Stability And Efficiency Of Passive Dynamic Walker With Torso And Simple Controller

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I hereby certify that this material, which I now submit for assessment on the programme of study leading to the award of PhD of Computer Applications is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within text of my work.

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Abstract

Passive dynamic walkers are a body of robots, both simulated and real-world, that can "walk" down a slightly inclined plane powered only by gravity and eventually acquire a stable periodic gait Of particular interest is the fact that the motion appears "human-like" Performance indicators such as efficiency, step period etc are also commendable Common to all previously modelled creatures is that a hip mass is utilised to represent a torso - an omission that is tackled here

An upper body, represented as an inverted pendulum, is added to a passive creature To keep the body in an upright position, a simple controller applies a varying torque as necessary Periodic gaits are achievable, both stable and unstable, where stability is contrived through the addition of a damper Performance indicators are as good as those of the body-less creatures indicating that the torso is not a hindrance Finally the addition of further dampers at the hip joint can improve performance

Acknowledgements

Throughout the four years it took to complete this body of work many people provided endless amounts of support and encouragement - all that was necessary to complement the academic work and make it bearable It is here that I wish to thank all - there are too many names to mention so I will just say "Thank you family (brother, sisters, niece, cousins, aunts, uncles) and friends (both in college and outside)!"

Special words of thanks go particularly to my parents, Brendan and Angela and grandparents Michael and Mary - who instilled in me the confidence to never give up

Finally thanks go to my supervisor Dr Michael Scott for all the academic advice and guidance - I think if I stayed in DCU any longer they would be dedicating a wing to me!

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Chapter One

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1.1 Introduction

Animating creatures, or articulated figures, can in essence be split up into two categories of approach *kinematic* and *dynamic* modelling

Kinematic Kinematic animation is concerned only with the specification of joint angles and velocities over time and does *not deal with the forces and torques* affecting a creature

Dynamic Physical based animation incorporates the rules of physics into the modelling process to generate realistic motion "*Realism*" here refers to behaviour consistent with a simulated model of the real world Incorporation of dynamics brings extra problems i e integration of the equations of motion over time is computationally expensive and cumbersome and the provision of control forces and torques to the creature is complex

There are broadly speaking two approaches to the method integrating physics into the creation of lifelike animation of creatures which are outlined in the next two sections

1.1.1 Trajectory based animation

The first poses the problem in terms of a *trajectory* through state-space and time, which is subject to the constraints of the desired motion Therefore a typical problem would deal with minimising a certain objective (e g minimum control energy) subject to certain constraints (e g be in position a at time t_0 and in position b at time t_1)

One restriction of dealing with motions as trajectories is that it is difficult to properly incorporate interactions with the environment Discontinuities in the motion, such as those caused by contact with the ground, pose difficulties for many optimisation techniques In addition, a new trajectory must be generated for each new desired motion Two advantages associated with this method however are that it relates well to the idea of key-framing, and that these techniques are also able to find the most plausible solution, even if no physical solution is possible (e g walking on water) A more detailed discussion of one method of posing the problem in a format based upon desired trajectory is outlined in [Ega96]

1.1.2 Control Algorithms

The second method is to utilise a *controller* or *control algorithm*, where a controller makes control decisions based upon a mechanical simulation and as such does not explicitly calculate a trajectory Therefore, the problem is one of the user providing the creatures construction and posing the question "How would it move?" The motion of a creature is thus made up of a sequence of control algorithms, with each control algorithm providing a particular type of motion e g walking, jogging, running etc *Physically built controllers*, require much user assistance and manual tweaking must be performed to provide correct motion In most cases the control system is decoupled and separate algorithms are needed to perform the various different kinds of motion required (e g hopping or skipping or walking or running etc)

It is therefore more useful to synthesis a controller and then maybe build one However synthesising controllers is not problem free Complex control algorithms utilising intricate algorithms such as neural-networks, genetic algorithms etc have been formulated providing realistic animation - (see [Ega97] for a more detailed discussion) A major drawback of these approaches is that researchers are able to provide motion to "certain" creatures in "certain" situations but are unable to provide widespread animation To provide a variation in gait e g changing from walking to running requires *reformulation* of the problem Also while controllers increase the autonomy of the creature thus reducing user input, they also reduce user control The *more* complex the control algorithm the *less* control the user has

1.2 Biped locomotion

Of primary concern in this thesis is bipedal locomotion - movement of two-legged creatures The main advantage of bipedal locomotion is its naturalness bipeds should be able to traverse whatever terrain they are in, much as a human might What follows is a brief discussion, not intended as a complete review, of some research in each area of the three disciplines given above

1.2.1 Human Motion analysis

The first area of bipedal research is purely medical based and involves capturing actual human data and analysing it Hurmuzulu's laboratory [Hur00] has been developing quantitative measures to assess the dynamic stability of human locomotion, where the analytical methodology is based on Floquet theory He carried out a study comparing the gait kinematics and dynamics of polio survivors with that of non-paralysed humans utilising graphical and analytical tools Phase plane portraits and first return maps were used as graphical tools to detect abnormal patterns in the sagittal kinematics of polio gait He concluded that polio patients walked less symmetrically than "normal" people did and that their motion was also less stable then "normal" people

1.2.2 Human Simulation

In her laboratory Hodgins et al [Hod00] are interested in providing animations, primarily of humans involved in various activities such as running, bicycling and diving The goal of their research is two-fold firstly realistic characteristic motion and secondly high level control by the animator and underlying simulation carried out by the machine These motions are achieved through application of control algorithms to the physically realistic model of the human that is being animated The physical model of a human is taken from the mass and inertia properties prevalent in the biomechanics literature The control algorithms involve the use of inverse kinematics, proportional-derivative control laws, state machines, active control laws and synergies - a complete published list is available on the web site [Hod00] In addition secondary motion and group behaviours have been added to the simulation to increase complexity and realism

Simulation is not without its difficulties and some of the problems that have been encountered are as follows adapting behaviours to new *actors* is difficult because a control system that is tuned for one character will not work on a character with different limb lengths, masses, or moments of inertia New *activities* need new controllers and also creating appropriate transitions from one behaviour (either existing or new) to the next can be a challenging problem While these problems have been solved the processes involved can be quite complex and may not lead themselves to a physical implementation in robotic form

1.2.3 Legged Robots

The body of work contained in this thesis falls primarily into the third and final area of research i e legged robots A list of biped robot researchers can be found at [Cal00], but what follows are examples of some of the more successful creatures that were built

The Massachusetts Institute of Technology [Mit00] has been successful in building legged robots for the past two decades Led by Marc Raibert [Rai86] the MIT Leg Laboratory explores active balance and dynamics in legged systems, robots and animals alike Activities for the robots are made up of a combination of simple algorithms that focus on support, posture and propulsion, thus providing balance and basic control A single set of control algorithms, modified in various ways, has successfully controlled numerous running machines as well as hopping, gymnastics etc Several simple algorithms currently under development have had promising results on walking machines According to the lab web-site *"the ability of simple algorithms to operate under these diverse circumstances suggests their fundamental nature"* [Mit00] A number of bipedal creatures in particular have been created including the spring turkey and planar biped

Again, there are a number of problems with the research partaken by the laboratory For each creature separate control algorithms must be formulated for each walking activity so reusability would be an issue Each creature also requires a relatively small, but in the long run, a considerable amount of power to keep in motion Finally taking for example the spring turkey, the construction costs i e $\approx$ \$100,000 are substantial

In Japan the Honda Corporation has successfully built a humanoid robot known as *P1* [Hon00] This robot with human-like appearance is versatile, capable of walking sideways as well as forwards and can traverse stairs and is robust enough to tolerate pushing Originally designed as a possible home robot several generations have evolved (the newest version available is *P3*) but still there are a number of problems in existence These are namely the high price tag (in the region of millions of dollars), low battery life (in the region of minutes) and limited intelligence (a person is constantly needed to operate the robot) Honda aims at improving performance and operability in future models

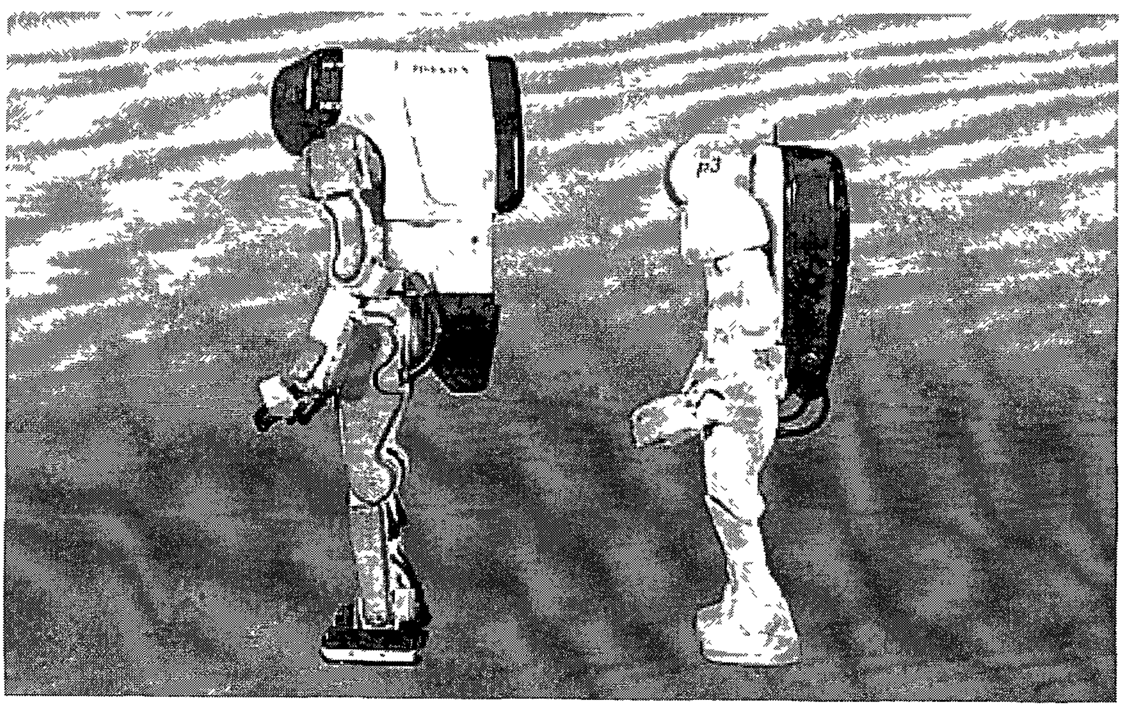


Fig 1.1: *The Honda robots (P2 and P3), © Honda Corporation Ltd*

Katoh and Mori constructed a biped with very simple dynamics and telescopically retractable legs [Kat84] It consists of a three degree of freedom model with independently adjustable leg lengths Mura and Shimoyama built a robot that generated gait by linear feedforward control, joint torque schedules were pre-calculated and played back on command [Miu84] Hurmuzulu created a kneeless biped with an additional body mass connected to the hip through a pelvic joint [Hur86] Particular attention was paid to the effect of the robots' impact with the ground and the impact conditions were justifiably considered as an integral part of the governing equations Central to the robots mentioned is that fact that all have some form of actuation Controlling this actuation, if applied, has involved the use of complex control algorithms

1.3 Passive Dynamics

Another topic of research is based upon bipedal creatures that have no actuation except the passive interaction of gravity, inertia and collisions and have no control system i e *passive dynamic* creatures

Def: A passive dynamic creature is one whose motion is fully determined by gravity, inertia and collisions and involves no control system [Gos96b]

The philosophy here is to solve a simple system to get a better insight into the underlying mechanics of complicated systems Then small amounts of power can be added in efficient ways to allow them to walk on level ground or up a hill and simple control mechanisms can be introduced to increase the stability of the motion

The rest of this thesis is organised as follows

- Chapter two introduces previous work on this topic and identifies the missing component common to all passive dynamic creatures namely the inclusion of a torso

- Chapter three outlines the mechanics of the creature In formulating the dynamics, the equations of motion for the creature along with the impact of the collision with the ground are taken into account
- Chapter four indicates how this motion will be analysed Poincaré maps are formulated and Newton's method is used to find fixed points These fixed points are then classified as either stable or unstable
- Chapter five gives the results attained for the creatures that are dealt with here The initial part of this chapter involves results that correlate with results for similar creatures i e for a body-less creature and the remaining gives previously unpublished results
- The final chapter identifies the conclusions gained and possibilities for future work

Chapter Two

Passive Dynamic Walkers

2.1 Passive Dynamic Walking

In 1980 Mochon and McMahon [Moc80] argued from electromyographic data that humans were not actively controlling most of their movements during walking. Other EMG studies, more recently published for instance in [Ros94], indicate that much of human walking may indeed be *passive* i.e. muscles are not used in significant quantities to provide movement. Inspired by the research [Moc80b] on ballistic walking (Ballistic walking is considered to be the most fundamental, and therefore the most revealing, approach to bipedal walking, involving creatures walking in a ballistic fashion i.e. legs swing and impact with the ground), Tad McGeer designed and analysed a passive dynamic walker [McG90]. This consisted of a simple rigid two-legged creature 'walking' down a shallow slope with no outside control or additional energy input i.e. it was powered by gravity alone. Thus the passive-walking pattern is determined by the natural frequency of the mechanical system. An interesting characteristic determined was that the creature achieved a *stable limit cycle* that looked almost human-like. One interpretation of a limit cycle means that one step only needs to be fully determined as all subsequent steps are just "copies" of it and stability indicates that any disturbance that occurs is rectified and the creature keeps walking. An extension given by McGeer [McG90b] was to include knees, which provided natural ground clearance, and again a stable limit cycle was achieved. These creatures were initially simulated and then later built.

In addition to pioneering the passive-dynamic approach to gait study, McGeer utilised a *Poincaré map* as a means of analysing the given simulation results. Other authors as shall be seen in section 2.3 have made improvements on the characteristics of passive creatures through the use of dampers and simple control laws. In addition the analysis of that motion has become more adept over the years.

2.2 Why study passive dynamic walking?

Designing and building biped robots is fuelled by the potential advantages they would provide Biped robots are better suited to working in hazardous environments, such as chemical spills, or exploration on unsuitable terrain such as on another planet and more especially in rehabilitation technology (i e as an alternative to wheel-chairs whereby paralysed people could actually walk again) Science fiction even dictates the possibility of front-line fighting in a war situation using bipeds At present one of the main obstacles to a wider application of legged robots is their lack of energy efficiency Much work has taken place on overall gait synthesis based upon brain control and muscle power leading to impressive but limited creatures The reasoning behind the study of passive dynamic walking can be summarised as follows

1 It makes for mechanical simplicity and relatively high efficiency McGeer's results and those of the researchers that emulated his work provide animation that is both humanlike and stable Trying to get a fundamental understanding of how humans walk from a mechanical point of view could prove useful in providing control later
2 The simplicity promotes understanding McGeer used the analogy of powered flight research [McG90] The Wright brothers began by studying and building gliders Once they fully understood the concepts of "unpowered" flight, adding power (i e engine) was only a minor change The concept therefore is to start with a machine with no active control and then the addition of control should be uncomplicated
3 Evidence exists (in the form of EMG results) that a minimal amount of control and actuation is necessary for some basic human motions, including gait [Gos98a] At the heart of these motions, the body is at or very close to a limit cycle As already outlined EMG studies have shown relative muscle inactivity during the swing phase of human motion [Ros94] that could be termed "passive" Of course an equally legitimate approach to achieving stable and efficient walking is to start with arbitrary amounts of control and actuation and then to gradually minimise their role

Central to the study of planar passive walking is the <u>simplicity</u> of the model being considered By disregarding complex additional characteristics to the idea of motion such as complex control algorithms, optimisation, external torques and forces etc, more insights can be gained on the fundamentals of bipedal motion – which is currently not fully understood

Rule: The general motto of passive walking could be epitomised therefore as *starting from the <u>bottom up</u>*

2.3 Collection of Passive Creatures

Passive dynamic toys are not a new phenomenon and a collection of pictures of antique patented toys is attainable at [Cor00] However the concept of passive dynamic creatures in terms of serious analysis and design is relatively fresh Therefore literature on the topic of passive dynamic walking is quite limited and predominantly contains the analysis of three very similar creatures designed by three authors, McGeer's original, Goswami's Compass model and Garcia's Point Mass model

2.3.1 McGeer's Original Passive Dynamic Walker

Mc Geer's [McG90] model, the original, has two rigid legs connected by a frictionless hinge at the hip Each leg has an arc-style structure at the base, which act as feet The arc-like semi-circular feet are used as a mathematical convenience rather than a physical necessity There is a point mass at the joint of the two legs i e the hip, which serves as being a "crude torso " The *stance leg* is in constant contact with the ground while the *swing leg* moves similar to a swinging pendulum - thus the complete system is akin to a double pendulum The complete system can therefore be modelled by four generalised co-ordinates one for each leg angle and angle velocity This creature is based on the ballistic walker of Mochon and McMahon [Moc80] – a bipedal toy that walks down shallow slopes by rocking sideways This model however doesn't rock from side to side In the solution method given by McGeer a rimless wheel model was analysed first to provide basic insights, followed by the more complex creature described above The wheel had only a centre point and spokes (no rim) and the analysis involved isolating two side by side spokes Finally note that as an extension knees were added in and this creature is represented in a simple form in Fig 2 1 The addition of knees leads to there being 8 states

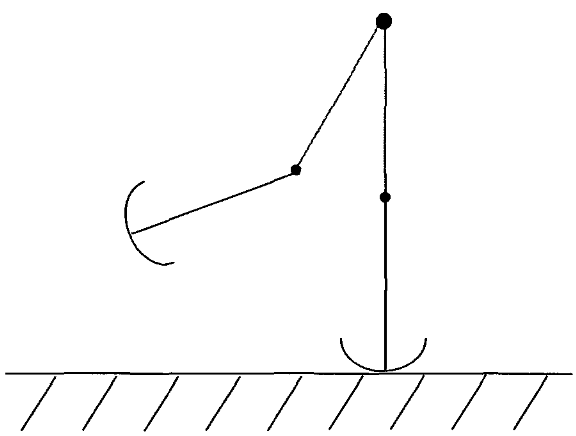


Fig 2.1: *Simple representation of McGeers' Passive Dynamic Walker with knees More details can be found at [McG90b]*

There are some general regulations that must be adhered too - but these are adopted by all models and as such are characteristic of passive creatures

- foot scuffing i e where the swing leg grazes the ground midway through its trajectory, is ignored
- collision of the feet with the ground is slipless plastic This means that the configuration of the creature stays the same and angular momentum is conserved
- finally foot transition (i e when one foot hits the ground and the roles of the legs are switched) is instantaneous

The solution process involved formulating the equations of motion of the swing phase, which are highly non-linear and a set of algebraic conditions to simulate heel-strike and the swapping of leg roles To solve the dynamics and to find limit cycles McGeer performed a linearisation about an equilibrium point i e the creature standing rigidly upright The flaw inherent in this method shall be outlined in section 2 4 Finally each step was modelled as a Poincaré map which could then be analysed for stability

The conclusions reached by McGeer are summarised as follows

- fixed points were found but these were not necessarily always stable
- efficiency can be measured as the minimum slope necessary to provide motion and the minimum angle γ found was 0 005 radians
- parameter changes were made and the effects noted scaling of leg mass, leg length and gravity may not destroy the limit cycle, moving centre of masses could destroy the limit cycle and addition of a hip mass improved efficiency

2.3.2 Compass Gait Creature

Others have adopted McGeer's original ideas Although the models that are used are not significantly different or improved from the original, it is the extent of analysis of passive walking that has advanced in recent years Goswami [Gos94] slightly modified the creature to form a compass-like biped This "compass-like" model is very similar in structure to that of McGeer's, except that there are no arcs present to resemble feet - instead there is just a point The problem of foot scuffing is avoided by including retractable mass-less lower legs (remember this is a simulation and those mass-less lower legs are plausible) The telescopic retraction of the leg solves the problem of foot clearance without affecting the robot dynamics The long-term motivation behind this study is to formulate a simple biologically inspired active control law of a 17-dof biped robot being built in project BIP co-ordinated by the INRIA laboratory in Grenoble, France [Bip00] The first prototype of this robot was built in March 2000 and successfully walks

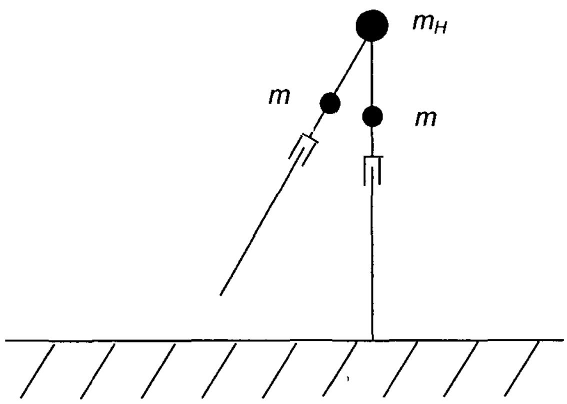


Fig 2.2: *Compass Gait Creature Note the absence of "feet" and use of retractable legs*

One notable augmentation in the solution method was the utilisation of the full non-linear equations As previously stated McGeer utilised a linearisation about an equilibrium point and this is discounted by Goswami In a later work [Gos96b] a comparison of both methods was carried out and this is outlined in section 2 5

Three parameters, namely the ground slope and normalised mass and length completely describe the creature Any continuous change in one of the parameters leads to an evolution of the steady gait through a regime of *bifurcations* leading to a chaotic state where no two steps are identical [Gos98] A bifurcation (or period doubling) indicates that each alternative step is repeated, and thus Goswami found that as the slope increases stable period one solutions transform into stable period two solutions and so on until eventually chaos is reached A necessary but not sufficient condition for the stability of such gaits is the contraction of the "phase fluid" volume and the volume contraction was thus computed Goswami added in passive dampers at the hip joint, to dissipate the energy build-up, and this results in a significant improvement in the stability and versatility of the gait (i e improving the maximum attainable stable slope) Finally Goswami also investigated the performance of several active control schemes which enlarged the basin of attraction of passive limit cycles and created new gaits [Gos97a] The notion of adding in dampers and outside control is addressed in section 2 7

In summary therefore the additional characteristics of passive walking found were

- possibility of using full non-linear equations
- period doubling (i e bifurcations) leading to a chaotic state
- addition of dampers at hip increase stability and versatility
- simple passivity mimicking laws can be added in

2.3.3 *Simplest Creature: Point Mass*

Garcia's "point-foot" [Gar98a] model is the most simplistic of all It is a deterministic generalisation of Alexander's non-deterministic theoretic "minimal" model [Ale95] This creature has no arcs for feet, instead having point masses (i e m) The hip-mass M is much larger than the foot mass m ($\cong$ 1000 times) so that the motion of a swinging foot does not affect the motion of the hip

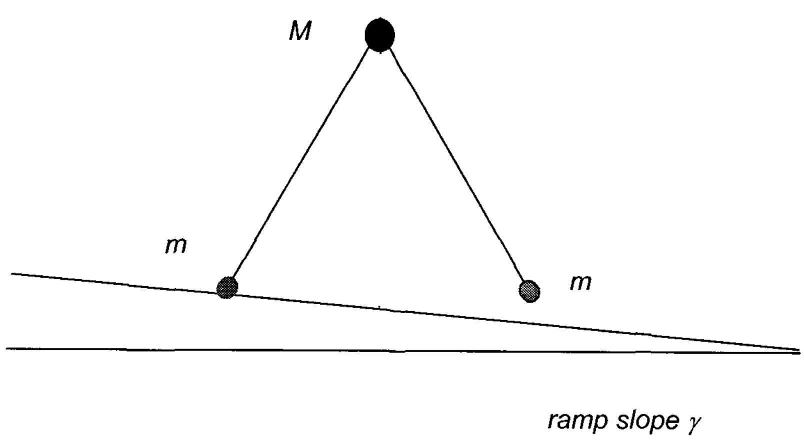


Fig 2.3: *Point mass creature*

This special mass distribution further simplifies the underlying mechanics and mathematics involved in the solution process This significant reduction also allows the author to perform analytical computation, estimate the initial conditions necessary and to form stability estimates of period one gaits After nondimensionalising the governing equations it was found that the only free parameter was the slope γ Again, similar to Goswami's solution method above, the full non-linear equations are utilised

The model displays two period one gait cycles, one of which is stable for $0 < \gamma < 0\ 015$ By increasing the slope γ beyond this value, stable cycles of higher periods appear, and the walking-like motions apparently become chaotic through a sequence of period doublings, which again agrees with the findings of Goswami

2.3.4 Other passive creatures of interest

Berkemeier and Smith [Ber97] extended the concept of passive dynamic walking from bipedal to quadrupedal locomotion The creature consisted of a pair of McGeer two-dimensional bipeds linked together by a 'spine' A rimless wheel model was analysed first to provide basic insights followed by a more complex model with free-swinging legs The gaits of the quadruped are more efficient than those of the biped but are unstable Future work was to evolve around stabilising this creature, but as of yet no results have been published

Camp [Cam97] demonstrated that a simple open-loop actuation/control scheme is all that is required to produce stable, powered, human-like walking motions in a set of roughly human-like legs By having a 'powered mode' the creature does not require a slope and can traverse level ground Stable and unstable gait limit cycles and period doubling, for a variety of structural, physical and control/actuation parameters were observed

The original passive walkers give a hip trajectory that is far from smooth However successful applications would require a smooth hip trajectory to protect the electronics of the creature from the large velocity changes due to ground collisions

Quint van der Linde [Qui98] showed that an actively adjustable stance leg compliance in combination with a viscous damping can result in smaller hip velocity changes

Work has also been carried out on motion in 3D McGeer's [McG91] numerical 3D studies only led to unstable period motions Garcia [Gar99] and Coleman [Col98] utilised a gradient search method to try to improve the unstable eigenvalues of McGeer's model Improvements were made but he still returned a maximum eigenvalue modulus that indicated instability (i e well above 1) Kuo numerically simulated a passive dynamic 3D model of walking but again did not find stable passive motions [Kuo98] Finally Coleman has a physical walker that walks and balances in 3D, but cannot stand still and does not yet know exactly which aspects of its physical description are needed to theoretically predict its stability with computer simulation [Col98]

Suggestions were given as to how to maybe stabilise models in three-dimensions and some of the suggestions include

- using ellipsoid or toroid feet [Gar99]
- using freely swinging arms [Gar99] - presumably a torso would be needed first!
- including ball-socket hips with torsional springs for stabilty [McG91]

2.4 Passive Dynamic Walker with Torso

Common to all passive creatures that have been developed up to now, is the omission of an extended torso and that is the primary goal of this body of work - to rectify that While addressing the issue of passive running McGeer [McG90a] indicated that a torso would *"have an important role as a torque-reaction partner, so this should be added to the model "* The added torso will be treated as another link, much akin to the well-known inverted pendulum problem Control will be needed to keep the body in an upright position and it is felt that the controller should be kept as simple as possible to preserve the simplicity of the creature The need to use this simple controller seems necessary and this fact is echoed by Ruina in [Rui97] *"the possibility that asymptotically stable balance can be achieved without control is somewhat* *unintuitive since top heavy upright things tend to fall down when standing still or more generally, since dynamical systems tend to fall down "*

In his proposal on future work in the area of passive dynamic walking Ruina [Rui97] suggested the importance of placing a torso onto a passive creature saying that *"Chopped at the waist theoretical and mechanical models may represent the motions of a more complete mechanism such a theoretical model might not have too great a relevance for healthy humans because the simulation of springs is most accurately accomplished with tiring co-contraction (which is often avoided by humans) But it does point towards the utility of passive measures for prosthetics and towards simple spring or damper simulating control laws "*

Finally note that in formulating the code involved in the solution for the bodied creature, a body-less creature shall also be considered The goal of this body-less creature is to attain the solutions previously published and as a building block for the "new" bodied creature

2.5 Linearisation versus full non-linear equations

Before the solution process begins one important decision must be made whether to use the full non-linear equations of motion, which shall be generated by the creature, or to perform some sort of linearisation

The process of solving the given problem has had two avenues of approach over the years McGeer [McG90] took the method of linearising the dynamic equations of the creature about an equilibrium point thus providing a simpler problem to deal with The *equilibrium point* was the creature standing perfectly upright, and this allowed explicit integration of the dynamical equations Next the collision equations with the ground were added and the conditions for the existence of a periodic solution of this coupled system were found In order to study the stability of this periodic solution a second linearisation about the periodic solution is necessary One means of determining efficiency for a creature is to determine the minimum slope attainable and McGeer numerically found walking motions for slopes as low as about 0 005 radians [McG90] The main problem with this approach is that the linear solution is valid only within a narrow region around the point of linearisation However for any real gait, significant deviation from this point is required [Gos96b]

The second approach as shall be adopted here is to utilise the full *non-linear* equations Advantages for this approach are outlined in the next paragraph though the main disadvantage is that you have to rely extensively on numerical simulations However, the computational burden is manageable as the robot model has a relatively small state space dimension

Goswami [Gos96b] used both techniques and compared them Apart from the fact that the non-linear approach has a much wider basin of attraction he found that the maximum slope attainable increases slightly This is due to the fact that for higher slopes, the robots dynamics involve larger state values (angles and velocities) which begin to render the linearisation (about an equilibrium point of the state vector being 0) invalid By comparing the linear and non-linear state vectors on equivalent slopes, he also found that the joint angles vary *less* sensitively than the joint velocities Finally the only energy source in the model, the mechanical energy, which comprises solely of the sum of the kinetic and potential energies i e $E = K E + P E$ also varies quite steeply between both methods Given that there is such variations he hypothesised that it would be more appropriate to use the full non-linear solution, which is in keeping with the approach of Garcia [Gar99]

2 6 Description of a stable passive period one gait.

It has been stated that passive dynamic creatures may possess stable limit cycles and it is this description that is now outlined For the purposes of outlining the motion of the creature *phase space* terminology shall be adopted Phase space is described as the space consisting of the generalised co-ordinate/generalised velocity variables i e state space [Gos96a] The phase space of the body-less creature is 4-dimensional (as shall be shown in chapter three) and for the bodied creature it is 6-dimensional, where the numbers correspond to the number of states present Since we cannot graphically visualise these high dimensional spaces, diagrams will be limited to the displacement and velocity of only <u>one</u> link This high-dimensionality also leads to problems in determining the size of the basin of attraction for the limit cycle

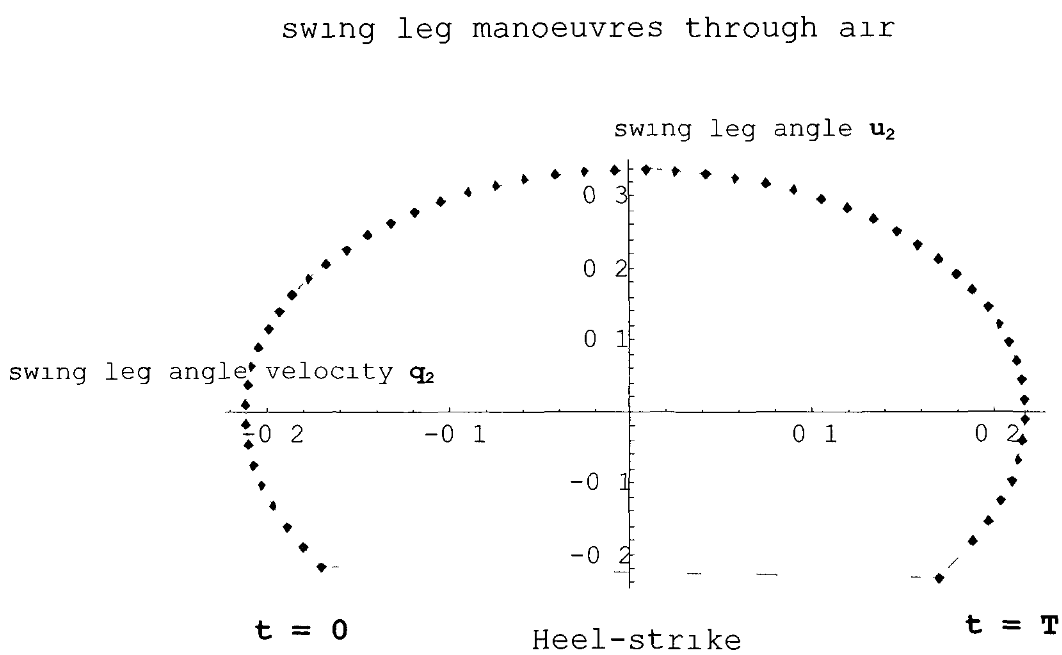


Fig 2.4: *Limit Cycle Note that this just represents the swing leg of the body-less creature The straight line represents the collision of the leg with the ground*

A *period one* gait is one, which repeats itself after every single time period, a period two is one which repeats itself after every two time periods and so on A loose definition of *stability* indicates that any disturbances to the gait get swallowed up and the creature keeps moving in an upright manner The phase space diagram in Fig 2 4 deals with the angle and angular velocity of the swing leg over time The step begins the moment after heel-strike has taken place At time $t = 0$, the pivot leg is in the stance position Immediately it becomes the swing leg, and the previous swing leg the stance leg, traverses up in the air, reaches a maximum point and descends At time $t = T$, the leg impacts with the ground (heel-strike) and a velocity jump is observed (i e the straight line in the diagram) Now the leg roles are <u>reversed</u> and next step continues The diagram shows the phase plane diagram for the body-less creature, with hipmass of 0, on a slope of 0 005

2.7 Poincaré Map

McGeer [Mc90] placed a step in terms of a Poincaré map - something which is prevalent in the non-linear dynamics literature e g [Ott93] What this basically means is that one step can be dealt with or encoded in terms of a complete function It is often useful to reduce a continuous dynamical system into a discrete one and this can be achieved through the use of a Poincaré map It is a tool developed by Henri Poincaré for a visualisation of the flow (i e continuous system) in a phase space of more than two dimensions If the phase space is N-dimensional then the Poincaré map has dimension N-1 Thus the Poincaré map represents a reduction of the N-dimensional flow to an N-1 dimensional map

The map itself is a carefully chosen (curved) surface in the phase space that is crossed by almost all orbits The Poincaré map maps the points of the Poincaré section onto itself For illustrative purposes take $N = 3$ with states $\{x_1, x_2, x_3\}$ The points A and B represent two successive crossings of the surface of section i e shaded region A can be used as an initial condition to find B and vice versa Thus the Poincaré map in Fig 2 5 shows the mapping of $\{x_1^{n}, x_2^{n}\}$ to $\{x_1^{n+1}, x_2^{n+1}\}$ and the Poincaré map section consists of the shaded region

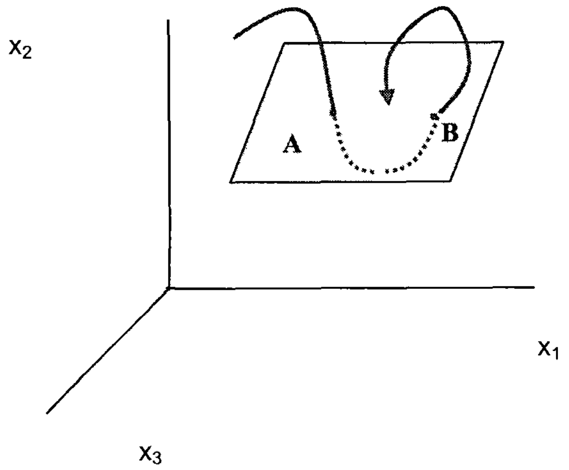


Fig 2.5 *Poincaré Map*

2 8 Active Control and Stabilisation Using Dampers

For the bodied creature as shall be outlined here, external torques are necessary to keep the body upright and to provide stability The use of these and passivity mimicking control laws enforced through dampers has been studied by Goswami [Gos97a] and McGeer [McG90a] primarily to increase performance The actual dampers and control laws that are used are highlighted in chapter four

Goswami's [Gos97a] <u>control laws</u> were founded on the mechanical energy principles of the system As the robot walks down on a slope its support point also shifts downward at every touchdown As it loses gravitational potential energy in this way its kinetic energy increases accordingly This is exactly the amount of kinetic energy that is to be absorbed at the end of each step by the impact By resetting the potential energy reference line to the line of touchdown (i e ignoring the slope which would lead to a decrease in potential energy), the total energy of the robot appears constant regardless of its downward descent The control law formulated attempted to bring the current energy level of the robot E to the target energy level $\hat{E}$ at an exponential rate What was introduced was a simple control law of the form $-\quad (u_1, u_2)$ The control law is implemented through the use of torques at either the ankle, or hip or both For instance the hip torque has the form $u_H = \frac{- \ (E-\hat{E})}{u_2 - u_1}$, where is a constant value (taken to be 0 1) The overall effect of this was a demonstration that the basin of attraction of the limit cycle could be significantly enlarged Another control law which attempted to maintain a specified average speed of progression based upon the velocity of the robot enables it to walk up a slope

McGeer [McG90a] employs <u>active control</u> to try to stabilise unstable cycles in his running creature He also states that the running cycle can be modulated to allow, for example, crossing unevenly-spaced stepping stones Step-to-step modulation is provided for by linearisation of the stride function Then active stabilisation is achieved through the use of the Linear Quadratic Regulator Algorithm

2.9 Goal of this research work

At this point it should be appropriate to highlight the purpose and eventual goals of this body of work Previous research as has been outlined in this chapter consists of body-less creatures and it is this omission that shall be tackled, as a torso leads to a more complete and realistic creature The primary reference or source model utilised shall be that of McGeer's - see section 2 3 1 and the objectives therefore are

- addition of extra link, i e torso into the creatures description
- keep this link upright but in accordance with the philosophy of passive dynamic walkers using as simple a controller as possible
- identify whether limit cycles exist, and the possibility of bifurcations leading to chaos
- analyse stability, efficiency, and performance indicators
- try and improve on performance through the addition of extra dampers

Chapter Three

The Solution Process

3.1 Introduction

Thıs chapter detaıls the solutıon process ınvolved ın generatıng the creatures motıon Thıs procedure has very defınıte ındıvıdual components and the complete method leads to the generatıon of a sıngle *step* The first component wıll be to determıne the confıguratıon of the creature beıng studıed Two creatures are consıdered ın thıs work, namely McGeer's orıgınal [McG90] and a new passıve dynamıc walker wıth a torso McGeer's model ıs utılısed as a means of formulatıng and codıng the solutıon process and of proofing the code ınvolved Some slıght ımprovements on hıs results were made, as shall be hıghlıghted ın chapter 5 All of the solutıon methods used on the body-less creature have been prevıously publıshed and were used as a framework ın the solutıon method for the bodıed creature

3.2 Creature Configurations

3.2.1 Body-less

Before the addıtıon of a torso, ıt was necessary to gaın an ınsıght ınto body-less motıon Therefore the first creature consıdered consısts, as McGeers' does [McG90], of the followıng parameters

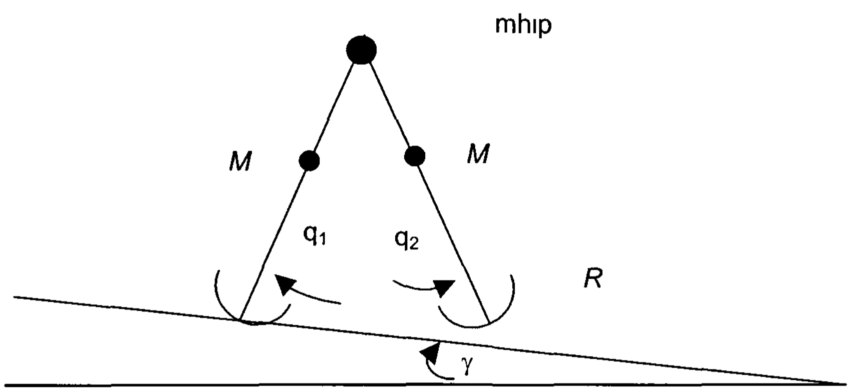


Fig 3.1: *Body-less Creature consisting of two legs and a hip mass on a slope* γ

Legs

The creature has two rigid legs, one in motion known as the *swing* leg and the other anchored to the ground, the *stance* leg Each leg is identical consisting of length L and of mass M with the centre of mass positioned at the point *comleg,* a vector given from the end point Each leg has an arc-style structure at the base of radius R, which act as feet The arc-like semi-circular feet are used as a mathematical convenience rather than a physical necessity and could be removed as necessary A more complete list of vectors influencing the creature is given in appendix A and in Fig 3 2 below Finally note that the centre of mass M is offset slightly and this is indicated by the variable w

Hip

The joint connecting both legs contains a mass, known as the *hip mass* and is represented by the parameter *mhip* The hip mass is attached to each leg and thus each leg has total mass $mhip + M$ The total mass of the robot is thus $2(mhip + M)$

Inertia

The moment of inertia is given by

$$I = Mass \times r_{gyr}^{2} \tag{3 1}$$

where r_{gyr} is the radius of gyration [Han88] and *Mass* is explained below The radius of gyration value used by McGeer [McG90], namely $r_{gyr} = 0\ 347$ is approximated here with an actual value of $r_{gyr}^{\ 2} = 0\ 121$ utilised The effects of varying this value will be highlighted in chapter five There are two different *Mass* values associated with equation 3 1 for each of the two legs for the swing leg $Mass = M + mhip$ and for the stance $Mass = M$

Length

At the time of heel-strike, since both legs are in contact with the ground, the robot configuration can be completely described by what Goswami [Gos96b] terms the inter-leg angle α Since the leg angles here are equal but opposite this inter-leg angle is simply twice the swing leg angle i e $2q_1$ The step length is then given by the following formula

$$Length = 2\,L\,Sin \qquad (3\ 2)$$

Stance Leg Only

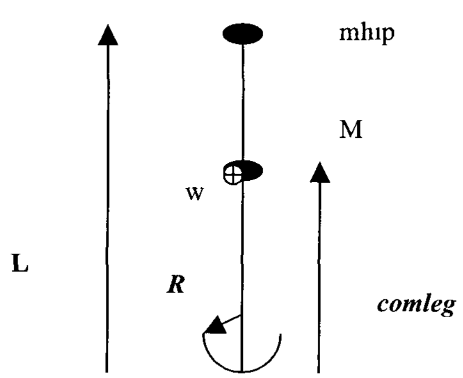


Fig 3.2: *A close up of just one leg This gives the vectors in the creature's configuration as given in the section above*

Generalised co-ordinates

The **gait** of the creature is given then by two stages *swing* where the swing leg moves while the stance is pivoted to the ground, and *transition* or collision whereby the roles of the two legs are swapped The combination of both processes leads to a **step** Angles are measured relative to the normal to the ground, and positive indicates a clockwise motion Changes in the shape of the creature are therefore specified through *generalised co-ordinates* (i e angles and speeds) and there are two of each

q_1	angle stance leg makes with the ground
u_1	speed at which angle is changing
q_2	angle swing leg makes with the stance leg
u_2	speed at which it is changing

(note that $u_1 = q_1$ and $u \ = q_2$)

Thus the state vector of the creature is

$$\theta(\mathbf{t}) = \{q_1, q_2, u_1, u_2\} \tag{3 3}$$

Note that at the start of a step the legs have equal and opposite angles i e $q_1 = - q_2$ Thus in equation 3 2 $\ = 2 \times q_1$ The final parameter involving an angle to be considered is the *slope* of the ground γ

Assumptions

Certain assumptions are also made namely

- the impact of the swing leg with the ground is inelastic and without sliding By being inelastic this means that there is no rebound This condition could be enforced in a physical model by placing dead rubber at the end of the feet These conditions lead to the robot configuration remaining the same throughout and to conservation of momentum before and after collision with the ground
- A knee-less creature would not be able to clear the ground as the swing leg manoeuvres and as such scuffing of the ground is ignored

These assumptions are not unusual and as such are common to the two other creatures currently been researched upon i e [Gar97] and [Gos96b]

External torques

Initially there are no external torques applied However in the next chapter it is shown that the addition of an external torque at the hip joint can be used in order to improve stability and versatility These dampers can be either linear or non-linear, with better performance gathered from the non-linear ones

3.2.1 Bodied Creature

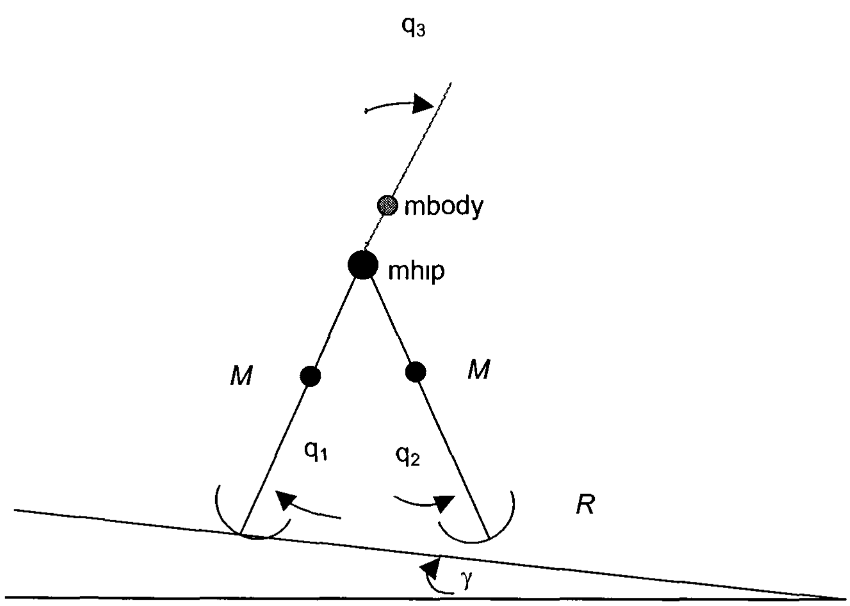


Fig 3.3: *Passive dynamic walker with torso*

Torso

The addition of a body is fairly straightforward the torso or *body* is treated as another rigid link added to the creature This link of length *lbody*, centre of mass *combody*, measured with respect to the end of the link, and mass *mbody*, is attached to the previous creature at the hip point, with another hinge joint

Generalised Co-ordinates

The body is initially in an upright position and therefore resembles an inverted pendulum It creates two new co-ordinates in relation to the upright normal, an angle q_3 and velocity u_3 and thus the state vector describing the system has six components namely

$$\theta(t) = \{q_1, q_2, q_3, u_1, u_2, u_3\} \tag{3 4}$$

Inertia

For the bodied creature the radius of gyration of each leg ${r_{gyr}}^2$ must be lowered to 0 09 in order for solutions to be found The moment of inertia of the body is given by $I = mbody \times {r_{gyrBOD}}^2$ where ${r_{gyrBOD}}^2$ is the radius of gyration of the body and initially has a value of 0 121

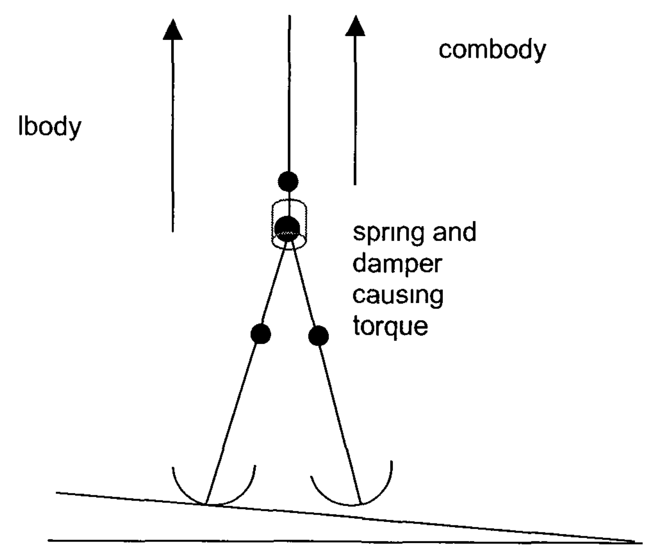


Fig 3.4: *Non-linear spring and damper at body joint*

External Torques

In order to keep the torso in an upright position an external torque is applied This torque is reacted off the stance leg and is incorporated into the equations of motion

3.3 Generating the Equations of motion for the creatures.

As the creatures involved consist of relatively few links and little or no external forces, formulation of the equations of motion is straightforward enough For the body-less creature consisting just of the two legs, the dynamic equations of the swing stage are similar to the well-known double pendulum equations Since the legs of the robot are assumed identical, the equations are similar regardless of the support leg considered Generation of the equations of motion can be achieved through numerous methods by hand, such as Newton-Euler integration, Lagrangian methods, Kane's method etc Alternatively they can be generated by machine Ideally two methods, one by machine and one by hand should be carried out to ensure accuracy

Goswami [Gos96a] uses the Lagrangian method and ends up with an equation involving a 2×2 inertia matrix, a 2×2 matrix with centrifugal terms and a 2×1 vector of gravitational torques The actual formulation of the equations was achieved using the freely available package Sci-lab [Sci00] Garcia [Gar97] generated his equations of motion using the special purpose generator AUTOLEV and correlated his results by working out the equations by hand His equations are in a similar format of a combination of matrices and vectors McGeer formulated his equations by hand with much of the solution method outlined in [McG90]

I have decided to use Kane's method, which is incorporated into a **Mathematica** ® package called the *Dynamics Workbench* [Kuo00] to produce the equations of motion incorporated here The Dynamics Workbench is a freely available Mathematica package for doing dynamics It enables the user to generate equations of motion primarily for rigid body mechanical systems Along with general Mathematica commands [Wol96] the overall equations of motion can be constructed

3.3.1 Dynamics Workbench

In formulating the velocities and forces applied to generate the equations of motion the dynamics workbench package is called upon The primary low-level commands that are used are briefly explained in this section While this section can be used as a reference the full Mathematica code used to generate the equations of motion for the body-less creature is given in Appendix B and for the bodied one in Appendix C

Reference Frames

The dynamics Workbench describes a mechanical system using *bodies* and reference *frames* where one or more bodies may be used to describe a rigid body and one or more reference frames may be attached to that body For instance a rigid body constituting a leg called "*legone*" will have a reference frame associated with it consisting of three axes **legone[1], legone[2], legone[3].** There is a single default body corresponding to the Newtonian reference frame called **ground** and therefore any initial body will be described in relation to the ground frame

Note: The bodied creature consists of 3 links and thus the reference frames involved here are as follows **sta[i]** (for the stance leg), **swi[i]** (for the swing leg), and **bod[i]** (for the body), where each *i* value corresponds to a certain axis and thus has value *1, 2* or *3* All the reference frames are outlined in Fig 3 5

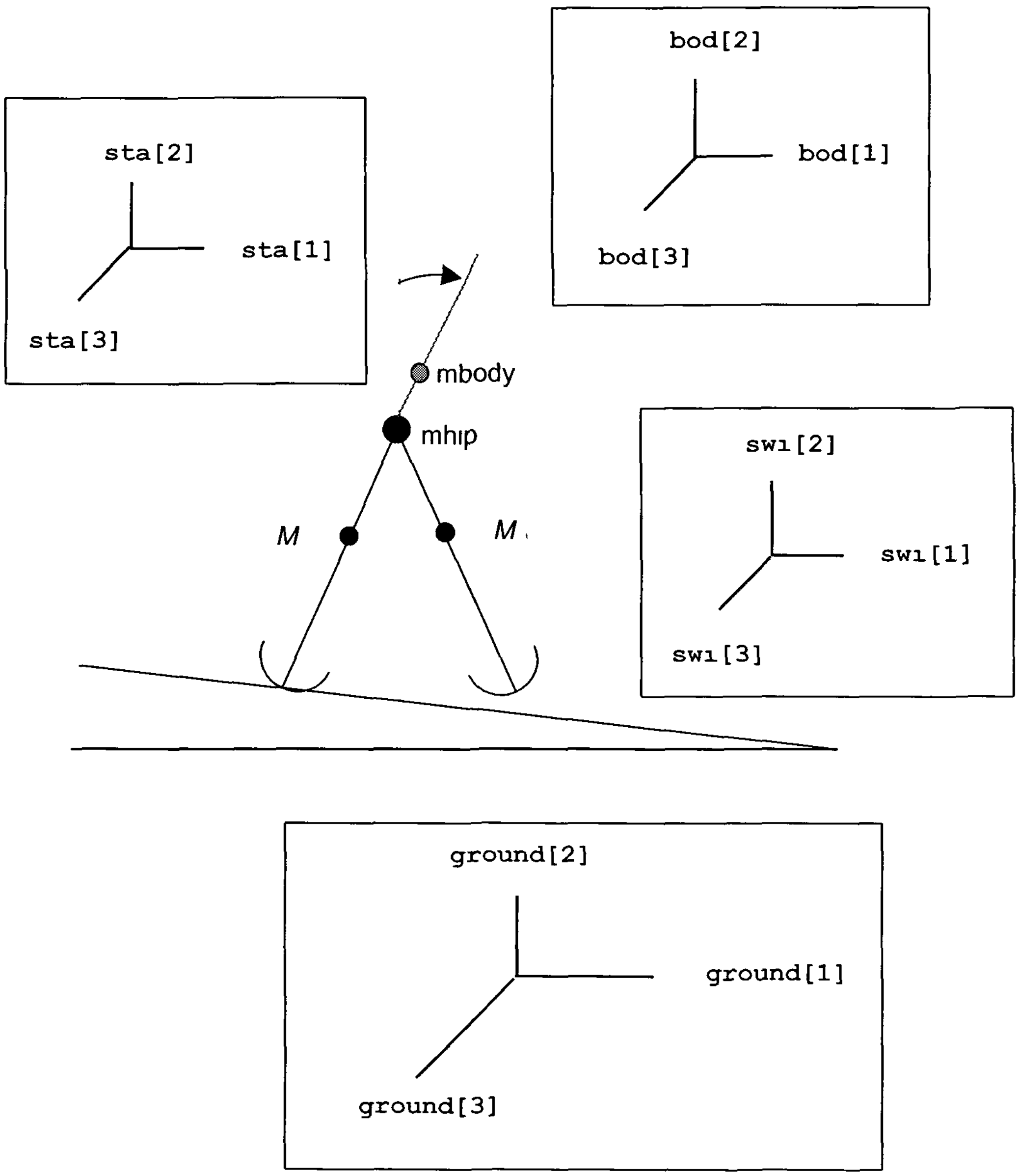


Fig 3.5: *Reference frames for each of the three rigid links in the bodied creature*

Connections

Each body is defined with respect to an inboard body, which precedes it, and are connected by a particular type of joint In this piece of work, joints that are considered are of one type only, *hinge* This means that the rigid body can only move in one direction only In describing say for instance a hinge joint between two bodies b_1 and b_2, on b_2 the vector *BodyToJnt* describes the joint location with respect to the body's centre of mass (i e *com*), and on the inboard body b_1, the vector *InbToJnt* describes the joint location with respect to the body's centre of mass This is shown in diagram format in Fig 3 6

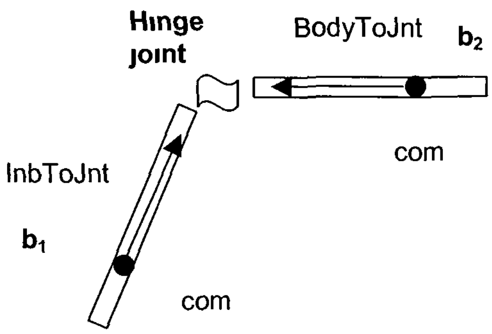


Fig 3.6: *Rigid links in Dynamics Workbench*

3.3.2 Dynamics Workbench - Some Generic Commands Used

This section gives some of the commands used that are particular to the Dynamics Workbench package These are the commands that are utilised to form the equations of motion and are included in the code given in Appendix B and C For a more complete tutorial on how to use the Dynamics Workbench see [Kuo00]

AddBody[new_body, inboard_body, joint_type] adds a body, new_body, to a previously defined inboard_body using a specified joint Joints used here are hinge joints

AppTrq[body, torque] applies a torque or moment specified by a vector torque to a body

PosPnt[point, body] returns as a vector, the position of the point attached to the body

Eom This command generates the equations of motion that describe the system

Inertia This gives the Inertia vector associated with the rigid body

3.3.3 Vectors describing motion

Upon setting up the description of the creature using some of the commands given in the previous section a group of vectors gives a portrayal of the bodied creature's movement These vectors for each leg and the torso involve the following velocity $\vec{v}_{COM}$ and acceleration $\vec{a}_{COM}$ of the centre of mass of the rigid link and angular rotation Ω of the rigid link Now the individual vectors for each leg, the torso and the hip point (where velocity $\vec{v}$ and $\vec{a}$ only are involved) are as follows (the reference frames below are outlined in Fig 3 5)

Stance Leg

$$\vec{\mathbf{v}}_{\mathrm{COM}} = (-(comleg - R)u_1)\mathbf{sta[1]} + (-Ru_1)\mathbf{ground[1]} \qquad (3\ 5)$$

$$\vec{\mathbf{a}}_{\mathrm{Com}} = (-(comleg - R)u_1^{\ 2})\mathbf{sta[1]} + (-(comleg - R)u_1')\mathbf{sta[1]} + (-Ru_1')\,\mathbf{ground[1]} \qquad (3\ 6)$$

$$\Omega_{\mathrm{sta}} = u_1'\ \mathbf{ground[3]} \qquad (3\ 7)$$

Hip joint

$$\vec{\mathbf{v}} = (-Ru_1)\mathbf{ground[1]} + ((-L + R)u_1)\mathbf{sta[1]} \qquad (3\ 8)$$

$$\vec{a} = ((-L + R)u_1^{\ 2})\ \mathbf{sta[2]} + (-Ru_1')\ \mathbf{ground[1]} + ((-L + R)u_1')\ \mathbf{sta[1]} \qquad (3\ 9)$$

Swing Leg

$$\vec{\mathbf{v}}_{\mathbf{COM}} = (-Ru_1)\,\mathbf{ground[1]} + ((-L+R)u_1)\mathbf{sta[1]} + (-(comleg - L)u_2)\,\mathbf{swi[1]}$$

(3 10)

$$\vec{\mathbf{a}}_{\mathbf{COM}} = ((-L+R)u_1{}^2)\,\mathbf{sta[2]} + (-(comleg - L)u_2{}^2)\,\mathbf{swi[2]} + (-Ru_1)\,\mathbf{ground[1]} + ((-L+R)u_1)\mathbf{sta[1]} + (-(comleg - L)u_2)\,\mathbf{swi[1]}$$

(3 11)

$$\dot{\mathbf{U}}_{\mathbf{swi}} = u_2{}'\ \mathbf{ground[3]} \qquad (3\ 12)$$

Body

$$\vec{\mathbf{v}}_{\mathbf{COM}} = (-Ru_1)\,\mathbf{ground[1]} + ((-L+R)u_1)\mathbf{sta[1]} + (-(-combody + lbody)u_3)\mathbf{bod[1]}$$

(3 13)

$$\vec{\mathbf{a}}_{\mathbf{COM}} = ((-L+R)u_1{}^2)\,\mathbf{sta[2]} + ((-combody - lbody)u_3{}^2)\,\mathbf{bod[2]} + (-Ru_1)\mathbf{ground[1]} + ((-L+R)u_1\ \mathbf{sta[1]} + ((combody - lbody)u_2)\,\mathbf{bod[1]}$$

(3 14)

$$\dot{\mathbf{U}}_{\mathbf{body}} = u_3{}'\ \mathbf{ground[3]} \qquad (3\ 15)$$

3.3.3.1 State vector

The vector descriptions given above in equations 3 5 to 3 15 along with the masses (i e hip, leg and body) and forces involved (i e gravity) are used to generate the equations of motion and the full code is given in the appendices However direct use of the Dynamics Workbench does not place the equations of motion in the required format, that of the state derivative The general form of the equations of motion (using Newtons law which states that Force is mass by acceleration) can be given as

$$A\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad (3\ 16)$$

where A is a matrix containing a mixture of all the terms involved and the vector of 0 values comes from the fact that the applied force is 0 Manipulation (incorporated directly through Mathematica and included in the final part of the code in Appendix C) of these terms in A can however lead to the following (for the equations of motion)

$$M\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} - R = 0 \qquad \underline{\text{or}} \qquad M\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = R \qquad (3\ 17)$$

where M is a 3×3 matrix containing the terms linear in the time-derivatives of generalised speeds from the equations of motion of the creature and R contains all the other values The matrices M and R for the body-less creature are given in appendix D but for the bodied creature (although comprehensive in size) are shown in equations 3 19 and 3 20

The state of the system is $\theta = \{q_1, q_2, q_3, u_1, u_2, u_3\}$, and thus the state derivative is

$$\theta(t) = \{ q_1, q_2, q_3, u_1, u_2, u_3 \} \qquad (3\ 18)$$

Calculation of the final three values in the state derivative vector is achieved by performing the following solving the linear equation (3 17) For this equation the matrices M and R are as follows

$M =$

$$
\begin{pmatrix}
\begin{gathered}[t] Ilsta + comleg\,2\,M + L^2\,Mbody + \\ comleg^2\,Mhip + L^2\,Mhip - \\ 2\,comleg\,M\,R - 2\,L\,M\,R - \\ 2\,L\,Mbody\,R - 2\,comleg\,Mhip\,R \\ -2\,L\,Mhip\,R + 4\,M\,R^2 + 2\,Mbody\,R^2 + \\ 2\,R\,(comleg\,(comleg\,(M + Mhip) + L(M + Mbody \\ + Mhip) - (2\,M + Mbody + 2\,Mhip)\,R)\,Cos\,q_1, \end{gathered} & \begin{gathered} (comleg - L)(M + Mhip) \\ ((L - R)Cos(q_1 - q_2) \\ + R\,Cos\,(q_2) \end{gathered} & \begin{gathered} -(comleg - lbody)(Mbody) \\ ((L - R)Cos(q_1 - q_3) + \\ R\,Cos\,(q_3) \end{gathered} \\
\begin{gathered} -(combody - lbody)\,Mbody\,((L - R) \\ Cos\,(q_1 - q_3) + RCos\,(q_3)\,(comleg - L) \\ (M + Mhip)\,(L - R)Cos(q1 - q2) + (comleg - L) \\ (M + Mhip)RCos(q2) - (combody - lbody)Mbody \\ ((L - R)Cos(q1 - q2) + RCos(q_3), \end{gathered} & \begin{gathered} Ilswi + (comleg - L)^2 \\ (M + Mhip) \end{gathered} & (combody - lbody)^2\,Mbody \\
0 & 0 & Inertiabody
\end{pmatrix}
$$

(3 19)

$R =$

$$
\begin{pmatrix}
\begin{gathered} g(M + Mhip)(-R + (-comleg + R)Cos(q_1))Sin(\gamma) + g\,Mbody(-R + (-L + R)\,Cos(q_1))\,Sin(\gamma) \\ + g(M + Mhip)(-R + (-L + R)Cos(q_1))Sin(\gamma) + g(M + Mhip)(comleg - R)Cos(\gamma)Sin(q_1) + \\ gMbody(L - R)Cos(\gamma)Sin(q_1) + (g + M + Mhip)(L - R)Cos(\gamma)Sin(q_1) + R(comleg(M + Mhip) \\ + L(M + Mbody + Mhip) - (2M + Mbody + 2Mhip)R)\,Sin(q_1)u_1^{\,2} + (comleg - L)(M + Mhip)((-L + \\ R)Sin(q_1 - q_2) + RSin(q_2))u_2^{\,2} - (combody - lbody)Mbody((-L + R)Sin(q_1 - q_3) + RSin(q_3))u_3^{\,2}, \end{gathered} \\
-(comleg - L)(M + Mhip)(g\,Sin(\gamma - q_2) - (L - R)\,Sin(q_1 - q_2)u_1^{\,2}), \\
\begin{gathered} g(combody - lbody)MbodySin(\gamma - q_3) - (combody - lbody)Mbody(L - R)Sin(q_1 - q_3)u_1^{\,2} + \\ frac(Klin - damp\;u_3) \end{gathered}
\end{pmatrix}
$$

(3 20)

Solvıng equatıon 3 17 usıng the matrıces ın 3 19 and 3 20 leads to the final three values ınvolved ın the formulatıon of the state derıvatıve $\theta(t)$ for the bodıed creature and thıs ıs gıven ın full ın Appendıx E

3.4 Collision detection - Transition Matrix

At heel-strıke, collısıon wıth the ground occurs and the two legs swıtch roles Thıs collısıon of the swıng leg ıs assumed to be ınelastıc and wıthout slıdıng Therefore the followıng rules must be observed

- the robot configuratıon must remaın unchanged
- The angular momentum of the creature about the ımpactıng foot as well as the angular momentum of the pre-ımpact support leg about the hıp are *conserved* These conservatıon laws lead to a dıscontınuous change ın robot velocıty

From the first rule above, the angles at transıtıon are just swapped ı e $q_2 = - q_1$ and ıt ıs assumed that the angle the body makes ı e q_3 remaıns the same The change ın the velocıty states ıs achıeved by the conservatıon of angular momentum gıven ın the second rule (angular momentum before and after collısıon are equal) Thus, where $^+$ ındıcates post heel-strıke, and $^-$ ındıcates pre heel-strıke,

$$\begin{pmatrix} u_1^+ \\ u_2^+ \\ u_3^+ \end{pmatrix} = T \begin{pmatrix} u_1^- \\ u_2^- \\ u_3^- \end{pmatrix} \qquad (3\ 21)$$

where T, the *transıtıon matrıx*, ıs formulated as follows let $AngM$ be the angular momentum Then

$$AngM^- = (AngM^+)T, \qquad (3\ 22)$$

and therefore T is found from linear solving the above equation, which again is implemented in Mathematica

Implementation of Algorithms

To implement the algorithms to find the limit cycles, the following was adopted As the availability of a dynamics package, such as AutoLev or Alias Wavefront was not viable due to financial constraints a free generator was searched for Goswami uses Sci-lab (available at [Sci00]) but I decided to use Arthur Kuo's Dynamics Workbench available at [Kuo00] The equations of motion and collision matrices were generated utilising this package and generic Mathematica terms are used to get the equations in the proper form to generate the fixed points of the Poincaré map

3.5 Generate complete step function i.e. Poincaré Map.

The movement of the creatures involved here consists of a swing phase and a transition stage, after which both legs exchange roles Each complete step is considered to be a *Poincare map*, or "stride function", as McGeer called it [McG90] - recall section 2 7 As a natural choice of the Poincaré section, the instant when the swing leg of the robot leaves the ground, is chosen Therefore a step will consist of the function $P(\theta^{+i})$ which takes as input θ^{+i}, the state vector at the beginning and returns $\theta^{+(i+1)}$, the state just after the following heel-strike Thus much information about a step will be encoded into the map $P(\theta)$ (where $P(\theta)$ is basically the combination of the equations of motion and transition equations grouped into one function)

The Poincaré map therefore consists of two components

- **Numerical integration of equations of motion to find heel-strike state:**
  First the swing leg manoeuvres upwards and then moves back down until heel-strike is reached The equations of motion fully describe this and, since to solve analytically for the state at heel-strike would be very cumbersome, if not impossible, a numerical integration technique is used The Runge-kutta technique

is used with a special stopping mechanism i e when heel-strike occurs, that is when the height of the swing leg above ground is zero (Note that the initial ground collision of foot scuffing occurs early on in the period and is ignored) To get a precise pre-heel-strike state value it is necessary to zoom in on the Runge-kutta stopping mechanism and thus a Newton-Raphson method is used to zero the swing foot height

- **Collision:**

  Once this process has achieved its goal, the state of the creature is known at the moment of heel-strike and the transition or collision component, as described in the previous section, can take place Post collision velocities are thus calculated by assuming angular momentum before and after impact, about various points

For the Runge-kutta algorithm utilised above the time step taken is *0 01* and the numerical tolerance is taken to be $1 \times e^{-8}$ The algorithm is coded to converge quadratically to abs(θ_i) < numerical tolerance / 1000 where the factor of 1000 is arbitrary chosen

3 6 Find limit cycle ... if it exists!

McGeer [McG90] demonstrated that a somewhat humanoid mechanism is capable of stable, human-like gait down a shallow slope with no external or internal forces (besides gravity) and no control His passive-dynamic theory of bipedal motion describes gait as a natural repetitive motion of a dynamical system, or in the language of non-linear dynamics, a limit cycle Therefore finding limit cycles is of vital importance

3.6.1 What is a stable limit cycle and how are they found?

A simple *period one* gait cycle, if it exists, corresponds to a set of initial values for the angles and rates which lead back to the same angles and rates after one complete step

Taking the Poincaré language approach the state vector $\hat{\theta}(t)$ is a period one gait cycle

if $P(\hat{\theta}(t)) = \hat{\theta}(t)$ (3 23)

where $\hat{\theta}(t)$ is what is known as a *fixed point* Higher period solutions do exist for the body-less creature [Gar97] [Gos98] as well as non-periodic ones but period one solutions are of central interest because they correspond to the important tasks of steady walking (bifurcations or period doubling although are mentioned later in chapter four)

A *Limit Cycle* is a periodic solution of a system and is represented by a closed loop in the phase space The difference between a simple periodic solution and a limit cycle is that the latter exerts its influence in its neighbourhood i e an attracting limit cycle will absorb all solutions towards itself that are in its neighbourhood, or *basin of attraction* as it is called

An attracting limit cycle is also called a *stable* limit cycle since small perturbations in the state of a system lying on the limit cycle reduce to zero in the long run

The periodic aspect of a limit cycle indicates that a limit cycle occurs if the output state is the same as the input state Thus if $\theta(t)$ is the initial state and as stated above one complete step consists of the function $P(\theta(t))$, where $P(\theta(t))$ is the Poincaré map, then

gait limit cycles correspond to fixed points of the map, or in other words the roots of the function

$$G(\hat{\theta}(t)) = P(\hat{\theta}(t)) - \hat{\theta}(t) \qquad (3\ 24)$$

where $\hat{\theta}(t)$ is known as a fixed point

Fixed points can be found by a separate Newton-Raphson search for zeros of 3 24 above

3.6.2 *Newton's Method*

To find the roots of the Poincaré map the well-known Newton-Raphson method is used Firstly a step can be thought of as an operator $P(\theta)$ (the stride function) which takes as input a vector of scalar values which represent the various angles and velocity rates at a definite point in the motion (i e just after ground collision) and returns the values of θ after the next ground collision

Starting with the initial step (i e initial state θ_0), each subsequent step is determined from the previous one The formula for the subsequent step is

$$\theta_{i+1} = \theta_i - \frac{P(\theta_i)}{P'(\theta_i)} \tag{3 25}$$

As it not practical with the equations involved to proceeds analytically, the numerical derivative of $P(\theta)$, namely,

$$P'(\theta) = \frac{P(\theta + \delta\theta) - P(\theta)}{\delta\theta} \tag{3 26}$$

is utilised Since it is the function $G(\hat{\theta}) = P(\hat{\theta}) - \hat{\theta}$ that we are looking at the numerical derivative is

$$G'(\theta) = \frac{(P(\hat{\theta} + \delta\theta) - (\hat{\theta} + \delta\theta)) - (P(\hat{\theta}) - \hat{\theta})}{\delta\theta} \tag{3 27}$$

As each step consists of multiple variables (i e 4 or 6 values in the state vector) P' is actually the Jacobian of the Poincaré map J, with respect to the state variables For six states this Jacobian J (i e is $P'(\theta)$) is

$$J = \begin{Bmatrix} \frac{\partial P_1}{\partial q_1} & \frac{\partial P_1}{\partial q_2} & \frac{\partial P_1}{\partial q_3} & \frac{\partial P_1}{\partial u_1} & \frac{\partial P_1}{\partial u_2} & \frac{\partial P_1}{\partial u_3} \\ \frac{\partial P_2}{\partial q_1} & \frac{\partial P_2}{\partial q_2} & \frac{\partial P_2}{\partial q_3} & \frac{\partial P_2}{\partial u_1} & \frac{\partial P_2}{\partial u_2} & \frac{\partial P_2}{\partial u_3} \\ \frac{\partial P_3}{\partial q_1} & \frac{\partial P_3}{\partial q_2} & \frac{\partial P_3}{\partial q_3} & \frac{\partial P_3}{\partial u_1} & \frac{\partial P_3}{\partial u_2} & \frac{\partial P_3}{\partial u_3} \\ \frac{\partial P_4}{\partial q_1} & \frac{\partial P_4}{\partial q_2} & \frac{\partial P_4}{\partial q_3} & \frac{\partial P_4}{\partial u_1} & \frac{\partial P_4}{\partial u_2} & \frac{\partial P_4}{\partial u_3} \\ \frac{\partial P_5}{\partial q_1} & \frac{\partial P_5}{\partial q_2} & \frac{\partial P_5}{\partial q_3} & \frac{\partial P_5}{\partial u_1} & \frac{\partial P_5}{\partial u_2} & \frac{\partial P_5}{\partial u_3} \\ \frac{\partial P_6}{\partial q_1} & \frac{\partial P_6}{\partial q_2} & \frac{\partial P_6}{\partial q_3} & \frac{\partial P_6}{\partial u_1} & \frac{\partial P_6}{\partial u_2} & \frac{\partial P_6}{\partial u_3} \end{Bmatrix} \tag{3 28}$$

(Remember that the map P, like θ is a vector and has 6 values with P_1 corresponding to the first value etc)

The combination of the individual components involved in finding the limit cycle solutions are given below as pseudocode

3.63 PseudoCode for full solution

```
//Physical Equations

Describe physical makeup of creature
        If necessary add in springs and dampers
Generate equations of motion
        Use to formulate state derivative
        Use to formulate transition matrix

// Poincaré Map

Give initial guess for algorithm
Use Rungekutta to numerically integrate state derivative
Ignore foot-scuffing and stop at heel-strike
Use Newton's method to zero in on heel-strike solution
```

Use Transition matrix for one complete step ***// Fixed Points i.e. Newton's Method*** *While fixed point not found*   *Use second Newton's method on Poincaré map*

3.64 Failure of Newton's method

Failure of Newton's method to converge to a solution can generally be caused by one of the following reasons

- the initial guess is not close enough to the solution
- there is no fixed point for the parameter family involved and this can only be rectified by changing at least one parameter
- the slope of one of the state variable vs parameter plots is approaching infinite slope This is known in the bifunfication literature as a "turning point", and is indicated by an unexpected zero value in the stride function Jacobian J

3.65 Initial Values

Estimating the initial values can be quite challenging in itself The choice of initial values, for the starting state is of vital importance, as only values within the basin of attraction will eventually converge to a limit cycle The shape and size of the basin of attraction of a limit cycle is in general a function of the robot parameters and are not directly amenable to analytical solutions

Various approaches have been adopted to estimate the initial values McGeer [McG90] indicated that the stance and swing angles of $\{q_1, q_2\} = \{0\ 3, -0\ 3\}$ correspond roughly to the "known" values of human gait The other values were formulated from known gaits but randomly generated values were used, as he hypothesised *"it is perhaps naive, but it is also unbiased, and so can reveal behaviour, which might otherwise go unnoticed "*

Goswami uses the initial conditions calculated from his initial linearised model and where these failed, the state vector corresponding to a known steady gait of a robot whose parameters were close to the robot under study were used [Gos96b]

Finally both McGeer[McG90] and Goswami [Gos98] pointed out explicitly that both robots that they simulated can accept without falling down a much larger change in the velocity states than the position states (i e there can be a change of $\approx 100°$ per second in the velocity of the angles but a change of 2° in the position takes the states out of the basin of attraction) This was also proved to be the case here for both the bodied and body-less creature as shall be highlighted in chapter five

The initial velocity values chosen here are twofold - for the body-less creature I have decided to use previously published values for a known solution i e

$$\theta_0 = (0\ 3015, -0\ 3015, -0\ 3763, -0\ 2822) \qquad (3\ 27)$$

These are similar to those published by McGeer [McG90] and used by Kuo and other researchers [Kuo00] For the bodied creature these values just have the initial body parameters appended to it, namely $q_3 = u_3 = 0$, giving

$$\theta_0 = (0\ 3015, -0\ 3015, 0, -0\ 3763, -0\ 2822, 0) \qquad (3\ 28)$$

Chapter Four

Control and Analysis

4.1 Introduction

For any creature under investigation, with or without a torso, once limit cycles have been identified an *analysis* stage begins Of prime importance is establishing whether or not the cycles or steps are stable or unstable Other aspects of analysis to be determined include efficiency of the creature, step period and velocity, energy utilised and the maximum slope attainable Improvements in some of these variables, most notably stability, can be gained through the addition of external torques and damper forces For some creatures, as found by both Goswami [Gos96b] and Garcia [Gar98] there is a period doubling route to chaos present and thus this is another characteristic that should be investigated All performance indicators mentioned above are outlined in this chapter along with the methods of implementation

As previously stated however some external torque is required to keep the torso of the bodied creature upright Feedback control in the form of a fuzzy logic controller is utilised and limit cycles are found The format of this controller is given along with the logic involved These limit cycles are then analysed using the same techniques as above

4.2 Analysis Terminology

In describing the passive dynamic walker system some terms that are common to non-linear dynamics are utilised These terms are now defined

4.2.1 Dynamic System

A dynamical system may be defined as a mathematical model determining the state of a system forward in time, where time can be discrete or continuous Therefore starting at time $t = 0$ at any subsequent time t_i the state can be determined

4.2.2 Hamiltonian System

Hamiltonian systems are a class of dynamical systems incorporating various properties such as mechanical systems in the absence of friction, the paths followed by magnetic field lines in plasma, the mixing of fluids and the ray equations describing the trajectories of propagating waves [Ott93] The main properties of these systems include

- energy is conserved for time-independent systems
- possibly do not have attractors in the usual sense This incompressibility of phase space volumes for Hamiltonian syatems is called Liouvilles theorem

4.2.3 Non-Holonomic System

A dynamical system can be classified as either holonomic or non-holonomic To determine which term applies you must examine the generalised co-ordinates If the coordinates satisfy the following two conditions then the system is *holonomic*, that is if the coordinate values determine the configuration of the system and secondly that the values may be varied arbitrarily and independently without violating the constraints of the system [Syn70]

An example of a holonomic system would be a robotic arm involved in the manufacture of cars The robot is in an initial state, performs its assigned task and returns to a final state This final state coincides with the initial one and therefore the robot does not change its location. Another example would be that of a scissors lying on a table An example of a non-holonomic system would involve a rigid sphere rolling without slipping on a fixed horizontal plane The system can be defined in terms of 5 generalised co-ordinates – the two horizontal Cartesian co-ordinates at the centre of the sphere and 3 Eulerian angles Since the plane is not smooth, two additional constraints are needed i e equating to 0 the horizontal velocity of the particle of the sphere at the point of contact These conditions are non-integrable and it is this non-integrability which make the system non-holonomic [Syn70] Other examples might be a wheelchair, bicycle or skateboard

The difference between holonomic and non-holonomic systems can be summarised as follows

- with a holonomic system return to the original internal configuration means a complete return to the original state i e the initial and final states are completely equal This however is not guaranteed for non-holonomic
- the system outcome for a non-holonomic system is path-dependent
- whereas holonomic kinematics can be expressed in terms of algebraic equations which constrain the internal, rotational co-ordinates of a robot to the absolute position/orientation of the body of interest, non-holonomic kinematics are expressible with differential relationships only

As is well known in dynamics systems theory, conservative holonomic (i e Hamiltonian) systems cannot have asymptotic stability since volume is conserved in their phase spaces Therefore only non-Hamiltonian systems have asymptotical stability Two mechanisms for losing the Hamiltonian structure of the governing equations are dissipation and non-holonomic constraints Passive dynamic creatures are non-holonomic by virtue of their intermittent contact with the ground and are moving along a particular path (i e down the slope) Also they are not conservative since energy is lost at every heel-strike Thus the existence of this dissipative element favours but does not guarantee the existence of a stable limit cycle Goswami investigated the contraction of phase space volume and found that the *"absolute value of the determinant of the transition matrix was always negative (i e inferior to 1) which indicates that phase space volumes are always contracted "* [Gos99]

Finally Ruina questions how stability can be gained stating that *"we know from our study of bicycle stability and the like that non-holonomic systems can have* *asymptotical stability even without dissipation Can legged mechanisms also be made stable without dissipation "* [Rui97]

4.2.4 Stability

For each steady motion the establishment of stability is of vital importance since it indicates whether the creature will keep walking indefinitely or will eventually collapse Due to the non-linearity of the creatures' dynamics, analytical methods of investigating the stability of the passive gaits cannot be utilised and therefore stability will be addressed using an *analytically guided numerical method* which involves finding the eigenvalues of the Jacobian matrix J of the Poincaré map This is due to the fact that the conventional definitions of stability of a system in the sense of Lyapunov, (around an equilibrium point) are not applicable to walking machines Therefore it is <u>orbital stability</u> that is investigated, where a solution of the dynamic system gives an orbit This method is also adopted by current researchers [Gar98] [Gos96b] and is explained fully in Section 4 3 The applicability of the numerical method practically guarantees that the limit cycle is stable as argued by Goswami [Gos96b] who states that *"unless we accidentally hit the exact states on an unstable limit cycle which will never be encountered in numerical trials"*

What do we mean though by saying that a cycle is <u>stable</u>?

Def: We may say a gait is stable *"if starting from a steady closed phase trajectory, any finite disturbance leads to another nearby trajectory of similar shape"* [Hur86]

Furthermore, if in spite of the disturbance, the system returns to the original cycle, the gait is <u>asymptotically</u> stable This is useful since it indicates that any disturbance to the creatures motion would be swallowed up and motion should therefore be infinite, so long as the required slope is present

Figure 4 1 presents the nature of a stable limit cycle in the phase plane of one rigid body link The effect of any disturbance to one of the states on the limit cycle is attracted and swallowed up Therefore a system starting from a state on a limit cycle will remain on it The complete shaded region in the diagram indicates the attracting region of the limit cycle and is known as the domain of the limit cycle or its basin of attraction Another indication of stability is the measure of the size of the basin of attraction but this method is not undertaken here

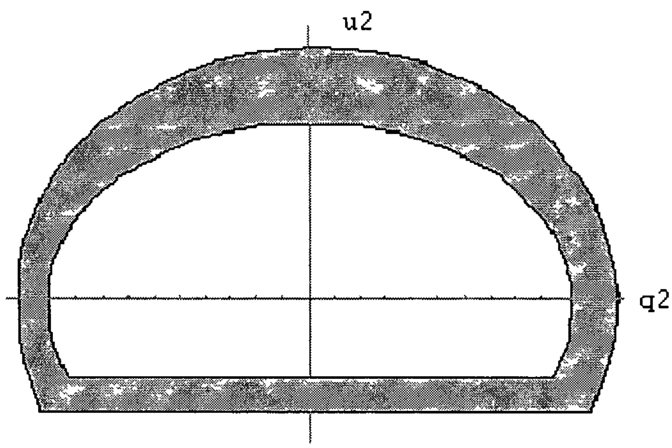


Fig 4.1: *Basin of attraction of a limit cycle Any point inside the shaded region would be in the basin of attraction and would eventually settle on the limit cycle which is also inside the region The limit cycle is not shown*

4.2.5 Bifurcation i.e. period doubling

A qualitative change in the dynamics which occurs as a system parameter varies is called a bifurcation There are a variety of types but the one of interest here is the *period doubling bifurcation* In this case a stable period one orbit bifurcates into a stable period two and an unstable period one orbit In practical terms here, taking for instance Goswami's creature [Gos96b] as the slope is increased, stable period one solutions bifurcate into stable period two solutions, stable period two solutions bifurcate into stable period four etc. An indication of bifurcation is achieved by inspection of the eigenvalues of the Jacobian of the Poincaré map in the neighbourhood of the limit cycle. These are identified for stability (see section 4.3) and should be all within the unit circle for stability. At a bifurcation point at least one of these eigenvalues crosses the unit circle.

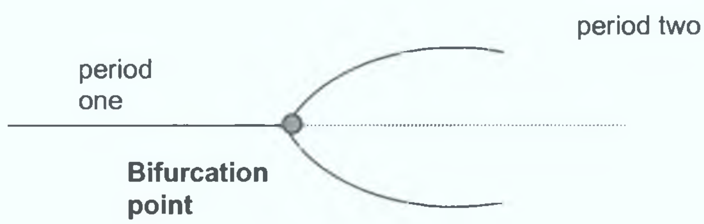


Fig 4.2: *Bifurcation. Initially there is a stable period one orbit (stability is indicated by a solid line) which bifurcates into stable period two and unstable period one.*

4.2.6 Chaos

It is easy to see and to formulate how dynamic systems settle into period motions (i.e. limit cycles) and steady states. Chaotic orbits can also appear at higher periods and they appear to be very complex and are usually described as wild or turbulent. They don't necessarily appear in very complex systems either. An example will be given to illustrate the concept which is taken from [Sha84].

Water drops from a tap continuously. A sensing device is used to time successive drops. Therefore the system of dropping water consists of time intervals $t_1, t_2, t_3, ...$ etc. where $\Delta t = t_{n+1} - t_n$. At a small flow rate the time intervals are equal. As the flow is increased slightly period two cycles or sequences are noted i.e. $t_a, t_b, t_a, t_b, t_a, ...$

etc As the flow increases further so too does the period frequency until at sufficiently large flow the sequence of time intervals has apparently no regularity This irregularity is due to chaotic dynamics [Sha84]

One thing that should be pointed out is there is a particular route to chaos i e in the above example there is a specific route of parameter t to chaos (t changes in a specific fashion)

4 3 Investigating local stability using a numerical method

To investigate the orbital stability of a limit cycle we again look at the Poincaré map This involves the state vector from just after heel-strike to just after the following heel-strike Again from section 3 6 1 a solution to the Poincaré map giving a limit cycle is known as a fixed point Therefore if $\hat{\theta}$ is a fixed point then by definition the following holds true

$$P(\hat{\theta}) = \hat{\theta} \qquad (4\ 1)$$

For a small perturbation $\partial\hat{\theta}$ around the limit cycle the non-linear mapping function P can be expressed in terms of the Taylor's series expansion as

$$P(\hat{\theta} + \delta\hat{\theta}) \approx P(\hat{\theta}) + (J)\delta\hat{\theta} \qquad (4\ 2)$$

where J is the Jacobian matrix of the map $P(\theta)$ with respect to the state variables (i e J is the matrix $\frac{\partial P}{\partial \theta}$ with components $\frac{\partial P_i}{\partial \theta_j}$) By rearranging the above equation the Jacobian can then be given as

$$J = \frac{P(\hat{\theta} + \delta\hat{\theta}) - P(\hat{\theta})}{\delta\hat{\theta}} \qquad (4\ 3)$$

It would not be practical to analytically calculate the matrix J in equation 4 3 and thus the numerical version is sought and utilised Construction of numerical version is achieved as follows the first state vector variable only i e q_1, is perturbed by a suitably small amount and the Poincaré map of the complete state is noted The mapping starting at this point will be close to, but not the same as, the original limit cycle The difference in the resulting Poincaré map of the perturbed state, minus the original fixed point and divided by the square root of the perturbation variable gives the first <u>column</u> of the Jacobian To get the second column perturb the second state vector variable i e q_2 and continue as above When all states have been dealt with the Jacobian is complete

Note that the size of the perturbation utilised here is 1×10^{-3}, and Garcia [Gar98a] used perturbations of the form 1×10^{-4} This would be one possibility for future work, to use even smaller perturbations to ensure improved accuracy Once the Jacobian has been formed stability can be measured through investigation of the eigenvalues of the matrix For the body-less creature there will be four eigenvalues, and for the bodied six

4.3.1 Eigenvalues

Def: An *eigenvector v* of a matrix B is a nonzero vector that does not rotate when B is applied to it i e $Bv = \lambda_i v$ where λ_i is an eigenvalue of B If $|\lambda_i| < 1$, then $B^i v = \lambda^i v$ will vanish as $i \to \infty$ If $|\lambda_i| > 1$, then $B^i v$ will grow to infinity [Ske94]

Therefore an eigenvalue indicates just how vulnerable to change the matrix is For asymptotical stability, which is required, all eigenvalues must be inside the unit circle If all the eigenvalues of the Jacobian J are thus <u>less</u> than one i e $|\lambda_i| < 1$, then all sufficiently small perturbations will decay to 0 and the system will approach its limit cycle However if any eigenvalue is greater than 1, then the corresponding eigenvector will bump the system divergently off the limit cycle An eigenvalue of exactly 1 indicates that the cycle is neutrally stable for perturbations along the relevant eigenvector and thus perturbations will neither shrink nor grow [Rui97] Commonly eigenvalues of magnitude 0 appear and these can be explained as follows the perturbation has been along the limit cycle and that the resulting trajectory corresponds to this perturbation along the same limit cycle [Gos96b] Frequently also, and as shall be shown in the next chapter, eigenvalues of magnitude 1 do appear appear and do not affect balance stability Many times persistent eigenvalues of magnitude one have some obvious physical significance, they can signify a one-parameter family of gait solutions for instance [Gar99] Also the indifference of most of the 3-D devices to direction of travel generates an eigenvalue of 1 in the map

4.3.2 Eigenvalue Examples

Body-less

Much headway in the analysis of the creatures' gait and the effect of various parameter changes can be made through dissection of the eigenvalues of the Jacobian For the creature consisting of four state variables, there are four eigenvalues and associated groups of eigenvectors, and therefore the system is four-dimensional (In actual fact it is only three-dimensional as one of the numerically calculated eigenvalues near zero is approximately zero - see below for an explanation as to why) McGeer [McG90] coined the term's *speed, totter* and *swing* for these eigenvalues and accordingly indicated the influence of each as follows

- ***Speed*** This is the convergence of the creature to a steady speed following that given perturbation i e the eigenvalue Or in other words it is the dissipation of speed appropriate for the slope in use
- ***Swing*** The eigenvector here is dominated by the swing angular speed The eigenvalue of this mode is usually small It is a rapid adjustment (i e eliminated immediately at the first support transfer) of the swing motion to a normal walking pattern

- ***Totter*** This is an oscillatory attempt to match step length with forward speed. This is explained by the fact that the initial angle must correspond somehow to the initial angular speed.

<u>Example:</u>

As an example, the fixed point, for a body-less creature with hip mass of 0, on a slope of 0.025 with an initial guess of {0.3015, -0.3015, -0.3763, -0.2822} turns out to be {0.30171, -0.30171, -0.376368, -0.282189}. The Jacobian J then constructed is:

$$\begin{bmatrix} -0.0000228 & -3.416\ 10^{-6} & -0.00001628 & 4.58391\ 10^{-7} \\ 0.0000128098 & -6.58372\ 10^{-6} & 0.0000168 & -4.58391\ 10^{-7} \\ 5.63304\ 10^{-6} & 1.68293\ 10^{-6} & -8.44291\ 10^{-7} & -9.11447\ 10^{-7} \\ 2.08611\ 10^{-7} & -1.63101\ 10^{-6} & 7.74861\ 10^{-7} & -0.0000107975 \end{bmatrix}$$

and finally the eigenvalues are as follows (which coincide with previously published values [Kuo00]:

speed	0.46223
swing	-0.398202
swing	-0.167559
totter	$7.47672\ 10^{-11}$

(Note: the names are associated by analysing the effects of the various angles and velocities as explained above).

Bodied

For the bodied creature there are two extra eigenvalues, thus giving a total of six eigenvalues. In practice and through the use of dampers etc. one eigenvalue stays at approximately 1 and two stay at approximately 0 Finally it is worth noting that the number of places counted in the eigenvalues depends on the perturbation size As highlighted in section 4 3 a size of 1×10^{-3} is used and thus 3 decimal places are counted,

Example

As an example, the fixed point, for a bodied creature with hip mass of 1 5 and body mass of 1 0, on a slope of 0 025, with damper coefficient of 0 55, length of body 0 8 and centre of mass 0 0795, and an initial guess of {0 3015, -0 3015, 0, -0 3763, -0 2822, 0} turns out to be
{0 283795, -0 283795, 0 0762199, -0 348054, -0 0943815, 0 00313977} The Jacobian J then constructed is

$$\begin{bmatrix} -1\ 7\times10^{-5} & -4\ 46\times10^{-6} & -1\ 07\times10^{-9} & -1\ 05\times10^{-5} & 6\ 74\times10^{-7} & -1\ 58\times10^{-8} \\ 7\ 08\times10^{-6} & -5\ 53\times10^{-6} & 1\ 07\times10^{-9} & 1\ 05\times10^{-5} & -6\ 74\times10^{-7} & 1\ 58\times10^{-8} \\ -3\ 17\times10^{-9} & 2\ 07\times10^{-9} & 4\ 11\times10^{-8} & -3\ 28\times10^{-9} & 4\ 29\times10^{-10} & 4\ 41\times10^{-7} \\ 9\ 64\times10^{-6} & 3\ 47\times10^{-7} & 3\ 73\times10^{-11} & 3\ 3\times10^{-6} & -1\ 13\times10^{-6} & 1\ 98\times10^{-8} \\ 8\ 4\times10^{-6} & -6\ 29\times10^{-6} & -7\ 14\times10^{-10} & 6\ 62\times10^{-6} & -9\ 84\times10^{-6} & 1\ 21\times10^{-8} \\ 2\ 89\times10^{-7} & -1\ 91\times10^{-7} & 2\ 02\times10^{-7} & 2\ 02\times10^{-8} & 1\ 72\times10^{-8} & -9\ 99\times10^{-6} \end{bmatrix}$$

and finally the eigenvalues are as follows

$\lambda_1 = 1\ 00411$

$\lambda_2 = 0\ 456093 + 0\ 518358I$

$\lambda_3 = 0\ 456093 - 0\ 518358I$

$\lambda_4 = 0\ 159779$

$\lambda_5 = -0\ 00001$

$\lambda_6 = 2\ 93\ 10^{-14}$

4.3.3 *Is stability vital?*

While stability is actively pursued and achieved in this body of work some researchers e g Garcia [Gar99] question whether it is crucial or not outlining the following

- slow instabilities (i e instabilities with a time scale of over a second or two) may not be important because humans do have control and need to exercise this control to go where they want e g a bicycle loses its passive stability at about 15 mph but this is not sensed by a rider since the time scale of the instability is long
- unstable period one gaits don't always lead to falls As is known in non-linear dynamics, systems which exhibit period-doubling and chaos can have a chaotic attractor which is bounded and stable in some sense, since the system does not leave the attractor if it starts on or near it Garcia therefore showed that some unstable period gaits did not fall down because of the stability of the higher-period gaits and the chaotic attractor

4.4 Other Performance monitors

4.4.1 *Step period*

The step period is the time taken for a complete step to occur Therefore this involves starting at just after heel-strike, the swing leg manoeuvring through the air, until heel-strike again when the two legs exchange roles

4.4.2 *Velocity*

The velocity of the creature should also be determined Step velocity or average speed per step (as determined in [Gos96b]) is given by the following formula

$$v = \frac{step\ length}{step\ period}, \qquad (4\ 4)$$

where *step length* is determined using equation 3 2

4.4.3 Efficiency

Energetic inefficiency is measured by the slope γ of the incline needed to sustain gait, with $\gamma = 0$ being perfectly efficient, since no energy is required for motion Why look for efficient locomotion? As argued clearly in e g [Ale95] both evolutionary pressure and individual motivation push for high efficiency in animal locomotion

In his research McGeer [McG90] numerically found walking motions for slopes as low as about 0 005 radians and utilising his model minimum slopes of 0 0005 were found here These will be fully outlined in Chapter 5 Garcia's creature reaches a minimum of zero where the dynamic solution approaches the static, parallel leg solution [Gar98a] Goswami does not directly address the issue of efficiency but the minimum slope he refers to is 0 25° [Gos96b]

4.4.3.1 Fundamental Questions about efficiency.

The results from the theoretical walking model pose fundamental theoretical questions according to Ruina [Rui97] Is it possible to have an asymptotically stable locomotion mechanism that is also perfectly efficient? The theory of Hamiltonian systems does not apply to walking machines because by virtue of their intermittent contact, they are non-holonomic Can legged mechanisms be made stable without dissipation?

4.5 Improving Performance

4.5.1 Tuning parameters

The creatures that are being studied involve various mass distributions, states and slopes By direct manipulation of these parameters improvements can be made in terms of efficiency, stability etc Formulation of the best parameters to use is evaluated using a brute force mechanism Parameter values are changed and effects noted until the best solution is found

4.5.1.1 Necessary conditions for Mass Distribution

Garcia finds that "*if the hip-mass were offset fore-aft from the legs, the gait cycles would approach a static solution at some non-zero slope which depended on this offset, and 'near-perfectly efficient' walking would not be possible So for this model and presumably for more complicated models, the existence of near-perfectly efficient gait depends on the details of the mass distribution*" [Gar98a] Some necessary conditions on the mass distribution for near-zero slope walkers therefore are found to be [Gar98b]

- If walking does occur at very small slopes then this motion will be very slow as the walker will be close to static equilibrium at all times
- As the slope goes to zero then the inter-leg angle at this instance also goes towards zero
- From the first two conditions the line from the hip through the body centre of mass must intersect the foot curve normally at the nominal contact point at zero-slope walking For circular feet this is equivalent to the co-linearity of the centre of mass of the whole body, the hip, and the foot centre

4.5.2 Add in external (passive) Springs and Dampers

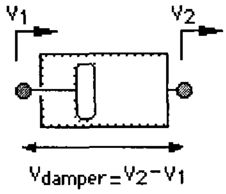


Fig 4.3: *A damper*

Def: A *damper* is a device, which associates each force to a velocity The velocity v of a damper is the rate at which it is lengthening and the force is applied to the damper by something else The velocity is relative and thus if the velocity of one end is v_1 and the velocity of the other end is v_2, the overall velocity v is $v_2 - v_1$ A positive force is an attempt to lengthen the damper, a negative one to shorten it Examples of dampers include shock absorbers and syringes The general constitutive law for a damper is

$$F_{damp} = bv \tag{4 5}$$

where b is the damping coefficient

Def: A spring is a device holds potential energy due to the way it is coiled Now consider a mass attached to a spring The spring exerts a force

$$F_{spring} = -k\,x \tag{4 6}$$

where x is the displacement of the mass from and equilibrium position and k is the spring constant

In McGeer's work [McG90a] it was suggested that the presence of springs and dampers, in particular at the hip joint could improve stability and even "convert" unstable cycles into stable ones Goswami [Gos98] continued this methodology by placing dampers at the hip joint significantly improving gait stability and overall gait versatility without violating the passiveness qualities of the creature The role of the damper as he states is to "*effect a continuous dissipation of energy in the robot in addition to the energy dissipated intermittedly during ground impact* " Although both linear and quadratic dampers were utilised, better results were achieved from the quadratic ones The effects of damper are easily imitated by a control law in an active robot and could be easily replaced by a motor implementing the same physical law A summary of the results obtained by Goswami for the effects of additional dampers include

- they do not alter the passive status of the robot
- the overall appearance of the gait however is altered i e the original cycle is modified to another cycle of different shape While the gait appearance is altered it is not necessarily destroyed This contradicts the claim by McGeer, who indicated that even a small amount of friction (i e hip damping) would destroy the stable limit cycle
- gives stable gaits for a much larger range of slopes (he mentions increases from 5° to 10°)
- the robot can possess extremely large limit cycle attraction basins

4.5.3 External Torques applied here

The external torques applied in this body of work are achieved through the use of torsional springs and dampers A linear torsional <u>spring</u> at a joint i will provide a restoring torque proportional to q_i A linear torsional <u>damper</u> at joint i will provide a negative torque proportional to u_i

As an example take a rigid body fixed to the ground with a hinge joint as in the diagram below A torsional spring and damper with coefficients s and d will generate a torque of $\tau = -sq_1 - du_1$

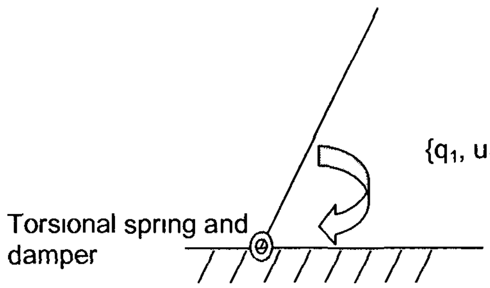


Fig 4.4: *Rigid body attached to ground with a hinge joint A torsional spring and damper is present at the joint*

Dampers are included here both at the hip joint and in conjunction with the torque applied to keep the body upright As shall be outlined in the next section a torque applied to the body acting off the stance leg can be utilised to keep the body upright However stability is not ensured and one method of providing stability is to provide a damper working in conjunction with this applied torque The damper utilised on the body is simply that of the general constitutive law (Equation 4 6) and so has the form

$$F_{damp} = damp\ u_3 \qquad (4\ 7)$$

where *damp* is the damper coefficient

Total Force on body

The total applied torque and damper applied to keep the body upright and to provide stability is given by the equation

$$F_{total} = Frac(Klin - damp \times u_3) \qquad (4\ 8)$$

where *damp* is the damper coefficient, *Klin* is the torque coefficient and *Frac* is a constant

Hip Dampers

Dampers at the hip are varied in form and their effects noted The general linear value is

$$F_{damp} = Ffric(u_2 - u_1) \qquad (4\ 9)$$

and the quadratic has the form

$$F_{damp} = Ffric(u_2 - u_1)^2 \qquad (4\ 10)$$

where $Ffric$ is the damping coefficient

4.6 Feedback Control For Upright Body

4.6.1 General Inverted Pendulum Problem.

The body will be treated as an inverted pendulum and that classic engineering problem is identified here The situation involves a dynamic system that consists of a cart with a stick hinged to its top The stick makes an angle ϕ with the normal The system is obviously unstable - the pendulum will not remain upright as the system is right now and an input force is required The objective of the problem is to identify a control system (i e feedback based) that will successfully maintain the pendulum in an upright position The problem is shown in the diagram below

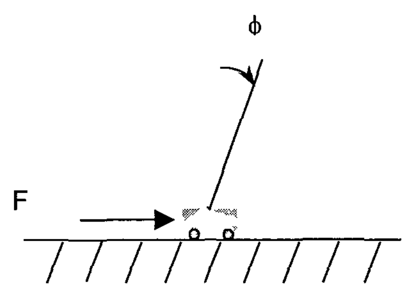


Fig 4.5: *Control of inverted pendulum*

There are many methods of controlling an inverted pendulum with most involving linearisation e g PID, exact pole placement [Gop84] and state space However utilising bang-bang logic (upon which the simple controller provided here i e bang - zero - bang) not only allows the full non-linear equations of motion to remain intact, but can be relatively simple to implement and it is this method that is adopted here

4.6.2 Bodied Creature

If the system is left as described then any reasonable body mass and length of body leads to failure of Newton's method in finding fixed points (i e it does not converge), as the link topples over Thus, to keep the link upright and to find fixed points, it is necessary to stabilise the upper body like an inverted pendulum using applied torques reacted against the stance leg The sytem here is not in essence the same as that of the general inverted pendulum as the issue of stability does not just affect the body (i e keeping the angle at 0) but involves the orbital stability of the <u>complete</u> system

A very simple *feedback control* law is adopted here, namely if the body is fully upright then no torque is applied If however there is an angle then a torque is utilised where this torque is reacted against the stance leg This torque changes value depending on the angle involved and thus the spring producing this torque is *non-linear* (the control curve in section 4 7 outlies why the curve is non-linear) This torque of magnitude *Klin* is applied to the body in the direction of **bod[3]** Each distinct creature has a specific control law whereby the various magnitudes of *Klin*, involved in the controller, are determined by the masses and lengths involved In physical terms these torques could be generated by a motor and a spring attached to it or maybe just a simple spring

4 6.4 Simple Controller Design

Creation of a simple controller and its underlining rules is achieved through analysing the control behaviours of the inverted pendulum The controller should reflect the relationship between the body variables (q_3, u_3) and the applied torque τ This relationship can be graphed as a non-linear control curve If in plotting the variation of applied torque due to the variables (q_3, u_3), a straight line at an angle appeared then the control would be linear However non-linearity appears here because the torque applied varies depending on the values associated with q_3 and u_3

In formulating the control curve the following considerations were adhered to

- It is desired to keep the body upright (i e q_3 is inside the range $|0\ 1|$ radians) This indicates that most τ will be applied within a small angle range At the desired fully upright position i e $q_3 = 0$, no torque is applied
- As a servo motor has a maximum output, the curve will have to reach a maximum value at a certain angle
- In keeping with the passive philosophy the variation of applied τ should be as simple as possible

It should be noted that in essence two controllers were investigated, one which depends solely on q_3 and one which depends on $\{q_3, u_3\}$ (the methodology behind this is to try and find the most simple controller which works effectively) The curve that is utilised for the simpler case is shown in Fig 4.7. The actual control curve for the control system is essentially unknown, as for any system and therefore this is just a simple representation of what it might be like, containing three states or applied torque values, *zero, small* and *large.* This representation was achieved through brute force analysis of various τ values and the effect caused i.e. the Runge-kutta algorithm was studied with various torque values applied and the angle and velocity noted. Because the controller is simple with just 2 unknown torque values *small* and *large,* and it was desired to keep the body fairly rigid, it was easy to identify potential values for the torque state values. Thus the controller utilised is a type of bang-zero-bang controller.

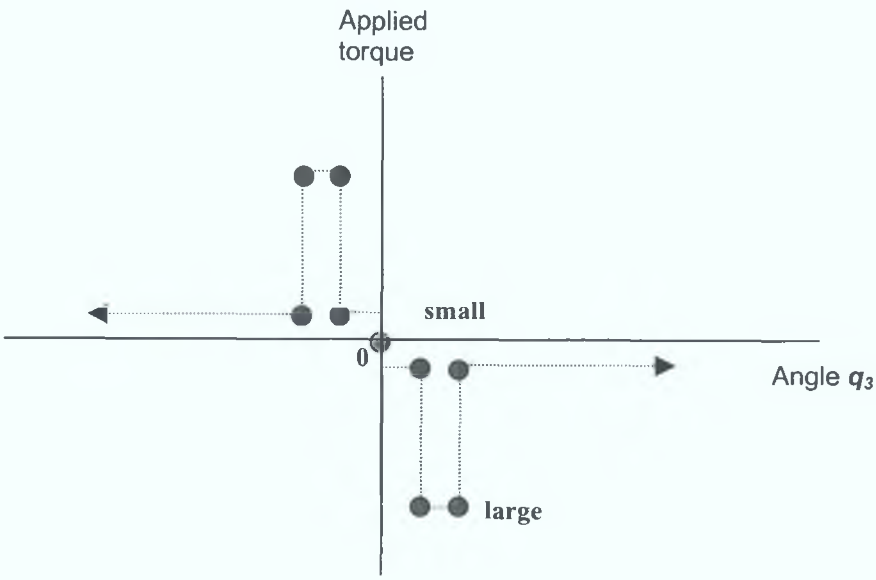


Fig 4.7: *Control curve for torso when the value of u_3 is unimportant.*

Formulation of the associated rules comes from the critical points on the curve The input for each rule is $\{q_3, u_3\}$ and initially it is assumed that the velocity u_3 is zero For the smallest positive angle the rule is

$$if\ (q_3 > 0\ and <= 0\ 01)\quad state = -small \tag{4 12}$$

The full set of rules for zero velocity is given in the Fig 4 8 When u_3 is not zero then the control curve is shifted This alters equation 4 12 giving a larger number if possibilities and the full table of values is also shown in Fig 4 9

Angle	**Torque**
Exactly 0	*zero*
0 00 - 0 01	*small*
0 01 - 0 02	*large*
> 0 02	*small*

Fig 4.8: *Table for control rules*

Values of u_3	$\leq$ -0.02	-0.01 $\rightarrow$ -0.02	0 $\rightarrow$ -0.01	0 $\rightarrow$ 0.01	0.01 $\rightarrow$ 0.02	$\geq$ 0.02
$<-1\times10^{-5}$	*small*	*small*	*small*	*small*	*large*	*small*
$\geq -1\times10^{-5}$ && $<-1\times10^{-4}$	*small*	*small*	*small*	*large*	*small*	*-small*
$\geq -1\times10^{-4}$	*small*	*small*	*large*	*small*	*-small*	*-large*
$=0$	*small*	*large*	*small*	*-small*	*-large*	*-small*
>0 && $<1\times10^{-5}$	*large*	*small*	*-small*	*-large*	*-small*	*-small*
$\geq 1\times10^{-5}$ && $<1\times10^{-4}$	*small*	*- small*	*-large*	*-small*	*-small*	*-small*
$\geq 1\times10^{-4}$	*- small*	*- large*	*-small*	*-small*	*-small*	*-small*

Fig 4.9: *Control rules when velocity u_3 is not 0.*

4.7 Energy

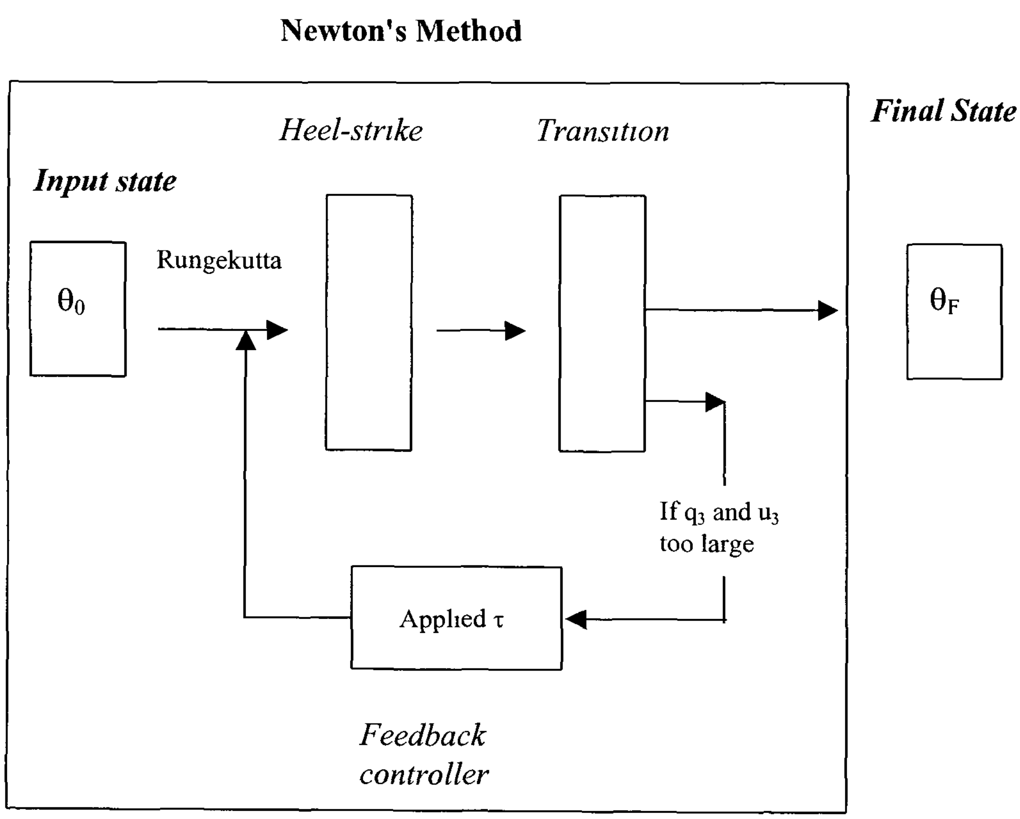
The energy involved in motion is now addressed. This consists of the mechanical energy the creature has and the applied external torques. The mechanical energy consists of the sum of the potential and kinetic energies. It is not addressed in this body of work but the equations involved are given in Appendix I.

Goswami addressed the issue of the change in the form of the components of the mechanical energy in [Gos96b]. He stated that, if the robot executes a periodic motion the energy of the system must return to its initial value after every cycle, and since the state values would be the exact same at the beginning as at the end, the potential and kinetic energy values should also be equivalent. Recall also that as the robot walks down on a slope its support point also shifts downward at every touchdown. As it loses gravitational potential energy in this way its kinetic energy increases accordingly. This is exactly the amount of kinetic energy that is to be absorbed at the end of each step by the impact. If we reset our potential reference line to the line of touchdown, the total energy of the robot appears constant regardless of its downward descent Failure to reset the potential reference line results in a slight loss of mechanical energy as the robot descends Although not explicitly shown in chapter five, when a sample of some mechanical energy values were configured this also proved to be the case here

With regards to the applied external torque, the value of this is just the sum of the individual forces in the time frame i e within each step It shall be shown in chapter five that the values involved are minute - indicating that although the passiveness of the creature may be compromised to keep the body upright, that cost is minimal

4.8 Block Diagram of complete system

To clarify how the complete system works , a block diagram is now shown



Chapter Five

Results

5.1 Introduction

The overall goal of this body of work is to enhance the pool of passive dynamic creatures through the simulation and analysis of a creature containing an *extended torso* Previous research has modelled a torso as just a hip point mass as outlined in [McG90] [Gos96] and [Gar97] Initial importance will be placed upon the effect of the addition of extra hip mass to a body-less creature as the torso will be an extension of this hip mass Therefore the first portion of this chapter deals with a body-less creature with varying hip-mass values (starting with $mhip = 0$) The results gained should, and do in fact, coincide with those of other body-less hip-massed creatures (as referenced above)

The second segment of this chapter focuses on the creature with the torso and in particular stability and performance issues It shall be outlined that the overall effect of the addition of a torso does not damage the creature's attributes, and in some situations can improve performance

Finally, the complete set of results, which are in part summarised here, are given in the appendices

5.2 Body-less creature results

5.2.1 *Initial Values and Basin of Attraction*

In order to find stable limit cycles parameter values should be wisely chosen as minor inaccuracies will lead to failure of convergence of Newton's Method There are two categories of parameter values, those which are held *constant* and those that are *varying* and numerical values are directly taken from known solutions given in previously published work e g [McG90] [Kuo00] The full list of parameter symbols with constant values and initial values for the variables which can be modified is given in Appendix F Variables such as *comleg* are those which can be modified and the effect of some of these parameter variations is outlined later

As the choice of the *initial state* values is crucial to the formulation of fixed point solutions, values for a <u>known</u> solution are incorporated This initial state is taken to be $\theta_0 = \{q_1, q_2, u_1, u_2\} = \{0\ 3015, -0\ 3015, -0\ 3763, -0\ 2822\}$, which is given in both [McG90] and [Kuo00] The allowance of variation in each state value can be summarised as follows (see Appendix G 2)

- the states q_1, q_2 can be altered by approximately 6%,
- the velocity u_1 is slightly more rigid and can be varied by approximately 5%
- The most flexible of all four states u_2 can be altered by up to approximately 75%

Taking all points together this shows that the basin of attraction is quite small for stable limit cycles

Finally it should be noted that the addition of a hip mass leads to a slight improvement in the versatility of q_1, q_2, in that if *mhip* is 1 2 or greater the range of values the angles take on increases by 0 01 radians (see Appendix G 2)

5.2.2 Limit Cycles

Once an initial state "guess" has been identified the process of finding a solution can begin Using the initial state values above, along with the constant values given in the Appendix F, Newton's method was implemented and solutions for the fixed points for a complete step i e solving equation 3 24 were sought Limit cycles as expected were indeed located and an example of one is given in Fig 5 1 More are given in Appendix G 1

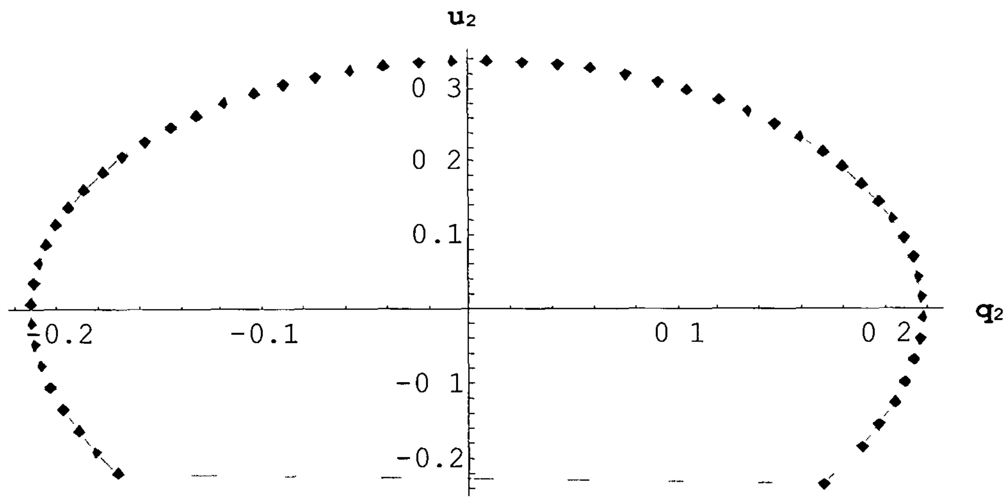


Fig 5.1: *The lımıt cycle for just the swıng leg ı e* $\{q_2, u_2\}$ *on* $\gamma = 0\ 005$ *radıans Note that mhıp = 0 ın thıs creatures descrıptıon*

5.3 Hip Mass effects

As the torso ın essence ıs an extensıon of the concept of the addıtıon of hıp mass to the creature, the effects of alterıng the hıp mass are of prımary ımportance Thıs sectıon hıghlıghts the effects of the addıtıon of ıncremental hıp mass values, partıcularly ın relatıon to performance ıssues, such as mınımum and maxımum attaınable slope

5.3.1 Varying the centre of mass of the leg

The centre of mass of the leg denoted by the scalar value *comleg* ıs gıven ın terms as dıstance from the end poınt of the rıgıd lınk to the centre of mass posıtıon It was found that the centre of mass could be moved by a small amount both away from and towards thıs end poınt The effect of the addıtıon of *extra* hıp mass has no profound effect on how much varıatıon the centre of mass can absorb Whıle the upper and lower bounds vary slıghtly the overall dıfference between both stays reasonably constant as ıs ıllustrated ın Fıg 5 2

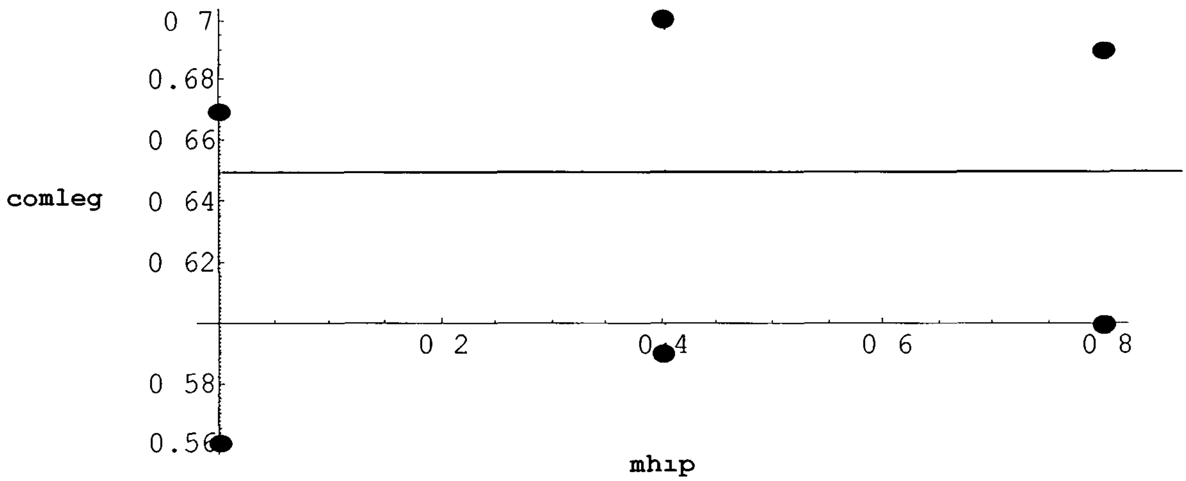


Fig 5.2: *The value of comleg is originally taken to be 0 645, the straight horizontal line Now for varying mhip i e 0, 0 4, and 0 8, the upper and lower bounds are shown here As can be seen, while the upper and lower bounds vary, the difference between both does not vary substantially*

5.3.2 Varying the foot radius R

With the variation of the size of the foot radius R there are two noteworthy outcomes Firstly there is a change in the slope needed for a limit cycle A small R needs a <u>steep</u> slope, whereas a large R needs a <u>small</u> slope As the goal for passive walking is efficiency i e slope as near to zero as possible, the larger the R, the more efficient the creature is However realism dictates that the value of R be kept rather small and thus the value $R = 0\ 3$ is utilised throughout this work, in the form of a constant value

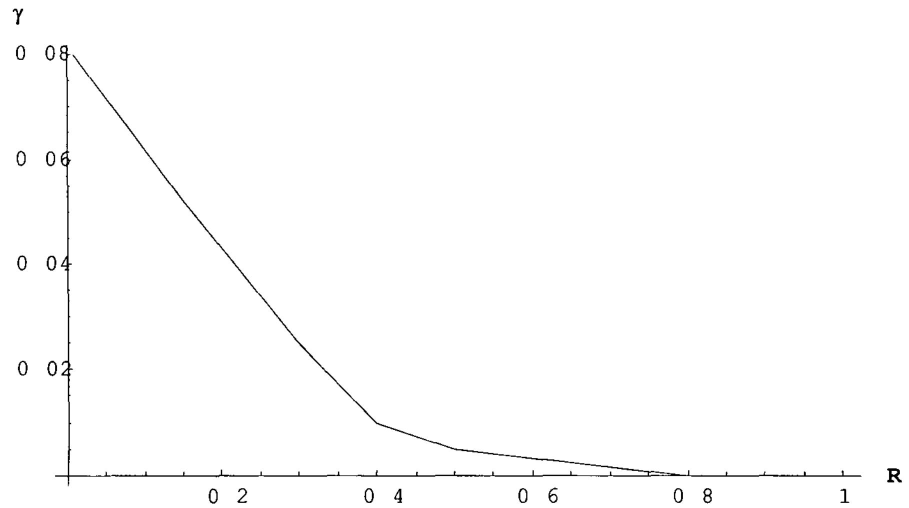


Fig 5.3: *Effect of varying R on γ needed For mhip = 0 8 this shows that γ needed to keep the stance leg angle q_1 at about 0 3015 is high as R approaches 0 and low as R approaches 1*

Secondly the value of R has a profound effect on the value of the speed eigenvalue i e λ_1 (recall that eigenvalues λ_i's were discussed in section 4 3 3) When R is in proximity to zero this value is also close to 0 and when in proximity to 1 it is also close to 1

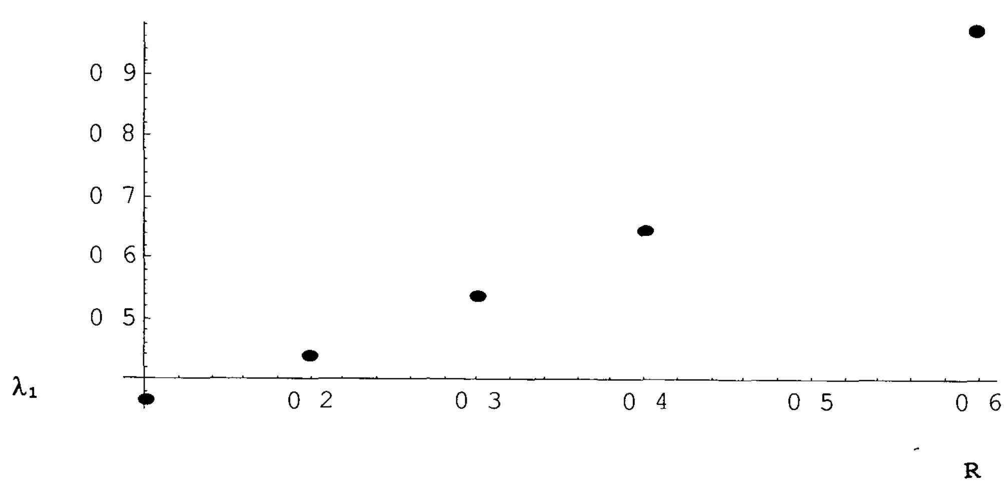


Fig 5.4: *The speed eigenvalue λ_1 as R increases*

5.3.3 Effect on leg angles - i.e. inter-leg angle

The inter-leg angle is defined as the angle between the stance and swing leg i e the angle at the hip joint Since both angles are equal and opposite the inter-leg angle is therefore just $2q_1$ For *mhip* = *0* as γ increases the inter-leg angle increases also i e the creature takes wider steps on larger γ

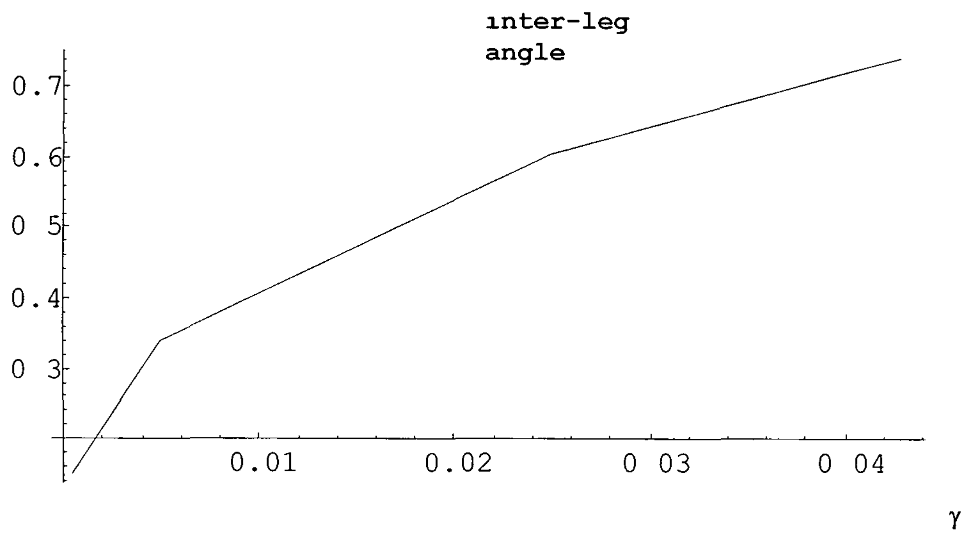


Fig 5.5: *Inter-leg angle increasing as γ increases*

As extra hip mass is added the effect again is for the inter-leg angle to widen, as illustrated in Fig 5 6

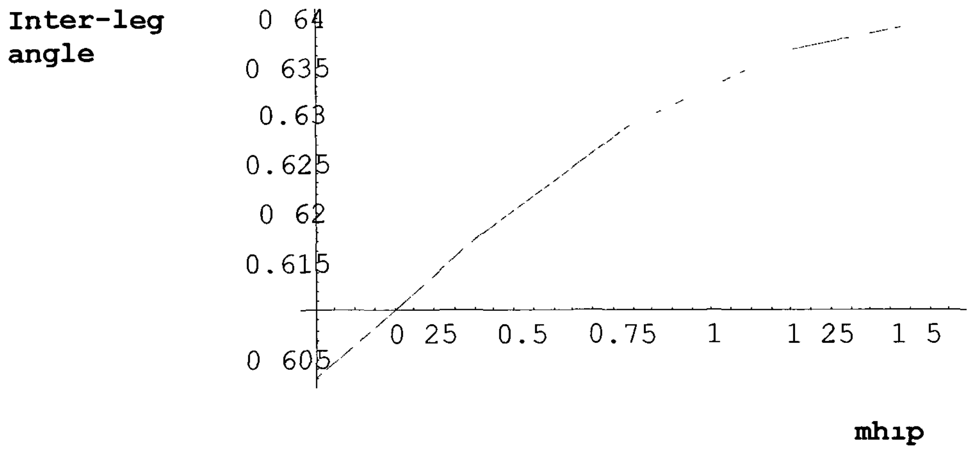


Fig 5.6: *As mhip increases (here from 0.2 to 1.6) so does the width of the inter-leg angle*

5.3.4 Slope - minimum and maximum and Stability

As slope is an indication of efficiency (the walker needs a slope to move and thus $\gamma = 0$ would be perfectly efficient) it is important to note effects on the minimum slope attainable by parameter changes. Firstly *mhip* is taken to be 0 and the limits of γ for period one gait are found to be from 0.002 to 0.043 radians. As hip mass is added and incremented, there is also initially a growth in efficiency. A hip mass, for instance of 0.8, leads to a decrease in minimum γ to 0.0005 radians. However if the hip mass is <u>too high</u> i.e. the creature has a heavy payload, then this efficiency gain seems to disappear. For example when *mhip* = 1.2 and *mhip* = 1.6 then the minimum γ attainable is 0.004 radians (as in Appendix G)

Concerning maximum attainable γ again the addition of *mhip* has a positive effect with the maximum attainable slope increasing as *mhip* is incremented. The maximum γ with *mhip* = *0* is 0.043, while with *mhip* = *1.2* a γ value of 0.058 radians is achievable

With regards to stability, changes in *mhip* adversely affect the speed eigenvalue i.e. λ_1. As the *mhip* value increases the value of λ_1 edges towards 1, as is shown in Fig

5 7 Slope also seems to affect this eigenvalue with low or high slope values bringing this close to instability and an example is illustrated in Fig 5 8

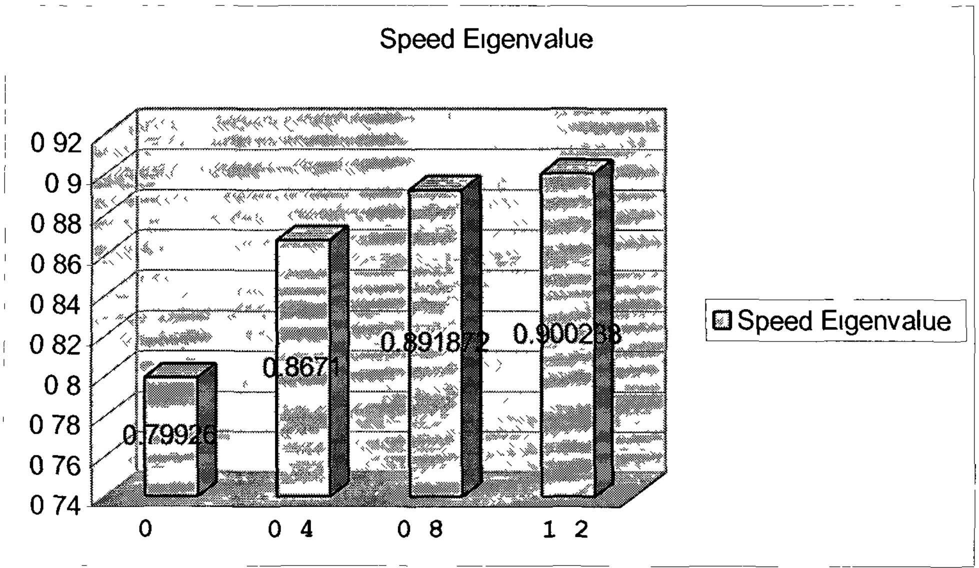


Fig 5.7: *Variation in λ_1 due to additional mhip*

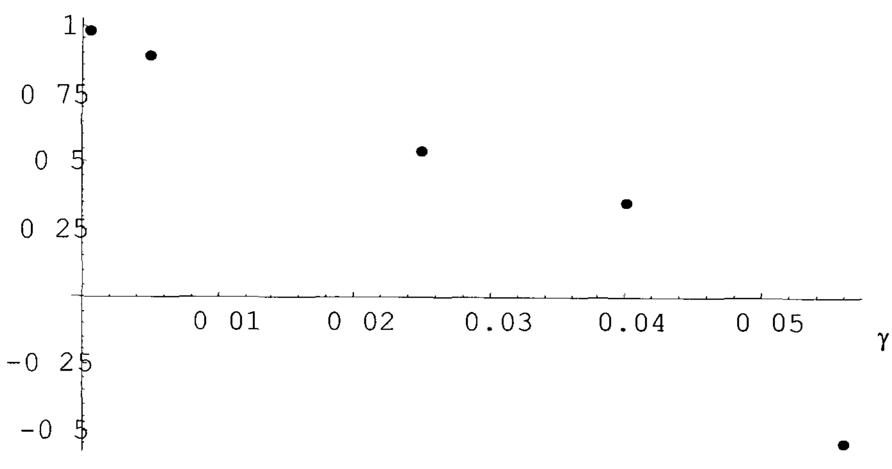


Fig 5.8: *Effect of variation of λ_1 due to increasing γ, for mhip = 0 8*

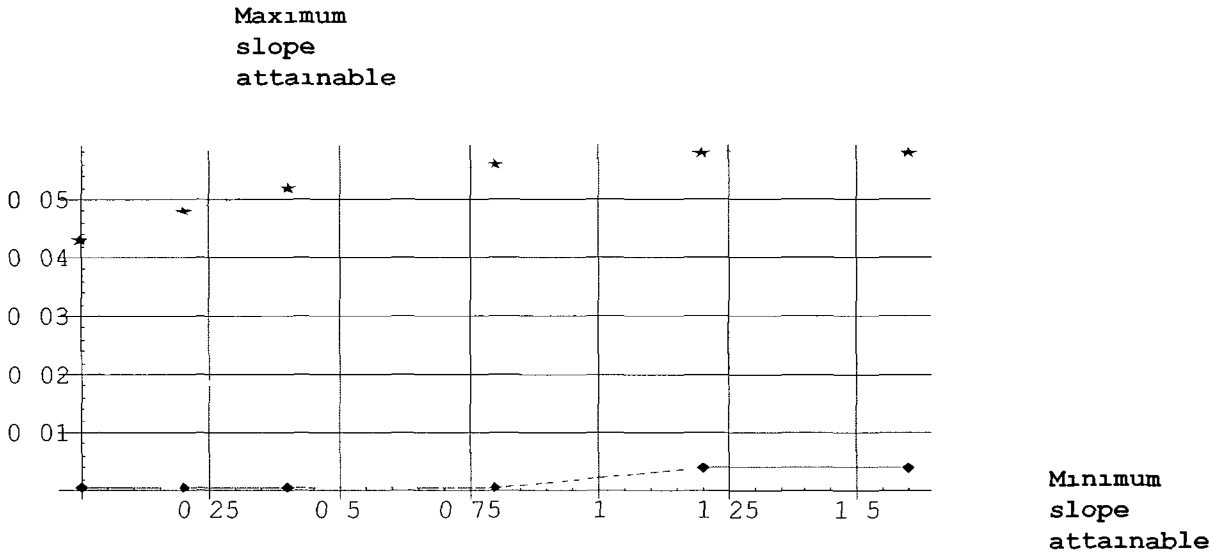


Fig 5.9: *Shows the variation in max and min γ as the value of mhip increases*

5.3.5 Effect on Step period

The step period is the time for one complete step to occur i e from heel-strike to heel-strike For a creature with no hip mass the step period increases as the slope increases i e as the slope gets larger the creature takes longer (i e section 5 3 3) slower (i e fewer) steps

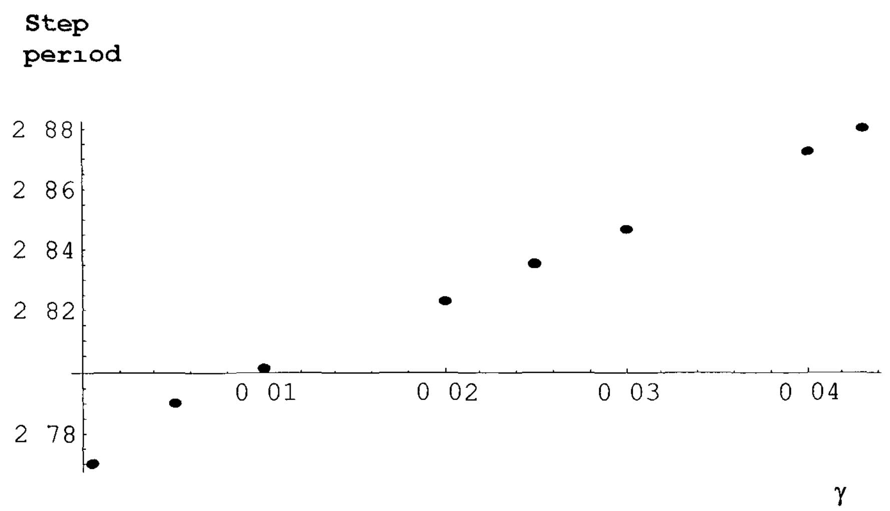


Fig 5.10: *The step period for the creature with mhip = 0 is shown, varying as γ increases*

The addition of varying hip mass does not alter the fact that the step period still increases as γ does However as the *mhip* value increases the overall step period decreases as is outlined in Fig 5 11

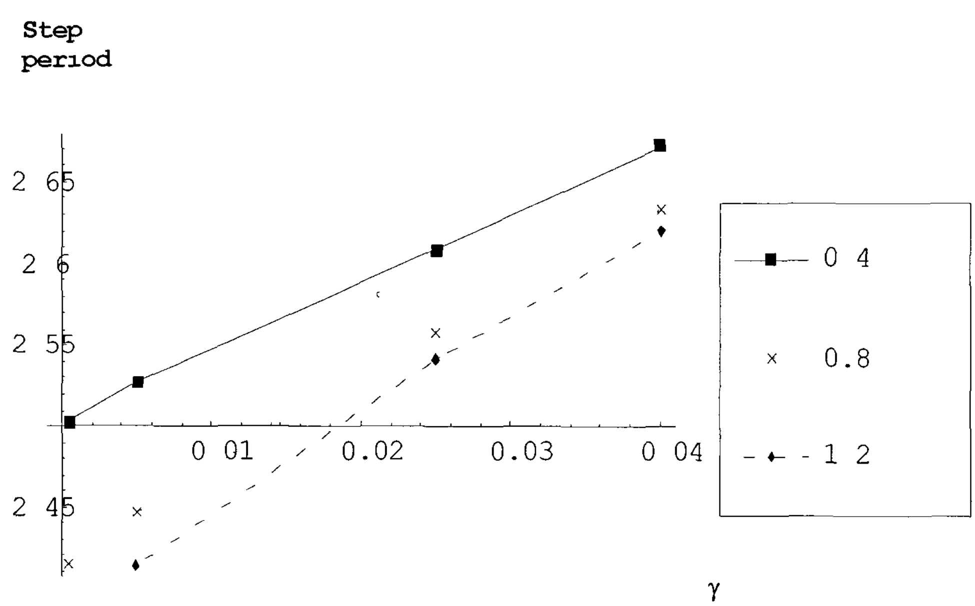


Fig 5.11: *Step period for mhip = 0 4, 0 8 and 1 2 over slope varying from $\gamma = 0$ to $\gamma = 0$ 04 radians*

5.3.6 Velocity and step length

Another means of determining performance is to determine the step length and velocity of the creature The step length as outlined in section 3 2 1 is given by $Length = 2 L Sin$ where is the inter-leg angle The velocity on the other hand of the robot over one step is

$$v = \frac{Length}{T}$$

i e equation 4 4

For the mass-less creature as γ increases so too does the step length and velocity

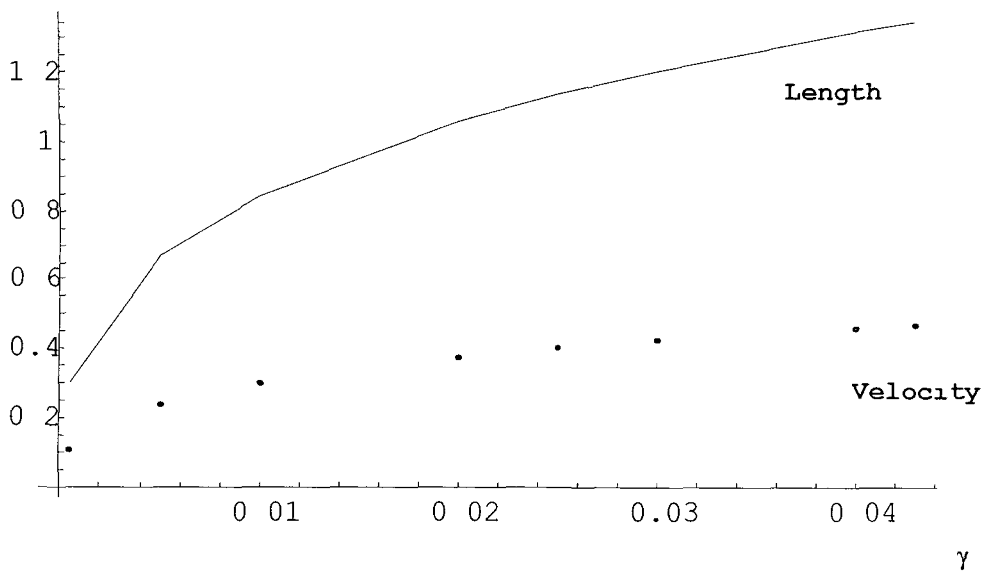


Fig 5.12: *Changes in velocity and length for the mass-less creature as γ increases*

The addition of a hip-mass does not alter the fact that both velocity and step length increase as slope increases Again though both numerical values are larger than that involved in the mass-less case i e the addition of extra *mhip* increases both the velocity and step length of the creature

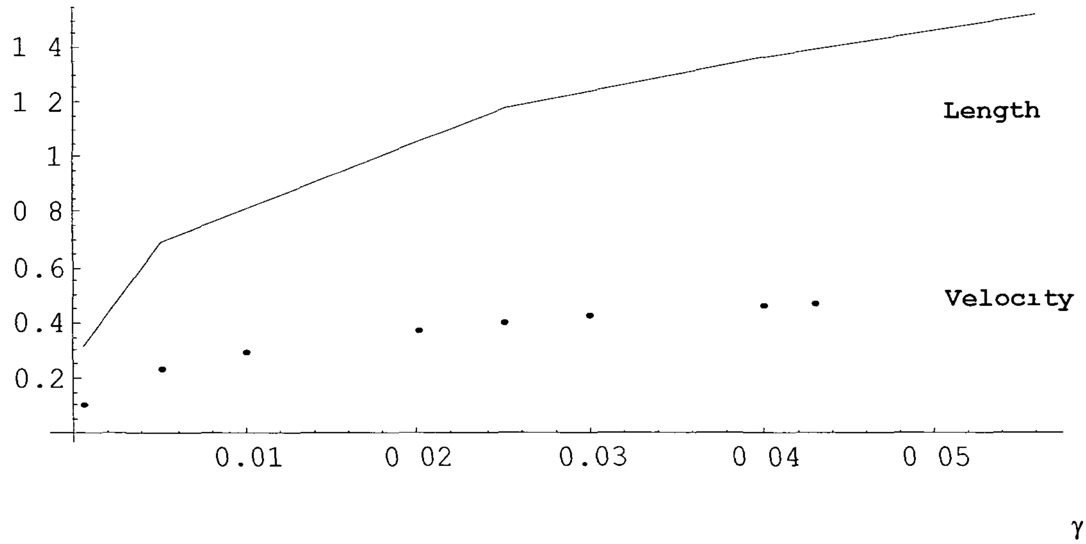


Fig 5.13: *Length and velocity for creature with mhip = 0 8 again as γ increases*

5.3.7 Addition of a damper

A damper is included at the hip joint in an effort to improve the maximum attainable slope The form of this damper is quadratic i e $Ffric(u_2 - u_1)^2$ where *Ffric* is the coefficient of damping (as was outlined in section 4 5 3) Addition of a damper has two main effects Firstly the versatility of the creature is inflated through the use of this applied damper This is outlined in Fig 5 14, which shows that the maximum attainable γ can be increased by using dampers of varying damper coefficient values The process of finding the best coefficient values is really through trial and error but it was found that only small values worked well here The coefficient values used with the various *mhip* values are {*mhip* = 0 4, *Ffric* = -0 008}, {*mhip* = 0 8, *Ffric* = -0 012} and {*mhip* = 1 2, *Ffric* = -0 02}

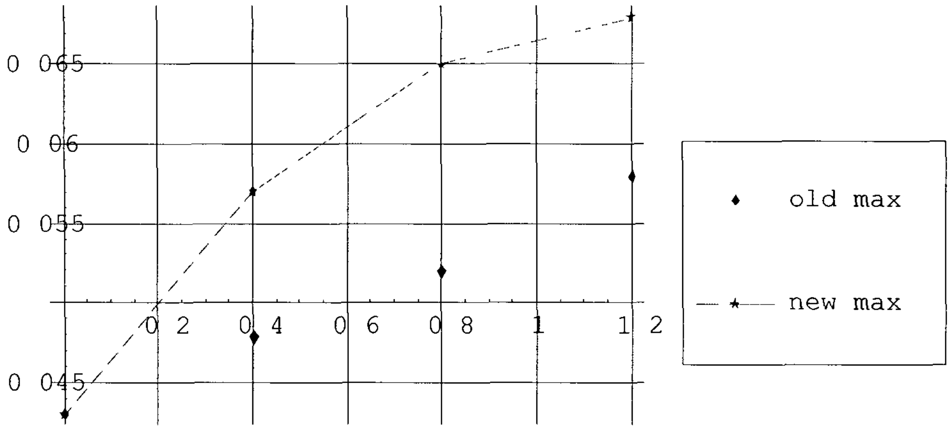


Fig 5.14: *Increased maximum slope attainable provided by the addition of a quadratic damper*

The second by-product of the utilisation of a hip damper is a change in the general appearance of the limit cycles This is illustrated in Fig 5 15 which shows the limit cycle for a damper-less model and for two different values of the damping coefficient

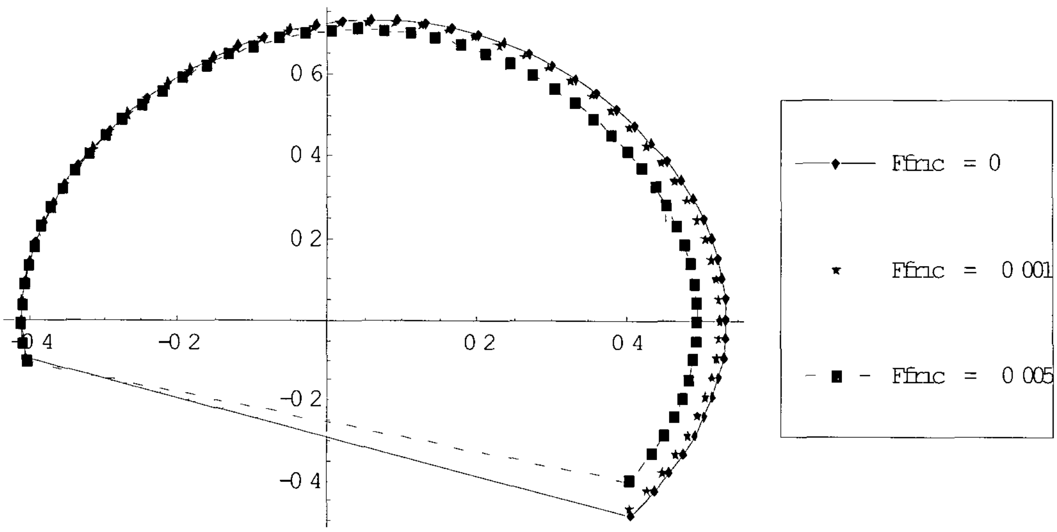


Fig 5.15: *Limit cycles for creature with mhip = 0 04 on γ = 0 052 radians*

5.3.8 Bifurcation

A bifurcation is a period doubling For one to occur period one solutions should disappear and stable period two solutions appear Other researchers [Gos96b] [Gar99] found them in their models as various parameters were brought towards a limit (i e bifurcation point) For instance stable period one solutions would exist up to a certain slope and would then bifurcate into stable period two solutions An indication of bifurcation is achieved by inspection of the eigenvalues of the Jacobian of the Poincaré map in the neighbourhood of the limit cycle These are identified for stability (see section 4 3) and should be all less than 1 for stability At a bifurcation point at least one of these eigenvalues crosses the unit circle Clearly on inspection in Appendix G (e g G 5) this is not the case here so bifurcation does not occur In [Gar98b] Garcia indicates that period doubling does occur for the model described here but in addition with knees Therefore the addition of knees causes bifurcation to arise

5.3.9 Summary

The following is a summary of the effects of increasing slope on the various parameters, firstly when the *mhip* is 0 and secondly when it is not

***mhip* = 0**

Slope	*Inter-leg angle*	*Velocity*	*Step Period*	*Step Length*
Increasing	Increases	Increases	Increases	Increases

***mhip* > 0**

Slope	*Inter-leg angle*	*Velocity*	*Step Period*	*Step Length*
Increasing	Bigger + Increases also	Increases	Increases	Increases

In terms of efficiency and maximum slope, as more *mhip* is added the efficiency (i e minimum slope γ) deteriorates and the maximum slope increases A damper placed at the hip can increase the maximum attainable slope

Finally this body of work deals with "human-like" motion and thus the goal would be for the creature to carry a fairly heavy payload Therefore if the results for the bodied creature can show an increase in efficiency and maximum slope attainable it can be deemed a success

5.4 Bodied Results

5.4.1 Initial Values and Basin of Attraction

The choice of parameter values and the initial state guess for θ_0 is of vital importance as invalid values may lead to failure in the discovery of fixed points As a starting point parameter values and state values where stable passive walking can be expected for the body-less creature are utilised With regard to the length and centre of mass of the torso, initial values were kept small until solutions were discovered Finally in conjunction with these values is the desired initial body position of uprightness i e

$q_3 = u_3 = 0$ This gives the guess value for the state as $\theta_0 = \{q_1, q_2, q_3, u_1, u_2, u_3\} = \{0\ 3015, -0\ 3015,\ 0, -0\ 3763, -0\ 2822, 0\}$

The original bang-zero-bang controller torque values were found through manual tweaking Through brute force various torque values were tested (within the Runge-kutta part of the solution process) and when the desired solution of a fairly rigid torso was found these were used as the initial values These then didn't change much and only 'considerable' changes in mass distributions (i e hip and body) required alteration of the applied torque state values

According to McGeer, a human has about 70% [McG90] of body mass above the hip and thus the goal would be to have a fairly heavy payload for the creature's legs to carry Therefore while some examples quoted in this chapter are for small hip and body masses, it is those concerning heavy payloads that are of primary interest

The addition of a torso leads to an increase in the basin of attraction for the state variable values Examples are shown in Appendix H 3 and shows that increases of up to 15 % are available on the state values for the body-less case As the controller is designed to quickly swallow up errors in the body states it is also worth noting that q_3 and u_3 don't need to be too accurate

5.4.2 Limit Cycles

Of primary concern was the discovery of fixed points i e limit cycles, if they existed at all Limit cycles were indeed found and an example is shown in Fig 5 16

As a means of testing the code involved in the solution method initial values concerning the body that are minuscule were chosen, and disregarding the bang-zero-bang control (since there is no body length), this gave a limit cycle similar to that of the body-less creature with the same parameters - see Appendix H 2

Initially, as shown in that diagram, the solutions are unstable, but this obstacle fades through the addition of a damper and "tweaking" of the parameter values as shall be indicated later

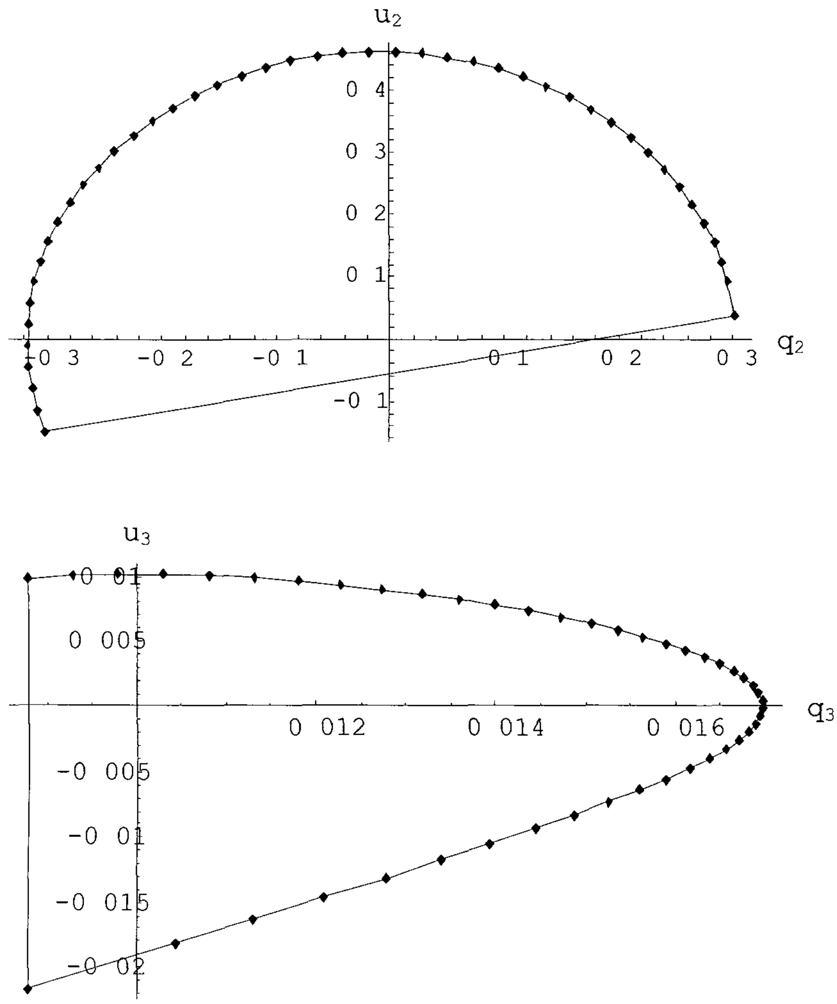


Fig 5.16: ***<u>Unstable</u>*** *solution The parameters involved are mbody = 0 8, mhip = 1, combody = 0 7, lbody = 0 8 and applied τ values {small = 0 003, large = 0 02}*

The goal throughout is to provide human-like motion and unrealistic solutions that were encountered, such as the body performing complete revolutions, are non-anthropomorphic and thus were discounted

5.4.3 Stability

Stabılıty ıs a contentıous ıssue wıth the presence of a just a torso and applıed torque leadıng to eıgenvalues well over the boundary lımıt of 1 Takıng the sıtuatıon ın Fıg 5 16 ı e an unstable solutıon, there are two eıgenvalues (ı e $_1$ and $_2$) outsıde the unıt cırcle and two are approxımately 0 (ı e $_5$ and $_6$) as ıllustrated ın Fıg 5 17

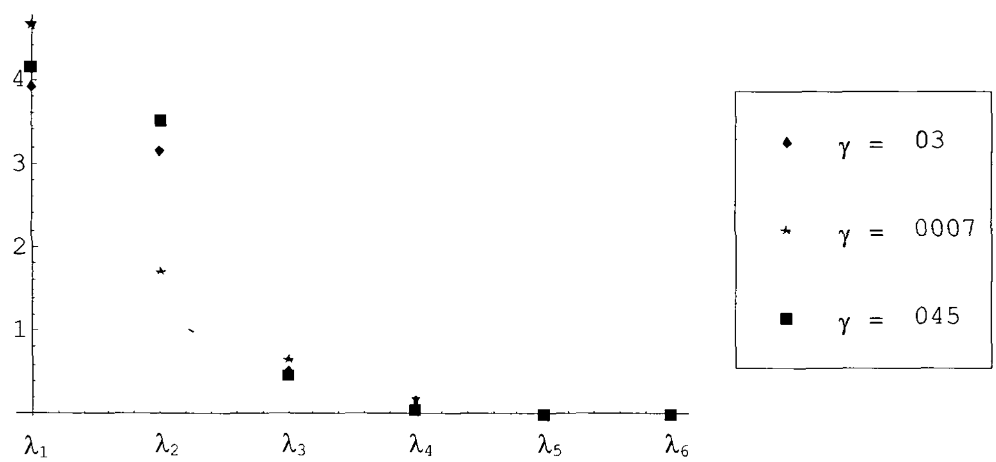


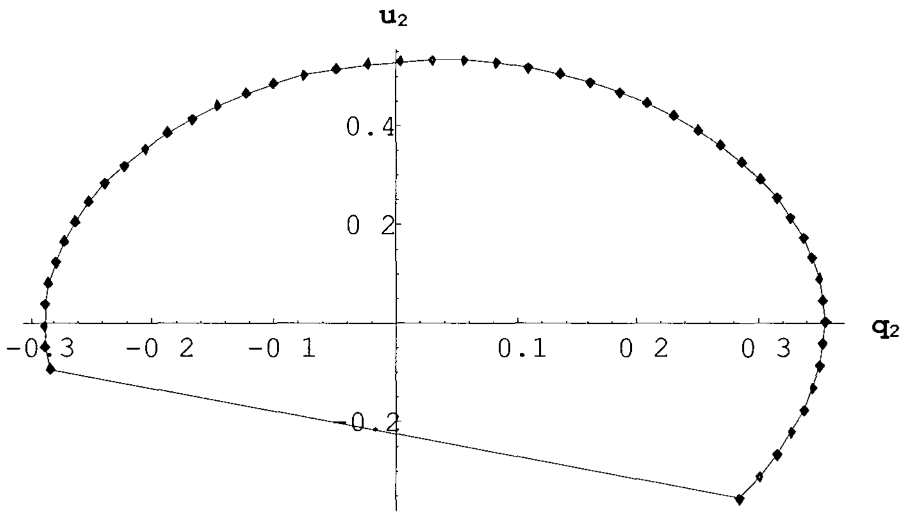
Fig 5.17: *Eıgenvalues for unstable solutıon on three dıfferent γ values The parameters agaın ınvolved are mbody = 0 8, mhıp = 1, combody = 0 7, lbody = 0 8 and applıed τ values {small = 0 003, large = 0 02}* <u>*Note*</u> *that ıt ıs the first two eıgenvalues that provıde ınstabılıty*

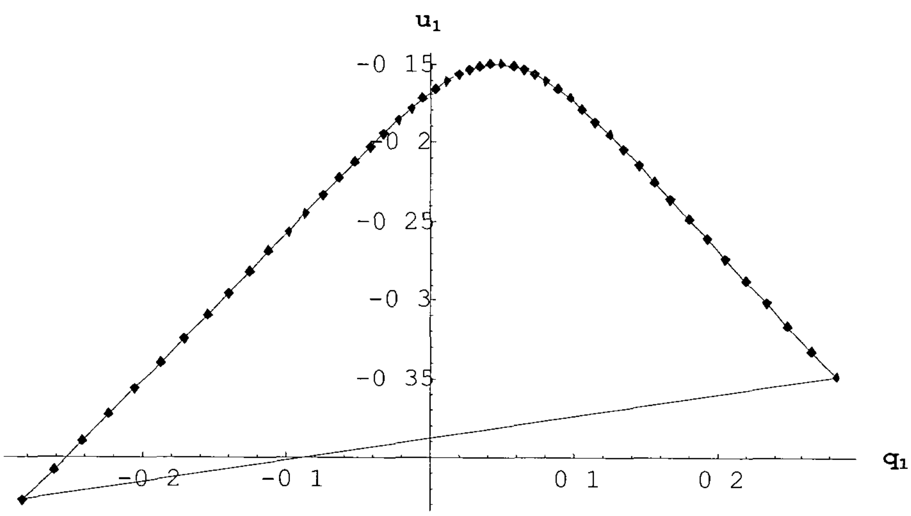
As prevıously descrıbed ın sectıon 4 5 3 ıt ıs necessary to utılıse a damper to provıde stabılıty Determınıng the type of damper used was achıeved through trıal and error Inıtıally a lınear damper was adopted and ın addıtıon a constant factorıng value had to be combıned to provıde a successful solutıon The overall applıed torque and damper ıs therefore gıven ın sectıon 4 5 3 ı e

$$F_{total} = Ffrac(Klın - damp \times u_3),$$

where *damp* is the damping coefficient, *Klin* is the applied torque and *Ffrac* is the constant factoring value

Once this was taken into consideration, stable solutions were discovered and can be seen in Appendix H





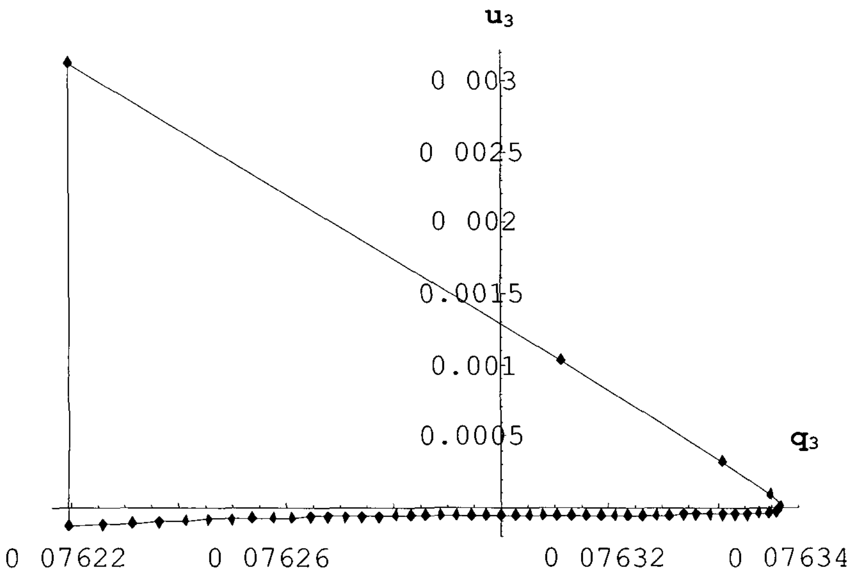


Fig 5.18: ***Stable*** *solution The following parameter choices are made mbody = 1, mhip =1 5, lbody =0 8, combody =0 795, damp = 0 55, Frac= 5, and applied τ values {small = 0 0008, large = 0 0009}*

A stable solution is now shown in Fig 5 18 One final thing to note is that the first eigenvalue remains at a value of approximately one This value of one indicates that the cycle is neutrally stable for perturbations along the relevant eigenvector and thus perturbations will neither shrink nor grow

It may be that another combination of applied torques and dampers may lead to a more "improved" class of solutions, in particular stability, and this is one option for future work

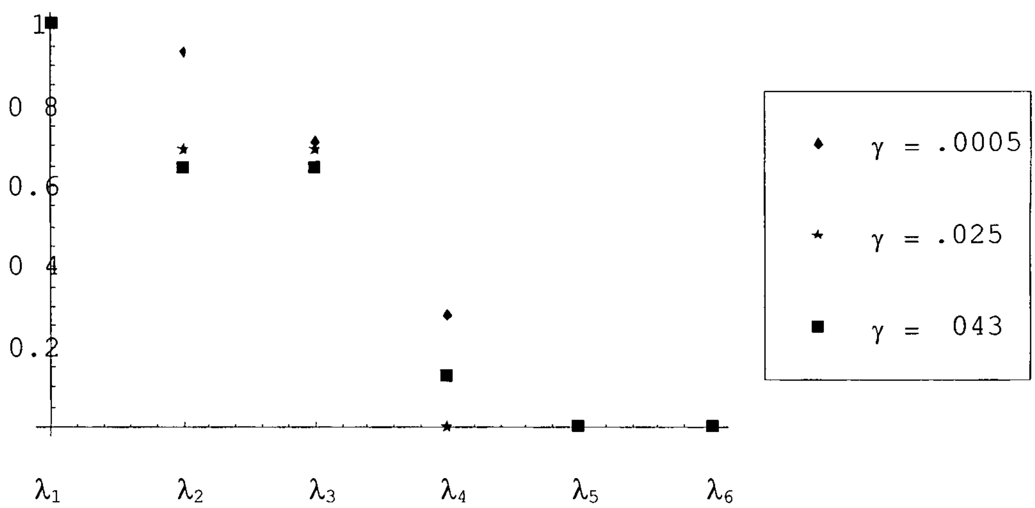


Fig 5.19 *Eigenvalues for stable solution in Fig 5 18 on three different γ values*

5.4.4 Effects of varying slope.

Angles

As the slope γ increases both q_1 and q_2 increase and thus does the inter-leg angle The angle the body makes i e q_3 however decreases as γ is enlarged i e the body tends towards "*straightening itself up*"

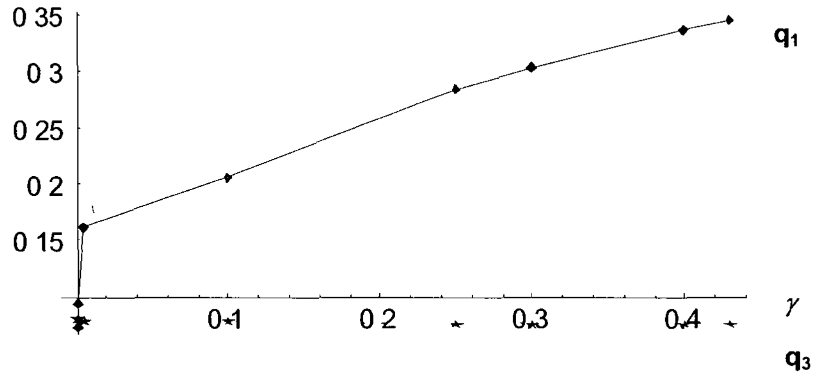


Fig 5.20: *Variation in angles as γ increases The case shown here is for the following parameters mbody = 1, mhip =1 5, lbody =0 8, combody =0 795, damp = 0 55, Frac= 5, and applied τ values {small = 0 0008, large = 0 0009} Note that the range of γ is from the minimum of 0 0005 to the maximum of 0 043 radians While it is hard to make out in the diagram q_3 actually decreases from 0 08 radians to 0 07*

Step period

Once again the step period is the time taken for one complete step i e from heel-strike to heel-strike and in common with the body-less creature the step period increases as the slope increases

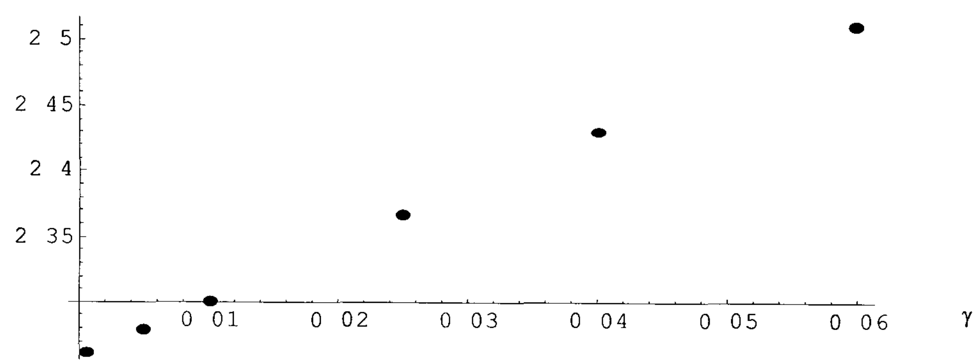


Fig 5.21: *Variation in step period as γ increases The case shown here is for the following parameters mbody = 0 4, mhip = 1, lbody = 0 8, combody = 0 795, damp = 0 39, Frac= 5, and applied τ values {small = 0 00002, large = 0 0001}*

Velocity

Velocity again determined by equation 4 4 1 e

$$v = \frac{Length}{T}$$

also increases

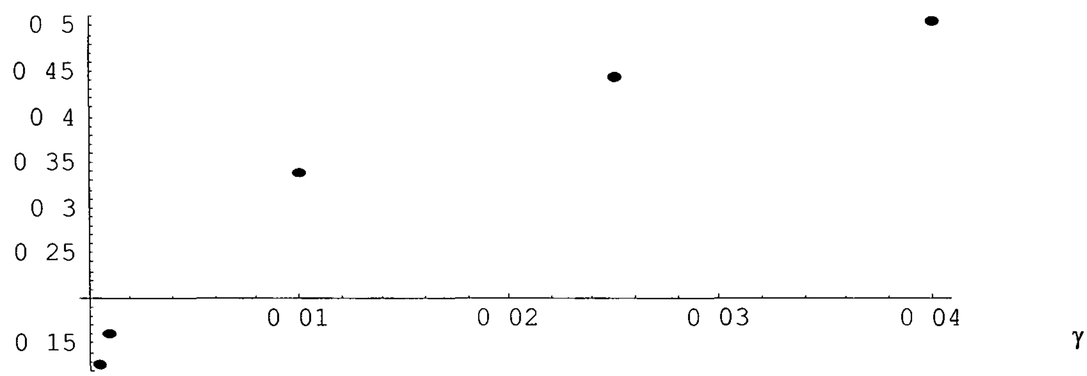


Fig 5.22: *Variation in velocity as γ increases The parameters are mbody = 0 8, mhip = 1, lbody = 0 8, combody = 0 795, damp = 0 39, Frac= 5, and applied τ values {small = 0 00005, large = 0 0005}*

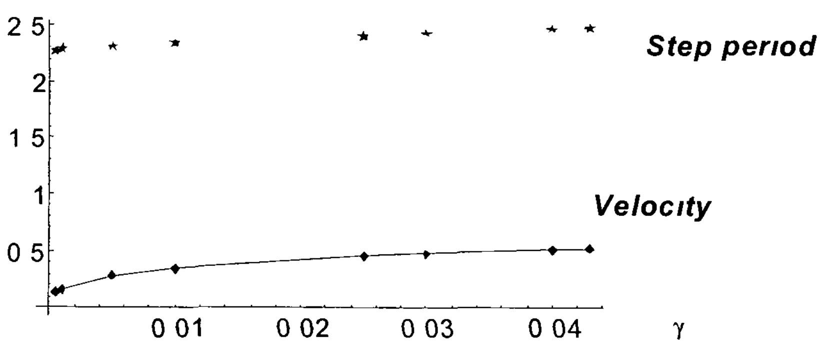


Fig. 5.23: *Effect of variation of γ on both step period and velocity The case shown*

here is for γ from the minimum of 0 0005 to the maximum of 0 043 radians The parameters involved are mbody = 1, mhip =1 5, lbody =0 8, combody =0 795, damp = 0 55, Frac= 5, and applied τ values {small = 0 0008, large = 0 0009}

Stability

As the slope increases the eigenvalues tend to approach zero until instability occurs An example of their structure is given in Fig 5 24

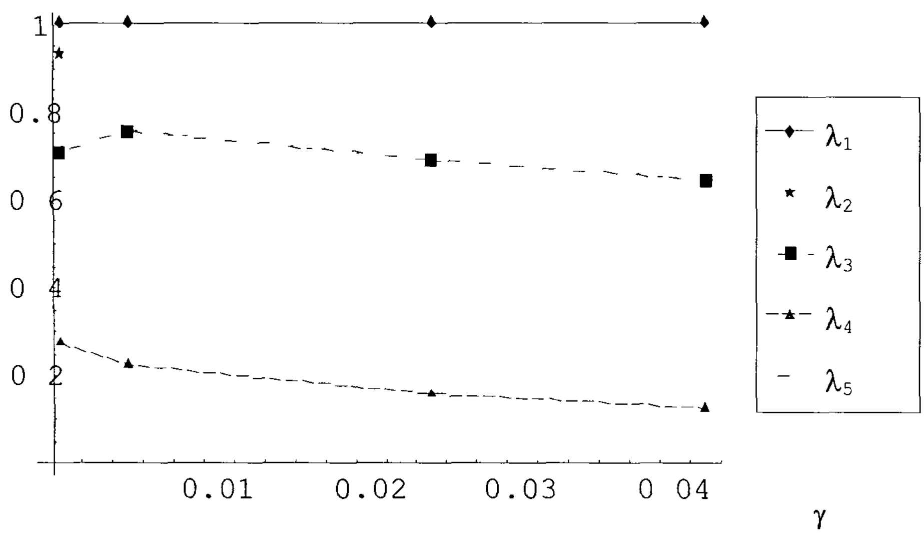


Fig 5.24: *Eigenvalues for the creature in Fig 5 23 as γ is varied Note that λ_5 and λ_6 are both approximately 0 and thus only one is shown*

5.4.6 Efficiency and Maximum slope

Efficiency is determined as the minimum slope attainable by the creature i e the minimum γ needed for continuous movement In section 5 3 4 it was concluded that the minimum slope needed for a creature with hip mass <u>included</u> was approximately 0 0005 radians In order therefore to claim that the bodied creature outlined does not significantly hamper efficiency, then values close to that of the body-less creature are desired Efficiency depends on the parameters involved (i e *mhip, mbody, combody,* *damp*) but as shown through all the examples outlined in appendix H, efficiency is good in general with minimum slopes equalling those of the body-less detected (e g creature with parameters *mbody = 1, mhip =1 5, lbody =0 8, combody =0 795, damp = 0 55, Frac= 5, and* τ *states {small = 0 0008, large = 0 0009}* has minimum γ of 0 0005 radians)

As for the maximum slope attainable this again depends on how the parameters are formulated but improvements can be made on that of the body-less creature For example a body-less creature of hip mass equal to 0 8 had a maximum feasible slope of 0 057 Now for a hip mass of 1 0 and body mass of 0 4 this could be increased to 0 06 radians for stable motions and as far as 0 81 radians for unstable ones Further increases are attainable through the addition of a hip damper, as shall be outlined in section 5 4 9

Finally it should be pointed out that further increases in maximum slope and efficiency may be attainable through a variation on the applied torque and damper used here, and this shall be addressed in the next chapter as possible future work

mhip	***mbody***	***lbody***	***combody***	***damp***	***minimum***	***maximum***
0 4	0 2	0 1	0 095	0 22	0 0006	0 06
1 0	0 4	0 8	0 795	0 3	0 0008	0 06
1 5	1 0	0 8	0 795	0 55	0 0005	0 043

Fig 5.25: *Table of some minimum and maximum* γ *values*

5.4.7 Varying the centre of mass of the body

As the centre of mass of the body is measured in relation to the end of the link, the higher the value of *combody* the closer the centre of mass is to the hip mass point For an unstable solution (i e no damper) the range of values for which fixed points can be found is fairly large In approximate terms the centre of mass can be moved from the end closest to the hip up to about the middle of the body link and still fixed points are found However all solutions are unstable The effects of moving the centre of mass up the body away from the hip are as follows all angles decrease (i e the inter-leg angle becomes smaller and the angle the body makes becomes more upright), the velocity u_3 increases and stability deteriorates

When the damper is added for stability the range of values of *combody* for which fixed point solutions can be found is diminutive As an example for the following creature parameters *mhip = 1 5, mbody = 1 0, lbody = 0 8 and damp = 0 55, combody* can be varied from 0 799 to 0 793 before instability occurs and after 0 75 no fixed point solutions are possible The same effects as above are also noted and a full set of solutions is given in Appendix H 5

This low centre of mass necessity is one flaw that needs to be eradicated and would form one major component of future work, possibly through the addition of more dampers This issue is addressed in the next chapter

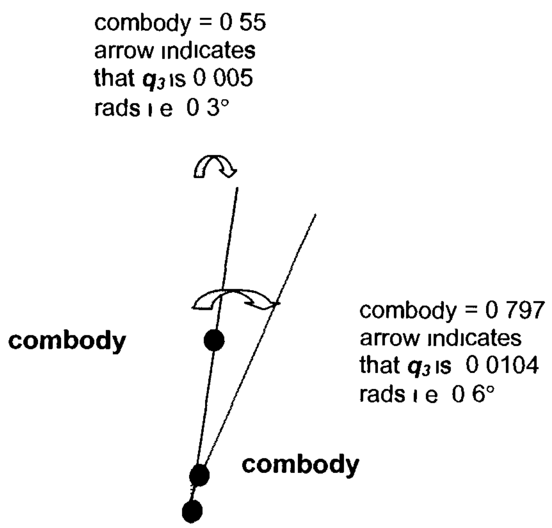


Fig. 5.26. *Effect of variation of combody on the body angle q_3 This is for the unstable case of parameters mbody = 0 8, mhip =1, combody =0 7, lbody =0 8 and applied τ values {small = 0 003, large = 0 02} on γ = 0 025*

5.4.8 Effect of varying body mass

The examples shown throughout this chapter have given a definite value to *mbody* What is addressed now is how much scope there is for variation to this value Once again variation depends on the parameters in question but in general there is scope for alteration The table below shows how much some values can be changed and the main detail that can be gathered from these results is the following the body mass must be <u>less</u> than the hip mass for stable limit cycles to occur Again this is an issue for further work and is addressed in the next chapter

mhip	*mbody*		*minimum*	*maximum*
1 5	1 0		0 4	1 2
1 0	0 4		0 3	0 7
0 4	0 2		0 2	0 35

Fig: 5.27: *Variation possible in mbody values*

5.4.9 Effect of varying radius of gyration

The radius of gyration for the body is given as a constant value of 0 121 - see Appendix F However this value can be altered - the estimation given is the maximum allowed but it can be decreased as far as 0 05 The effect of diminishing the value is as follows all angles decrease m size

The radius of gyration for the leg is also given as a constant value of 0 09 For the body-less creature a value of 0 121 was used but this only leads to failure of Newton's method to converge if used here The range of possible values is from 0 1 back to 0 04 Again the effect of diminishing the value is to decrease the angular values

5.4.10 Hip Damper

Taking cue from the body-less creature and in an effort to improve versatility a damper was placed at the hip joint As outlined in section 4 5 3 both a linear and quadratic were tested but it was the quadratic one that provided initially impressive results Therefore the equation used for the applied damper is $Ffric(u_2 - u_1)^2$, where $Ffric$ is the damping coefficient and u_1 and u_2 are the velocities of the stance and swing angles Individual values utilised for $Ffric$ are given in Appendix H 7

Again there are two outcomes of note with regard to the addition of the quadratic damper described Firstly there is an increase in the maximum slope attainable The table in Fig 5 28 highlights the increases for a few examples It is worth highlighting that with the body-less creature and additional hip damper, the maximum slope found for a realistic creature was 0 07 radians, which is increased on slightly here

Parameters	***mhip***	***mbody***	***Old max γ***	***New max γ***	***Increase***
	1 0	0 4	0 06	0 073	≅ 21 %
	0 8	0 4	0 055	0 065	≅ 18 %
	1 0	0 8	0 04	0 051	≅ 27 %
	1 5	1 0	0 043	0 055	≅ 27 %

Fig 5.28: *Increases in maximum slope through addition of damper*

Secondly the addition of a hip damper alters the general appearance of the limit cycle created Addition of a damper, may slightly shrink or magnify the format of the limit cycle

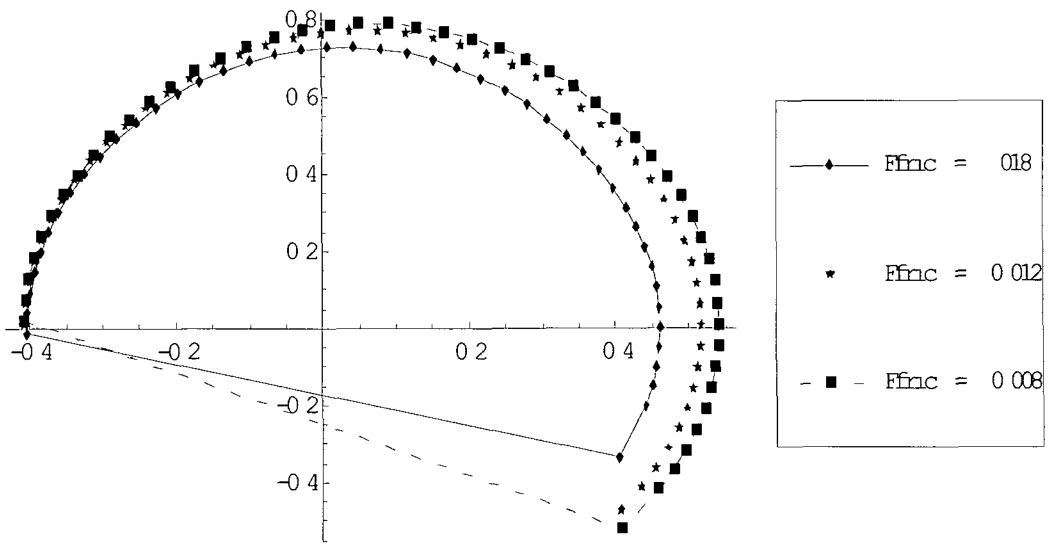


Fig 5.29: *Limit cycles for various hip damper coefficient values The creature parameters are mbody = 0 4, mhip = 1 0, damp = 0 39, Frac= 5, lbody = 0 8 all on a slope of γ = 0 065 It is worth noting also that previous to this damper addition the maximum attainable γ was 0 055 radians*

5.4.11 Total applied torque in each step.

To keep the body upright and stable it has been established that external torque values are required How significant is the value of this external torque? The complete external force applied to the creature is given in section 4 5 3 i e $F_{total} = Frac(Klin - damp \times u_3)$ This is per iteration and per step involves summation but is diminutive For example, for the creature outlined in Fig 5 23 on $\gamma = 0\ 025$ it is -0 0289354 N/m per step The applied torque increases as the γ increases and is shown in Fig 5 30 and Fig 5 31 Finally more values are given in Appendix H 8

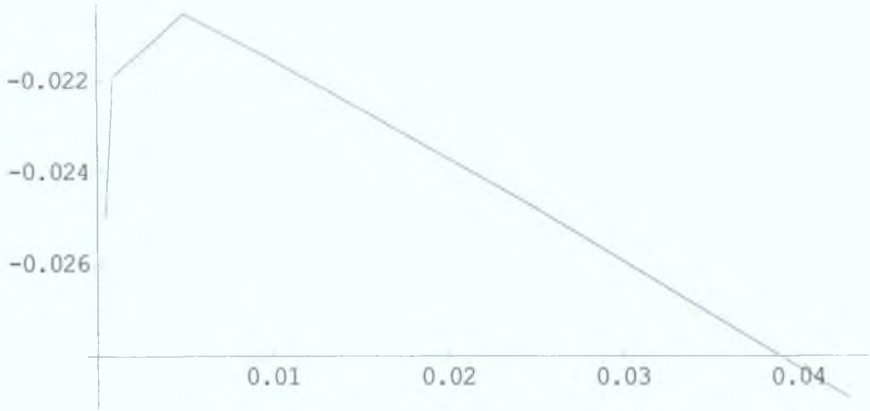


Fig 5.30: *Total applied torque as γ increases for one step for the creature with mhip = 1.5 and mbody = 1.0. Note that the applied torque is shown as having a negative value - this means that it is a restoring force.*

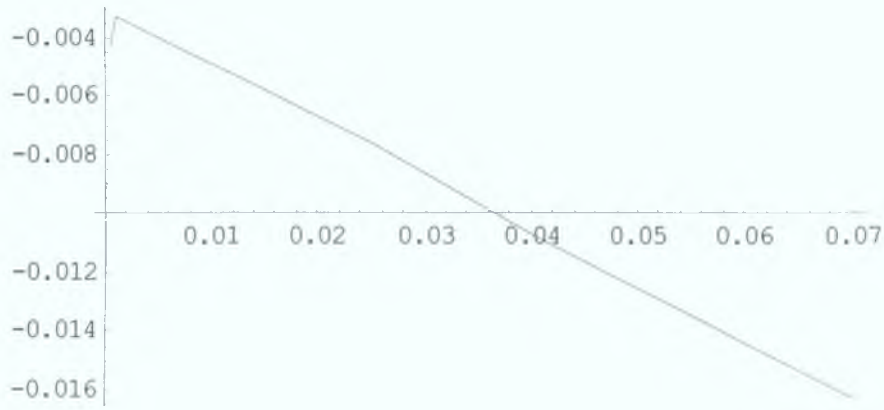


Fig 5.31: *Total applied torque as γ increases for one step for the creature with mhip = 0.8 and mbody = 0.2.*

5.4.12 Controller Issues

The purpose of this work was the addition of a torso with as simplistic a controller as possible, thus trying to sustain as much as possible the basic premise of passive dynamic walkers i.e. little or no control. Most of the results prior to now have involved the controller based solely on the q_3 value and given in Fig 4.8. If however the controller given in Fig 4.9 is utilised there are no major changes in performance.

The only dıfference ıs a change ın the appearance of q_3 - now ıt leans backwards slıghtly Some results ıllustrate this and are gıven ın Appendıx H 10

5.4.13 Summary

The followıng ıs a summary of the effects of the varıatıon of the slope on the varıous parameters whıch ıs ın-lıne wıth that of the body-less creature

Slope	*Inter-leg angle*	*Angle q_3*	*Velocity*	*Step Period*	*Step Length*
Increasıng	Increases	Decreases	Increases	Increases	Increases

In terms of effıcıency values equallıng those of the body-less creature have been found Improvements ın maxımum slope attaınable, ın partıcular ın conjunctıon wıth a damper placed at the hıp have been ıdentıfied

Chapter Six

Conclusions And Future Work

6.1 Achievements

Previous work [McG90] [Gos96] and [Gar97] has well established that a passively engineered biped can "walk" down a slightly inclined plane powered only by gravity and eventually acquire a stable periodic gait Thus the passive-walking pattern is determined by the natural frequency of the mechanical system An interesting characteristic was that the creatures involved achieved a stable limit cycle that looked human-like Common to all creatures involved was that a hip mass was utilised to represent a torso and it is this exclusion of an extended torso that has been addressed here

McGeer's creature [McG90] was used as the foundation with an extra inverted link representing the torso Keeping this link in an upright position can be achieved through the use of a simple fuzzy logic controller without violating the inherent simplicity of the model The solution process involved formulation of the equations of motion and transition equations and then fixed point solutions were sought and these provided the limit cycles

Once limit cycles were found it needed to be determined if they were stable or unstable While not immediately available, stability could be achieved through the addition of a damper and manual tweaking of the variable values involved Finally it was shown that the addition of a hip damper could improve on the previous results gamed

6.2 Performance issues

The main performance issues regarding passive creatures can be summarised as *stability*, *efficiency*, *maximum slope*, *velocity* and *step period* With regard to practical performance issues, the creature of McGeer (used as the foundation of this research) achieved a minimum slope of 0 005 radians and a maximum of 0 06 radians [McG90] Thus to deem the bodied creature a "success" values similar in stature were sought As stated previously, stability was achieved although after some tweaking There is a slight increase in the basin of attraction indicating that a larger error in the initial state vector is acceptable as compared to the body-less case McGeer utilised a linearised solution process in his work and thus improvements in efficiency should be gained, and this was found to be the case with solutions existing for slopes as low as 0 0005 radians - this is similar to the result found here for the body-less case Improvements were also made in the maximum slope achievable The velocity and step period values are in keeping with those of the body-less creature i e increasing as the slope increases Finally the applied torque utilised in keeping the body upright, an external force which may be problematic in a physical implementation of the creature, is shown to be minute per step taken Thus the applied torque necessary to keep the body upright would not require a large power source

6.3 Future Work

6.3.1 Creature configuration

The simulated creature outlined here consists of three rigid links, two representing legs and one a torso, connected via hinge joints This type of joint has limit degrees of freedom and thus keeps the structure of the creature simple Addition of ball and socket joints would aid realism (more human-like), and may positively effect some of the performance issues and should be simulated

This creature requires a low centre of mass for stable solutions to be detected However higher centre of mass positions would be necessary in a real world environment One method of combating this problem may be to add in extra mass components in specific locations

Addition of a torso is just one development necessary for a chopped at the waist bipedal creature Any realistic creature (either simulated or real) will be required to perform some sort of tasks other than just movement Thus some sort of freely moving gripping arms need to be added

In [McG90b] McGeer updated his bipedal robot through the addition of knees This was accomplished by splitting each leg into two, a thigh and a shank and placing a stop at each knee to prevent hyperextension Once again stable limit cycles were found and as possible future work knees should be incorporated into the creature containing a torso described here

6.3.2 Dampers

Addition of dampers have been very useful here, providing stability and giving more versatile solutions Those modelled have a linear form to keep the torso upright and a quadratic form at the hip joint Different types of dampers, other than those mentioned here may further increase the performance of the creature In particular addition of extra springs and dampers may lead to improvements in the positioning of the centre of mass of the torso

6.3.3 Controller

Central to the research in the area of passive ballistic walkers is the notion of simplicity To remain true to this motto as elementary a controller as possible was utilised As previously outlined there were two versions of controller used, each had three states and one took into account the value of the angle velocity u_3 While solutions were found other slightly more complex controllers (however not too complex!) might be used to keep the body upright and provide better performance

Other forms of external control (such as the passivity mimicking laws used by Goswami [Gos97a]) might also be added into the solution process to provide better solutions

6.3.4 Optimisation

Many performance gauges were emphasised throughout chapter five However instead of just finding a solution it would be best to find the best solution Therefore the solution process should be optimised to find for example the least energetic cost in movement, the fastest step etc Within this also is the notion of improving performance values e g trying to get the most efficient creature etc

6.3.5 Physical implementation

The whole process outlined in this is work is based upon simulation Thus a real model should be constructed and investigated One of the main issues would be how to implement the controller - as a non-linear spring or as an actuator? Obviously the most energy efficient solution should be sought giving the best performances

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Appendix A:

Vectors associated with the Bodied Creature.

Height of 'Points' above ground

It will be useful to have the heights of various points on the creature at particular time intervals Therefore the vector values for these points are

$$foot-height = (L - R)Cos[q_1]\,\mathbf{sta[2]} + (-L + R)Cos[q_2]\,\mathbf{swi[2]}$$

$$hip-height = (L - R)\,\mathbf{sta[2]} + R\,\mathbf{ground[2]}$$

Vectors

***Bodytojnt* is the vector from the new body's COM to the joint connecting it**

- *Stance to ground*

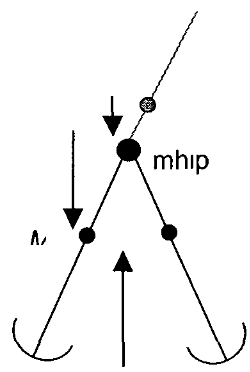
$-R\mathbf{ground[2]} - (comleg - R)\ \mathbf{sta[2]}$

- *Swing to stance*

$(L - comleg)\ \mathbf{swi[2]}$

- *Body to hip*

$-(lbody - combody)\ \mathbf{bod[2]}$



Inbtojnt is the vector from the Inboard bodys COM to the joint connecting it to the hew body

- Stance to ground

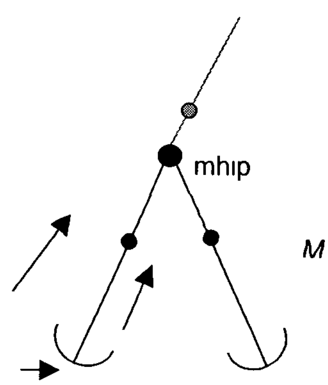
0

- Swing to stance

$(L - comleg)\ \mathbf{sta[2]}$

- Body to stance

$(L - comleg)\ \mathbf{sta[2]}$



Appendix B:

Mathematica Code for equations of motion i.e. state derivative sta for the body-less creature. Note that code for transition matrix is only given for the bodied creature.

```
(* Include dynamics Workbench package *)

<<c \DynamicsWorkbench m

(* Clear the internal variables storing parameters of the model *)

NewModel

(*  Add in the two leg reference frames sta and swi *)

AddFrame[sta, ground, Hinge, Axis->ground[3]],
AddFrame[swi, ground, Hinge, Axis->ground[3]],
udofs = {1, 2},

(* Kinematics of the legs *)

AngVel[ sta ] = u[1] ground[3],
AngVel[ swi ] = u[2] ground[3],
Bodies = {sta, swi},
Inboard[ sta ] = ground,
Inboard[ swi ] = sta,
rank = R ground[2],
vank = Cross[ AngVel[sta], rank],

(* hip is positioned at joint joining the two legs *)

(*NOTE THAT Cl IS USED FOR comleg here *)

hip = PosPnt[(L-Cl) sta[2], sta] ground[2],

(* Velocities of legs *)

VelCOM[ sta ] = vank + Cross[ AngVel[sta], (Cl-R) sta[2]],
AccCOM[ sta ] = Dt[VelCOM[sta],t, Constants->{R,Cl,lp}] /
                  {q[1]'->u[1],q[2]'->u[2]},
AngAcc[ sta ] = Dt[AngVel[sta],t],
Kinematics = {u[1] == qt[1]'[t], u[2] == qt[2]'[t] },
Force[sta] = {},
Torque[sta] = {},
Force[swi] = {},
Torque[swi] = {},
VelJnt[ swi ] = Simplify[ VelCOM[ sta ] + Cross[AngVel[sta],
                  (L-Cl) sta[2]] ]
VelCOM[ swi ] = VelJnt[swi] + Cross[ AngVel[swi],
                  -(L-Cl) swi[2] ]
AngAcc[swi] = Dt[AngVel[swi],t],
AccJnt[swi] = Simplify[ AccCOM[sta] + Cross[AngVel[sta],
                  Cross[AngVel[sta], (L-Cl) sta[2] ] ] + Cross[
AngAcc[sta], (L-Cl) sta[2]], Trig->False],
AccCOM[swi] = Simplify[ AccJnt[swi] + Cross[AngVel[swi],
Cross[AngVel[swi], -(L-Cl) swi[2] ] ] + Cross[
```

```
                          AngAcc[swi], -(L-Cl) swi[2] ] ],
BodyToJnt[sta] = -rank - (Cl-R) sta[2],
BodyToJnt[swi] = (L-Cl) swi[2],
InbToJnt[sta] = 0,
InbToJnt[swi] = (L-Cl) sta[2],

(* Mass of legs including hip mass *)
Mass[ swi ] = M + Mhip, Mass[ hip ] = Mhip, Mass[ sta ] = M + Mhip,
Inertia[ sta ] = Il sta[1]**sta[1]+Il sta[3]**sta[3],
Inertia[ swi ]=Ilswi swi[1]**swi[1]+Ilswi swi[3]**swi[3],


(* Forces applied to the legs i e  gravity *)

AppFrc[ sta, Mass[sta] grav, 0],
AppFrc[ swi, Mass[swi] grav, 0],

(* Torque applied by spring and damper at the hip joint *)

AppTrq[swi , Ffric((u[2]-u[1])^2) ground[3]],

grav = g (-Cos[gamma] ground[2] + Sin[gamma] ground[1]),

(* Generate the equations of motion *)
eom = EOM

(* next need to split up eom so that state derivative can be found *)

test=Simplify[MassMatrix[eom]]


(*------------------------------------------------------------------
*)

Print[StringForm["Value for the 2*2 matrix M"]]

m11=Coefficient[eom[[1]][[1]], (u[1])', 1],
m12=Coefficient[eom[[1]][[1]], (u[2])', 1],
m21=Coefficient[eom[[2]][[1]], (u[1])', 1],
m22=Coefficient[eom[[2]][[1]], (u[2])', 1],

(* Matrix M in section 3 3 3*)

MatM[2,2]={{-m11,-m12},{-m21,-m22}}

Print[StringForm["right hand side values"]]

rest1 = eom[[1]][[1]] - (Coefficient[ eom[[1]][[1]], (u[1])',
1]*u[1]') - (Coefficient[ eom[[1]][[1]], (u[2])', 1]*u[2]'),

rest2 = eom[[2]][[1]] - (Coefficient[ eom[[2]][[1]], (u[1])',
1]*u[1]') - (Coefficient[ eom[[2]][[1]], (u[2])', 1]*u[2]'),

(* Matrix R in section 3 3 3*)

Rmat[1,2]={rest1,rest2}

(* Linear solve to get last values for state derivative *)

uveldot = LinearSolve[MatM[2,2],Rmat[1,2]]
```

```
(*------------------------------------------------------------------------
*)

(* Linear solving above gives the state vector sta *)

sta = {u[1],u[2],uveldot[[1]],uveldot[[2]]}
sta >> c \stateder ma
```

Appendix C:

Mathematica Code for equations of motion i.e. state derivative sta for the <u>bodied</u> creature and the transition equation.

```
(* Include DynamicsWorkbench package *)

<<c \DynamicsWorkbench m

NewModel

(*-----------------------------------------------------------------*)

(* Setting up frames of reference *)

AddFrame[sta, ground, Hinge, Axis->ground[3]],
AddFrame[swi, ground, Hinge, Axis->ground[3]],
AddFrame[bod, ground, Hinge, Axis->ground[3]],

udofs={1, 2, 3},

AngVel[ sta ] = u[1] ground[3],
AngVel[ swi ] = u[2] ground[3],
AngVel[ bod ] = u[3] ground[3],
Bodies = {sta, swi, bod},

Inboard[ sta ] = ground,
Inboard[ swi ] = sta,
Inboard[ bod ] = sta,

(* hdpos is the vector from the com upwards  - means back towards the
hip *)

hip = PosPnt[(L-comleg) sta[2], sta] ground[2],

rank = R ground[2],
vank = Cross[ AngVel[sta], rank],
VelCOM[ sta ] = vank + Cross[ AngVel[sta], (comleg-R) sta[2]],
AccCOM[ sta ] = Dt[VelCOM[sta],t, Constants->{R,comleg,lp}] /
{q[1]'->u[1],
_q[2]'->u[2]},
AngAcc[ sta ] = Dt[AngVel[sta],t],
Kinematics = {u[1] == qt[1]'[t], u[2] == qt[2]'[t], u[3] == qt[3]'[t]
},

Force[sta] = {},
Torque[sta] = {},
Force[swi] = {},
Torque[swi] = {},
Force[bod] = {},
Torque[bod] = {},

VelJnt[ swi ] = Simplify[ VelCOM[ sta ] + Cross[AngVel[sta],
_(L-comleg) sta[2]] ],
VelCOM[ swi ] = VelJnt[swi] + Cross[ AngVel[swi],
_-(L-comleg) swi[2] ],
AngAcc[swi] = Dt[AngVel[swi],t],
AccJnt[swi] = Simplify[ AccCOM[sta] + Cross[AngVel[sta],
_Cross[AngVel[sta], (L-comleg) sta[2] ] ] + Cross[
```

```
_AngAcc[sta], (L-comleg) sta[2]], Trig->False];
AccCOM[swi] = Simplify[ AccJnt[swi] + Cross[AngVel[swi],
_Cross[AngVel[swi], -(L-comleg) swi[2] ] ] + Cross[
_AngAcc[swi], -(L-comleg) swi[2] ] ];


(*------------------------------------------------------------------
-*)

VelCOM[bod] = VelJnt[swi] + Cross[AngVel[bod], (lbody-combody) bod[2]
];
AngAcc[bod] = Dt[AngVel[bod],t];
AccCOM[bod] = Simplify[ AccJnt[swi] + Cross[AngVel[bod],
_Cross[AngVel[bod], (lbody-combody) bod[2] ] ] + Cross[
_AngAcc[swi], (lbody-combody) bod[2]], Trig->False];


BodyToJnt[sta] = -rank - (comleg-R) sta[2];
BodyToJnt[swi] = (L-comleg) swi[2];
BodyToJnt[bod] = (lbody-combody) bod[2];
InbToJnt[sta] = 0;
InbToJnt[swi] = (L-comleg) sta[2];
InbToJnt[bod] = (L-comleg) sta[2];

(* Note that each leg contains the hip mass *)

Mass[ swi ] = M + Mhip;
Mass[ hip ] = Mhip;
Mass[ sta ] = M + Mhip;
Mass[ bod ] = Mbody;


Inertia[ sta ] = Ilsta sta[1]**sta[1]+Ilsta sta[3]**sta[3];
Inertia[ swi ] = Ilswi swi[1]**swi[1]+Ilswi swi[3]**swi[3];
Inertia[ bod ] = Inertiabody bod[1]**bod[1]+Inertiabody
bod[3]**bod[3];

AppFrc[ sta, Mass[sta] grav, 0];
AppFrc[ swi, Mass[swi] grav, 0];
AppFrc[ bod, Mass[bod] grav, 0];

(* Torsional Springs position and force - added to try to improve
stability *)

AppTrq[ swi ,  Ffric((u[2]-u[1])^2) swi[3]];

(* Inverted pendulum torque - spring and damper with damping coeff
damp and torque Klin and constant Frac *)

AppTrq[ bod, Frac(Klin - damp*u[3]) bod[3]];

grav = g (-Cos[gamma] ground[2] + Sin[gamma] ground[1]);

eom = EOM;

test=Simplify[MassMatrix[eom]];

(* Again need to get into form for linear solving *)

(* Matrix M in section 3.3.3*)
M[3,3]={test[[1]][[1]], test[[1]][[2]], test[[1]][[3]]};
```

(* Matrix R in section 3 3 3*)

```
R[1,3]=test[[2]],

uveldot=LinearSolve[M[3,3],R[1,3]],
Print[StringForm["State Vector is  "]],

sta={u[1],u[2],u[3],uveldot[[1]],uveldot[[2]],uveldot[[3]]}

sta >> c \statemat nb
```

****

TRANSITION EQUATIONS (SECTION 3.4)- SAME CODE AS ABOVE UNTIL EQUATIONS OF MOTION ARE FORMED

```
eom = EOM,

amawhole = AngMom[{sta, swi}, 0],(*+
___AngMom[{bod}, PosPnt[ (L-comleg) sta[2], sta]]*)
amaswi = AngMom[{swi}, PosPnt[ (L-comleg) swi[2], swi]],
amabod = AngMom[{bod}, PosPnt[ (L-comleg) sta[2], sta]],

M[3,3]={{Coefficient[amawhole sta[3],
u[1]],Coefficient[amawhole sta[3], u[2]],
___Coefficient[amawhole sta[3], u[3]]},
{Coefficient[amaswi sta[3], u[1]],Coefficient[amaswi sta[3], u[2]],
Coefficient[amaswi sta[3], u[3]]},_
{Coefficient[amabod sta[3], u[1]],Coefficient[amabod sta[3], u[2]],
Coefficient[amabod sta[3], u[3]]}},

(*Angular Momemtum before*)

ambefwhole = AngMom[{sta, swi}, PosPnt[-(comleg-R) swi[2] -R
ground[2],
___swi ]], (*+ AngMom[{bod}, PosPnt[ (L-comleg) sta[2], sta]]*)

ambsta = AngMom[{sta}, PosPnt[ (L-comleg) sta[2], sta]],
ambbod = AngMom[{bod}, PosPnt[ (L-comleg) sta[2], sta]],

Bef[1,3]={ambefwhole sta[3],ambsta sta[3],ambbod sta[3]},

cond1=LinearSolve[M[3,3],Bef[1,3]]

cond1 >> c \artkuo\mar3\transmat nb
```

Appendix D

Equations of motion matrices for the body-less creature

The equatıons of motıon for the body-less creature can be wntten as

$$M\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} - R\begin{pmatrix} u_1^2 \\ u_2^2 \end{pmatrix} = 0,$$

where the matnces M and N are

$$M = \begin{bmatrix} Il + (comleg^2 + L^2)(2m + mhip) & (comleg - L)L(2m + mhip)\,Cos(q_1 - q_2) \\ (comleg - L)L(2m + mhip)\,Cos(q_1 - q_2) & Ilswi + (comleg - L)^2(2m + mhip) \end{bmatrix}$$

$$R = \begin{bmatrix} -(2m + mhip)(g(comleg + L)\,Sin(\gamma - q_1) + (comleg - L)L\,Sin(q_1 - q_2)u_2^2 \\ (comleg - L)(2m + mhip)(-g\,Sin(\gamma - q_2) + L\,Sin(q_1 - q_2)u_1^2 \end{bmatrix}$$

Appendix E:

State derivative for Bodied Creature

$\theta(t) =$

```
{u[1], u[2], u[3], -(((-((comleg - L)*(combody - lbody)^2*
Mbody*(M + Mhip)*((L - R)*Cos[q[1] - q[2]] + R*Cos[q[2]])) -
(combody - lbody)*Mbody*(Ilswi + (comleg - L)^2*(M + Mhip))*
((L - R)*Cos[q[1] - q[3]] + R*Cos[q[3]]))*
(g*(combody - lbody)*Mbody*Sin[gamma - q[3]] -
(combody - lbody)*Mbody*(L - R)*Sin[q[1] - q[3]]*
u[1]^2 + frac*(Klin - damp*u[3])))/(Inertiabody*((Ilswi + (comleg -
L)^2*(M + Mhip))*(Ilsta + comleg^2*M + L^2*M + L^2*Mbody +
comleg^2*Mhip +
L^2*Mhip - 2*comleg*M*R - 2*L*M*R - 2*L*Mbody*R -
2*comleg*Mhip*R - 2*L*Mhip*R + 4*M*R^2 + 2*Mbody*R^2 +
4*Mhip*R^2 + 2*R*(L*M + L*Mbody + L*Mhip +
comleg*(M + Mhip) - 2*M*R - Mbody*R - 2*Mhip*R)*
Cos[q[1]]) -(comleg - L)*(M + Mhip)*
((L - R)*Cos[q[1] - q[2]] + R*Cos[q[2]])*
((comleg - L)*(M + Mhip)*(L - R)*Cos[q[1]-q[2]]+
(comleg - L)*(M + Mhip)*R*Cos[q[2]] -
(combody - lbody)*Mbody*
((L - R)*Cos[q[1] - q[3]] + R*Cos[q[3]])))) +
(-((comleg - L)*(M + Mhip)*((L - R)*Cos[q[1] - q[2]] +
R*Cos[q[2]])*(g*(-comleg + L)*(M + Mhip)*
Sin[gamma - q[2]] + Khip/(q[1] - q[2]) +
(comleg - L)*(M + Mhip)*(L - R)*Sin[q[1] - q[2]]*
u[1]^2 + Ffric*Sin[u[1] - u[2]]*
(u[1] - u[2])^2)) +(Ilswi + (comleg - L)^2*(M + Mhip))*
(g*(M + Mhip)*(-R + (-comleg + R)*Cos[q[1]])*Sin[gamma] +
g*Mbody*(-R + (-L + R)*Cos[q[1]])*Sin[gamma] +
g*(M + Mhip)*(-R + (-L + R)*Cos[q[1]])*Sin[gamma] +
g*(M + Mhip)*(comleg - R)*Cos[gamma]*Sin[q[1]] +
g*Mbody*(L - R)*Cos[gamma]*Sin[q[1]] +
g*(M + Mhip)*(L - R)*Cos[gamma]*Sin[q[1]] +
R*(L*M + L*Mbody + L*Mhip + comleg*(M + Mhip) - 2*M*R -
Mbody*R - 2*Mhip*R)*Sin[q[1]]*u[1]^2 +
(comleg - L)*(M + Mhip)*
((-L + R)*Sin[q[1] - q[2]] + R*Sin[q[2]])*u[2]^2 \
- (combody - lbody)*Mbody*((-L + R)*Sin[q[1] - q[3]] +
R*Sin[q[3]])*u[3]^2))/((Ilswi + (comleg - L)^2*(M + Mhip))*
(Ilsta + comleg^2*M + L^2*M + L^2*Mbody + comleg^2*Mhip + L^2*Mhip -
2*comleg*M*R - 2*L*M*R - 2*L*Mbody*R - 2*comleg*Mhip*R -
2*L*Mhip*R + 4*M*R^2 + 2*Mbody*R^2 + 4*Mhip*R^2 +
2*R*(L*M + L*Mbody + L*Mhip + comleg*(M + Mhip) - 2*M*R -
Mbody*R - 2*Mhip*R)*Cos[q[1]]) -(comleg - L)*(M + Mhip)*((L -
R)*Cos[q[1] - q[2]] +R*Cos[q[2]])*((comleg - L)*(M + Mhip)*(L - R)*
Cos[q[1] - q[2]] +(comleg - L)*(M + Mhip)*R*Cos[q[2]] -
(combody - lbody)*Mbody*((L - R)*Cos[q[1] - q[3]] + R*Cos[q[3]]))),
(g*(-comleg + L)*(M + Mhip)*Sin[gamma - q[2]] +
Khip/(q[1] - q[2]) +(comleg - L)*(M + Mhip)*(L - R)*Sin[q[1] - q[2]]*
u[1]^2 + Ffric*Sin[u[1] - u[2]]*(u[1] - u[2])^2)/
(Ilswi + (comleg - L)^2*(M + Mhip)) -((combody -
lbody)^2*Mbody*(g*(combody - lbody)*Mbody*Sin[gamma - q[3]] -(combody
- lbody)*Mbody*(L - R)*Sin[q[1] - q[3]]*u[1]^2 + frac*(Klin -
```

```
damp*u[3])))/(Inertiabody*(Ilswi + (comleg - L)^2*(M + Mhip))) -
(((comleg - L)*(M + Mhip)*(L - R)*Cos[q[1] - q[2]] +
(comleg - L)*(M + Mhip)*R*Cos[q[2]] -(combody - lbody)*Mbody*
((L - R)*Cos[q[1] - q[3]] + R*Cos[q[3]]))*
(-(((-((comleg - L)*(combody - lbody)^2*Mbody*(M + Mhip)*
((L - R)*Cos[q[1] - q[2]] + R*Cos[q[2]])) \
- (combody - lbody)*Mbody*(Ilswi + (comleg - L)^2*(M + Mhip))*
((L - R)*Cos[q[1] - q[3]] + R*Cos[q[3]]))*
(g*(combody - lbody)*Mbody*Sin[gamma - q[3]] -
(combody - lbody)*Mbody*(L - R)*Sin[q[1] - q[3]]*
u[1]^2 + frac*(Klin - damp*u[3])))/(Inertiabody*((Ilswi + (comleg -
L)^2*(M + Mhip))*(Ilsta + comleg^2*M + L^2*M + L^2*Mbody +
comleg^2*Mhip +
L^2*Mhip - 2*comleg*M*R - 2*L*M*R - 2*L*Mbody*R -
2*comleg*Mhip*R - 2*L*Mhip*R + 4*M*R^2 + 2*Mbody*R^2 +
4*Mhip*R^2 +2*R*(L*M + L*Mbody + L*Mhip + comleg*(M + Mhip) -
2*M*R - Mbody*R - 2*Mhip*R)*Cos[q[1]]) -(comleg - L)*(M + Mhip)*((L -
R)*Cos[q[1] - q[2]] + R*Cos[q[2]])*((comleg - L)*(M + Mhip)*(L - R)*
Cos[q[1] - q[2]] +(comleg - L)*(M + Mhip)*R*Cos[q[2]] -
(combody - lbody)*Mbody*((L - R)*Cos[q[1] - q[3]] + R*Cos[q[3]])))))
\
+ (-((comleg - L)*(M + Mhip)*((L - R)*Cos[q[1] - q[2]] +
R*Cos[q[2]])*(g*(-comleg + L)*(M + Mhip)*Sin[gamma - q[2]] +
Khip/(q[1] - q[2]) +(comleg - L)*(M + Mhip)*(L - R)*Sin[q[1] - q[2]]*
u[1]^2 +Ffric*Sin[u[1] - u[2]]*(u[1] - u[2])^2)) +
(Ilswi + (comleg - L)^2*(M + Mhip))*(g*(M + Mhip)*(-R + (-comleg +
R)*Cos[q[1]])*Sin[gamma] +g*Mbody*(-R + (-L +
R)*Cos[q[1]])*Sin[gamma] +
g*(M + Mhip)*(-R + (-L + R)*Cos[q[1]])*Sin[gamma] +
g*(M + Mhip)*(comleg - R)*Cos[gamma]*Sin[q[1]] +
g*Mbody*(L - R)*Cos[gamma]*Sin[q[1]] +g*(M + Mhip)*(L -
R)*Cos[gamma]*Sin[q[1]] +R*(L*M + L*Mbody + L*Mhip + comleg*(M +
Mhip) - 2*M*R -Mbody*R - 2*Mhip*R)*Sin[q[1]]*u[1]^2 +
(comleg - L)*(M + Mhip)*((-L + R)*Sin[q[1] - q[2]] + R*Sin[q[2]])*
u[2]^2 - (combody - lbody)*Mbody*((-L + R)*Sin[q[1] - q[3]] +
R*Sin[q[3]])*
u[3]^2))/((Ilswi + (comleg - L)^2*(M + Mhip))*
(Ilsta + comleg^2*M + L^2*M + L^2*Mbody + comleg^2*Mhip +
L^2*Mhip - 2*comleg*M*R - 2*L*M*R - 2*L*Mbody*R -
2*comleg*Mhip*R - 2*L*Mhip*R + 4*M*R^2 + 2*Mbody*R^2 +
4*Mhip*R^2 + 2*R*(L*M + L*Mbody + L*Mhip + comleg*(M + Mhip) - 2*M*R
-
Mbody*R - 2*Mhip*R)*Cos[q[1]]) -(comleg - L)*(M + Mhip)*
((L - R)*Cos[q[1] - q[2]] + R*Cos[q[2]])*((comleg - L)*(M + Mhip)*(L
- R)*Cos[q[1] - q[2]] +(comleg - L)*(M + Mhip)*R*Cos[q[2]] -
(combody - lbody)*Mbody*((L - R)*Cos[q[1] - q[3]] + R*Cos[q[3]])))))/
(Ilswi + (comleg - L)^2*(M + Mhip)),(g*(combody -
lbody)*Mbody*Sin[gamma - q[3]] -(combody - lbody)*Mbody*(L -
R)*Sin[q[1] - q[3]]*u[1]^2 +frac*(Klin - damp*u[3]))/Inertiabody}
```

Transition matrix for Bodied Creature

```
cond =
{(Ilsta*u[1] + (comleg - L)*(M + Mhip)*(comleg - R)*u[1] +
      (comleg - L)*(M + Mhip)*R*Cos[q[1]]*u[1])/
    (-((comleg - L)*(M + Mhip)*(-L + R)*Cos[q[1] - q[2]]) +
      (comleg - L)*(M + Mhip)*R*Cos[q[2]]) -
   ((Ilswi + (comleg - L)^2*(M + Mhip))*
```

```
        (-((Ilsta + (M + Mhip)*R*(R + (comleg - R)*Cos[q[1]]) +
              (M + Mhip)*(comleg - R)*(comleg - R + R*Cos[q[1]]) -
              (M + Mhip)*(-L + R)*
               (L - R + R*Cos[q[1]] +
                 (comleg - L)*Cos[q[1] - q[2]]) +
              (M + Mhip)*R*(R + (L - R)*Cos[q[1]] +
                 (comleg - L)*Cos[q[2]]))*
            (Ilsta*u[1] + (comleg - L)*(M + Mhip)*(comleg - R)*
               u[1] + (comleg - L)*(M + Mhip)*R*Cos[q[1]]*u[1])) \
+ (-((comleg - L)*(M + Mhip)*(-L + R)*Cos[q[1] - q[2]]) +
            (comleg - L)*(M + Mhip)*R*Cos[q[2]])*
          (Ilsta*u[1] - (M + Mhip)*(-L + R)*
             (R*Cos[q[1]] + (comleg - R)*Cos[q[1] - q[2]])*
             u[1] + (M + Mhip)*(comleg - R)*
             (comleg - L + R*Cos[q[1]] +
               (L - R)*Cos[q[1] - q[2]])*u[1] +
            (M + Mhip)*R*(R + (comleg - R)*Cos[q[2]])*u[1] +
            (M + Mhip)*R*(R + (comleg - L)*Cos[q[1]] +
               (L - R)*Cos[q[2]])*u[1] + Ilswi*u[2] +
            (comleg - L)*(M + Mhip)*(comleg - R + R*Cos[q[2]])*u[2]))
)/((-((comleg - L)*(M + Mhip)*(-L + R)*Cos[q[1] - q[2]]) +
          (comleg - L)*(M + Mhip)*R*Cos[q[2]])*
        (-((Ilswi + (comleg - L)^2*(M + Mhip))*
            (Ilsta + (M + Mhip)*R*(R + (comleg - R)*Cos[q[1]]) +
              (M + Mhip)*(comleg - R)*(comleg - R + R*Cos[q[1]]) -
              (M + Mhip)*(-L + R)*
               (L - R + R*Cos[q[1]] +
                 (comleg - L)*Cos[q[1] - q[2]]) +
              (M + Mhip)*R*(R + (L - R)*Cos[q[1]] +
                 (comleg - L)*Cos[q[2]]))) +
         (-((comleg - L)*(M + Mhip)*(-L + R)*Cos[q[1] - q[2]]) +
            (comleg - L)*(M + Mhip)*R*Cos[q[2]])*
          (Ilswi + (comleg - L)*(M + Mhip)*
             (comleg - L + (L - R)*Cos[q[1] - q[2]] + R*Cos[q[2]])
))), (-((Ilsta + (M + Mhip)*R*(R + (comleg - R)*Cos[q[1]]) +
            (M + Mhip)*(comleg - R)*(comleg - R + R*Cos[q[1]]) -
            (M + Mhip)*(-L + R)*
             (L - R + R*Cos[q[1]] +
               (comleg - L)*Cos[q[1] - q[2]]) +
            (M + Mhip)*R*(R + (L - R)*Cos[q[1]] +
               (comleg - L)*Cos[q[2]]))*
          (Ilsta*u[1] + (comleg - L)*(M + Mhip)*(comleg - R)*u[1] +
            (comleg - L)*(M + Mhip)*R*Cos[q[1]]*u[1])) +
       (-((comleg - L)*(M + Mhip)*(-L + R)*Cos[q[1] - q[2]]) +
          (comleg - L)*(M + Mhip)*R*Cos[q[2]])*
        (Ilsta*u[1] - (M + Mhip)*(-L + R)*
           (R*Cos[q[1]] + (comleg - R)*Cos[q[1] - q[2]])*u[1] \
+ (M + Mhip)*(comleg - R)*(comleg - L + R*Cos[q[1]] +
             (L - R)*Cos[q[1] - q[2]])*u[1] +
          (M + Mhip)*R*(R + (comleg - R)*Cos[q[2]])*u[1] +
          (M + Mhip)*R*(R + (comleg - L)*Cos[q[1]] +
             (L - R)*Cos[q[2]])*u[1] + Ilswi*u[2] +
          (comleg - L)*(M + Mhip)*(comleg - R + R*Cos[q[2]])*u[2]))/
     (-((Ilswi + (comleg - L)^2*(M + Mhip))*
          (Ilsta + (M + Mhip)*R*(R + (comleg - R)*Cos[q[1]]) +
            (M + Mhip)*(comleg - R)*(comleg - R + R*Cos[q[1]]) -
            (M + Mhip)*(-L + R)*(L - R + R*Cos[q[1]] +
               (comleg - L)*Cos[q[1] - q[2]]) +
            (M + Mhip)*R*(R + (L - R)*Cos[q[1]] +
               (comleg - L)*Cos[q[2]]))) +
       (-((comleg - L)*(M + Mhip)*(-L + R)*Cos[q[1] - q[2]]) +
```

```
        (comleg - L)*(M + Mhip)*R*Cos[q[2]])*
      (Ilswi + (comleg - L)*(M + Mhip)*
         (comleg - L + (L - R)*Cos[q[1] - q[2]] + R*Cos[q[2]]))),
  -(((-((combody - lbody)*Mbody*(-L + R)*Cos[q[1] - q[3]]) +
          (combody - lbody)*Mbody*R*Cos[q[3]])*
        ((Ilsta*u[1] + (comleg - L)*(M + Mhip)*(comleg - R)*
             u[1] + (comleg - L)*(M + Mhip)*R*Cos[q[1]]*u[1])/
          (-((comleg - L)*(M + Mhip)*(-L + R)*Cos[q[1] - q[2]]) +
            (comleg - L)*(M + Mhip)*R*Cos[q[2]]) -
          ((Ilswi + (comleg - L)^2*(M + Mhip))*
            (-((Ilsta + (M + Mhip)*R*(R + (comleg - R)*Cos[q[1]]) +
                    (M + Mhip)*(comleg - R)*
                     (comleg - R + R*Cos[q[1]]) -
                    (M + Mhip)*(-L + R)*
                     (L - R + R*Cos[q[1]] +
                       (comleg - L)*Cos[q[1] - q[2]]) +
                    (M + Mhip)*R*
                     (R + (L - R)*Cos[q[1]] +
                       (comleg - L)*Cos[q[2]]))*
                  (Ilsta*u[1] +
                    (comleg - L)*(M + Mhip)*(comleg - R)*u[1] +
                    (comleg - L)*(M + Mhip)*R*Cos[q[1]]*u[1])) +
               (-((comleg - L)*(M + Mhip)*(-L + R)*
                    Cos[q[1] - q[2]]) +
                  (comleg - L)*(M + Mhip)*R*Cos[q[2]])*
                (Ilsta*u[1] -
                  (M + Mhip)*(-L + R)*
                   (R*Cos[q[1]] +
                     (comleg - R)*Cos[q[1] - q[2]])*u[1] +
                  (M + Mhip)*(comleg - R)*
                   (comleg - L + R*Cos[q[1]] +
                     (L - R)*Cos[q[1] - q[2]])*u[1] +
                  (M + Mhip)*R*(R + (comleg - R)*Cos[q[2]])*u[1] +
                  (M + Mhip)*R*
                   (R + (comleg - L)*Cos[q[1]] +
                     (L - R)*Cos[q[2]])*u[1] + Ilswi*u[2] +
                  (comleg - L)*(M + Mhip)*(comleg - R + R*Cos[q[2]])*
                   u[2])))/
          ((-((comleg - L)*(M + Mhip)*(-L + R)*
                  Cos[q[1] - q[2]]) +
               (comleg - L)*(M + Mhip)*R*Cos[q[2]])*
             (-((Ilswi + (comleg - L)^2*(M + Mhip))*
                  (Ilsta + (M + Mhip)*R*(R + (comleg - R)*Cos[q[1]])
+
                    (M + Mhip)*(comleg - R)*
                     (comleg - R + R*Cos[q[1]]) -
                    (M + Mhip)*(-L + R)*
                     (L - R + R*Cos[q[1]] +
                       (comleg - L)*Cos[q[1] - q[2]]) +
                    (M + Mhip)*R*
                     (R + (L - R)*Cos[q[1]] +
                       (comleg - L)*Cos[q[2]]))) +
               (-((comleg - L)*(M + Mhip)*(-L + R)*
                    Cos[q[1] - q[2]]) +
                  (comleg - L)*(M + Mhip)*R*Cos[q[2]])*
                (Ilswi + (comleg - L)*(M + Mhip)*
                   (comleg - L + (L - R)*Cos[q[1] - q[2]] +
R*Cos[q[2]]))))))/
      (Inertiabody + (combody - lbody)*(-combody + lbody)*Mbody)) +
    (-((combody - lbody)*Mbody*(-L + R)*Cos[q[1] - q[3]]*u[1]) +
      (combody - lbody)*Mbody*R*Cos[q[3]]*u[1] +
```

```
      Inertiabody*u[3] + (combody - lbody)*(-combody + lbody)*Mbody*
       u[3])/(Inertiabody + (combody - lbody)*(-combody +
lbody)*Mbody)}
```

Appendix F:

Parameters, constants and initial values used with the Bodied Creature.

The following is a list of the parameters involved in the creatures' description and movement Any parameters that were treated as constants have their values shown

Symbol	*Description*	*Value*
q_1	Stance leg angle	
q_2	Swing leg angle	
q_3	Body angle	
u_1	Speed at which stance angle is changing	
u_2	Speed at which swing angle is changing	
u_3	Speed at which body angle is changing	
γ	Slope angle of ground	
L	Length of leg	1
R	Radius of foot	0 3
Cl	Distance from foot to centre of mass of leg	0 645
lbody	Length of the body	
combody	Distance from end of body to centre of mass	
g	Gravity	1
M	Mass of leg	0 4
Mhip	Mass of hip	
Mbody	Mass of body	
Ffric	Coefficient of friction	
Khip	Coefficient of damping	
Klin	Torque applied to body	
tol	Numerical tolerance of the integrator	0 00001
Frac	Torque constant	
Ilswi	Inertia of swing leg	0 121 * (M + Mhip)
Ilsta	Inertia of stance leg	0 121 * M
Inertiabody	Inertia of body	
w	leg axis -> mass centre offset	

Appendix G.

Body-less creature results

G.1 Limit Cycles

The first issue was the discovery of limit cycles Here is a selection of those that were discovered and are for various values of *mhip*, on divers γ values Finally note that the phase plane diagram is for <u>one</u> leg only

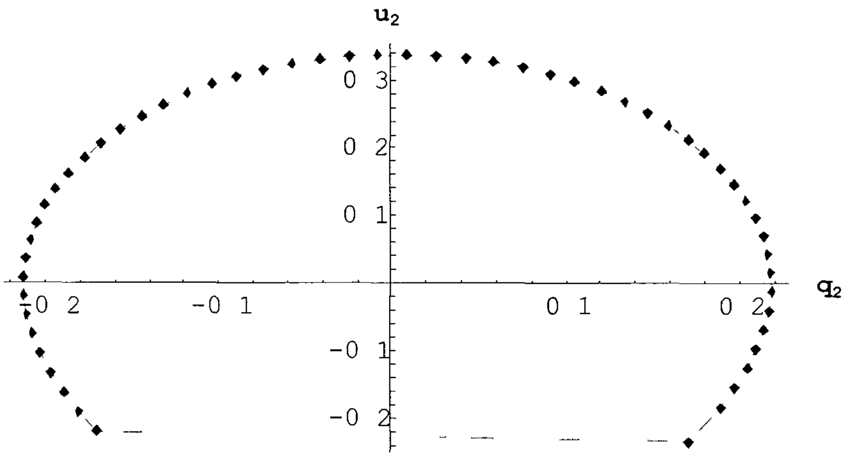


Fig G.1: $\gamma = 0\ 025$, *mhip* = *0 2*

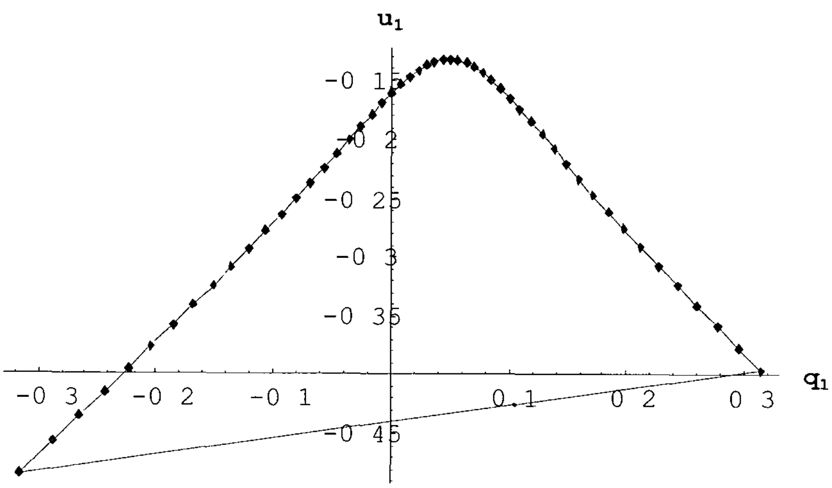


Fig G.2: $\gamma = 0\ 025$, *mhip* = *0 08*

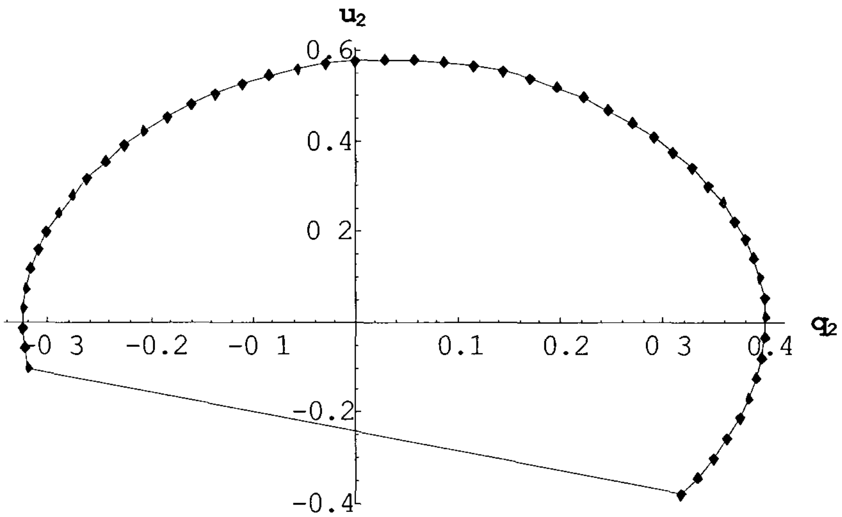


Fig G.3: $\gamma = 0\ 025\ mh\imath p = 1\ 2$

G.2 Basin of Attraction

The ınıtıal values for the state vector are $\theta_0 = \{q_1, q_2, u_1, u_2\} = \{0\ 3015, -0\ 3015, -0\ 3763, -0\ 2822\}$ The basın of attractıon ındıcates how much each of the four values can be altered by ındıvıdually and stıll gıve a fixed poınt solutıon These possıble mutatıons are as follows

mhip = 0.4

Original	**Maximum**	**Minimum**	**Average dıfference**	**Percentage**
0 3015	0 32	0 28	$\pm 0\ 02$	$\approx 6\ 6\%$
-0 3763	-0 4	-0 36	$\pm 0\ 015$	$\approx 3\ 9\%$
-0 2822	-0 45	0	$\pm 0\ 22$	$\approx 77\%$

Fınally note that for a *mhıp* value of 1 2 then the q_1 and q_2 angles can be altered from 0 27 to 0 32 radıans

G.3 Results for mhip = 0.0

G.3.1 Max and Min slope

This set of results deals with the minimum and maximum slope attainable by the body-less, **hipmass-less** walker. Note that the initial state guess is taken to be {0.3015, -0.3015, -0.3763, -0.2822} and the slope range is from 0.0005 to 0.043 radians, which corresponds to previous published results, and the various values are:

Note: Maximum values are given in bold and min in italics

Slope	q_1	q_2	u_1	u_2
0.0005	0.075262	-0.075262	-0.10977	-0.108068
0.005	0.170507	-0.170507	-0.237502	-0.218184
0.010	0.21709	-0.21709	-0.292437	-0.25407
0.020	0.278032	-0.278032	-0.355112	-0.279307
0.025	0.301571	-0.301571	-0.376368	-0.282189
0.030	0.322498	-0.322498	-0.393858	-0.281540
0.040	0.359002	-0.359002	-0.421196	-0.273343
0.043	0.368913	-0.368913	-0.427929	-0.269614

G.3.2 Eigenvalues

Slope	*speed*	*totter*	*swing*	*swing*
0.0005	0.957501	-0.520021	-0.092922	≈ 0
0.005	0.79926	-0.484081	-0.105031	≈ 0
0.010	0.687616	-0.443237	-0.123194	≈ 0
0.020	0.526698	-0.397183	-0.156886	≈ 0
0.025	0.462247	-0.398299	-0.167495	≈ 0
0.030	-0.419282	0.405872	-0.17061	≈ 0
0.040	-0.517468	0.314626	-0.158651	≈ 0
0.043	-0.559012	0.292647	-0.15272	≈ 0

G.3.3 Step Length, Period and Velocities

Slope	*Step Length*	*Step Period*	*Velocity* = $\frac{Length}{Period}$
0.0005	0.299912	2.7699070	0.10827511
0.005	0.668821	2.790077	0.23971417
0.010	0.841333	2.8014572	0.30031977

0 020	1 055694	2 8235438	0 37388971
0 025	1 134231	2 8349733	0 40008524
0 030	1 202390	2 8468138	0 42236341
0 040	1 315765	2 8717991	0 45816749
0.043	1 345361	2 8796142	0 46720182

G.4 Results for mhip = 0.2

G.4.1 Max and Min slope

Slope	q_1	q_2	u_1	u_2
0 0005	0 0753826	-0 0753826	-0 111317	-0 091232
0 005	0 171816	-0 171816	-0 242286	-0 180288
0 025	0 305022	-0 305022	-0 384336	-0 210171
0 04	0 363234	-0 363234	-0 4292	-0 183763
0.048	0 389026	-0 389026	-0 445726	-0 164839

G.5 Results for mhip = 0.4

G.5.1 Max and Min slope

Slope	q_1	q_2	u_1	u_2
0 0005	0 0752312	-0 0752312	-0 112021	-0 0825862
0 005	0 173445	-0 173445	-0 24629	-0 16124
0 025	0 308686	-0 308686	-0 38977	-0 170418
0 04	0 367128	-0 367128	-0 433815	-0 133593
0.052	0 404767	-0 404767	-0 456631	-0 0980575

G.5.2 Eigenvalues

Slope	*speed*	*totter*	*swing*	*swing*
0 0005	0 975364	0 280297	0 280297	≈ 0
0 005	0 8671	0 273757	0 273757	≈ 0
0 025	0 537091	0 285454	0 285454	≈ 0
0 04	0 352319	0 311417	0 311417	≈ 0
0.052	0 327514	0 327514	0 265992	≈ 0

G.5.3 Step Length, Period and Velocities

Slope	*Step Length*	*Step Period*	*Velocity* = $\frac{Length}{Period}$
0 0005	0 2997906	2 5031640	0 119764665
0 005	0 6799490	2 5270688	0 269066279
0 025	1 1577885	2 6079568	0 443944661
0 04	1 3400700	2 6719546	0 501531725
0.052	1 4479315	2 7221394	0 531909387

G.6 Results for mhip = 0.8

G.6.1 Max and Min slope

Slope	q_1	q_2	u_1	u_2
0 0005	0 0773091	-0 0773091	-0 116113	-0 075824
0 005	0 177204	-0 177204	-0 253079	-0 141855
0 025	0 314512	-0 314512	-0 396331	-0 125782
0 04	0 372531	-0 372531	-0 438615	-0 077983
0.056	0 429037	-0 429037	-0 466206	-0 021889

G.6.2 Eigenvalues

Slope	*speed*	*totter*	*swing*	*swing*
0 0005	0 981897	0 296308	0 296308	≈ 0
0 005	0 891872	0 283191	0 283191	≈ 0
0 025	0 53887	0 287592	0 287592	≈ 0
0 04	0 353086	0 309621	0 309621	≈ 0
0.056	-0 542341	0 251882	-0 191078	≈ 0

G.6.3 Step Length, Period and Velocities

Slope	*Step Length*	*Step Period*	*Velocity* = $\frac{Length}{Period}$
0 0005	0 3080057	2 4144430	0 127568014
0 005	0 6940704	2 4470927	0 283630611
0 025	1 1767116	2 5589101	0 459848745
0 04	1 3560347	2 6334398	0 514929067
0.056	1 5131691	2 7781639	0 544665165

G.7 Results for mhip = 1.2

G.7.1 Max and Min slope

Slope	q_1	q_2	u_1	u_2
0 0005	N/A	N/A	N/A	N/A
0 004	0 167281	-0 167281	-0 241439	-0 127301
0 005	0 180855	-0 180855	-0 25859	-0 131708
0 025	0 31832	-0 31832	-0 399955	-0 100985
0 04	0 37577	-0 37577	-0 440996	-0 0478034
0.058	0 429081	-0 429081	-0 470442	0 0197021

G.7.2 Eigenvalues

Slope	*speed*	*totter*	*swing*	*swing*
0 0005	N/A	N/A	N/A	N/A
0 004	0 918128	0 288104	0 288104	≈ 0
0 005	0 900238	0 285847	0 285847	≈ 0
0 025	0 531771	0 287683	0 287683	≈ 0
0 04	0 351469	0 306511	0 306511	≈ 0
0 058	-0 742184	0 246388	-0 133312	≈ 0

G.7.3 Step Length, Period and Velocities

Slope	*Step Length*	*Step Period*	*Velocity* = $\frac{\text{Length}}{\text{Period}}$
0 0005	N/A	N/A	N/A
0 005	0 7077482	2 4133982	0 293257946
0 025	1 1889940	2 5416899	0 467796641
0 04	1 3655295	2 6201817	0 521158322
0.058	1 5132842	2 7055385	0 559328281

G.8 Results for mhip = 1.6

G.8.1 Max and Min slope

Slope	q_1	q_2	u_1	u_2
0 0005	N/A	N/A	N/A	N/A
0 004	0 170311	-0 170311	-0 245814	-0 121734
0 025	0 32087	-0 32087	-0 402197	-0 0851974
0 04	0 377863	-0 377863	-0 442398	-0 0288882
0.058	0 430805	-0 430805	-0 471316	0 0411057

G.8.2 Eigenvalues

Slope	*speed*	*totter*	*swing*	*swing*
0 0005	N/A	N/A	N/A	N/A
0 004	0 921018	0 289166	0 289166	≈ 0
0 025	0 525414	0 287315	0 287315	≈ 0
0 04	0 350121	0 303839	0 303839	≈ 0
0 058	-0 818211	0 248165	-0 116589	≈ 0

G.9 Effect of varying the legs centre of mass position

All results are based on γ = 0 025 radians

mhip = 0.0

comleg	q_1	q_2	u_1	u_2
Crashes before this value for comleg				
0.56	0 277749	-0 277749	-0 391155	-0 307288
eigenvalues	-0 798382	0 53773	-0 0948024	≈ 0
0 645	0 301571	-0 301571	-0 376368	-0 282189
eigenvalues	0 462247	-0 398299	-0 167495	≈ 0
0.067	0 308731	-0 308731	-0 372087	-0 276276
eigenvalues	0 424337	-0 314878	-0 195367	≈ 0
Only unstable solutions exist after this value				

mhip = 0.4

comleg	q_1	q_2	u_1	u_2
Crashes before this value for comleg				
0.59	0 292008	-0 292008	-0 397874	-0 200875
eigenvalues	0 601379	0 300718	0 300718	≈ 0
0 645	0 308686	-0 308686	-0 38977	-0 170418
eigenvalues	0 537091	0 285454	0 285454	≈ 0
0.70	0 325127	-0 325127	-0 381431	-0 143181
eigenvalues	0 449546	0 263785	0 263785	≈ 0
Only unstable solutions exist after this value				

mhip = 0.8

comleg	q_1	q_2	u_1	u_2
Crashes before this value for comleg				
0.06	0 300216	-0 300216	-0 402682	-0 155974
eigenvalues	0 604919	0 271268	0 271268	≈ 0
0 645	0 314512	-0 314512	-0 396331	-0 125782
eigenvalues	0 353086	0 309621	0 309621	≈ 0
0.069	0 328305	-0 328305	-0 389756	-0 098244
eigenvalues	0 459957	0 300466	0 300466	≈ 0
Only unstable solutions exist after this value				

G.10 Effects of varying R

If R is at zero then the creature is similar to a point-footed model and the creature needs a rather steep slope At R = 1 we have a synthetic wheel rolling on a level ground i e slope is 0 Results gathered for various values of R are now given
Note that where necessary the γ value changes (i e at large and small R values) and this is also highlighted

mhip is 0.4 and initially = 0.025

R	q_1	q_2	u_1	u_2
0 1 $\gamma = 0\ 035$	0 269148	-0 269148	-0 353544	-0 129867
	0 366008	0 366008	0 267224	≈ 0
0 2	0 270485	-0 270485	-0 356631	-0 15801
	0 440723	0 311663	0 311663	≈ 0
0 3	0 301571	-0 301571	-0 376368	-0 282189
	0 537091	0 285454	0 285454	≈ 0
0 4	0 359682	-0 359682	-0 431959	-0 184419
	0 634328	0 261995	0 261995	≈ 0
0 6	0 375468	-0 375468	-0 427439	-0 238985
	0 867302	0 242081	0 242081	≈ 0
0 7 $\gamma = 0\ 001$	0 21022	-0 21022	-0 242615	-0 17222
	0 971823	-0 492455	-0 140326	≈ 0

mhip is 0.8 and initially = 0.025

R	q_1	q_2	u_1	u_2
0 2	0 274961	-0 274961	-0 361261	-0 115642
	0 439079	0 313502	0 313502	≈ 0
0 3	0 314512	-0 314512	-0 396331	-0 125782
	0 53887	0 287592	0 287592	≈ 0
0 4	0 366941	-0 366941	-0 441002	-0 13739
	0 649096	0 263418	0 263418	≈ 0
0 6 $\gamma = 0\ 001$	0 162229	-0 162229	-0 204301	-0 12943
	0 969924	0 288947	0 288947	≈ 0

G.11 Effect of adding in a damper

<u>Note</u> that damper doesn't work on a creature wıth no hıp mass - a slıght damper coeffıcıent tended to destroy the lımıt cycle and thus gıve no solutıon

A Quadratıc damper ıs admınıstered wıth applıed torque of $Ffric(u_2 - u_1)^2$

mhip = 0.4

Ffric	γ	q_1	q_2	u_1	u_2
-0 001	0 053	0 406592	-0 406592	-0 458499	-0 0974121
		0 328889	0 328889	0 261396	≈ 0
-0 005	0 054	0 404075	-0 404075	-0 461585	-0 106188
		0 337496	0 337496	0 254777	≈ 0
-0 005	0 055	0 40702	-0 40702	-0 463183	-0 103013
		0 339252	0 330252	0 248098	≈ 0
-0 008	0 056	0 403	-0 403	-0 466745	-0 11422
		0 407078	-0 407078	0 184229	≈ 0
-0 008	**0.06**	0 415236	-0 415236	-0 472814	-0 100185
		0 399425	0 399425	0 17696	≈ 0

mhip = 0.8

Ffric	γ	q_1	q_2	u_1	u_2
-0 008	0 058	0 420472	-0 420472	-0 471493	-0 0302886
		0 334499	0 334499	0 235099	≈ 0
	0 062	0 431414	-0 431414	-0 476747	-0 0156495
		0 336215	0 336215	0 217427	≈ 0
-0 012	**0.065**	0 435402	-0 435402	-0 482552	-0 0148927
		0 364051	0 364051	0 183449	≈ 0

mhip = 1.2

Ffric	γ	q_1	q_2	u_1	u_2
-0 015	0 062	0 431466	-0 431466	-0 479804	0 011745
		0 341678	0 341678	0 207332	≈ 0
-0 015	0 064	0 436833	-0 436833	-0 482208	0 0196097
		0 341111	0 341111	0 200562	≈ 0
-0 02	**0.068**	0 443128	-0 443128	-0 489323	0 0241565
		0 386533	-0 386533	0 152497	≈ 0

Appendix H:

Bodied creature results

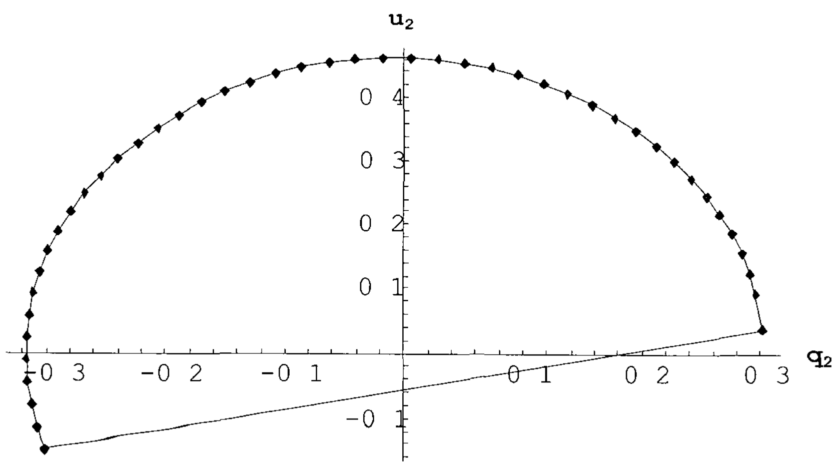
H.1 Limit Cycles - Unstable creature

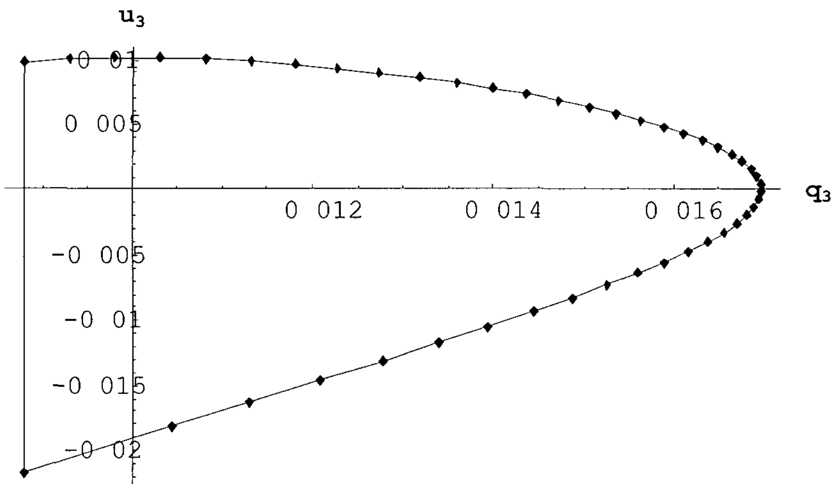
UNSTABLE RESULTS

Initially unstable solutions were encountered, but these were not desirable As they were undesirable solutions, only one example is included here The parameters for this solution are as follows

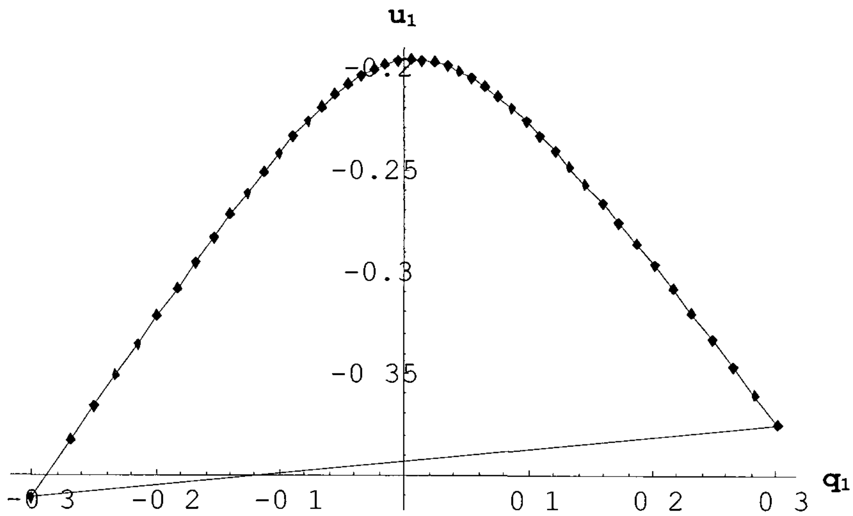
mbody = 0 8, mhip =1, combody =0 7, lbody =0 8 and τ *= (0 003, 0 02, 0 003)*

slope	q_1	q_2	q_3	u_1	u_2	u_3
0 0007	0 0617144	-0 0617144	0 0027611	-0 0830726	-0 0560537	-0 00154928
0 001	0 0716057	-0 0716057	0 0037038	-0 0968041	-0 0653969	-0 00203824
0 01	0 221512	-0 221512	-0 0067466	-0 266008	-0 124314	0 0107297
0 03	0 301618	-0 301618	0 0087996	-0 375297	-0 149613	0 00982882
0 04	0 33431	-0 33431	0 0090379	-0 409796	-0 137195	0 0148455
0 45	0 348545	-0 348545	0 0092159	-0 424534	-0 129626	0 0172924
Eigen-values	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
0 0007	4 64345	1 71251	0 662563	0 169514	-0 00007	$\cong 0$
0 001	4 5159	1 23109	0 86754	0 0904947	-0 000078	$\cong 0$
0 01	5 81933	0 435251	0 435251	0 342155	-0 00008	$\cong 0$
0 03	3 90528	3 15237	0 520259	0 0643784	-0 00007	$\cong 0$
0 04	3 94581	3 4856	0 484671	0 0560412	-0 00007	$\cong 0$
0 45	4 12756	3 49796	0 470406	0 0525742	-0 00007	$\cong 0$





u3
0 01
0 005
0 012
0 014
0 016
q3
-0 005
-0 01
-0 015
-0 02

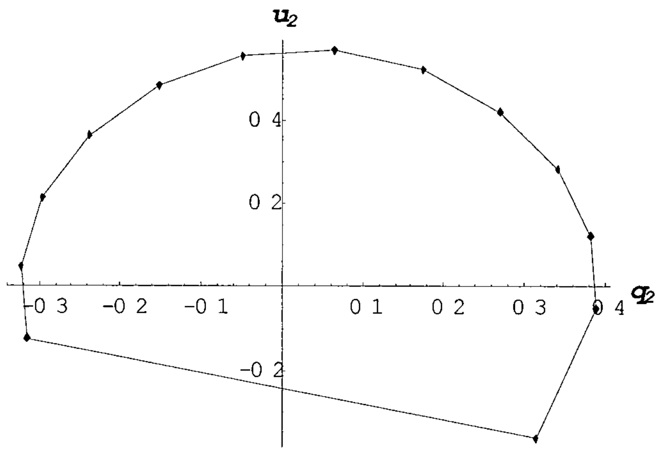


u1
-0.2
-0.25
-0.3
-0 35
q1
-0 3
-0 2
-0 1
0 1
0 2
0 3

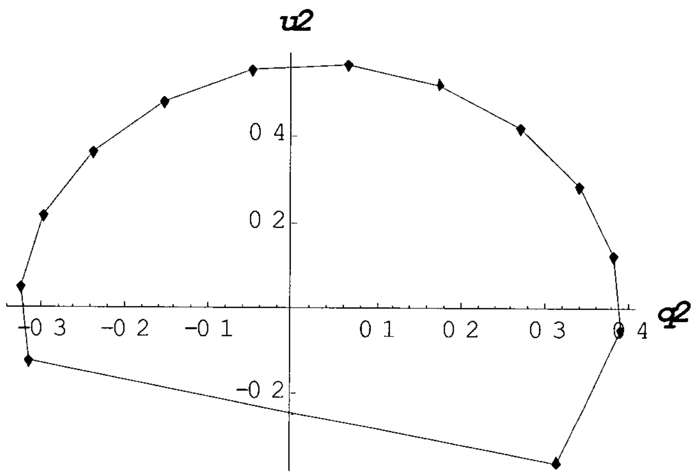
H.2 Body-less vs. Bodied

As a means of testing the code written for the bodied creature, a comparison was made between a bodied creature with minute parameters and a body-less one Both creatures have mhip = 0 4 and are on $\gamma = 0\ 025$ radians The parameters for the bodied creature are all approximately 0

This the limit cycle for the **body-less** creature



This is the limit cycle for the **bodied** creature



H.3 Basin of Attraction

Initial values are
$\theta_0 = \{q_1, q_2, q_3, u_1, u_2, u_3\} = \{0\ 3015, -0\ 3015, 0, -0\ 3763, -0\ 2822, 0\}$ There is no variation between no hip mass and one of 0 4 Now for each variable the max and min value are

mhip = 1.5, mbody = 1.0

Original	Maximum	Minimum	Average difference	Percentage
0 3015	0 36	0 29	$\pm 0\ 035$	$\approx 8\ 6\ \%$
0				
-0 3763	-0 39	-0 33	$\pm 0\ 03$	$\approx 12\ 5\ \%$
-0 2822	-0 5	0 1	$\pm 0\ 3$	$\approx 94\ \%$
0				

mhip = 0.8, mbody = 0.4

Original	Maximum	Minimum	Average difference	Percentage
0 3015	0 34	0 28	$\pm 0\ 03$	$\approx\ 10\ \%$
0				
-0 3763	-0 4	-0 34	$\pm 0\ 03$	$\approx 12\ 5\ \%$
-0 2822	-0 5	0 1	$\pm 0\ 3$	$\approx 94\%$
0				

H.4 Results for bodied creature - stable

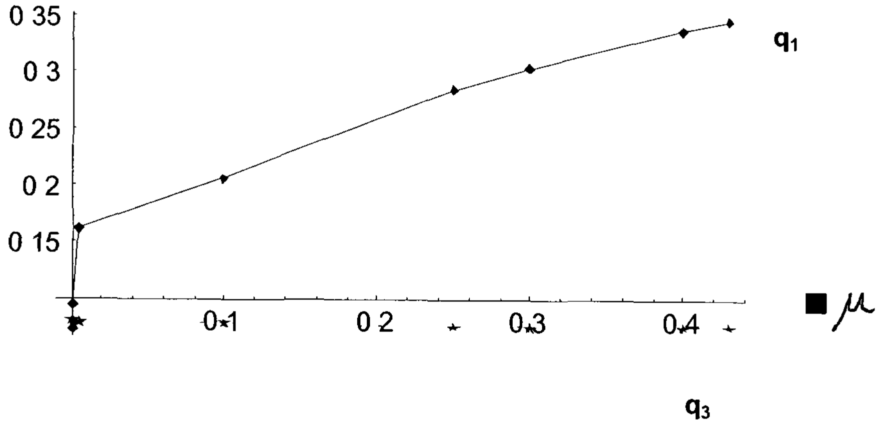
H.4.1 mbody = 1.0, mhip = 1.5

lbody = 0.8, combody = 0.795, damp = 0.55, Frac= 5
$\tau\ states = \{small = 0\ 00008, l\arg e = 0\ 0009\}$.

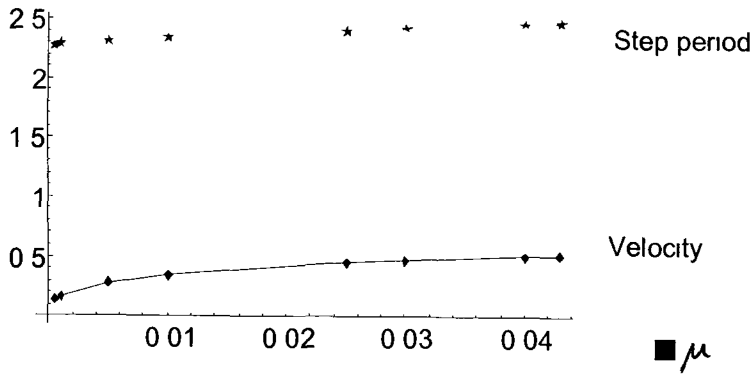
	q_1	q_2	q_3	u_1	u_2	u_3
0 0005	0 0732531	-0 0732531	0 080648	-0 105581	-0 0691833	0 0004858
0 001	0 0939995	-0 093995	0 0802105	-0 133665	-0 0841465	0 0001813
0 005	0 162181	-0 162181	0 07952	-0 222382	-0 120715	0 0006229
0 01	0 2058	-0 2058	0 0784453	-0 272871	-0 126115	0 00126436
0 025	0 283795	-0 283795	0 0762199	-0 348054	-0 0943815	0 00313797
0 03	0 302953	-0 302953	0 0759478	-0 363439	-0 0784396	0 00373738
0 04	0 33631	-0 33631	0 0761449	-0 387401	-0 0434981	0 00489863
0 043	0 345367	-0 345367	0 0763266	-0 393305	-0 032517	0 0052378

Eigenvalue	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
0 0005	1 00399	0 934877	0 709489	0 276596	-0 00001	$\cong 0$
0 001	1 00406	0 870775	0 643534	0 279627	-0 00001	$\cong 0$
0 005	1 00406	0 756813	0 756813	0 225386	-0 00001	$\cong 0$
0 01	1 00407	0 740568	0 740568	0 200875	-0 00001	$\cong 0$
0 025	1 00411	0 690446	0 690446	0 159779	-0 00001	$\cong 0$
0 03	1 00413	0 67654	0 67654	0 14968	-0 00001	$\cong 0$
0 04	1 00416	0 651676	0 651676	0 13227	-0 00001	$\cong 0$
0 043	1 00417	0 644786	0 644786	0 127619	-0 00001	$\cong 0$

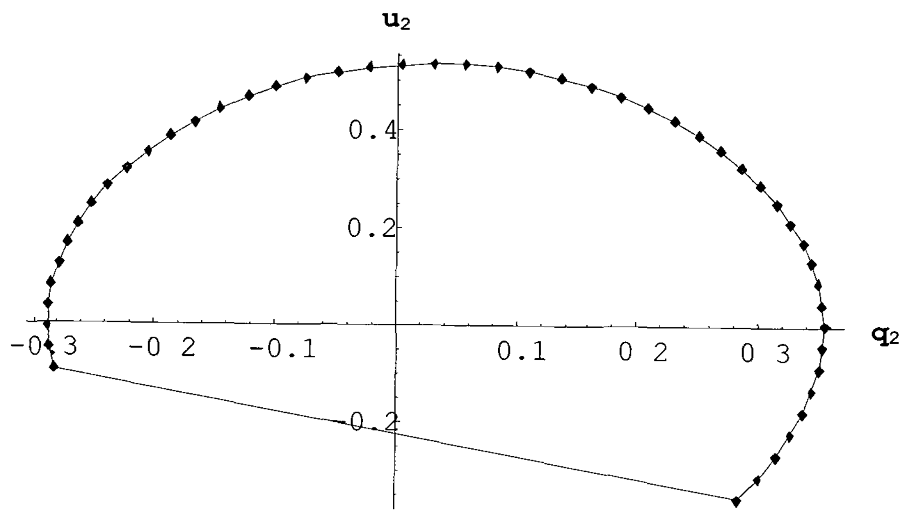
slope	*step period*	*length*	*Velocity*
0 0005	2 2650957353	0 29196531	0 1288975
0 001	2 2911910094	0 37378705	0 1631409
0 005	2 3117506184	0 63740823	0 2757253
0 01	2 3353536755	0 80015242	0 3426257
0 025	2 4007477684	1 07520300	0 4478617
0 03	2 4211080232	1 13901405	0 4704515
0 04	2 4604336279	1 24607493	0 5064452
0 043	2 4719707128	1 27420621	0 5154616



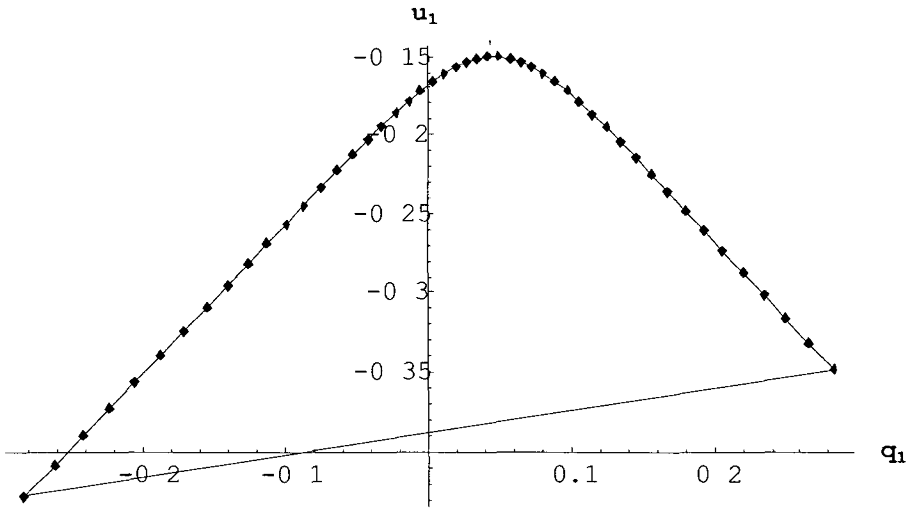
0 35
0 3
0 25
0 2
0 15
q1
0 1
0 2
0 3
0 4
μ
q3



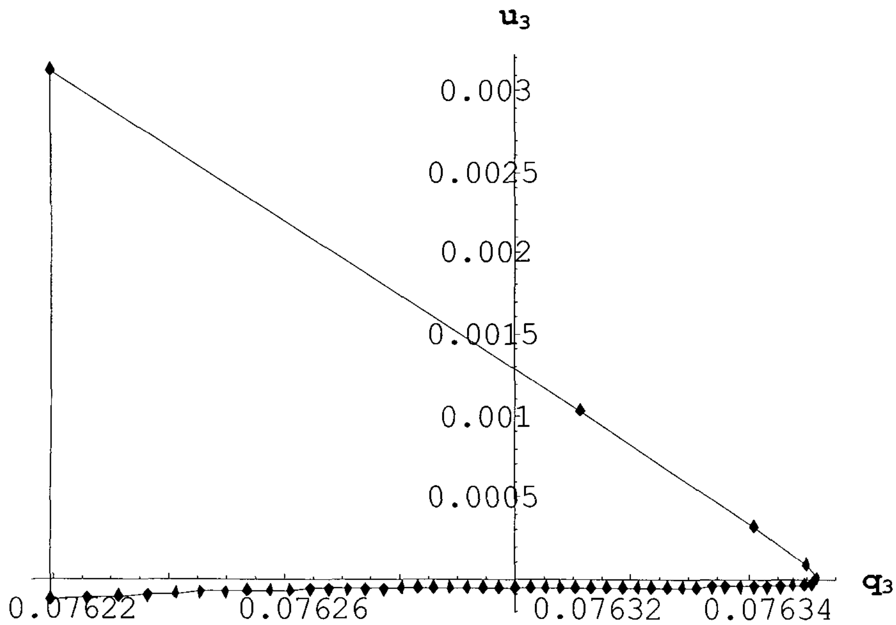
2 5
2
1 5
1
0 5
Step penod
Velocıty
0 01
0 02
0 03
0 04
μ



u2
0.4
0.2
-0.2
-0 3
-0 2
-0.1
0.1
0 2
0 3
q2



u1
-0 15
-0 2
-0 25
-0 3
-0 35
q1
-0 2
-0 1
0.1
0 2



u3
0.003
0.0025
0.002
0.0015
0.001
0.0005
q3
0.07622
0.07626
0.07632
0.07634

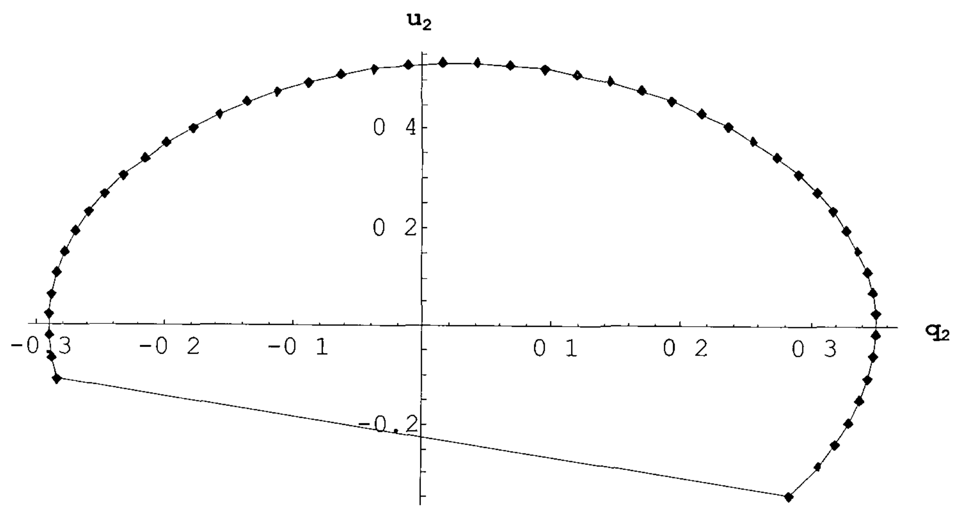
H.4.2 mbody = 0.8, mhip = 1.0

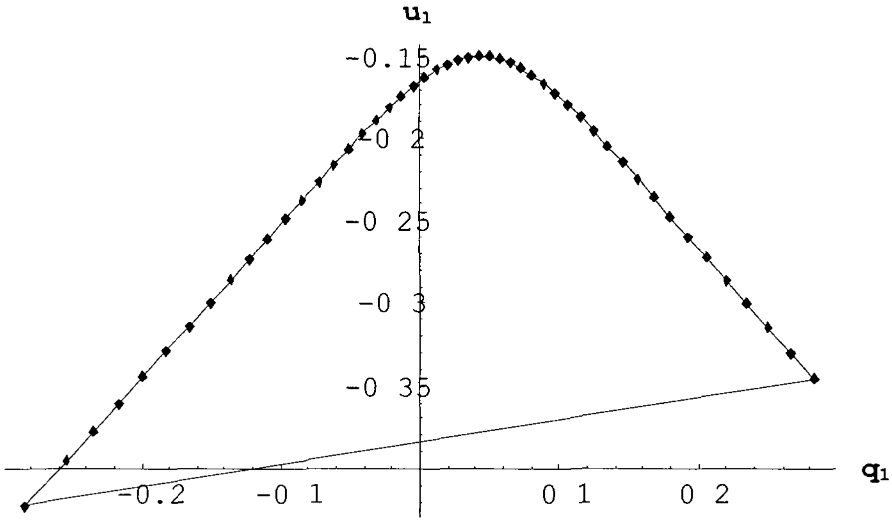
damp = 0.39, Frac= 5, lbody = 0.8, combody = 0.795.
$\tau\, states = \{small = 0\ 00005, l\arg e = 0\ 0005\}$

	q_1	q_2	q_3	u_1	u_2	u_3
0 0005	0 0739573	-0 0739573	0 0629932	-0 10524	-0 0721221	0 0004
0 001	0 0940243	-0 0940243	0 062665	-0 132227	-0 087613	0 0001667
0 01	0 205645	-0 205645	0 0614322	-0 270616	-0 136102	0 00118242
0 025	0 28356	-0 28356	0 0597739	-0 346021	-0 110307	0 00297124
0 04	0 336056	-0 336056	0 0599388	-0 385662	-0 0634686	0 00467344

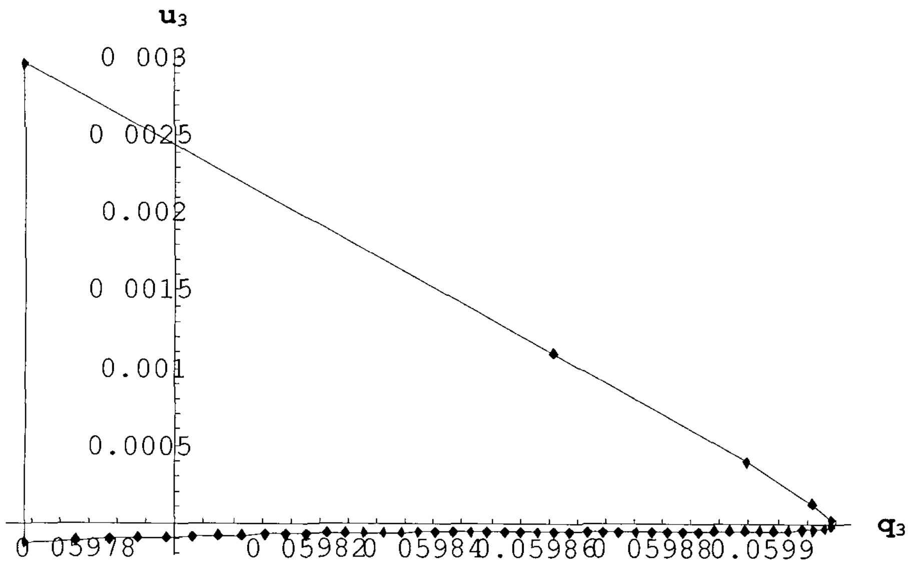
Eigenvalue	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
0 0005	1 00459	0 891317	0 713645	0 275695	-0 00001	$\cong 0$
0 001	1 00465	0 830715	0 668302	0 273414	-0 00001	$\cong 0$
0 01	1 00468	0 712002	0 712002	0 151918	-0 00001	$\cong 0$
0 025	1 00472	0 673993	0 673993	0 126536	-0 00001	$\cong 0$
0 04	1 00464	0 756596	0 75696	0 191054	-0 00001	$\cong 0$

slope	*step period*	*length*	*Velocity*
0 0005	2 3057505310	0 29475165	0 127833278
0 001	2 327261659	0 37388455	0 160654263
0 01	2 3622067996	0 79958416	0 338490331
0 025	2 4215065021	1 07441027	0 443694979
0 04	2 4780760816	1 24528006	0 502518898





u1
-0.15
-0 2
-0 25
-0 3
-0 35
q1
-0.2
-0 1
0 1
0 2



u3
0 003
0 0025
0.002
0 0015
0.001
0.0005
q3
0 05978
0 05982
0 05984
0.05986
0 05988
0.0599

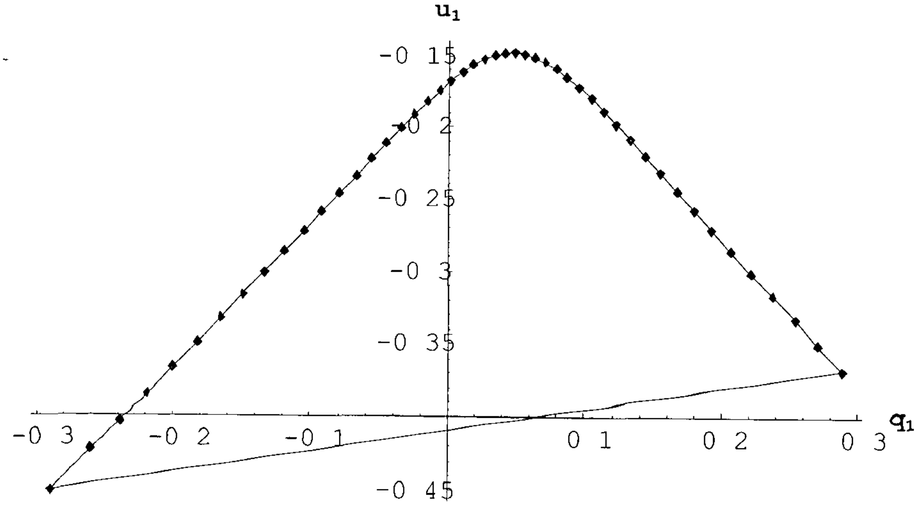
H.4.3 mbody = 0.4, mhip = 1.0

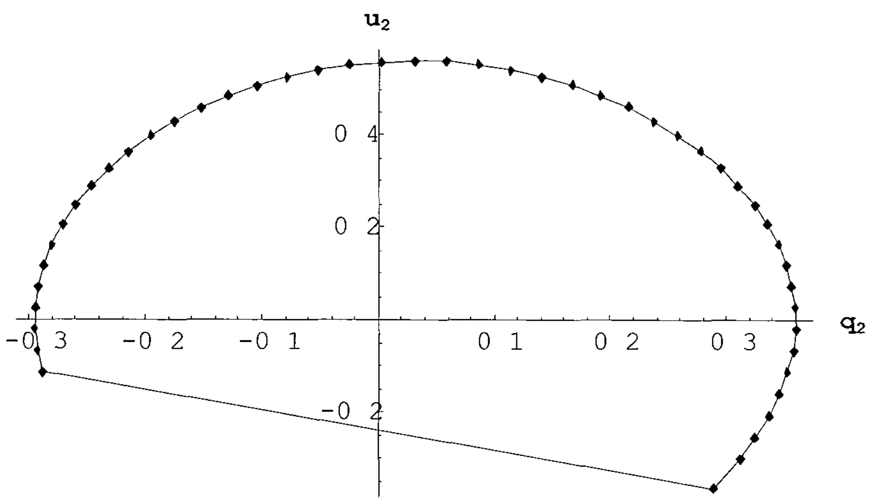
damp = 0.39, Frac= 5, lbody = 0.8, combody = 0.795.
$\tau\, states = \{small = 0\ 00005, l\arg e = 0\ 0003\}$

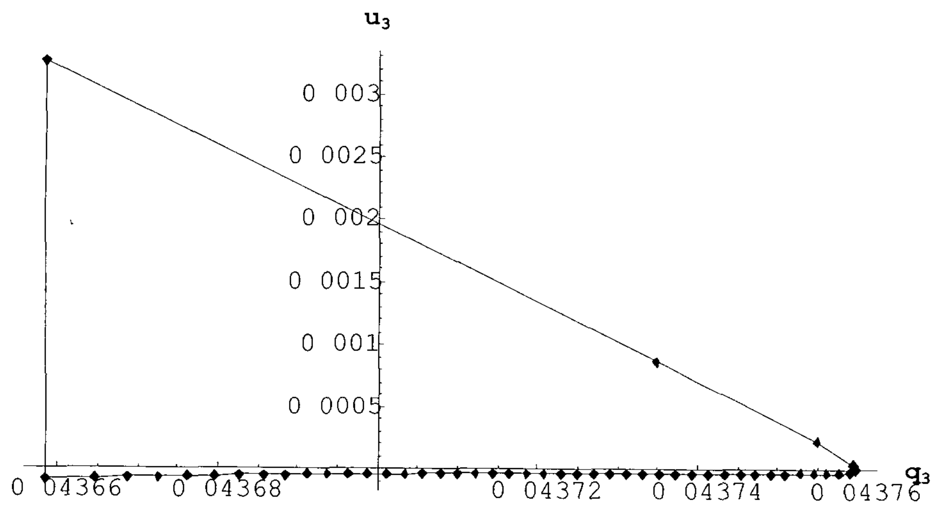
	q_1	q_2	q_3	u_1	u_2	u_3
0 0008	0 0873017	-0 0873017	0 0500665	-0 130485	-0 0872932	0 00025708
0 005	0 163876	-0 163876	0 0490164	-0 235228	-0 136209	0 00062824
0 01	0 208629	-0 208629	0 0474583	-0 28909	-0 144412	0 00128692
0 025	0 288673	-0 288673	0 0436584	-0 368168	-0 11355	0 00326011
0 04	0 342181	-0 342181	0 0423192	-0 408314	-0 0609854	0 00511136
0 045	0 357288	-0 357288	0 0424135	-0 417904	-0 0419738	0 0056998
0 06	0 397601	-0 397601	0 0441499	-0 440095	0 0166736	0 00738824

Eigenvalue	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
0 0008	1 00295	0 966749	0 403191	0 403191	-0 00001	$\cong 0$
0 005	1 00295	0 799045	0 399281	0 399281	-0 000001	$\cong 0$
0 01	1 00295	0 566881	0 440263	0 440263	-0 00001	$\cong 0$
0 025	1 00298	0 560751	0 560751	0 242892	-0 00001	$\cong 0$
0 04	1 00301	0 553457	0 553457	0 184287	-0 00001	$\cong 0$
0 045	1 00302	0 548016	0 548016	0 171176	-0 00001	$\cong 0$
0 06	1 00306	0 528229	0 528229	0 140815	-0 00001	$\cong 0$

slope	*step period*	*length*	*Velocity*
0 0005	2 261732610	0 3474351627	0 15361460553
0 005	2 279620999	0 6438310164	0 28242897248
0 01	2 301328022	0 8105104668	0 35219249887
0 025	2 367861499	1 0916040749	0 46100841428
0 04	2 430688604	1 2643576068	0 52016436984
0 045	2 450916405	1 3105943964	0 53473647396
0 06	2 509863499	1 4280100957	0 56895926664




u2
0 4
0 2
-0 2
-0 3
-0 2
-0 1
0 1
0 2
0 3
q2


u3
0 003
0 0025
0 002
0 0015
0 001
0 0005
0 04366
0 04368
0 04372
0 04374
0 04376
q3

H 4 4 mbody = 0 4, mhıp = 0 8

damp = , Frac= 5, lbody = 0.1 com = 0.095

$\tau\, states = \{small = 0\ 00002, l\arg e = 0\ 0002\}$

	q_1	q_2	q_3	u_1	u_2	u_3
0 0006	0 0778613	-0 0778613	0 0502617	-0 115391	-0 0806356	0 0003386
0 001	0 0942551	-0 0942551	0 050072	-0 13863	-0 0947573	0 0001783
0 01	0 207839	-0 207839	0 0482123	-0 285069	-0 149873	0 0012138
0 025	0 287522	-0 287522	0 0452662	-0 363961	-0 123864	0 0030943
0 04	0 340931	-0 340931	0 0443998	-0 404379	-0 0747838	0 0048811
0 055	0 383591	-0 383591	0 0456764	-0 429943	-0 0194907	0 0065592

Eıgenvalue	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
0 0006	1 00404	0 980972	0 409205	0 409205	-0 0001	$\cong 0$
0 001	1 00407	0 941525	0 399171	0 399171	-0 0001	$\cong 0$
0 01	1 00406	0 53232	0 53232	0 385078	-0 0001	$\cong 0$
0 025	1 00408	0 594693	0 594693	0 217646	-0 0001	$\cong 0$
0 04	1 00413	0 581812	0 581812	0 169606	-0 0001	$\cong 0$
0 055	1 00417	0 561825	0 561825	0 139369	-0 0001	$\cong 0$

slope	***step perıod***	***length***	***Velocıty***
0 0006	2 284634911	0 3101879921	0 1357713617
0 001	2 293852043	0 3747913920	0 1633895233
0 01	2 325821714	0 8076205730	0 3472409635
0 025	2 386921268	1 0877434292	0 4557098065
0 04	2 446703529	1 2604795471	0 5517461439
0 055	2 504432417	1 3882188099	0 5543047600

H.4.5 mbody = 0.2, mhip = 0.8

damp = 0.22, Frac= 5, lbody =0.1 combody = 0.095.

$\tau\, states = \{small = 0\ 00001, l\arg e = 0\ 0001\}$

	q_1	q_2	q_3	u_1	u_2	u_3
0 0007	0 0821395	-0 0821395	0 0501706	-0 126733	-0 0880368	0 0007518
0 001	0 0941341	-0 0941341	0 0500382	-0 14446	-0 0988131	0 00034861
0 01	0 209401	-0 209401	0 0474405	-0 298216	-0 156367	0 00129217
0 025	0 290784	-0 290784	0 0430628	-0 379984	-0 126879	0 0033278
0 04	0 345019	-0 345019	0 0411814	-0 420769	-0 0735953	0 0052468
0 06	0 400898	-0 400898	0 042569	-0 452363	0 00528144	0 0075980
0 07	0 424867	-0 424867	0 0446988	-0 463066	0 0453224	0 0086966

Eigenvalue	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
0 0007	1 00199	0 984819	0 39847	0 39847	-0 0001	$\cong 0$
0 001	1 002	0 967022	0 39338	0 39338	-0 0001	$\cong 0$
0 01	1 002	0 724261	0 389101	0 389101	-0 00009	$\cong 0$
0 025	1 00202	0 465806	0 465806	0 35324	-0 0001	$\cong 0$
0 04	1 00204	0 492255	0 492255	0 23423	-0 0001	$\cong 0$
0 06	1 00207	0 482722	0 482722	0 170188	-0 0001	$\cong 0$
0 07	1 00209	0 471818	0 471818	0 150138	-0 0001	$\cong 0$

slope	*step period*	*length*	*Velocity*
0 0007	2 242348709	0 3270821614	0 145865877
0 001	2 247302146	0 3743159553	0 166562362
0 01	2 281866721	0 8133325603	0 356432982
0 025	2 348415539	1 0986696719	0 467834441
0 04	2 412943795	1 2731329782	0 527626453
0 06	2 494759878	1 4372124370	0 576092492
0 07	2 534482328	1 5022096460	0 592708668

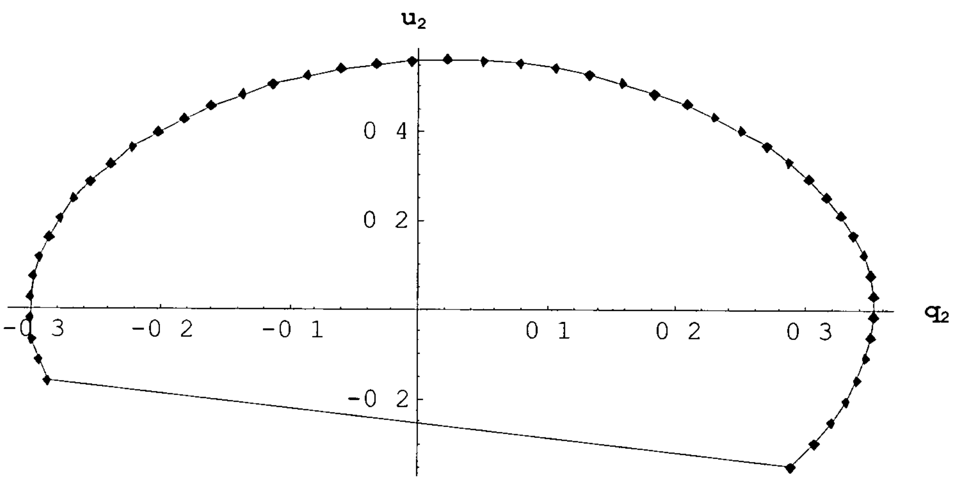
H 4 6 mbody = 0.2, mhip = 0.4

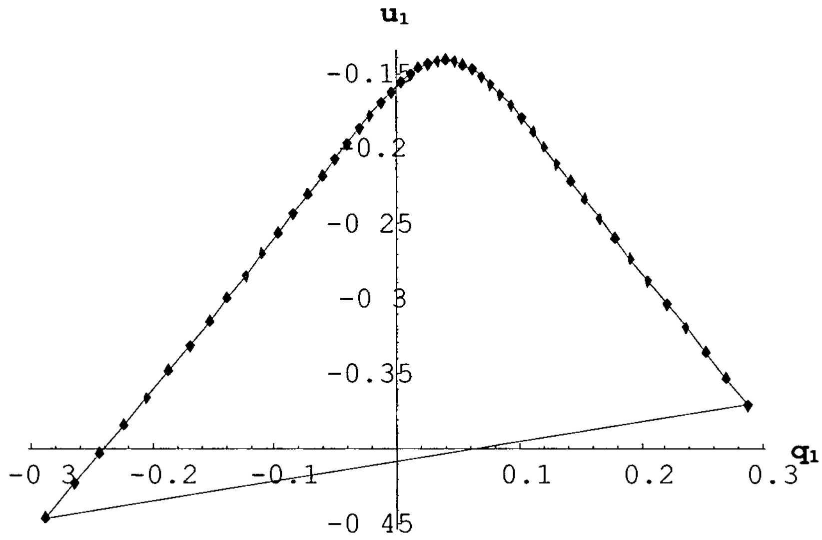
damp = 0.22, Frac= 5, lbody = 0.1 combody = 0.095
$\tau\ states = \{small = 0\ 00001, l\arg e = 0\ 0001\}$

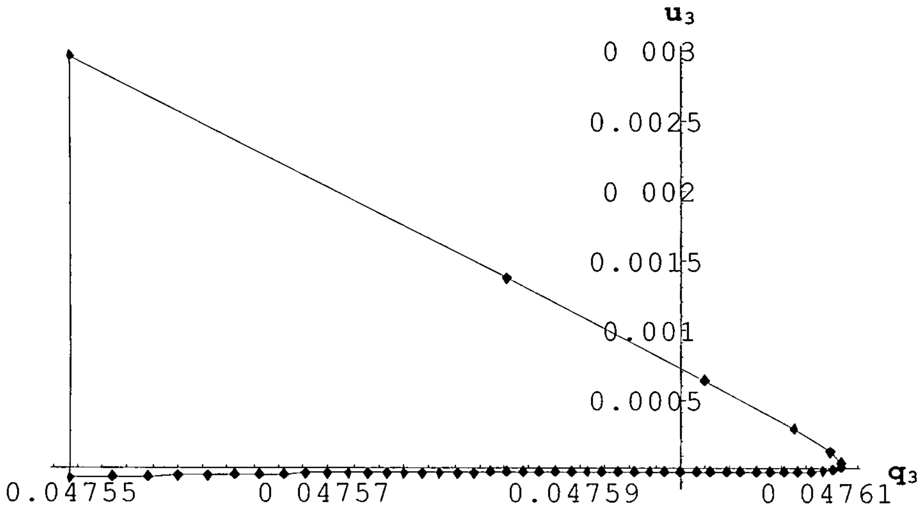
	q_1	q_2	q_3	u_1	u_2	u_3
0 0006	0 0774912	-0 0774912	0 0502639	-0 116283	-0 0894548	0 00109891
0 001	0 0940019	-0 0940019	0 0501563	-0 140062	-0 105833	0 00034438
0 01	0 207599	-0 207599	0 0493676	-0 289011	-0 176563	0 00114208
0 025	0 287975	-0 287975	0 0475515	-0 370686	-0 162563	0 00296023
0 04	0 342044	-0 342044	0 0471085	-0 412598	-0 120271	0 00474351
0 06	0 298029	-0 298029	0 0494079	-0 445566	-0 0518869	0 00699127

Eigenvalue	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
0 0007	1 00206	0 988903	0 3919	0 39847	-0 00010	$\cong 0$
0 001	1 00209	0 944972	0 380886	0 380886	-0 0001	$\cong 0$
0 01	1 00207	0 631141	0 407733	0 407733	-0 0001	$\cong 0$
0 025	1 00207	0 532047	0 532047	0 272277	-0 0001	$\cong 0$
0 04	1 00209	0 550366	0 550366	0 194262	-0 0001	$\cong 0$
0 06	1 00211	0 539927	0 539927	0 14573	-0 0001	$\cong 0$

slope	*step period*	*length*	*Velocity*
0 0006		0 3087254205	
0 001	2 3370165	0 3737964862	0 15994601
0 01	2 3570127	0 8067422310	0 34227317
0 025	2 4056726	1 0892635556	0 45278960
0 04	2 4598844	1 2639329566	0 51381803
0 06	2 5328199	1 4292082130	0 56427549




u1
-0.15
-0.2
-0 25
-0 3
-0.35
-0 45
-0 3
-0.2
-0.1
0.1
0.2
0.3
q1


u3
0 003
0.0025
0 002
0.0015
0.001
0.0005
0.04755
0 04757
0.04759
0 04761
q3

H.5. Varying Centre of mass

Thıs sectıon outlınes the effect of varyıng *combody* on the solutıon process Once agaın ıt ıs broken up ınto two sectıons stable and unstable, where unstable solutıons allow much varıatıon and stable ones do not

Unstable

Paramters: mbody = 0.8, mhip =1
combody =0.7, lbody =0.8 and τ states {small = 0.003, large = 0.02} on = 0.025

combody	q_1	q_2	q_3	u_1	u_2	u_3
0 55	0 275166	-0 275166	0 00546444	-0 342053	-0 161562	0 01681355
λ_i's	13 7286	4 27536	0 536872	0 0745697	-0 00008	≅ 0
0 6	0 278483	-0 278483	0 0067958	-0 346876	-0 15866	0 0132771
λ_i's	9 54331	3 69205	0 536617	0 0720775	-0 00008	≅ 0
0 7	0 282361	-0 282361	0 00871669	-0 35448	-0 153748	0 00727246
λ_i's	3 96888	2 90849	0 542962	0 0695022	-0 00007	≅ 0
0 795	0 283357	-0 283357	0 0104471	-0 359796	-0 150245	0 0004434
λ_i's	2 45145	1 10343	0 551493	0 0678219	≅ 0	≅ 0
0 797	0 28335	-0 28335	0 0104869	-0 359887	-0 150184	0 0002674
λ_i's	2 44335	1 0616	0 55168	0 0677896	≅ 0	≅ 0

Stable

Parameters: mbody = 0.2, mhip = 0.4
damp = 0.22, Frac= 5, lbody = 0.1 com = 0.095
τ states {small = 0.00001, large = 0.0001} , on = 0.025

combody	q_1	q_2	q_3	u_1	u_2	u_3
0 09	0 288068	-0 288068	0 0215296	-0 370461	-0 162407	0 0058998
λ_i's	1 00416	0 531239	0 531239	0 273064	-0 00009	≅ 0
0 093	0 288013	-0 288013	0 0326807	-0 370597	-0 1625	0 00413606
λ_i's	1 00291	0 531716	0 531716	0 272591	-0 0001	≅ 0
0 095	0 287975	-0 287975	0 0475515	-0 370686	-0 162563	0 00296023
λ_i's	1 00207	0 532047	0 532047	0 272277	-0 0001	≅ 0
0 097	0 287935	-0 287935	0 0822982	-0 370774	-0 162627	0 0017831
λ_i's	1 00124	0 532353	0 532353	0 271964	-0 0001	≅ 0
0 099	0 287893	-0 287893	0 25852	-0 370862	-0 162693	0 0005
λ_i's	1 0004	0 532674	0 532674	0 271642	-0 0001	≅ 0

Parameters: mhip = 1.5, mbody = 1.0
lbody = 0.8, combody = 0.795, damp = 0.55, Frac= 5.
τ states {small = 0.00008, large = 0.0009}, on = 0.025

combody	q_1	q_2	q_3	u_1	u_2	u_3
0 797	0 283719	-0 283719	0 131882	-0 348172	-0 094408	0 0018901

λ_i's	1 00245	0 691179	0 691179	0 159356	-0 00001	$\cong 0$
0 795	0 283795	-0 283795	0 0762199	-0 348054	-0 0943815	0 00313797
λ_i's	1 00411	0 690446	0 690446	0 159779	-0 00001	$\cong 0$
0 793	0 283867	-0 283867	0 0524644	-0 347933	-0 094357	0 00438008
λ_i's	1 00577	0 689713	0 689713	0 160203	-0 00001	$\cong 0$
0 79	0 283969	-0 283969	0 0346825	-0 347749	-0 0943236	0 00623843
λ_i's	1 00827	0 688606	0 688606	0 160844	$\cong 0$	$\cong 0$
0 75	0 284636	-0 284636	0 00147668	-0 344857	-0 0942744	0 0309918
λ_i's	1 04285	0 676311	0 676311	0 170046	$\cong 0$	$\cong 0$

H 6. Effect of varying mbody

mhip = 1.5, mbody = 1.0,
lbody = 0.8, combody = 0.795, damp = 0.55, Frac= 5
$\tau\, states = \{small = 0\ 00008, large = 0\ 0009\}$.

mbody	q_1	q_2	q_3	u_1	u_2	u_3
0 4	0 29134	-0 29134	0 198439	-0 375901	-0 095529	-0 00005
λ_i's	1 00264	0 513655	0 513655	0 277827	0 00009	$\cong 0$
0 7	0 287131	-0 287131	0 110182	-0 360442	-0 0950866	0 00334149
λ_i's	1 00284	0 610819	0 610819	0 202481	-0 00001	$\cong 0$
1 0	0 283795	-0 283795	0 0762199	-0 348054	-0 0943815	0 00313797
λ_i's	1 00411	0 690446	0 690446	0 159779	-0 00001	$\cong 0$
1 2	0 281921	-0 281921	0 0633571	-0 341073	-0 0938582	0 0030274
λ_i's	1 00496	0 731726	0 731726	0 142845	-0 00001	$\cong 0$
unstable then until 1 6 when no solutions begin to exist						

mhip = 1.0, mbody = 0.4,
damp = 0.39, Frac= 5, lbody = 0.8, combody = 0.795.
$\tau\, states = \{small = 0\ 00005, large = 0\ 0003\}$

mbody	q_1	q_2	q_3	u_1	u_2	u_3
0 3	0 290297	-0 290297	0 0600032	-0 375222	-0 114438	0 00337426
λ_i's	1 00222	0 504217	0 504217	0 299433	-0 00001	$\cong 0$
0 6	0 285882	-0 285882	0 0280669	-0 356052	-0 111855	0 00307297
λ_i's	1 00451	0 647992	0 647992	0 182785	-0 00001	$\cong 0$
0 8	0 286776	-0 286776	0 0312723	-0 360719	-0 158734	0 00284804
λ_i's	1 00605	0 711942	0 711942	0 151935	-0 00001	$\cong 0$
No solutions begin to exist after this						

mhip = 0.4, mbody = 0.2,
damp = 0.22, Frac= 5, lbody = 0.1 com = 0.095
τ states {small = 0.00001, large = 0.0001} , on = 0.025

mbody	q_1	q_2	q_3	u_1	u_2	u_3
No solutions exist for *mbody* lower than 0 2						
0 3	0 286776	-0 286776	0 0312723	-0 360719	-0 158734	0 00284804
λ_i's	1 00314	0 615799	0 615799	0 202611	-0 0001	$\cong 0$
0 35	0 28622	-0 28622	0 0267937	-0 356316	-0 157047	0 00279888
λ_i's	1 00368	0 64938	0 64938	0 181939	-0 0001	$\cong 0$

H.7 Hip Damper

A **Quadratic damper** is used with applied torque of $Ffric(u_2 - u_1)^2$

H.7.1 mbody = 0.4, mhip = 1.0

Same parameters as H.4.3.

This table gives the effect of various *Ffric* values on slope = **0.065** where the *Ffric* values that gave the best results are shown.

Ffric	q_1	q_2	q_3	u_1	u_2	u_3
-0.008	0.403418	-0.403418	0.0543739	-0.469985	-0.0137412	0.00634116
-0.012	0.407565	-0.407565	0.0495499	-0.457063	0.0122486	0.0071346
-0.018	0.408641	-0.408641	0.047823	-0.452929	0.0218136	0.00744112

Eigenvalues

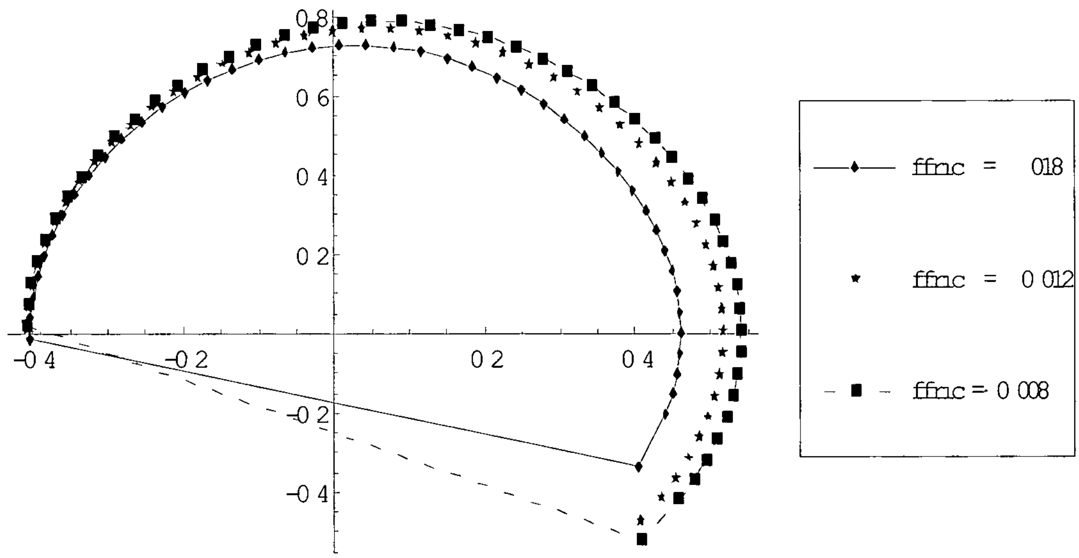
Ffric	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
-0.008	1.00274	0.756131	0.756131	0.0786171	-0.00001	$\cong 0$
-0.012	1.00291	0.611246	0.611246	0.103464	-0.00001	$\cong 0$
-0.018	1.00297	0.568417	0.568417	0.115422	-0.00001	$\cong 0$

Ffric	*step period*	*length*	*Velocity*
-0.008	2.45625196	1.458577708	0.593821729
-0.012	2.41267131	1.455629510	0.603326903
-0.018	2.28896185	1.444203959	0.630942782

Maximum slopes attainable

Ffric	q_1	q_2	q_3	u_1	u_2	u_3
-0.012	0.419505	-0.419505	0.0503852	-0.462592	0.0329884	0.00768644
γ = **0.07**						
-0.018	0.423758	-0.423758	0.0541803	-0.475091	0.0261558	0.00741166
γ = **0.073**						

Ffric	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
-0.012	1.00292	0.5942	0.5942	0.0994319	-0.00001	$\cong 0$
γ = **0.07**						
-0.018	1.0028	0.682079	0.682079	0.0782041	-0.00001	$\cong 0$
γ = **0.073**						



Thıs dıagram shows the Lımıt Cycles on γ = 0 065

H.7 2 mbody = 0.8, mhip = 1.0

Same parameters as H.4.2.

Thıs table gıves the effect of varıous *Ffrıc* values on slope = **0.045** where the *Ffrıc* values that gave the best results are shown

Ffrıc	q_1	q_2	q_3	u_1	u_2	u_3
-0 008	0 351355	-0 351355	0 0631716	-0 403639	-0 0590579	0 00480934
-0 012	0 351146	-0 351146	0 0653776	-0 409957	-0 0691161	0 00449811

Eigenvalues

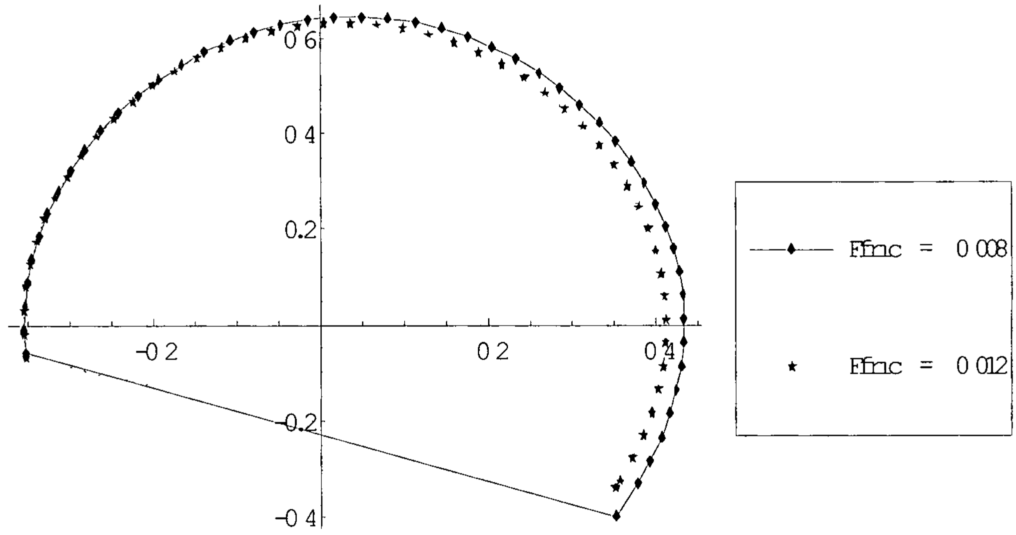
Ffrıc	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
-0 008	1 00458	0 724368	0 724368	0 104909	-0 00001	$\cong 0$
-0 012	1 00446	0 785412	0 785412	0 0946992	-0 00001	$\cong 0$

Ffrıc	*step perıod*	*length*	*Velocıty*
-0 008	2 4225913267	1 2925760828	0 5335510238
-0 012	2 3663344714	1 2919380237	0 5459659398

Maxımum slopes attainable

Ffrıc	*q1*	*q2*	*q3*	*u1*	*u2*	*u3*
-0 008	0 357066	-0 357066	0 0633256	-0 407093	-0 0518172	0 0050292
γ = **0 047**						
-0 012	0 367983	-0 367983	0 0655578	-0 419202	-0 0459992	0 00518607
γ = **0 051**						

Ffrıc	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
-0 008	1 00459	0 716684	0 716684	0 103059	-0 00001	$\cong 0$
γ = **0 047**						
-0 012	1 0045	0 750494	0 750494	0 091013	-0 00001	$\cong 0$
γ = **0 051**						



This diagram shows the Limit Cycles on $\gamma = 0\ 045$

H.7.3 mbody = 0.4, mhip = 0 8

Same parameters as H.4.4.

Thıs table gıves the effect of varıous *Ffrıc* values on slope μ = **0.06** where the *Ffrıc* values that gave the best results are shown

Ffrıc	q_1	q_2	q_3	u_1	u_2	u_3
-0 003	0 373504	-0 373504	0 0733276	-0 487863	-0 118508	0 003575
	3 44766	1 00326	0 491696	0 0655008	-0 00001	$\cong 0$
-0 008	0 395018	-0 395018	0 0499126	-0 445407	-0 0180766	0 0065185
	1 00402	0 621116	0 621116	0 110058	-0 00001	$\cong 0$

<u>μ = 0.062</u>

Ffrıc	q_1	q_2	q_3	u_1	u_2	u_3
-0 008	0 400022	-0 400022	0 0501982	-0 447795	-0 0102048	0 0067368
	1 00402	0 615099	0 615099	0 108142	-0 00001	$\cong 0$

<u>μ = 0.065</u>

Ffrıc	q_1	q_2	q_3	u_1	u_2	u_3
-0 012	0 405943	-0 405943	0 0529754	-0 457614	-0 0108365	0 00666508
	1 00392	0 667797	0 667797	0 0917404	-0 00001	$\cong 0$

H.8 Applied Torque

mbody = 1, mhip = 1 5	*Torque*		*mbody = 0 8 mhip =1 0*	*Torque*
0 0005	-0 0250161		0 0005	-0 0158356
0 001	-0 0204931		0 001	-0 0139283
0 005	-0 0204931		0 01	-0 0135017
0 01	-0 021497		0 025	-0 0157188
0 025	-0 0289354		0 04	-0 0180096
0 043	-0 0289354			

mbody = 0 4 mhip = 1	*Torque*		*mbody = 0 4 mhip =0 8*	*Torque*
0 0008	-0 00578143		0 0006	-0 00615093
0 005	-0 00549235		0 001	-0 00552428
0 01	-0 00611214		0 01	-0 00557509
0 025	-0 00833589		0 025	-0 00695179
0 04	-0 0105034		0 04	-0 00833714
0 06	-0 0132937		0 055	-0 00966618

mbody = 02 mhip = 0 8	*Torque*			Torque
0 0007	-0 00422771			
0 001	-0 00327594			
0 025	-0 00759446			
0 04	-0 0106775			
0 07	-0 016279			

H.9 Varying Radius of gyration

mhip = 1.5, mbody = 1.0

Parameters:
lbody = 0.8, combody = 0.795, damp = 0.55, Frac= 5, = 0.025.
$\tau\ states = \{small = 0\ 00008, l\arg e = 0\ 0009\}$.

radius gyr	q_1	q_2	q_3	u_1	u_2	u_3
bod = 0 121	0 283795	-0 283795	0 0762199	-0 348054	-0 0943815	0 00313797
λ_i's	1 00411	0 690446	0 690446	0 159779	-0 00001	$\cong 0$
bod = 0 09	0 283791	-0 283791	0 0762169	-0 348051	-0 0943829	0 0042597
λ_i's	1 00413	0 690442	0 690442	0 159781	-0 00001	$\cong 0$
bod = 0 05	0 283669	-0 283669	0 0718488	-0 348274	-0 0944591	0 246121
λ_i's	1 00402	0 764968	0 764968	0 164953	$\cong 0$	$\cong 0$

radius gyr	q_1	q_2	q_3	u_1	u_2	u_3
leg = 0 1	0 291521	-0 291521	0 0773395	-0 349971	-0 0919796	0 00313335
λ_i's	1 00425	0 63606	0 63606	0 160012	-0 00001	$\cong 0$
leg = 0 09	0 283791	-0 283791	0 0762169	-0 348051	-0 0943829	0 0042597
λ_i's	1 00413	0 690442	0 690442	0 159781	-0 00001	$\cong 0$
leg = 0 04	0 239549	-0 239549	0 0699325	-0 341002	-0 117388	0 00303788
λ_i's	1 00335	0 970099	0 970099	0 197147	-0 00001	$\cong 0$

H.10 Complex Controller

On slope γ = 0 025

mhıp = 1 5 mbody = 1 0

q_1	q_2	q_3	u_1	u_2	u_3
0 283787	-0 283787	-0 0899587	-0 348033	-0 0943614	0 00305087
1 00409	0 690225	0 690225	0 159846	-0 00001	$\cong 0$

mhip = 1 0 mbody = 0 8

q_1	q_2	q_3	u_1	u_2	u_3
0 283553	-0 283553	-0 0699312	-0 346004	-0 11029	0 00290952
1 00466	0 71182	0 71182	0 151968	-0 00001	$\cong 0$

mhip = 1 0 mbody = 0 4

q_1	q_2	q_3	u_1	u_2	u_3
0 288672	-0 288672	-0 0604298	-0 368161	-0 113541	0 00319901
1 00297	0 560653	0 560653	0 242955	-0 00001	$\cong 0$

mhıp = 0 8 mbody = 0 4

q_1	q_2	q_3	u_1	u_2	u_3
0 28752	-0 28752	-0 0587354	-0 363954	-0 123854	0 00304068
1 00407	0 594581	0 594581	0 217706	-0 00001	$\cong 0$

mhıp = 0 8 mbody = 0 2

q_1	q_2	q_3	u_1	u_2	u_3
0 290784	-0 290784	-0 0611985	-0 37998	-0 126872	0 00326162
1 00201	0 465768	0 465768	0 353277	-0 00001	$\cong 0$

mhıp = 0 4 mbody = 0 2

q_1	q_2	q_3	u_1	u_2	u_3
0 287974	-0 287974	-0 0564755	-0 370681	-0 162555	0 00290384
1 00207	0 531955	0 531955	0 272335	-0 00001	$\cong 0$

Appendix I:

Mechanical Energy

Body-less:

Potential Energy

$PE = m \times g \times h$ and the total PE for the creature is

$$\begin{aligned} PE = & \; M \times g \times height\ of\ swiCOM\ above\ ground \; + \\ & M \times g \times height\ of\ sta\ COM\ above\ ground \; + \\ & Mhip \times g \times height\ of\ hip\ COM\ above\ ground \end{aligned}$$

Kinetic Energy

Since the creature is similar to that of a rolling wheel, the total kinetic energy is composed of two parts, the kinetic energy of the translation of the centre of mass, and the kinetic energy of rotation about the centre of mass This gives that the kinetic energy is given by $KE = (0.5 \times M \times v^2) + (0.5 \times Il \times \omega^2)$ and thus is

$$\begin{aligned} KE = & \; 0.5 \times M \times \| \vec{v}_{COM} swi \|^2 \; + \\ & 0.5 \times M \times \| \vec{v}_{COM} sta \|^2 + \\ & 0.5 \times Mhip \times \| \vec{v}_{COM} hip \|^2 + \\ & 0.5 \times Ilsta \times \| u_1 \|^2 + 0.5 \times Ilswi \times \| u_2 \|^2 \end{aligned}$$

Bodied:

Potential Energy

$$\begin{aligned} PE = & \; M \times g \times height\ of\ swiCOM\ above\ ground \; + \\ & M \times g \times height\ of\ sta\ COM\ above\ ground \; + \\ & Mhip \times g \times height\ of\ hip\ COM\ above\ ground \; + \\ & Mbody \times g \times height\ of\ bodyCOM\ above\ ground \end{aligned}$$

Kınetic Energy

$$
\begin{aligned}
K\,E \;=\;& 0\,5 \times M \times \| \vec{v}_{COM} swı \|^2 \; + \\
& 0\,5 \times M \times \| \vec{v}_{COM} sta \|^2 \; + \\
& 0\,5 \times Mhıp \times \| \vec{v}_{COM} hıp \|^2 \; + \\
& 0\,5 \times Mbody \times \| \vec{v}_{COM} body \|^2 \; + \\
& 0\,5 \times Ilsta \times \| u_1 \|^2 \; + 0\,5 \times Ilswı \times \| u_2 \|^2 \; + 0\,5 \times Inertıabody \times \| u_3 \|^2
\end{aligned}
$$