VALUING PROJECT RISK AND FLEXIBILITY IN MINING RESOURCE DEVELOPMENT

by

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Abstract

The Real Options valuation approach is used as an alternative to Discounted Cash Flow (DCF) and Decision Tree Analysis (DTA) to value the decision to invest in the development of gold mining resources and to capture the value of risk and managerial flexibility in the decision to invest in mining resource development.

The relative merits of DCF, DTA and Real Options valuation approaches for managerial investment decisions are discussed. Two valuation methods (DCF and Real Options) are applied to data from a completed gold mining development project by Ashanti Goldfields Company in Bibiani-Ghana to evaluate the managerial decision to invest.

A detailed comparison and evaluation of the two methods is presented to support the claim that DCF understates the value of mining investments and the use of Real Options valuation is capable of partially remedying the situation.

Thesis Supervisor: Professor John B. Miller

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CHAPTER 1: INTRODUCTION AND PROBLEM STATEMENT

Corporate planners and finance executives concerned with capital budgeting or strategic planning, particularly face a lot of uncertainty in investment planning. Most corporate investment decisions today are based on Discounted Cash Flow (DCF) techniques. While DCF remains a useful approach for evaluating investment decisions, it has a number of drawbacks that leaves corporate planners unsatisfied. Amram and Kulatilaka (1999) mention that senior management misses important value-increasing opportunities because tactics, strategy and valuation are considered separately by current tools like DCF.

Historically, DCF was a tool developed for the valuation of safe financial investments like stocks and bonds where the investor remained passive and had no flexibility to defer or abandon the investment. But there is value in flexibility especially for irreversible investments and today's managers respond proactively to manage investments by changing subsequent plans in response to market conditions. Application of DCF to value irreversible investments will undervalue such investments because it will fail to capture the value associated with managerial flexibility and strategy.

DCF also relies on present day estimates of future uncertain variables and it requires a one-time decision to accept or reject a project based on its Net Present Value (NPV). It makes implicit assumptions concerning an “expected scenario” of cash flows and presume management’s passive commitment to a certain static “operating strategy” (e.g. to initiate a capital project immediately, and to operate it continuously at base scale until the end of its pre-specified expected useful life). Since forecast of uncertain outcomes tend to be incorrect at times, there is always the likelihood of accepting projects when they should indeed be rejected and rejecting projects when they should be accepted.

DCF techniques make inappropriate adjustment for risk. Standard NPV valuation uses a discount rate to discount future cash flows which is determined using an equilibrium model like the Capital Asset Pricing Model (CAPM). The CAPM assumes that only systematic risk[1] should be priced and non-systematic risk[2] can be diversified. Meanwhile we know that uncertainty for mining projects is often project specific. The CAPM states that the appropriate rate of return to holding any risky asset can be obtained as a linear function of the riskless rate of return, r^f, and the return on the market portfolio, r^m. With management flexibility to alter investment decisions, we don't expect the rate of return on investment to be linear. Indeed projects are constantly redefined as time passes. Management's flexibility to adapt its future actions in response to altered future market conditions and competitive reactions expands a capital-investment opportunity's value by improving its upside potential while limiting downside losses relative to the initial expectations of a passive management.

Combined, the above pitfalls clearly show a gap between the actions of managers and the traditional DCF tools used for most corporate investment valuation. A valuation method that will provide a framework to link managerial strategic planning with corporate finance is needed to address the pitfalls.

[1] Systematic risks, also known as market risk, are economywide risks that affect all businesses.

Black, Scholes and Merton (1973) realized that for irreversible investments, since conditions can change as new information arrives, the option to wait has an economic value, which should be priced. Their work on pricing financial options[3] is the foundation of Real Option[4] valuation.

Real Options has been known in academia for over fifteen years and has been applied to valuation of drug development projects in the pharmaceutical industry [2], infrastructure investment [2], land development [3], natural resource projects [3] and improving the valuation of research and development in organizations [14].

The evolution of the literature on Real Options started when Dean (1951), Hayes and Abernathy (1980), and Hayes and Garvin (1982) recognized that standard DCF criteria often undervalued investment opportunities, leading to myopic decisions, underinvestment, and eventual loss of competitive position, because they had ignored or did not properly value important strategic considerations. Decision scientist maintained that the problem lied in the application of the wrong valuation techniques, proposing instead, the use of simulation and decision tree analysis (Hertz 1964; Magee 1964) to capture the value of future operating flexibility associated with many projects. Myers (1987), while confirming that part of the problem arises from various misapplications of the underlying theory, acknowledges that DCF methods have inherent limitations when it comes to valuing investments with significant operating or strategic options (e.g., in

[2] Non-systematic risk, also known as unique or idiosyncratic risk, is risks that are particular to an individual company or to an individual project.
[3] Financial Options refer to discretionary investments in financial assets.

capturing the sequential interdependencies among investments over time), suggesting that option pricing holds the best promise of valuing such investments. Trigeorges and Mason (1987) clarify that option valuation can be seen operationally as a special, economically corrected version of decision-tree analysis that is better suited to valuing a variety of corporate operating and strategic options.

Real Options valuation provides a framework that combines corporate strategic vision with information from financial markets to value corporate investment decisions. There is a close relationship between investment opportunities and financial options. The owner of a discretionary investment opportunity has the right but not the obligation to acquire the gross present value of expected cash flow by making an investment outlay on or before the anticipated date when the investment opportunity will cease to exist. Similarly, a Call option on an asset gives the right, with no obligation, to acquire the underlying asset by paying a pre-specified price on or before a given maturity. Similarly a Put option gives the right to sell the underlying asset and receive the exercise price. The asymmetry deriving from having the right but not the obligation to exercise the option lies value.

Managerial operating flexibility can also be likened to financial options. Options are contingent decisions. It provides opportunities to make a decision as events unfold. On the decision date if events turn out well, one decision is made and if they turn out poorly, another decision is made. Therefore options approach to capital budgeting has the

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[4] Real Options are similar to financial assets and they refer to discretionary investments in real (tangible) assets.

potential to conceptualize and quantify the value of options from active management and strategic interactions. This value is typically manifest as a collection of "Real Options" embedded in capital investment opportunities, having as underlying asset the gross project value of discounted expected operating cash inflows. Many of these Real Options, (e.g. to defer, contract, mothball, shutdown, or abandon a capital investment) occur naturally; others may be planned and built in at some extra cost at the outset.

Real Options are discretionary rights, with no obligation to acquire or exchange an asset for a specified alternative price. The insights and techniques derived from option pricing are capable of quantifying the elusive elements of managerial operating flexibility and strategic interactions, which are often ignored or underestimated by the conventional NPV and other quantitative approaches.

Real Options are not needed for valuation of incredibly valuable investments whose key strategic variables are known or can be forecasted safely or those investments whose risks and uncertainty are so small that they can be ignored. For such projects, DCF valuation is still a good tool for investment decisions and it will be shown that for such projects the outcomes of both valuation methods are the same. DCF works well when there are no options at all, or there are options but very little uncertainty [2]. Real Options theory is also not needed as a valuation tool for those investments that are worthless and whose outcomes are unlikely to be changed by Real Options. In such cases, the outcomes of DCF are consistent with those of Real Options. Real Options is valuable for those marginal investments that lie in the gray area requiring hard-headed thinking. For these marginal investments, Real Options valuation can increase the NPV provided by DCF valuation by including the value associated with managerial operating strategy and flexibility. Real Options are needed when there are contingent investment decisions, when uncertainty is large enough that it is sensible to wait for more information, avoiding regret for irreversible investments, when uncertainty is large enough to make flexibility a consideration, and when there will be project updates and mid-course corrections.

This thesis will only look at valuing the risk and managerial flexibility in the decision to invest in gold mining development and extraction and evaluate the results obtained with static DCF NPV approach and to show that traditional DCF misses the value associated with managerial flexibility. A method used by Nathalie Moyen, Margaret Slade and Raman Uppal [13] will be applied to data from a recently developed mining tract of Ashanti Goldfields Corporation (a gold mining firm) in Ghana. While important, this work will not look at managerial flexibility during operations. Additionally, while the key uncertain variables include the cost of extraction and the quantity of reserves, this work has been limited to the price of gold to simplify the problem. Future work would consider multiple sources of uncertainty. Real Option valuations that consider multiple sources of uncertainty are admittedly complex and computationally difficult as the number of sources of uncertainty increases beyond three of four, and it is advisable in practice to spend resources on only the key uncertain variables that are important for the investment decision [2].

This report has been organized into five chapters. Chapter 1 provides a brief introduction and the problem statement. Chapter 2 discusses the different valuation methods, their strengths and their weaknesses. Chapter 3 looks at Real Option valuation methodology. Chapter 4 presents the Real Option valuation of the Bibiani gold project and compares results obtained with a DCF valuation method. Chapter 5 is a conclusion of the main findings.

CHAPTER 2: VALUATION METHODS

Introduction

This chapter discusses the merits and demerits of three valuation methods as a tool for real investment decisions. The three methods are:

- Discounted cash flow (also called the traditional valuation method);
- Decision tree analysis;
- Real Options.

Discounted Cash Flow

DCF analysis based upon the time value of money, and accordingly weights near term cash flows more highly than future cash flows. The discount rate determines the weights to be applied to the cash flows. The basic NPV relationship is

$$NPV = -C_o + \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$$

Where C_o is the initial investment capital and C_t is the expected cash flow of the project in year t, T is the planning horizon and r is the discount rate. A project is accepted if the NPV is positive and rejected if the NPV is negative. Although there are other economic measures of project viability like the Internal Rate of Return (IRR), Discounted Payback (DPB), Overall Rate of Return (ORR), Simple Payback (SPB), the NPV criterion has been the most widely used [5] [10].

The DCF approach is a useful valuation tool for investment when future changes are certain and predictable and although very simple to use, it has a number of drawbacks when applied to investments in a world of uncertainty. These drawbacks are the following:

- Cash flows must be forecasted over the entire life of the project. One relies on estimates of key uncertain variables e.g. input and output prices, exchange rates, material and processing costs, taxation policy, government regulation and political climate.

- DCF assumes passive managers because it assumes future investment decisions are fixed from the outset but real managers introduce a lot of flexibility in choosing the scale and timing of projects, and in updating and revising investment plans as the world changes and make important decisions through strategic considerations.

- The risk-adjusted discount rate is also estimated. Theoretically this is accomplished using CAPM which prices systematic risk. CAPM works on the assumption that only systematic risk needs to be priced and idiosyncratic risk can be diversified by holding a portfolio of risky assets whose returns are imperfectly correlated.

- DCF improperly value investments, which contain future embedded contingent investment decisions.

- Flexibility in investment is difficult to capture with DCF because the value to switching depends on the current status.

While we know that under certainty, there is no risk and we need to adjust cash flows for timing only using the risk-free interest rate, under uncertainty a future variable is characterized not by a single value but by a probability distribution of its possible outcomes. The amount of dispersion or its variability is a measure of how risky that uncertain variable is. Risk in an investment project arise from errors in forecasts and estimates of key primary variables of future cash flows, such as expected inflation, project lifetime, effective tax rate, growth rates, profit margins etc., that are used to calculate NPV.

There are a number of ways to deal with uncertainty in DCF applications. One approach is to use a risk-adjusted discount rate to account for risk and time value of money. A risk adjusted opportunity cost of capital can be interpreted as the percentage reduction in present value as a result of risk, being the proportion of present value of expected cash flow discounted for both time value of money and risk. The risk-adjusted discount rate is assumed to be constant in each period or that it increases at a constant rate over time. The CAPM is used to estimate the risk adjusted discount rate.

DCF also uses other approaches to deal with uncertainty. These are sensitivity analysis, scenario analysis and Monte-Carlo (traditional) simulation. Sensitivity analysis starts with a base case scenario where management determines the base-case (usually the most likely) estimates of the key primary variables from which the base NPV is calculated. Then while keeping all the other variables equal to their base case, each variable is changed by a certain percentage below and above its base case value.

Whether a variable is crucial or not would indicate whether it is worth investing additional time and money to gather additional information that could reduce the uncertainty surrounding the variable, or whether we should bother with estimating the probability distribution describing that uncertainty in the first place. Sensitivity analysis also indicates how bad a misrepresentation or how large a forecast error of a variable can be before the investment becomes acceptable.

Scenario analysis sets key uncertain variables to their optimistic and pessimistic values and the resulting perturbed NPV values can give a picture of the possible variation in or sensitivity of NPV when a given risky variable is misestimated.

Sensitivity and scenario analysis have their limitations. Sensitivity analysis and scenario analysis considers the effect on NPV of only one error in a variable at a time, or the best and worst cases of outcomes, thus ignoring the many combinations of errors in all the variables simultaneously.

A method that considers the impact of all possible combinations of variables is achieved through Monte Carlo simulation but it is a method that still relies on today's estimates of uncertain future variables.

Decision Analysis

Decision Tree Analysis (DTA) is another approach that attempts to account for uncertainty and the possibility of later decisions by management. DTA helps management structure the decision problem by mapping out all feasible alternative managerial actions contingent on the possible states of nature in a hierarchical manner. As such it is particularly useful for analyzing complex sequential investment decisions when uncertainty is resolved at distinct points in time. Whereas conventional NPV analysis might be misused by managers inclined to focus only on the initial decision to accept or reject the project at the expense of subsequent decisions that are dependent on the initial decision, DTA forces management to bring to the surface its implied operating strategy and to recognize explicitly the interdependencies between the initial decision and subsequent decision[3]. The major drawback of DTA is that:

- It uses a risk-adjusted discount rate to adjust payoffs period by period;
- The determination of probabilities for each outcome poses a practical problem. Probabilities are often estimated based on prior experience and upon subjective judgement [3].

Therefore whiles DTA is able to account for various managerial choices and flexibility, it inappropriately adjusts for risk [3].

Real Options Valuation

Real Options are the extension of financial option theory to options on real (non-financial) assets. Real Options deal with tangible investments and it brings the discipline in financial markets to internal strategic investment decisions.

An option is defined as the right, without an associated symmetric obligation, to buy (if a call) or to sell (if a put) a specified asset (e.g. common stock) by paying a pre-specified price (the exercise or strike price) on or before a specified date (the expiration or maturity date). If the option can be exercised before maturity, it is called an American option; if only at maturity, a European option [2] [8] [13].

Options are contingent decisions. It is the opportunity to make decisions as events unfold. In mining, the decision to explore gives you the right, not the obligation, to develop and the decision to develop gives you the right, not the obligation, to extract. Options are aligned with financial market valuations. Options uses financial market inputs to value complex payoffs across all types of real assets. The current value of derivative securities such as financial options are determined using some variant of Contingent Claims Analysis (CCA) developed by Black and Scholes (1973) and Merton. The idea behind contingent claims analysis is to value an option not on its own, but as part of a riskless portfolio. This method is an entirely different approach which gets around the interest rate dilemma that plagues the DCF method and the decision tree analysis method [2].

A tracking portfolio of securities, which has the same payoffs as the option, is constructed and from the law of one price, the two securities which have the same future payoffs must have the same current value. The CCA uses the "no arbitrage" condition to dynamically ensure that the value of the option equals the value of the tracking portfolio as the stock price evolves - a phenomenon known as dynamic tracking [2]. For example, suppose that the underlying security is a stock, e.g. gold. It is possible to take a long position[5] in the derivative security (the stock option) and a short position in the underlying asset (the stock/ the tracking portfolio). Both positions are affected by the same source of uncertainty i.e. the stock price. Because they are offsetting positions, the change in value of the option will equal the change in value of the tracking portfolio i.e. the capital gains of one investment is completely offset by the losses of the other. The value of the hedged positions is independent of the fluctuations in the underlying asset. Therefore the rate of return of the combined position is riskless and should equal the risk free rate[6].

The Black Scholes equation below is a solution to a differential equation that relates the expected future value of the derivative security (the stock option) to the price of the underlying security (the tracking portfolio e.g. the price of gold) and the riskless rate of return. This differential equation can be solved for the option's current value, which is its market price.

$$V = N(d_1)A - N(d_2)Xe^{-rT}$$

[5] To take a long position in an asset is to buy or own an asset.

[6] The risk-free return is the interest rate earned by entities (like governments) that are entirely credit worthy during the period of a loan. In practice, the rate of return used is the government treasury bill rate.

where V is the current value of the call option, A is the current value of the underlying asset, X is the cost of investment, r is the risk-free rate of return, T is time to expiration, σ is the volatility of the underlying asset, $N(d1)$ and $N(d2)$ are the values of the normal distribution at d_1 and d_2. d_1 and d_2 are given by the relation

$$d_1 = [\ln\left(A/X\right) + \left(r + 0.5\sigma^2\right)T] / \sigma\sqrt{T}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Relationship Between Option Valuation and Investment Opportunities: Options thinking can be used to design and manage strategic investments proactively. The beneficial asymmetry deriving from the right to exercise an option only if it is in the option holders interest to do so, and with no obligation to do so if it is not, lies at the heart of an option's value. The value of an option depends directly on the payoffs to the contingent decision, the length of time to the decision date, and volatility. Higher volatility leads to a higher chance of a bad outcome, but in the case of options, losses are limited. Higher volatility leads to a higher outcome, creating value. Volatility or equivalently, total risk is a critically important determinant of option value.

The value of managerial adaptability, and the resulting asymmetry, can better be captured by the expanded (strategic) NPV rule [3], i.e.:

Expanded (Strategic) NPV = Static (passive) NPV + Option Premium

Where "Static (passive) NPV" is the NPV without the value of management's flexibility to adapt and revise later decisions obtained using DCF valuation. This equation holds true because Trigeorges (1996) shows that when there is little or no value to managerial operating strategy and flexibility risk, the valuation of an investment using DCF yields the same value as the Real Option approach. Therefore the option premium in the above equation accounts for the value of managerial flexibility which the DCF method fails to capture. The option premium is the non-negative value of management's flexibility to adapt and revise later decisions as and when uncertainty is resolved and as future events turn out differently from what management expected at the outset. The option premium reflects the basic inadequacy of the NPV and DCF approaches to capital budgeting, because they ignore or cannot properly capture it.

<u>Differences Between Real and Financial Options:</u> Although there are similarities between financial and Real Options, there are also notable differences. One area of difference is the source of uncertainty. The financial option pricing formula was derived based on the ability to use traded underlying security with riskless borrowing in a dynamic portfolio that replicates the payoff of the option in any state of the world and thereby allowing risk neutral valuation. For example, In the case of financial options on stocks, the tracking portfolio is often the underlying asset, which is the stock itself, resulting in perfect tracking. In Real Options valuation, the underlying asset is not often traded and there may be multiple sources of uncertainty and a mix of systematic and non-systematic risks. In Real Option valuation, the underlying asset is not traded. Nonetheless, any contingent claim on an asset, whether traded or non-traded, can be priced in a world with systematic

risk by replacing expected cash flow with a certainty-equivalent rate (by subtracting a risk premium that would be appropriate in a market equilibrium) and then behaving as if the world were risk neutral. Another difference is the leakage in value of Real Options in the form of cash flow or convenience yields to the holder from the underlying asset. Leakage can arise from positive cash flows (e.g. dividends, interest, royalties, etc.) and negative cash flows (storage costs, taxes, etc.). The leakage in value change how the value of the underlying real asset evolves and hence affect the value of the option and the timing of the optimal investment decision. The Real Option solution methodology is shown in the next chapter.

Similarity between mining and financial options

Investing in mining projects draws a lot of similarity with exercising a financial option. Both are at least partially irreversible, i.e. once an option has been exercised, it is dead; once a project is initiated, sunk costs cannot be recouped. For both investments, timing is critical i.e. it is rarely optimal to exercise an American call option as soon as share prices rise above the strike price. Similarly it is not optimal to invest at the outset if the $NPV>I$[7] since delaying can result in new market information about prices and costs.

[7] I denotes the initial investment capital

CHAPTER 3: REAL OPTION VALUATION METHODOLOGY

Introduction

There is a three-step approach to structuring and solving an option [2]. The steps are:

- Identification and definition of the Real Option;
- Establishment of the mathematical representation;
- Selection of the solution method.

Identification and definition of the Real Option

This step involves framing and writing a description of the decision, what the contingent decision is and what observable variable triggers the decision. The source(s) of uncertainty must be identified. For example the decision to invest in developing a mine and extracting gold is to invest if the value of the mine exceeds the investment cost. The source of uncertainty in this case may be the price of gold, the cost of extraction and the quantity of reserves.

Establishment of a mathematical representation

A simple mathematical expression is written for the decision rule. For example the right but not the obligation to invest if the project value exceeds the necessary investment. So the option to defer the investment is identical to a call option on the gross project value V, with an exercise price equal to the required outlay I. This translates into the right to choose the maximum of the project value minus the required investment or zero i.e.

$$Max\,[\,V - I\,,\,0\,]$$

management will either invest if $V>I$ or do nothing.

Selection of a solution method

There are three standard solution methods used to value options. With each solution method, there are alternative mathematical techniques to solve the mathematical models. The methods are:

- The partial differential equation approach which equates the change in option value with the change in the value of the tracking portfolio;
- The dynamic programming approach which lays out possible future outcomes and folds back the value of the optimal strategy;
- The simulation approach that averages the value of the decision date for thousands of possible outcomes.

The Partial Differential Equation Approach: The partial differential equation is a mathematical equation that relates the continuously changing value of the option to observable changes in market securities. Boundary conditions specify the particular option to be valued. There are two ways to solve the partial differential equation. There is the analytical solution and the numerical solution. In the analytical solution the option value is written in one equation as a direct function of the inputs. The analytical solution is the easiest and fastest way to obtain the option value when applicable. The Black-Scholes equation is an analytical solution to a partial differential equation with a set of boundary conditions that define a European Call option. The analytical solution is not applicable to every Real Option problem. Sometimes a modified partial differential equation is solved to obtain an analytical approximation to the option value.

An example of the analytical solution method is given by McDonald and Siegel (1986) in their study of the optimal timing of investing in an irreversible project where the gross project value follows a stochastic process of the form

$$\frac{dV}{V} = \alpha dt + \sigma dz$$

Where V is the gross present value of (appropriately discounted) cash flows, α is the instantaneous expected return on the project, σ is the instantaneous standard deviation of project value, and dz is an increment of a standard Wiener diffusion process[8]. In a risk neutral world, α is replaced by $(\mu - \delta)$. Where μ is the expected return on the gross project value and δ is the payout rate on the project as a fraction of the project value.

Numerical solutions are used when an analytical solution is not possible. It is based on converting the partial differential equation into a set of equations that must hold for short time intervals. The numerical solution starts with a rollout of the possible values of the underlying asset for the life of the option. The value of the option is then obtained at any point in the grid of rolled over values of the underlying asset by solving the set of equations. The disadvantage of the numerical technique is that it gets very complex as the number of sources of uncertainty increases.

<u>The Dynamic Programming Method:</u> The dynamic programming solution method rolls out possible values of the underlying asset during the life of the option and then folds

back the value of the optimal decisions in the future. The risk neutral approach[9] to valuation is used. With dynamic programming, given the choice of initial strategy, the optimal strategy in the next period is the one chosen if the entire analysis were to begin in the next period. The solution gives the optimal strategy in a backward recursive fashion, discounting the future values and cash flows into the current decision. Dynamic programming allows intermediate results and decisions can be determined within the life of the option. Dynamic programming can handle complex decision structures, complex relationships between the value of the option and the underlying asset.

Simulation Method: The simulation method rolls out possible paths of values of the underlying asset from the present to the final decision date in the option. A Monte Carlo simulation approach is used to determine the optimal investment strategy at the end of each possible path.

Standard Option Valuation Assumptions:

Option valuation relies on a set of standard assumptions. It assumes that:

- Financial markets for stocks bonds and options are frictionless i.e. there are no transaction costs and no restrictions on short sales;
- Shares of all securities are infinitely divisible;
- Borrowing and lending are not restricted;

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[8] A Wiener diffusion process is identical to the geometric Brownian motion.

[9] The risk neutral approach to valuation introduced by Cox, Ross, and Rubinstein in 1976 is based on the argument that underlies the option valuation model. Because the hedged position i.e. the combination of the option and the tracking portfolio earns a risk free rate of return, it would have the same value under any preferences of risk. Hence for valuation purposes, everyone is assumed to be risk neutral and that eliminates the need to estimate any sort of risk premium.

- The risk free interest rate is constant over the life of the option.

Stock prices follow a stochastic diffusion process. When the uncertain variable is allowed to evolve continuously, it is modeled as a continuous-time stochastic process. Most diffusion processes are modeled as log normal diffusion processes[10] (also called Geometric Brownian Motion) or as a mean reverting diffusion process[11].

[10] The characteristic of lognormal diffusion processes is that stock prices cannot fall below zero and that the distribution of outcomes at the final decision date has a long tail to the right. The log normal distribution for stock prices is consistent with the use of bell shape normal distribution for stock returns. The range of possible paths for the log normal diffusion process widens with the length of the time horizon. Mathematically, the variance of the stock returns grows with the time horizon and the volatility of the stock returns grows with the square root of the length of the time horizon.

[11] The mean reverting diffusion process describes the evolution of commodity prices or other assets that experience short-run shocks that move prices away from the long-run level and market forces of demand and supply push them back.

CHAPTER 4. OPTION VALUATION AND MINING DEVELOPMENT - THE BIBIANI GOLD PROJECT

Introduction

Ashanti Goldfields Company Limited is an African-based international gold mining and exploration group with operations in twelve countries in Africa. Ashanti operates four mines (as at 1997) with two others in their final stages of commissioning and another in feasibility study. The largest of its operations, the hundred-year-old Obuasi mine in Ghana is one of the top five producing gold mines in the world. Ashanti has exploration programs in most of the significant gold belts of sub-Saharan Africa. The company is listed on six international stock exchanges.

The Bibiani Mine

The Bibiani mine is located in the gold belt in Ghana's Ashanti region and close to the Obuasi mine. In late 1996 the management of Ashanti took the decision to invest in the development and subsequent extraction of gold from the Bibiani mine. The DCF valuation method was used as a basis for the investment decision and in this chapter we shall use Real Option valuation techniques to evaluate the decision to invest and comment on any significant differences in both valuation methods. The relevant dates for the development and operation of the mine used for the DCF analysis is given in Table 4.1. The dates in Table 4.1 were also used for the Real Option valuation of the Bibiani mine.

Table 4.1 Program for Mine Development and Operation

ACTIVITY	DATE
Start Financing	1997.00
Expected Processing Plant Completion	1998.00
Start Mine Operation	1998.00
End of Mine Operation	2005.00

The DCF Valuation Approach for the Bibiani Mine

Ashanti used the DCF NPV model for its project valuation of the Bibiani mine and subsequently for the investment decision. Ashanti's operations can be modeled like a US-based firm whose mining operations are reasonably independent of the country location of the mine for the following reasons:

- Ashanti's gold production is sold to the industrialized countries and revenues are in US dollars. The bulk of Ashanti's capital and operating expenditure are also in US dollars because most of the inputs needed for the development and operation of the mine have to be imported from industrialized countries. It therefore suffices to say that the bulk of Ashanti's financial transactions are not in the local currency of the countries in which they operate but in US dollars.
- Ashanti also syndicates its debt capital from foreign banks in Europe and United States at prime rates without punitive costs for country risks

because Ashanti has the reputation of a well managed firm with a lot of landed assets, and it is fairly well insulated from domestic country risks.

- Ashanti also deposits a significant amount of its revenues from gold sales in foreign banks.

Inflation rates and the risk free interest rates used for valuation of its investments are the ruling US rates. Appropriately, the US dollar was the currency used in the DCF model. A simplified discussion of the model is given below. For proprietary reasons, detailed discussions of the model were avoided in this report. The DCF model includes the following relevant line items:

- Operating costs;
- Revenues;
- Capital expenditures;
- Working capital;
- Royalties and taxes.

Operating Costs: Operating costs in mining consists of actual mining costs, fixed and variable processing costs, refining costs and a markup for contingency (usually 5%).

Revenues: Revenues are incomes accruing from gold sales. Ashanti sells a smaller percentage of its production at the ruling spot gold price and the bulk of its gold production is sold at a hedged price. For the purposes of comparison with the Real Option valuation, a single gold price is assumed in the DCF model.

Capital Expenditure: Gold mining is a capital intensive and a high fixed cost business. The bulk of the capital expenditure goes into the development of the mine which includes construction costs, pre-production mining costs, process plant pre-production costs, provision for working capital, Owner's cost and a markup for contingency. After the development phase, the other capital expenditure required during operations is sustaining capital for tailings disposal and reclamation (where necessary), additional mine equipment and infrastructure and insurance costs.

Royalties and Taxes: The last line item in the DCF is royalty and taxes. Royalty is paid to government on an annual basis and it is a percentage (about 3%) of annual revenue. Taxes are paid on declared taxable income based on the corporate tax regimes in the country of operation. The corporate tax rate used for the Bibiani project in Ghana was 35%.

An 8% discount rate has been used to discount future cash flows and a US inflation rate of 3% has been assumed. Inflation in the country of operation is treated exogenously. Table 4.2 and shows the "static NPV" obtained for varying spot gold prices using the DCF approach. The detailed DCF is shown in Table 4.4a

Table 4.2b Static NPV for Different Prices of Gold

Gold Price (US $)	Static NPV (US$/oz)
250	-55.29
260	-47.76
265	-43.99
270	-40.23
275	-36.46
280	-32.69
285	-28.93
290	-25.16
295	-21.39
300	-17.63
305	-13.86
310	-10.09
315	-6.32
320	-2.56
325	1.21
330	4.98
335	8.74
340	12.51
345	16.28
350	20.04
355	23.81

Real Option Valuation of the Bibiani Project

The inputs required for the option valuation are:

- The current value of the underlying asset (stock price);
- The time to the decision date (time to maturity);
- The investment cost or exercise price (strike price);
- The risk-free rate of interest;
- The volatility of the underlying asset;

In the Real Option valuation model, the above parameters are modeled as shown in Table 4.3.

Table 4.3 Option Valuation Parameters

Call option on stock	**Real Option on mining project**
Current value of underlying asset (stock)	Product of current gold price and quantity of reserves
Exercise price	Investment cost
Time to expiration	Time until opportunity disappears
Risk free rate of interest	Risk free rate of interest
Stock value uncertainty	Project value uncertainty

The Current Value of the Underlying Asset: The value of the underlying asset is the gross project value. Unlike financial options were the value of the underlying asset is usually a traded stock, Real Options usually suffer from the fact that the underlying stock is often not traded. Even when they are, markets are very thin [13]. Suppose there was a market for trading operating mines, then an average market value for developed mine could be easily obtained however transactions in developed mines are very few and the fact that prices are rarely disclosed compounds the problem. Within this constraint, another method was sought for the value of the operating mine (underlying asset). The current value of the underlying asset (the developed and operating mine) is equivalent to calculating the NPV of the mine using the current spot price of gold but excluding the development cost. The initial development cost is modeled as the strike price because it is the outlay needed (the action that has to be taken) to get the asset (the developed mine). The NPV obtained was used as a proxy for the current value of the underlying asset. The use of NPV to determine the value of the underlying asset makes us lose one of the benefits of using Real Options valuation over DCF. The spot price of gold and the discount rate used in the static NPV is used in determining the gross developed and operating mine value. See Table 4.4a. The value obtained for the gross project value for different values of spot gold prices is given in Table 4.4b.

Table 4.4a Discounted Cash Flow Analysis

Discount Rate	8	%								
Spot Gold Price US$/oz		340.00								
Development Cost/oz		60.30								
Operating Cost/oz		183.34								
Revenues US$/oz		256.15								
NPV US$/oz		12.51								
Project Value US$/oz		72.81								
		1997	1998		1999		2000		2001	
		Pre-Production	Year 1		Year 2		Year 3		Year 4	
		Period 1	Half 1	Half 2	Half 1	Half 2	Half 1	Half 2	Half 1	Half 2
Production Cost Analysis										
Gold Production	oz	0.00	67,657.38	67,657.38	129,030.75	129,030.75	131,835.79	131,835.79	94,510.79	91,582.51
Operating Costs	US$	105,000.00	16,973,377.63	18,894,033.40	24,283,946.38	24,176,158.38	24,518,623.81	24,518,623.81	23,756,436.95	23,524,666.27
Total Costs	US$	105,000.00	16,973,377.63	18,894,033.40	24,283,946.38	24,176,158.38	24,518,623.81	24,518,623.81	23,756,436.95	23,524,666.27
Total Revenue	US$	0.00	23,003,510.57	23,003,510.57	43,870,453.33	43,870,453.33	44,824,169.18	44,824,169.18	32,133,666.99	31,138,054.67
Capital Expenditure	US$	82,400,000.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Sustaining Capital	US$	0.00	1,343,817.00	1,343,817.00	272,453.00	272,453.00	295,900.00	295,900.00	519,514.00	519,514.00
Royalty Paid	US$	0.00	795,405.32	795,405.32	1,421,413.60	1,421,413.60	1,569,366.72	1,569,366.72	1,094,375.82	1,014,375.82
Tax Payable	US$	0.00	0.00	0.00	0.00	0.00	1,255,413.84	1,255,413.84	2,725,793.65	2,725,793.65
Project Cash Flow Before Financing (Interest, etc	US$	(82,505,000.00)	3,890,910.63	1,970,254.86	17,892,640.35	18,000,428.35	17,184,864.81	17,184,864.81	4,037,546.57	3,353,704.94
Discount Factor		1.00	1.08	1.08	1.17	1.17	1.26	1.26	1.36	1.36
Present Values	US$	(82,505,000.00)	3,602,695.03	1,824,310.05	15,340,055.17	15,432,466.01	13,641,899.74	13,641,899.74	2,967,717.26	2,465,073.25
Present Values of Costs	US$	82,505,000.00	17,696,851.80	19,475,236.77	22,271,787.53	22,179,376.69	21,940,970.93	21,940,970.93	20,651,487.25	20,422,326.50
Present Value of Revenues	US$	0.00	21,299,546.83	21,299,546.83	37,611,842.70	37,611,842.70	35,582,870.67	35,582,870.67	23,619,204.51	22,887,399.74
Static NPV	US$	17,116,292.21								
NPV of Operation Costs	US$	250,845,700.47								
NPV of Revenues	US$	350,466,992.68								
Project Value as at 1997	US$	99,621,292.21								
Project Value/oz as at 1997		72.81072542								

Table 4.4a Discounted Cash Flow Analysis (Continued)

		2002		2003		2004		2005		
		Year 5		Year 6		Year 7		Year 8		Total
		Half 1	Half 2	Half 1	Half 2	Half 1	Half 2	Half 1	Half 2	Pre-Prod - Year 7
Production Cost Analysis										
Gold Production	oz	116,082.15	116,082.15	98,876.78	98,876.78	51,623.70	43,540.06	0.00	0.00	1,368,222.77
Operating Costs	US$	21,892,353.79	21,892,353.79	16,318,359.06	16,318,359.06	8,023,987.30	5,677,023.22	0.00	0.00	270,873,302.84
Total Costs	US$	21,892,353.79	21,892,353.79	16,318,359.06	16,318,359.06	8,023,987.30	5,677,023.22	0.00	0.00	270,873,302.84
Total Revenue	US$	39,467,931.96	39,467,931.96	33,618,105.69	33,618,105.69	17,552,058.25	14,803,619.43	0.00	0.00	465,195,740.79
Capital Expenditure	US$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	82,400,000.00
Sustaining Capital	US$	259,374.00	259,374.00	234,327.00	132,000.00	-4,698,566.00	0.00	0.00	0.00	1,049,877.00
Royalty Paid	US$	2,751,509.26	2,751,509.26	3,152,609.79	3,152,609.79	1,751,329.01	1,751,329.01	0.00	0.00	24,992,019.05
Tax Payable	US$	5,687,682.70	5,687,682.70	5,615,234.18	5,615,234.18	2,984,435.02	2,984,435.02	0.00	0.00	36,537,118.78
Project Cash Flow Before Financing (Interest, etc	US$	8,877,012.21	8,877,012.21	8,297,575.65	8,399,902.65	9,490,872.92	4,390,832.17	0.00	0.00	49,343,423.12
Discount Factor		1.47	1.47	1.59	1.59	1.71	1.71	1.85	1.85	
Present Values	US$	6,041,545.35	6,041,545.35	5,228,880.15	5,293,363.52	5,537,833.19	2,562,008.40	0.00	0.00	
Present Values of Costs	US$	20,819,665.96	20,819,665.96	15,956,228.97	15,891,745.60	4,703,624.21	6,075,761.35	0.00	0.00	
Present Value of Revenues	US$	26,861,211.31	26,861,211.31	21,185,109.12	21,185,109.12	10,241,457.41	8,637,769.75	0.00	0.00	

Table 4.4b Project Value at Different Gold Prices

Spot Gold Price	Project Value US$/oz
240	-2.53
250	5.01
260	12.54
265	16.31
270	20.07
275	23.84
280	27.61
285	31.38
290	35.14
295	38.91
300	42.68
305	46.44
310	50.21
315	53.98
320	57.74
325	61.51
330	65.28
335	69.04
340	72.81
345	76.58
350	80.34

Gold Price Volatility (Volatility in Project Value): Since the only uncertain variable in the option valuation was limited to the price of gold, the volatility of gold was used as a measure to reflect the volatility in the project value. In the option valuation model, historical data of monthly average closing price of gold compiled from 1976 to 1996 from the LME[12] was used to determine the volatility in gold prices. The data is presented in Tables 4.5a and 4.5b and Fig. 4.1 shows a chart of the data.

Volatility in uncertain variables, e.g. gold prices, are either described as "log normal" or "mean reverting." Prices are said to be mean reverting when commodity prices fluctuate in the short run but are driven towards a long-run value by market forces such as demand and supply. The log normal distribution of prices is based on the premise that stock prices cannot fall below zero and that the distribution of outcomes on the final decision date has a long tail to the right. Margaret Slade (1998) concludes from her work, in valuing risk and flexibility in copper mines in Canada, that prices of precious metals are mean reverting. Amram and Kulatilaka (1999) also mention that gold prices are mean reverting but argue that specifying the form of the uncertain variable as to whether it is mean reverting or it is log normal is irrelevant to the option valuation for the following reasons:

- With dynamic tracking, the value of a financial option contract on a security whose price followed a mean-reverting process, is exactly mimicked by the tracking portfolio, so the form of uncertainty doesn't enter the option valuation result;

[12] London Metals Exchange.

TABLE 4.5a London Gold Bullion Market

Monthly Average Price (1976 - 1996)

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEPT	OCT	NOV	DEC	AVG
1976	131.49	131.07	132.58	127.94	126.94	125.71	117.76	109.93	114.15	116.14	130.48	133.88	**124.84**
1977	132.26	136.29	148.22	149.16	146.60	140.77	143.39	144.95	149.52	158.86	162.10	160.45	**147.71**
1978	173.17	178.15	183.66	175.27	176.30	183.75	188.72	206.30	212.07	227.39	206.07	207.83	**193.22**
1979	227.27	245.67	242.04	239.16	257.61	279.06	294.73	300.81	355.11	391.65	391.99	455.08	**306.68**
1980	675.30	665.32	553.58	517.41	513.82	600.71	644.28	627.14	673.62	661.14	623.46	594.92	**612.56**
1981	557.38	499.76	498.76	495.80	479.69	464.76	409.28	410.15	443.58	437.75	413.36	410.09	**460.03**
1982	384.38	374.13	330.04	350.34	333.82	314.98	338.97	364.23	435.76	422.15	414.91	444.30	**375.67**
1983	481.29	491.96	419.70	432.93	438.08	412.84	422.72	416.24	411.80	393.58	381.66	389.36	**424.35**
1984	370.90	386.33	394.33	381.36	377.40	377.67	347.45	347.70	341.09	340.17	341.19	320.14	**360.48**
1985	302.74	299.10	304.17	324.74	316.64	316.83	317.38	329.33	324.25	325.93	325.22	320.81	**317.26**

Table 4.5b London Gold Bullion Market - Monthly Average Price (1976 - 1996)

(Continued)

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEPT	OCT	NOV	DEC	AVG
1986	345.38	338.89	345.71	340.44	342.56	342.57	348.54	376.60	417.73	423.51	398.81	391.23	**367.66**
1987	408.26	401.12	408.91	438.35	460.23	449.59	450.52	461.15	460.20	465.36	467.57	486.31	**446.46**
1988	476.58	442.07	443.61	451.55	451.01	451.33	437.63	431.31	412.79	406.78	420.17	418.49	**436.94**
1989	404.01	387.78	390.15	384.06	371.00	367.60	375.04	365.37	361.75	366.88	394.26	409.39	**381.44**
1990	410.11	416.83	393.07	374.27	369.19	352.33	362.53	394.73	388.41	380.74	381.73	378.16	**383.51**
1991	383.64	363.83	363.33	358.39	356.82	366.72	367.68	356.23	348.74	358.69	360.17	361.06	**362.11**
1992	354.45	353.89	344.35	338.50	337.23	340.80	353.05	342.96	345.55	344.38	335.87	334.80	**343.82**
1993	329.01	329.35	330.08	342.07	367.18	371.89	392.19	378.84	355.27	364.18	373.83	383.35	**359.77**
1994	386.88	381.91	384.13	377.27	381.26	385.64	385.49	380.35	391.58	389.77	384.39	379.29	**384.00**
1995	378.55	376.64	382.12	391.03	385.12	387.56	386.23	383.81	383.05	383.14	385.30	387.44	**384.17**
1996	400.27	404.79	396.25	392.83	391.86	385.27	383.47	387.46	383.14	381.07	377.85	369.00	**387.77**

Fig 4.1 Monthly Average Gold Prices (1976-1996)

- Statistical tests have little power to discriminate between mean reversion and lognormal specification leaving the matter to judgement.

Calculating the Volatility: The data used in the first three columns of Table 4.6 in the Appendix was obtained from Tables 4.5a and 4.5b. The Price Ratio in the fourth column of Table 4.6 was calculated using the relation

$$M_t / M_{t-1}$$

Where, M, is the monthly average closing gold price and, t, is the time period in months. The monthly return in column five of Table 4.6 is determined by taking the natural log of the price ratio.

$$\textbf{Monthly Return} = \ln \textbf{(Price Ratio)}$$

The monthly volatility is calculated by taking the standard deviation of the monthly returns. The quarterly volatility is calculated from the monthly volatility using the equation below:

$$\text{quarterly volatility} = \text{monthly volatility} * \sqrt{3}$$

Similarly, the annual volatility can be determined from the monthly or quarterly volatility as follows:

$$\text{annual volatility} = \text{quarterly volatility} * \sqrt{4}$$

$\text{annual volatility} = \text{monthly volatility} * \sqrt{12}$

The Strike Price (The Investment Cost): The strike price is the cost of developing the mine. The full development of the mine was completed in a year and payment for all development costs were disbursed in that year. The investment cost was obtained from the static NPV calculation and it was $60.30 per ounce of gold. With the flexibility to defer the investment, the investment cost changed each period commensurate with inflation. The quarterly ruling US inflation rate as at the end of 1996 was 0.61% (i.e. computed from an annual inflation rate of 2.47%) and this value was used. An assumption in the model is that the inflation rate used remains constant throughout the life of the investment opportunity.

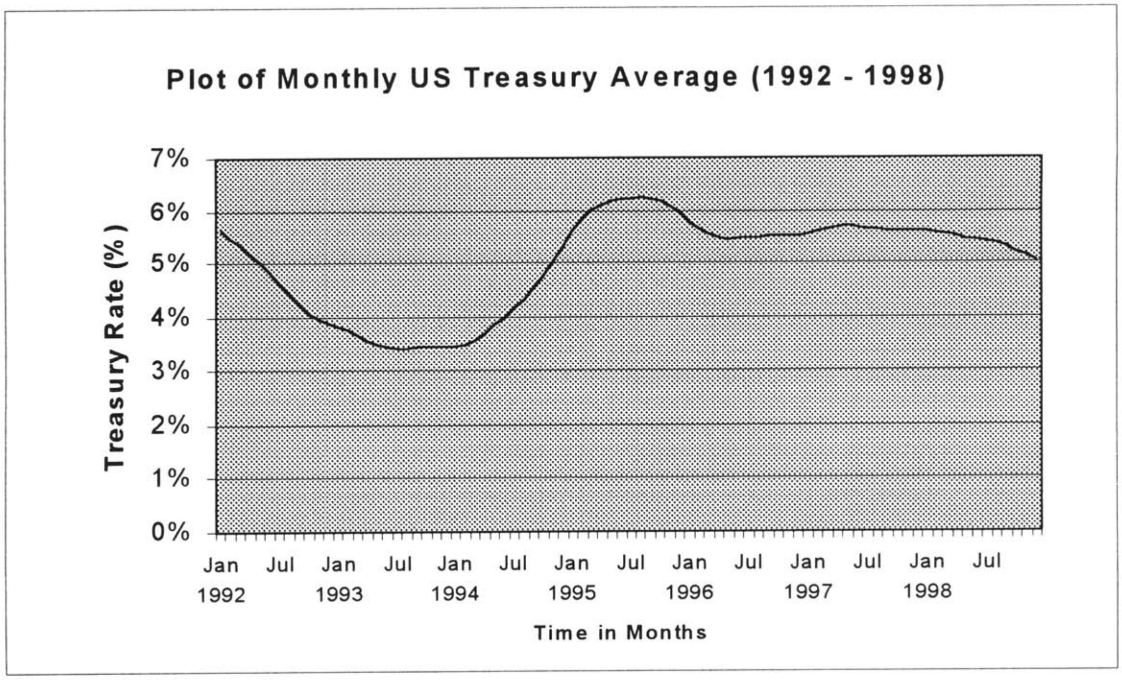
Time to Expiration (Planning Horizon): This is the waiting time for the decision to invest and it is also the period of the option (the investment opportunity). It is usually the time until the expiration of the license to mine. In the option valuation model this period is assumed to span thirty-six quarters i.e. from the year 1997 till 2005.

Risk Free Interest Rate: Because all the original NPV calculations were done in US dollars and because of reasons aforementioned about Ashanti's operations, the return on US government treasury securities was used as the riskfree rate. The data on US government securities for the period 1992 to 1996 is presented in Table 4.7 and Fig.4.2 shows a chart of the data. During the period from 1992 to 1996, US government interest rates averaged 4.808%. A more realistic choice was to use the rate in December, 1996 the ruling rate in the month before the project was initiated. The rate in December, 1996 was 5.513%. Because investors are interested in after tax return, the appropriate riskless after tax return on treasury securities was used. Using a marginal tax rate of 35% yields a real risk free after tax return of approximately 3.78%. The quarterly riskfree rate is a fourth of the annual rate and this quarterly rate was used to determine the expanded NPV's and the option premiums.

Table 4.7. 12 Month Treasury Average

12 MONTH TREASURY AVERAGE							
Month	1992	1993	1994	1995	1996	1997	1998
Jan	5.651%	3.835%	3.438%	5.603%	5.785%	5.556%	5.599%
Feb	5.486%	3.760%	3.478%	5.838%	5.638%	5.605%	5.581%
Mar	5.338%	3.652%	3.560%	6.014%	5.548%	5.643%	5.547%
Apr	5.177%	3.563%	3.692%	6.135%	5.487%	5.681%	5.496%
May	5.015%	3.494%	3.854%	6.193%	5.457%	5.700%	5.460%
Jun	4.833%	3.442%	3.998%	6.223%	5.471%	5.690%	5.437%
Jul	4.607%	3.431%	4.166%	6.223%	5.493%	5.664%	5.422%
Aug	4.414%	3.428%	4.343%	6.248%	5.486%	5.655%	5.393%
Sep	4.215%	3.443%	4.543%	6.237%	5.503%	5.629%	5.325%
Oct	4.046%	3.451%	4.769%	6.193%	5.500%	5.622%	5.213%
Nov	3.945%	3.443%	5.016%	6.101%	5.499%	5.625%	5.136%
Dec	3.889%	3.434%	5.310%	5.948%	5.513%	5.630%	5.052%

Fig. 4.2 Monthly US Treasury Average



The Option Valuation Model

The binomial option valuation approach was used as a solution method in the model. The binomial option valuation relies on the dynamic programming method. It is based on a simple representation of the evolution of the value of the underlying asset (the gross project value) based on the implied volatility of the underlying asset and subsequently on the probabilities of the upward and downward movements. See Table 4.9.

In each time period the underlying asset can take only one of two possible values. Thus the asset, V, can either move to a higher value, Vu, in the next period or a lower value, Vd. Where u and d are the probabilities of an upward or downward movement respectively and are determined from the volatility of the gold prices (the uncertain variable) as follows.

$$u = e^{\sigma} \text{ and } d = 1 - u$$

where σ is the volatility of gold prices. Because the periods used in the model are quarters, the volatility of gold price applicable is the quarterly volatility.

In the risk neutral world, the expected return on the underlying asset is the risk free rate of interest, r, and its volatility will be the same as σ. With continuous compounding, the expected return during each period is given by

$$e^{r} = \frac{pAu + (1-p)Ad}{A}$$

Where p is the probability that weights the outcomes to obtain the risk free rate of return and is called the risk neutral probability. Similarly equating the variance of the return from the binomial model to that of the observed normal distribution of outcomes gives

$$pu^2 + (1-p)d^2 - [pu + (1-p)d]^2 = \sigma^2$$

One solution to the above equations that assumes the underlying asset has symmetric up and down movement i.e. $u=d^{-1}$ is given by

$$U = e^{\sigma};\ D = e^{-\sigma}$$

$$P = (e^{r} - d) / (u - d)$$

Where P is the risk neutral probability of an upward movement. The model inputs are shown in Tables 4.8a and 4.8b.

Rolling Forward the Value of the Underlying Asset: Table 4.9 shows the roll out of the binomial tree for the value of the mine. Each period shows the possible average values of the mine based of the implied quarterly volatility in the stock price of gold. Each average mine value has two possible changes of state in the following period - an upward state or a downward state. For example in the first quarter of 1997,the value of the underlying asset is $72.81. In the next quarter, the possible values of the underlying asset are $79.82/oz if there is an upward movement in the price of gold, or $66.42/oz if there is a downward movement. The symmetric upward and downward movement is calculated using the afore-explained equations.

Table 4.8a Input Data

Data Inputs				
Implied Volatility in Gold Prices (quarterly)	Volatility	σ	9.19	%
Risk Free Interest Rate per quarter	Risk Free Rate	**R**	0.95	%
Planning Horizon	Time to Maturity	**T**	36	Quarters
Net Present Value of Investment Opportunity	Current Stock Price	**A**	72.81	US$/oz
Inflation Rate (quarterly)	Inflation	**I**	0.61	%
Present Value of Investment Costs	Strike Price	**X**	60.30	US$/oz

Table 4.8b Intermediate Data Inputs

Calculated Inputs		
Probability of an Upward Movement	**U**	1.0963
Probability of a Downward Movement	**D**	0.9122
Risk Neutral Probability of an Upward Movement	**P**	0.53
Risk Neutral Probability of a Downward Movement	**1-p**	0.47

Table 4.9 Rollout of the Value of the Underlying Asset

ROLLOUT OF INVESTMENT VALUE BASED ON IMPLIED VOLATILITY OF GOLD PRICES

Year	1997				1998				1999				2000				2001			
	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Quarter 1	Quarter 2	Quarter 3	Quarter 4
	72.81	79.82	87.50	95.92	105.16	115.28	126.38	138.54	151.88	166.49	182.52	200.09	219.35	240.46	263.61	288.98	316.80	347.29	380.72	417.37
		66.42	72.81	79.82	87.50	95.92	105.16	115.28	126.38	138.54	151.88	166.49	182.52	200.09	219.35	240.46	263.61	288.98	316.80	347.29
			60.59	66.42	72.81	79.82	87.50	95.92	105.16	115.28	126.38	138.54	151.88	166.49	182.52	200.09	219.35	240.46	263.61	288.98
				55.27	60.59	66.42	72.81	79.82	87.50	95.92	105.16	115.28	126.38	138.54	151.88	166.49	182.52	200.09	219.35	240.46
					50.41	55.27	60.59	66.42	72.81	79.82	87.50	95.92	105.16	115.28	126.38	138.54	151.88	166.49	182.52	200.09
						45.99	50.41	55.27	60.59	66.42	72.81	79.82	87.50	95.92	105.16	115.28	126.38	138.54	151.88	166.49
							41.95	45.99	50.41	55.27	60.59	66.42	72.81	79.82	87.50	95.92	105.16	115.28	126.38	138.54
								38.27	41.95	45.99	50.41	55.27	60.59	66.42	72.81	79.82	87.50	95.92	105.16	115.28
									34.91	38.27	41.95	45.99	50.41	55.27	60.59	66.42	72.81	79.82	87.50	95.92
										31.84	34.91	38.27	41.95	45.99	50.41	55.27	60.59	66.42	72.81	79.82
											29.05	31.84	34.91	38.27	41.95	45.99	50.41	55.27	60.59	66.42
												26.50	29.05	31.84	34.91	38.27	41.95	45.99	50.41	55.27
													24.17	26.50	29.05	31.84	34.91	38.27	41.95	45.99
														22.05	24.17	26.50	29.05	31.84	34.91	38.27
															20.11	22.05	24.17	26.50	29.05	31.84
																18.35	20.11	22.05	24.17	26.50
																	16.73	18.35	20.11	22.05
																		15.27	16.73	18.35
																			13.92	15.27
																				12.70

Table 4.9 (Continued) Rollout of the Value of the Underlying Asset

2002				2003				2004				2005			
Quarter 1	Quarter 2	Quarter 3	Quarter 4	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Quarter 1	Quarter 2	Quarter 3	Quarter 4
457.54	501.58	549.86	602.79	660.81	724.41	794.14	870.58	954.38	1046.25	1146.95	1257.35	1378.38	1511.05	1656.50	1815.95
380.72	417.37	457.54	501.58	549.86	602.79	660.81	724.41	794.14	870.58	954.38	1046.25	1146.95	1257.35	1378.38	1511.05
316.80	347.29	380.72	417.37	457.54	501.58	549.86	602.79	660.81	724.41	794.14	870.58	954.38	1046.25	1146.95	1257.35
263.61	288.98	316.80	347.29	380.72	417.37	457.54	501.58	549.86	602.79	660.81	724.41	794.14	870.58	954.38	1,046.25
219.35	240.46	263.61	288.98	316.80	347.29	380.72	417.37	457.54	501.58	549.86	602.79	660.81	724.41	794.14	870.58
182.52	200.09	219.35	240.46	263.61	288.98	316.80	347.29	380.72	417.37	457.54	501.58	549.86	602.79	660.81	724.41
151.88	166.49	182.52	200.09	219.35	240.46	263.61	288.98	316.80	347.29	380.72	417.37	457.54	501.58	549.86	602.79
126.38	138.54	151.88	166.49	182.52	200.09	219.35	240.46	263.61	288.98	316.80	347.29	380.72	417.37	457.54	501.58
105.16	115.28	126.38	138.54	151.88	166.49	182.52	200.09	219.35	240.46	263.61	288.98	316.80	347.29	380.72	417.37
87.50	95.92	105.16	115.28	126.38	138.54	151.88	166.49	182.52	200.09	219.35	240.46	263.61	288.98	316.80	347.29
72.81	79.82	87.50	95.92	105.16	115.28	126.38	138.54	151.88	166.49	182.52	200.09	219.35	240.46	263.61	288.98
60.59	66.42	72.81	79.82	87.50	95.92	105.16	115.28	126.38	138.54	151.88	166.49	182.52	200.09	219.35	240.46
50.41	55.27	60.59	66.42	72.81	79.82	87.50	95.92	105.16	115.28	126.38	138.54	151.88	166.49	182.52	200.09
41.95	45.99	50.41	55.27	60.59	66.42	72.81	79.82	87.50	95.92	105.16	115.28	126.38	138.54	151.88	166.49
34.91	38.27	41.95	45.99	50.41	55.27	60.59	66.42	72.81	79.82	87.50	95.92	105.16	115.28	126.38	138.54
29.05	31.84	34.91	38.27	41.95	45.99	50.41	55.27	60.59	66.42	72.81	79.82	87.50	95.92	105.16	115.28
24.17	26.50	29.05	31.84	34.91	38.27	41.95	45.99	50.41	55.27	60.59	66.42	72.81	79.82	87.50	95.92
20.11	22.05	24.17	26.50	29.05	31.84	34.91	38.27	41.95	45.99	50.41	55.27	60.59	66.42	72.81	79.82
16.73	18.35	20.11	22.05	24.17	26.50	29.05	31.84	34.91	38.27	41.95	45.99	50.41	55.27	60.59	66.42
13.92	15.27	16.73	18.35	20.11	22.05	24.17	26.50	29.05	31.84	34.91	38.27	41.95	45.99	50.41	55.27
11.59	12.70	13.92	15.27	16.73	18.35	20.11	22.05	24.17	26.50	29.05	31.84	34.91	38.27	41.95	45.99
	10.57	11.59	12.70	13.92	15.27	16.73	18.35	20.11	22.05	24.17	26.50	29.05	31.84	34.91	38.27
		9.64	10.57	11.59	12.70	13.92	15.27	16.73	18.35	20.11	22.05	24.17	26.50	29.05	31.84
			8.79	9.64	10.57	11.59	12.70	13.92	15.27	16.73	18.35	20.11	22.05	24.17	26.50
				8.02	8.79	9.64	10.57	11.59	12.70	13.92	15.27	16.73	18.35	20.11	22.05
					7.32	8.02	8.79	9.64	10.57	11.59	12.70	13.92	15.27	16.73	18.35
						6.68	7.32	8.02	8.79	9.64	10.57	11.59	12.70	13.92	15.27
							6.09	6.68	7.32	8.02	8.79	9.64	10.57	11.59	12.70
								5.55	6.09	6.68	7.32	8.02	8.79	9.64	10.57
									5.07	5.55	6.09	6.68	7.32	8.02	8.79
										4.62	5.07	5.55	6.09	6.68	7.32
											4.22	4.62	5.07	5.55	6.09
												3.85	4.22	4.62	5.07
													3.51	3.85	4.22
														3.20	3.51
															2.92

The Decision Rule: Recall that the decision rule is to invest in the development and extraction of gold at any time during the life of the investment opportunity if the value of the investment opportunity is *max[V - I, 0]*. The decision rule is applied to the possible outcomes of the project value in each quarter to obtain new values $V_{(i,t)}$, where *i*, is the row number and *t*, is the period number. The true value of the investment cost (the strike price), *I*, grows each quarter at the riskless rate as shown in Table 4.10. The value of the investment opportunity in the preceding quarters are determined in a backward recursive fashion as explained in the next paragraph.

Folding Back the Values: The next step in the model is to bring back future values to the present. In period *T-1*, the value of the optimal decision at time *T* is given by

$$V_{(i,t-1)} = \text{Max}\ \{V_{(i,t-1)},\ [V_{(i,t)} * p + V_{(i+1,t)} * (1 - p)] / e^{r}\}$$

$$t = 1, 2, \ldots.., T$$

Where $V_{(i,t-1)}$ is the value of the investment opportunity in the *ith* row and in the *(t-1)th* column of the binomial tree. See Table 4.10. The present value of the option to wait for the optimal time to invest anytime within the lifetime of the investment opportunity is the value $V_{(1,1)}$ in Table 4.10.

Model Results: Table 4.11 shows the option premiums and the expanded NPV's obtained from the option valuation model for different values of spot gold prices. These results show that for lower spot gold prices the expanded NPV of the project is positive even

Table 4.10 Folding Back the Option Value

Year	**1997**				**1998**				**1999**			
Period	**Quarter 1**	**Quarter 2**	**Quarter 3**	**Quarter 4**	**Quarter 1**	**Quarter 2**	**Quarter 3**	**Quarter 4**	**Quarter 1**	**Quarter 2**	**Quarter 3**	**Quarter 4**
Index	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Investment cost (Strike Price)	60.30	60.87	61.45	62.03	62.61	63.20	63.80	64.40	65.01	65.63	66.25	66.87
Option Values	**25.12**	30.37	36.45	43.43	51.37	60.35	70.45	81.73	94.28	108.19	123.57	140.52
		19.73	24.17	29.36	35.39	42.33	50.25	59.21	69.30	80.58	93.13	107.04
			15.15	18.83	23.19	28.33	34.31	41.20	49.09	58.04	68.12	79.40
				11.34	14.31	17.90	22.19	27.26	33.19	40.05	47.91	56.85
					8.23	10.58	13.45	16.95	21.16	26.16	32.03	38.85
						5.78	7.56	9.80	12.58	15.98	20.10	25.03
							3.89	5.20	6.88	9.01	11.68	14.99
								2.50	3.42	4.63	6.20	8.22
									1.52	2.13	2.96	4.06
										0.87	1.25	1.78
											0.46	0.68
												0.22

Table.4.10 (Continued) Folding Back the Option Value

Year	2000				2001				2002			
Period	**Quarter 1**	**Quarter 2**	**Quarter 3**	**Quarter 4**	**Quarter 1**	**Quarter 2**	**Quarter 3**	**Quarter 4**	**Quarter 1**	**Quarter 2**	**Quarter 3**	**Quarter 4**
Index	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00	21.00	22.00	23.00
Investment cost (Strike Price)	67.51	68.14	68.79	69.44	70.09	70.76	71.42	72.10	72.78	73.47	74.16	74.86
Option Values	159.18	179.71	202.28	227.06	254.29	284.19	317.02	353.06	392.63	436.05	483.71	536.01
	122.42	139.37	158.03	178.55	201.10	225.88	253.10	282.99	315.81	351.84	391.39	434.80
	91.96	105.87	121.25	138.20	156.85	177.37	199.91	224.68	251.88	281.76	314.57	350.59
	66.92	78.20	90.76	104.68	120.06	137.01	155.65	176.16	198.69	223.45	250.65	280.51
	46.69	55.62	65.69	76.98	89.55	103.47	118.85	135.79	154.44	174.93	197.46	222.20
	30.84	37.62	45.44	54.36	64.44	75.73	88.31	102.24	117.62	134.56	153.20	173.68
	19.01	23.86	29.61	36.35	44.15	53.07	63.16	74.47	87.06	101.00	116.38	133.31
	10.77	13.96	17.89	22.65	28.34	35.04	42.83	51.74	61.85	73.18	85.78	99.73
	5.52	7.41	9.84	12.91	16.72	21.40	27.02	33.69	41.46	50.39	60.51	71.87
	2.51	3.50	4.84	6.60	8.89	11.82	15.52	20.09	25.65	32.28	40.05	49.00
	1.00	1.45	2.08	2.96	4.16	5.77	7.91	10.70	14.26	18.73	24.21	30.81
	0.34	0.51	0.77	1.14	1.67	2.43	3.49	4.95	6.92	9.54	12.95	17.30
	0.10	0.15	0.24	0.37	0.56	0.86	1.29	1.93	2.84	4.12	5.91	8.34
		0.04	0.06	0.10	0.15	0.25	0.39	0.61	0.94	1.45	2.20	3.30
			0.01	0.02	0.03	0.05	0.09	0.15	0.24	0.40	0.64	1.02
				0.00	0.00	0.01	0.02	0.03	0.05	0.08	0.13	0.23
					0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03
						0.00	0.00	0.00	0.00	0.00	0.00	0.00
							0.00	0.00	0.00	0.00	0.00	0.00
								0.00	0.00	0.00	0.00	0.00
									0.00	0.00	0.00	0.00
										0.00	0.00	0.00
											0.00	0.00
												0.00

Table 4.10 (Comtinued) Folding Back Option Value

Year	2003				2004				2005			
Period	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Index	24	25	26	27	28	29	30	31	32	33	34	35
Investment cost (Strike Price)	75.57	76.29	77.01	77.73	78.47	79.21	79.96	80.71	81.48	82.25	83.02	83.81
Option Values	593.39	656.36	725.44	801.23	884.37	975.57	1,075.60	1,185.33	1,305.67	1,437.66	1,582.41	1,741.15
	482.45	534.73	592.11	655.06	724.13	799.91	883.03	974.22	1,074.24	1,183.95	1,304.28	1,436.26
	390.13	433.53	481.16	533.43	590.80	653.74	722.80	798.56	881.67	972.85	1,072.86	1,182.55
	313.31	349.31	388.84	432.23	479.85	532.11	589.46	652.39	721.43	797.18	880.29	971.45
	249.38	279.24	312.02	348.01	387.53	430.90	478.51	530.76	588.10	651.02	720.05	795.78
	196.19	220.93	248.10	277.94	310.71	346.69	386.19	429.56	477.15	529.39	586.71	649.61
	151.93	172.41	194.91	219.63	246.79	276.62	309.37	345.34	384.83	428.18	475.76	527.99
	115.11	132.03	150.65	171.11	193.60	218.31	245.45	275.27	308.01	343.97	383.44	426.78
	84.49	98.44	113.82	130.74	149.34	169.79	192.26	216.96	244.09	273.89	306.62	342.57
	59.16	70.54	83.18	97.14	112.51	129.41	148.00	168.44	190.90	215.58	242.70	272.49
	38.59	47.58	57.79	69.21	81.86	95.82	111.17	128.06	146.64	167.06	189.51	214.18
	22.71	29.29	37.10	46.14	56.41	67.86	80.53	94.47	109.81	126.69	145.25	165.66
	11.58	15.79	21.13	27.70	35.56	44.69	55.03	66.52	79.17	93.10	108.43	125.29
	4.88	7.09	10.13	14.18	19.44	26.04	34.00	43.26	53.67	65.14	77.78	91.70
	1.60	2.49	3.83	5.78	8.57	12.44	17.63	24.30	32.45	41.88	52.28	63.74
	0.38	0.63	1.05	1.71	2.77	4.40	6.87	10.51	15.65	22.53	31.06	40.48
	0.06	0.10	0.18	0.32	0.57	0.99	1.71	2.93	4.96	8.25	13.41	21.13
	0.00	0.01	0.01	0.03	0.05	0.10	0.20	0.38	0.72	1.38	2.63	5.02
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
				0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
						0.00	0.00	0.00	0.00	0.00	0.00	0.00
							0.00	0.00	0.00	0.00	0.00	0.00
								0.00	0.00	0.00	0.00	0.00
									0.00	0.00	0.00	0.00
										0.00	0.00	0.00
											0.00	0.00
												0.00

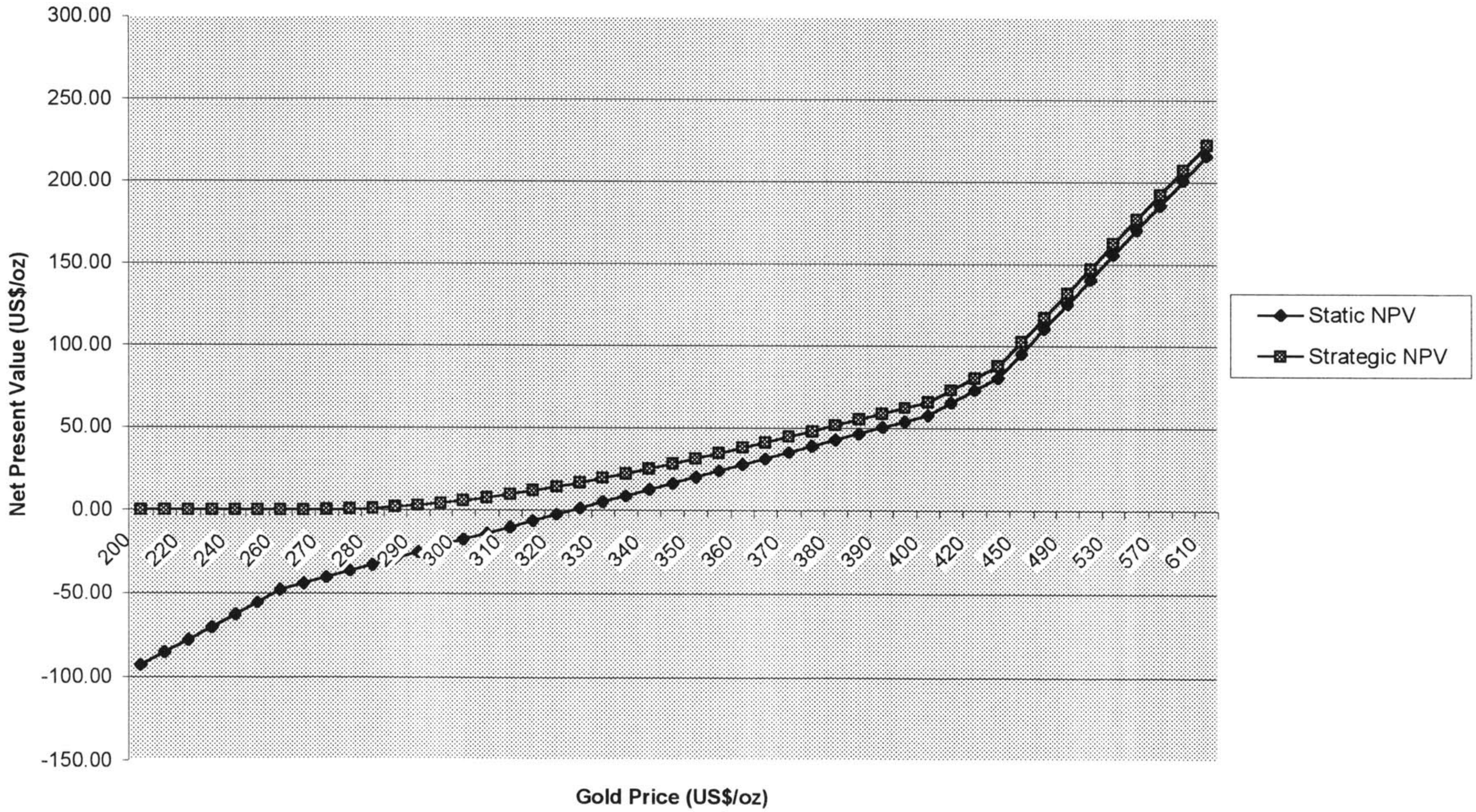
Table 4.11. Results from Option Valuation Model

Gold Price (US $)	Project Value US$/oz	Static NPV US$/oz	Expanded NPV US$/oz	Option Value US$/oz
200	-32.66	-92.96	0.00	92.96
210	-25.13	-85.43	0.00	85.43
220	-17.59	-77.90	0.00	77.90
230	-10.06	-70.36	0.00	70.36
240	-2.53	-62.83	0.00	62.83
250	5.01	-55.29	0.00	55.29
260	12.54	-47.76	0.01	47.77
265	16.31	-43.99	0.06	44.06
270	20.07	-40.23	0.22	40.44
275	23.84	-36.46	0.53	36.99
280	27.61	-32.69	1.02	33.71
285	31.38	-28.93	1.79	30.72
290	35.14	-25.16	2.84	28.00
295	38.91	-21.39	4.04	25.43
300	42.68	-17.63	5.67	23.30
305	46.44	-13.86	7.36	21.22
310	50.21	-10.09	9.45	19.54
315	53.98	-6.32	11.65	17.97
320	57.74	-2.56	13.97	16.52
325	61.51	1.21	16.63	15.43
330	65.28	4.98	19.30	14.33
335	69.04	8.74	22.06	13.31
340	72.81	12.51	25.12	12.61
345	76.58	16.28	28.18	11.90
350	80.34	20.04	31.24	11.20
355	84.11	23.81	34.46	10.65
360	87.88	27.58	37.82	10.24
365	91.65	31.34	41.17	9.82
370	95.41	35.11	44.52	9.41
375	99.18	38.88	47.90	9.02
380	102.95	42.64	51.45	8.80
385	106.71	46.41	54.99	8.58
390	110.48	50.18	58.54	8.36
395	114.25	53.95	62.08	8.14
400	118.01	57.71	65.63	7.91
410	125.55	65.25	72.93	7.68
420	133.08	72.78	80.25	7.47
430	140.61	80.31	87.57	7.25
450	155.68	95.38	102.42	7.04
470	170.75	110.45	117.30	6.85
490	185.82	125.52	132.29	6.77
510	200.88	140.58	147.28	6.70
530	215.95	155.65	162.31	6.66

when the DCF method gives a negative value. This result show that managerial flexibility and strategic interactions to defer the investment decision in the event of unfavorable market situations has value and this value can be captured by the expanded NPV approach. As the spot price of gold increases, the NPV's obtained from both methods approach the same value. This results show that option valuation is indeed able to capture the value due to managerial flexibility in strategically managing investments that DCF techniques fail to capture. By having the option to wait before investing, management is able to add value to what would otherwise have been without waiting. The results also show that option valuation is valuable for those investment that lie in the gray area for which managerial operating strategy is required. For very low gold prices the benefit of option valuation is to limit loses by doing nothing and for higher gold prices where it is very valuable to invest, option valuation yields about the same NPV as DCF methods. Fig 4.3 shows a chart of the comparison of the payoffs from an expanded (strategic) NPV approach to investments and a static NPV approach for different values in spot gold price.

Sensitivity Analysis: In the option valuation model, the implied volatility of historical gold prices was used as a good estimate to forecast the possible future states of the investment opportunity. In this model the implied volatility was computed from historical gold prices from 1976 - 1996. Because different implied volatilities can be obtained based on a richer set of data on historical gold prices a sensitivity analysis of the model results to volatility is presented in Fig. 4.4. We observe that option valuation is more important when uncertainty is very high because it is during those instances that managerial strategic decisions are most important. The results also show that there is value in uncertainty. The higher the uncertainty, the greater the value in flexibility and the greater the option. We also observe that the Strategic NPV gets closer to the Static NPV as volatility decreases. When there is absolutely no volatility i.e. when the investment is safe and there is no need for active management, option valuation yields the same result as DCF. The only deference being that with option valuation, the downside risk minimized by doing nothing.

Fig.4.3. Plot of Strategic & Static NPV for different values of Gold Price



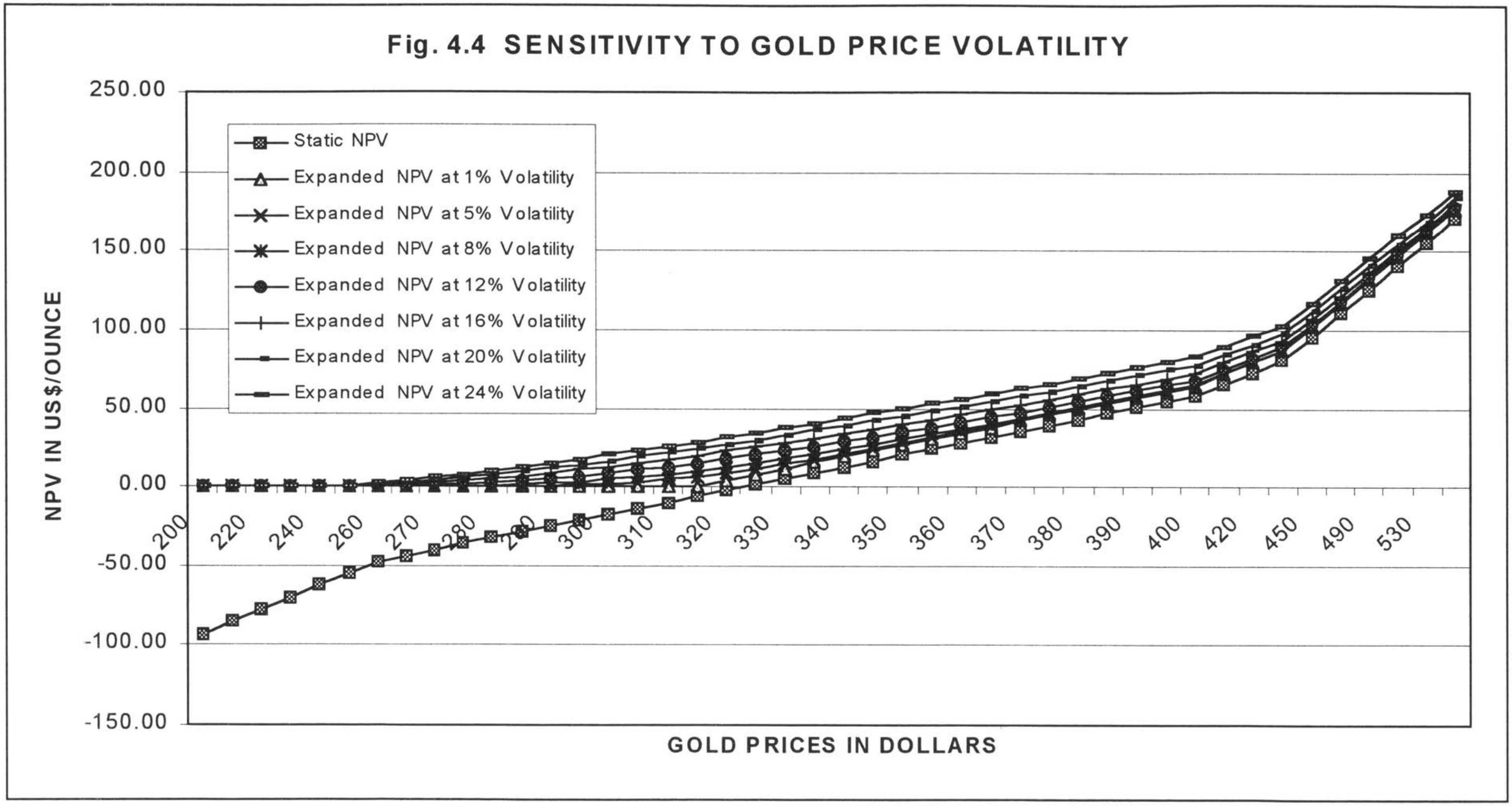


Fig. 4.4 SENSITIVITY TO GOLD PRICE VOLATILITY
250.00
200.00
150.00
100.00
50.00
0.00
-50.00
-100.00
-150.00
NPV IN US$/OUNCE
Static NPV
Expanded NPV at 1% Volatility
Expanded NPV at 5% Volatility
Expanded NPV at 8% Volatility
Expanded NPV at 12% Volatility
Expanded NPV at 16% Volatility
Expanded NPV at 20% Volatility
Expanded NPV at 24% Volatility
200
220
240
260
270
280
300
310
320
330
340
350
360
370
380
390
400
420
450
490
530
GOLD PRICES IN DOLLARS

CHAPTER 5: CONCLUSION

Corporate planners and managers involved with capital budgeting decisions have often expressed dissatisfaction with traditional methods like DCF especially in cases where there are contingent investment decisions or when uncertainty is large enough to make flexibility a consideration. Additionally when there will be project updates or mid-course corrective action, DCF cannot be used effectively to value such investments. While DCF is still a useful valuation method especially for safe investments, it also makes inappropriate adjustment for risk especially in the case of mining projects. What practitioners want from valuation is the opportunity to value projects together with their implementation strategy.

This thesis used a Real Option valuation method to value the decision to invest in gold mining resource development. The valuation results obtained were compared to results obtained using DCF. It has been shown from results in Table 11 and from the chart of the two valuation methods in Fig. 4.3 that valuation based on Real Options quantifies more value especially for marginal investments. The difference in value between the DCF method and the Real Options method is greatest between the range of gold prices from $300 to $360. Its also been shown that the DCF method yields a positive NPV for gold prices above $325/oz whiles the Real Option valuation yields a positive NPV for gold prices as low as $260/oz. This result underscores the introductory statement that DCF methods undervalue mining investments. The difference in the two methods is the value due to flexibility and managerial operating strategy. This is the value that the DCF method fails to capture from the onset. The results also show that when gold prices are high (above $400/oz) and the investment is very valuable, Real Option valuation may not be needed because the result from DCF may not differ much from that of option valuation. Under such circumstances, either method would effectively assist practitioners in the decision making. For gold prices below $260/oz, Real Options cannot help because no amount of sophistication in managerial strategy would be able to make the investment profitable. The sensitivity analysis results in Fig 4.4 support the claim by Amram and Kulatilaka (1999) and Trigeorges (1996) that there is value uncertainty and that uncertainty is good for todays managers. Fig. 4.4 shows that the investment value increases with higher volatility in gold prices.

For almost fifteen years, academic literature has suggested that the methods used to price financial options can be adapted to price real assets and their option values. However the industry decision makers, although unsatisfied with current valuation methods, have not yet adopted this method. In this thesis, the case of investing in gold mines has been used to demonstrate that there is value in flexibility and strategic managerial interactions, which should be priced. Two valuation methodologies have been applied to value the same mining investment. When uncertainty is high, traditional DCF undervalues mining assets. For other capital budgeting decisions where there is significant risk and uncertainty and where managerial flexibility and strategic decisions can affect project outcomes, Real Options valuation methodology should be used. Real Option valuation captures the value in flexibility and managerial strategic decisions to defer investments and wait for favorable market outcomes. Options valuation is also able to capture the total risk associated with a particular investment and gets around the interest rate dilemma that plagues DCF valuation.

Future Work

This work was limited to only one uncertain variable, which was the price of gold. However we know that in mining operations there are other key uncertain variables like the quantity of reserves and costs for which it will be worthwhile to investigate and to incorporate in the option valuation model. Future work should focus on incorporating these additional uncertain variables. Additionally this work was limited to valuing the managerial flexibility in the decision to invest. Options valuation can also be used to value managerial operational flexibility to abandon, contract, mothball or expand an operating mine.

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APPENDIX

Table 4.5

MONTHLY AVERAGE PRICES (LONDON GOLD BULLION MARKET)

YEAR	MONTH	MONTHLY AVE. PRICE	Price Ratio	Daily Return
1976	Jan	131.49		
	Feb	131.07	0.997	-0.320%
	Mar	132.58	1.012	1.145%
	Apr	127.94	0.965	-3.562%
	May	126.94	0.992	-0.785%
	Jun	125.71	0.990	-0.974%
	Jul	117.76	0.937	-6.533%
	Aug	109.93	0.934	-6.880%
	Sept	114.15	1.038	3.767%
	Oct	116.14	1.017	1.728%
	Nov	130.48	1.123	11.642%
	Dec	133.88	1.026	2.572%
1977	Jan	132.26	0.988	-1.217%
	Feb	136.29	1.030	3.002%
	Mar	148.22	1.088	8.391%
	Apr	149.16	1.006	0.632%
	May	146.60	0.983	-1.731%
	Jun	140.77	0.960	-4.058%
	Jul	143.39	1.019	1.844%

YEAR	MONTH	MONTHLY AVE. PRICE	Price Ratio	Daily Return
	Aug	144.95	1.011	1.082%
	Sept	149.52	1.032	3.104%
	Oct	158.86	1.062	6.059%
	Nov	162.10	1.020	2.019%
	Dec	160.45	0.990	-1.023%
1978	Jan	173.17	1.079	7.629%
	Feb	178.15	1.029	2.835%
	Mar	183.66	1.031	3.046%
	Apr	175.27	0.954	-4.676%
	May	176.30	1.006	0.586%
	Jun	183.75	1.042	4.139%
	Jul	188.72	1.027	2.669%
	Aug	206.30	1.093	8.907%
	Sept	212.07	1.028	2.758%
	Oct	227.39	1.072	6.975%
	Nov	206.07	0.906	-9.845%
	Dec	207.83	1.009	0.850%
1979	Jan	227.27	1.094	8.942%
	Feb	245.67	1.081	7.785%
	Mar	242.04	0.985	-1.489%
	Apr	239.16	0.988	-1.197%

YEAR	MONTH	MONTHLY AVE. PRICE	Price Ratio	Daily Return
	May	257.61	1.077	7.431%
	Jun	279.06	1.083	7.998%
	Jul	294.73	1.056	5.463%
	Aug	300.81	1.021	2.042%
	Sept	355.11	1.181	16.595%
	Oct	391.65	1.103	9.794%
	Nov	391.99	1.001	0.087%
	Dec	455.08	1.161	14.924%
1980	Jan	675.30	1.484	39.468%
	Feb	665.32	0.985	-1.489%
	Mar	553.58	0.832	-18.386%
	Apr	517.41	0.935	-6.757%
	May	513.82	0.993	-0.696%
	Jun	600.71	1.169	15.624%
	Jul	644.28	1.073	7.002%
	Aug	627.14	0.973	-2.696%
	Sept	673.62	1.074	7.150%
	Oct	661.14	0.981	-1.870%
	Nov	623.46	0.943	-5.868%
	Dec	594.92	0.954	-4.686%
1981	Jan	557.38	0.937	-6.518%

YEAR	MONTH	MONTHLY AVE. PRICE	Price Ratio	Daily Return
	Feb	499.76	0.897	-10.912%
	Mar	498.76	0.998	-0.200%
	Apr	495.80	0.994	-0.595%
	May	479.69	0.968	-3.303%
	Jun	464.76	0.969	-3.162%
	Jul	409.28	0.881	-12.712%
	Aug	410.15	1.002	0.212%
	Sept	443.58	1.082	7.836%
	Oct	437.75	0.987	-1.323%
	Nov	413.36	0.944	-5.733%
	Dec	410.09	0.992	-0.794%
1982	Jan	384.38	0.937	-6.475%
	Feb	374.13	0.973	-2.703%
	Mar	330.04	0.882	-12.539%
	Apr	350.34	1.062	5.969%
	May	333.82	0.953	-4.830%
	Jun	314.98	0.944	-5.809%
	Jul	338.97	1.076	7.340%
	Aug	364.23	1.075	7.187%
	Sept	435.76	1.196	17.931%
	Oct	422.15	0.969	-3.173%

YEAR	MONTH	MONTHLY AVE. PRICE	Price Ratio	Daily Return
	Nov	**414.91**	0.983	-1.730%
	Dec	**444.30**	1.071	6.844%
1983	Jan	**481.29**	1.083	7.997%
	Feb	**491.96**	1.022	2.193%
	Mar	**419.70**	0.853	-15.886%
	Apr	**432.93**	1.032	3.104%
	May	**438.08**	1.012	1.183%
	Jun	**412.84**	0.942	-5.934%
	Jul	**422.72**	1.024	2.365%
	Aug	**416.24**	0.985	-1.545%
	Sept	**411.80**	0.989	-1.072%
	Oct	**393.58**	0.956	-4.525%
	Nov	**381.66**	0.970	-3.075%
	Dec	**389.36**	1.020	1.997%
1984	Jan	**370.90**	0.953	-4.857%
	Feb	**386.33**	1.042	4.076%
	Mar	**394.33**	1.021	2.050%
	Apr	**381.36**	0.967	-3.344%
	May	**377.40**	0.990	-1.044%
	Jun	**377.67**	1.001	0.072%
	Jul	**347.45**	0.920	-8.340%

YEAR	MONTH	MONTHLY AVE. PRICE	Price Ratio	Daily Return
	Aug	347.70	1.001	0.072%
	Sept	341.09	0.981	-1.919%
	Oct	340.17	0.997	-0.270%
	Nov	341.19	1.003	0.299%
	Dec	320.14	0.938	-6.368%
1985	Jan	302.74	0.946	-5.588%
	Feb	299.10	0.988	-1.210%
	Mar	304.17	1.017	1.681%
	Apr	324.74	1.068	6.544%
	May	316.64	0.975	-2.526%
	Jun	316.83	1.001	0.060%
	Jul	317.38	1.002	0.173%
	Aug	329.33	1.038	3.696%
	Sept	324.25	0.985	-1.555%
	Oct	325.93	1.005	0.517%
	Nov	325.22	0.998	-0.218%
	Dec	320.81	0.986	-1.365%
1986	Jan	345.38	1.077	7.380%
	Feb	338.89	0.981	-1.897%
	Mar	345.71	1.020	1.992%
	Apr	340.44	0.985	-1.536%

YEAR	MONTH	MONTHLY AVE. PRICE	Price Ratio	Daily Return
	May	342.56	1.006	0.621%
	Jun	342.57	1.000	0.003%
	Jul	348.54	1.017	1.728%
	Aug	376.60	1.081	7.743%
	Sept	417.73	1.109	10.365%
	Oct	423.51	1.014	1.374%
	Nov	398.81	0.942	-6.009%
	Dec	391.23	0.981	-1.919%
1987	Jan	408.26	1.044	4.261%
	Feb	401.12	0.983	-1.764%
	Mar	408.91	1.019	1.923%
	Apr	438.35	1.072	6.952%
	May	460.23	1.050	4.871%
	Jun	449.59	0.977	-2.339%
	Jul	450.52	1.002	0.207%
	Aug	461.15	1.024	2.332%
	Sept	460.20	0.998	-0.206%
	Oct	465.36	1.011	1.115%
	Nov	467.57	1.005	0.474%
	Dec	486.31	1.040	3.930%
1988	Jan	476.58	0.980	-2.021%

YEAR	MONTH	MONTHLY AVE. PRICE	Price Ratio	Daily Return
	Feb	442.07	0.928	-7.517%
	Mar	443.61	1.003	0.348%
	Apr	451.55	1.018	1.774%
	May	451.01	0.999	-0.120%
	Jun	451.33	1.001	0.071%
	Jul	437.63	0.970	-3.082%
	Aug	431.31	0.986	-1.455%
	Sept	412.79	0.957	-4.389%
	Oct	406.78	0.985	-1.467%
	Nov	420.17	1.033	3.239%
	Dec	418.49	0.996	-0.401%
1989	Jan	404.01	0.965	-3.521%
	Feb	387.78	0.960	-4.100%
	Mar	390.15	1.006	0.609%
	Apr	384.06	0.984	-1.573%
	May	371.00	0.966	-3.460%
	Jun	367.60	0.991	-0.921%
	Jul	375.04	1.020	2.004%
	Aug	365.37	0.974	-2.612%
	Sept	361.75	0.990	-0.996%
	Oct	366.88	1.014	1.408%

YEAR	MONTH	MONTHLY AVE. PRICE	Price Ratio	Daily Return
	Nov	394.26	1.075	7.198%
	Dec	409.39	1.038	3.766%
1990	Jan	410.11	1.002	0.176%
	Feb	416.83	1.016	1.625%
	Mar	393.07	0.943	-5.869%
	Apr	374.27	0.952	-4.901%
	May	369.19	0.986	-1.367%
	Jun	352.33	0.954	-4.674%
	Jul	362.53	1.029	2.854%
	Aug	394.73	1.089	8.509%
	Sept	388.41	0.984	-1.614%
	Oct	380.74	0.980	-1.994%
	Nov	381.73	1.003	0.260%
	Dec	378.16	0.991	-0.940%
1991	Jan	383.64	1.014	1.439%
	Feb	363.83	0.948	-5.302%
	Mar	363.33	0.999	-0.138%
	Apr	358.39	0.986	-1.369%
	May	356.82	0.996	-0.439%
	Jun	366.72	1.028	2.737%
	Jul	367.68	1.003	0.261%

YEAR	MONTH	MONTHLY AVE. PRICE	Price Ratio	Daily Return
	Aug	**356.23**	0.969	-3.164%
	Sept	**348.74**	0.979	-2.125%
	Oct	**358.69**	1.029	2.813%
	Nov	**360.17**	1.004	0.412%
	Dec	**361.06**	1.002	0.247%
1992	**Jan**	**354.45**	0.982	-1.848%
	Feb	**353.89**	0.998	-0.158%
	Mar	**344.35**	0.973	-2.733%
	Apr	**338.50**	0.983	-1.713%
	May	**337.23**	0.996	-0.376%
	Jun	**340.80**	1.011	1.053%
	Jul	**353.05**	1.036	3.531%
	Aug	**342.96**	0.971	-2.900%
	Sept	**345.55**	1.008	0.752%
	Oct	**344.38**	0.997	-0.339%
	Nov	**335.87**	0.975	-2.502%
	Dec	**334.80**	0.997	-0.319%
1993	**Jan**	**329.01**	0.983	-1.745%
	Feb	**329.35**	1.001	0.103%
	Mar	**330.08**	1.002	0.221%
	Apr	**342.07**	1.036	3.568%

YEAR	MONTH	MONTHLY AVE. PRICE	Price Ratio	Daily Return
	May	367.18	1.073	7.084%
	Jun	371.89	1.013	1.275%
	Jul	392.19	1.055	5.315%
	Aug	378.84	0.966	-3.463%
	Sept	355.27	0.938	-6.424%
	Oct	364.18	1.025	2.477%
	Nov	373.83	1.026	2.615%
	Dec	383.35	1.025	2.515%
1994	Jan	386.88	1.009	0.917%
	Feb	381.91	0.987	-1.293%
	Mar	384.13	1.006	0.580%
	Apr	377.27	0.982	-1.802%
	May	381.26	1.011	1.052%
	Jun	385.64	1.011	1.142%
	Jul	385.49	1.000	-0.039%
	Aug	380.35	0.987	-1.342%
	Sept	391.58	1.030	2.910%
	Oct	389.77	0.995	-0.463%
	Nov	384.39	0.986	-1.390%
	Dec	379.29	0.987	-1.336%
1995	Jan	378.55	0.998	-0.195%

YEAR	MONTH	MONTHLY AVE. PRICE	Price Ratio	Daily Return
	Feb	**376.64**	0.995	-0.506%
	Mar	**382.12**	1.015	1.444%
	Apr	**391.03**	1.023	2.305%
	May	**385.12**	0.985	-1.523%
	Jun	**387.56**	1.006	0.632%
	Jul	**386.23**	0.997	-0.344%
	Aug	**383.81**	0.994	-0.629%
	Sept	**383.05**	0.998	-0.198%
	Oct	**383.14**	1.000	0.023%
	Nov	**385.30**	1.006	0.562%
	Dec	**387.44**	1.006	0.554%
1996	**Jan**	**400.27**	1.033	3.258%
	Feb	**404.79**	1.011	1.123%
	Mar	**396.25**	0.979	-2.132%
	Apr	**392.83**	0.991	-0.867%
	May	**391.86**	0.998	-0.247%
	Jun	**385.27**	0.983	-1.696%
	Jul	**383.47**	0.995	-0.468%
	Aug	**387.46**	1.010	1.035%
	Sept	**383.14**	0.989	-1.121%
	Oct	**381.07**	0.995	-0.542%

YEAR	MONTH	MONTHLY AVE. PRICE	Price Ratio	Daily Return
	Nov	**377.85**	0.992	-0.849%
	Dec	**369.00**	0.977	-2.370%
			Monthly Volatility	5.31%
			Quarterly Volatility	9.19%
			Annual Volatility	18.38%