Calculation of Acoustic Shielding at Full-Scale Aircraft Configurations

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11. Oktober 2012

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- Summary and Outlook

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- Complex sound sources are difficult to handle

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$$\frac{d\boldsymbol{x}}{ds} = \frac{\boldsymbol{g}}{|\boldsymbol{g}|}, \quad \frac{d\boldsymbol{g}}{ds} = 0, \quad \frac{d\psi}{ds} = \frac{1}{c}$$

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- Amplitude A by solution of transport equation ▸ Jacobian ▸ Initial Conditions

$$\frac{A^2 D}{\varrho c^2 |\boldsymbol{g}|^2} = \text{const. along a ray}, \qquad D = \frac{d\boldsymbol{x}}{ds} \cdot \left(\frac{\partial \boldsymbol{x}}{\partial \alpha} \times \frac{\partial \boldsymbol{x}}{\partial \beta} \right)$$

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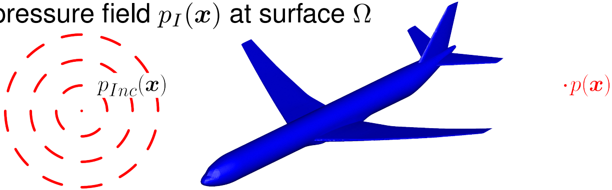

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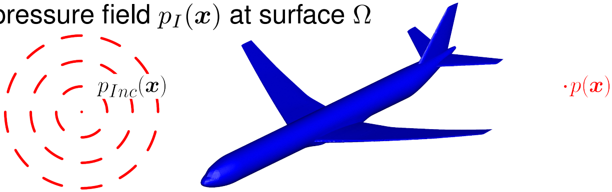

- Kirchhoff integral: $p(\boldsymbol{x})$ by integral over surface pressure $p(\boldsymbol{y})$

$$c\, p(\boldsymbol{x}) - \frac{1}{4\pi}\int_\Omega p(\boldsymbol{y}) \frac{\partial G(\boldsymbol{x},\boldsymbol{y})}{\partial n_y} d\Omega_y + \frac{1}{4\pi}\int_\Omega \frac{\partial p(\boldsymbol{y})}{\partial n_y} G(\boldsymbol{x},\boldsymbol{y}) d\Omega_y = p_I(\boldsymbol{x})$$

Green's Function $G(\boldsymbol{x},\boldsymbol{y}) = \frac{e^{ik|\boldsymbol{x}-\boldsymbol{y}|}}{|\boldsymbol{x}-\boldsymbol{y}|}$

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- Complex sources (CRORs, Fans, etc.) are possible

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  - Model has been extended to CRORs with different rotational speed

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FMM – Scattering at a Sphere

Shielding Examples

- Unit sphere $R = 1$

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- $\lambda = 0.020$ – 9 elements per wavelength

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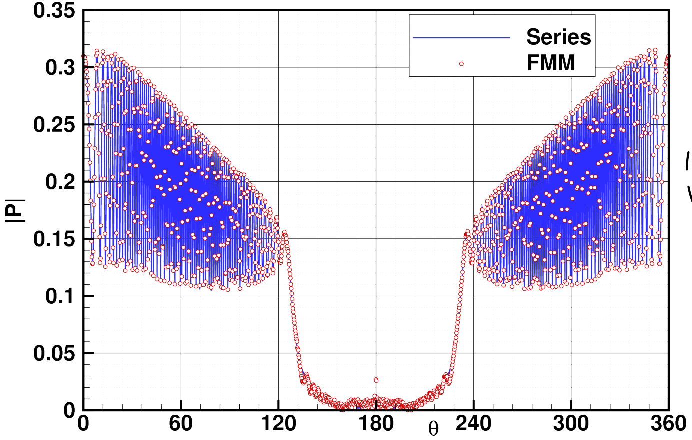
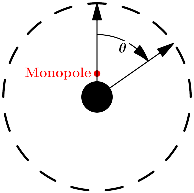
Sphere – 2.7M – 6 PPW





- Modulus of pressure on circle with $R = 6$
- 6 Elements per wavelength

Illuminated

Shadow

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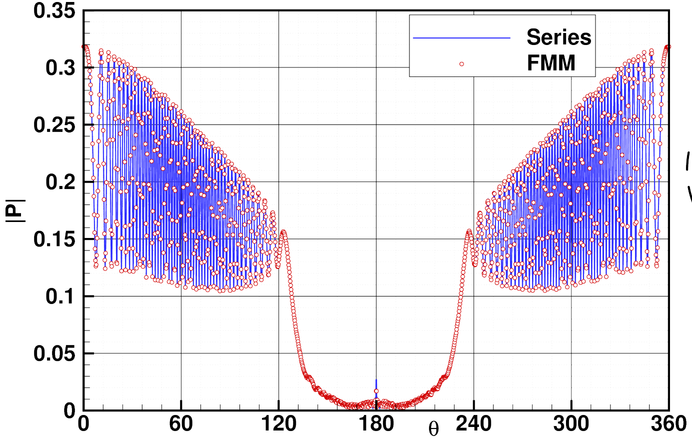

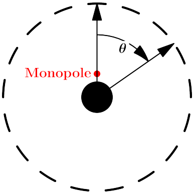

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Comparison RTM – FMM

Shielding Examples

- Monopole above unit sphere

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- Monopole above unit sphere
- Monopole shielding by Low-Noise-Aircraft (LNA)

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- Monopole above unit sphere
- Monopole shielding by Low-Noise-Aircraft (LNA)
- 'Shielding' of rear mounted CROR at a conventional Z08 aircraft configuration

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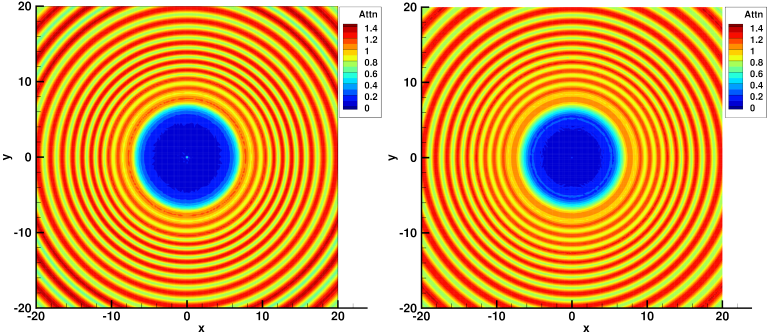
Sphere RTM – FMM – 6 PPW

- Monopole at $R = 1.5$ above unit sphere
- Attenuation factor $\left|\frac{p_{Shielded}}{p_{Incident}}\right|$ in plane centered at $R = 6$ below sphere
- Wavelength $\lambda = 0.013$, 2.7×10^6 triangles 6 elements per wavelength



(a) Ray-Tracing (b) FMM

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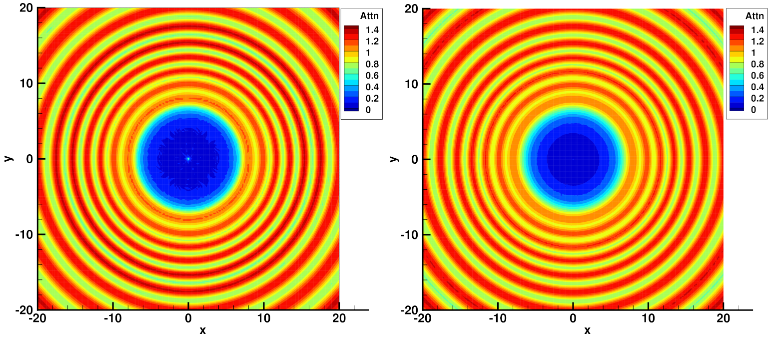

(c) Ray-Tracing (d) FMM

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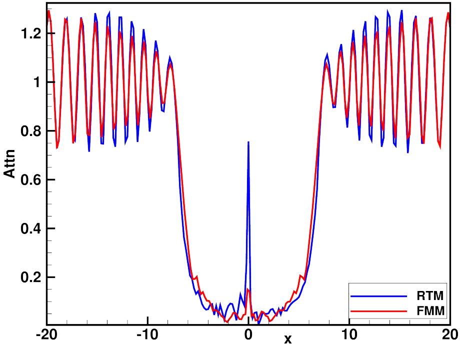
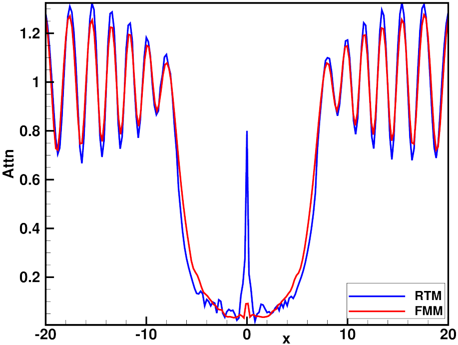
- Attenuation factor $\left|\frac{p_{Shielded}}{p_{Incident}}\right|$ in plane centered at $R = 6$ below sphere
- Wavelength $\lambda = 0.013, 0.020$
- Arago spot visible



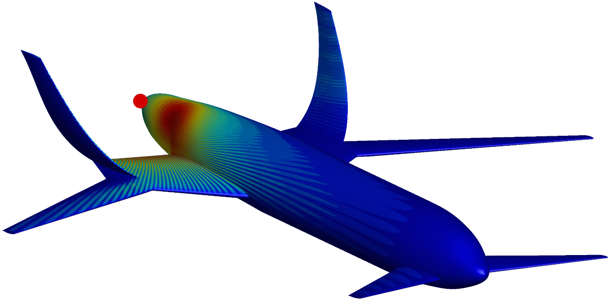
(e) Attenuation 6 ppw



(f) Attenuation 9 ppw

Low-Noise-Aircraft (LNA) Configuration

- DLR Low-Noise-Aircraft (LNA) – fuselage length ca. 50 m



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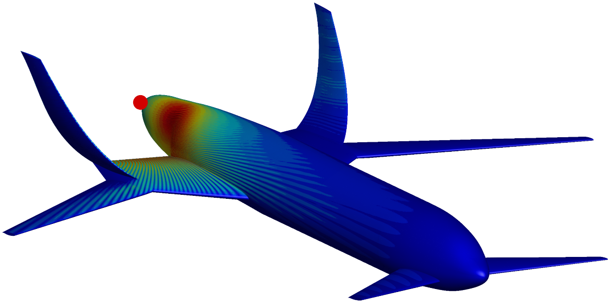

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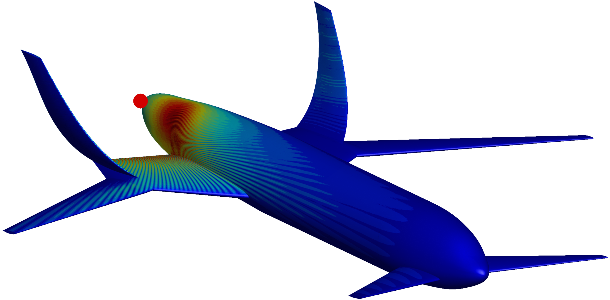

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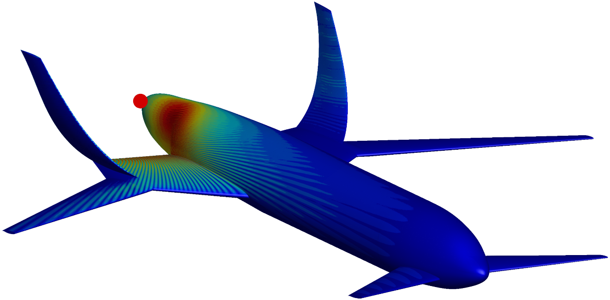

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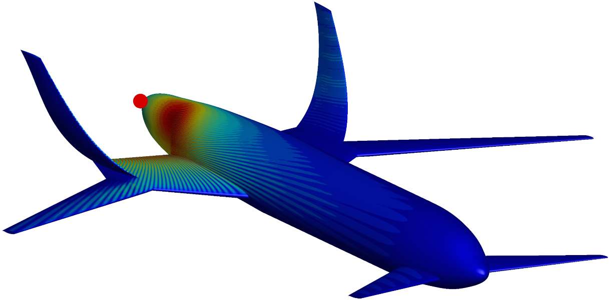

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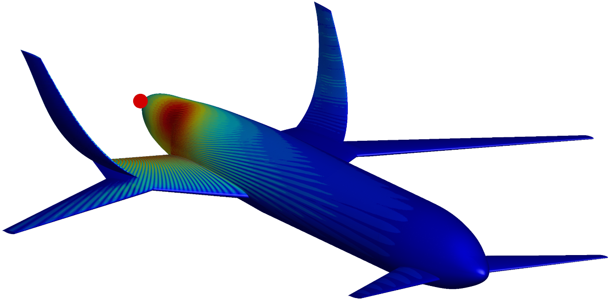
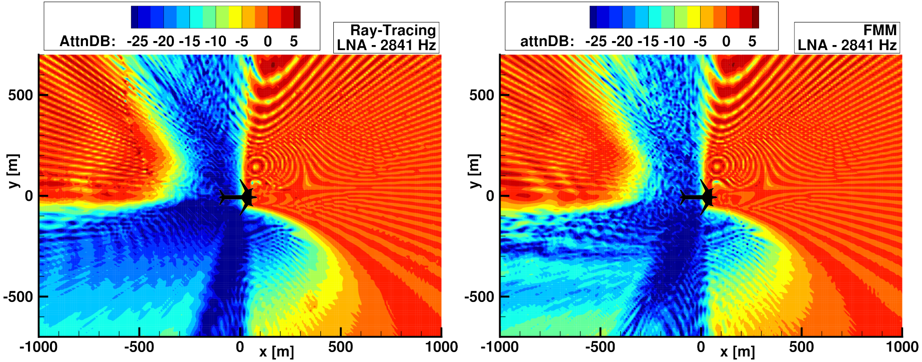
- 4×10^6 triangles – frequency 2841 Hz

>



LNA - FMM/RTM

- Attenuation in dB
- Monopole above right wing trailing edge
- a/c shown not scaled!

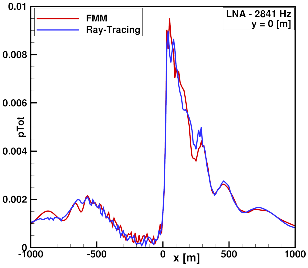


(g) Ray-Tracing

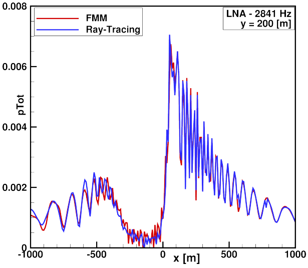
(h) FMM

LNA - FMM/RTM

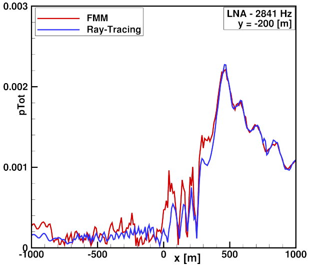
- Modulus of pressure 120 m below flight path ($y = 0$ m) and along $y = \pm 200$ m (y-axes scaled differently!)



(i) Below flight path y=0m



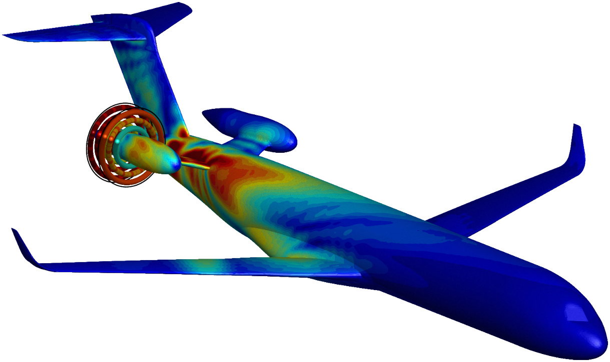
(j) Illuminated side y=200m



(k) Shadow side y=-200m

Z08 - Asymmetry - Left/Right installed CROR

- Pusher CRORs are 'faked': inflow undisturbed by pylon [Cror model]



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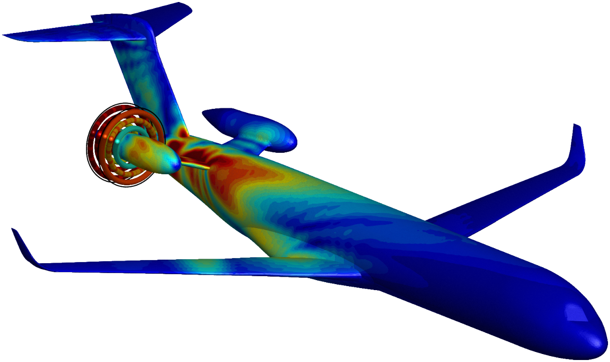
- Combination mode $B_F + B_R$ – Frequency 308 Hz



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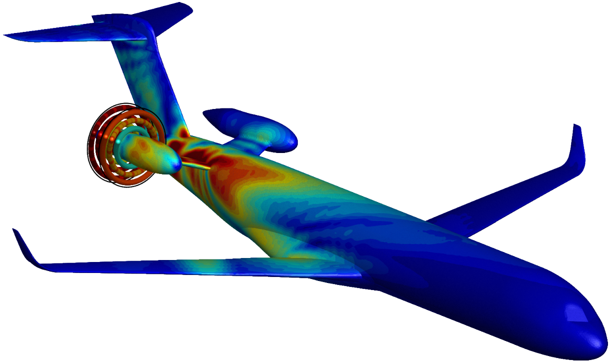

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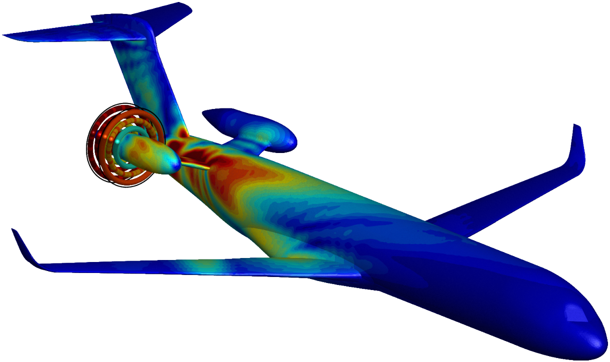
- Combination mode $B_F + B_R$ – Frequency 308 Hz
- Different diffraction pattern for left/right installed CROR by interaction of near field with geometry
- Can not be obtained by Ray-Tracing!

>



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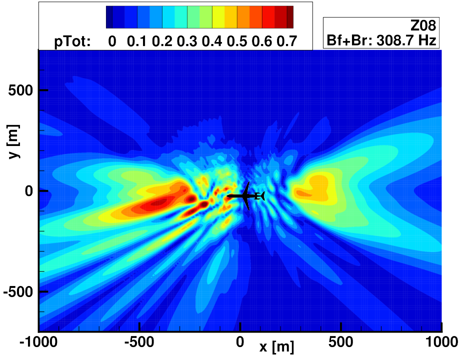
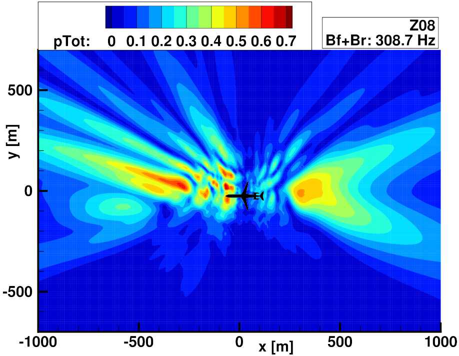


(l) Left installed CROR



(m) Right installed CROR

Summary and Outlook

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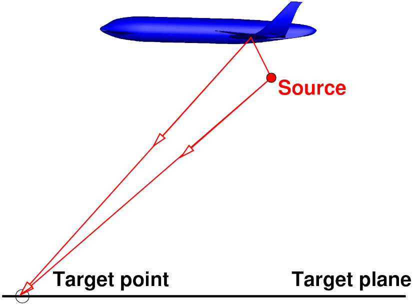
    - Formulated for point sources so far

      >

Thank you for your attention!

Shooting to Target Point

Determination initial conditions of rays to a target point



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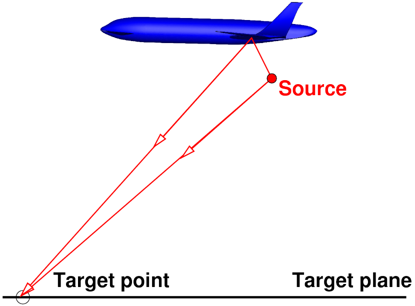

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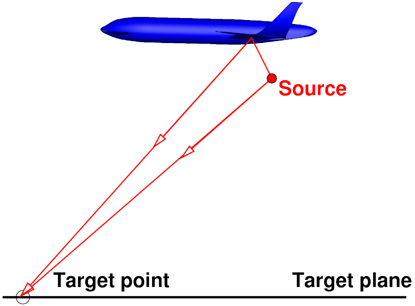

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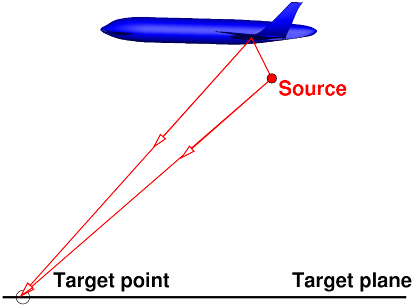

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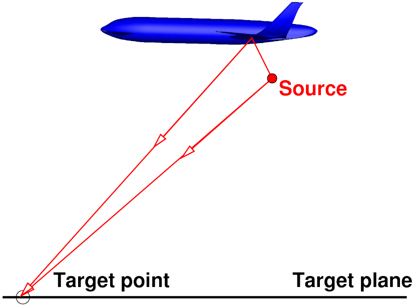

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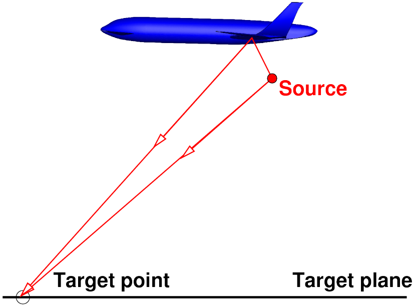

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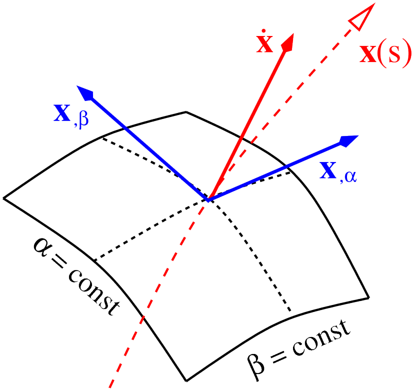
Jacobian of Rays



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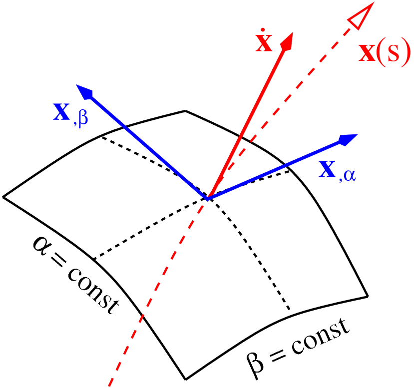

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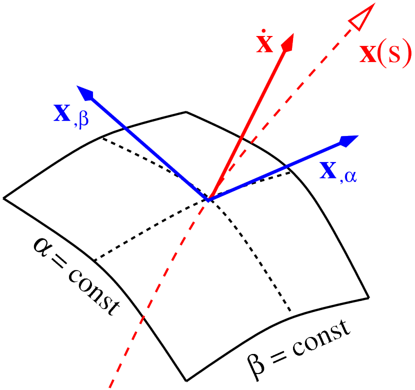

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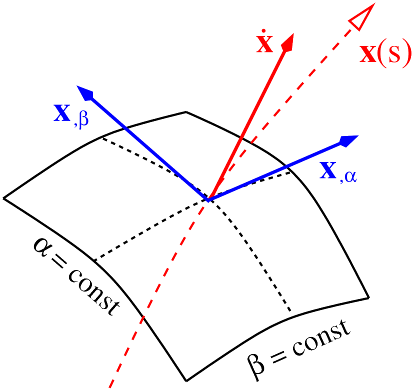

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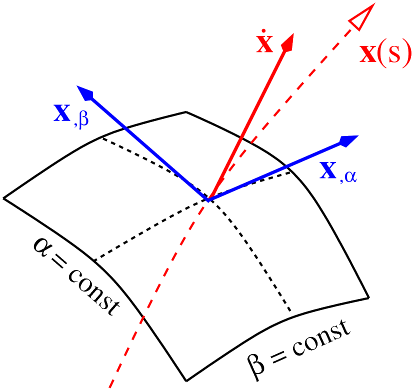

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Diffraction Correction

Kirchhoff diffraction for aperture/obstacle

- Kirchhoff diffraction: Calculate diffracted field by area integral over incident field in aperture

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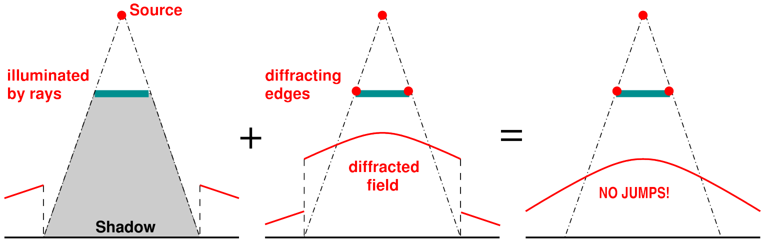
- 

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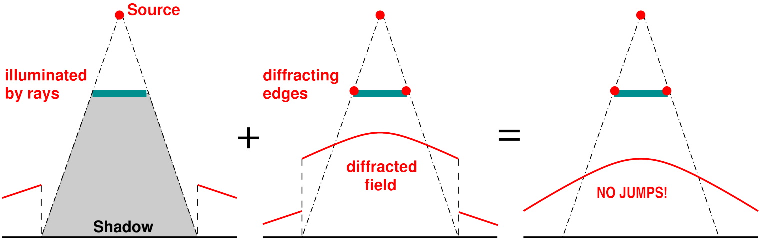
- 

- General geometries: Perform line integral along the shadow boundaries on the body ▸ Shadow Boundary

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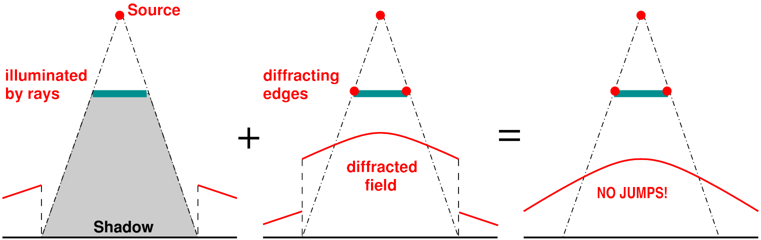
- 

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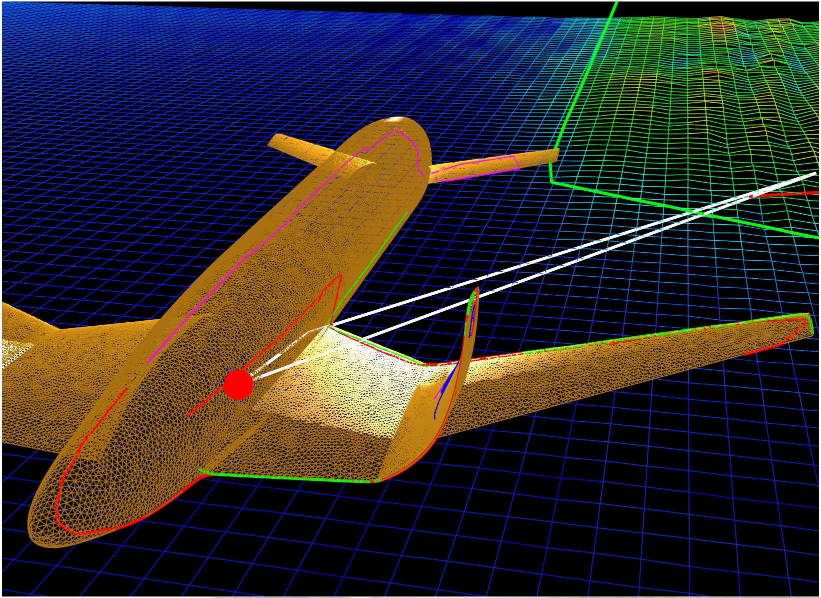
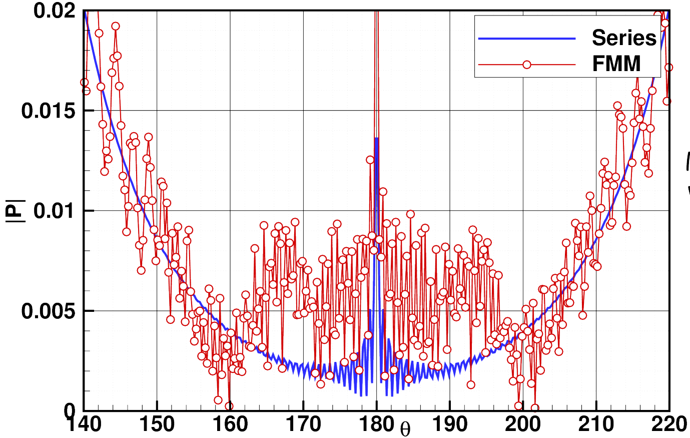
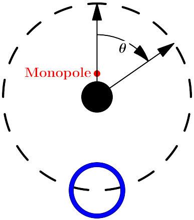
Shadow Boundaries



Return

Shadow Region – 2.7M – 6 PPW





- 6 Elements per wavelength

Return

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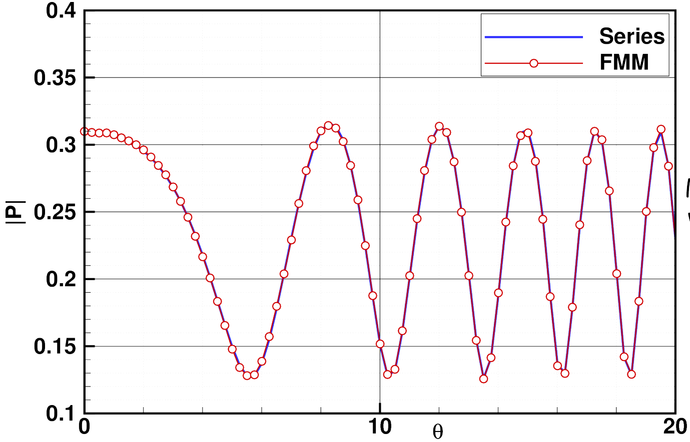

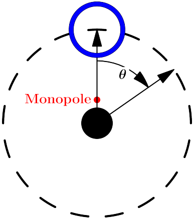

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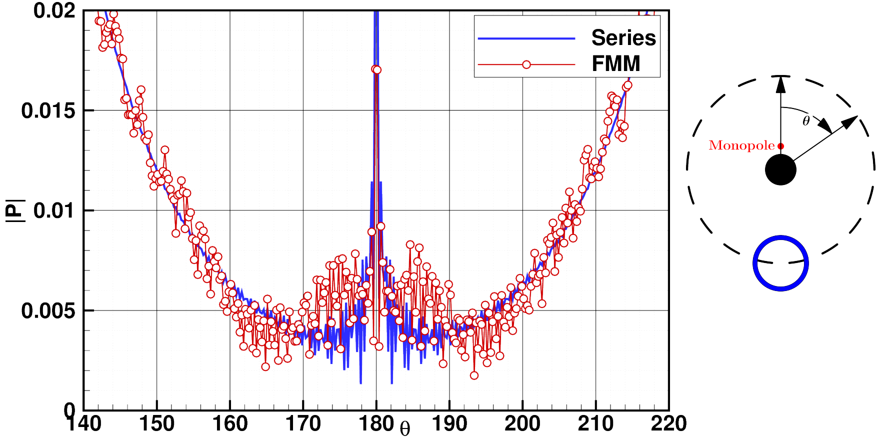
Shadow Region – 2.7M – 9 PPW



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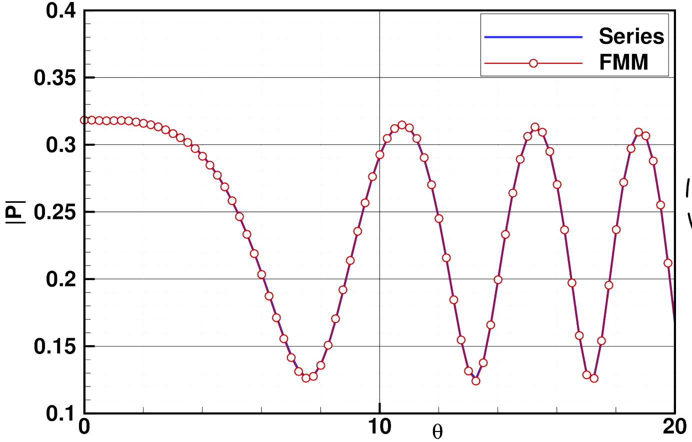

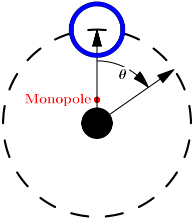

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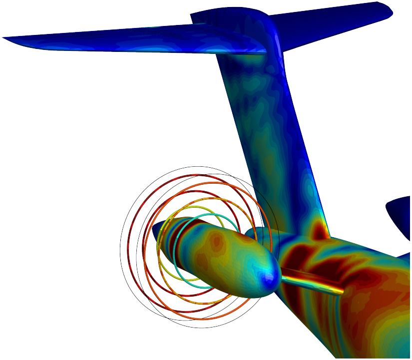
CROR Model

DLR-CROR where unsteady CFD-data are easily available



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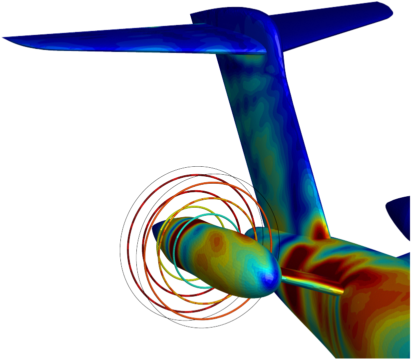

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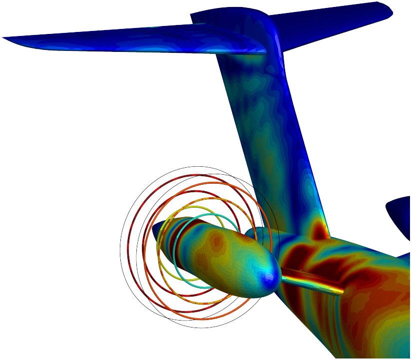
CROR Model

DLR-CROR where unsteady CFD-data are easily available



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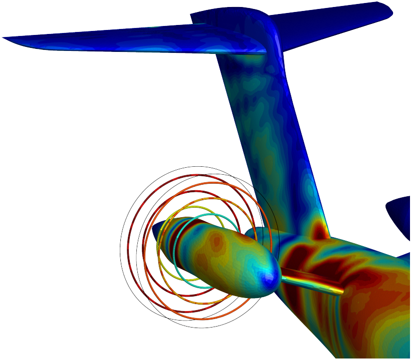

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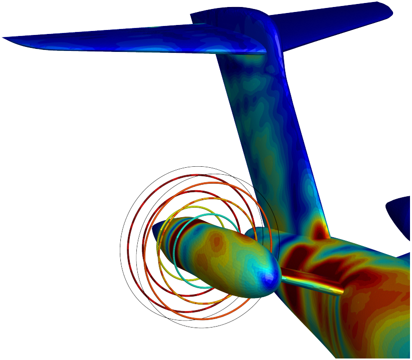

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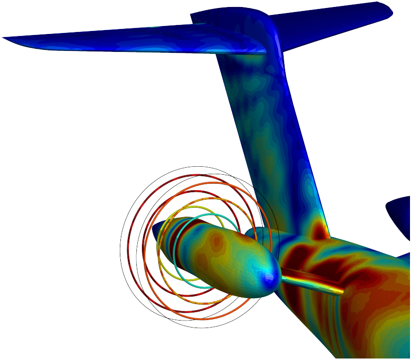

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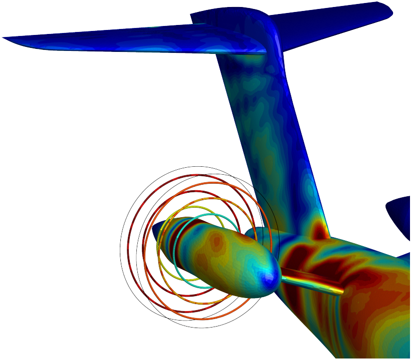

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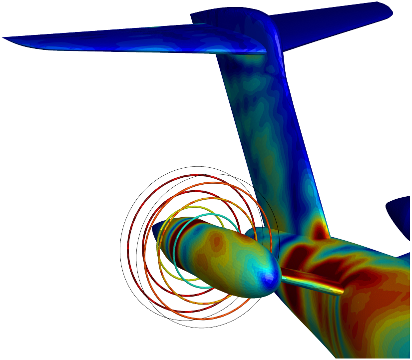

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Taylor Transformation

- Low Mach number potential mean flow field
- Convected wave equation for acoustic velocity potential
- Calculation of velocity potential from CFD data necessary

Taylor Transformation

Taylor (1978), Agarwal & Dowling (2007)

- Convected wave equation (Mean flow potential $\boldsymbol{u} = \nabla\overline{\Phi}$)

$$\left(\frac{\partial}{\partial t} + \nabla\overline{\Phi}\cdot\nabla\right)^2 \phi_g - c^2\nabla^2\phi_g = -\delta(\boldsymbol{x}-\boldsymbol{x}_0)e^{-i\omega t}$$

- Acoustic velocity potential $\phi_g(\boldsymbol{x},t)$

$$\phi_g(\boldsymbol{x},t) = \hat{\Phi}_h(\boldsymbol{x},\omega)e^{ik\frac{\overline{\Phi}(\boldsymbol{x}_0)-\overline{\Phi}(\boldsymbol{x})}{c}}e^{-i\omega t}$$

$$(\omega^2 + c^2\nabla^2)\hat{\Phi}_h = -\delta(\boldsymbol{x}-\boldsymbol{x}_0), \qquad \boldsymbol{n}\cdot\nabla\hat{\Phi}_h = 0$$

- Acoustic pressure

$$p(\boldsymbol{x},t) = c\overline{\varrho}\left(ik\hat{\Phi}_h - \frac{\nabla\overline{\Phi}}{c}\cdot\nabla\hat{\Phi}_h\right)e^{ik\frac{\overline{\Phi}(\boldsymbol{x}_0)-\overline{\Phi}(\boldsymbol{x})}{c}}e^{-i\omega t}$$

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