

ESTIMATING
POTENTIAL EVAPOTRANSPIRATION

by

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Abstract

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Water balance studies require knowledge of evapotranspiration, which as a counterpart of precipitation determines the water resources of a region. Evapotranspiration is difficult to measure and indirect procedures through correlations with meteorological factors have been generally used for estimations. This latter procedure resorts to estimates of potential evapotranspiration, an assumed condition of no water deficit, and these values reduced on the basis of available moisture to indicate the actual evapotranspiration.

The requirement of obtainable data has necessitated the use of temperature as an index to potential evapotranspiration. Available methods require complicated computational procedures or the applications of corrective factors that vary with season and location. Investigations in this

study led to the development of a simple procedure to estimate potential evapotranspiration, utilizing the saturated water vapor concentration at the mean temperature adjusted by a daylength factor squared. The daylength factor operates to account for plant response, duration of turbulence, and net radiation. General applicability seems justified in view of the obtained correspondence between observed and computed values of potential evapotranspiration both on a yearly and seasonal basis for a number of widely displaced areas.

TABLE OF CONTENTS

	Page
Title Page	
Abstract.....	i
Table of Contents.....	iii
List of Tables.....	iv
List of Figures.....	v
I. INTRODUCTION.....	1
II. EVAPORATION	
The Evaporation Process.....	4
Measurement of Evaporation.....	7
Methods to Compute Evaporation.....	10
III. EVAPOTRANSPIRATION	
Elements of Evapotranspiration.....	17
Measurement of Evapotranspiration.....	22
Indirect Methods of Estimation.....	23
IV. DEVELOPMENT OF METHOD TO ESTIMATE POTENTIAL EVAPOTRANSPIRATION.....	27
V. EVALUATION.....	39
VI. CONCLUSIONS.....	57
Bibliography.....	58
Appendix.....	66

LIST OF TABLES

		Page
Table 1.	Computed average annual potential evapotranspiration as compared with the Thornthwaite method.	38
Table 2.	Data of observed water loss and computed average potential evapotranspiration for Fig. 2.	41
Table 3.	Computed average daily lake evaporation and average daily potential evapotranspiration for Hartford, Conn. (Bradley Field).	46
Table 4.	Computed average monthly potential evapotranspiration for Asheville, North Carolina, by several available methods.	48
(Appendix)		
Table I.	Mean possible duration of sunlight in units of 12 hours.	66
Table II.	Daylength factor. Square of the mean 12-hour units of possible sunshine.	67
Table III.	Vapor concentration (absolute humidity) in saturated air.	68

LIST OF FIGURES

	Page
Figure 1. Net radiation as a function of temperature and daylength.	31
Figure 2. Consumptive use for basins compared with computed potential evapotranspiration.	40
Figure 3. Seasonal distribution of consumptive use and potential evapotranspiration.	40
Figure 4. Potential evapotranspiration from water balance for basins.	43
Figure 5. Evapotranspirometer measurement of evapotranspiration and computed potential evapotranspiration.	44
Figure 6. Yearly water loss, in inches, obtained from precipitation and runoff.	49
Figure 7. Computed yearly lake evaporation in inches.	50
Figure 8. Computed average annual potential evapotranspiration in inches.	51
Figure 9. Computed average annual potential evapotranspiration in inches.	53
Figure 10. Computed average (January) potential evapotranspiration in inches.	54
Figure 11. Computed average (July) potential evapotranspiration in inches.	55

I. INTRODUCTION

The transfer of moisture from the earth's surface back to the atmosphere, through the process of evaporation from water, soil and snow surfaces and of transpiration from plants is called "evapotranspiration". This process serves to replenish the moisture in the atmosphere that precipitation extracts and to equalize temperatures. Large quantities of solar energy are consumed in the conversion of water into vapor and this heat is then exchanged back to the atmosphere wherever condensation occurs.

Evaporation from a free-water or a wet surface proceeds in accordance with the combined effects of net radiation, temperature of the evaporating surface and of the air, humidity, and wind. The evapotranspiration from land-areas is dependent upon the available soil moisture, in addition to the meteorological factors, with the type of vegetation and soils further altering the response to evaporation opportunity.

Measurement of evaporation from small exposures, such as pans, is easily accomplished, and for a particular locality satisfactory estimates of evaporation from lakes and reservoirs are obtained through use of appropriate conversion factors. For localities where evaporation is

not measured, empirical and analytical methods based on meteorological factors may be employed, depending on available data. In contrast, the measurement of evapotranspiration is extremely difficult due to the lack of available water under natural conditions and the effects of plants and soil. To avert the complications of deficit water in attempting to compute evapotranspiration, Thornthwaite (14)* introduced the concept of potential evapotranspiration which assumes a condition of adequate moisture supply.

Temperature has been used as an index to potential evapotranspiration on the basis of its conservativeness and because a fixed relation exists between net radiation used for heating the air and that used in evaporation under potential conditions. Estimates of actual evapotranspiration are obtained by applying adjustment factors based on soil moisture to the indicated potential evapotranspiration.

Present methods of estimating potential evapotranspiration either require generally unavailable meteorological data; complicated procedures when utilizing temperature as an index; or coefficients that are highly variable with season and location. The task set forth in this study is

* Numbers in parenthesis refer to bibliography.

that of examining procedures for the estimation of evapotranspiration and through the consideration of parameters involved, devise a new procedure that will have general applicability requiring only readily obtainable meteorological data and fashioned such that computations will be straightforward and simple.

The trend of expanded utilization of water resources must surely accentuate studies of the water balance over large areas for such apparent purposes as drought evaluation, irrigation feasibility and control, runoff of water into streams and rivers, flux of moisture between areas, soil shrinkage due to excessive drying, moisture infiltration, soil tractionability and the estimation of agricultural productivity as related to climate.

II. EVAPORATION

The Evaporation Process

The term "evaporation" is generally accepted by engineers and hydrologists as representing the loss of water, in excess of condensation, to the atmosphere from water surfaces, snow and soil. Water transferred to the atmosphere by vegetation is considered as resulting from transpiration, although the same phase change (water to vapor) is involved. The combined evaporation from all surfaces is termed total evaporation or evapotranspiration over a vegetated area.

A plausible explanation of evaporation is to be found in the kinetic theory of matter which requires that all matter be composed of molecules in motion when the temperature is above absolute zero, -273°C . As the temperature of water is increased, the mean motion of the molecules becomes faster and some acquire sufficient energy to escape into the adjacent air. The molecules of water vapor in the air are also in constant motion, and some penetrate the water surface and remain within the liquid. The change in state from liquid to vapor is called vaporization; and the reverse process, condensation.

The continued transfer of water vapor to the surrounding air requires a condition which favors the escape of molecules from the surface of the water to form vapor in excess over those returning and a mechanism for the removal of the vapor from the area. Those environmental factors that contribute to such favorable conditions are: wind, radiation, air and water temperatures, water vapor in the air adjacent to the saturated layer at the water surface, atmospheric pressure, and quality of the water. Since solar radiation is an important factor, evaporation varies with latitude, season, time of day, and sky conditions. Although these controlling factors are known, the determination of their relative effectiveness is difficult due to the interrelations.

Solar radiation is the ultimate source of heat with portions used in evaporation, heating of the earth's surface and air - mostly indirect - and in driving the atmospheric motions. Radiation takes a prominent place in methods to estimate evaporation by the heat-balance method. The mean temperature of an area, as we shall see later, must be adjusted to indicate the net radiation.

The exact role of temperature has not been firmly established, but it is known that the emission of molecules

from liquid water is a function of temperature - the higher the temperature, the greater the rate of emission. Due to the interrelation between temperature and other climatic factors, it has been used in many empirical relations to represent evaporation and, particularly, evapotranspiration.

The effect of vapor pressure upon evaporation was first formulated by Dalton in 1802. He stated that the rate of evaporation from a free water surface, other factors being constant, depends upon the difference between the vapor pressure at saturation for the temperature of the water and the vapor pressure actually existing in the air above the water, expressed as $(e_s - e_a)$. This principle has been adopted by most investigators. There is evidence, however, that temperature enters this vapor pressure difference relation. Hinus (17) found the evaporation to be proportional to $(e_s - e_a)^{0.83}$ which is proportional to the difference in vapor concentration rather than vapor pressure difference. Another empirical equation (21) based on data from evaporation pans at a number of stations resulted in a proportionality of evaporation and $(e_s - e_a)^{0.88}$. Millar (31) found for constant wind conditions that evaporation was proportional to the dif-

ference in vapor concentration or $(\frac{e_s}{T_s} - \frac{e_a}{T_a})$, where T_s and T_a are absolute temperatures of the water and air, respectively.

In view of the kinetic theory of heat, wind becomes an indirect cause of evaporation. Turbulence increases with the wind speed and therefore evaporation, since turbulence acts to remove from the water surface the moisture-laden air and replace it by drier air.

Other factors play a minor role in controlling evaporation. The most significant is the effect of salinity and in sea water this may reduce the evaporation by 2 to 3 percent.

Measurement of Evaporation

The direct approach to the determination of evaporation from a water surface, such as lakes and reservoirs, would be to obtain the residual from inflow, outflow, precipitation, and seepage. Such a water budget determination though can rarely be used since seepage cannot be determined. Indirect methods are generally used which measure the evaporative power of the environment as water lost from some retaining instrument rather than the true evaporation. Such evaporation is accentuated markedly in a dry climate.

Instruments used for measuring the rate of the apparent evaporation may take the form of tanks or pans, porous porcelain bodies, and wet paper surface. Such instruments are commonly called atmometers.

The use of evaporation pans has gained the greatest prominence as an indicator of evaporation. Of the various sizes and shapes employed, the Weather Bureau Class A land pan has been most widely used. This is a four-foot diameter pan, 10 inches deep, and exposed 6 inches above ground, with the water maintained within 2 - 3 inches of the top. The Lake Hefner Study (22) has related the evaporation as obtained from different pans to that of the Class A pan and also pan evaporation to lake evaporation. In this study, the lake evaporation was found to be 0.7 that of the Class A pan. For individual months, the conversion factor or pan coefficient showed an extensive swing from low values in late winter to high values in late fall. When 3-month averages are taken of the observed coefficients, a sinusoidal change from about 0.4 in February and March to that of near 1.0 in October and November resulted. The yearly pan coefficient is usually considered to range from 0.6 to 0.8 but a much greater variance has been observed. Reports (29) indicate

that the actual evaporation for a 3-year period from the Salton Sea (California) was exceeded by more than two and one-half times by an inland ground pan.

A principal cause for the seasonal variation in the pan coefficient is the difference in the heat storage. Heat received at the surface of large water bodies during the warm season serves to warm the water to considerable depths. This stored heat provides additional energy for evaporation during the cold season. The pan, in contrast, has a small volume of water and the evaporation is more in line with heat supply.

The yearly pan coefficient and the seasonal variance from lake evaporation is much more conservative in humid areas than in dry regions. Radiational differences between the pan and natural waters in humid climates account for the major increase in pan evaporation over lake evaporation. Where the surrounding area is relatively moist, advected heat to the pan in contrast to that over lakes is not too different. In dry climates, this is not the case, since a much larger portion of the net radiation is diverted to heating the air which becomes available by advection and turbulence for increased evaporation. Also, the moisture

deficit in this advected air tends to accentuate the evaporation. In areas of high humidity, the size of the evaporating area is of minor importance, whereas under low humidity conditions the evaporation varies inversely with the area of the evaporating surface.

Methods to Compute Evaporation

For each gram of water evaporated, there must be supplied around 590 calories of heat. This quantity of heat must be supplied by radiation and conduction from the overlying air or at the expense of energy stored below the surface. Therefore, if a strict accounting could be made of all the energy transfers from and to a water surface, the evaporation could be determined. Such a procedure, known as the energy-budget method, like the water budget, employs a continuity equation and obtains evaporation as the residual required to maintain a balance.

The Lake Hefner Study (22) investigated the use of the energy-budget for determining lake evaporation. The balance is expressed by

$$Q_s - Q_r - Q_b - Q_h - Q_e - Q_o - Q_v$$

where Q_s is sun and sky radiation incident at the water

surface; Q_r , reflected short-wave radiation; Q_b , net energy lost by the water body through exchange of long-wave radiation with the atmosphere; Q_h , sensible-heat transfer (conduction) to the atmosphere; Q_e , energy used for evaporation; Q_o , the increase in energy stored in the water body; Q_v , net energy advected into the water body - all in calories per square centimeter. Letting H_v represent the latent heat of vaporization and R the ratio of heat loss by conduction to heat loss by evaporation (Bowen ratio), the preceding relation becomes

$$E = \frac{Q_s - Q_r - Q_b - Q_v - Q_o}{PH_v (1 + R)}$$

where E is the evaporation in centimeters and P is the density of water. The Bowen ratio (5) can be computed from the equation

$$R = 0.61 \frac{T_s - T_a}{e_s - e_a} \frac{p}{100}$$

where p is the atmospheric pressure; T_a and e_a , the temperature and vapor pressure of the air, respectively; T_s , the water surface temperature; e_s , the saturation vapor pressure corresponding to T_s ; and all temperatures and pressures are in degrees centigrade and millibars.

The mass-transfer approach to the determination of reservoir evaporation has received considerable attention.

Such methods as advanced by Prandtl, Schmidt and also of Taylor have been tested at Lake Hefner (22) and Lake Meade (23). Tests of the equations derived by Sverdrup and Sutton gave good results at Lake Hefner but were considered inadequate when applied to Lake Meade. Authorities (27) state that there is reason to believe that the Thornthwaite-Holzman equation (41) would give satisfactory results with instrumentation meeting the exacting requirements. This method depends upon the vertical gradient of water vapor in the atmosphere and the coefficient of turbulent exchange. Rate of change in state is assumed to equal the movement of moisture vapor from the surface.

The differential equation for the rate of evaporation on the above basis is

$$E = A \frac{dq}{dh}$$

in which A is the coefficient of turbulent exchange and $\frac{dq}{dh}$ is the gradient of vapor concentration with respect to altitude. Researches by von Karman and Rossby have provided means of evaluating the coefficients and arriving at a formula that can be used with observational data. Assuming an adiabatic atmosphere and logarithmic distribution of wind speed and moisture in the vertical, the

derived equation (41) is

$$E = \frac{833 K^2 (e_1 - e_2) (V_1 - V_2)}{(T - 459.4) \log_e \left(\frac{Z_2}{Z_1}\right)^2}$$

where E is the evaporation in inches per hour; K, von Karman's constant (0.4); e, the vapor pressure in inches of mercury; V₁, the wind speed in miles per hour; and T, the mean temperature in degrees Fahrenheit of the layer between the lower Z₁ and the upper level Z₂.

Numerous empirical formulas have been derived which express evaporation as a function of atmospheric elements and which are similar in some respects to the mass-transfer approach. Such a formula, depending on vapor pressure differential and wind is that of Meyer (30) in which

$$E = C(e_s - e_a) \left(1 + \frac{V}{10}\right)$$

where e_s and e_a are the vapor pressure of the water surface and overrunning air in inches of mercury, and V is the wind speed in miles per hour. The coefficient C has a value of about 0.36 when the formula is applied to daily data for an ordinary lake, provided the wind and humidity measurements are about 25 feet above the surface.

Pan evaporation can be satisfactorily estimated from

such empirical formulas from knowledge of wind, dew point, and water temperature with proper coefficients to get yearly lake evaporation. In most cases where water temperature is available the actual pan evaporation is also observed so that this method is limited.

By assuming the changes in heat storage of the water-body and the heat conducted through the container walls to be negligible, Penman (36) derived the equation

$$E = \frac{dQ_n + rE_a}{d + r}$$

where d is the slope of the saturation-vapor-pressure versus temperature curve at the air temperature T_a ; E_a , the evaporation given by an empirical mass-transfer equation, assuming water temperature T_s to equal to T_a ; Q_n , the net radiant-energy exchange expressed in the same units as evaporation E ; and r , defined by the Bowen ratio equation

$$R = r \frac{T_s - T_a}{e_s - e_a}$$

This approach has been correlated for pan data in the United States (21) and the value of r found to be about two-and-one-half times the theoretical value. Such a difference is due mainly to advected energy and sensible heat transfer across the pan walls. An empirical relation

(27) is available to estimate the heat transfer through the pan from observations of air and water temperatures, wind movement, and atmospheric pressure. Adjustments are still needed for any net heat advection.

As an outgrowth of the Lake Hefner Study and by utilizing the Penman equation, the concept of a "theoretical" pan (27) has been utilized to estimate lake evaporation. This considers the use of the theoretical value of r in the Bowen ratio and a conversion coefficient from pan to lake by assuming negligible net advection and sensible heat transfer through the pan. This equation is

$$E = 0.7 \frac{dQ_n + rE_a}{d + r}$$

where E is the daily lake evaporation in inches; d , the slope of the saturation-vapor-pressure in inches of mercury versus the temperature curve at the air temperature in degrees Fahrenheit; Q_n , the net radiation, usually obtained in langley's per day and converted to inches by division of 1500 cal. per in. per sq. cm.; r , a constant of .0105; and E_a obtained from the relation

$$E_a = (e_s - e_a)^{0.88} (.37 + .0041W)$$

where E_a is an estimate of pan evaporation; e_s and e_a , the

vapor pressures in inches of mercury for the condition of saturation at the air temperature (standard instrument shelter) and the actual vapor pressure, respectively. The daily wind movement, W , is that at the 2-foot level.

A graphical solution for the theoretical pan equation as discussed above is available (27). It is considered that the conversion coefficient of 0.7 for conversion of such computed pan evaporation to lake evaporation is, for practical adaptability, a constant.

III. EVAPOTRANSPIRATION

Elements of Evapotranspiration

The term "consumptive use" has been adopted from usage in the western portion of the United States in connection with irrigation studies and was originally intended to mean water lost by evaporation from cultivated soil and by transpiration from crops. This meaning has been extended to include similar losses from land and natural vegetation. Other terms used synonymously with consumptive use are "water loss" and "evapotranspiration". The term "evapotranspiration" is now most commonly used.

The rate of evaporation from a saturated soil is approximately the same for a free water surface of the same temperature (27). In the case of soil evaporation, as the soil begins to dry, the evaporation decreases and its temperature rises to maintain the heat balance. Without replenishment of soil water, evaporation will eventually cease when the soil is unable to transport moisture to the surface. Therefore, the rate of evaporation from soil surfaces is limited by the availability of water or the evaporation opportunity.

Direct evaporation from land areas also occurs from

water that is intercepted by the vegetation but the principal mechanism by which water is transferred to the atmosphere is through the process of transpiration. The small amount retained by plants is of negligible quantity.

The same meteorological factors that control evaporation from water surfaces are effective in evapotranspiration and in addition the physiological factors become important since transpiration is now the main mechanism involved in water loss.

Radiation still retains its prime importance as the energy source for an area but transpiration and plant growth depend more directly upon the temperature.

Water is principally transferred to the atmosphere through the stomata of the plant leaves which have the characteristic of closing at night. Therefore, transpiration is directly related to daylength. Thus, while 75 to 90 percent of daily soil evaporation occurs between sunrise and sunset (24), about 95 percent of daily transpiration occurs during the daylight hours (25).

New information as to the effect of radiation on transpiration, other than a heat source, may be gained from the illuminating work by Wald (48). He has demon-

strated that chlorophyll - the pigment universally used in photosynthesis - absorbs heat at the ends of the spectrum of sunlight, where energy falls off steeply from the maximum around the middle of the spectrum. Since the action of chlorophyll depends on light and not on other radiant energy, daylength acquires special significance in plant growth. On the other hand, plant growth is proportional to transpiration as water is a necessary constituent in the photosynthetic process. The essence of the photosynthetic process is the use of the energy of light to split water. The hydrogen from the water is used to reduce carbon dioxide while oxygen is released to the atmosphere.

Optimum temperature and radiation for maximum growth vary with plant species with all growth ceasing at some minimum temperature, usually a mean temperature of 40 - 43° F. Plant growth is considered to increase proportionally to the van't Hoff principle of chemical activity, doubling for each rise in 10° C or 18° F. Such a growth rate may be approximated from the minimum growth temperature to the optimum growth temperature and then growth diminishes with a further increase in temperature, ceasing at temperature of around 110° F. These temperature points of growth are known as cardinal temperatures.

The response of plant growth to temperature indicates its significant role in transpiration. Plants may exhibit the ability to wilt under conditions of evaporation opportunity that exceeds the functioning of the plant at its temperature. Thus, excess wind and low humidities may cause the plant to conserve moisture.

Transpiration is limited by the availability of water in the soil. Water is transported to the soil surface by capillary action only to depths of 8 inches or so, depending upon the soil, but plant roots extend capillary tubes to depths in the soil. Plants obtain most of their water from the upper one to two feet of soil where the main root system is concentrated but some plant roots, for such plants as alfalfa, may extend many feet into the soil.

The decrease in evapotranspiration in relation to available soil moisture is viewed differently by various investigators. Some (44) feel that water is extracted at a constant rate to the wilting point in the root zone. A widely used method (29) considers that the available energy used in evaporation is proportional to the soil moisture. Another procedure (16, 18, 36) for adjustment of evapotranspiration is one that uses a constant availability of moisture in the top layer of soil, with increased unavailability for moisture at lower depths.

The available water for evapotranspiration is considered to be that moisture in the root zone ranging from field capacity (after excess gravitational water has drained) to the wilting point. The amount varies with soil type but generally ranges from 0.5 inch per foot in sand to 2 inches or more per foot for clay loam.

Methods have been developed for computing evaporation from a water surface with certain assumptions and the heat-budget method is a logical procedure to determine evapotranspiration. This method is of little practical application because of the difficulty in obtaining the necessary observation of sufficient accuracy. To circumvent this obstacle and to determine the evapotranspiration from a land surface where the rate of water loss is dependent on the soil moisture, Thornthwaite (40) introduced the concept of potential evapotranspiration.

Potential evapotranspiration is defined as the evapotranspiration that would occur were there an adequate moisture supply at all times with other conditions satisfied; such as, the albedo of the evaporating surface a standard, rate of evapotranspiration not influenced by the advection

of moist or dry air, and the ratio of the energy utilized in evaporation to that heating the air must remain constant.

Measurement of Evapotranspiration

The measurement of both potential and actual evapotranspiration is extremely difficult. As in the case of lakes or reservoirs, the water-budget method may be utilized for a basis over a period of time where accounting can be made of storage, inflow and outflow, and precipitation measured to estimate the evapotranspiration. More direct but often misleading determinations are made from soil containers, variously known as lysimeters (16) or evapotranspirometers (29). Various types of vegetation and soil may be placed in the containers upon a sand and gravel subsoil and placed so that the soil surface is level with the surrounding. Water may be admitted to the gravel stratum at the bottom of the pan or added to the surface in quantities to produce percolation. A water balance of the tank reveals the evapotranspiration. This method is analogous to the evaporating pan with free water surface, with added soil and plant variables. The greatest defect is the lack of similarity between the evapotranspirometer and natural conditions in the field. An indication of potential evapotranspiration may be obtained from such installation when

watered sufficiently to maintain optimum evapotranspiration and no water surplus in the tank, and when exposed to a homogenous plant cover composing a buffer area that also has sufficient available moisture. Operational size areas may practically eliminate the air humidity differential but advected heat may still be substantial from greater distances due to excess heating of the air under deficit soil moisture conditions.

Indirect Methods of Estimation

Early work by Hedkl produced an equation for estimating consumptive use (evapotranspiration) in terms of the heat available for plant growth. Hedkl's equation was

$$U = K Q$$

in which U is the consumptive use in feet; Q, the effective heat of the area expressed in day degrees and taken as the difference between the mean monthly temperature and the germinating temperature for each crop, the difference being multiplied by the number of days in the growing period; and K, the proportionality coefficient.

Using yearly data in the same fashion, Lowry and Johnson (28) found high correlation between consumptive use and accumulated degree days, base of 32° F., during the growing season.

Blaney and Criddle (4) correlated actual measurements of consumptive use with monthly mean temperature and daylength to obtain applicable coefficients for different cropping conditions in the West. The average coefficients vary from around 0.6 for field crops to 1.2 over the growth period for rice when applied to semi-arid and arid conditions. The coefficient for grapefruit in the vicinity of Phoenix, Arizona, ranged from 0.55 in January to 0.75 in August.

A method to estimate potential evapotranspiration has been devised by Thornthwaite (40) by correlating mean monthly temperature with evapotranspiration as determined from the water balance for valleys when sufficient moisture was available. The equation takes the form

$$E = 1.6 \left(\frac{10t}{J} \right)^a$$

where E is the unadjusted monthly evapotranspiration in centimeters; t, the mean monthly temperature in °C.; J, a heat index determined by the summation of the 12 monthly indices (i), where $i = (t/5)^{1.514}$ and the value of a is obtained from

$$a = 0.000000675J^3 - 0.0000771J^2 + 0.01792J + 0.49239.$$

Adjusted values of E are obtained by multiplying by the average monthly daylength expressed as units of 12 hours.

In commenting on the formula, Thornthwaite has stated that its workability requires the use of monograms and that the chief obstacle in developing a rational equation is the lack of understanding of why potential evapotranspiration corresponding to a given temperature is not the same everywhere. Even with the apparent drawback, the equation has been widely used in water balance studies. Reliable estimates on a yearly basis are obtained but it is conceded (29) the formula overestimates in the summer and underestimates in the winter. This is inherent since no evapotranspiration is determined for temperatures below freezing. In effect, the formula is a better estimate of transpiration than for evapotranspiration.

The Penman equation for determining pan evaporation has been widely used in England for evapotranspiration estimates by applying coefficients that range from 0.6 in the winter to 0.8 in the summer. As with converting pan evaporation to lake evaporation, these coefficients will depend on the climate of an area. Von Bavel (45)

has assumed a coefficient of 0.7 for water balance studies to predict agricultural drought in the southeastern states.

Blaney (2) indicates that observations confirm the assumption that evaporation from an open water surface may be used to estimate evapotranspiration by vegetative cover. Others (27) suggest that annual lake evaporation is approximately equal to potential evapotranspiration. Under conditions of unimpeded growth and transpiration of plants, the albedo of the two surfaces, water (.07) and vegetation ranging from .10 for forests to .25 for some crops, would result in evapotranspiration slightly less than that of a free-water surface.

IV. DEVELOPMENT OF METHOD TO ESTIMATE POTENTIAL EVAPOTRANSPIRATION

In seeking a relationship to represent potential evapotranspiration as a function of temperature, due consideration must be given to the variability of temperature and radiation in the climatic and geographic sense. Thornthwaite, in devising his empirical function, found it necessary to develop a heat index based on the mean monthly temperatures at the station for adjustment of the temperature. Since net radiation is the heat source for an area, aside from advected heat, we must obtain a correspondence between temperature and net radiation.

Average incoming solar radiation, as shown in charts prepared by Fritz and MacDonald (12), varies little over the eastern United States during June and July, the peak radiation period. This correspondence is further improved when adjustments are made for variable cloudiness. In a study (15) of latitude and percent of possible sunshine as related to incoming radiation, it was found that incoming radiation for clear skies is a constant quantity at the summer solstice for latitude 25 to 50 degrees north over North America. Average daily temperatures (9), however,

in July over the eastern United States vary from 80 - 85 degrees in the southern states to 65 - 70 degrees over the northern tier states.

Net radiation has not been observed except at scattered localities during recent years. The necessity then exists for an indirect determination of net radiation from the observed value of incoming radiation and other climatic data. Brunt (6) proposed an expression for computing total long-wave nocturnal atmospheric radiation in the form,

$$R/\sigma T^4 = a + be^{\frac{1}{2}}$$

in which R is the total long-wave downcoming atmospheric radiation under a cloudless sky; σT^4 , outgoing black-body radiation at the surface temperature; e, mean monthly local vapor pressure in millibars, and a and b are constants. The net long-wave flux leaving the surface, R_0 , is just, $\sigma T^4 - R$, making

$$R_0 = \sigma T^4(1 - a - b \sqrt{e})$$

Adopting average constants which vary with air mass as listed (13) and the relations presented by Penman (34) which includes a correction for cloudiness, the following

expression for net long-wave radiational loss results

$$R_o = \sigma T^4 (0.5 - .07\sqrt{e}) (0.1 + .9S)$$

In terms of net radiation, R_n , we may write

$$R_n = R_I (1 - r) - \sigma T^4 (.5 - .07\sqrt{e}) (0.1 + 0.9S)$$

where

R_n = Average daily net radiation (Langleys/Day)

R_I = Average daily incoming radiation (Langleys/Day)

r = Reflective coefficient (albedo)

e = Mean local vapor pressure (in millibars)

σ = Stefan-Boltzman constant -

$(8.132 \times 10^{-11}) \times 1440$ (Langleys/ T^4 /Day)

T = Temperature in absolute degrees (Kelvin)

S = Percent of possible clear skies

Values of the albedo have been listed by Houghton (19) and additional data is available (42). In computing the net radiation for stations scattered over the United States, the following albedos for various surfaces were used: grass and crops, .20; forest and bushes, .10; dry sand, .25; wet soil, .09; new snow, .70; old snow, 0.55. In general, the albedo of water is considered as .07.

Computed values of net radiation have been related to the product of the average daylength in units of 12

hours and the temperature, °F. Plotting of the values, all obtained from monthly mean data, demonstrate a linear relation, Fig. 1, approximated by

$$R_n = 5(DT - 27) \quad (\text{Eq. 1})$$

The variability between stations and the seasonal bias, high values of computed net radiation in the Spring and low values in the Fall, must be viewed in the light of average constants in the long-wave relation and the use of vapor pressure and temperature as observed in the instrument shelter. The total moisture content in the air above the station and the surface temperature are the actual factors involved. In the Spring, the earth is relatively cooler with respect to the air than in the Fall, therefore acting as a heat sink. A verification of the linear relation of the product of the day-length factor and temperature with observed monthly net radiation (8) is seen in the insert of Fig. 1 for Raleigh, N. Carolina. Reference to this relation will be made in development of a relation to estimate potential evapotranspiration.

Earlier, the equations for estimating evaporation were noted and the formulas based on meteorological elements

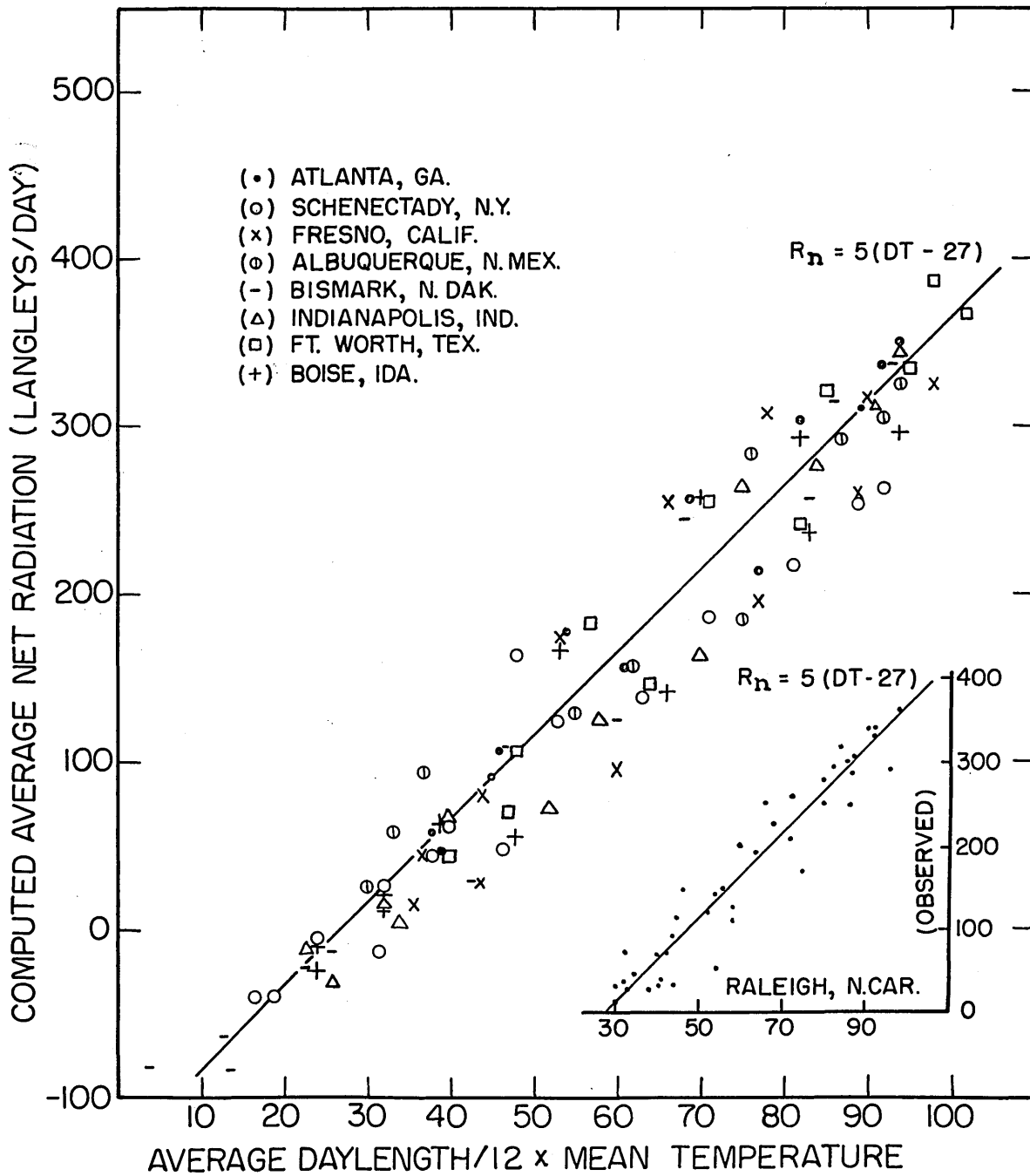


FIGURE I. NET RADIATION AS A FUNCTION OF TEMPERATURE AND DAYLENGTH

vary from the simple expression given by Dalton in 1802, $E = c (e_s - e_d)$, to the rather complex formulas involving the energy-budget and mass-transfer approaches.

A theoretical approach to the climatological estimation of potential evapotranspiration has been made by Halstead (14). He started with the basic equation of mass transfer of water vapor

$$E = -D \frac{dp}{dz}$$

where, E is evaporation, D the coefficient of diffusion of water vapor through air and $\frac{dp}{dz}$ the vertical gradient of the water vapor concentration (absolute humidity).

Assuming that for air in turbulent motion, the coefficient of diffusion is increased with distance from the surface and taking a constant change in water vapor concentration with elevation, the equation reduced to

$$E = -CL(P_1 - P_0)$$

where C represents a turbulence parameter; L, the day-length; P_1 , the water vapor concentration or vapor density at some distance above the surface and P_0 the same quantity at the surface.

To evaluate the effect of a variable turbulence or wind, reference is made to the results of studies by Linsley, Kohler, and Paulhus (27). They show that for a daily wind of 50 to 100 miles per day the percent error in evaporation with percent change in wind speed was of the order of 0.25 percent, or a 25 percent change in the wind speed produces a 6 percent change in evaporation. Meyer (30) has reduced the wind observations over the United States to comparable speeds at an elevation of 25 feet. The mean daily wind at 133 stations east of the Rockies shows an average variation of only 9 percent from the average of the stations. It then appears justifiable to consider the wind as a constant factor in a climatological estimate of potential evapotranspiration.

The daylength term is a proper adjustment since turbulence occurs principally during the daytime in the absence of a temperature inversion and plants restrict most of their transpiration to daylight hours when the stomata are open.

Adhering to the concept of potential evapotranspiration and by assuming constant turbulent air flow under similar lapse rates of temperatures, another logical assumption would be to consider a constant departure in water vapor concentration between the

saturated surface and that at some elevation. In so doing, we may substitute a mean water vapor concentration and for convenience a choice is made of the saturated water vapor concentration at the mean daily temperature. The above assumption, of course, must be applied to a period of time and average value. In estimating potential evapotranspiration, the accumulated value is of main concern where water balance studies are involved. We may now write

$$E_p = KDP_t \quad (\text{Eq. 2})$$

where E_p is the potential evapotranspiration; D , a day-length factor; P_t , the saturated water vapor concentration at the mean temperature; and K , a proportionality coefficient.

As we have noted, any relation which is a function of temperature has neglected the disparity between net radiation and mean temperature as noted earlier. The combination of energy and mass-transfer approaches to estimate pan evaporation was successfully accomplished by Penman (36) and if we use the form of his equation and substitute the derived empirical-climatological estimates for the two approaches from Eq's. 1 and 2,

we obtain

$$E_p = (f) \frac{d \times 5(DT-27) + rKDP_t}{1500 \times d + 5}$$

By comparing with results of the graphical solution of the Penman equation converted to represent lake evaporation (27) for station in eastern United States, we obtain an apparent workable relation from readily available climatic data by choosing $K = 0.01$ and where f is 0.75 rather than 0.7. The division by 1500 of the net radiation term converts the net radiation into equivalent inches of water evaporated. The equation for potential evapotranspiration becomes

$$E_p = .75 \frac{d \times 5(DT-27) + .01 rDP_t}{1500 \times d + r} \quad (\text{Eq. 3})$$

where E_p is the potential evapotranspiration in inches per day; D , the hours of possible daily sunlight in units of 12 hours; T , the mean temperature in $^{\circ}\text{F}$; P_t , the saturated water vapor density in gm/M^3 at the mean temperature; d , the slope of the saturation-vapor-pressure in inches of mercury versus temperature curve at the air temperature T ; r , the theoretical constant of the Bowen ratio of 0.0105.

We now have a relation, Eq. 3, to estimate potential evapotranspiration in term of quantities which are a function of temperature and a daylength factor which varies with time and latitude. Such an adaptation of the Penman equation results in an equation far more complicated than desired for practical use.

In estimating net radiation from temperature, we found that a daylength factor was required as in the adjustment of the mass-transfer relation. This later adjustment was on the basis of the duration of turbulence and the stomata behavior of the plant. Correspondingly, we may theorize that another daylength factor is required to adjust for net energy available. Also, chlorophyll action which has a control over transpiration is dependent upon light rather than other sources of energy. From these considerations, it seems feasible to formulate a simplified expression for potential evapotranspiration, represented by

$$E_p = CD^2P_t \quad (\text{Eq. 4})$$

where, E_p is the potential evapotranspiration in inches per day; D , the possible hours of daily sunshine in units of 12 hours; P_t , the saturated water vapor concentration

at the mean temperature in grams/ M^3 ; and C a constant of .0055 obtained by choosing a level of yearly potential evaporation to correspond to those obtained by the Thornthwaite method and assumed reliable (See Table 1 below). Values of D, D^2 , and P_t have been tabulated in Tables I, II, and III, respectively, of the Appendix.

Table 1. Computed average annual potential evapotranspiration as compared with the Thornthwaite Method.

Station	P.E. ^{a)}	Ep ^{b)}	$\frac{P.E.}{E_p}$	Station	P.E. ^{a)}	Ep ^{b)}	$\frac{P.E.}{E_p}$
	(in.)	(in.)			(in.)	(in.)	
Portland, Me.	23.3	22.1	1.05	Raleigh, N.C.	33.4	33.8	.99
Lansing, Mich.	24.7	25.0	.99	Marked Tree, Ark.	34.7	34.8	1.00
Madison, Wisc.	25.1	25.6	.98	Memphis, Tenn.	35.6	36.5	.98
Minneapolis, Minn.	25.2	26.4	.95	Columbia, S.C.	37.2	37.1	1.00
Columbus, Ohio	28.1	28.0	1.00	Augusta, Ga.	38.5	37.4	1.03
Lincoln, Neb.	28.3	29.4	.96	Houston, Tex.	43.1	41.8	1.03
Philadelphia, Pa.	29.2	29.9	.98	Gainesville, Fla.	43.3	41.9	1.03
Columbia, Mo.	29.7	30.7	.97	Tampa, Fla.	46.5	44.4	1.05
Louisville, Ky.	31.8	31.7	1.00	Miami, Fla.	51.1	48.1	1.06
Knoxville, Tenn.	31.9	31.9	1.00				
Richmond, Va.	32.3	32.2	1.00		Average		1.00

a) Computed by Thornthwaite Method.

b) Computed by Equation. 4.

V. EVALUATION

Yearly values of potential evapotranspiration as computed by Eq. 4 are practically identical to those obtained by the Thornthwaite method (43) with only a consistent departure of about 5 percent noted for latitudes of southern Florida. A further evaluation on yearly data has been made by relating computed value of potential evapotranspiration to the data of consumptive use for valleys as obtained by Lowry and Johnson (28) and additional data of Williams (49). The relation is shown in Fig. 2 with the observed and computed values approaching a one-to-one relation. The observed values are for irrigated valleys and other valleys adjusted to represent optimum moisture conditions except for the data of Williams which represent actual water for locations in New England. The locations of station with corresponding numbers in Fig. 2 are found in Table 2.

An important test of the reliability of Eq. 4 is whether such a simplified procedure is adequate in representing the seasonal variation of potential evaporation. Several comparisons have been made.

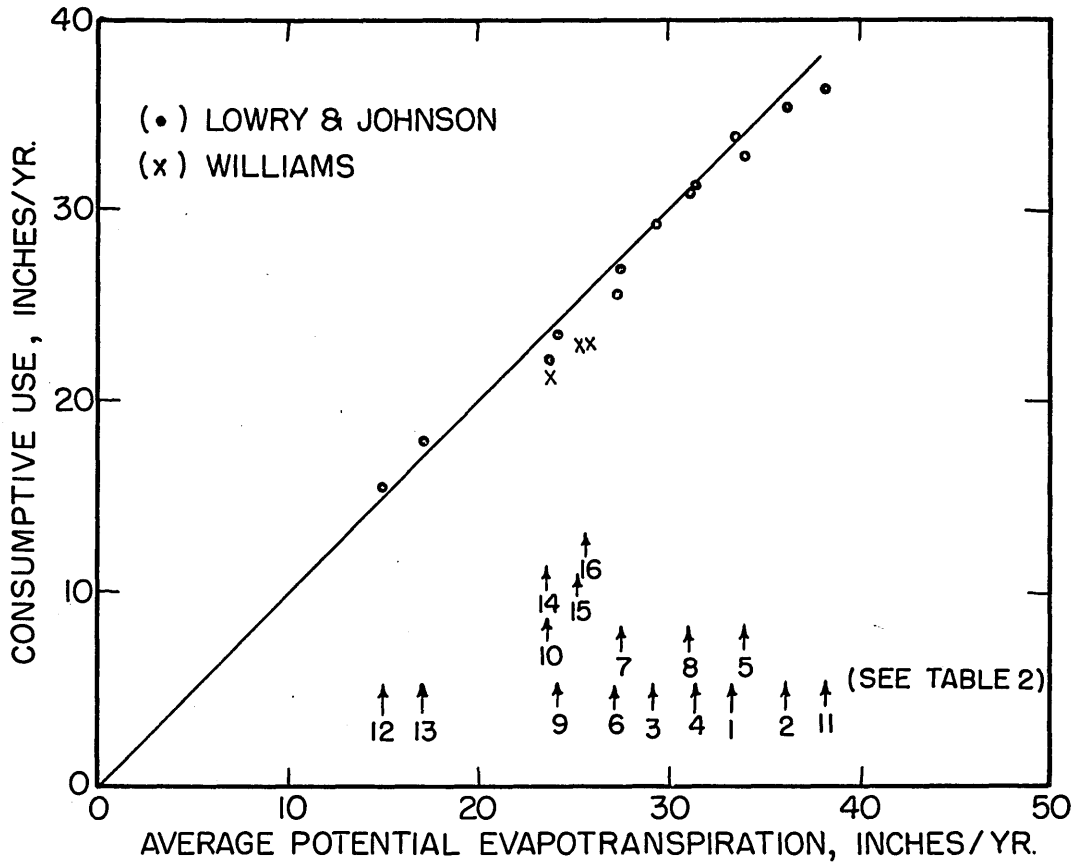


FIGURE 2. CONSUMPTIVE USE FOR BASINS COMPARED WITH COMPUTED POTENTIAL EVAPOTRANSPIRATION

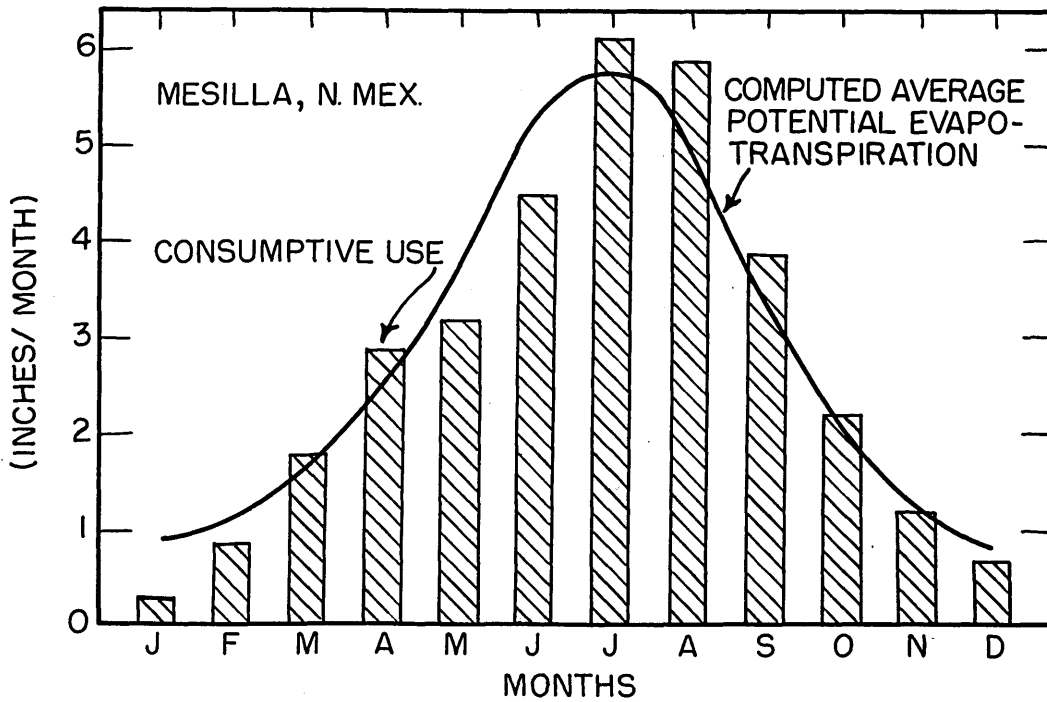


FIGURE 3. SEASONAL DISTRIBUTION OF CONSUMPTIVE USE AND POTENTIAL EVAPOTRANSPIRATION (SOURCE: LOWRY & JOHNSON)

Table 2. Data of observed water loss and computed average potential evapotranspiration for Fig. 2.

Valley and State	Area (acres)	Length of Record (years)	Mean Water Loss (inches)	E_p by Eq. 4 (inches)
1. Mesilla, N.Mex.	109,000	13	34.0	33.4
2. Pecos, N. Mex.	37,850	18	35.3	36.3
3. Sangawon R., Ill.	1,640,000	16	29.2	29.3
4. Green R., Ky.	5,000,000	6	31.4	31.4
5. Tallapoosa R., Ala.-Ga.	1,060,000	13	33.0	34.0
6. Mad R., Ohio	307,000	13	25.8	27.3
7. Skunk R., Iowa	1,890,000	16	27.0	27.6
8. N.Fork of White R., Mo.	755,000	14	31.0	31.1
9. N.Platte, Wyo.-Neb.	462,000	9	23.8	24.1
10. Black R., Wisc.	494,000	13	22.2	23.9
11. Cypress Crk., Tex.	545,000	13	36.2	38.3
12. Wagon Wheel Gap, Colo.	222	14	15.6	15.0
13. Mich.-Ill,R., Colo.	43,000	3	18.0	17.2
14. West R., Vt.		8	21.5	23.9
15. L.Cochituate, Mass.		30	23.2	25.1
16. Swift R., Mass.		15	23.1	25.5

Monthly value of water loss from an irrigated valley, Mesilla Valley in New Mexico, was obtained by Lowry and Johnson (28). The correspondence of these observed values of water loss to computed average potential evapotranspiration is satisfactory as seen in Fig. 3.

An analysis of the average water loss for periods of adequate water supply has been obtained by Fox (11) for basins in Wisconsin by the water-balance method. His determined values of water loss and the curve of average potential evaporations from Eq. 4 compare favorably in Fig. 4.

Observed values of evapotranspiration are available at Seabrook, N.J., from evapotranspirometer under conditions assumed to approximate potential conditions (29). Observations made during one year have been extracted and presented in Fig. 5. The tanks with water added from above daily showed a considerable increase in water loss over those with a constant water table at 35 cm., a condition which supplies adequate water. This departure of the two observations indicates that the tanks are losing water in excess of potential losses; the tanks with water at the surface functioning almost as

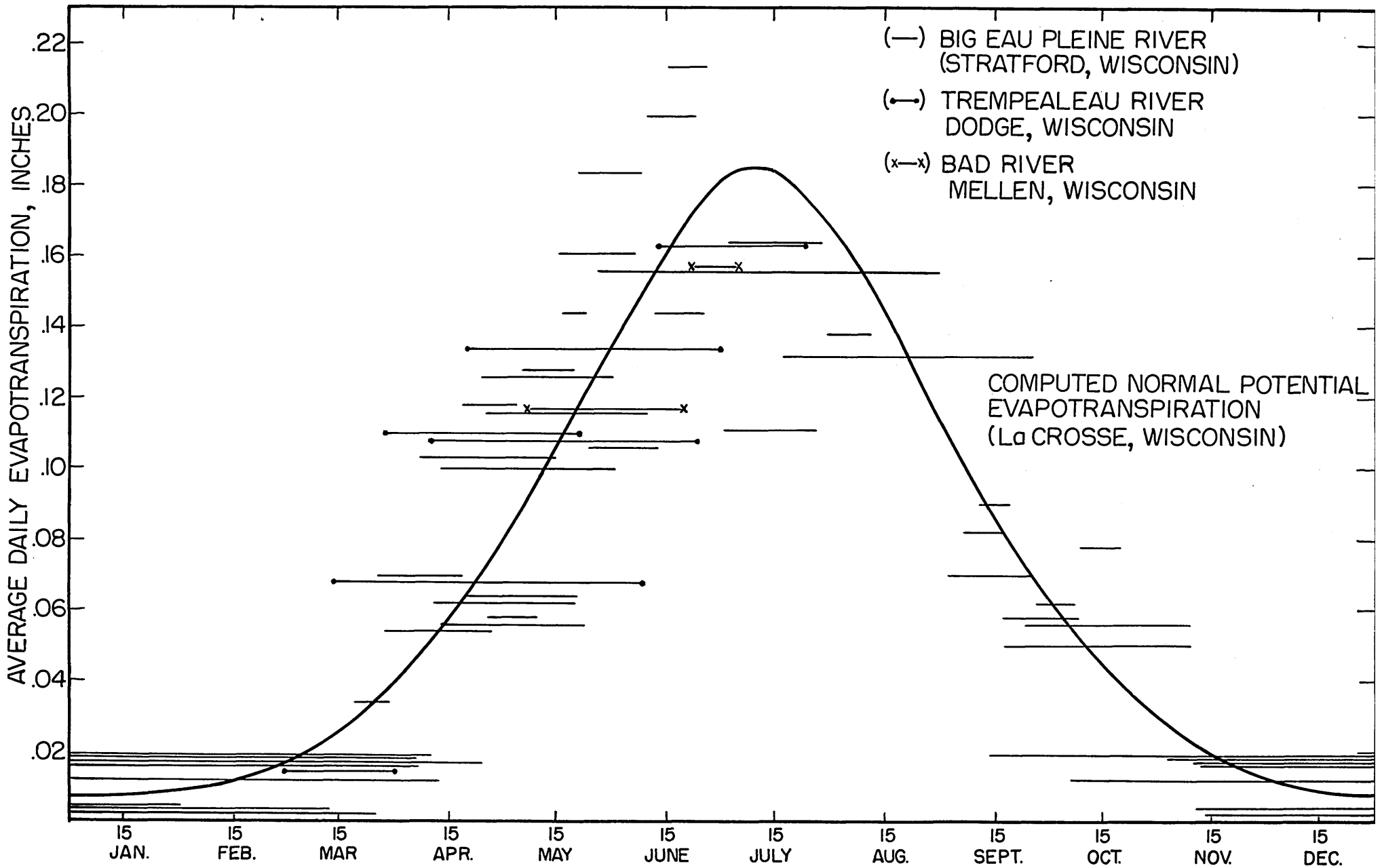


FIGURE 4. POTENTIAL EVAPOTRANSPIRATION FROM WATER BALANCE FOR BASINS

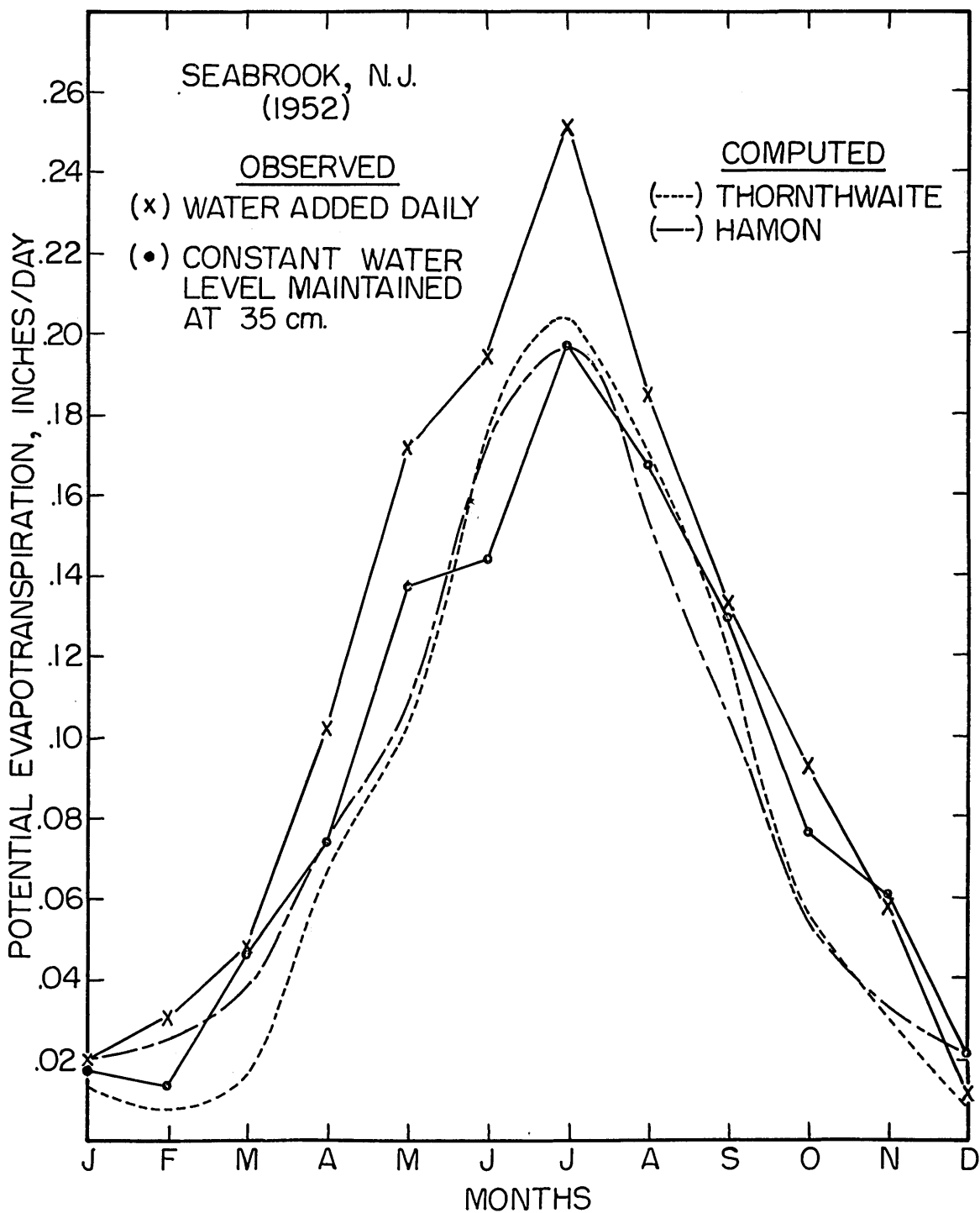


FIGURE 5. EVAPOTRANSPIROMETER MEASUREMENT OF EVAPOTRANSPIRATION AND COMPUTED POTENTIAL EVAPOTRANSPIRATION

a free water surface, or pan evaporation. Results from Eq. 4 for the first six months of the year give results very satisfactory and during this period conditions are more nearly that required for potential evapotranspiration. During the fall season computed values are lower than observed values for during this period climatic conditions are unfavorable for measurement of potential losses. The corresponding values computed by the Thornthwaite method are also indicated in Fig. 5. The corrections obtained by Eq. 4 are in the right direction; higher in winter and lower in summer.

Computed values of lake evaporation at Hartford, Conn., by the Penman equation as used in a graphical procedure (27) requiring radiation, temperature, humidity and wind data have been obtained. These values are compared with estimates of potential evapotranspiration obtained by Eq. 3 and Eq. 4 as tabulated in Table 3.

The computed values by the three methods are of the same order of magnitude. Lake evaporation should normally exceed that of potential evapotranspiration by only a small fraction for the climate and vegetation

Table 3. Computed average daily lake evaporation and average daily potential evapotranspiration for Hartford, Conn. (Bradley Field)

Months	Lake ^{a)}	Ep ^{b)}	Ep ^{c)}
January	.020	.009	.014
February	.025	.014	.018
March	.050	.035	.032
April	.090	.069	.059
May	.130	.116	.106
June	.160	.157	.158
July	.175	.176	.179
August	.135	.148	.143
September	.090	.105	.090
October	.055	.061	.049
November	.030	.031	.025
December	.015	.010	.014

a) Graphical solution of Penman equation

b) Eq. 3

c) Eq. 4

of New England. Apparently, the values obtained by converting the Penman relation to a solution requiring only a knowledge of temperature and daylength, Eq. 3, gives satisfactory results. Of more interest and significance is the equivalent solution by the extremely simple Eq. 4.

Penman (35) and Blaney (2) have computed values of potential evapotranspiration by their respective methods for Asheville, N. Carolina. For comparison, computations have been made by the Thornthwaite method and by Eq. 4. Results from Eq. 4 are fully satisfactory in respect to the other procedure as seen in Table 4.

Since an extensive investigation has been made of the water loss and lake evaporation for New England and New York (20), computed values of potential evapotranspiration have been obtained for the same area by use of Eq. 4. The water loss chart has been reproduced in Fig. 6 and the lake evaporation, obtained by Penman's formula as adopted from the study, has been reconstructed as Fig. 7. Results from Eq. 4 are shown in Fig. 8. The general configuration of isopleths representing lake evaporation corresponds with those of potential evapotranspiration.

Table 4. Computed average monthly evapotranspiration for Asheville, North Carolina, by several methods.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Yrly	Source
(Inches)													
1.1	1.1	2.0	2.4	3.7	4.8	5.1	4.7	3.4	2.2	1.3	1.1	32.9	Blaney ^{a)}
0.7	1.1	2.0	2.7	4.0	4.8	4.8	4.0	2.4	1.7	0.8	0.6	29.6	Penman ^{b)}
0.3	0.4	1.0	2.1	3.5	4.9	5.4	4.9	3.6	2.1	0.8	0.4	29.4	Thorntwaite ^{c)}
0.7	0.9	1.4	2.3	3.5	4.7	5.1	4.4	3.1	1.8	1.0	0.7	29.6	Eq. 4

a) After Blaney (2)

b) After Penman (35)

c) Computed by Thorntwaite Method.



FIGURE 6. YEARLY WATER LOSS IN INCHES OBTAINED FROM PRECIPITATION AND RUNOFF
(SOURCE: GEOLOGICAL SURVEY)

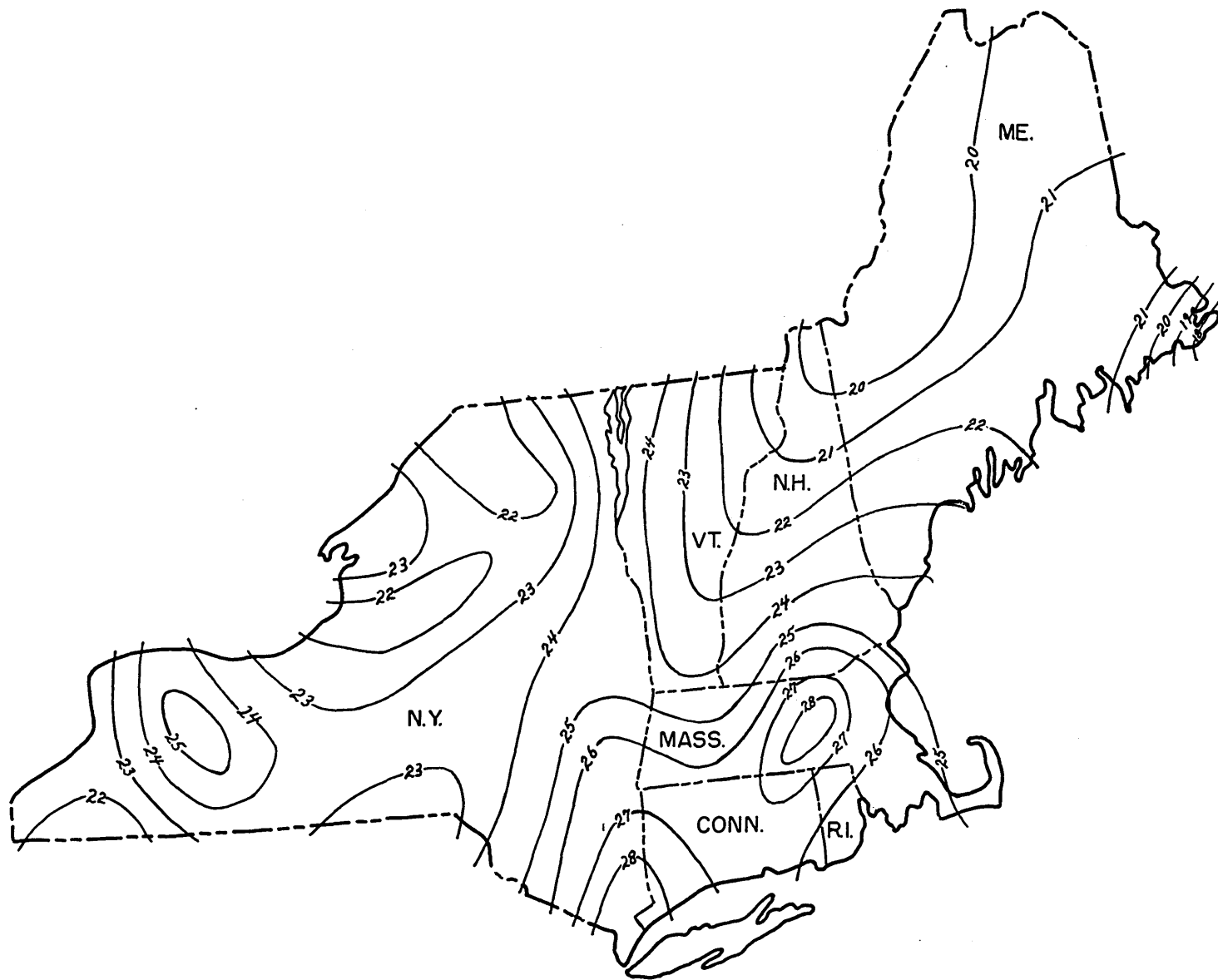


FIGURE 7. COMPUTED YEARLY LAKE EVAPORATION IN INCHES
(SOURCE: GEOLOGICAL SURVEY)

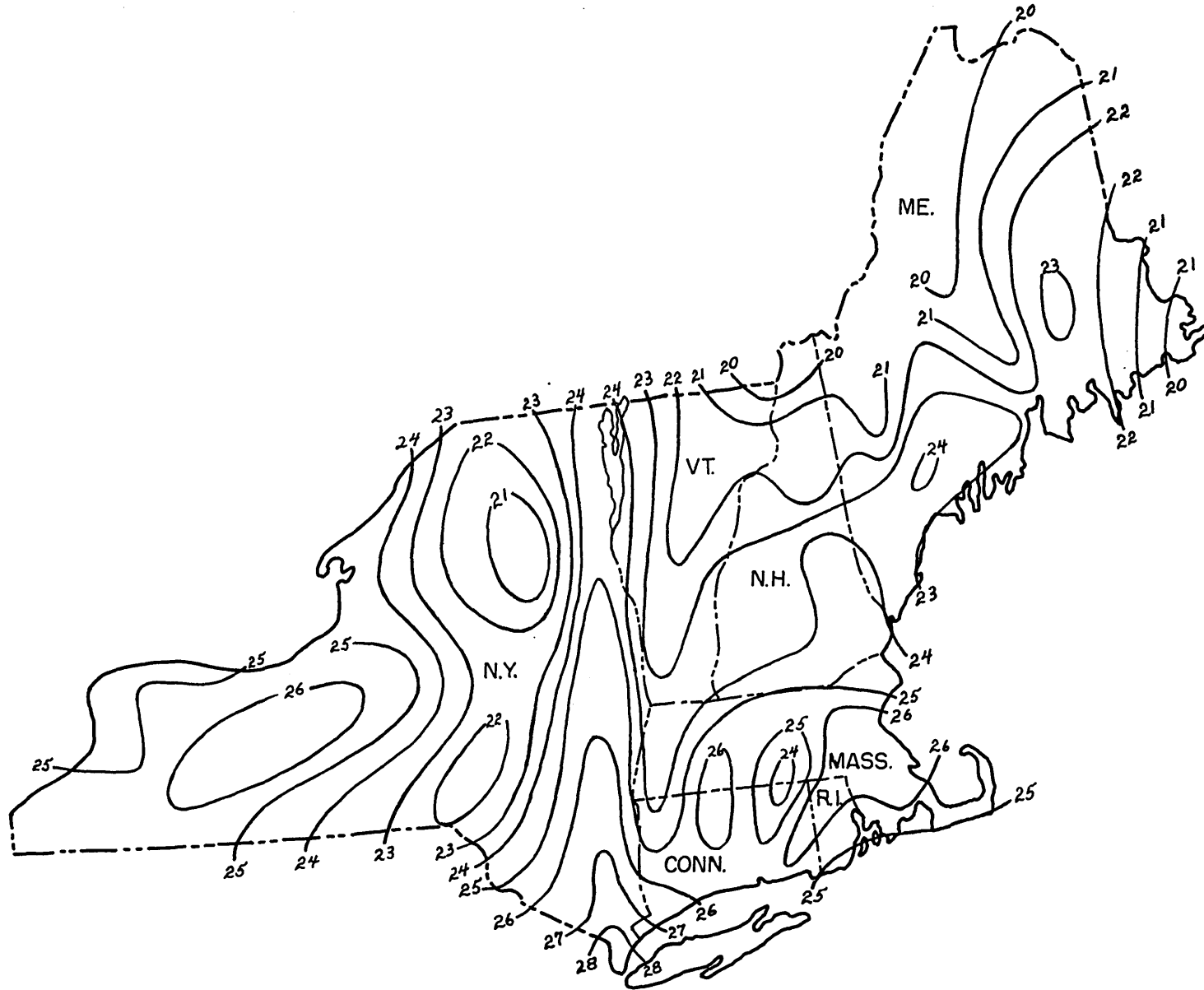


FIGURE 8. COMPUTED AVERAGE ANNUAL POTENTIAL EVAPOTRANSPIRATION IN INCHES

Since Eq. 4 has shown every indication of representing a unique and simple procedure for obtaining useful estimates of potential evapotranspiration, computations have been made using monthly mean data of the United States, east of the Rockies. A map of generalized yearly average values is shown in Fig. 9, while average January values are presented in Fig. 10, and likewise the values for July in Fig. 11.

As noted before, the yearly values of computed potential evaporation are somewhat lower than those obtained by the Thornthwaite formula in southern Florida and southern Texas. There is evidence to indicate that the results of Eq. 4 are closer to the truth. Computations (42) have been made on the basis of net solar energy available for evapotranspiration as an indication of the approximate maximum rates that might be expected. The results for Miami, Florida, for a grass surface take a value for June of 5.4 inches. The corresponding value from Eq. 4 is 5.73 inches and the Thornthwaite formula results in a value of 6.26 inches. In another study of evaporation and transpiration (7) of large cane areas, the water loss at Belle Glade, Florida, is estimated as between 42 and 45 inches per year. In this case, Eq. 4 gives a value of 44.3 inches and the Thornthwaite formula, 47.3 inches.

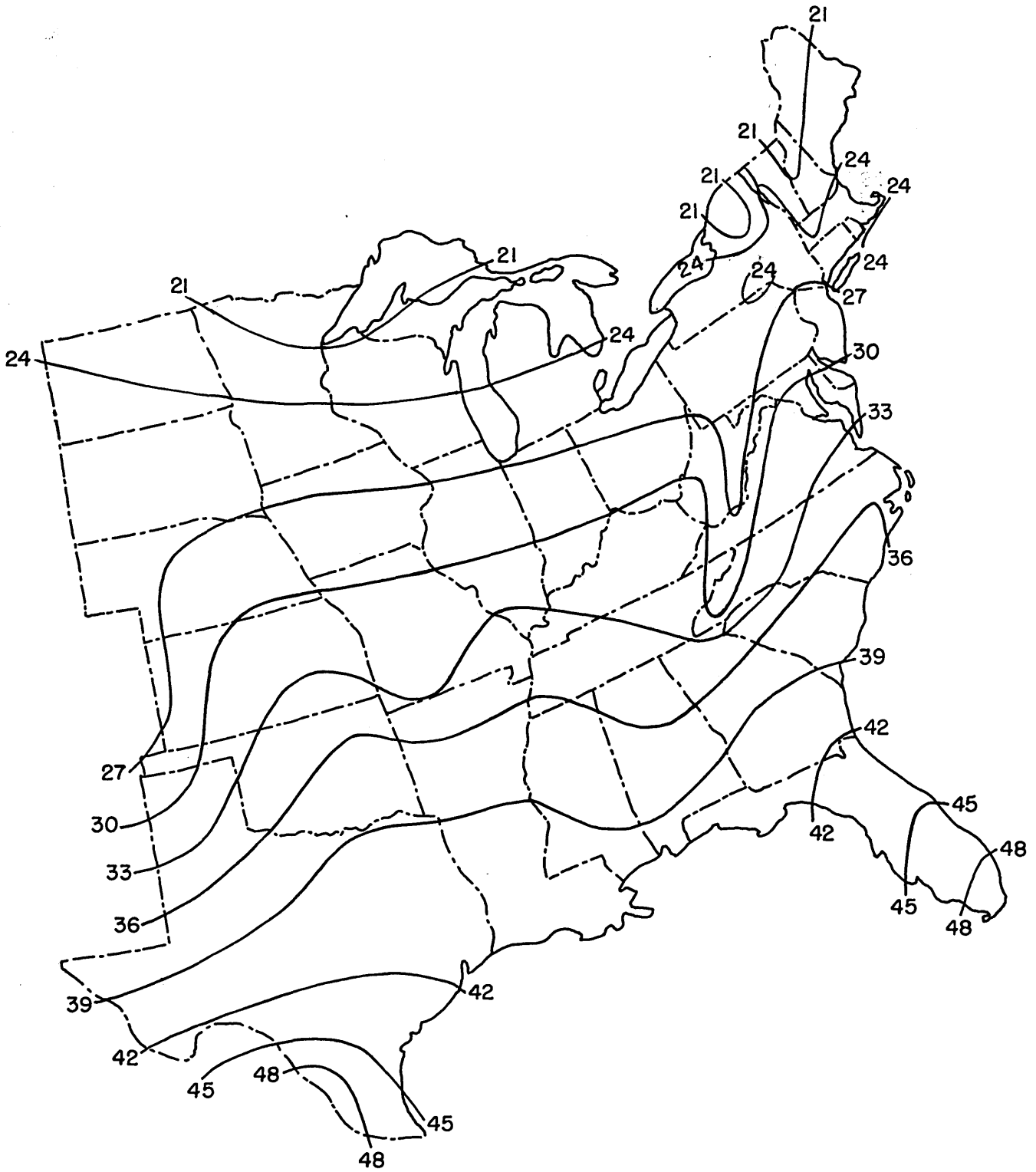


FIGURE 9. COMPUTED AVERAGE ANNUAL POTENTIAL EVAPOTRANSPIRATION IN INCHES

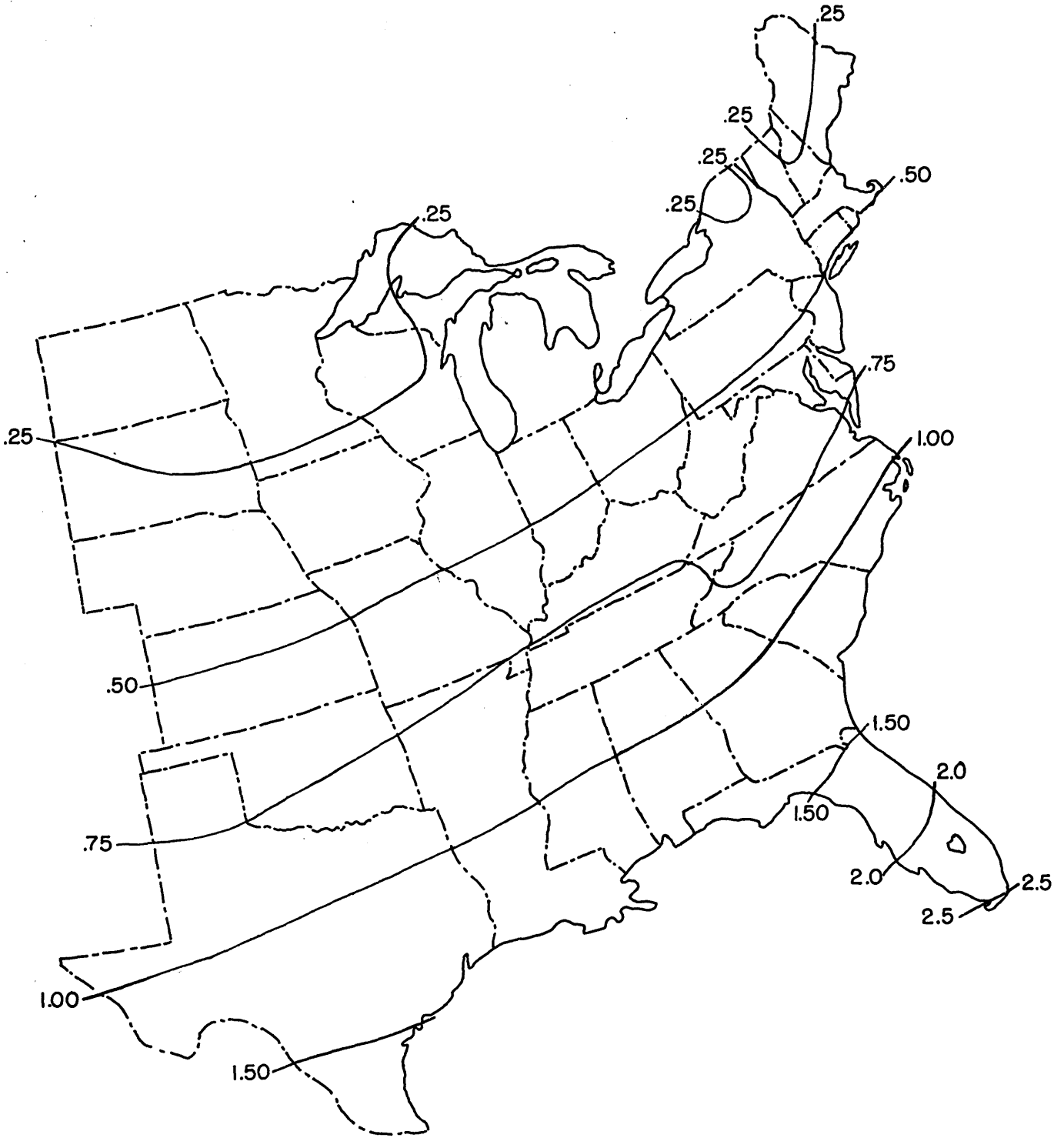


FIGURE 10. COMPUTED AVERAGE (JANUARY) POTENTIAL EVAPOTRANSPIRATION IN INCHES

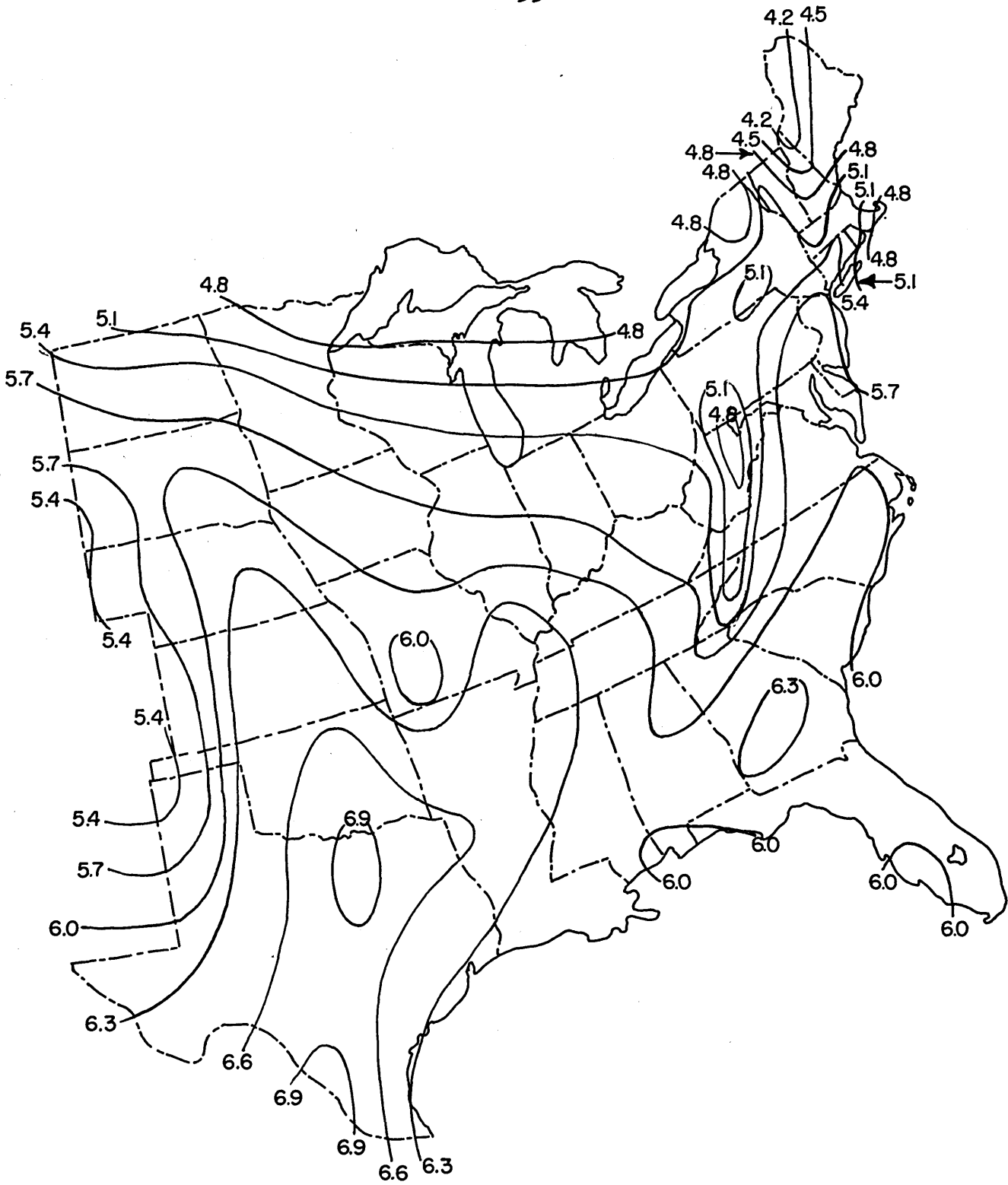


FIGURE II. COMPUTED AVERAGE (JULY) POTENTIAL EVAPOTRANSPIRATION IN INCHES

Williams (49) has prepared a generalized map of actual water loss for the area covering the Great Plains eastward. In general over the eastern United States where precipitation on the yearly basis exceeds the potential evapotranspiration, the correspondence with Fig. 9 is good with the actual water-loss values generally exceeded by 2 - 4 inches, which should be expected due to a normal shortage of soil moisture during the summer months.

The generalized January values of potential evaporation obtained by Eq. 4 in Fig. 10 are of the same order of magnitude as computed values of lake evaporation by the Meyer formula (30). This formula which is empirical and based only on mass-transport, gives values which range from .25 inch in central Minnesota to 1.5 inches in Louisiana and from 0.50 inch in Central New York to 3.0 inches at the tip of Florida. The computed values for July, Fig. 11, corresponds closely with the Meyer values east of the Mississippi River where potential conditions are approached.

VI. CONCLUSIONS

The equation developed, Eq. 4, page 36, to estimate potential evapotranspiration adheres to the desirable features of requiring only readily available meteorological data expressed in an easily computable fashion. Saturated water vapor concentration at the mean temperature adjusted by a daylength factor squared to account for plant response, duration of turbulence, and net radiation apparently is proportional to potential evapotranspiration. General applicability seems justified in view of the correspondence between observed and computed values of potential evapotranspiration, both on a yearly and seasonal basis for widely scattered localities.

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APPENDIX

Table I. Mean possible duration of sunlight in units of 12 hours.
(Source: Smithsonian Meteorological Tables)

N. Lat.	J	F	M	A	M	J	J	A	S	O	N	D
25	.90	.95	1.00	1.06	1.12	1.15	1.13	1.08	1.02	.97	.91	.88
26	.89	.94	1.00	1.06	1.12	1.15	1.14	1.08	1.02	.97	.90	.88
27	.89	.94	1.00	1.07	1.13	1.15	1.15	1.09	1.02	.97	.90	.87
28	.88	.94	1.00	1.07	1.13	1.16	1.15	1.09	1.02	.96	.90	.87
29	.88	.93	1.00	1.07	1.14	1.16	1.16	1.09	1.03	.96	.90	.86
30	.87	.93	1.00	1.08	1.14	1.17	1.16	1.10	1.03	.96	.89	.85
31	.87	.93	1.00	1.08	1.14	1.18	1.17	1.10	1.03	.96	.89	.85
32	.86	.92	1.00	1.08	1.15	1.19	1.17	1.11	1.03	.95	.88	.84
33	.85	.92	1.00	1.09	1.15	1.20	1.18	1.11	1.03	.95	.87	.83
34	.84	.92	1.00	1.09	1.16	1.20	1.18	1.12	1.03	.95	.87	.83
35	.83	.91	1.00	1.09	1.17	1.21	1.19	1.12	1.03	.95	.86	.82
36	.83	.91	1.00	1.09	1.17	1.22	1.20	1.13	1.03	.95	.86	.81
37	.82	.90	1.00	1.10	1.18	1.23	1.21	1.13	1.04	.94	.85	.80
38	.82	.90	1.00	1.10	1.19	1.24	1.21	1.14	1.04	.94	.84	.80
39	.81	.90	1.00	1.11	1.19	1.24	1.22	1.14	1.04	.94	.84	.79
40	.80	.89	1.00	1.11	1.20	1.25	1.23	1.15	1.04	.93	.83	.78
41	.79	.89	1.00	1.11	1.21	1.26	1.24	1.16	1.04	.93	.82	.77
42	.78	.88	1.00	1.12	1.21	1.27	1.25	1.16	1.04	.92	.82	.76
43	.77	.88	.99	1.12	1.22	1.28	1.26	1.17	1.04	.92	.81	.75
44	.77	.87	.99	1.13	1.23	1.29	1.27	1.17	1.04	.92	.80	.74
45	.76	.87	.99	1.13	1.23	1.29	1.28	1.18	1.04	.91	.79	.73
46	.75	.87	.99	1.13	1.24	1.31	1.29	1.18	1.04	.91	.79	.72
47	.74	.86	.99	1.14	1.25	1.32	1.30	1.19	1.04	.90	.78	.71
48	.73	.86	.99	1.14	1.26	1.33	1.31	1.20	1.05	.90	.77	.70
49	.72	.85	.99	1.14	1.27	1.34	1.32	1.20	1.05	.90	.76	.69
50	.71	.84	.99	1.15	1.28	1.36	1.33	1.21	1.06	.89	.76	.68

TABLE II. Daylength factor. Square of the mean 12-hour units of possible sunshine.

LAT.	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
24	.81	.88	1.00	1.14	1.22	1.29	1.28	1.17	1.06	.96	.85	.78
25	.80	.88	1.00	1.14	1.23	1.30	1.29	1.18	1.06	.95	.84	.77
26	.79	.88	1.00	1.14	1.25	1.32	1.30	1.19	1.06	.94	.83	.77
27	.78	.88	1.00	1.14	1.26	1.33	1.31	1.20	1.06	.93	.82	.76
28	.77	.88	1.00	1.14	1.28	1.35	1.32	1.21	1.06	.92	.81	.75
29	.76	.87	1.00	1.15	1.29	1.36	1.33	1.21	1.06	.92	.80	.74
30	.76	.86	1.00	1.17	1.30	1.37	1.35	1.21	1.06	.92	.79	.72
31	.75	.86	1.00	1.16	1.31	1.39	1.36	1.22	1.06	.91	.78	.71
32	.74	.85	1.00	1.17	1.32	1.42	1.37	1.23	1.06	.90	.77	.71
33	.72	.85	1.00	1.18	1.33	1.43	1.38	1.24	1.06	.90	.76	.70
34	.71	.85	1.00	1.19	1.35	1.44	1.39	1.25	1.06	.90	.76	.69
35	.70	.84	1.00	1.19	1.36	1.46	1.41	1.26	1.07	.89	.74	.67
36	.69	.83	1.00	1.19	1.37	1.49	1.44	1.28	1.08	.88	.73	.66
37	.68	.82	.99	1.20	1.39	1.50	1.45	1.29	1.08	.88	.72	.65
38	.67	.81	.99	1.21	1.42	1.51	1.46	1.30	1.08	.88	.71	.64
39	.65	.80	.99	1.21	1.43	1.53	1.48	1.30	1.08	.87	.70	.62
40	.64	.79	.99	1.22	1.44	1.56	1.51	1.31	1.08	.86	.69	.61
41	.63	.78	.98	1.23	1.45	1.58	1.53	1.33	1.08	.85	.68	.59
42	.62	.77	.98	1.24	1.47	1.61	1.56	1.35	1.08	.85	.67	.58
43	.60	.76	.98	1.25	1.49	1.63	1.58	1.36	1.08	.85	.66	.56
44	.59	.76	.98	1.26	1.51	1.66	1.61	1.37	1.08	.85	.65	.55
45	.57	.75	.98	1.27	1.52	1.69	1.63	1.38	1.09	.84	.63	.53
46	.56	.74	.98	1.28	1.54	1.72	1.66	1.39	1.10	.83	.62	.52
47	.54	.74	.98	1.29	1.56	1.76	1.69	1.41	1.10	.83	.60	.50
48	.53	.74	.98	1.30	1.59	1.80	1.72	1.44	1.10	.83	.59	.49
49	.52	.73	.98	1.31	1.61	1.82	1.74	1.45	1.11	.82	.58	.47
50	.50	.72	.98	1.32	1.64	1.85	1.77	1.46	1.12	.81	.58	.46

Table III. Vapor Concentration (absolute humidity) in saturated air. (Source: Smithsonian Meteorological Tables)

°F	gm/M ³	°F	gm/M ³	°F	gm/M ³	°F	gm/M ³
-10	.81	-6.0	.98	-2.0	1.18	2.0	1.41
-9.9	.81	-5.9	.98	-1.9	1.18	2.1	1.41
-9.8	.82	-5.8	.99	-1.8	1.19	2.2	1.42
-9.7	.82	-5.7	.99	-1.7	1.19	2.3	1.43
-9.6	.83	-5.6	1.00	-1.6	1.20	2.4	1.43
-9.5	.83	-5.5	1.00	-1.5	1.20	2.5	1.44
-9.4	.83	-5.4	1.00	-1.4	1.21	2.6	1.44
-9.3	.84	-5.3	1.01	-1.3	1.21	2.7	1.45
-9.2	.84	-5.2	1.02	-1.2	1.22	2.8	1.46
-9.1	.85	-5.1	1.03	-1.1	1.22	2.9	1.46
-9.0	.85	-5.0	1.03	-1.0	1.23	3.0	1.47
-8.9	.85	-4.9	1.03	-0.9	1.23	3.1	1.47
-8.8	.86	-4.8	1.04	-0.8	1.24	3.2	1.48
-8.7	.86	-4.7	1.04	-0.7	1.24	3.3	1.49
-8.6	.87	-4.6	1.05	-0.6	1.25	3.4	1.49
-8.5	.87	-4.5	1.05	-0.5	1.26	3.5	1.50
-8.4	.87	-4.4	1.05	-0.4	1.27	3.6	1.50
-8.3	.88	-4.3	1.06	-0.3	1.27	3.7	1.51
-8.2	.88	-4.2	1.06	-0.2	1.28	3.8	1.52
-8.1	.89	-4.1	1.07	-0.1	1.28	3.9	1.53
-8.0	.89	-4.0	1.07	0	1.29	4.0	1.54
-7.9	.89	-3.9	1.07	.1	1.29	4.1	1.54
-7.8	.90	-3.8	1.08	.2	1.29	4.2	1.55
-7.7	.90	-3.7	1.08	.3	1.30	4.3	1.55
-7.6	.91	-3.6	1.09	.4	1.31	4.4	1.56
-7.5	.91	-3.5	1.10	.5	1.32	4.5	1.57
-7.4	.92	-3.4	1.10	.6	1.32	4.6	1.58
-7.3	.92	-3.3	1.11	.7	1.33	4.7	1.59
-7.2	.93	-3.2	1.11	.8	1.34	4.8	1.59
-7.1	.94	-3.1	1.12	.9	1.35	4.9	1.60
-7.0	.94	-3.0	1.12	1.0	1.35	5.0	1.61
-6.9	.94	-2.9	1.13	1.1	1.36	5.1	1.62
-6.8	.95	-2.8	1.13	1.2	1.37	5.2	1.62
-6.7	.95	-2.7	1.14	1.3	1.37	5.3	1.63
-6.6	.96	-2.6	1.15	1.4	1.38	5.4	1.64
-6.5	.96	-2.5	1.15	1.5	1.38	5.5	1.64
-6.4	.96	-2.4	1.16	1.6	1.39	5.6	1.65
-6.3	.97	-2.3	1.16	1.7	1.39	5.7	1.66
-6.2	.97	-2.2	1.17	1.8	1.40	5.8	1.66
-6.1	.98	-2.1	1.17	1.9	1.40	5.9	1.67

$^{\circ}\text{F}$	gm/M^3	$^{\circ}\text{F}$	gm/M^3	$^{\circ}\text{F}$	gm/M^3	$^{\circ}\text{F}$	gm/M^3
6.0	1.68	10.0	1.99	14.0	2.36	18.0	2.78
6.1	1.68	10.1	2.00	14.1	2.37	18.1	2.79
6.2	1.69	10.2	2.01	14.2	2.38	18.2	2.80
6.3	1.70	10.3	2.02	14.3	2.39	18.3	2.81
6.4	1.70	10.4	2.03	14.4	2.40	18.4	2.83
6.5	1.71	10.5	2.04	14.5	2.41	18.5	2.84
6.6	1.72	10.6	2.05	14.6	2.42	18.6	2.85
6.7	1.73	10.7	2.06	14.7	2.43	18.7	2.86
6.8	1.73	10.8	2.06	14.8	2.44	18.8	2.87
6.9	1.74	10.9	2.07	14.9	2.45	18.9	2.89
7.0	1.75	11.0	2.08	15.0	2.46	19.0	2.90
7.1	1.76	11.1	2.09	15.1	2.47	19.1	2.91
7.2	1.77	11.2	2.09	15.2	2.48	19.2	2.92
7.3	1.77	11.3	2.10	15.3	2.49	19.3	2.93
7.4	1.78	11.4	2.11	15.4	2.50	19.4	2.95
7.5	1.79	11.5	2.12	15.5	2.51	19.5	2.96
7.6	1.80	11.6	2.13	15.6	2.52	19.6	2.97
7.7	1.81	11.7	2.14	15.7	2.53	19.7	2.98
7.8	1.82	11.8	2.15	15.8	2.54	19.8	2.99
7.9	1.82	11.9	2.16	15.9	2.55	19.9	3.01
8.0	1.83	12.0	2.17	16.0	2.56	20.0	3.02
8.1	1.84	12.1	2.18	16.1	2.57	20.1	3.03
8.2	1.84	12.2	2.19	16.2	2.58	20.2	3.04
8.3	1.85	12.3	2.20	16.3	2.59	20.3	3.06
8.4	1.86	12.4	2.21	16.4	2.61	20.4	3.07
8.5	1.87	12.5	2.22	16.5	2.62	20.5	3.08
8.6	1.88	12.6	2.23	16.6	2.63	20.6	3.09
8.7	1.89	12.7	2.24	16.7	2.64	20.7	3.10
8.8	1.89	12.8	2.25	16.8	2.65	20.8	3.12
8.9	1.90	12.9	2.25	16.9	2.66	20.9	3.13
9.0	1.91	13.0	2.26	17.0	2.67	21.0	3.14
9.1	1.92	13.1	2.27	17.1	2.68	21.1	3.15
9.2	1.93	13.2	2.28	17.2	2.69	21.2	3.17
9.3	1.93	13.3	2.29	17.3	2.70	21.3	3.18
9.4	1.94	13.4	2.30	17.4	2.72	21.4	3.19
9.5	1.95	13.5	2.31	17.5	2.73	21.5	3.21
9.6	1.96	13.6	2.32	17.6	2.74	21.6	3.22
9.7	1.97	13.7	2.33	17.7	2.75	21.7	3.23
9.8	1.98	13.8	2.34	17.8	2.76	21.8	3.24
9.9	1.98	13.9	2.35	17.9	2.77	21.9	3.26

°F	gm/M ³	°F	gm/M ³	°F	gm/M ³	°F	gm/M ³
22.0	3.27	26.0	3.84	30.0	4.49	34.0	5.23
22.1	3.28	26.1	3.86	30.1	4.51	34.1	5.25
22.2	3.30	26.2	3.87	30.2	4.53	34.2	5.27
22.3	3.31	26.3	3.89	30.3	4.54	34.3	5.29
22.4	3.33	26.4	3.90	30.4	4.56	34.4	5.31
22.5	3.34	26.5	3.92	30.5	4.58	34.5	5.33
22.6	3.35	26.6	3.93	30.6	4.60	34.6	5.35
22.7	3.37	26.7	3.95	30.7	4.62	34.7	5.37
22.8	3.38	26.8	3.96	30.8	4.63	34.8	5.39
22.9	3.40	26.9	3.98	30.9	4.65	34.9	5.41
23.0	3.41	27.0	3.99	31.0	4.67	35.0	5.43
23.1	3.42	27.1	4.01	31.1	4.69	35.1	5.45
23.2	3.44	27.2	4.02	31.2	4.71	35.2	5.47
23.3	3.45	27.3	4.04	31.3	4.72	35.3	5.49
23.4	3.47	27.4	4.05	31.4	4.74	35.4	5.51
23.5	3.48	27.5	4.07	31.5	4.76	35.5	5.54
23.6	3.49	27.6	4.09	31.6	4.78	35.6	5.56
23.7	3.51	27.7	4.11	31.7	4.80	35.7	5.58
23.8	3.52	27.8	4.12	31.8	4.81	35.8	5.60
23.9	3.54	27.9	4.14	31.9	4.83	35.9	5.62
24.0	3.55	28.0	4.15	32.0	4.85	36.0	5.64
24.1	3.56	28.1	4.17	32.1	4.87	36.1	5.66
24.2	3.58	28.2	4.18	32.2	4.89	36.2	5.68
24.3	3.59	28.3	4.20	32.3	4.91	36.3	5.71
24.4	3.61	28.4	4.21	32.4	4.93	36.4	5.73
24.5	3.62	28.5	4.23	32.5	4.95	36.5	5.75
24.6	3.63	28.6	4.25	32.6	4.96	36.6	5.77
24.7	3.65	28.7	4.27	32.7	4.98	36.7	5.79
24.8	3.66	28.8	4.29	32.8	5.00	36.8	5.82
24.9	3.68	28.9	4.30	32.9	5.02	36.9	5.84
25.0	3.69	29.0	4.32	33.0	5.04	37.0	5.86
25.1	3.71	29.1	4.34	33.1	5.06	37.1	5.88
25.2	3.72	29.2	4.35	33.2	5.08	37.2	5.90
25.3	3.74	29.3	4.37	33.3	5.10	37.3	5.93
25.4	3.75	29.4	4.38	33.4	5.12	37.4	5.95
25.5	3.76	29.5	4.40	33.5	5.14	37.5	5.97
25.6	3.78	29.6	4.42	33.6	5.15	37.6	5.99
25.7	3.80	29.7	4.44	33.7	5.17	37.7	6.01
25.8	3.81	29.8	4.46	33.8	5.19	37.8	6.04
25.9	3.83	29.9	4.47	33.9	5.21	37.9	6.06

°F	gm/M ³	°F	gm/M ³	°F	gm/M ³	°F	gm/M ³
38.0	6.08	42.0	7.05	46.0	8.15	50.0	9.40
38.1	6.10	42.1	7.08	46.1	8.18	50.1	9.43
38.2	6.13	42.2	7.10	46.2	8.21	50.2	9.47
38.3	6.15	42.3	7.13	46.3	8.24	50.3	9.50
38.4	6.17	42.4	7.15	46.4	8.27	50.4	9.54
38.5	6.20	42.5	7.18	46.5	8.30	50.5	9.57
38.6	6.22	42.6	7.21	46.6	8.33	50.6	9.60
38.7	6.24	42.7	7.23	46.7	8.36	50.7	9.64
38.8	6.26	42.8	7.26	46.8	8.39	50.8	9.67
38.9	6.29	42.9	7.28	46.9	8.42	50.9	9.71
39.0	6.31	43.0	7.31	47.0	8.45	51.0	9.74
39.1	6.33	43.1	7.34	47.1	8.48	51.1	9.78
39.2	6.36	43.2	7.36	47.2	8.51	51.2	9.81
39.3	6.38	43.3	7.39	47.3	8.54	51.3	9.85
39.4	6.41	43.4	7.42	47.4	8.57	51.4	9.88
39.5	6.43	43.5	7.45	47.5	8.61	51.5	9.92
39.6	6.45	43.6	7.47	47.6	8.64	51.6	9.95
39.7	6.48	43.7	7.50	47.7	8.67	51.7	9.99
39.8	6.50	43.8	7.53	47.8	8.70	51.8	10.02
39.9	6.53	43.9	7.55	47.9	8.73	51.9	10.06
40.0	6.55	44.0	7.58	48.0	8.76	52.0	10.09
40.1	6.58	44.1	7.61	48.1	8.79	52.1	10.13
40.2	6.60	44.2	7.64	48.2	8.82	52.2	10.16
40.3	6.63	44.3	7.66	48.3	8.85	52.3	10.20
40.4	6.65	44.4	7.69	48.4	8.88	52.4	10.23
40.5	6.68	44.5	7.72	48.5	8.92	52.5	10.27
40.6	6.70	44.6	7.75	48.6	8.95	52.6	10.30
40.7	6.73	44.7	7.78	48.7	8.98	52.7	10.34
40.8	6.75	44.8	7.80	48.8	9.01	52.8	10.37
40.9	6.78	44.9	7.83	48.9	9.04	52.9	10.41
41.0	6.80	45.0	7.86	49.0	9.07	53.0	10.44
41.1	6.83	45.1	7.89	49.1	9.10	53.1	10.48
41.2	6.85	45.2	7.92	49.2	9.14	53.2	10.51
41.3	6.88	45.3	7.95	49.3	9.17	53.3	10.55
41.4	6.90	45.4	7.98	49.4	9.20	53.4	10.59
41.5	6.93	45.5	8.01	49.5	9.23	53.5	10.63
41.6	6.95	45.6	8.03	49.6	9.27	53.6	10.66
41.7	6.98	45.7	8.06	49.7	9.30	53.7	10.70
41.8	7.00	45.8	8.09	49.8	9.33	53.8	10.74
41.9	7.03	45.9	8.12	49.9	9.37	53.9	10.77

$^{\circ}\text{F}$	gm/M ³	$^{\circ}\text{F}$	gm/M ³	$^{\circ}\text{F}$	gm/M ³	$^{\circ}\text{F}$	gm/M ³
54.0	10.81	58.0	12.40	62.0	14.20	66.0	16.20
54.1	10.85	58.1	12.44	62.1	14.25	66.1	16.25
54.2	10.89	58.2	12.49	62.2	14.29	66.2	16.31
54.3	10.92	58.3	12.53	62.3	14.34	66.3	16.36
54.4	10.95	58.4	12.57	62.4	14.39	66.4	16.42
54.5	10.99	58.5	12.62	62.5	14.44	66.5	16.47
54.6	11.03	58.6	12.66	62.6	14.48	66.6	16.52
54.7	11.07	58.7	12.70	62.7	14.53	66.7	16.58
54.8	11.11	58.8	12.74	62.8	14.58	66.8	16.63
54.9	11.15	58.9	12.79	62.9	14.62	66.9	16.69
55.0	11.19	59.0	12.83	63.0	14.67	67.0	16.74
55.1	11.23	59.1	12.88	63.1	14.72	67.1	16.80
55.2	11.27	59.2	12.92	63.2	14.77	67.2	16.85
55.3	11.31	59.3	12.97	63.3	14.82	67.3	16.91
55.4	11.35	59.4	13.01	63.4	14.87	67.4	16.96
55.5	11.39	59.5	13.06	63.5	14.92	67.5	17.02
55.6	11.42	59.6	13.10	63.6	14.97	67.6	17.08
55.7	11.46	59.7	13.15	63.7	15.02	67.7	17.13
55.8	11.50	59.8	13.19	63.8	15.07	67.8	17.19
55.9	11.54	59.9	13.24	63.9	15.12	67.9	17.24
56.0	11.58	60.0	13.28	64.0	15.17	68.0	17.30
56.1	11.62	60.1	13.33	64.1	15.22	68.1	17.36
56.2	11.66	60.2	13.37	64.2	15.27	68.2	17.41
56.3	11.70	60.3	13.42	64.3	15.32	68.3	17.47
56.4	11.74	60.4	13.46	64.4	15.37	68.4	17.53
56.5	11.78	60.5	13.51	64.5	15.43	68.5	17.59
56.6	11.82	60.6	13.55	64.6	15.48	68.6	17.64
56.7	11.86	60.7	13.60	64.7	15.53	68.7	17.70
56.8	11.90	60.8	13.64	64.8	15.58	68.8	17.76
56.9	11.94	60.9	13.69	64.9	15.63	68.9	17.81
57.0	11.98	61.0	13.73	65.0	15.68	69.0	17.87
57.1	12.02	61.1	13.78	65.1	15.73	69.1	17.93
57.2	12.06	61.2	13.82	65.2	15.78	69.2	17.99
57.3	12.11	61.3	13.87	65.3	15.84	69.3	18.04
57.4	12.15	61.4	13.92	65.4	15.89	69.4	18.10
57.5	12.19	61.5	13.97	65.5	15.94	69.5	18.16
57.6	12.23	61.6	14.01	65.6	15.99	69.6	18.22
57.7	12.27	61.7	14.06	65.7	16.04	69.7	18.28
57.8	12.32	61.8	14.11	65.8	16.10	69.8	18.33
57.9	12.36	61.9	14.15	65.9	16.15	69.9	18.39

°F	gm/M ³	°F	gm/M ³	°F	gm/M ³	°F	gm/M ³
70.0	18.45	74.0	20.97	78.0	23.79	82.0	26.91
70.1	18.51	74.1	21.04	78.1	23.87	82.1	26.99
70.2	18.57	74.2	21.10	78.2	23.94	82.2	27.08
70.3	18.63	74.3	21.16	78.3	24.02	82.3	27.16
70.4	18.69	74.4	21.23	78.4	24.09	82.4	27.24
70.5	18.76	74.5	21.30	78.5	24.17	82.5	27.33
70.6	18.82	74.6	21.36	78.6	24.24	82.6	27.41
70.7	18.88	74.7	21.43	78.7	24.32	82.7	27.49
70.8	18.94	74.8	21.50	78.8	24.39	82.8	27.57
70.9	19.00	74.9	21.57	78.9	24.47	82.9	27.66
71.0	19.06	75.0	21.64	79.0	24.54	83.0	27.74
71.1	19.12	75.1	21.71	79.1	24.62	83.1	27.83
71.2	19.18	75.2	21.78	79.2	24.69	83.2	27.91
71.3	19.25	75.3	21.85	79.3	24.77	83.3	28.00
71.4	19.31	75.4	21.92	79.4	24.85	83.4	28.08
71.5	19.37	75.5	21.99	79.5	24.93	83.5	28.17
71.6	19.43	75.6	22.05	79.6	25.00	83.6	28.26
71.7	19.49	75.7	22.12	79.7	25.08	83.7	28.34
71.8	19.56	75.8	22.19	79.8	25.16	83.8	28.43
71.9	19.62	75.9	22.26	79.9	25.23	83.9	28.51
72.0	19.68	76.0	22.33	80.0	25.31	84.0	28.60
72.1	19.74	76.1	22.40	80.1	25.39	84.1	28.69
72.2	19.81	76.2	22.47	80.2	25.49	84.2	28.78
72.3	19.87	76.3	22.55	80.3	25.55	84.3	28.86
72.4	19.94	76.4	22.62	80.4	25.63	84.4	28.95
72.5	20.00	76.5	22.69	80.5	25.71	84.5	29.04
72.6	20.06	76.6	22.76	80.6	25.78	84.6	29.13
72.7	20.13	76.7	22.83	80.7	25.86	84.7	29.22
72.8	20.19	76.8	22.91	80.8	25.94	84.8	29.30
72.9	20.26	76.9	22.98	80.9	26.02	84.9	29.39
73.0	20.32	77.0	23.05	81.0	26.10	85.0	29.48
73.1	20.39	77.1	23.12	81.1	26.18	85.1	29.57
73.2	20.45	77.2	23.20	81.2	26.26	85.2	29.66
73.3	20.52	77.3	23.27	81.3	26.34	85.3	29.75
73.4	20.58	77.4	23.35	81.4	26.42	85.4	29.84
73.5	20.65	77.5	23.42	81.5	26.51	85.5	29.93
73.6	20.71	77.6	23.49	81.6	26.59	85.6	30.02
73.7	20.78	77.7	23.57	81.7	26.67	85.7	30.11
73.8	20.84	77.8	23.64	81.8	26.75	85.8	30.20
73.9	20.91	77.9	23.72	81.9	26.83	85.9	30.29

°F	gm/M ³	°F	gm/M ³	°F	gm/M ³	°F	gm/M ³
86.0	30.38	90.0	34.23	94.0	38.50	98.0	43.22
86.1	30.47	90.1	34.33	94.1	38.61	98.1	43.34
86.2	30.57	90.2	34.44	94.2	38.73	98.2	43.47
86.3	30.66	90.3	34.54	94.3	38.84	98.3	43.59
86.4	30.75	90.4	34.64	94.4	38.95	98.4	43.72
86.5	30.85	90.5	34.75	94.5	39.07	98.5	43.84
86.6	30.94	90.6	34.85	94.6	39.18	98.6	43.96
86.7	31.03	90.7	34.95	94.7	39.29	98.7	44.09
86.8	31.12	90.8	35.05	94.8	39.40	98.8	44.21
86.9	31.22	90.9	35.16	94.9	39.51	98.9	44.34
87.0	31.31	91.0	35.26	95.0	39.63	99.0	44.46
87.1	31.41	91.1	35.37	95.1	39.74	99.1	44.59
87.2	31.50	91.2	35.47	95.2	39.86	99.2	44.71
87.3	31.60	91.3	35.58	95.3	39.98	99.3	44.84
87.4	31.69	91.4	35.68	95.4	40.10	99.4	44.97
87.5	31.79	91.5	35.79	95.5	40.22	99.5	45.10
87.6	31.89	91.6	35.89	95.6	40.33	99.6	45.22
87.7	31.98	91.7	36.00	95.7	40.45	99.7	45.35
87.8	32.08	91.8	36.10	95.8	40.57	99.8	45.48
87.9	32.17	91.9	36.21	95.9	40.68	99.9	45.60
88.0	32.27	92.0	36.31	96.0	40.80	100.0	45.73
88.1	32.37	92.1	36.42	96.1	40.92	100.1	45.86
88.2	32.46	92.2	36.53	96.2	41.04	100.2	45.99
88.3	32.56	92.3	36.63	96.3	41.16	100.3	46.13
88.4	32.66	92.4	36.74	96.4	41.28	100.4	46.26
88.5	32.76	92.5	36.85	96.5	41.40	100.5	46.39
88.6	32.85	92.6	36.96	96.6	41.51	100.6	46.52
88.7	32.95	92.7	37.07	96.7	41.63	100.7	46.65
88.8	33.05	92.8	37.17	96.8	41.75	100.8	46.79
88.9	33.14	92.9	37.28	96.9	41.87	100.9	46.92
89.0	33.24	93.0	37.39	97.0	41.99	101.0	47.05
89.1	33.34	93.1	37.50	97.1	42.11	101.1	47.19
89.2	33.44	93.2	37.61	97.2	42.24	101.2	47.32
89.3	33.54	93.3	37.72	97.3	42.36	101.3	47.46
89.4	33.64	93.4	37.83	97.4	42.48	101.4	47.59
89.5	33.74	93.5	38.94	97.5	42.61	101.5	47.73
89.6	33.83	93.6	38.05	97.6	42.73	101.6	47.86
89.7	33.93	93.7	38.16	97.7	42.85	101.7	48.00
89.8	34.03	93.8	38.27	97.8	42.97	101.8	48.13
89.9	34.13	93.9	38.39	97.9	43.10	101.9	48.27

°F	gm/M ³
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102.0	48.40
102.1	48.54
102.2	48.67
102.3	48.81
102.4	48.95
102.5	49.09
102.6	49.22
102.7	49.36
102.8	49.50
102.9	49.63

103.0	49.77
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104.0	51.19
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105.0	52.65
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106.0	54.12
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107.0	55.64
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108.0	57.18
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109.0	58.77
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110.0	60.36
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