

Wolfgang von Ohnesorge

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This manuscript got started when one of us (GHM) presented a lecture at the Institute of Mathematics and its Applications at the University of Minnesota. The presentation included a photograph of Rayleigh and made frequent mention of the Ohnesorge number. When the other of us (MR) enquired about a picture of Ohnesorge, we found out that none were readily available on the web. Indeed, little about Ohnesorge is available from easily accessible public sources. A good part of the reason is certainly that, unlike other “numbermen” of fluid mechanics, Ohnesorge did not pursue an academic career.

The purpose of this article is to fill the gap and shed some light on the life of Wolfgang von Ohnesorge. We shall discuss the highlights of his biography, his scientific contributions, their physical significance, and their impact today.

1 Biography of Wolfgang von Ohnesorge

1.1 Family background and early years

Wolfgang von Ohnesorge was born on September 8, 1901 in Potsdam. His full name was Wolfgang Feodor Hermann Alfred Wilhelm [1]. His family had extensive land holdings in the area of Poznan, then part of Prussia. His father, Feodor von Ohnesorge, was a career military officer. As mentioned in his brother's wedding announcement [2], Feodor was a great-great-grandson of Gebhard Leberecht von Blücher, the Prussian general who, in conjunction with an allied army under the Duke of Wellington, defeated Napoleon at Waterloo¹. Feodor was decorated in World War I and retired in 1925 at the rank of Major General. He died only a year and a half later. Wolfgang attended the Augusta-Viktoria Gymnasium in Poznan and subsequently the Klosterschule Roßleben in Thuringia, where he graduated in 1921. The "Klosterschule" is a prestigious private boarding school which was once affiliated with a monastery. The monastery was dissolved during the reformation, but the school continued and kept its name.

1.2 Student years

After graduating from high school, Wolfgang was admitted to the University of Freiburg, intending to study music and art history. Although he retained a love of music throughout his life, he changed his mind about pursuing it as a career. He did not attend the University of Freiburg. After working as a trainee in the mining and steel industries, he enrolled as a student of mechanical engineering at the Technical University (then called Technische Hochschule) in Berlin in the fall of 1922. He graduated with a diploma in April 1927. After a brief return to industry, he became an assistant at the Technical University in 1928, in the institute directed by Hermann Föttinger. He retained this position until 1933. Wolfgang later commented to his family that the experiments done for his dissertation were tedious and difficult,

¹Feodor's brother, Baron Johannes Leopold von Ohnesorge, was a lieutenant in the German Army and married in New York City in 1898 to Miss Natalie Conkling, the daughter of the pastor of the Rutgers Presbyterian Church. After the wedding, Johannes returned with his bride (now Baroness von Ohnesorge) to live in Saxe-Weimar, Germany. It does not appear that Feodor joined him on the trip to New York. The announcement mentions that the engagement ring worn by the bride was a family heirloom that had once been given to Blücher's wife.

and there were many initial failures. He submitted his doctoral thesis [3] on November 14, 1935 and was awarded a doctorate the following year. He presented a lecture based on his doctoral research on September 25, 1936 at the GAMM conference in Dresden, and a paper appeared in the proceedings [4]. Other participants at the GAMM conference included Prandtl, Schlichting, Tollmien, and Weber [5]. Web of Science shows more than 90 citations of Ohnesorge’s paper since 1975. A more condensed version of the paper was published in 1937 in the *Zeitschrift des Vereins deutscher Ingenieure* [6].

Wolfgang married Antonie von Stolberg-Wernigerode on April 20, 1929. A daughter, Gisela, was born in 1930. The marriage ended in divorce in 1934.

1.3 Post-graduation years, second marriage and World War II

Wolfgang’s original post-dissertation plan had been to work for Borsig, at the time the leading manufacturer of locomotives in Germany. However, the Great Depression derailed this plan. Wolfgang took up a position in the Eichverwaltung (Bureau of Standards). In this position he worked first in Berlin and then in Reichenberg (Liberec) in Bohemia [7].

On September 2, 1939, Wolfgang married Sigrid von Büнау. Sigrid’s mother Hildegard was from the Crevese branch of the Bismarck family.² The marriage led to five children [1, 8]: Johannes-Leopold (born 1940), Reinhild (born 1942), Elisabeth (born 1943), Wolfgang (born 1949), and Sigrid (born 1953).

Wolfgang remained in his position in the Eichverwaltung for a part of the war, but was eventually drafted into the army. He served first in France and then in Russia, where he was wounded. Because of this circumstance and his relatively advanced age, he was assigned [10], for three months in 1944, to guard duty at the Plaszow concentration camp near Cracow.³ He eventually

²The ancestry of the Bismarck family is extremely well documented [8]. The earliest known ancestor is Herbold von Bismarck (1200-1280). Friedrich “the Permutator” von Bismarck (1513-1589) got his nickname because of a land trade with the Elector of Brandenburg [9]. Crevese and Schönhausen were among the villages the Bismarck family obtained in this trade. The Bismarcks living today are descendants of one of two sons of Friedrich: Pantaleon (the “Crevese” line) and Ludolph (the “Schönhausen” line). The most well-known Bismarck (Otto von Bismarck) was from the Schönhausen line.

³This is the camp where Schindler recruited his work force.



Figure 1: Wolfgang von Ohnesorge in 1937.

succeeded in getting transferred back to regular military service. At the end of the war, he held the rank of lieutenant, was taken prisoner of war by the Soviets, but managed to escape [7]. He reunited with his family and moved west.

1.4 Work on behalf of the Johanniter order

After the war, Wolfgang and his family settled in Cologne. From 1951 to 1966 he was director of the Landeseichdirektion (Bureau of Standards) of the newly created state of Nordrhein-Westfalen [7]. During this period, the Jo-

hanniter order⁴ became a very important part of his life. The Johanniter are a Lutheran offshoot of the Knights Hospitaller with extensive historic connections to the Prussian royal family and the Prussian state. In modern times, they are mostly known for their sponsorship of a number of charities. The most visible one is the Johanniter-Unfallhilfe, which assists victims of traffic accidents (there is an analogue called the St. John Ambulance in English speaking countries). Wolfgang von Ohnesorge became a member of the order in 1951 (his father had also been a member [11]), and in 1952 he cofounded and became the first president of the Johanniter-Hilfsgemeinschaft. The goal of this organization was to assist families in need; in the post war years this meant primarily those affected by the war. Wolfgang chaired the organization until 1958. In recognition of his merits he was named Ehrenkommendator. A “Kommendator” is a regional leader of the order; an Ehrenkommendator is an honorary rank meant to be equivalent, but without the duties of an active Kommendator. In later years, Wolfgang became leader of the Subkommende (local district) of the Johanniter in Cologne and represented the order on the board of directors of an affiliated hospital.

Wolfgang von Ohnesorge died in Cologne on May 26, 1976.

2 Ohnesorge’s work and its significance

2.1 Research contribution of Wolfgang von Ohnesorge

Ohnesorge’s thesis was entitled “Application of a cinematographic high frequency apparatus with mechanical control of exposure for photographing the formation of drops and the breakup of liquid jets” and was carried out under the guidance of Profs Föttinger and Stenger at the Technische Hochschule Berlin (now TU Berlin). The key technical contribution was a sophisticated spark flash timing and variable exposure system that could be used to take magnified images of dripping and jetting phenomena with high temporal resolution. A representative sequence of images from the thesis is shown in Figure 3 at an imaging frequency of 300 Hz. His imaging system was also able to resolve the dynamics associated with more complex phenomena such as jetting as well as quasiperiodic transitions close to the onset of jetting such as ‘double dripping’ (see for example Abbildung 30, p. 67 of reference

⁴or, more formally, “The Bailiwick of Brandenburg of the Chivalric Order of Saint John of the Hospital at Jerusalem”

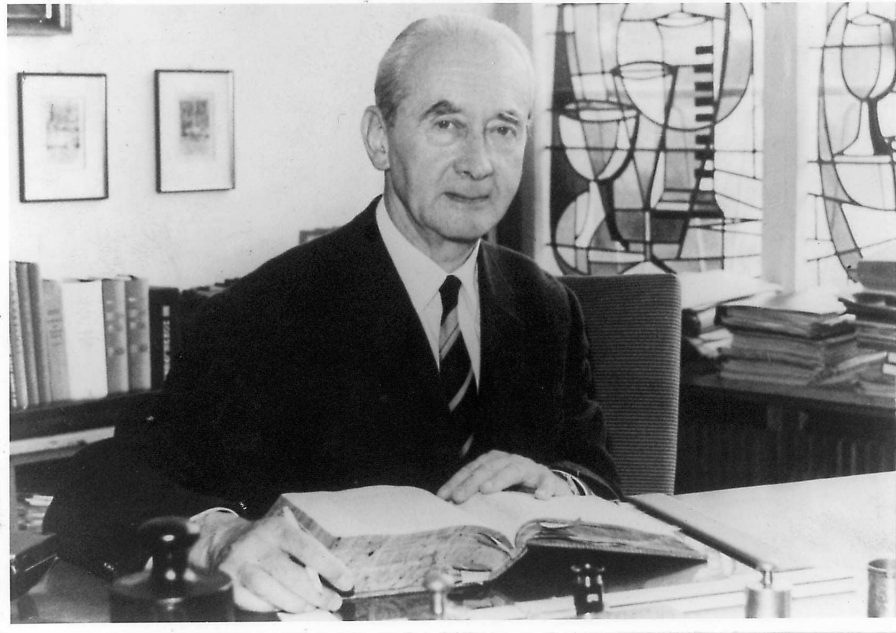


Figure 2: Wolfgang von Ohnesorge on the occasion of his retirement from the Landeseichdirektion in 1966.

[3]).

By varying the physical properties of the fluid exiting from the nozzle (water, aniline, glycerin and two hydrocarbon oils were studied in the thesis) as well as the speed of the exiting fluid stream, Ohnesorge showed that there were four important regimes, which were labeled 0 - III in the thesis and described as follows:

- 0) Slow dripping from the nozzle under gravity with no formation of a jet.
- I) Breakup of a cylindrical jet by axisymmetric perturbations of the surface (according to Rayleigh [12, 13]).
- II) Breakup by screw-like perturbations of the jet (wavy breakup according to Weber-Haenlein [14, 15]).⁵

⁵The name of A. Haenlein is now largely forgotten. In 1931 he published an extensive photographic study of jet breakup for a range of viscous fluids which motivated the parallel theoretical analysis of Weber. These experiments documented the dramatic decrease in

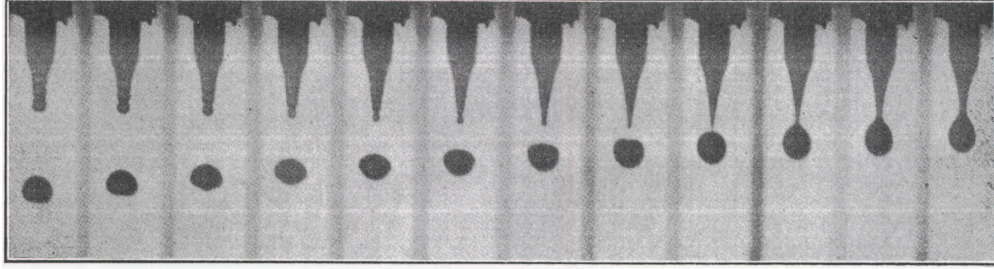


Abbildung 26. „Statische“ Tropfenbildung: $\frac{r}{a} = 0,52$. Bildfrequenz: 300 sek.⁻¹ *).

Figure 3: “Static” drop breakup (reproduced from Abbildung 26, p. 66 of reference [3]). The nozzle diameter ($d = 2r$) is expressed in terms of the ratio $r/a = 0.52$, where $a = \sqrt{\sigma/(\rho g)}$ is the capillary length (or “Laplace constant” as Ohnesorge refers to it) of the dripping fluid stream. The sequence of image frames plays from right to left as was customary in German hydrodynamic literature of the era.

III) Atomization of the jet.

An extensive discussion of dimensional analysis forms a large part of the thesis, and Ohnesorge investigated the relative importance of fluid inertia, viscosity and surface tension in controlling the transitions between the different documented modes of jet breakup in terms of the Reynolds number for the jet $Re = \rho V d / \eta$, the Weber number $We = \rho V^2 d / \sigma$, as well as several other nondimensional groupings. The thesis concludes with a demonstration that the clearest way of delineating the boundaries of the distinct operating regimes for the jet breakup problem is by defining a new *Kennzahl* or dimensionless group given by

$$Z = \frac{\eta}{\sqrt{\rho \sigma d}}. \quad (1)$$

Here η is the shear viscosity of the fluid, σ is the surface tension, ρ is the density, d is the jet diameter and V is the fluid speed. This is the original

the breakup length of a jet that accompanies transition from Rayleigh mode breakup to screw-symmetric or wavy perturbations in which aerodynamic effects play a significant role.

definition of what is now referred to commonly as the Ohnesorge number. The central findings of the thesis were published in the ZAMM article of 1936 [4] that featured a slightly expanded operating diagram (reproduced here in Figure 4) with data for two additional fluids (“gas oil”, i.e. diesel or heating oil and “ricinus”, i.e. castor oil) beyond those studied in the thesis.

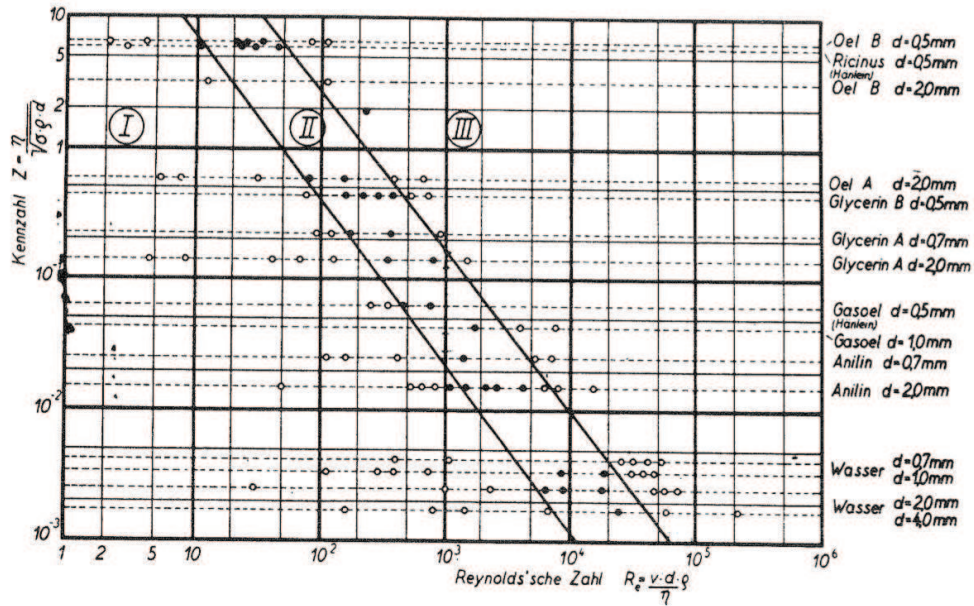


Figure 4: The operating diagram developed by Ohnesorge in his thesis to distinguish between the critical conditions for transition between different modes of breakup for a cylindrical jet exiting from an orifice. The dimensionless number or Kennzahl on the ordinate axis is now referred to as the Ohnesorge number. (Reproduced with permission from Wiley Interscience.)

2.2 Physical interpretation

Representing the experimental results on an operating diagram of the form in Figure 4 clearly delineates the transitions between different modes of breakup and it is immediately apparent that there appears to be a simple power law relationship between the critical Reynolds number and the corresponding

value of the dimensionless number Z , although Ohnesorge never gave such an expression (as is discussed further below). The dimensionless grouping of variables captured in the parameter Z can be best understood as a ratio of two time scales, the Rayleigh timescale for breakup of an inviscid fluid jet, $t_R \sim \sqrt{\rho d^3/\sigma}$, and the viscocapillary time scale $t_{visc} \sim \eta d/\sigma$ that characterizes the thinning dynamics of a viscously-dominated thread ([16, 17]):

$$Z = \frac{t_{visc}}{t_R} = \frac{\eta d/\sigma}{\sqrt{\rho d^3/\sigma}}. \quad (2)$$

The Ohnesorge number thus provides a ratio of how large each of these timescales is for a fluid thread or jet of diameter d , given knowledge of the fluid viscosity, density and surface tension. In typical jets (with $d \sim 1\text{mm}$) of low viscosity fluids (such as water or aniline), the Ohnesorge number is very small, $Z \ll 1$; in viscous liquids such as glycerin or machine oils, the Ohnesorge number can exceed unity.

Another very useful way to consider the physical significance of this grouping is to recognize that an appropriate Reynolds number for self-similar breakup of a fluid thread (in which there is no external forcing scale such as an imposed jet velocity V) is to use the capillary thinning velocity $V_{cap} \sim d/t_{visc}$ as the relevant velocity scale. This leads to an effective Reynolds number

$$Re = \frac{\rho(\sigma/\eta)d}{\eta} = \frac{\rho\sigma d}{\eta^2} = Z^{-2}. \quad (3)$$

Typically, as the characteristic length scale reduces, this effective Reynolds number decreases and viscous effects become increasingly important and the self-thinning process crosses over into a “universal regime” in which surface tension, viscosity and inertial effects are all equally important [16, 18]. However for special choices such as liquid mercury, inertially-dominated pinchoff (corresponding to $Z \ll 1$) can be observed even at nanometer length scales [19]. For further discussion of the resulting similarity solutions that govern breakup when $Re \ll 1$ and $Re \gg 1$ see Eggers [20].

2.3 Subsequent confusion and rediscovery

Ohnesorge’s paper was published in one of the leading mechanics journals of its time, and also presented at the GAMM conference in 1936. Nevertheless, the results were not as widely appreciated or uniformly incorporated into

the broader literature as they might have been, and this has led to some subsequent confusion and rediscovery. The reasons for this may be related to the complex geopolitical issues of the late 1930s.

A shortcoming of Ohnesorge’s paper is that he did not provide a quantitative expression for the functional form $Z = Z(Re)$ that is immediately apparent from Figure 4. Richardson [22] nevertheless attributes one to him. However, the expression given by Richardson, $Z \sim 2000Re^{-4/3}$ for the transition from Regime I (Rayleigh breakup) to Regime II (screw symmetric breakup), is inconsistent with Ohnesorge’s figure. A factor 200 gives a reasonable fit, so this could be a typo. Becher [21] notes the error and also the incorrect attribution to Ohnesorge. He comments that although the paper is written in “the turgid academic German of its period,” Richardson should have noticed the absence of the equation he attributes to it. This remark may be more of a sarcastic jab at Richardson than an actual comment on Ohnesorge’s paper. Becher’s paper does not give a corrected formula, but a later erratum suggests $Z \sim 50Re^{-4/3}$. This is also inconsistent with Ohnesorge’s plot. A closer inspection of the lines drawn by Ohnesorge on Figure 4 shows that they are actually not very well described by a power-law slope of $-4/3$, but instead by an exponent closer to $-5/4$. For the transition from Region I to Region II we would have a numerical relationship $Z \sim 125Re^{-5/4}$. A qualitatively similar functional form governs the onset of splashing on drop on demand printing applications as we discuss further below. Additional numerical confusion in describing these boundaries can also easily arise depending on the choice of radius or diameter $d = 2r$ in the characteristic scales for the Reynolds number and the Rayleigh timescale. Great care must be taken in comparing values from different literature.

When looking up Ohnesorge’s work in Web of Science, he suffers from a “nobleman’s curse.” His paper [4] appears in the Web of Science separately under both the entry “Vonohnesorge W” and “Ohnesorge W” while the shorter second paper [6] appears under the entry “von Ohnesorge W”. As a humorous aside, “Zerfall flüssiger Strahlen,” which means “breakup of liquid jets,” is translated by Web of Science as “decomposition of liquid irradiance.” It seems that automatic translation software still needs a little fine tuning. Overall, his 1936 ZAMM article has had approximately 90 citations since 1975.

The Laplace number is a quantity closely related to the Ohnesorge number, specifically, $La = Oh^{-2}$. Laplace, in conjunction with Young, is honored as one of the pioneers in the field of surface tension and capillary phenomena

[23, 24]; however, his work does not specifically involve dimensionless constants. The Laplace number figures prominently in Weber’s 1931 paper [14]; for instance the abscissa on his Figure 14 is equal to $2/9 La$. Weber does not name this dimensionless combination or introduce a symbol for it. Of course Weber’s name is already attached to a different dimensionless number. In a very widely cited work on liquid-liquid dispersion processes, Hinze [25] defines the same grouping of variables as eq. (1) simply as a “viscosity group.”

In the broader literature, the Laplace number has also been named the Suratman number. The review article of Boucher and Alves [26] cites Riley [27] as the source. Riley’s paper does not explain the origin of the term. Elsewhere in the paper, Riley cites a 1955 publication [28], in which one of the authors is named Suratman. Indeed, this is the only paper in the field which we were able to find and which has an author by that name. However, this paper does not introduce the “Suratman number” per se. Compilations of dimensionless variables, see e.g. [26, 29] now often give definitions of both the Suratman number and Ohnesorge number. Broadly speaking, it appears that in the jet breakup and atomization literature the name of Ohnesorge is more recognized, whereas in the emulsification and drop breakup literature the Suratman name is more familiar.

2.4 Present day applications

In present usage, the dimensionless grouping given by (1) is often referred to as the Ohnesorge number and given the symbol Oh . It provides a convenient way of capturing the relative magnitudes of inertial, viscous and capillary effects in any free surface fluid mechanics problem. With the rapid explosion in drop-on-demand and continuous ink-jet printing processes, understanding the relative balance of time scales captured by the Ohnesorge number, Oh (or equivalently the relative magnitude of forces in the equation of motion given by Oh^{-2}) is of central importance in understanding the dynamic processes controlling breakup as well as the shape and size of the droplets that are formed [20].

Of particular importance is the fact that the Ohnesorge number is independent of the external forcing dynamics (e.g. the flow rate Q or jet velocity V). It is solely a reflection of the thermophysical properties of the fluid and the size of the nozzle. Experiments with a given fluid and given geometry thus correspond to constant values of Oh , i.e. horizontal trajectories through

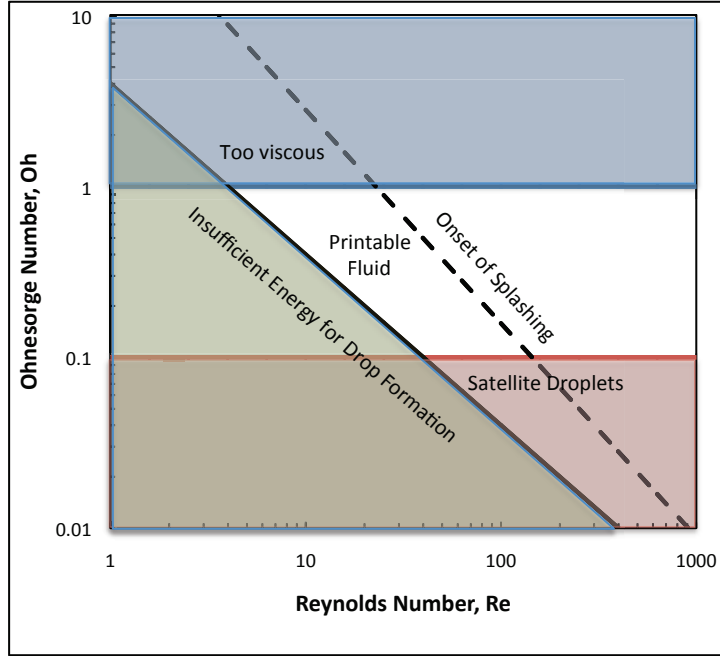


Figure 5: Figure 5. A schematic diagram showing the operating regime for stable operation of drop-on-demand inkjet printing. The diagram is redrawn from [33] using the Ohnesorge number as the ordinate axis in place of the Weber number $We = (ReOh)^2$. The criterion for a drop to possess sufficient kinetic energy to be ejected from the nozzle is given by Derby as $We_{crit} \geq 4$ or $Re \leq 2/Oh$. The criterion for onset of splashing following impact is given by Derby [33] as $OhRe^{5/4} \geq 50$.

an operating space (which may be unknown *a priori*) such as the one sketched originally by Ohnesorge. By contrast, numerical simulations of such processes typically employ more familiar dynamical scalings in terms of the Reynolds number and the Weber number, which both vary with the dynamical forcing (V). Experiments and simulations for a specific fluid thus correspond to fixed values of the ratio $\sqrt{We}/Re = Oh$ (as recognized for example by Kroesser and Middleman, [30]). By keeping this ratio constant, numerical simulations can be used to systematically explore transitions between different drop pinchoff regimes, drop sizes and formation of satellite droplets; see, for example [31, 32]. Successful operation of drop on demand inkjet printing

operations becomes increasingly difficult as the fluid becomes more viscous, or the droplet size becomes smaller so that the Ohnesorge number exceeds unity. The Ohnesorge number also plays an important role in droplet deposition processes when fluid droplets impact the substrate on which they are being printed/deposited. A recent review of this field has been presented by Derby [33]⁶ and the existing knowledge of relevant transitions in terms of critical values of We , Re and Oh can again be succinctly summarized in terms of an operating diagram reminiscent of Ohnesorge’s figure above (Fig. 4). If the axes are selected to be the Reynolds number and the Ohnesorge number this operating diagram takes the form shown in Figure 5. In Ohnesorge’s terms, the lower diagonal line marks the boundary between regimes 0) and I) (not plotted by Ohnesorge), while the second diagonal line is of the same functional form as the boundaries separating regimes (I), (II) and (III) in Fig. 4. In this respect, both operating diagrams capture a number of the key physical boundaries that constrain the operation of a particular commercial process.

As inkjet processes become increasingly sophisticated and fluids with more complex rheology (e.g. biological materials, polymer solutions or colloidal dispersions) are deployed, additional dimensionless groupings must also become important in fully defining the operating conditions for a particular process. Following Ohnesorge’s lead, it makes physical sense to isolate dynamical effects into a single dimensionless variable (e.g. a Reynolds number, Re or if preferred a Weber number or capillary number) and then group the remaining material properties in terms of ratios of relevant time scales; for example in drop pinchoff and jetting of polymer fluids, the ratio of the polymer relaxation time to the Rayleigh time gives rise to a Deborah number $De = \lambda/\sqrt{\rho d^3/\sigma}$ [34].

In this short note we hope to have provided some interesting historical background on Wolfgang von Ohnesorge and also clarified his specific contributions to the research literature on atomization and jet breakup. The continuous growth in the importance of inkjet printing processes [35, 36] in a wide variety of commercial and manufacturing fields is likely to keep his

⁶In this review article [33], Derby defines an Ohnesorge number as $Oh = \eta/\sqrt{\rho\sigma d}$ (this is consistent with our equation (1), but he then also denotes this as $Oh = 1/Z$ (resulting in his parameter Z being the inverse of the original definition of Ohnesorge). The source of this additional confusion is attributed to Fromm [31], but in fact Fromm does not introduce any dimensionless parameter Z or Oh , but presents his results directly in terms of ratios of the square root of the Weber number and Reynolds number.

name relevant for the foreseeable future.

Acknowledgment

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