adversely affect the variation in another. An assumption of independence between quality characteristics leads to independent adjustment of control factors to adjust one quality characteristics at a time. This may also make the computation of a joint performance measure easy but the results obtained are going to be sub-optimal. Systems with multiple quality characteristics require that performance measures account for correlation among quality characteristics and, in this way, lead to a balanced solution..

3.5 Preference for Certain Quality Characteristics

Customers of a product value each quality characteristic differently. It may be more important for a television viewer to have less variation in picture quality than in audio quality. Or in terms of the case study, the variation in the value of the low-pass filter pole may be less important than variation in the full-scale deflection of the light beam. This preference for certain quality characteristics may also be explained in terms of economic loss to society. The costs associated with variation in each quality characteristics of a system would be different in general. For example, variation in inside temperature of a car is much less important than variation in its braking distance. Therefore, it is important that a performance measure be able to capture the relative preference for certain quality characteristics over others in its structure.

4. Performance Measures for Systems with a Single Quality Characteristic

4.1 Robust Design

In designing robust products/ processes, Taguchi methods have been applied successfully in many industries over the years and have resulted in improved product quality and manufacturing processes. The purpose of these methods is to make the product/ process insensitive to uncertain and uncontrollable operating environments.

Even a cursory look at the Taguchi methods of robust design reveals a "systems" approach to experimental design. This approach sometimes leads to a blind application of mechanistic routines without paying attention to the underlying processes, assumptions and limitations of the method. Because of this reason (among others), this approach has invited a lot of criticism, especially from the statistics community. On the other hand, the practitioners and advocates of Taguchi methods seem to be so enthralled that they take this approach to be the one and only true answer to every design problem. It is important that a realistic perspective be maintained on this approach. Undoubtedly, Taguchi methods have been successfully applied in a variety of industrial settings. However, one must be careful in applying these techniques without knowledge of the underlying assumptions. Another criticism on Taguchi methods has been on his use of quadratic quality loss and then signal-to-noise ratios as estimates of performance measures. This section will attempt to state the assumptions, whether implicit or explicit, that are made when using quadratic loss or signal-to-noise ratio as a performance measures. If these assumptions hold for most situations in product development, then we can safely state that quadratic loss or signal-

to-noise ratio is a good performance measure. However, if the assumptions hold for only a limited class of product development problem, then these are not good performance measures. I do not dispute that statistics may provide more accurate methods to analyze the problem but, as an engineer, I must be aware of the practicality and cost of these methods.

Before I discuss the quality loss functions and signal-to-noise ratio, I would like to quote Gunter [1988] who, in my view, put the use of Taguchi methods in its true perspective. He said, "Do we really believe that we can write down exact mathematical loss functions and thereby provide meaningful and explicit minimization procedures? I doubt it. Moreover, I believe that such a mathematical formulation trivializes the engineering strategies that Taguchi has proposed and confuses statistical estimates with known parameters... I believe that the power of Taguchi's optimization strategy is as an engineering heuristic, not as a statistical procedure. It seems to me that the discussions of PerMIA's (performance measure independent of adjustment), aim-off factors, and the like are little more than mathematical sophistry. Do we really think that the loss functions are exactly quadratic and that the models are so accurately known that such statistical adjustments affect the practical outcome?"

4.1.1   Quality Loss

In most practical manufacturing, product acceptance is binary where a part is either accurate enough or it is defective. Taguchi correctly points out that the distinction between a part slightly outside the tolerance range and a part slightly within the range is arbitrary. For this reason Taguchi has encouraged managers and engineers to think beyond this "goal post" mentality.

As an alternative to the "goal post" approach, Taguchi suggests using a quadratic loss

function. If Y is the quality characteristic and m the target value for the quality characteristic, the quadratic quality loss is given by

$$L(Y) = C(Y - m)^2 \qquad (4.1)$$

where C is a constant called the quality loss coefficient. Let $m \pm \Delta_o$ be the tolerance limits on the quality characteristic. Suppose the loss at $m \pm \Delta_o$ is A_o. Then, by substitution into Equation 4.1, we obtain

$$C = \frac{A_o}{\Delta_o^2} \qquad (4.2)$$

Because of variations in the input parameters and the process, the quality loss as given by Equation 4.1 will vary from unit to unit. The expected value of quality loss can be derived as

$$E(L(Y)) = C\left((\mu - m)^2 + \sigma^2\right) \qquad (4.3)$$

where μ and σ^2 are the mean and the variance of Y, respectively.

Questions have been raised about the quadratic form of the loss function. There is no particular reason to assume a quadratic quality loss function except that it is a simple, continuous function that meets the boundary conditions, i.e., quality loss must be zero when the dimension equals the target dimension and equals A_o at the upper and lower tolerance limits. Taguchi's quality loss function encourages managers and engineers to improve continuously by making them recognize that the closer the product is to its target dimensions the better it performs and thus creates more value for the society in general and the customer in particular. Thus, the quality

loss function can be a useful tool for evaluating performance of a system whether in the design/development stage or in its manufacturing phase.

As mentioned before, one should exercise some caution in using Taguchi's quality loss function. The proponents of Taguchi methods often present the quadratic loss function as a universal law of nature whereas it is just a convenient local approximation using Taylor series. The main flaw in Taguchi quality loss function is that it continues to rise above A_o outside the tolerance band. It is unclear what physical meaning or dollar value to attach to a "quality loss" that exceeds the loss due to scrap. It is likely that if a defect lies far outside the tolerance band, it will be detected and scrapped. Therefore, it may be more reasonable to bound the loss function above by A_o.

4.1.2   Signal-to-Noise Ratio

Signal-to-noise ratio is a performance measure used in Taguchi method of robust design to minimize quadratic quality loss. Taguchi defines several signal-to-noise ratios for different types of quality characteristics and suggests a two step procedure to obtain the control factor levels that maximize the signal-to-noise ratio (and minimize quality loss). The following table summarizes the signal-to-noise ratios for the most common cases of quality characteristics.

Case	Signal-To-Noise Ratio
Smaller-the-better	$\eta = -10\log\left[\dfrac{1}{n}\sum_{i=1}^{n} Y_i^2\right]$
Larger-the-better	$\eta = -10\log\left[\dfrac{1}{n}\sum_{i=1}^{n} \dfrac{1}{Y_i^2}\right]$
Nominal-the-best, type I	$\eta = 10\log\left[\dfrac{\mu^2}{\sigma^2}\right]$
Nominal-the-best, type II	$\eta = -10\log\left[\sigma^2\right]$

Table 4.1 Signal-to-noise ratios for different types of quality characteristics

As before, Y is the response, μ is the mean response, and σ^2 is the variance of response. Most of the research to date has been on nominal-the-best, type II signal-to-noise ratio. The following paragraphs concentrate on this ratio as well.

For nominal-the-best, type II quality characteristics, it is assumed that standard deviation is directly proportional to the mean of the response, i.e., as the mean increases the standard deviation also increases and vice versa. An input variable that has no effect on the variance but affects only the mean, can be used to bring the mean to target. This input variable can be adjusted by a factor of μ_o/μ to get the mean on target. The predicted standard deviation after adjusting the mean on target would be $(\mu_0/\mu)\sigma$. So we have

$$E(L(Y)) = C\left[\frac{\mu_o}{\mu}\sigma\right]^2 \qquad (4.4)$$

We can rewrite the above equation as

$$E(L(Y)) = C\mu_o^2 \left[\frac{\sigma^2}{\mu^2} \right]$$ (4.5)

Since in a given system C and μ_o are constants, only the expression within brackets needs to be minimized. The inverse of the expression within brackets is called the signal-to-noise ratio because σ^2 is the effect of noise factors and μ^2 is the desirable part of the response data. The ratio σ/μ is also known as the coefficient of variation. Maximizing signal-to-noise ratio is equivalent to minimizing the quality loss after adjustment, given by the above equation. It also results in minimizing sensitivity to the known noise factors. For improved additivity of the control factor effects, log transform can be employed and the signal-to-noise ratio can be expressed in decibels as:

$$\eta = -10 \log_{10} \left(\sigma^2 / \mu^2 \right)$$ (4.6)

Taguchi's two step procedure

Taguchi suggests the following two step procedure to obtain the control factor levels that maximize the signal-to-noise ratio (and minimize quality loss).

1. By plotting the signal-to-noise ratio at different levels of control factors, determine factors which have a significant effect on the signal-to-noise ratio. For each control factor, choose the level with the highest signal-to-noise ratio. Thus the overall signal-to-noise ratio is maximized.

2. Select a factor which has the smallest effect on the signal-to-noise ratio among all factors that

have a significant effect on the mean. Ideally this factor should have no effect on the signal-to-noise ratio. Adjust the level of this factor so that the mean response is on target.

4.1.3   Role of Data Transformations in Robust Design

As discussed in the previous section, the use of signal-to-noise ratio helps separate the control factor that affects only the mean response from other control factors that affect the variability in the response. Box [1988] has suggested that it is better to study the mean and the variance separately rather than combine them into a single signal-to-noise ratio as Taguchi suggests. By estimating the location and dispersion effects of the factors separately, one can gain insight into the behavior of the various factors and can adjust the settings of the factor levels to minimize whatever measure is appropriate, including the mean square error. This is attractive, since it presents a single unified approach for the analysis of data from static parameter-design experiments. However, this approach is more difficult to use; and since it cannot be directly related to an economic loss, difficult to motivate among managers.

The main assumption behind the use of signal-to-noise ratio to minimize quality loss is that the standard deviation of a response is directly proportional to its mean. However, that may not always be true. Box (1988) states, " ... for measurements having a natural origin at 0 (such as height, weight, area, tensile strength, yield, absolute temperature, and reaction time) it is often true that the standard deviation increases with the mean. In the absence of anything better, therefore, tacitly assuming a proportional increase—a constant coefficient of variation—would often make more sense than to assume no dependence (see, in particular, Gaddum 1945)…however, such rigid assumptions are unnecessary. Evidence from the data themselves as

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well as from the probability setup may be taken into account in making the appropriate analysis."

For the reasons stated above, Box [1988] states that the universal use of "signal-to-noise" ratios for choice of criteria is unconvincing. According to him, Taguchi methods do not appreciate or exploit the importance of data transformation in achieving a much more efficient and valid analysis. He points out that the coefficient of variation σ/μ is, over extensive ranges, almost proportional to the standard deviation of $\ln Y$. Let us define the transformed response ψ as

$$\psi = \ln Y \tag{4.7}$$

Also, let μ_ψ and σ_ψ denote the mean and standard deviation of the transformed response ψ, respectively. If ψ is normally distributed then it can be shown that

$$\mu = \exp\left(\mu_\psi + \frac{1}{2}\sigma_\psi^2\right) \tag{4.8}$$

$$\sigma = \mu\sqrt{\exp\left(\sigma_\psi^2\right) - 1} \tag{4.9}$$

which can be rewritten as

$$\frac{\sigma}{\mu} = \sqrt{\exp\left(\sigma_\psi^2\right) - 1} \tag{4.10}$$

It is clear that the coefficient of variation [signal-to-noise ratio] is a monotonically increasing [decreasing] function of σ_ψ. Therefore, analysis in terms of signal-to-noise ratio is equivalent to

analysis in terms of standard deviation of the transformed response. We can thus do Taguchi analysis in terms of the following signal-to-noise ratio

$$\eta' = -10\log\sigma_\psi^2 \qquad (4.11)$$

which would give identical results to that of Taguchi's traditional nominal-the-best, type II signal-to-noise ratio. Note that we have retained the use of the term "ratio" although the expression in the above equation does not truly represent a ratio. This is done to conform to the convention in Taguchi methods. A better term for the measure would be "performance measure" or "performance statistic" as recommended by Kackar (1985) or Box (1988).

It is also noted that through a transformation of the response data, a nominal-the-best, type II quality characteristics can be analyzed in terms of a nominal-the-best, type I signal-to-noise ratio.

Taguchi's two-step procedure achieves independence between the signal-to-noise ratio and the mean response. It has been shown that this separation is possible only if $\psi = \log Y$ is normally distributed. Therefore, Taguchi methods implicitly assume that the transformation $\psi = \log Y$ achieves independence between the signal-to-noise ratio and the mean response, and furthermore, that ψ is normally distributed. It has also been shown that the use of η' is equivalent to the use of η. Box (1987) states that, "Now SN$_T$ is a function only of σ/μ, the coefficient of variation of Y, which is, as is well known, nearly proportional to $\sigma_{\ln Y}$. Thus the circumstances in which the procedure discussed previously will work are precisely those in which, after a log transformation of the response Y, the desired separation of the measures μ and $\sigma_{\ln Y}$ (or of $\mu_{\ln Y}$ and $\sigma_{\ln Y}$) will occur. The assumptions underlying all of this are, therefore, that after a log transformation of the data the standard deviation will be independent of the

mean, and that it is then more likely that the design factors will separate into a few that affect variation and some others that affect location without changing variation."

From the above discussion, it is clear that data transformations can be utilized to satisfy the statistical assumptions (such as independence of mean and standard deviation, and normality of response) and in selecting appropriate performance measures (signal-to-noise ratios). For the specific transformation of $\psi = \ln Y$, Box (1988) states that this transformation will lead to independence of cell mean and cell variance, exactly for normal distribution and approximately otherwise.

Box transformation can be combined with Taguchi method to carry out robust design. I propose an extension of the two step procedure used in robust design by incorporating the Box transformation. The result is the following three-step procedure:

1. Transform the response variable by the Equation 4.7 ($\psi = \ln Y$)

2. Formulate the signal-to-noise ratio as in Equation 4.11 ($\eta' = -10 \log \sigma_\psi^2$). By plotting signal-to-noise ratio at different levels of control factors, determine factors that have a significant effect on the signal-to-noise ratio. For each control factor, choose the level with the highest signal-to-noise ratio. Thus the overall signal-to-noise ratio is maximized.

3. Select a factor that has the smallest effect on the signal-to-noise ratio among all factors that have a significant effect on the mean. Ideally this factor should have no effect on the signal-to-noise ratio. Adjust the level of this factor so that the mean transformed response μ_ψ is on the transformed target M, where $M = \ln m$.

Note that the last two steps are identical to the Taguchi two-step procedure except for minor changes. An additional first step in data transformation has been added to improve the statistical analysis. We will see how this transformation and the new signal-to-noise ratio in terms of the transformed response variable may lead to additional insight into the use of signal-to-noise ratio. This procedure enables the designer to take clear-cut decisions for improving a product or a process. Therefore, the original spirit of the Taguchi methods is maintained.

4.2 Information Content in Axiomatic Design

Suh [1990] delves into the fundamental principles of engineering design and presents the theory of axiomatic design. Although Appendix A gives a more thorough overview of Axiomatic design, the following paragraph is intended only to give a perspective on the information content as defined in the Axiomatic Design theory.

Suh [1990] presents Axiomatic Design as a decision support mechanism during the design process. The core of this method is a pair of axioms upon which corollaries and theorems are built which may be applied within a structured, multi-domain mapping process. The design axioms are stated as follows:

Axiom 1   *The Independence Axiom*

An optimal design always maintains the independence of functional requirements.

Axiom 2   *The Information Axiom*

The best design is a functionally uncoupled design that has the minimum information content.

The functional requirements are the output or response variables of a system, or in the language of this thesis, quality characteristics. The design parameters are the input variables in the physical domain. Design is defined as the mapping process between the quality characteristics in the functional domain and design parameters in the physical domain.

Axiom 2 deals with the complexity of a design that is associated with the difficulty of achieving a task. According to Suh [1995], "Information is the measure of knowledge required to satisfy a given functional requirement at a given level of the functional requirement hierarchy." The knowledge required to achieve a task depends on the probability of success. Therefore, the notion of information is very closely related to the probability of achieving the functional requirement. For a single quality characteristic system, the Information I may be expressed as

$$I = \log_2 (1/p)$$ (4.12)

where p is the probability of design parameters satisfying the specified functional requirement or the desired quality characteristic as given by

$$p = \Pr(m - \Delta_o \leq Y \leq m + \Delta_o)$$ (4.13)

where m and Δ_o represent the desired value and the desired tolerance of the functional requirement Y. Let $f(y)$ be the probability density function for the quality characteristic Y. Equation 4.12 can then be written as

$$I = -\log_2 \int_{m-\Delta_o}^{m+\Delta_o} f(y) dy$$ (4.14)

In engineering design, the inputs most often are in the form of specifications on components or dimensions on parts. Because of manufacturing variations, the specifications or dimensions vary and a Gaussian distribution is normally used to represent this variation. The probability density function of a normally distributed multivariate is

$$f(\mathbf{x}) = \frac{1}{\left(\sqrt{2\pi}\right)^m |\mathbf{K}_x|^{1/2}} e^{-\frac{1}{2}(\mathbf{x}-\mu_x)^T \mathbf{K}_x^{-1}(\mathbf{x}-\mu_x)} \qquad (4.15)$$

where $\mathbf{K}_x$ is the covariance matrix and μ_x is a vector comprising the mean values of the input variables. For the single quality characteristic, the design equation would be

$$Y = \mathbf{a}\mathbf{X} \qquad (4.16)$$

where $\mathbf{a}$ is a row vector. The probability distribution of the quality characteristic Y would also Gaussian with the variance σ^2 given as $\mathbf{a}\mathbf{K}_x\mathbf{a}^T$. Equation 4.14 can be rewritten as

$$I = -\log_2 \int_{m-\Delta_o}^{m+\Delta_o} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}} dy \qquad (4.17)$$

It is unfortunate that no closed form solution exists for Equation 4.17. Therefore, Equation 4.17 must either be numerically integrated or probability tables be used to obtain a value for the information content in a given design.

4.3 Performance Measures in Manufacturing

So far we have concentrated on the performance measures that are used to capture variation in engineering design. However, it becomes quite clear that measures of variation in

manufacturing are no different than in engineering design. In fact one might propose to use them interchangeably! The following sections describe the performance measures as used in manufacturing and their similarity to engineering design performance measures.

4.3.1   Process Capability Index

Manufacturing processes are "engineering systems" and like any other system exhibit some variation (dimensional or otherwise) and bias in their outputs. These outputs are themselves systems called products. No one disputes the existence of variations in inputs. Despite variations in inputs, managers and engineers strive to reduce variation in output products and employ measures to assess the capability of a process to manufacture acceptable products. One of the most common measures employed to assess the capability of a manufacturing system is the process capability index, C_p. It is a dimensionless ratio of the amount of variation that can be tolerated and the amount of variation present. In mathematical terms

$$C_p = \frac{\Delta_o}{3\sigma}$$ (4.18)

where, as before, Δ_o is the symmetric tolerance below and above the desired specification that can be tolerated and σ is the standard deviation of the quality characteristic. Manufacturers try to maximize the process capability index by reducing σ and increasing the tolerance band Δ_o, where practical.

4.3.2   Performance Index

Note that the process capability index does not take bias into account. Bias is defined as the difference between the mean value of the quality characteristics and the center of the tolerance band. Bias in dimensional variations or other quality characteristics of a system degrades the performance of a system. In manufacturing systems, bias tends to reduce yield. Manufacturing engineers, therefore, use a modified process capability index called the performance index, C_{pk} [Kane, 1986]. It is defined as

$$C_{pk} = C_p \left(1 - k\right) \tag{4.19}$$

where k is a dimensionless ratio of the absolute value of the bias and tolerance range

$$k = \frac{|\mu - m|}{\Delta_o} \tag{4.20}$$

Equation 4.19 can be rewritten in the following form to show its similarity to performance measures such as quality loss or signal-to-noise ratio.

$$C_{pk} = \frac{\Delta_o - |\mu - m|}{3\sigma} \tag{4.21}$$

The performance index gives an indication of the expected performance of a manufacturing system when both bias and variation are present in the output product quality characteristics.

4.3.3   First Time Yield

The probability that a quality characteristic meets its tolerances is sometimes called the first time yield, Y_{FT}. In mathematical terms

$$Y_{FT} = \Pr\left(m - \Delta_o \le Y \le m + \Delta_o\right) \tag{4.22}$$

Let $f(y)$ be the probability density function for the quality characteristic Y. Equation 4.22 can then be written as

$$Y_{FT} = \int_{m-\Delta_o}^{m+\Delta_o} f(y)\,dy \tag{4.23}$$

Comparing Equation 4.23 with the expression for information content as given in Equation 4.14, we can write

$$I = -\log_2 Y_{FT} \tag{4.24}$$

Therefore, maximizing the first time yield is equivalent to minimizing information content of an engineering system!

If the probability density function $f(y)$ is Gaussian, the first time yield can be solved for as a function of the process capability index and bias factor [Frey, 1997].

$$Y_{FT} = \frac{1}{2}\left[erf\left(\frac{3\sqrt{2}}{2}C_p(1-k)\right) + erf\left(\frac{3\sqrt{2}}{2}C_p(1+k)\right)\right] \tag{4.25}$$

Note that evaluating this expression necessitates use of numerical methods or look-up tables. Therefore, the use of this expression as an easy-to-compute performance measure is questionable.

4.4 Differential Entropy

The concept of entropy appears in numerous technical fields but so far has not been applied in the domain of engineering design. Suh [1990] does talk about similarities and differences between entropy and information content in Axiomatic Design. However, he leaves the subject for further research and, instead, pursues the concept of information content as summarized in Section 4.2 of this chapter. In this section, I examine the concept of entropy and propose it as a more general performance measure for engineering systems. I will borrow extensively from the concept of entropy as used in information theory.

Entropy, as defined in information theory, is a measure of the uncertainty of a random variable; it is a measure of the amount of information required on the average to describe the random variable. Entropy has many properties that agree with the intuitive notion of what a measure of information should be and as such determines the complexity of a given system. In engineering design, given the inadequacies of a model and some level of non-repeatability for a process, the quality characteristics of a product or a process can be treated as random variables in most cases. As stated earlier, the variation in these quality characteristics must be measured in order to manage or minimize them. Since variation and uncertainty are closely related, entropy offers good promise as a measure of variation or uncertainty in quality characteristics, which can be modeled as random variables.

With a given amount of data, the probability of success of a design (achieving a desired quality characteristic) for each set of manufacturing conditions (operators, components,

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machines, tools, environment, etc.) will be different. In this case, the definition of probability of achieving a desired quality characteristic can be extended to compute the average probability of success as

$$\text{Average probability of success} = -\sum_i p_i \log_2 p_i \qquad (4.26)$$

where the subscript i denotes the i th produced unit. The right hand side of the Equation 4.29 is the entropy as defined in information theory. Thus, it can be stated that the average probability of achieving a desired quality characteristic is equal to the entropy of that quality characteristic. In this case, we are treating the quality characteristic as a discrete random variable. Let Y be the discrete random variable with probability mass function $p(y)$, $y \in \mathcal{Y}$, where $\mathcal{Y}$ is the support set of the random variable Y, then

$$H(Y) = -\sum_{y \in \mathcal{Y}} p(y) \log_2 p(y) \qquad (4.27)$$

It is clear that

$$H(Y) = E\left(\log_2 \frac{1}{p(Y)} \right) \qquad (4.28)$$

where E denotes expectation and Y is drawn according to the probability mass function $p(y)$. These and the following results about entropy are well known in information theory and the interested reader is referred to Cover and Thomas [1991] for further details.

Next the concept of differential entropy is introduced. Differential entropy is the entropy of a continuous random variable and is related to the shortest description length (information), and is

similar in many ways to the entropy of a discrete random variable. But there are some important differences and one should use care in using this concept. However, it will become apparent that differential entropy is better suited as a performance measure in engineering systems.

The differential entropy $h(Y)$ of a continuous random variable Y with a density $f(y)$ is defined as

$$h(Y) = -\int_S f(y) \log_2 f(y) dy \tag{4.29}$$

where S is the support set of the random variables. It is easy to see that differential entropy represents the expected value of the information content in a continuous random variable. As in the discrete case, the differential entropy depends only on the probability density function.

For a normally distributed quality characteristic, Equation 4.29 can be integrated to obtain

$$h(Y) = \frac{1}{2} \log_2 \left(2\pi e \sigma^2\right) \tag{4.30}$$

Note that the units of differential entropy as in Equation 4.30 are bits. However, since differential entropy can only be used as a relative measure, the base of the logarithm is immaterial as long as it is consistent.

As in Section 4.2, for the single quality characteristic, the design equation would be $Y = \mathbf{a}\mathbf{X}$, where $\mathbf{a}$ is a row vector. If the input parameters have a normal distribution with the covariance matrix $\mathbf{K}_x$, the probability distribution of the quality characteristic Y would also be Gaussian with the variance $\sigma^2 = \mathbf{a}\mathbf{K}_x\mathbf{a}^T$. Equation 4.30 can be rewritten as

$$h(Y) = \frac{1}{2}\log_2\left(2\pi e \mathbf{aK}_x \mathbf{a}^T\right)$$

(4.31)

Equation 4.31 provides an easy to compute measure for a system where the input parameters are normally distributed. In addition, the concept is based on sound theoretical footing. Because of this reason, I propose to use differential entropy rather than signal-to-noise ratio for use in robust design.

4.4.1 Robust Design Using Differential Entropy

Note that the expression in Equation 4.30 is very similar to the nominal-the-best type, I signal-to-noise ratio. In addition, it was shown in Section 4.1.3 that data transformations make possible the use of a nominal-the-best, type I signal-to-noise ratio even when one is working with nominal-the-best, type II quality characteristics. Therefore, the modified three step procedure proposed in Section 4.1.3 would now be

1. Transform the response variable by the Equation 4.7 ($\psi = \ln Y$)

2. Formulate the entropy

$$h = \frac{1}{2}\log_2\left(2\pi e \sigma_\psi^2\right)$$

(4.32)

By plotting h at different levels of control factors, determine factors that have a significant effect on the differential entropy, h. For each control factor, choose the level with the lowest h. Thus the overall complexity of the system is minimized.

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3. Select a factor that has the smallest effect on h among all factors that have a significant effect on the mean. Ideally this factor should have no effect on h. Adjust the level of this factor so that the mean transformed response μ_ψ is on the transformed target M, where $M = \ln m$.

This approach of using entropy as a performance measure in robust design will be extended for systems with multiple characteristics in Chapter 5.

It was noted earlier that differential entropy has some important differences from the better-known and more-used discrete entropy. The discrete entropy measures the average information content of a discrete random variable in absolute terms whereas in the case of differential entropy, the measurement of information depends on the units or the coordinate system chosen. If we change the units, the differential entropy will in general change. However, for a given coordinate system, differential entropy will provide a relative measure between two different designs (or sets of random variables). For the same reason, the lowest possible value of discrete entropy is zero whereas no lower bound exists on the differential entropy. Therefore, differential entropy can be used only as a relative measure.

4.5 Summary

The expressions for the various performance measures defined for systems with a single quality characteristic are summarized in the following table.

Performance Measure	Expression
Expected Quality Loss	$E(L(Y)) = C((\mu - m)^2 + \sigma^2)$
Signal-to-Noise Ratio	$\eta = -10\log_{10}(\sigma^2/\mu^2)$

Information Content	$I = -\log_2 \int\limits_{m-\Delta_o}^{m+\Delta_o} \dfrac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}}\, dy$
Process Capability Index	$C_p = \dfrac{\Delta_o}{3\sigma}$
Performance Index	$C_{pk} = \dfrac{\Delta_o - \lvert \mu - m \rvert}{3\sigma}$
First Time Yield	$Y_{FT} = \int\limits_{m-\Delta_o}^{m+\Delta_o} \dfrac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}}\, dy$
Differential Entropy	$h(Y) = \dfrac{1}{2}\log\left(2\pi e\, \sigma^2\right)$

Table 4.2 Summary of Performance Measures for Systems with a Single Quality Characteristic

These expressions can be evaluated for the case study given in Chapter 2. First we assume

that the quality characteristic of interest is the value of the filter pole frequency, ω_c. The values

for the performance measures for the four design options are then calculated and are tabulated in

Table 4.3.

	SG(A)	SG(B)	LVDT(A)	LVDT(B)
$E(L)$	5.2943E-04	5.2430E-04	5.1988E-04	5.1987E-04
η	2.4732E+01	2.4753E+01	2.4771E+01	2.4771E+01
I	4.3350E-03	4.0670E-03	3.9020E-03	3.9010E-03
C_p, C_{pk}	1.2920E+02	1.3046E+02	1.3157E+02	1.3157E+02
Y_{FT}	0.9970	0.9972	0.9973	0.9773
h	-3.3945E+00	-3.4016E+00	-3.4077E+00	-3.4077E+00

Table 4.3 Evaluation of performance measures for quality characteristic ω_c

Table 4.4 summarizes the calculated values for the performance measures if we assume that

the quality characteristic of interest is the full scale deflection, D.

	SG(A)	SG(B)	LVDT(A)	LVDT(B)
$E(L)$	5.0048E-05	8.6178E-07	9.8256E-05	1.7636E-11
η	2.6274E+01	3.5094E+01	2.4809E+01	5.8539E+01
I	1.6082E+00	1.7770E+00	2.0720E+00	≈ 0
C_p, C_{pk}	5.9943E+02	3.4812E+04	3.0533E+02	1.7011E+09
Y_{FT}	0.328	0.999	0.238	≈ 1
h	-5.0961E+00	-8.0260E+00	-4.6095E+00	-1.5814E+01

Table 4.4 Evaluation of performance measures for quality characteristic D

Note that since there is no bias assumed in the system, the values of C_p and C_{pk} are identical. The optimum design according to each of the performance measures is highlighted in the tables.

If ω_c is the dominant quality characteristic, the optimum design according to all the performance measures is equally LVDT(a) and LVDT(b). However, for D being the dominant quality characteristic, the design LVDT(b) is by far the best one. In general, the two quality characteristics can drive the optimum design in two different directions. Note that for systems with a single quality characteristic, all performance measures give the same result irrespective of the tolerance band specified on the quality characteristics. I will show in the next chapter that it is not the case if both quality characteristics are to be simultaneously considered. Furthermore, correlation between quality characteristics would require more than an ad hoc approach of taking a weighted average of the performance measures corresponding to the quality characteristics.

5. Performance Measures for Systems with Multiple Quality Characteristics

I do not agree with Taguchi's emphasis on a performance statistic based on a single quality characteristic. It is rare that a quality characteristic dominate the design of a system. Usually there are more than one quality characteristic of interest to the designer or user of the system. An ad hoc approach of looking at the responses and balancing them using engineering sense is not attractive for obvious reasons. However, using linear programming or multivariate optimization is equally unattractive since it makes the problem tedious and cumbersome. To me, it seems more logical to combine various responses in an overall "performance measure" instead of considering them separately or in cumbersome analysis.

The following sections extend the concepts presented in the previous chapter to systems with multiple quality characteristics.

5.1 Quality Loss

Note that Equation 4.1 gives the quality loss function for a single quality characteristic system. For a system with multiple quality characteristics, the quality loss function can be written as [Pignatiello, 1993]

$$L(\mathbf{Y}) = [\mathbf{Y} - \mathbf{m}]^T \mathbf{C}[\mathbf{Y} - \mathbf{m}] \qquad (5.1)$$

where $\mathbf{Y}$ and $\mathbf{m}$ are the quality characteristics vector and the target vector, respectively, and $\mathbf{C}$ is the quality loss coefficient matrix. Note that if $\mathbf{C}$ is a diagonal matrix, then the quality loss is

the sum of single quality characteristic loss functions. For non-diagonal C, Pignatiello [1993] states that the off-diagonal terms are related to societal losses that are incurred when pairs of quality characteristics are simultaneously off-target. Suppose the loss at the point $m_i \pm \Delta_{oi}$ and $m_j \pm \Delta_{oj}$ is A_{oij}. Then, by substitution into Equation 5.1, we obtain

$$C_{ij} = \frac{A_{oij}}{\Delta_{oi}\Delta_{oj}} \tag{5.2}$$

Clearly, the matrices C and A_o are symmetric. Equation 5.2 can be used to obtain values to the C matrix. However, it is unclear to me how one would assign values to the elements in matrix A_o.

The expected value of this multiple quality characteristic loss function can be derived and is given by

$$E(L(Y)) = trace[CK] + [\mu - m]^T C[\mu - m] \tag{5.3}$$

where μ is a vector containing the means of the quality characteristics and K is the covariance matrix of the quality characteristics vector.

5.2 Signal-to-Noise Ratio

Taguchi method is applicable for single quality characteristic systems only. In the case study undertaken by Filippone [1988] (the same as the case study in this thesis) there are two quality characteristics. Filippone treats both responses equally and maximizes the sum of the two signal-to-noise ratios. It is interesting to note that Taguchi also suggests adding the signal-to-noise

ratios for systems with multiple response variables [Pignatiello, 1993]. Therefore, another measure that can be used to quantify variation in a design is the modified signal-to-noise ratio as defined by

$$\eta_1 = -\sum_{i=1}^{n} 10 \log_{10}\left(\sigma_i^2 / \mu_i^2\right) \tag{5.4}$$

where the subscript i denotes the sample mean or variance associated with the i th quality characteristic.

Equation 5.4 assumes that all quality characteristics are equally important. For the general case of certain quality characteristics preferred over others, Equation 5.4 can be modified as

$$\eta_2 = -\sum_{i=1}^{n} 10 \log_{10}\left(C_i \frac{\sigma_i^2}{\mu_i^2} \right) \tag{5.5}$$

where C_i's are the relative costs of the quality characteristics.

It is noted that unlike the single response signal-to-noise ratio that was derived from the single quality characteristic loss function of Equation 4.3, these equations have not been derived from the multiple quality characteristic loss function of Equation 5.3. However, the underlying assumption is that loss to society is incurred from single quality characteristic deviations only and not from any combination of quality characteristics being simultaneously off-target.

I next examine the problem of deriving a general signal-to-noise ratio for multiple quality characteristic directly from the expected value of the quality loss function. Equation 5.3 can be expressed in scalar form as

$$E(L(\mathbf{Y})) = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} \left[K_{ij} + \left(\mu_i - m_j \right) \left(\mu_j - m_i \right) \right] \qquad (5.6)$$

Following the pattern of derivation of single quality characteristic signal-to-noise ratio, it is assumed that standard deviation is directly proportional to the mean of the quality characteristics. In addition, the same relationship is assumed between the off-diagonal terms of the covariance matrix and the product of the respective quality characteristics means. If $m > n$, assume that n input variables or control factors can be found that have no effect on the variance or covariance but affect only the mean values of n quality characteristics. Then each of the n control factors can be adjusted to bring the n means to their desired values. So we obtain

$$E(L(\mathbf{Y})) = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} K_{ij} \frac{m_i m_j}{\mu_i \mu_j} \qquad (5.7)$$

For improved additivity of the control factor effects, log transform can be employed and the signal-to-noise ratio can be expressed in decibels as

$$\eta_3 = -10 \log_{10} \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \left(C_{ij} K_{ij} \frac{m_i m_j}{\mu_i \mu_j} \right) \right] \qquad (5.8)$$

The advantage of Equation 5.8 over Equation 5.5 lies in the fact that correlation among the quality characteristics is clearly reflected. However, the assumptions made in the derivation are much more demanding and it is debatable whether a large percentage of engineering systems would satisfy such assumptions. It is surprising that this easily derivable result has not been presented in literature on robust design so far!

5.3 Information Content

For an uncoupled design, the Information I may be expressed as [Suh, 1995]

$$I = \sum_{i=1}^{n} \left[\log_2 \left(1/p_i \right) \right] \qquad (5.9)$$

where n is the number of quality characteristics and p_i is the probability of X_i delivering Y_i within stated tolerances, assuming that X_i s are uncorrelated. In mathematical terms

$$p_i = \Pr\left(m_i - \Delta_i \leq Y_i \leq m_i + \Delta_i \right) \qquad (5.10)$$

where m_i denotes the desired value and Δ_i represents the desired tolerance of the i th quality characteristic.

A procedure for computing information content of a design based on Suh's definition is now presented with the additional relaxation that the quality characteristics can be correlated. The definition in Equation 5.9 can be generalized for an arbitrary design (uncoupled, decoupled or coupled)

$$I = \log_2 \left(1/p \right) \qquad (5.11)$$

where p is the probability of design parameters satisfying all of the specified quality characteristics. Let $f\left(y_1, y_2, ..., y_n \right)$ be the probability density function. Equation 5.11 can then be written as

$$I = -\log_2 \int\limits_{m_1-\Delta_1}^{m_1+\Delta_1} \int\limits_{m_2-\Delta_2}^{m_2+\Delta_2} \cdots \int\limits_{m_n-\Delta_n}^{m_n+\Delta_n} f(y_1, y_2, \ldots, y_n) dy_1 dy_2 \ldots dy_n \qquad (5.12)$$

where m_i and Δ_i represent the desired value and the desired tolerance of the functional requirement Y_i.

As before, we let the variability in the design parameters be represented by a Gaussian probability distribution with $\mathbf{K}_x$ as the covariance matrix. If $\mathbf{Y} = \mathbf{AX}$, then the covariance matrix of $\mathbf{Y}$ is given as $\mathbf{K} = \mathbf{AK}_x\mathbf{A}^T$ and the information content can be represented as

$$I = -\log_2 \int\limits_{m_1-\Delta_1}^{m_1+\Delta_1} \int\limits_{m_2-\Delta_2}^{m_2+\Delta_2} \cdots \int\limits_{m_n-\Delta_n}^{m_n+\Delta_n} \frac{1}{\left(\sqrt{2\pi}\right)^n |\mathbf{K}|^{1/2}} e^{-\frac{1}{2}(y-\mu)^T \mathbf{K}^{-1}(y-\mu)} dy_1 dy_2 \ldots dy_n \qquad (5.13)$$

It is unfortunate that no closed form solution exists for Equation 5.13. Therefore, Equation 5.13 must be numerically integrated to obtain a value for the information content in a given design.

5.4 Process Capability Matrix

Frey [1997] proposes and defines a process capability matrix as an extension of the process capability index. Let $\mathbf{C}_p$ represent the $n \times m$ process capability matrix. In the terminology of this thesis and with the system represented as $\mathbf{Y} = \mathbf{AX}$, we can write

$$C_{pij} = \frac{3\sigma_{xj} A_{ij}}{\Delta_{oi}} \qquad (5.14)$$

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where σ_{xj} is the standard deviation of the j th input and Δ_{oi} is the tolerance limit above and below the desired quality characteristic value. This matrix is quite useful in predicting rolled throughput yield and in giving insight into the most promising tolerances to widen. However, since this is a different approach than using one joint performance measure for a multiple quality characteristics system, I will not pursue this concept further.

5.5 Rolled Through-put Yield

Rolled throughput yield is defined as the probability that every quality characteristic of a system is simultaneously met. In the previous chapter, first time yield was defined as the probability that a single quality characteristic meets its tolerances. In six sigma producibility analysis, it is suggested that rolled throughput yields be computed by multiplying the individual first time yields [Harry and Lawson, 1992].

$$Y_{RT} \approx \prod_{i=1}^{n} Y_{FT_i} \tag{5.15}$$

This formula is simple and easy to apply. However, it assumes statistical independence among the quality characteristics of the products. It is similar to Suh's information content for the case of an uncoupled design that was given as Equation 5.9.

Frey [1997] showed that this unwarranted assumption of statistical independence among quality characteristics may result in estimates of rolled throughput yield that are in error by over two orders of magnitude. Correlation is the rule rather than the exception in manufacture and can significantly affect rolled throughput yield. Frey [1997] further suggests that rolled throughput yield be estimated as

$$Y_{RT} = \Pr\left(m_1 - \Delta_1 \leq y_1 \leq m_1 + \Delta_1, \ldots, m_n - \Delta_n \leq y_n \leq m_n + \Delta_n\right) \qquad (5.16)$$

As before, we let $f\left(y_1, y_2, \ldots, y_n\right)$ be the joint probability density function for the quality characteristics of a product. Equation 5.16 can then be written as

$$Y_{RT} = \int_{m_1 - \Delta_1}^{m_1 + \Delta_1} \int_{m_2 - \Delta_2}^{m_2 + \Delta_2} \ldots \int_{m_n - \Delta_n}^{m_n + \Delta_n} f(y_1, y_2, \ldots, y_n) dy_1 dy_2 \ldots dy_n \qquad (5.17)$$

Comparing Equation 5.12 and 5.17, we can relate the rolled throughput yield to the information content as

$$I = -\log_2 Y_{RT} \qquad (5.18)$$

5.6 Differential Entropy

The differential entropy of a vector $\mathbf{X}$ of m random variables with probability density function $f\left(x_1, x_2, \ldots, x_m\right)$ is defined as

$$h(\mathbf{X}) = -\int_S f(x_1, x_2, \ldots, x_m) \log_2 f(x_1, x_2, \ldots, x_m) dx_1 dx_2 \ldots dx_m \qquad (5.19)$$

Equation (2.2) ($\mathbf{Y} = \mathbf{AX}$) represented the design process which mapped quality characteristics to design parameters. For the case of $\mathbf{A}$ being a square matrix, and from the definition of joint differential entropy in Equation 5.19, it can be easily shown [Cover and Thomas, 1991] that

$$h(\mathbf{Y}) = h(\mathbf{AX}) = h(\mathbf{X}) + \log_2|\mathbf{A}| \qquad (5.20)$$

where $|.|$ denotes the absolute value of the determinant. Equation 5.20 expresses the average information content (or entropy) of the quality characteristics as a function of the average information content in input design parameters and the design matrix very concisely.

The entropy of a multivariate normal distribution is given as

$$h(\mathbf{X}) = \frac{1}{2}\log_2 (2\pi e)^m |\mathbf{K}_x| \qquad (5.21)$$

where, as before, $\mathbf{K}_x$ is the covariance matrix for the vector $\mathbf{X}$. Given $\mathbf{Y} = \mathbf{AX}$, we can write the differential entropy for the quality characteristic vector

$$h(\mathbf{Y}) = \frac{1}{2}\log_2 (2\pi e)^n |\mathbf{K}| \qquad (5.22)$$

where $\mathbf{K} = \mathbf{AK}_x\mathbf{A}^T$ is the covariance matrix of $\mathbf{Y}$.

5.6.1   Robust Design Using Differential Entropy

Note that unlike single quality characteristics systems, the expressions for signal-to-noise ratio and differential entropy are quite different in the case of systems with multiple quality characteristics. It seems to me that differential entropy should provide a better measure of variation because of its inherent relationship to information and complexity in a system. In Section 4.4.1, I outlined a procedure for robust design of single response systems based on differential entropy. That approach can be extended very simply for systems with multiple quality characteristics. I propose the following procedure for robust design of systems with multiple quality characteristics:

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1. Transform each of the response variable by Equation 4.7 ($\psi = \ln Y$)

2. Formulate the entropy of random vector ψ according to Equation 5.22. By plotting h at different levels of control factors, determine factors that have a significant effect on the differential entropy, h. For each control factor, choose the level with the lowest h. Thus the overall complexity of the system is minimized.

3. For each quality characteristic, select a factor that has the smallest effect on h among all factors that have a significant effect on the mean. Ideally this factor should have no effect on h. Adjust the level of each of these factors so that the mean transformed response vector, $\mathbf{\mu}_\psi$ is on the transformed target vector $\mathbf{M}$, where $M_i = \ln m_i$.

5.7 Summary

The expressions for the various performance measures defined for systems with multiple quality characteristics are summarized in the following table.

Performance Measure	Expression
Expected Quality Loss	$E(L(\mathbf{Y})) = trace[\mathbf{CK}] + [\mu - \mathbf{m}]^T \mathbf{C}[\mu - \mathbf{m}]$
Signal-to-Noise Ratio	$\eta_1 = -\sum_{i=1}^{n} 10\log_{10}\left(\sigma_i^2 / \mu_i^2\right)$ $\eta_2 = -\sum_{i=1}^{n} 10\log_{10}\left(C_i \dfrac{\sigma_i^2}{\mu_i^2}\right)$ $\eta_3 = -10\log_{10}\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\left(C_{ij}K_{ij}\dfrac{m_i m_j}{\mu_i \mu_j}\right)\right]$

Information Content	$I = -\log_2 \displaystyle\int_{m_1-\Delta m_1}^{m_1+\Delta m_1}\int_{m_2-\Delta m_2}^{m_2+\Delta m_2}\cdots\int_{m_n-\Delta m_n}^{m_n+\Delta m_n} \frac{1}{\left(\sqrt{2\pi}\right)^n |\mathbf{K}|^{1/2}} e^{-\frac{1}{2}(\mathbf{y}-\mu)^T \mathbf{K}^{-1}(\mathbf{y}-\mu)} dy_1 dy_2 \ldots dy_n$	
Rolled Throughput Yield	$Y_{RT} = \displaystyle\int_{m_1-\Delta m_1}^{m_1+\Delta m_1}\int_{m_2-\Delta m_2}^{m_2+\Delta m_2}\cdots\int_{m_n-\Delta m_n}^{m_n+\Delta m_n} \frac{1}{\left(\sqrt{2\pi}\right)^n |\mathbf{K}|^{1/2}} e^{-\frac{1}{2}(\mathbf{y}-\mu)^T \mathbf{K}^{-1}(\mathbf{y}-\mu)} dy_1 dy_2 \ldots dy_n$	
Differential Entropy	$h(\mathbf{Y}) = \frac{1}{2}\log_2 (2\pi e)^n |\mathbf{K}|$	

Table 5.1 Summary of Performance Measures for Systems with Multiple Quality Characteristics

Note that all performance measures presented above are relative and can only be used for comparing different designs. In such a case, the base of log becomes irrelevant since the base can be conveniently changed by a constant multiplicative factor. Also note that evaluation for expected quality loss and signal-to-noise ratio requires knowledge of the cost matrix, $\mathbf{C}$. Let us first calculate the performance measures for the case study given in Chapter 2 for the case when $\mathbf{C}$ is an identity matrix. The values for the performance measures are tabulated in Table 5.2 and the optimum design according to each of the performance measures is highlighted in the table.

	SG(A)	SG(B)	LVDT(A)	LVDT(B)
$E(L)$	5.2943E-04	5.2430E-04	5.1988E-04	5.1987E-04
η_1, η_2	1.0201E+02	1.1969E+02	9.9161E+01	1.6662E+02
η_3	3.2370E+01	3.2797E+01	3.2089E+01	3.2841E+01
I	1.6101E+00	5.8347E-03	2.0878E+00	3.9127E-03
Y_{RT}	0.3276	0.9960	0.2352	0.9973
h	-9.8295E+00	-1.2759E+01	-9.3429E+00	-2.0548E+01

Table 5.2 Evaluation of performance measures for $\mathbf{C} = \mathbf{I}$

As mentioned before, the performance measures I and Y_{RT} are dependent on the desired range of the quality characteristics. I ran a few experiments to observe the effect of change in desired range of quality characteristics on the information content I (the effect is going to be the same on Y_{RT} since they are directly related). The results are summarized in Table 5.3.

ΔD	$\Delta \omega_c$	SG(a)	SG(b)	LVDT(a)	LVDT(b)
0.1%	0.1%	3.6911	2.0919	4.1561	2.0843
1%	0.1%	2.0970	2.0902	2.0758	
0.1%	1%	1.6101	5.8347E-03	2.0878	3.9127E-03
1%	1%	4.3053E-03	4.0600E-03	7.4784E-03	

Table 5.3 Information Measure I for different desired specifications on ω_c and D

The low values in the last row of Table 5.3 indicate that with the given tolerances on design parameters, the desired specifications on quality characteristics will almost always be met. The values for two sets of specifications could not be determined because the solution was not converging in the software package being used. We observe that changing the desired specifications of D and ω_c changes the relative information content of different designs. In general, one would expect the design choice to change depending on the tolerances.

As a last exercise in this chapter, let us assume the following cost matrix:

$$\mathbf{C} = \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix} \tag{5.22}$$

The performance measures are recalculated for the above matrix and the results are summarized in Table 5.4.

	SG(A)	SG(B)	LVDT(A)	LVDT(B)
$E(L)$	7.4528E-04	5.3276E-04	8.2028E-04	5.1987E-04
η_1	1.0201E+02	1.1969E+02	9.9161E+01	1.6662E+02
η_2	9.9001E+01	1.1668E+02	9.6151E+01	1.6361E+02
η_3	3.1186E+01	3.2694E+01	3.1381E+01	3.2841E+01
I	1.6101E+00	5.8347E-03	2.0878E+00	3.9127E-03
Y_{RT}	0.3276	0.9960	0.2352	0.9973
h	-9.8295E+00	-1.2759E+01	-9.3429E+00	-2.0548E+01

Table 5.4 Evaluation of performance measures for sample **C**

The values for η_1, I, Y_{RT}, and h remain unchanged since they do not depend on the cost matrix. The values of other performance measures have changed as one might expect. However, the optimum design choice has not changed. I have tried numerous values for the **C** matrix but the optimum design remains the same. This raises an interesting question whether there is any value in assigning preference among quality characteristics for this problem. In general, one would expect that the optimum solution would change depending on the relative preference assigned to various quality characteristics.

6. Conclusion

In this thesis, I have attempted to provide a rationale in defining performance measures for systems with inputs or outputs that can be modeled as random variables. By laying out the various performance measures in one place and in one terminology, it becomes easier to examine their relative merits. I have proposed the use of a standard set of criteria for comparing the performance measures. The desired properties a performance measure must possess are outlined below:

1. **Economic Loss**: It is an indicator of economic loss resulting from deviation of quality characteristics from the desired specifications.

2. **Easily Computable**: It is easily computable from the information regarding variations in input variables or alternatively from observed statistical properties of the quality characteristics.

3. **Minimum Assumptions**: It makes a minimum of assumptions about the problem structure (probability distribution, heteroscadicity, etc.).

4. **Correlation**: It accounts for correlation among quality characteristics.

5. **Relative Preference**: It allows incorporation of preference for certain quality characteristics over others.

These desired properties can provide an important tool in the choice of a performance measures that capture variation in engineering systems whether they are in the design/ development stage or being manufactured.

The following sections summarize the "performance" of the performance measures described in Chapters 4 and 5.

6.1 Systems with a Single Quality Characteristic

Table 6.1 below tabulates the performance measures for systems with a single quality characteristic presented in Chapter 4 and whether they possess the desired properties.

	Economic Loss	Easily Computable	Minimum Assumptions	Correlation	Relative Preference
$E(L)$	✔	✔	?	N/A	N/A
η	✔	✔	✖	N/A	N/A
I	✖	✖	✔	N/A	N/A
C_p	✖	✔	✔	N/A	N/A
C_{pk}	✖	✔	✔	N/A	N/A
Y_{FT}	✖	✖	✔	N/A	N/A
h	✔	✔	✔	N/A	N/A

Table 6.1 Summary of criteria for single quality characteristic systems

The last two columns corresponding to Correlation and Relative Preference do not apply to single quality characteristic systems but are included for completion. The main assumption behind quality loss is that it is in a quadratic form. This is clearly an approximation based on Taylor series expansion of the function. It is debatable whether this limits the use and validity of a quadratic quality loss function. Signal-to-noise ratio is derived from this quality loss function with the additional assumption that standard deviation and mean of a sample are proportional to each other. Suh's information content and the first time yield both do not account for

economic loss and neither are they easily computable since they involve evaluation of integrals. Evaluation of process capability and performance indices did indicate the same design preference as other economic loss measures. However, it must be kept in mind that these indices have been defined based on long years of manufacturing experience and have not been derived from a function of economic loss nor do they have any theoretical basis. In addition, they can vary depending upon the tolerance levels. Differential entropy for systems with a single quality characteristic satisfies all three criteria. It measures complexity of a design and therefore, indicates economic loss. One might even suppose that it is a better measure of economic loss than the quadratic loss function since it is not a local approximation but is based on minimizing overall variation in the system. The assumption behind the form of entropy that has been used in the case study is that of a normal probability distribution for input design parameters and quality characteristics. This is a reasonable assumption for almost all engineering systems with variation. If this were not the case, the basic form of entropy given in Equation 4.29 would have to be used which may involve the use of numerical integration and thus may render the problem difficult to compute.

6.2 Systems with Multiple Quality Characteristics

Each of the performance measures for systems with multiple quality characteristics were presented and discussed in Chapter 5. The following table summarizes the performance measures as they relate to the desired properties.

	Economic Loss	Easily Computable	Minimum Assumptions	Correlation	Relative Preference
$E(L)$	✔	✔	✖	✔	✔
η_1	✔	✔	✖	✖	✖
η_2	✔	✔	✖	✖	✔
η_3	✔	✔	✖	✔	✔
I	✖	✖	✔	✔	✖
Y_{FT}	✖	✖	✔	✔	✖
h	✔	✔	✔	✔	✖

Table 6.2 Summary of criteria for systems with multiple quality characteristics

The quadratic quality loss function for systems with multiple quality characteristics is also a local approximation to an economic loss. As mentioned before, the validity of this assumption is debatable. However, it has considerable advantages for multiple quality characteristics systems since it captures correlation and can also incorporate relative preference among quality characteristics. A simple addition or weighted average of the signal-to-noise ratios of individual quality characteristics is clearly not suitable for system with multiple quality characteristics. A signal-to-noise ratio derived from the quality loss function makes additional assumptions about the structure of the problem. Suh's information content and the rolled throughput yield vary as a function of the desired tolerance on the quality characteristics and are difficult to compute. It is debatable whether a performance measure should be viewed as an inherent property of a design or can vary depending on the desired tolerance on the outputs. In my view, a performance measure should be viewed as an inherent property of a design, i.e., the design must be such that it minimizes deviation from the target. The differential entropy as a performance measures for

systems with multiple quality characteristics is quite attractive. The form given in this thesis assumes a normal probability distribution for quality characteristics, which, as mentioned before, is a reasonable assumption for most engineering systems. The only drawback of differential entropy is that it cannot incorporate relative preference for quality characteristics. It is hoped that with proper data transformations, this measure can be used as a performance measure. In summary, the quadratic loss and differential entropy offer the best promise as performance measures to be used in minimizing variations in engineering systems.

In this thesis, my intention has been that industrial practitioners will gain access to tools and techniques that can be used in multi-response system design in an elegant and practical way. I hope that my arguments will provide a stronger conceptual foundation for making the design insensitive to noise and provide a framework for improving the quality of a process or a product with multiple quality characteristics. At the same time, I hope that researchers will benefit from the argument through an enhanced ability to formulate focused research questions around these issues.

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A. Notes on Axiomatic Design

Axiomatic design theory is based on two axioms with the aim of putting the design process on a more scientific footing. The two axioms are based on experience and proposed as universal principles behind any "good" design. In this appendix, I examine the nature of and the interdependence between the two axioms.

In the next section, an overview of Axiomatic Design is provided. The nature of axiomatic design and how well it fits the models of other engineering sciences is looked at in the following section. Section A.4 looks at the interdependence of the two axioms in a qualitative manner. Then the same interdependence is examined quantitatively by using measures of coupling and information content. It is shown that the two axioms may lead a design in opposite directions necessitating a trade-off between minimizing coupling and minimizing information content. In other words, it may be possible that coupling can reduce the information content of a design and vice versa. A case study is presented which demonstrates that minimizing information content of a design and minimizing coupling among the functional requirements can be conflicting requirements and there is no inherent reason to prefer one axiom over another.

A.1 Review of axiomatic design

Suh [1990] delves into the fundamental principles of engineering design and presents the theory of axiomatic design. Axiomatic design provides a decision support mechanism during the design process. The core of this method is a pair of axioms upon which corollaries and theorems are built which can be applied within a structured, multi-domain mapping process. The two axioms are supposed to provide a designer with objective criteria when deciding between

alternative engineering designs. The design axioms are stated as follows:

Axiom 1   *The Independence Axiom*

   An optimal design always maintains the independence of functional requirements.

Axiom 2   *The Information Axiom*

   The best design is a functionally uncoupled design that has the minimum information content.

These axioms are proposed as universal principles that govern any good design. Axiom 1 deals with the relationship between functions and physical variables. The design process must be such that as we go from the FRs in the functional domain to the DPs in the physical domain, the mapping must be one to one, i.e., a perturbation in a particular DP must affect only its corresponding FR. Axiom 2 deals with the complexity of design and is interpreted to state that, among all the designs that satisfy the first axiom, the one with the minimum information content is the best design.

Based on the clarifications above, Suh [1990] gives the following alternative statements for Axioms 1 and 2:

Axiom 1   *The Independence Axiom*

   Alternate Statement 1: An optimal design always maintains the independence of FRs.

   Alternate Statement 2: In an acceptable design, the DPs and the FRs are related in such a way that specific DP can be adjusted to satisfy its corresponding FR without affecting other functional requirements.

Axiom 2   *The Information Axiom*

Alternate Statement: The best design is a functionally uncoupled design that has

the minimum information content.

A.2 Axiomatic Method

According to the Cambridge Dictionary of Philosophy: "It is a method for reorganizing the

accepted propositions and concepts of an existent science in order to increase certainty in the

propositions and clarity in the concepts. Application of this method was thought to require the

identification of (1) the "universe of discourse" (domain, genus) of entities constituting the

primary subject matter of science, (2) the "primitive concepts" that can be grasped immediately

without the use of definition, (3) the "primitive propositions" (or "axioms"), whose truth is

knowable immediately, without the use of deduction, (4) an immediately acceptable "primitive

definition" in terms of primitive concepts for each non-primitive concept, and (5) a deduction

(constructed by chaining immediate, logically cogent inferences ultimately from primitive

propositions and definitions) for each non-primitive accepted proposition." Each of these

categories could be examined in detail with respect to the axiomatic design theory. However, we

will concentrate on the third category, i.e., whether the truth of the two axioms is knowable

immediately without the use of deduction. In other words, we look at the universality of the two

axioms.

As Suh points out, axioms, by definition, are fundamental truths that are always observed to

be valid and for which there are no counterexamples or exceptions. Furthermore, axioms are

formal statements either of what people already know explicitly, or of the inherent implicit

knowledge imbedded in their actions and thoughts. Axioms are the bare minimum set of

principles upon which a set of self-consistent logic can be built which leads to correct solutions to all problems within a given domain.

The axiomatic approach to design assumes that a fundamental set of principles does exist which determines good design practice. Similar to axioms (laws?) in thermodynamics or in classical mechanics that were based on observations of the physical phenomena, the design axioms are founded upon physical observations. The only way to refute the axiomatic design theory would be to uncover "good" designs that conflict with the two axioms.

In the arguments presented by Suh, the "goodness" of the design is supposed to be apparent to everyone. In reality, the "goodness" is quite subjective and is in the eyes of the beholder. Unlike the thermodynamic case where a machine cannot be built which violates the basic axioms, an infinite number of designs are possible which violate the design axioms. Suh postulates that among all the possible designs, the "best" design is the one that is uncoupled and has the minimum information content.

A.3 Interdependence of the Two Axioms

The Independence Axiom states that an uncoupled design is the best design. It is not immediately clear whether that is the case. An example in point is the water-mixing faucet. The FRs are (a) to regulate the flow of water and (b) to control the temperature of water. The old design was to use two faucets, one for the hot water and one for the cold water. By adjusting the two faucets, a person would get the required flow and the desired temperature. The newer designs follow the axiomatic design principles where the controls correspond to the FRs, i.e., the design has been uncoupled. However, some people would argue that the older design, although coupled, was better. One could also argue that the information content in the older design is less than in

the newer design (the probability of success in achieving the desired flow and the desired temperature has decreased because of added complications and increased number of parts in the design) and, therefore, the older design is to be preferred. In this case, which axiom is given preference is upto the judgement of individuals.

It is not clear to me whether the basic axioms in any axiomatic system need to stand on their own and be of equal weight or not. The examples from mechanics or thermodynamics seem to indicate that the basic axioms (or laws) must be independent of each other and carry equal weight. For example, Newton's three laws are equally valid and equally true, i.e, one law is not more true than the others. However, Suh's axiomatic design theory seems to suggest that the first axiom is more true than the second one. The independence axiom is more important since the information axiom depends on it. The axioms suggest that if a coupled design has less information content than an uncoupled design, the uncoupled design is to be preferred. However, Suh also states as a corollary to the axioms:

Corollary 7 (Uncoupled Design with Less Information)

> Seek an uncoupled design that requires less information than coupled designs in satisfying a set of FRs.

Suh goes on to say that, "Corollary 7 states that there is always an uncoupled design that involves less information than a coupled design (authors' emphasis). This corollary is a consequence of Axioms 1 and 2. If this corollary were not true, then Axioms 1 and 2 must be invalid. The implication of this corollary is that if a designer proposes an uncoupled design which has more information content than a coupled design, then the designer should return to the "drawing board" to develop another uncoupled or decoupled design having less information

content than the coupled design." If we are interpreting this statement right then Suh is claiming

that there is always an uncoupled design with less information than a coupled design. This

proposition is hard to refute since it is so general. In cases where a coupled design seems to be the

lowest information content solution, someone can always claim that a better design exists which

is uncoupled, however, it has not yet been discovered!

In addition to the corollary stated above, Suh says the following about the relationship

between information content and functional independence, "Information content associated with

FRs of an uncoupled design can be obtained by simply adding the information associated with

each of the FRs at each level of the FR hierarchy. However, in the case of a coupled design, any

one DP can affect all other FRs. Therefore, the information content cannot be defined a priori

since the information content depends on the particular path followed in varying the DPs.

Consequently, the information associated with a coupled process is greater than that of a

decoupled process, which in turn is greater than that of an uncoupled process (authors'

emphasis)." This statement indicates that minimizing coupling and minimizing information

content go hand in hand.

However, later in the book, Suh states that, "it is wiser to keep these two axioms as two

independent propositions. For example, in the absence of Axiom 1, one might choose a coupled

design that happened to have less information than a particular uncoupled design, rather than to

seek another uncoupled design having less information content (see Corollary 7). Conversely, in

the absence of Axiom 2, there is no way in which we can choose the best design among

uncoupled designs satisfying Axiom 1." He goes on to say, "In an actual design process, one

always starts out with Axiom 1 and seeks an uncoupled design. Only after several designs that

satisfy Axiom 1 are proposed can we apply Axiom 2 to determine which is the best among

those proposed (authors' empahsis)." It is not immediately obvious as to what is meant by "satisfying Axiom 1." However, the two axioms have been interpreted to have a lexicographic relationship [Bras and Mistree, 1995]. The solution of a multi-objective scenario, as described by Axiomatic Design theory, would involve determining the lexicographic minimum of a deviation function. Ignizio [1985] defined the lexicographic minimum as follows:

Lexicographic Minimum

Given an ordered array $\mathbf{f} = (f_1, f_2, \ldots f_n)$ of nonnegative elements f_k's, the solution given by $\mathbf{f}^{(1)}$ is preferred to $\mathbf{f}^{(2)}$ if $f_k^{(1)} < f_k^{(2)}$ and all higher order elements (i.e. $f_1, \ldots f_{k-1}$) are equal. If no other solution is preferred to $\mathbf{f}$, then $\mathbf{f}$ is the lexicographic minimum.

Completely uncoupled designs are rare even for small systems and large scale uncoupled systems are even more so. As the scale of the systems increases, some degree of coupling always appears. This is also observed through the DSM literature where even with rearrangement, only block diagonal and not strictly diagonal DSM's can be obtained. Since completely uncoupled systems are so rare, we can state that degree of coupling will vary between competing designs. Similarly, the information content of each competing design will be different in general. Therefore, the difference between competing designs is in their degrees of coupling and of relative information content. A lexicographic relationship between the two axioms would imply that the minimum coupling design is always chosen. This would be okay if minimizing coupling and minimizing information content resulted in the same design. However, we will show that the two requirements may conflict with each other. In these cases it seems natural that a trade-off between coupling and information content would be required.

The interdependence of the two axioms can also be inferred from the interdependence of their consequences. The degree of coupling makes the development work more organized and makes future modifications of a system easier. By allowing concurrent design, independence among FRs can also reduce time to market. The information content, on the other hand, increases the robustness of a product and thus its probability of success. It is difficult, if not impossible, to optimize both objectives simultaneously. Therefore, for any practical engineering design project, a balance between the two must be achieved.

To examine the relationship of information content with functional independence, we next study the mathematical representation of the axiomatic theory.

A.4 Mathematical Representation of the Axioms

Design is defined as the mapping process between the FRs in the functional domain and DPs in the physical domain. This mapping process can be represented mathematically. The set of FRs can be represented by a vector **FR** with m components, where m is the number of FRs. Similarly, the DPs in the physical domain may be represented by a vector **DP** with n components, where n is the number of DPs. For the ideal design case $m = n$. The design process involves choosing the right set of DPs to satisfy the given FRs, which may be expressed as the Design Equation:

$$\{FR\} = [A]\{DP\} \tag{A.1a}$$

where $\{FR\}$ is the functional requirement vector, $\{DP\}$ is the design parameter vector, and $[A]$ is the design matrix. To simplify, we let $Y = \{FR\}$ and $X = \{DP\}$. The design equation can

now be written as

$$\mathbf{Y} = \mathbf{AX}$$ (A.1b)

The structure of the design matrix determines the degree of independence of the FRs. Measures like semangularity and reangularity [Rinderle, 1982] can be employed to quantify the degree of coupling between the functional requirements. Jahangir and Frey [1999] propose differential entropy as a measure of information content and show its superiority over Suh's [1990] information measure and Taguchi's [1990] signal-to-noise ratio. In this paper we use semangularity/ reangularity as a measure of functional independence and differential entropy as a measure of information content. The next sections summarize the main points and calculation procedure for the two measures. These measures would then be used to examine the case study presented in Chapter 2.

A.4.1   Semangularity/ Reangularity as a Measure of Coupling

For a square design matrix, the semangularity and reangularity give identical values. For a system with two functional requirements and two design parameters, the measure is given as:

$$S = \frac{A_{11}A_{22}}{\sqrt{\left(A_{11}^2 + A_{12}^2\right)\left(A_{21}^2 + A_{ss}^2\right)}}$$ (A.2)

where A_{ij} is the i, j th element of the design matrix $\mathbf{A}$.

A.4.2 Differential Entropy as a Measure of Information Content

Differential entropy is a quantitative measure of information content which depends on the units or the coordinate system chosen. If we change the units, the differential entropy will in general change. For the same reason, no absolute lower bound exists on differential entropy. Therefore, for a given coordinate system, information content will provide only a relative measure of information content between two or more different designs (or sets of random variables).

Section 5.6 presented the mathematical expressions for the differential entropy. For the case study, differential entropy is

$$h(\mathbf{Y}) = \frac{1}{2}\log_2 (2\pi e)^2 |\mathbf{K}| \qquad (A.3)$$

A.4.3 Case Study

Matrices $\mathbf{A}$ and $\mathbf{K}$ were computed in Chapter 2. They values of these matrices can be substituted in Equations A.2 and A.3 to obtain the following:

	SG(a)	SG(b)	LVDT(a)	LVDT(b)
S	0.9821	0.7071	1.0000	0.7071
h	-9.8295E+00	-1.2759E+01	-9.3429E+00	-2.0548E+01

Table A.1: Summary of coupling and information measures for the four design options

Clearly, if the relationship between the two axioms is lexicographic, LVDT(a) would be picked since its design is uncoupled. However, we note that its information content (as

represented by differential entropy) and hence its probability of success is lower than that of other designs. The design LVDT(b) has the minimum value of entropy and thus possesses the maximum probability of success. One can also relate this to complexity and say that it is the least complex among the four designs.

A.5 Conclusion

It has been demonstrated that minimizing information content and minimizing coupling can be opposing objectives in a design. Axiomatic design clearly prefers uncoupling over minimizing complexity in a design. The choice of the "best" design would be subjective since the requirements of uncoupling and simplicity in design would vary from one application to another and from one designer to another. In most situations, a trade-off between the two would be necessary to arrive at an "optimum" solution.

B. Robust Architecture

Robust design deals with reducing the sensitivity of a product or a process to various uncontrollable noise factors such as variations in operating conditions and in manufactured parts. Robust design has been very successful in improving quality of products by considering robustness earlier in the design cycle as opposed to being done after the product parameters had been fixed. The robust design process as currently practiced takes place in the detailed design phase after a product architecture has been defined and the functional requirements of each of the sub-systems have been fixed. In order to improve robustness of the product, it would be advantageous to start earlier than the detailed design phase. This paper proposes a scheme to determine functional requirements of sub-systems. The scheme draws on similarities between information content and signal-to-noise ratio of robust design. The information content is defined as the quantitative measure of robustness and the scheme relies on minimizing the total information content of a system. By comparing information content of competing functional decomposition schemes, a more robust architecture for products can be chosen.

B.1 Motivation

Phadke [1989] defines *Robust Design* as an engineering methodology for improving productivity during research and development so that high-quality products can be produced quickly and at low cost. More specifically, robust design deals with making a product's performance insensitive to noise factors. The method of robust design can best be illustrated with the help of a P Diagram:
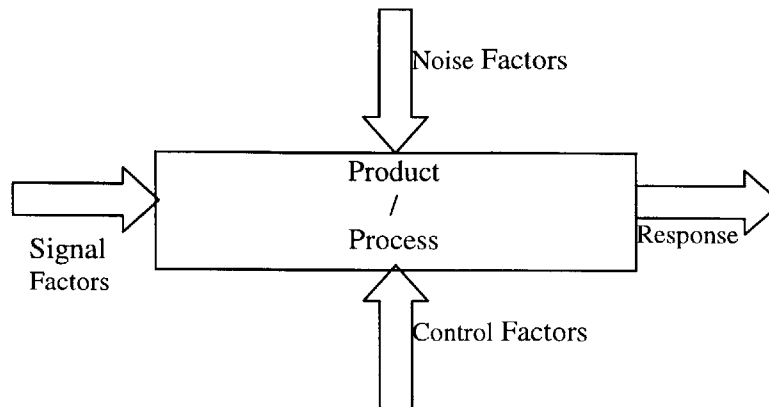
Figure B.1 Block diagram of a product/process: P Diagram

Response variables are the quality characteristics that are observed for the purpose of evaluating loss to the society caused by low quality of the product or the process. Signal factors are the parameters set by the user of operator of the product. Noise factors are any uncontrollable variables that cause a functional characteristic or response to deviate from its target value. Control factors are the parameters that can be specified by the designer with the object of minimizing effect of noise factors on the response variables. In other words, control factors are chosen to minimize loss to the society that can occur if the product deviates from its ideal characteristics.

The general process of engineering design has been divided into the following five phases by Taguchi [1993]:

> ➢ System Selection

> ➢ Parameter Design

> ➢ Tolerance Design

> ➢ Tolerance Specifications

> ➢ Quality Management for the Production Process

Robust design deals with the second step of the engineering design process when the system architecture has already been defined and only numerical values of the control factors remain to be assigned. Dan Frey [1998] has mapped Ulrich and Eppinger's [1995] steps in the design process to the robust design process as in the following diagram:
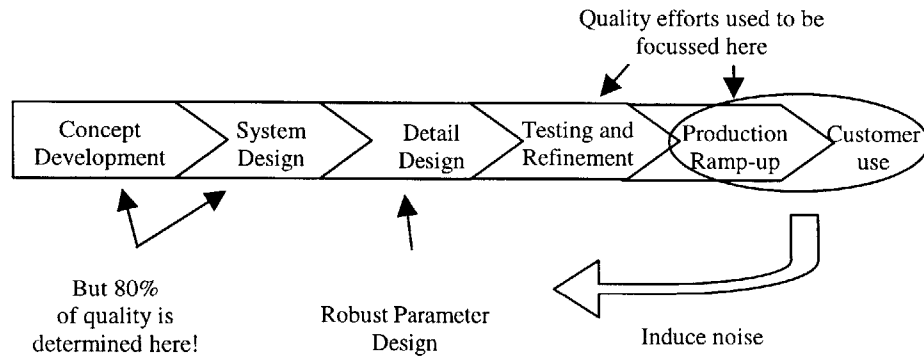
Quality efforts used to be
focussed here

| Concept Development | System Design | Detail Design | Testing and Refinement | Production Ramp-up | Customer use |

But 80%
of quality is
determined here!

Robust Parameter
Design

Induce noise

Figure B.2 Design Process

It is clear that robust design results in increased productivity by moving the quality considerations earlier in the design phase. Moving quality considerations (making product insensitive to noise) even earlier in the design phase should improve the productivity to an even greater extent. Rechtin [1991] points out that "in architecting a new [aerospace] system, by the time of the first design review, performance, cost, and schedule have been pre-determined. One might not know what they are yet, but to first order all the critical assumptions and choices have been made which will determine those key parameters." Therefore, the greatest potential for improving productivity and minimizing societal loss lies in the concept development and system design phases. Ford and Barkan [1994] illustrate the point with the following diagram where it is seen that greatest opportunities for product improvement are at the start of the product life cycle.
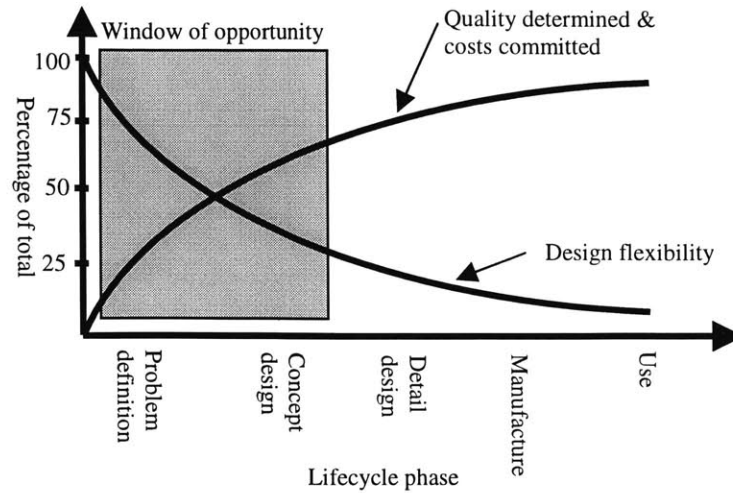
85

Figure B.3 Opportunities to affect cost in product development

Ulrich and Eppinger [1995] place the design of a product architecture into the system design phase. We will follow this guideline and limit the scope of the paper to System Design when a concept has been selected and functional requirements for each of the lowest-level decomposed physical chunks are being determined. Nevertheless, we will suggest a method of comparing competing product architectures on the basis of a quantitative criterion.

According to Ulrich and Eppinger [1995], *Product Architecture* is the scheme by which the functional elements of a product are arranged into physical chunks and by which the chunks interact. Combining the concept of robustness with the definition of product architecture, robust architecture can be defined as:

Robust Architecture is the scheme by which the functional elements of a product are arranged into physical chunks and by which the chunks interact such that sensitivity to noise factors and uncertainty in model parameters is minimized.

It may be noted that conventional robust design does not deal with uncertainty in a system model since a physical product is assumed to be in existence and its behavior under controlled experiments is observed to arrive at optimal parameter levels. However, in the early stages of design, only models of the system are present and modeling uncertainties must be taken into account.

B.2 A Proposed Framework

In a strictly top-down architecture, form follows function. In a top-down system architecture, the overall function of a system is specified and then this main function is separated into several sub-functions. This process is carried out until the function at the lowest level can be given a manageable form as a physical chunk. By aggregating the physical chunks from the lowest level to the top through the use of their interconnections, the overall system takes physical shape.
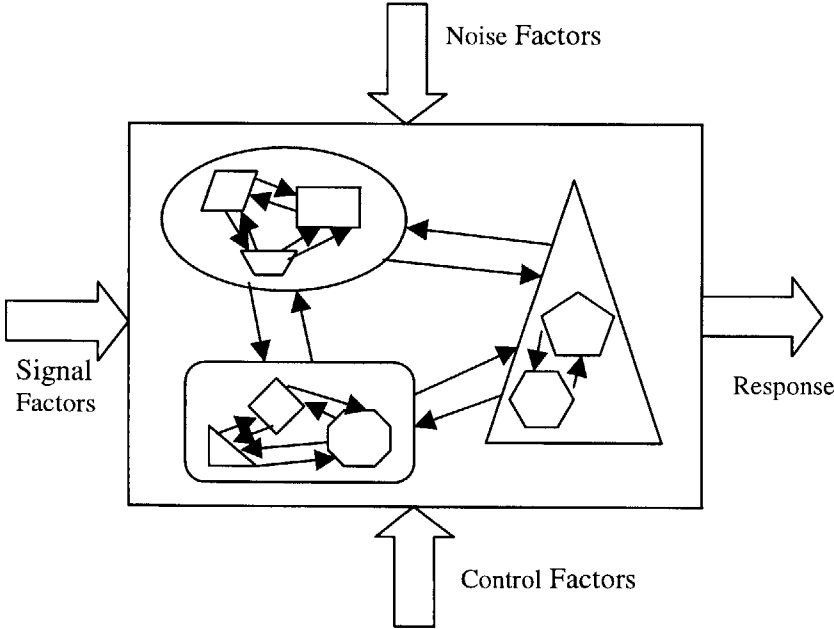

Figure B.4 System Architecture and Robust Design

Robust architecture would involve making the overall system less sensitive to noise by a judicious choice of physical chunks and their interfaces.

The process of robust architecture would start with generation of multiple schemes of functional decomposition and conceptual form of each of the decomposed functions. For each functional decomposition scheme, the problem of robust architecture would be defined as deciding on the nominal values of the functional requirements for each of the functionally decomposed physical chunks such that the overall system form is insensitive to or tolerant of any noise in environmental conditions. The sensitivity of the system with respect to noise must be quantifiable. This performance metric, which when maximized, would result in minimizing loss to the society. This process can be repeated for every alternate decomposition scheme. The choice between multiple functional decomposition schemes can be made by looking at the relative magnitudes of the overall performance metric.

Appendix A gave an overview of Axiomatic Design. Axiomatic design attempts to make architecture robust by making a design uncoupled. In other words it stresses the minimization of interfaces between physical chunks (Design Structure Matrices can be used to make the design uncoupled). Once the design is made as uncoupled as possible, it would seem that the information content of each of the sub-systems can be minimized independently which would also minimize information content of the overall system. However, in practice the sub-systems would be coupled to varying extent and reducing information content in one sub-system can increase information content in another sub-system and vice-versa.

Jahangir and Frey [1998] show a similarity between the concepts of minimizing information content of a design and maximizing signal-to-noise ratio in the practice of robust parameter design. For the purposes of this paper, we assume that the two approaches are practically the

same and minimizing the information content would result in maximizing the signal-to-noise ratio and thus making the design robust.

The last diagram showed a product decomposed down two levels. Each of the smallest blocks represents the physical chunk associated with the lowest layer of the functional tree. In order for any analysis to take place, a model of each of the physical chunks must be available. We assume this model of the form:

$$Y = f(U, W) \tag{B.1}$$

where Y is the output (or response or functional requirement), U is the input vector to the physical chunk, and W is the noise vector affecting the output. An input vector in a physical chunk would be composed of the elements of output vectors of other physical chunks. In general, the order of each physical chunk model would be different. In the presence of model uncertainty and noise, we can treat the output vector as composed of random variables.

From information theory it is known that information content of a system is represented by entropy. In Chapter 5, the following form of differential entropy was given which assumes a normal probability distribution for the functional requirements:

$$h(Y) = \frac{1}{2} \log(2\pi e)^n |K| \tag{B.2}$$

where K is the covariance matrix of the FR vector. Each sub-system would have an entropy or information content associated with it. We represent this as $h(Y^i)$ where Y^i is the functional requirement vector of the i th sub-system. Note also that for each sub-system, the dimension of the functional requirement vector may be different.

If $\mathbf{Y}^i$ s are independent (all the sub-systems have been uncoupled) then the information content of the overall system is the sum of the entropies of the sub-systems. In the case of some coupling, the information content of the overall system can still be represented to first order as a sum of the entropies of the sub-systems or physical chunks (actually the sum represents the upper bound on the information content in this case). A system dynamic type of model can be built which relates the dependencies of one sub-system to another. The object in robust architecture would be to choose the levels of the functional requirements (outputs of physical chunks) which minimize the total information content.

$$\min H \approx \sum_i h(\mathbf{Y}^i) \qquad (B.3)$$

This can be carried out using mathematical programming. This optimization would be done under the cost and schedule constraints of the project.

The above scheme is proposed assuming that only one scheme of functional decomposition exists and only the functional requirements must be determined for each sub-system. If there are multiple schemes of functional decomposition, a similar analysis can be carried out on each scheme. The functional decomposition scheme resulting in the minimum total information content would be expected to be the most robust.

B.3 Conclusion

An attempt has been made to bring the robust design process as early in the design cycle as possible. In this regard, we have proposed a scheme that can be used during the System Design Phase as defined by Ulrich and Eppinger [1995]. This scheme consists of the following steps:

- ➢ Generate multiple functional decomposition schemes

- ➢ For each functional decomposition:

  - ➢ Model each sub-system in the physical decomposition

  - ➢ Determine the information content of each sub-system as a function of the inputs

  - ➢ Minimize the sum of the information content from all sub-systems

- ➢ Choose the functional decomposition scheme with the minimum overall information content