



On a Dynamical Origin for Fermion Generations

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Abstract

We investigate a proposal to address several outstanding shortcomings of the perturbative Standard Model (SM) of particle physics, specifically a common, dynamical origin for the number of fermion generations, the spectrum of fermion masses and for Charge-Parity (CP) violating processes. The appeal of this proposal is that these features are a manifestation of the non-perturbative sector of the SM, requiring no assumptions about new physics beyond presently attainable experimental limits.

In this thesis we apply non-perturbative techniques, which have been used to investigate dynamical symmetry breakdown in other quantum field theories, to the $U(1)$ hypercharge sector of the SM, peculiar among chiral gauge theories in that the same gauge field mediates both interacting sectors.

We consider two models, a toy 4-fermion model containing explicit chiral symmetry-breaking terms (of an anomalous origin) and the quenched hypercharge gauge interaction, which complement each other. The key difference between this theory and studies of “conventional” breaking (such as in the gauged Nambu Jona Lasinio model) is the realisation that here fermion pairing terms associated with dynamical chiral symmetry breakdown are a necessary but not sufficient requirement for dynamical mass generation, analogous to the pseudogap phenomenon observed in systems of strongly-correlated electrons.

Understanding of how the mass, generations and CP-violation might arise are first investigated in the toy 4-fermion model. It is shown that different scale-invariant 4-fermion operators are present for the three subspaces of the full theory, enabling self-consistent introduction of three fermion generations.

The second part of the thesis is concerned with dynamical fermion mass generation in the quenched hypercharge interaction. In particular we follow the successful procedure developed for QED, developing a 1-loop renormalisable vertex ansatz for solution of the fermion self-energy Dyson-Schwinger equation. In the absence of dynamical fermion-antifermion bound states it is found there exist two mass “gaps” corresponding to two types of condensate. These “gaps” cannot be interpreted as physical fermion mass. It is suggested that only after the incorporation of the composite scalars does the self-energy equation admit multiple (physical) solutions. An alternative

possibility, that of a rearrangement of fermionic degrees of freedom analogous to spin-charge separation (SCS) in condensed matter physics, is also briefly outlined.