# OPTIMIZATION OF MULTI-ECHELON INVENTORY DEPLOYMENT IN A FINISHED GOODS NETWORK

by

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## Abstract

The object of the thesis is to develop a methodology to help managers of finished goods to deploy an optimized multi-echelon inventory. In a multi-echelon inventory system, Distribution Requirement Planning (DRP) is widely used as a scheduling tool to solve the finished goods dispatch problem. Consumer demand is the ultimate factor which drives the DRP solution. Each individual location in the multi-echelon system including the retail level receives an efficient product flow within the distribution network. However, a DRP system has drawbacks in that DRP does not explicitly include cost considerations such as transportation costs, holding costs, lost sales costs, or order costs in the solution.

To provide optimized inventory deployment, we begin by outlining how a multi-echelon inventory system works. Then we present a discussion of how a DRP procedure schedules the system. We then optimize the scheduled DRP solution by considering transportation costs, backorder costs, and lost sale costs. This generates an optimal planned solution based on the forecasted demanded for known future demand periods. Finally, we will provide a methodology to help the manager respond quickly to deviations in demand from the forecast which were used in the plan. For example, how should the retail manager deal with a sudden increase in demand after having sent his order to the manufacturers, considering that manufacturing needs a long lead-time in order to fill the requirement? Should the manager respond to a changed demand in multi-echelon system by rescheduling the DRP plan, using express delivery the product, or by losing the sale? The procedure developed solves this problem optimally by considering all the options and costs involved.

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# 1 Background and Motivation

The incentive to optimize the multi-echelon inventory methodology originates in common inventory shortage problems. Such problems frequently occur at the retail level which is located at the end of a supply chain where retailers directly face customers. Inventory is typically stocked at many points in a supply chain, and finished goods might be stocked at all levels of the distribution network. The varieties of stock will include raw materials, semi-finished goods, finished goods, and scraps among as many as hundreds of thousands of retailers, distribution centers, and trucks. Recently, material deployment techniques have been widely used at both manufacturing sites and finished goods distribution sites. This technique, a schedule-planning model, is called Material Requirement Planning (MRP) or Distribution Requirement Planning (DRP). This technique is called MRP when it deploys the scheduling model on the manufacturing's materials requirement, and it is called DRP when it dispatches the finished goods to the consumers' locations by storing them in intermediate locations such as warehouses or distribution centers. Since the supply chain widely uses these techniques, DRP and MRP can be integrated as a useful schedule-planning tool. Once the consumers' demand is forecasted or provided, information about the finished goods that have been consumed can be directly transferred into the purchasing orders of raw materials. As a result, DRP drives MRP by providing information about demand.

There are three motives to develop on optimization of multi-echelon inventory deployment in a finished goods network. First, the difficulties of forecasting demand accurately will frequently cause rescheduling. There is a need for managers who control finished goods in multi-echelon network to have a simple easy methodology to do this rescheduling. Second, the lead-time issue also increases the complexity involved in scheduling the supply chain. The different lead-times between manufacturers and distributors must be considered carefully in order to avoid shortages in the distribution channel. Third, there is a strong possibility that the distribution channel will stock too many finished goods. A higher than necessary stockage level will slow the inventory turnover and increase the product cycle time. Excess inventory is a major factor that effects the return on investment.

The shortage problems commonly occur on the retailers' side because of difficulties in forecasting consumer demand. The difficulties of forecasting may come from retailers' short-term promotions forced by competitors' unexpected price reduction, holiday or seasonality effects. When the unexpected increasing demand is more than the stock level at a retail store, the manager must quickly respond to the situation and decide to either accept the order or lose the sales. When accepting the order, the manager must ask for an expedited transfer from other available locations such as manufacturers, distribution centers, or other retailers. These available locations must carry enough stock to provide for both themselves and the location which is experiencing the shortage. Otherwise, this unexpected increased demand will become a lost sale and might result in an unfulfilled customer order. Providing managers with a handy methodology to respond precisely and quickly to such problems is one of the motivations for this thesis.

Second, the lead-time gap between manufacturing sites and distribution sites also increases the possibility of inventory shortages in the supply chain. Within the supply chain, the distribution centers are the intermediate locations connecting manufacturers and retailers and also providing flexibility in the multi-echelon network based on different lead-times. The distribution centers can serve as a buffer area to deal with short lead-time on the retailers' side and long lead-time on the manufacturer's side. The major component of lead-time for the retailer is the shipping time for the product. On the other hand, the major component of lead-time for the manufacturer includes the time to source raw materials, the manufacturing process, and packaging the product. Because MRP and DRP scheduling methodologies are widely used in planning, the availability of inventory through the whole supply chain is important. If retailers change the order quantity after the orders have been released and manufacturers must respond within their lead-time period, the MRP will experience difficulty catching up with the DRP rescheduling. As a result, inventory stockage level for each location in the distribution network becomes a critical issue. How managers can benefit most from the multi-echelon inventory network is a second motive for this thesis.

Third, either a business or supply chain wants to increase its profitability by reducing its own inventory level. Inventory reduction could provide a benefit from cost saving. However, a lower inventory stock level increases the possibility of shortage. So, the inventory stock level decision should be made carefully. The major benefits come from increasing the Net Income and reducing the inventory level in the financial statement. The Return on Investment (ROI) of a business or a supply chain is the major profitability measure for top management, stockholders, and financial analysts. Return on Asset (ROA) is a very important measure which is calculated by dividing the Net Income (NI) by the Total Asset from financial statement. Another similarly important measure is Gross ROA, which is calculated by dividing the Earning before Income Tax (EBIT) by the Total Asset.

$$ROA = \frac{Net \quad Income}{Total \quad Asset}$$

$$Gross \quad ROA = \frac{EBIT}{Total \quad Asset}$$

From the day-to-day business perspective, the fastest and most common way to increase Return on Investment (ROI) is to try to increase the sales amount as well as reduce the inventory level at the same time. The action of increasing sales simultaneously increases proportionally the Net Income or EBIT in the numerator of ROA equations. The action of reducing inventory level causes reduction of the Total Asset in the denominator of ROA equations. Whenever inventory is discussed in a business, the inventory includes all physical materials such as raw materials, working in process, semi-finished products, and finished goods.

From the supply chain point of view, inventory is one of the most important issues. The inventory level directly affects the product cost and customer service level which are the two core components which determine a supply chain competitive advantage. Each business unit in the supply chain must make a demand forecast for its customer. The forecast errors will increase dramatically if the supply chain suffers from information that is isolated and segmented. Also, examples can be easily found in all kinds of business, such as the "Beer Game" or "Bull Whip" effect caused by manufacturers and retailers. The Beer Game effect occurs when demand forecasts or marketing focus is targeted only on the immediate or direct customers, instead of end users. Unshared information will inflate the perceived demand through the whole supply chain. Therefore, inventory levels will be unrealistic on all levels of the supply chain no matter what kind of state-of-the-art information technology (IT) system is used.

With increasing availability of information technology, inventory stockage levels of the entire business unit could be calculated easily and updated daily. The effect of IT can be more powerful if there is IT collaboration within the supply chain. This means data can be precisely calculated whenever DRP is rescheduled and can be updated at any location within the pipeline. In this way all the demand forecasts and marketing systems can focus on the identical subjects, i.e., end consumers.

However, in spite of the power of IT, the supply chain is still constrained by the issues of demand variations and lead-time. this thesis will propose a method by which DRP systems can deal with demand fluctuation within system lead-times to optimize inventory utilization in the supply chain.

## 2 Literature Review

A review of the literature on multi-echelon models reveals that most of the past research focus on where to locate the inventory within different echelon levels.

Stephen Graves states:[4]

> "Most of the work considers a two-echelon distribution system with identical retail sites and Poisson demand, and then develops an approximate model of system cost or performance as a function of stockage levels; a simulation is used to evaluate the approximate model."

More and more papers suggest that "build to order" and shipping directly to customer is the trend for the future. Dis-intermediation is a very popular topic for studying how to eliminate levels in a multi-echelon distribution network. Because of today's Just In Time (JIT) world, the multi-echelon inventory for finished goods network may appear to be obsolete. Many people now believe that multi-echelon inventory deployment is a problem source instead of a solution provider.

A good example of multi-echelon inventory is one in which companies provide the service of repairing and supplying low-demand, recoverable and repairable items to customers. Companies require a multi-echelon inventory network to ensure reliable service. Take an automotive windshield provider for example. It is very critical for its customers to have spare parts available on an emergency basis. Also, the time to deliver the service is a major concern for customers. For a business that implements a JIT system or dis-intermediation ideas, the inventory only behold at either the manufacturers' or local retailers' facility. With JIT implementation, the retailer will spend time waiting for the replacement parts and risk the possibility that the customer may seek alternative suppliers. Instead, the local retailers must maintain a very high level inventory availability if they want to keep good customer service.

The major benefit of the multi-echelon inventory network is that it supports the solution of the physical layout problem. The following conditions provide the rationale:

- The long delivery time required from the sources of raw material to manufacturing plants.
- The lead-time and fabrication time required in plants.
- Delivering finished goods to a customer takes a fixed amount of shipping time.
- Expedited shipment may be possible for finished goods but not for raw materials or working in process.

Another benefit of the multi-echelon inventory network is that adequate stockage at each echelon will provide flexibility for management in supply chain. Because of the complicated and difficult demand forecasting task, the multi-echelon inventory network would streamline the supply and order processes and reduce delay and shortages. A well-designed multi-echelon inventory is a good management tool for business to use to provide good customer service. Managing and operating a multi-echelon inventory system is a strategic as well as a tactical issue.

In the field of multi-echelon inventory networks, most of the past research focuses on the inventory stockage problem. The cases examined in these papers reveal the characteristics of each multi-echelon model, but none of these models allow the inventory to be cross-transferred after the schedule is fixed.

A.J. Clark identifies several characteristics for multi-echelon models. These characteristics can distinguish various multi-echelon models. Each multi-echelon system has the following characteristics:

- Product: Single product or Multi products;
- Demand: Deterministic or Stochastic;
- Usage: Stationary or Nonstationary;
- Review: Continuous review or Periodic review;
- Category: Consumable product or Repairable product;
- Shortage: Backlog or No backlog;

For different types of multi-echelon systems, the research papers examine many different approaches. For instance, somc focus on the decision of how to make the stockage standard for each echelon; others use a heuristic approach to find the best solution with simulation models; others modify the inventory stock policies for different scenarios, etc.

One approach focuses on the optimal stead-state stockage levels in a multi-echelon inventory system. Grave's paper modeling the multi-echelon inventory system tries to find the optimal stock levels at each location. The major issue presented in this paper is to provide the proper stockage levels at each echelon under the assumption of known deterministic demand. With the known information, the paper provides a model to characterize service performance that estimates the expected shortage. Each different stockage level in the multi-echelon system results in a different service level. The model allows managers to experiment with different combinations of stockage levels and to decide what is the best at the moment. This kind of approach is a preventive inventory shortage methodology and has a number of constraints that the model has to follow: 1) Forecast pattern should be determined and assumed to be precise. 2) Overall service performance does not have cost associated with it. 3) It is a one-time approach but dynamically adjusts as the environment changes.

Another approach to multi-echelon inventory problems is to decide whether the inventory is better controlled by central levels or by local levels. If the central levels such as manufacturers' warehouses and center distribution centers control the inventory, the management at the central levels must have more sufficient market information to decide when the best time is to move finished goods to retail stores. This procedure is usually employed in a system where frequent replenishment is possible. On the contrary, if the local levels such as retail stores and customer service centers control the inventory, the management at the local levels has the power to respond to the changing market, and this may provide higher customer service levels than centralized control. This technique is usually found in a system where local levels must stock high levels of finished goods. There are many businesses choosing a hybrid of these two models. Simulation of stockage policies is the methodology used to helping the management to make the stockage decision. This methodology gives the decision maker a way to find the best stockage combination by adopting a heuristic approach. Because of the heuristic approach, there is no optimization solution for the particular business case. The simulation methodology may suffer from drawbacks: 1) it could be a time consuming process; 2) there could be a lot of variables on a simulation model; 3) the assumptions may not be realistic.

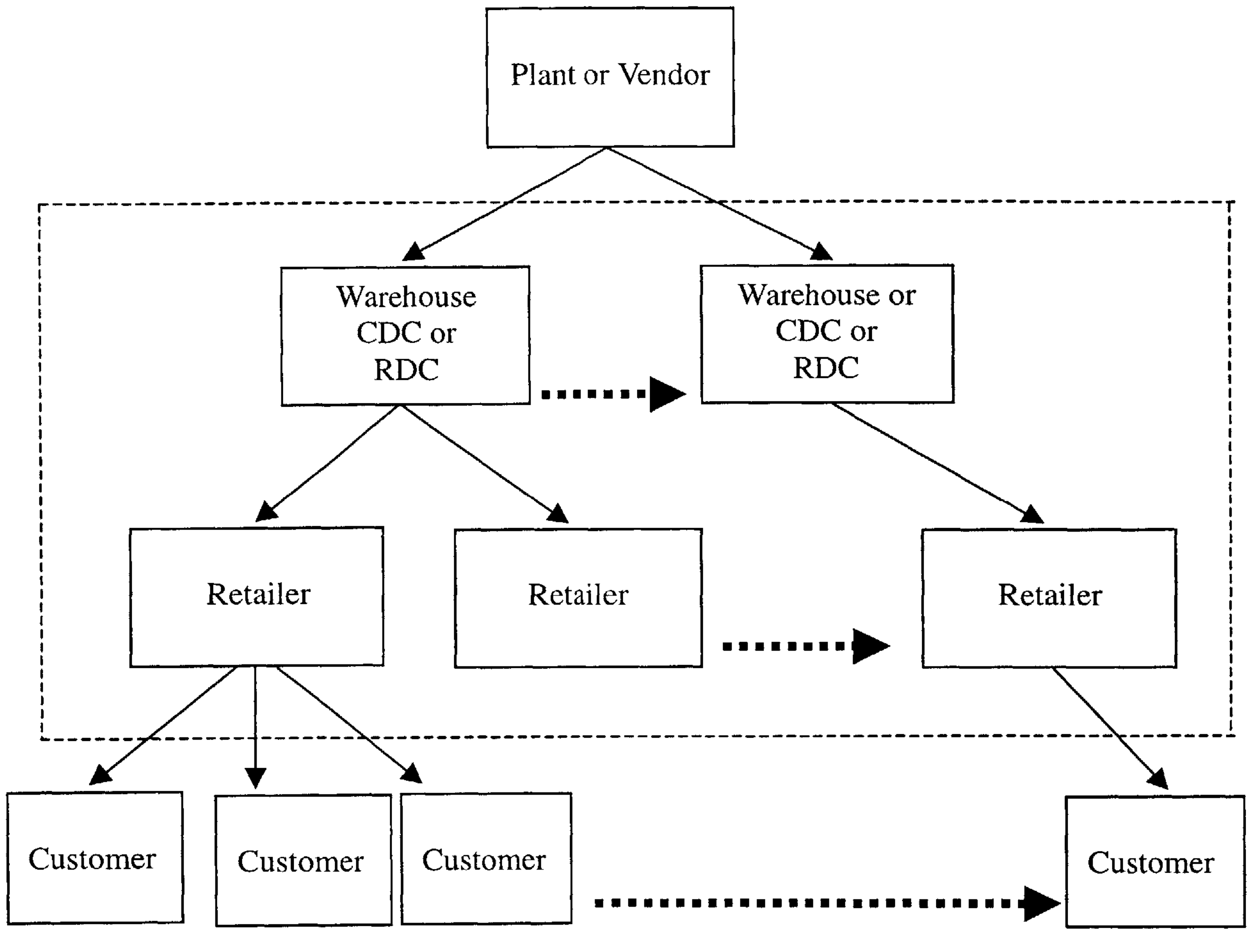
## 3 Formulation Development

For developing an optimized model based on a DRP multi-echelon inventory system, we begin with a discussion of the nature of the components which are used in developing the optimization methodology. The process of development begins with a multi-echelon inventory system model, followed by the DRP system model, then by optimization with transportation cost trade-off model. All three previously discussed models provide an environment in which a methodology can be developed for a quick response to increased consumer demand at specific locations. This methodology can provide a manager in a post-optimized multi-echelon DRP deployment environment with a quick way to deal with inventory shortages caused by an increased demand. In consideration of the time factor, inventory at each location may be transferred to other locations at appropriate times and by means of appropriate methods depending on the stockage availability within the time periods. The manager may decide whether to expedite goods from other locations, to lose the sale, or to backorder in order to keep costs at a minimum.

## 3.1 Multi-echelon Model

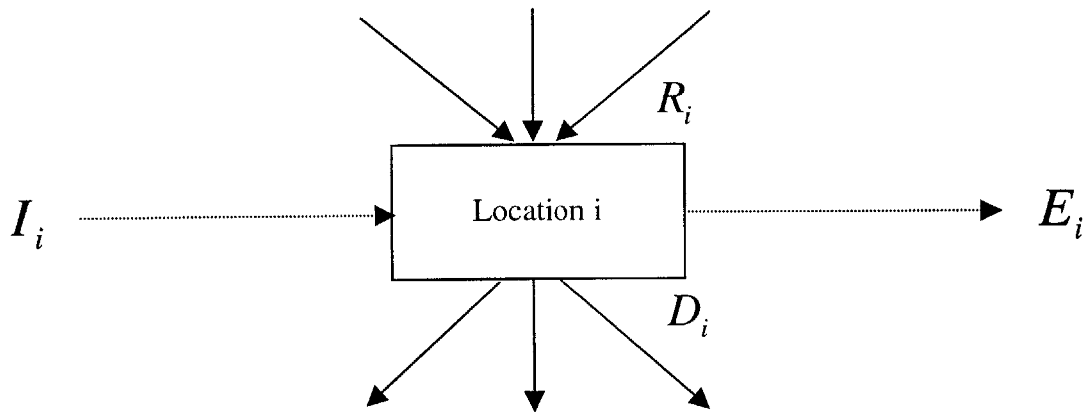
Multi-echelon inventory models are commonly used in distribution systems. Multi-echelon models have been used in distribution systems suffering from geographical constraints where retailers are widely spaced, and where there are economies of scale involving transportation costs, market-driven service locations, or maintenance spare part stock level requirements. By establishing a multi-echelon model, distribution systems can provide better customer service, inventory replenishment, and less transportation time than direct shipment from manufacturers can provide. DRP is a scheduling technique application that is often employed by multi-echelon models. The scope of the multi-echelon system model is shown on Figure 1, which depicts the system from the end of the production line to the delivery of the goods into consumer's hand. The multi-echelon system could be as simple as one supplier and one retailer or so complex that it would cover ten or twenty of levels.

**Figure 1. A multi-echelon Inventory System**




The inventory process at one location can be expanded into a multi-echelon system that is appropriate in many kinds of situations. At a single location, the two major transactions on inventory are the inflow and outflow of finished goods. The central idea would be that the inflow is always equal to the outflow plus the stockage amount. When the situation allows a backorder or a lost sale at a location, a backorder or lost sale quantity can represent a fulfillment of the inflows in maintaining a balance in the inventory equation at the single location. Figure 2 gives a graphic depiction of how to visualize the inventory relationships.

**Figure 2. Inventory Transshipment in Single Location**




To build the single location model, we made the following assumptions on this location: 1) known demand, 2) one period of time, 3) no same level inventory transfer. Any demand that exceeds the location stock level will become lost sales without backorder or expediting.

$I_i$ Beginning inventory at location i;

$E_i$ Ending inventory at location i;

$R_i$ Planned receipt amount at location i from upper echelon;

$D_i$ Demand at location i from lower echelon;

The transaction in this location at one period of time will be:

$$E_i \geq I_i + R_i - D_i \tag{1}$$

The right side of equation (1) maybe smaller than zero; this indicates the lost sales amount. The left side of equation (1) will be larger than or equal to zero. Following the algorithm, the locations on the same level can be formulated as:

$$\sum_i E_i \geq \sum_i I_i + \sum_i R_i - \sum_i D_i \tag{2}$$

Equation (2) is the expansion of equation (1) and has the same general form. The right side of the equation may be larger than zero, but some of the individual locations may occur lost sales which would provide a negative number. The situation shows the imbalance of inventory stockage within the same echelon.

Total demand for this echelon is:

$$\sum_i D_i$$

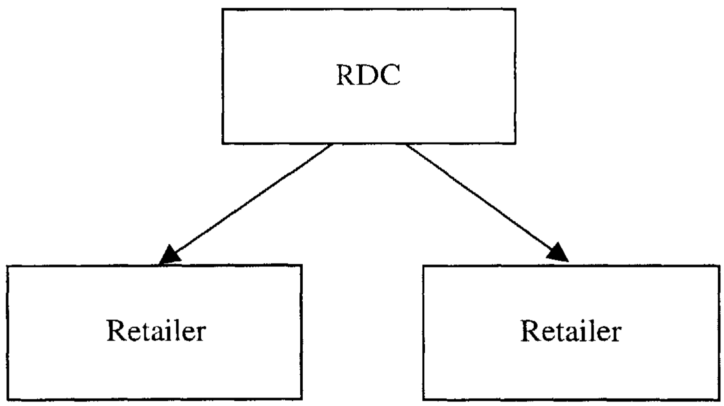
Total amount received for this echelon is:

$$\sum_i R_i$$

## 3.2 Distribution Requirement Planning

DRP deals well with a deterministic demand situation, but does adapt well to stochastic demand scheduling. The reason is that DRP requires future demand to be known.

**Figure 3. Two-echelon System**




In this simple two-layer multi-echelon Distribution Requirement Planning model, RDC supplies two regional retailers. The information required is 1) order lot size, 2) safety stock, 3) lead-time, and 4) beginning inventory level. In addition to these requirements, the future demand for the lower levels, or retailers, should be known in advance. Figure 4 depicts an example set of original DRP tables which could be used to deploy the retailer demand. The required data inputs are marked in *Italics*. The deployed DRP tables are shown in Figure 5.

For Regional Distribution Center, the cells in Figure 4 require the inputs Q, SS, LT, and Begin Inventory. For Retailers, the cells require the inputs Period Usage, Q, SS, LT, and Begin Inventory. Let us define:

| | |
|---|---|
| Q | The required order size; |
| SS | Safety stock desired at the location; |
| LT | Lead-time to transfer stock from the next higher echelon; |
| Gross Requirement | Gross Requirement equals to Period Usage plus the safety stock; |
| $I_i = E_{i-1}$ | Begin Inventory in period i is equal to the previous period Ending Inventory; |

Net Requirement = Gross Requirement - Begin Inventory

Net Requirement is equal to the Gross Requirement minus Begin Inventory;

$$\text{Plan Receipt} \begin{cases} 0; \text{ if Net Requirement equals to } 0; \\ Q; \text{ if Net Requirement greater than } 0; \end{cases}$$

End Inventory = Plan Receipt + Begin Inventory - Period Usage

Plan Order Release $_i$ = Plan Receipt $_{i+LT}$

Plan Order Release at period i equals to the Plan Receipt at period ( i plus lead-time);

**Figure 4. Distribution Requirement Planning Setup Data**

## Regional Distribution Center

Q= *200* SS= *0* LT= *2*

| | Now | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Period Usage | | | | | | | | | |
| Gross Requirement | | | | | | | | | |
| Begin Inventory | | | | | | | | | |
| Schedule Receipt | | | | | | | | | |
| Net Requirement | | | | | | | | | |
| Plan Receipt | | | | | | | | | |
| End Inventory | *150* | | | | | | | | |
| Planned Order Release | | | | | | | | | |

## Retailer A

Q= *50* SS= *15* LT= *1*

| | Now | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Period Usage | | *25* | *25* | *25* | *25* | *25* | *25* | *25* | *25* |
| Gross Requirement | | | | | | | | | |
| Begin Inventory | | | | | | | | | |
| Schedule Receipt | | | | | | | | | |
| Net Requirement | | | | | | | | | |
| Plan Receipt | | | | | | | | | |
| End Inventory | *50* | | | | | | | | |
| Planned Order Release | | | | | | | | | |

## Retailer B

Q= *30* SS= *10* LT= *1*

| | Now | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Period Usage | | *10* | *10* | *10* | *10* | *10* | *10* | *10* | *10* |
| Gross Requirement | | | | | | | | | |
| Begin Inventory | | | | | | | | | |
| Schedule Receipt | | | | | | | | | |
| Net Requirement | | | | | | | | | |
| Plan Receipt | | | | | | | | | |
| End Inventory | *20* | | | | | | | | |
| Planned Order Release | | | | | | | | | |

**Figure 5. The Completed Distribution Requirement Planning Tables**

## Regional Distribution Center

Q= *200* SS= *0* LT= *2*

| | Now | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Period Usage | | 80 | 0 | 50 | 30 | 50 | 0 | 80 | 0 |
| Gross Requirement | | 80 | 0 | 50 | 30 | 50 | 0 | 80 | 0 |
| Begin Inventory | | 150 | 70 | 70 | 20 | 190 | 140 | 140 | 60 |
| Schedule Receipt | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Net Requirement | | 0 | 0 | 0 | 10 | 0 | 0 | 0 | 0 |
| Plan Receipt | | 0 | 0 | 0 | 200 | 0 | 0 | 0 | 0 |
| End Inventory | *150* | 70 | 70 | 20 | 190 | 140 | 140 | 60 | 60 |
| Planned Order Release | | 0 | 200 | 0 | 0 | 0 | 0 | 0 | 0 |

## Retailer 1

Q= *50* SS= *15* LT= *1*

| | Now | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Period Usage | | *25* | *25* | *25* | *25* | *25* | *25* | *25* | *25* |
| Gross Requirement | | 40 | 40 | 40 | 40 | 40 | 40 | 40 | 40 |
| Begin Inventory | | 50 | 25 | 50 | 25 | 50 | 25 | 50 | 25 |
| Schedule Receipt | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Net Requirement | | 0 | 15 | 0 | 15 | 0 | 15 | 0 | 15 |
| Plan Receipt | | 0 | 50 | 0 | 50 | 0 | 50 | 0 | 50 |
| End Inventory | *50* | 25 | 50 | 25 | 50 | 25 | 50 | 25 | 50 |
| Planned Order Release | | 50 | 0 | 50 | 0 | 50 | 0 | 50 | 0 |

## Retailer 2

Q= *30* SS= *10* LT= *1*

| | Now | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Period Usage | | *10* | *10* | *10* | *10* | *10* | *10* | *10* | *10* |
| Gross Requirement | | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| Begin Inventory | | 20 | 10 | 30 | 20 | 10 | 30 | 20 | 10 |
| Schedule Receipt | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Net Requirement | | 0 | 10 | 0 | 0 | 10 | 0 | 0 | 10 |
| Plan Receipt | | 0 | 30 | 0 | 0 | 30 | 0 | 0 | 30 |
| End Inventory | *20* | 10 | 30 | 20 | 10 | 30 | 20 | 10 | 30 |
| Planned Order Release | | 30 | 0 | 0 | 30 | 0 | 0 | 30 | 0 |

### 3.3 Optimized DRP with Transportation Cost Trade-off

The DRP system demonstrates an effective way for all locations within the multi-echelon inventory deployment network to interact level by level. Once the consumer demand pattern is known, DRP then could calculate the transshipment schedule according to order lot sizes, lead-times, and safety stocks at each location within the multi-echelon inventory network. The schedule considers the benefit that on-time shipments would reduce potential holding costs. However, DRP does not explicitly consider the cost of the product shipment. Usually, the transportation rate is decided by several major factors, including the product weight and volume, the length of shipping time, and the way the product is shipped. The lighter and smaller product will be cheaper than the heavy and bigger one. A Truck Load (TL) would be cheaper than a Less Truck Load (LTL). Shipping once per week would be more economical than three times per week. Regular shipping would receive a better freight rate than an expedited rate. The transportation cost is a major proportion of total cost when we consider whether the scheduled DRP plan is realistic to use or not.

From Federick S. Hiller's[7] inventory model, the components of a total inventory cost model includes

1) the costs of ordering or manufacturing,
2) holding cost,
3) shortage costs,
4) revenue,
5) salvage costs,
6) discount rates of the product.

When total cost models are compared, the first three components are usually treated as decision variables in approaching a solution. The other three components are usually hold constant when comparison occurs in a same time frame. As a result, the total cost formulation for finished goods is written as follows:

$$\text{Total Cost} = \text{Production Cost} + \text{Order Cost} + \text{Holding Cost} + \text{Shortage Cost} \qquad (3)$$

The total cost of a finished good could be composed production cost, order cost, holding cost, and shortage cost. Production cost includes all the costs that finished goods require such as raw material cost, labor cost, and overhead. Order cost sometimes may be referred to as setup cost in manufacturing which is a one-time cost whenever the order is released to suppliers. Holding cost, also called carrying cost, is the cost of storing the finished goods until sold or used. Shortage cost occurs when the amount of finished goods demanded exceeds the available stock.

When we optimize a multi-echelon inventory network, the components of total cost in equation (3) that could be ignored are production cost, and order cost. In many cases the production cost will not change based on where or when the product is located. Order cost in a distribution system would only refer to transaction fees rather than to setup costs in the manufacturing environment. In addition, holding cost is often insignificant due to the short lead-time requirement. Stuart J Allen's[10] paper explains that the holding cost in lot sizing decisions will continue to be de-emphasized in modern manufacturing theory. Also, order and holding costs become a small portion if we calculate the total cost by unit.

On the other hand, shortage cost often becomes the major issue when we try to optimize the scheduled DRP. The components of shortage cost include backorder cost, lost sale cost, and expedited shipment cost depending on the situation.

Figure 6 demonstrates a simple two level multi-echelon inventory network in which two RDCs, RDC A and RDC B, supply finished goods to 3 retailers, Retailer 1, Retailer 2, and Retailer 3. In this multi-echelon system, basically RDC A supplies Retailer 1 and Retailer 2 while RDC B supplies Retailer 3 regularly. In some cases, RDC A can also supplies Retailer 3 finished goods, but it will cost a higher rate than RDC B provides because of reasons such as geographical difficulty. When demand suddenly exceeds stockage at one of Retailer locations, all the locations at higher echelon other than the shortage location will become the potential suppliers of each other on an express shipment basis.

Furthermore, if we allow more flexibility in the multi-echelon inventory network, then we may permit all the locations in the network to be potential finished goods providers. Figure 7 demonstrates these relationships. After a location is structured or planned by DRP, the location can change its role in the multi-echelon inventory network when we try to find the optimal solution based on cost. All the echelon-style organizations can be discarded, and each single location can be thought of as at the same level. Consequently, the optimization for this "one-echelon" representation can be calculated. The trade-off to get the optimized minimum cost is to choose the lowest cost among expediting shipment, backorder, and lost sale. Figure 7 also depicts the location relationships of how the finished goods are transshipped in a multi-echelon inventory network. In this section, we develop a methodology that can optimize the multi-echelon inventory network by considering the fact that finished goods can be transferred in different periods. Figure 8 shows the 3-dimensional representation of how the system could be modeled. The three dimensions represent space and time. All the demand in a specific period defined by DRP tables can be fulfilled or allocated using regular transshipment, expediting shipment, backorder, or lost sale in the same period of time.

**Figure 6. A Two Level of Multi-Echelon Distribution System**

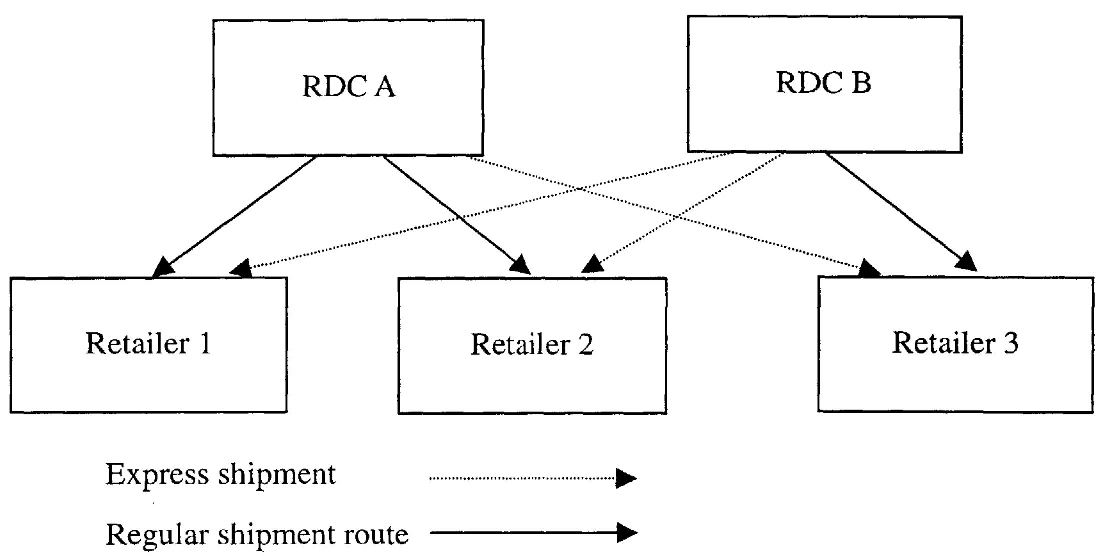


**Figure 7. The Possible Ways of Transshipment in a Distribution Network**

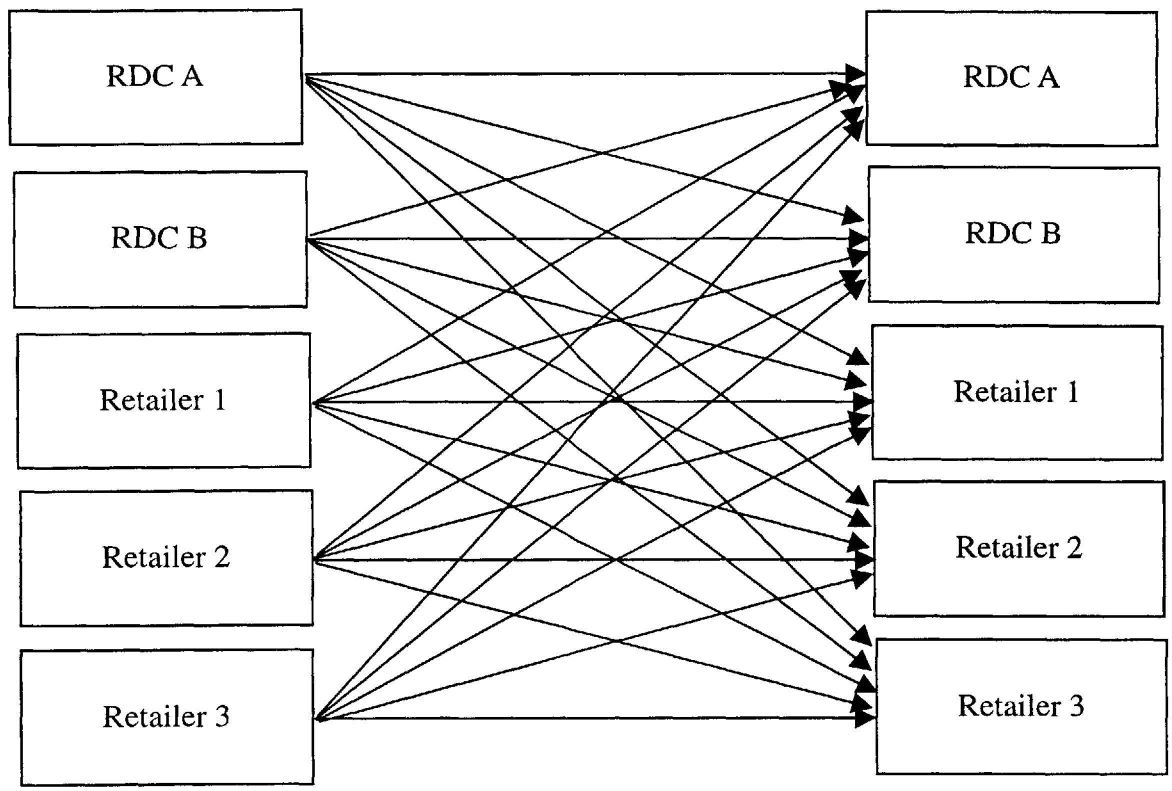
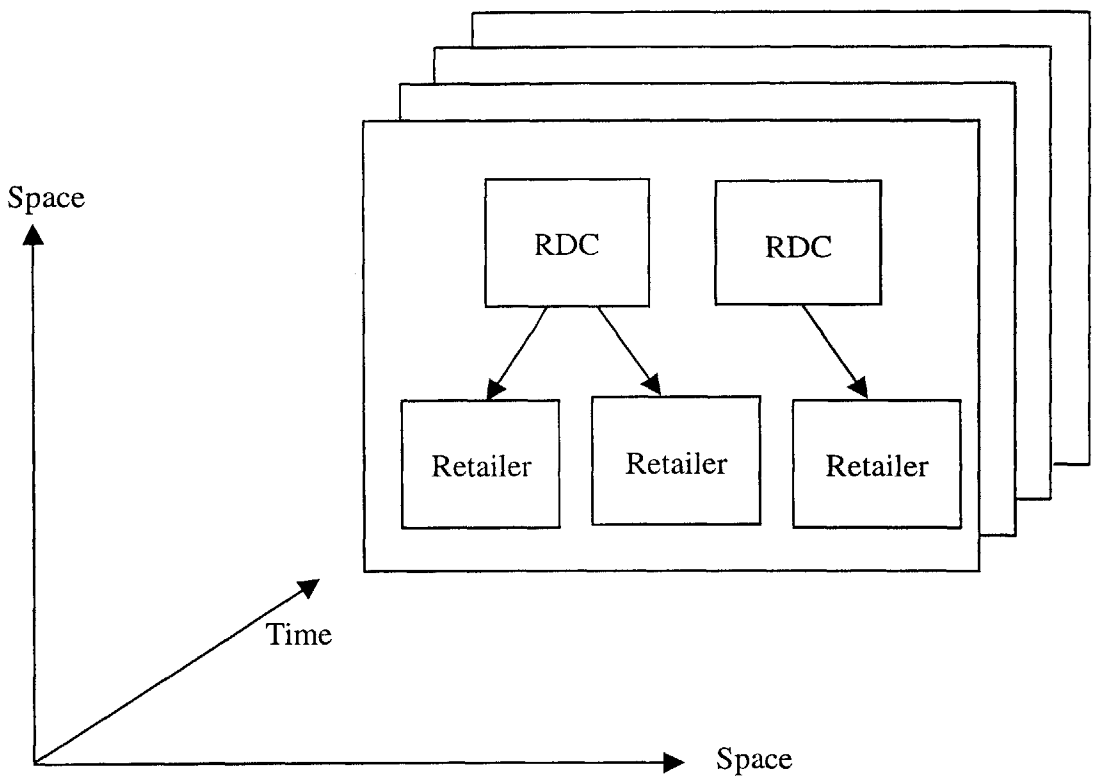


**Figure 8. Multi-Echelon Inventory System Considering Inventory Transfer in Different Time Periods**




The optimized solution is recalculated at the time when total the location demands can not be fulfilled from routinely planned stock flows. In this section, we would like to provide more flexible choices for managers in the multi-echelon network. First thing we can immediately change is the transfer routes within the network. Unlike Figure 6, Figure 7 shows there are no constraints on locations to prohibit transfer to another location. The new routes may cost higher freight rate than the planed routine routes which are depicted in Figure 6. The reason to include more transfer routes in Figure 7 is that we will make a comparison on the cost associated with transportation, backorder, and lost sale when we try to minimize the multi-echelon network cost. At the location facing stock shortage, the most expensive expediting rate may still cost less than a lost sale or backorder. On the other hand, if we provide more flexibility in the solution options, the complexity of the problem will also increase.

Figure 1 demonstrates the relationship of how a finished goods network connects with its manufacturers and consumers. A multi-echelon network receives or forecasts its consumers' demand at the locations at the lowest echelon where these locations and retailers directly face the consumers. For these known demands, the lowest echelon transmits this future demand information to the highest echelon via the information system. Usually, the locations of the highest echelon are national central warehouses, central distribution centers, or the manufacturers' warehouses. The highest echelon cooperates with manufacturers to schedule the transportation date and amount according to the manufacturers' lead-time. When the consumers' future demands are known and the manufacturers' production plans have been provided, the management of this network then determines the stockage level at each location for these future periods. The managers in the multi-echelon network will decide how these demands then can be fulfilled by the network's stock based on pursuing the minimum network cost. The information transmission processes and response mechanisms become important issues when a business tries to obtain competitive advantage through providing high customer satisfaction.

When we begin to optimize the multi-echelon inventory, we will make assumptions that these consumers' demands and manufacturers' transportation schedules are now known and precise. In addition, the beginning inventory level in the first period at each location will be given. Those exogenous data requirements are the data used to optimize the network system flows. The notations that will be used in the optimization formulations are as follows:

## COST NOTATIONS

$CB_{j,t}$ Backorder Cost at location j, period t (dollars/unit/period);

$CH_{j,t}$ Holding Cost at location j, period t (dollars/unit/period);

$CL_{j,t}$ Lost Sale Cost at location j, period t (dollars/unit);

$CT_{i,j,s,t}$ Regular shipment cost from supply location i to receipt location j and from origin period s to destination period t (dollars/unit);

$CX_{i,j,s,t}$ Expediting shipment Cost from supply location i to receipt location j and from origin period s to destination period t (dollars/unit);

## VARIABLES NOTATIONS

### Decision variables:

$B_{j,t}$ Backorder amount at location j, period t (unit);

$L_{j,t}$ Lost Sale amount at location j, period t (unit);

$T_{i,j,s,t}$ Regular shipment amount from supply location i to receipt location j originating in period s to arrive in period t (unit);

$X_{i,j,s,t}$ Expedited shipment amount from supply location i to receipt location j originating in period s to arrive in period t (unit);

**Auxiliary variables:**

$E_{j,t}$ Ending inventory at location j, period t (unit);

$I_{j,t}$ Beginning inventory at location j, period t (unit);

$D_{j,t}$ Demand amount at location j, period t;

$R_{j,t}$ Planned receipt amount at location j, period t (unit);

**Data:**

$RST_{i,j}$ Regular shipping time from location i to j;

$EST_{i,j}$ Expedited shipping time from location i to j;

**MODEL**

**Objective function**

**Min**

$$\sum_{t=1}^{n}\sum_{s=1}^{n}\sum_{j=1}^{m}\sum_{i=1}^{m}\left(CT_{i,j,s,t} * T_{i,j,s,t} + CX_{i,j,s,t} * X_{i,j,s,t}\right) + \sum_{t=1}^{n}\sum_{j=1}^{m}\left(CB_{j,t} * B_{j,t} + CL_{j,t} * L_{j,t} + CH_{j,t} * E_{j,t}\right) \quad (5)$$

**Subject to**

$$E_{j,t} = I_{j,t} + R_{j,t} - D_{j,t} - B_{j,t-1} + B_{j,t} + L_{j,t} \quad (6)$$

for all j, t;

$$R_{j,t} = \sum_{s=1}^{t}\sum_{i=1}^{m} T_{i,j,s,t} + \sum_{s=1}^{t}\sum_{i=1}^{m} X_{i,j,s,t} \quad (7)$$

for all j $\neq$ the highest echelon locations; for all t;

$R_{j,t}$ = data or given (8)

for all j = the highest echelon locations; for all t;

$$D_{i,s} = \sum_{t=s}^{n}\sum_{j=1}^{m} T_{i,j,s,t} + \sum_{t=s}^{n}\sum_{j=1}^{m} X_{i,j,s,t} \quad (9)$$

for all i $\neq$ the lowest echelon locations; for all s;

$D_{i,s}$ = data or given (10)

for all j = the lowest echelon locations; for all s;

$$I_{j,t} = E_{j,t-1} \quad (11)$$

for all j, t > 1;

$I_{j,1}$ = data or given (12)

for all j, the beginning inventory of all locations in the first period;

$T_{i,j,s,t} = 0$ if s-t$\neq$ $RST_{i,j}$ (13)

for all i, j, s, t

$X_{i,j,s,t} = 0$ if s-t$\neq$ $EST_{i,j}$ (14)

for all i, j, s, t

$B_{j,t}$, $L_{j,t}$, $T_{i,j,s,t}$, $X_{i,j,s,t} \geq 0$

locations i = j = 1, 2, 3, ….m;

period s = t = 1, 2, 3, …..n;

There are some specific variable values that must be known when we establish the formulation:

$RST_{i,j}$ the regular transshipment time required for shipment from origin i to destination j;

$EST_{i,j}$ the expedited transshipment times for shipment from origin i to destination j;

$I_{j,1}$ the beginning inventory at all locations in the first period;

$R_{j,t}$ planned receipt amounts at the highest echelon locations for all t;

$D_{j,t}$ demand amount at the lowest echelon locations for all t;

We will interpret the formula of which equations are used to solve the multi-echelon inventory optimization problems.

In the objective function equation (5), the objective function is to minimize the costs that are composed by five components: the regular transfer cost, the expediting costs, the backorder cost, the lost sale cost, and the holding costs.

For solving the optimized cost for the multi-echelon network, there are several constraints that define the boundaries and relationships of the variables with each other. As a consequence, from defining the variables of the objective function, the boundaries of these variables have to be defined by each constraint.

Equation (6) describes the transactions at each location in each period of time. For example, the Ending inventory, $E_{j,t}$, at location j in the period of t is equal to

- the Beginning inventory at location j in the period of t, plus
- the Received shipments at location j in the period of t, minus
- the Demand amount at location j in the period of t, minus

- the Backorder amount form the prior period, plus
- the Backorder amount for this period time plus
- the Lost Sale amount for this period time.

Equation (7) is the constraint for receipt amount. The amount received at location j in the period of t is equal to the sum of regular and expedited shipments from any locations to j, from period one through period t. Here is a note for Equation (7), the t could be equal to 1 which means the same period transshipment by either regular or expedited shipment. Besides, the origin i and destination j could be the same location at different periods which means the location holds the inventory till another period. Equation (8) is the known value which is the planned receipt amount for the highest echelon locations throughout the m periods. These are fixed and known as input data.

Equation (9) is the constraint for demands or requirements. The "demand" occurring at location i in the period s is equal to the sum of regular and expedited shipment from location i to any other locations beginning at the period of t to arrive in period t or later. This equation has the same characteristic with Equation (7). Namely, the origin location could be the same as destination location and the origin shipping time could be the same with destination time while each means holding inventory till future period and same period shipment. In addition, Equation (10) is the known value which is the customer demand amount at the lowest echelon locations for each period, which is assumed to be known.

Equation (11) is the constraint for beginning inventory. Beginning inventory at location j in the period t is equal to the ending inventory of the last period, except that the beginning inventory at the first period of time, which we assume to be known, as in

Equation (12).

Equation (13) and (14) are the constraints of shipping time. Every origin/destination pair has one regular shipment time and one expedited shipment time. This is a reasonable assumption because the geographical spread on multi-echelon network location. The transshipment time will be varied according to the distance on origin/destination pair. If there are more than two ways of shipment, the variables for shipment may be increased.

Finally, we set the constraints that all variables are non-negative real variables or integer variables. Moreover, all variables are set to be integer value.

## 4 Discussion

For more clarification, there is one simple example of the many situations that could happen at a single Retailer 2 location in Figure 6. This finished good has an 8-week lead-time for DRP model and there is a 12-week lead-time for the manufacturer site to schedule its MRP model. Once the feasible DRP table is made, the production order will be fixed and cannot be changed when the order has been released. The Table One in Figure 9 is the scheduled DRP model with an 8-week period usage. The order has been released to the manufacturer for a 12-week period and the receipts within the next 8-week period have been fixed and cannot change. At this moment, there are two situations could happen when demand is increasing; one situation is presented in Table Two and the other situation is presented on Table Three in Figure 9. In contrast to Table One, the customers' demand in Table Two in period 3 suddenly increases to 35 in period 3. Changing the Period Usage cell from 25 to 35 in period 3 of Table Two will also change the following periods' stockage level. But the amount of demand fluctuation in Table Two is smaller than the safety stock at Retailer 2, so the whole multi-echelon network will keep the same solution and there will not be any need to expedite inventory from other locations. Once the numbers have been changed in period 3, all the following periods have to be adjusted justified based on the availability of the stockage. If there is any unexpected increasing demand on the following periods such as period 4, this is the time we may apply the methodology we have developed in section 3.3 because of the shortage of inventory. Table Three follows the algorithm from Table Two and depicts the demand increases in period 4 from 25 to 30 units. The change in period 4 also revises the following periods which show there is now insufficient inventory at period 4. We can discover that several replenishments will arrive later and these replenishments will eventually cover the shortage of the period 4, but the customers will have to wait. At this moment, the manager in Retailer 2 has to decide whether to expedite the order from other locations, or to backorder from the network, or to lose this sale. The model in section 3.3 can help the manager to make a quick decision about how to fulfill the demand.

This example seems very straightforward and easy. However, this example demonstrates one item only. Whereas the managers in a retail store have to deal with hundred or thousands of SKU, and if there are many echelons as well as many retail stores, the management of SKU will become far more complex than a manpower can solve. This quick response methodology will help them save time and make the right decisions. The methodology not only benefit managers, it also helps the business save time and money on expediting shipments costs and shortages costs caused by unexpected demand fluctuations.

When there is more than one SKU to manage, there can often be a reduction in the transportation costs whose more than one finished good from the same supplier can be consolidated into the same trucks. This consolidated transshipment can provide a cost saving benefit for the whole system but it will increase the complexity of the calculation. To solve the problem with varieties of shipment combinations, we could introduce new transportation costs associated with each kind of SKU combination and replace the old transportation cost with the new blended item transportation cost. As a consequence, the maximum cost saving situation might be a shipment by less than truck load rather than as a full truck load.

**Figure 9. DRP Tables on Retailer B**

TABLE One

**Retailer 2**

Q= *30* SS= *10* LT= *1*

| | Now | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Period Usage | | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 |
| Gross Requirement | | 35 | 35 | 35 | 35 | 35 | 35 | 35 | 35 |
| Begin Inventory | | 20 | 25 | 30 | 35 | 10 | 15 | 20 | 25 |
| Schedule Receipt | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Net Requirement | | 15 | 10 | 5 | 0 | 25 | 20 | 15 | 10 |
| Plan Receipt | | 30 | 30 | 30 | 0 | 30 | 30 | 30 | 30 |
| End Inventory | 20 | 25 | 30 | 35 | 10 | 15 | 20 | 25 | 30 |
| Planned Order Release | | 30 | 30 | 0 | 30 | 30 | 30 | 30 | 0 |

TABLE Two

**Retailer 2**

Q= *30* SS= *10* LT= *1*

| | Now | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Period Usage | | 25 | 25 | 35 | 25 | 25 | 25 | 25 | 25 |
| Gross Requirement | | 35 | 35 | 45 | 35 | 35 | 35 | 35 | 35 |
| Begin Inventory | | 20 | 25 | 30 | 25 | 0 | 5 | 10 | 15 |
| Schedule Receipt | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Net Requirement | | 15 | 10 | 15 | 10 | 35 | 30 | 25 | 20 |
| Plan Receipt | | 30 | 30 | 30 | 0 | 30 | 30 | 30 | 30 |
| End Inventory | 20 | 25 | 30 | 25 | 0 | 5 | 10 | 15 | 20 |
| Planned Order Release | | 30 | 30 | 0 | 30 | 30 | 30 | 30 | 0 |

TABLE Three

**Retailer 2**

Q= *30* SS= *10* LT= *1*

| | Now | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Period Usage | | 25 | 25 | 35 | 30 | 25 | 25 | 25 | 25 |
| Gross Requirement | | 35 | 35 | 45 | 40 | 35 | 35 | 35 | 35 |
| Begin Inventory | | 20 | 25 | 30 | 25 | -5 | 0 | 5 | 10 |
| Schedule Receipt | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Net Requirement | | 15 | 10 | 15 | 15 | 40 | 35 | 30 | 25 |
| Plan Receipt | | 30 | 30 | 30 | 0 | 30 | 30 | 30 | 30 |
| End Inventory | 20 | 25 | 30 | 25 | -5 | 0 | 5 | 10 | 15 |
| Planned Order Release | | 30 | 30 | 0 | 30 | 30 | 30 | 30 | 0 |

## 5 Conclusion and Recommendation

This paper has developed a methodology that provides a procedure to help managers work within the supply chain. These managers can closely collaborate with each other by implementing this methodology which provides a series of benefits, including a DRP scheduling, a minimized cost strategy from a system point of view, and an easy planning tool that responds to changing conditions. This model is based on DRP whose planed receipts and demand amounts are known and fixed. From the given information, a firm using a multi-echelon inventory network and its partners can pursue the minimum cost based strategy considering transportation costs (both the regular transfer cost and expediting costs), backorder costs, lost sale costs, and holding costs. In addition, the model can easily be modified when there is any demand or receipt fluctuation, and thus can keep the optimal plan up-to-date. In addition to obtain the minimum cost, the management can use sensitivity analysis to determine which shipping tactics the best portfolio to fit the market and inventory strategies.

The methodology and procedures are developed in an attempt to help build a framework which can be practically used in multi-echelon inventory deployment, especially in a finished goods network. This multi-echelon inventory deployment optimization methodology can be applied to a variety of industries, including auto parts, appliances, electronics and apparel. Further research may needed to investigate the collaboration with finished goods providers and demand forecast structures. The finished goods providers may include manufacturers, plants, or third party logistics. The research on demand forecast structures could provide a more precise forecasting methodology that more accurately reflects the consumer pattern. In order to pursue high performed collaboration of supply chain operations, these further studies could be integrated into a supply chain optimization methodology which provides a core decision tool to the business world.

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