

DESIGNS OF LOW DELAY COSINE MODULATED FILTER BANKS AND SUBBAND AMPLIFIERS

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ABSTRACT

This paper proposes a design of a low delay cosine modulated filter bank and subband amplifier coefficients for digital audio hearing aids denoising applications. The objective of the design is to minimize the delay of the filter bank. Specifications on the maximum magnitude of both the real and the imaginary parts of the transfer function distortion and the aliasing distortion of the filter bank are imposed. Also, the constraint on the maximum absolute difference between the desirable magnitude square response and the designed magnitude square response of the prototype filter over both the passband and the stopband is considered. The subband amplifier coefficients are designed based on a least squares training approach. The average mean square errors between the noisy samples and the clean samples is minimized. Computer numerical simulation results show that our proposed approach could significantly improve the signal-to-noise ratio of digital audio hearing aids.

1. INTRODUCTION

There is a huge demand and market for digital audio hearing aids. The net audio hearing instrument sales in the United States in 2004 have surpassed a two million unit mark [1] in which the sales of digital audio hearing instruments rose to constitute four-fifths (83%) of the market. More than 90% of the gross revenues came from digital audio hearing aids.

Cosine modulated filter banks are widely used in digital audio signal processing because digital signals can be decomposed into different frequency bands via a prototype filter. However, as humans are highly intelligent, the perfect reconstruction of the filter banks is not essential. Instead, humans are more sensitive to the aliasing distortions of the filter banks, delays of the signals and noises [2]. Hence, low delay filter banks with low maximum aliasing distortions serving denoising purposes are preferred. Nevertheless, most

of existing cosine modulated filter bank designs are based on the perfect reconstruction condition. Only few results found in the literature have addressed the above issues [2].

To tackle these issues, this paper proposes a design a cosine modulated filter bank with subband amplifier coefficients. The design of the prototype filter of the cosine modulated filter bank is formulated as an optimization problem without considering the subband amplifier coefficients. The optimization problem is to minimize the delay of the filter bank subject to specifications on the maximum magnitude of both the real and the imaginary parts of the transfer function distortion and the aliasing distortion of the filter bank as well as on the maximum absolute difference between the desirable magnitude square response and the designed magnitude square response of the prototype filter. The subband amplifiers are designed based on a least squares training approach. The average mean square errors between the noisy samples and the clean samples is minimized. Computer numerical simulation results show that our proposed approach could significantly improve the signal-to-noise ratio of digital audio hearing aids.

The outline of this paper is as follows. In Section 2, the design of the prototype filter of a cosine modulated filter bank is formulated as a nonsmooth functional inequality constrained optimization problem. In Section 3, the subband amplifier coefficients are designed based on a least squares training approach. Computer numerical simulation results are presented in Section 4. Finally, conclusions are drawn in Section 5.

2. COSINE MODULATED FILTER BANK DESIGN

In this Section, we consider a design of a uniform maximally decimated cosine modulated finite impulse response filter bank.

Let

$$\mathbf{x} \equiv [p(0) \ \dots \ p(N-1)]^T$$

be the vector of the prototype filter coefficients, where the superscript T , N and $p(n)$ for $n=0, \dots, N-1$ denote, respectively, the transposition operator, the length and the impulse response of the prototype filter of the cosine modulated filter bank. The magnitude square response of the prototype filter can be given as

$$|P(\omega)|^2 = \mathbf{x}^T (\mathbf{u}_c(\omega) \mathbf{u}_c^T(\omega) + \mathbf{u}_s(\omega) \mathbf{u}_s^T(\omega)) \mathbf{x},$$

where

$$\mathbf{u}_s(\omega) \equiv [0 \quad \sin \omega \quad \dots \quad \sin((N-1)\omega)]^T$$

and

$$\mathbf{u}_c(\omega) \equiv [1 \quad \cos \omega \quad \dots \quad \cos((N-1)\omega)]^T$$

denote the frequency response kernels of the prototype filter. Let \mathcal{B}_p and \mathcal{B}_s be the desirable passband and the desirable stopband of the prototype filter, respectively. Also, let $D(\omega)$ and δ_f be the desirable magnitude square response and the specification on the maximum absolute difference between the desirable magnitude square response and the designed magnitude square response of the prototype filter over both the passband and the stopband, respectively. Then, the magnitude square response of the prototype filter is constrained to

$$|\mathbf{x}^T (\mathbf{u}_c(\omega) \mathbf{u}_c^T(\omega) + \mathbf{u}_s(\omega) \mathbf{u}_s^T(\omega)) \mathbf{x} - D(\omega)| \leq \delta_f \quad \forall \omega \in \mathcal{B}_p \cup \mathcal{B}_s.$$

Let M be the total number of channels of the cosine modulated filter bank. Denote by $h_m(n)$ and $f_m(n)$ for $m=1, \dots, M$ and $n=0, \dots, N-1$ the impulse responses of the m^{th} analysis filter and the m^{th} synthesis filter of the filter bank, respectively. Then, we have

$$h_m(n) = 2p(n) \cos\left(\frac{\pi}{M} \left(m - \frac{1}{2}\right) \left(n - \frac{N}{2}\right) + (-1)^{m-1} \frac{\pi}{4}\right)$$

and

$$f_m(n) = 2p(n) \cos\left(\frac{\pi}{M} \left(m - \frac{1}{2}\right) \left(n - \frac{N}{2}\right) - (-1)^{m-1} \frac{\pi}{4}\right)$$

for $m=1, \dots, M$ and $n=0, \dots, N-1$. Let

$$\omega_m \equiv \frac{\pi}{M} \left(m - \frac{1}{2}\right) \quad \text{for } m=1, \dots, M$$

be the modulated frequency of the m^{th} analysis filter and the m^{th} synthesis filter. Also, let

$$\theta_m \equiv (-1)^{m-1} \frac{\pi}{4} - \frac{\pi N}{2M} \left(m - \frac{1}{2}\right)$$

and

$$\phi_m \equiv (-1)^{m-1} \frac{\pi}{4} - \frac{\pi N}{2M} \left(m - \frac{1}{2}\right)$$

for $m=1, \dots, M$ be and the phase shift of the m^{th} analysis filter and the m^{th} synthesis filter, respectively. Denote by $H_m(\omega)$ and $F_m(\omega)$ for $m=1, \dots, M$ the frequency responses of the m^{th} analysis filter and the m^{th} synthesis filter of the filter bank, respectively. Then, we have

$$H_m(\omega) = \cos \theta_m (P(\omega - \omega_m) + P(\omega + \omega_m)) \\ + j \sin \theta_m (P(\omega - \omega_m) - P(\omega + \omega_m))$$

and

$$F_m(\omega) = \cos \phi_m (P(\omega - \omega_m) + P(\omega + \omega_m)) \\ + j \sin \phi_m (P(\omega - \omega_m) - P(\omega + \omega_m))$$

for $m=1, \dots, M$. Let c and $\tau(\omega)$ be the desirable gain and the desirable delay of the filter bank. Also, let δ_t^R and δ_t^I be the bounds imposed on the real and the imaginary parts of the transfer function distortion of the filter bank. Similarly, let δ_a^R and δ_a^I be the bounds imposed on the real and the imaginary parts of the aliasing distortion of the filter bank. Then, we have

$$\left| \operatorname{Re} \left(\frac{1}{M} \sum_{m=1}^M H_m(\omega) F_m(\omega) \right) - c \cos(\tau\omega) \right| \leq \delta_t^R,$$

$$\left| \operatorname{Im} \left(\frac{1}{M} \sum_{m=1}^M H_m(\omega) F_m(\omega) \right) + c \sin(\tau\omega) \right| \leq \delta_t^I,$$

$$\left| \operatorname{Re} \left(\frac{1}{M} \sum_{m=1}^M H_m \left(\omega - \frac{2\pi k}{M} \right) F_m(\omega) \right) \right| \leq \delta_a^R$$

and

$$\left| \operatorname{Im} \left(\frac{1}{M} \sum_{m=1}^M H_m \left(\omega - \frac{2\pi k}{M} \right) F_m(\omega) \right) \right| \leq \delta_a^I$$

for $k=1, \dots, M-1$. Let

$$\mathbf{H}_m^R(\omega) \equiv \cos \theta_m (\mathbf{u}_c(\omega - \omega_m) + \mathbf{u}_c(\omega + \omega_m)) \\ + \sin \theta_m (\mathbf{u}_s(\omega - \omega_m) - \mathbf{u}_s(\omega + \omega_m)),$$

$$\mathbf{H}_m^I(\omega) \equiv -\cos \theta_m (\mathbf{u}_s(\omega - \omega_m) + \mathbf{u}_s(\omega + \omega_m)) \\ + \sin \theta_m (\mathbf{u}_c(\omega - \omega_m) - \mathbf{u}_c(\omega + \omega_m)),$$

$$\mathbf{F}_m^R(\omega) \equiv \cos \phi_m (\mathbf{u}_c(\omega - \omega_m) + \mathbf{u}_c(\omega + \omega_m)) \\ + \sin \phi_m (\mathbf{u}_s(\omega - \omega_m) - \mathbf{u}_s(\omega + \omega_m))$$

and

$$\mathbf{F}_m^I(\omega) \equiv -\cos \phi_m (\mathbf{u}_s(\omega - \omega_m) + \mathbf{u}_s(\omega + \omega_m)) \\ + \sin \phi_m (\mathbf{u}_c(\omega - \omega_m) - \mathbf{u}_c(\omega + \omega_m))$$

for $m=1, \dots, M$. Since

$$P(\omega) = \mathbf{u}_c^T(\omega) \mathbf{x} - j \mathbf{u}_s^T(\omega) \mathbf{x},$$

we have

$$H_m(\omega) = (\mathbf{H}_m^R(\omega))^T \mathbf{x} + j (\mathbf{H}_m^I(\omega))^T \mathbf{x}$$

and

$$F_m(\omega) = (\mathbf{F}_m^R(\omega))^T \mathbf{x} + j (\mathbf{F}_m^I(\omega))^T \mathbf{x}$$

for $m=1, \dots, M$. Let

$$\mathbf{A}_0^R(\omega) \equiv \frac{1}{M} \sum_{m=1}^M \mathbf{H}_m^R(\omega) (\mathbf{F}_m^R(\omega))^T - \frac{1}{M} \sum_{m=1}^M \mathbf{H}_m^I(\omega) (\mathbf{F}_m^I(\omega))^T,$$

$$\mathbf{A}_0^I(\omega) \equiv \frac{1}{M} \sum_{m=1}^M \mathbf{H}_m^R(\omega) (\mathbf{F}_m^I(\omega))^T + \frac{1}{M} \sum_{m=1}^M \mathbf{H}_m^I(\omega) (\mathbf{F}_m^R(\omega))^T,$$

$$\mathbf{A}_k^R(\omega) \equiv \frac{1}{M} \sum_{m=1}^M \mathbf{H}_m^R \left(\omega - \frac{2\pi k}{M} \right) (\mathbf{F}_m^R(\omega))^T \\ - \frac{1}{M} \sum_{m=1}^M \mathbf{H}_m^I \left(\omega - \frac{2\pi k}{M} \right) (\mathbf{F}_m^I(\omega))^T$$

and

$$\mathbf{A}_k^I(\omega) \equiv \frac{1}{M} \sum_{m=1}^M \mathbf{H}_m^R \left(\omega - \frac{2\pi k}{M} \right) (\mathbf{F}_m^I(\omega))^T + \frac{1}{M} \sum_{m=1}^M \mathbf{H}_m^I \left(\omega - \frac{2\pi k}{M} \right) (\mathbf{F}_m^R(\omega))^T$$

for $k=1, \dots, M-1$. Then, the constraints on both the real and the imaginary parts of the transfer function distortion and the aliasing distortion of the filter bank can be expressed as follows:

$$\begin{aligned} |\mathbf{x}^T \mathbf{A}_0^R(\omega) \mathbf{x} - c \cos(\tau\omega)| &\leq \delta_t^R, \\ |\mathbf{x}^T \mathbf{A}_0^I(\omega) \mathbf{x} + c \sin(\tau\omega)| &\leq \delta_t^I, \\ |\mathbf{x}^T \mathbf{A}_k^R(\omega) \mathbf{x}| &\leq \delta_a^R \end{aligned}$$

and

$$|\mathbf{x}^T \mathbf{A}_k^I(\omega) \mathbf{x}| \leq \delta_a^I$$

for $k=1, \dots, M-1$. Combining the constraint on the magnitude square response of the prototype filter, the cosine modulated filter bank design problem with low delay can be formulated as the following optimization problem:

Problem (P)

$$\min_{\mathbf{x}} J(\mathbf{x}) = \tau$$

$$\text{subject to } g_0(\omega, \mathbf{x}) \equiv |\mathbf{x}^T \mathbf{A}_0^R(\omega) \mathbf{x} - c \cos(\tau\omega)| \leq \delta_t^R$$

$$\forall \omega \in [-\pi, \pi],$$

$$g_1(\omega, \mathbf{x}) \equiv |\mathbf{x}^T \mathbf{A}_0^I(\omega) \mathbf{x} + c \sin(\tau\omega)| \leq \delta_t^I$$

$$\forall \omega \in [-\pi, \pi],$$

$$g_{2,k}(\omega, \mathbf{x}) \equiv |\mathbf{x}^T \mathbf{A}_k^R(\omega) \mathbf{x}| \leq \delta_a^R$$

$$\forall \omega \in [-\pi, \pi] \text{ and } k=1, \dots, M-1,$$

$$g_{3,k}(\omega, \mathbf{x}) \equiv |\mathbf{x}^T \mathbf{A}_k^I(\omega) \mathbf{x}| \leq \delta_a^I$$

$$\forall \omega \in [-\pi, \pi] \text{ and } k=1, \dots, M-1$$

$$\text{and } g_4(\omega, \mathbf{x}) \equiv |\mathbf{x}^T (\mathbf{u}_c(\omega) \mathbf{u}_c^T(\omega) + \mathbf{u}_s(\omega) \mathbf{u}_s^T(\omega)) \mathbf{x} - D(\omega)| \leq \delta_f$$

$$\forall \omega \in \mathcal{B}_p \cup \mathcal{B}_s,$$

where $J(\mathbf{x})$ is the objective function, and $g_0(\mathbf{x}, \omega)$, $g_1(\mathbf{x}, \omega)$, $g_4(\mathbf{x}, \omega)$, $g_{2,k}(\mathbf{x}, \omega)$ and $g_{3,k}(\mathbf{x}, \omega)$ for $k=1, \dots, M-1$ are the nonsmooth functional inequality constraints. This nonsmooth optimization problem can be approximated by a smooth optimization problem using our recently proposed method [3] and the global optimal solution of the corresponding nonconvex optimization problem can be found using our recently proposed modified filled function method [4].

3. OPTIMAL SUBBAND AMPLIFIER COEFFICIENTS DESIGN

In this Section, subband amplifier coefficients are designed based on the prototype filter obtained in Section 2. The block diagram of the proposed digital audio hearing aids is shown in Figure 1. Assume that there are K training signals and they are processed by unknown operators and corrupted by noises with unknown distributions. In this case, conventional maximally likelihood estimation approaches,

minimum variance unbiased estimation approaches and Bayesian estimation approaches are not applied for solving the problem. Here, the problem formulation is based on a least squares approach. Let $u_q(n)$ and $y_q(n)$ for $q=0,1, \dots, K-1$ be the unprocessed and clean training signals as well as the corresponding processed and noisy signals, respectively. Assume that $u_q(n)$ for $q=0,1, \dots, K-1$ are finite impulse response signals. Suppose that the length of all the training signals are the same and equal to T . Let

$$\mathbf{u}_q \equiv [u_q(0) \ \dots \ u_q(T-1)]^T \text{ for } q=0,1, \dots, K-1.$$

Also, let w_m for $m=1, \dots, M$ be the m^{th} subband amplifier coefficient. Let

$$\mathbf{w} \equiv [w_1 \ \dots \ w_M]^T$$

and

$$s_{m,q}(n) \equiv \sum_{d=0}^{\text{ceil}\left(\frac{T+N-1}{M}\right)-1} \sum_{k=0}^{T-1} y_q(k) h_m(dM-k) f_m(n-dM)$$

for $m=1, \dots, M$ and $q=0,1, \dots, K-1$, where $\text{ceil}(\cdot)$ denotes the rounding operator towards plus infinity. Define

$$L = N-1 + M \text{ceil}\left(\frac{T+N-1}{M}\right).$$

Then, L is the length of $s_{m,q}(n)$ for $m=1, \dots, M$ and $q=0,1, \dots, K-1$. Let

$$\mathbf{s}_{m,q} \equiv [s_{m,q}(0) \ \dots \ s_{m,q}(L-1)]^T$$

for $m=1, \dots, M$ and $q=0,1, \dots, K-1$,

$$\mathbf{S}_q \equiv [\mathbf{s}_{1,q} \ \dots \ \mathbf{s}_{M,q}] \text{ for } q=0,1, \dots, K-1$$

and

$$\hat{\mathbf{y}}_q \equiv \mathbf{S}_q \mathbf{w} \text{ for } q=0,1, \dots, K-1.$$

Define

$$\hat{\mathbf{I}} \equiv \begin{bmatrix} \mathbf{0}_{N' \times (N-1)} & \mathbf{I}_{N' \times N'} & \mathbf{0}_{N' \times (L-N-N'+1)} \end{bmatrix},$$

where $\mathbf{0}_{a \times b}$ is the $a \times b$ zero matrix and $\mathbf{I}_{a \times a}$ is the $a \times a$ identity matrix. Thus, $\hat{\mathbf{I}} \mathbf{S}_q \mathbf{w}$ for $q=0,1, \dots, K-1$ is the vector containing the response of the q^{th} reconstructed signal. The total least squares error introduced by the system becomes

$$e \equiv \sum_{q=0}^{K-1} \left\| \hat{\mathbf{I}} \mathbf{S}_q \mathbf{w} - \mathbf{u}_q \right\|^2.$$

The optimal subband amplifier coefficients are

$$\mathbf{w}^* \equiv \left(\sum_{q=0}^{K-1} \mathbf{S}_q^T \hat{\mathbf{I}}^T \hat{\mathbf{I}} \mathbf{S}_q \right)^{-1} \left(\sum_{q=0}^{K-1} \mathbf{S}_q^T \hat{\mathbf{I}}^T \mathbf{u}_q \right).$$

4. COMPUTER NUMERICAL SIMULATION RESULTS

Since humans are very sensitive to the delay of the filter bank, the length of the prototype filter should be short. However, a prototype filter with short length cannot achieve good performances of the cosine modulated filter bank, such as low values on δ_f , δ_t^R , δ_t^I , δ_a^R and δ_a^I . Similarly, large value of M can decompose the input signal

into very narrow frequency bands, but the transition bandwidth of the prototype filter will become very narrow. In general, there are tradeoffs between the specifications on Δ , N , M , δ_f , δ_i^R , δ_i^I , δ_a^R and δ_a^I , where Δ is defined as the transition bandwidth of the prototype filter. In this paper,

$$\begin{aligned} \Delta &= 0.1\pi, \\ N &= 16, \\ M &= 4, \\ \delta_f &= -25 \text{ dB for the stopband,} \end{aligned}$$

$$\delta_i^R = -11 \text{ dB,}$$

$$\delta_i^I = -10.5 \text{ dB,}$$

$$\delta_a^R = -14.5 \text{ dB}$$

and

$$\delta_a^I = -14.5 \text{ dB}$$

are chosen because these values are commonly used in the signal processing community [2]. Without the loss of generality, the gain of the filter bank is set to one, i.e., $c = 1$. Based on the above specifications, the problem formulation discussed in Section 2 and applying our recently developed methods discussed in [3] and [4], a prototype filter is obtained. Figure 2 shows both the real and the imaginary parts of the transfer function distortion of the filter banks designed by our proposed approach and the approach discussed in [5]. Similarly, Figure 3 shows both the real and the imaginary parts of the aliasing distortion of the filter banks designed by our proposed approach and the approach discussed in [5]. Finally, Figure 4 shows the absolute difference between the desirable magnitude square response and the designed magnitude square response of the prototype filter over both the passband and the stopband designed by our proposed approach and the approach discussed in [5]. Table 1 shows the performances of the cosine modulated filter banks designed by our proposed approach and the approach discussed in [5].

Our proposed design (dB)	Designed discussed in [5] (dB)	Design Specification (dB)
$\delta_i^R = -11.0780$	$\delta_i^R = -11.0780$	$\delta_i^R = -11$
$\delta_i^I = -10.6506$	$\delta_i^I = -10.1503$	$\delta_i^I = -10.5$
$\delta_a^R = -14.8234$	$\delta_a^R = -14.8234$	$\delta_a^R = -14.5$
$\delta_a^I = -14.7051$	$\delta_a^I = -13.9948$	$\delta_a^I = -14.5$
$\delta_f = -25.4137$	$\delta_f = -25.4137$	$\delta_f = -25$ for the stopband

Table 1. Performances of the cosine modulated filter banks designed by our proposed approach and the approach discussed in [5].

It can be seen from the above that the our proposed design satisfies the required specifications, while the design discussed in [5] does not satisfy the specifications on the imaginary part of both the transfer function distortion and the aliasing distortion of the filter bank. In general, if the feasible set of the optimization problem is nonempty, our proposed design can guarantee to find the solution satisfying all the constraints. Also, the delay of the filter bank designed by our proposed approach is 15.39726, which is

lower than that by the approach discussed in [5]. Thus, our proposed design has a slightly improvement on the delay of the filter bank.

To illustrate how to apply our results to a practical denoising application in digital audio hearing aids, we record 12 sets of utterances of English letters which each set of utterances of English letters contains 4 samples. Hence,

$$K = 48.$$

Since difference utterance samples have different lengths, we pack zeros at the end of the utterance samples so that they have the same length. Each utterance sample is normalized to the unit energy. An additive white Gaussian noise is added to each sample so that the signal-to-noise ratio is 5 dB. Figure 5 plots the mean square errors of the corrupted signals and the signals reconstructed by our designed filter bank. It can be seen from the figure that the average mean square errors of the signals reconstructed by our designed filter bank is -56 dB, while that of the corrupted signals is -53.3105 dB. Obviously, our proposed filter bank can significantly improve the signal-to-noise ratio of the system.

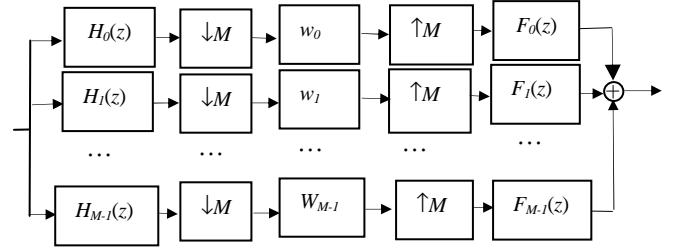


Figure 1. Block diagram of our proposed digital audio hearing aids.

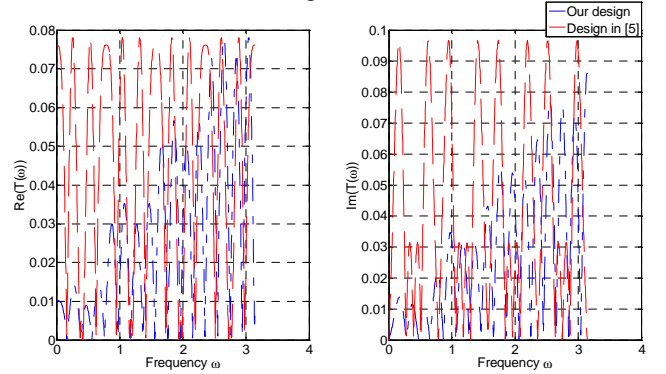


Figure 2. The real and the imaginary parts of the transfer function distortion of the filter banks designed by our proposed approach and the approach discussed in [5].

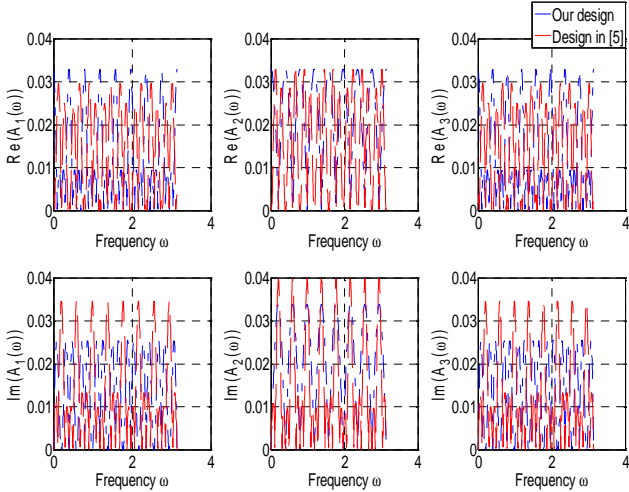


Figure 3. The real and the imaginary parts of the aliasing distortion of the filter banks designed by our proposed approach and the approach discussed in [5].

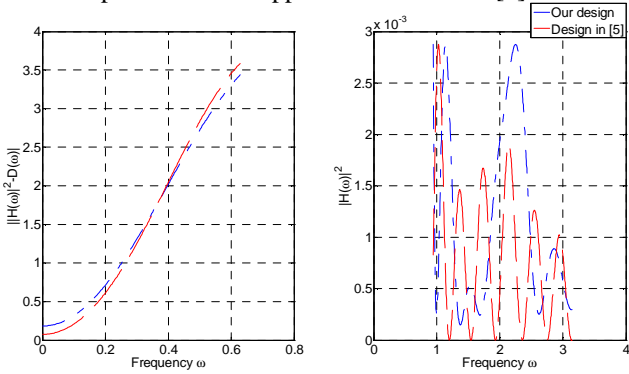


Figure 4. The absolute difference between the desirable magnitude square response and the designed magnitude square response of the prototype filter over both the passband and the stopband designed by our proposed approach and the approach discussed in [5].

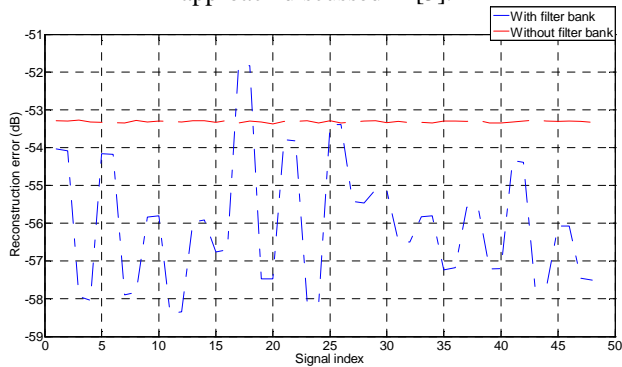


Figure 5. The mean square errors of the corrupted signals and the signals reconstructed by our designed filter bank.

5. CONCLUSIONS

This paper proposes a design of a low delay cosine modulated filter bank and subband amplifier coefficients for digital audio hearing aids denoising applications. The prototype filter is designed such that the delay of the filter bank is minimized subject to specifications on the maximum magnitude of both the real and the imaginary parts of the transfer

function distortion and the aliasing distortion of the filter bank as well as on the maximum absolute difference between the desirable magnitude square response and the designed magnitude square response of the prototype filter. The design problem is a nonsmooth functional inequality constraint optimization problem. The problem is approximated by a smooth optimization problem using our recently proposed method and the global optimal solution of the corresponding nonconvex optimization problem is found using our recently proposed modified filled function method. The subband amplifier coefficients are designed based on a least squares training approach. An analytical formulae is derived. Computer numerical simulation results show that our proposed filter bank can significantly improve the signal-to-noise ratio of the system and hence it is very useful for digital audio hearing aids denoising applications.

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