Electron States and Quasienergy Spectrum of the Graphene Exposed to the Electromagnetic Wave

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The effective spectrum and wave functions of electron states in graphene under the electromagnetic wave with circular polarization are found. The band gap in the graphene spectrum induced by the electromagnetic wave is calculated. Occurrence of such gap is shown to have the nature of the parametric resonance. The effective energy of the electron state in graphene is proved to be the energy averaged over the period of the electromagnetic wave.

1. INTRODUCTION

The investigations of electronic and optical properties of graphene are very intensive last time [1-8]. The relativistic character of the graphene electron spectrum leads to the unusual electrodynamical properties of the graphene based structures which are presently studied both experimentally and theoretically [9-17]. In [11, 12] the high-frequency conductivity of the graphene is calculated. The nonlinear electromagnetic (EM) response of the graphene is studied in [13-15] where the frequency multiplication effects were predicted. The possibility of the propagation of the solitary EM wave in the graphene superlattice was shown in [16, 17]. In [18-23] the dynamical modification of the graphene band structure under the high-frequency EM radiation was investigated.

The nonlinear electrodinamical properties of graphene open new opportunities in optoelectronic engineering. The possible applications of the predicted optical effects of this material were discussed in [14, 23-26]. The graphene dispersion law can be modified by the action of the electromagnetic wave. This dynamical modification can leads to new optical features of graphene such as induced selftransparency [28]. Below we suggest the graphene exposed to the electromagnetic wave with circular polarization and find the wave functions and the effective energy of electron in this case using the Klein-Gordon-Fock equation.

2. DYNAMICAL OCCURRENCE OF THE GAPS IN THE GRAPHENE SPECTRUM

The circularly polarized EM radiation is supposed to fall on the graphene plane (Fig. 1). The electron spectrum of graphene near the so-called Dirac point has the form [1-5]:

$$\varepsilon(p) = \upsilon_{\mathrm{F}} p, \qquad (1)$$

where υ_{F} is the velocity on the Fermi surface. The vector potential of EM wave has the harmonic form:

$$\mathbf{A}(t) = \frac{cE}{\omega}\left\{-\sin\omega t, \ \cos\omega t\right\}, \qquad (2)$$

where E and ω are the intensity and the frequensy of the EM wave correspondingly. To find the wave function of the electron state in the graphene we use the Dirac equation [6]:

$$i\hbar\frac{\partial\psi}{\partial t} = \upsilon_{\mathrm{F}}\hat{\boldsymbol{\sigma}}\cdot\left(\hat{\mathbf{p}} + \frac{e}{c}\mathbf{A}\right)\psi, \qquad (3)$$

where $\hat{\boldsymbol{\sigma}} = (\hat{\sigma}_x, \hat{\sigma}_y)$ are Pauli matrixes, $\hat{\mathbf{p}} = -i\hbar\nabla$, υ_{F} is the velocity on the Fermi surface, ε – electron energy, ψ is the spinor function:

$$\psi = \begin{pmatrix} \psi_1(\mathbf{r},t) \\ \psi_2(\mathbf{r},t) \end{pmatrix}, \qquad (4)$$

functions $\psi_1(\mathbf{r},t)$ and $\psi_2(\mathbf{r},t)$ describe the electron states in two nondependent crystal sublattices, of graphene [1-4]. From (3) we derive:

$$-\frac{\hbar^2}{\upsilon_{\mathrm{F}}^2}\frac{\partial^2\psi_1}{\partial t^2} = \left(-i\hbar\nabla + \frac{e}{c}\mathbf{A}(t)\right)^2\psi_1. \qquad (5)$$

The equation for the function ψ_2 has the same form as that of the function ψ_1. The solution of (5) is found in the next form:

$$\psi_1(\mathbf{r},t) = \chi(t)e^{\frac{i\mathbf{p}\cdot\mathbf{r}}{\hbar}}. \qquad (6)$$

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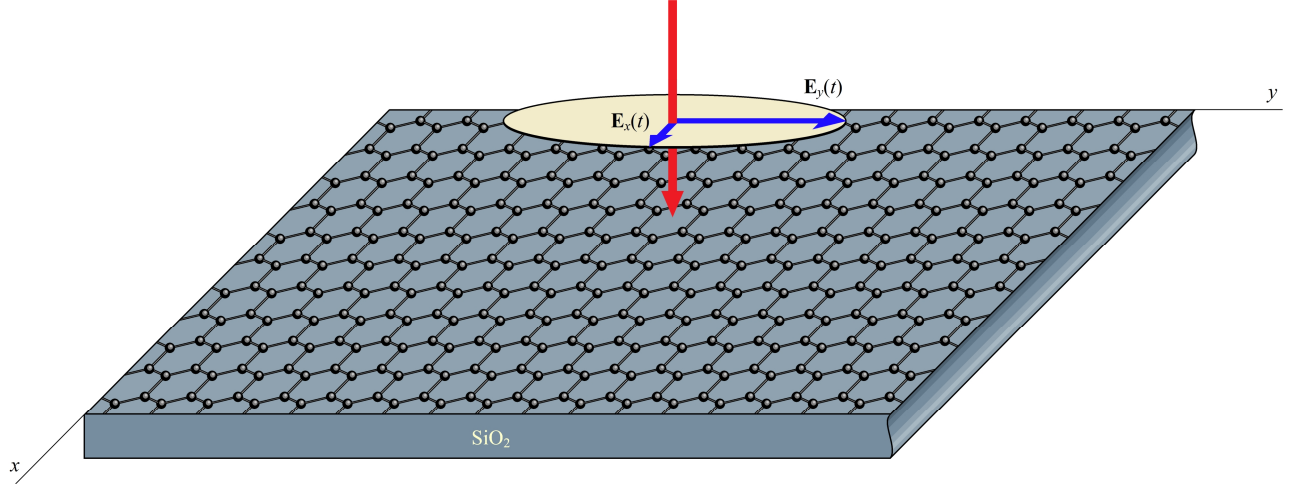† eikuhar@yandex.ru
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Fig. 1 – Schematic of the process

Substituting (6) in (5) we receive:

$$-\frac{\mathrm{d}^2\chi}{\mathrm{d}t^2} + \frac{\omega^2 q}{2}\chi(t)\sin(\omega t - \theta) = \Omega_0^2 \chi(t) \qquad (7)$$

where $\theta = \mathrm{arctg}\left(p_y/p_x\right)$, the parameters Ω_0 and q are defined by the formulas:

$$\hbar^2\Omega_0^2(p) = \upsilon_{\mathrm{F}}^2\left(p_E^2 + p^2\right), \quad q = \frac{4\upsilon_{\mathrm{F}}^2 p_E p}{\hbar^2\omega^2}, \quad p_E = \frac{eE}{\omega}. \qquad (8)$$

We introduce a new variables: $2\xi = \omega t - \theta - \pi/2$, $\Gamma = 4\Omega_0^2/\omega^2$. Then instead of (7) we derive the Mathieu equation [28]:

$$-\frac{\mathrm{d}^2\chi}{\mathrm{d}\xi^2} + 2q\chi\cos 2\xi = \Gamma\chi, \qquad (9)$$

The solution of (9) is the function satisfying the Floquet-Bloch theorem:

$$\chi(t) = P_n(t)e^{-\frac{i\varepsilon_{\mathrm{eff}}t}{\hbar}}, \quad P_n\left(t + \frac{2\pi}{\omega}\right) = P_n(t). \qquad (10)$$

The parameter $\varepsilon_{\mathrm{eff}}$ is determined as a qusienergy [27]. If the qusienergy is the whole number of $\hbar\omega$ then $\chi(t)$ is the periodic Mathieu function [28]:

$$\chi(t) = \sqrt{\frac{\omega}{\pi}}\,\mathrm{ce}_k(t,q), \quad \chi(t) = \sqrt{\frac{\omega}{\pi}}\,\mathrm{se}_k(t,q), \qquad (11)$$

where $k = 0, 1, 2\ldots$ Near the values of the quasimomentum p_n which satisfy the following condition:

$$2\Omega_0(p_n) = n\omega, \quad n = 1, 2\ldots, \qquad (12)$$

the exponential increase in the amplitude of the wave function occurs [27,29,30] (parametric resonance). This leads to a series of gaps arising in the energy spectrum of originally gapless graphene (Fig. 2). From (12) we derive for the quasimomentum p_n:

$$p_n = \sqrt{\frac{n^2\hbar^2\omega^2}{4\upsilon_{\mathrm{F}}^2} - p_E^2}. \qquad (13)$$

The appearance of the gap with number n is seen from (8) and (13) to be possible if the next condition is true: $2\upsilon_{\mathrm{F}}p_E < n\hbar\omega$.
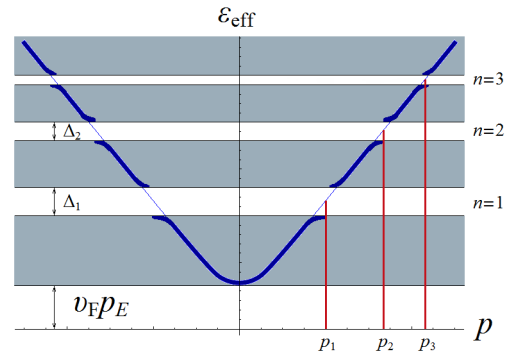

Fig. 2 – The scheme of the effective energy spectrum of the graphene

3. GAP SEMIWIDTH IN THE EFFECTIVE SPECTRUM OF GRAPHENE

If the next inequality is performed:

$$q\omega^2 << \Omega_0^2, \qquad (14)$$

then the wave function $\psi_1(\mathbf{r},t)$ has the next approximate form:

$$\psi_1(\mathbf{r},t) = e^{i\left(\frac{\mathbf{p}\cdot\mathbf{r}}{\hbar} - \Omega_0 t - \frac{\omega q}{4\Omega_0}\cos(\omega t - \theta)\right)}. \qquad (15)$$

In the situation (14) the gap semiwidth in the effective energy spectrum, corresponding to the case $n = 1$, is:

$$\Delta_1 = \frac{2\upsilon_{\mathrm{F}}p_E}{\hbar\omega}\sqrt{\hbar^2\omega^2 - 4\upsilon_{\mathrm{F}}^2 p_E^2}. \qquad (16)$$

The width of the energy gaps corresponding to the

cases $n \geq 2$ decreases as $\left(q\omega^2/\Omega_0^2\right)^n$ [29, 30]. The condition (14) can be rewritten as the next inequality $p_E p \ll p_E^2 + p^2$ which is true when $p \ll p_E$ or $p \gg p_E$.

4. QUASICLASSICAL LIMIT

If the next two inequalities are performed:

$$\Omega_0^2 \gg \omega^2, \ q \gg 1 \tag{17}$$

then the function $\varepsilon_{\text{eff}}(\Gamma, q)$ is equal approximately [28]:

$$\varepsilon_{\text{eff}}(\Gamma, q) = \frac{\hbar\omega}{2\pi} \int_0^\pi \sqrt{\Gamma - 2q\cos 2z}\,dz . \tag{18}$$

Substituting (8) in (18) we obtain:

$$\varepsilon_{\text{eff}}(p) = \frac{\upsilon_{\text{F}}}{\pi} \int_0^\pi \sqrt{p_E^2 + p^2 + 2p_E p\cos x}\,dx , \tag{19}$$

After integration in (19) we have:

$$\varepsilon_{\text{eff}}(p) = \frac{2\upsilon_{\text{F}}(p_E + p)}{\pi} \text{E}\left(\frac{2\sqrt{p_E p}}{p + p_E}\right). \tag{20}$$

where $E(x)$ is the complete elliptic integral.

Conditions (17) lead to the following correlation: $p_E p \sim p_E^2 + p^2 \gg \hbar^2\omega^2/\upsilon_{\text{F}}^2$ which is performed for EM waves of high intensities and low frequencies: $\upsilon_{\text{F}} p_E \gg \hbar\omega$. The last inequality means that electron free path over the wave period $(\lambda_e \sim 2\pi\upsilon_{\text{F}}/\omega)$ is much more than the de Broglie wavelength $(\lambda_E \sim 2\pi\hbar/p_E)$. Thus the dispersion law (20) corresponds to the quasi-classical approximation of the electron dynamics in the graphene under the EM wave [31]. In this situation the qusienergy $\varepsilon_{\text{eff}}(p)$ can be interpreted as the electron energy averaged over the EM wave period. Indeed, the average energy of the electron in the graphene under the presence of the EM wave is:

$$\bar{\varepsilon}(\mathbf{p}) = \frac{\omega\upsilon_{\text{F}}}{2\pi} \int_0^{2\pi/\omega} \left|\mathbf{p} + \frac{e}{c}\mathbf{A}(t)\right| dt . \tag{21}$$

After substitution (2) in (21) and after integrating we obtain for the electron spectrum of the graphene under the EM wave: $\varepsilon_{\text{eff}}(p) = \bar{\varepsilon}(\mathbf{p})$ [31]. The dependences of the effective energy (20) on the quasimomentum are shown on the figure 3 for the several values of the amplitude of the electric field intensity E. Near the minimum $(p \ll p_E)$ the spectrum (20) can be approximated as:

$$\varepsilon_{\text{eff}}(p) = \Delta_E + \frac{p^2}{2m_E} , \tag{22}$$

where $m_E = 2p_E/\upsilon_{\text{F}}$ is the electron effective mass, $\Delta_E = \upsilon_{\text{F}} p_E$ is the band gap semiwidth. The band gap is seen from (22) to arise under the HF electric field. The effective mass becomes nonzero. The magnitudes of these parameters can be regulated by changing of the amplitude of the EM wave.

5. DISCUSSIONS

The EM wave is shown above to induce the series of band gaps in the spectrum of originally gapless graphene. The appearance of such gaps is due to conditions (12) performing, where Ω_0 depends on the parameters of the graphene band structures and wave. So the gap appearance has the parametric nature. The band gap semiwidth is seen from (16), (20) and (22) to be regulated by the EM wave intensity changing.
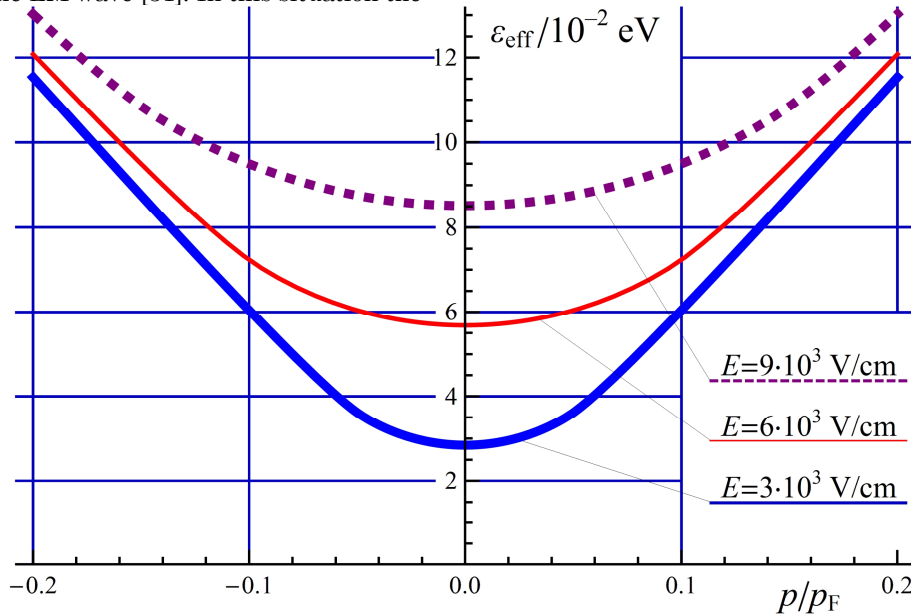

Fig. 3 – The graphs of the dispersion law (20), $p_{\text{F}} = \hbar/a$ is the charge carrier quasimomentum on the Fermi surface, $a = 0.246$ nm is the lattice constant of graphene [1, 2]

The effective mass m_E dependence on the EM wave intensity can lead to the so-called effect of induced selftransparency. This effect is explained by the plasma frequency ω_{pl} changes due to effective mass changes with the wave intensity E. So the condition $\omega > \omega_{pl}$ arises instead of initial condition $\omega < \omega_{pl}$. As a result weak wave reflects from the surface, and the intensive wave passes through.

Now we make the numerical estimates of value of the band gap semiwidth. For the next values of the parame-

ters: $\upsilon_F = 10^8$ cm/s, $\omega = 10^{13}$ s^{-1}, $E = 3 \cdot 10^3$ V/cm, we have $\Delta_E = 0.03$ eV, which is quite measurable experimentally.

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