A Stronger Bell Argument for Quantum Non-Locality

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It is widely accepted that the violation of Bell inequalities excludes local theories of the quantum realm. This paper presents a stronger Bell argument which even forbids certain non-local theories. The remaining non-local theories, which can violate Bell inequalities, are characterised by the fact that at least one of the outcomes in some sense probabilistically depends both on its distant as well as on its local parameter. While this is not to say that parameter dependence in the usual sense necessarily holds, it shows that the received analysis of quantum non-locality as 'outcome dependence or parameter dependence' is deeply misleading about what the violation of Bell inequalities implies.

Contents

1. Introduction

Bell's argument (1964; 1971; 1975) establishes a mathematical no-go theorem for theories of the micro-world. In its standard form, it derives that theories which are local (and fulfil certain auxiliary assumptions) cannot have correlations of arbitrary strength between events which are space-like separated. An upper bound for the correlations is given by the famous Bell inequalities. Since certain experiments with entangled quantum objects have results which violate these inequalities (EPR/B correlations), it concludes that the quantum realm cannot be described by a local theory. Any correct theory of the quantum realm must involve some kind of non-locality, a 'quantum non-locality'. In some sense, entangled quantum objects fundamentally depend on another, even when they are space-like separated.

While this general result is widely accepted, there is a large debate about *what kind of non-locality exactly* follows from the violation of Bell inequalities. There are at least four levels of discussion:

(1) Probabilistic level: Which probabilistic dependences (between space-like separated variables) follow from the violation of Bell inequalities?

(2) Causal level: Which (space-like separated) variables influence another?

(3) Space-time level: Is quantum non-locality (as characterised in (1) and (2)) compatible with a relativistic space-time structure?

(4) Metaphysical level: What kind of metaphysical relation is quantum non-locality?

There are extensive debates on any of these levels. In this paper we shall be concerned exclusively with the probabilistic level. Epistemically, it is the most fundamental one: drawing logical consequences from the empirical violation of Bell inequalities, it is the level closest to the empirical data; the discussions on the other levels are based on its results. The main question on this level is what the correct probabilistic characterisation of quantum non-locality is: *which variables of the experiments have to depend probabilistically on another given that Bell inequalities are violated?* There is an apparent conflict between the two main answers to this question. We shall now shortly present these answers, before we say how we are going to resolve the conflict in this paper.

Since in its most general form, Bell's theorem is formulated in probabilistic language, the standard answer comes in terms of *probabilistic* dependences. In its probabilistic version, the conclusion of Bell's argument is that the total probability distribution of the experiments does not factorise into local terms. This failure of 'local factorisation', as we shall call it (often denoted 'local causality' or just 'factorisation') is the probabilistic characterisation of quantum non-locality according to the standard view. It was Jarrett's idea (1984) to make this characterisation more explicit: he analysed the failure of local factorisation as the disjunction of two pairwise probabilistic dependences, 'outcome dependence or parameter dependence'.[1] This result provides a more detailed insight into the probabilistic nature of quantum non-locality: there must be either a specific dependence between the measurement outcomes or a specific dependence between at least one of the measurement outcomes and the distant measurement setting ('parameter'), or both. We should note that Jarrett's distinction has had an enormous impact on the discussion of quantum non-locality on the other levels.[2]

There is a second answer which does not agree with Jarrett's. Instead of analysing the failure of local factorisation in terms of probabilistic dependences, Maudlin (1994, ch. 6) develops an information theoretic account of how the EPR/B correlations come about. He shows that quantum non-locality *necessarily* requires that one of the outcomes depends on *information* about the distant setting:

> *Bell's inequality can reliably be violated only when the response of one of the particles depends (at least sometimes) on the question asked its partner.* [...] [D]ependence on the distant polariser setting is crucial. Jarrett's division of theories into those that violate outcome independence and those that violate parameter independence is again seen to be misleading: *any* successful theory must postulate some influence of a distant 'parameter' (i.e. the polariser angle) on the response of a local photon. Without such dependence the quantum statistics cannot be recovered. (p. 167)

In a recent paper by Pawlowski et al. (2010) this point is strengthened to the fact that one of the outcomes must also depend on information about the other outcome:

> ...it is impossible to model a violation [of the Bell inequalities] without having information in one laboratory about both the setting and the outcome at the distant

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[1] Jarrett's original names for the probabilistic conditions are 'completeness' (for what today is usually called 'outcome independence') and 'locality' (for 'parameter independence'). Here we use the names which are common in the debate, not least because Jarrett's names have been criticised to be misleading (Shimony, 1984; Norsen, 2009).

[2] On the space-time level, many philosophers of science have argued that it is outcome dependence and not parameter dependence which holds because parameter dependence is incompatible with relativity, while outcome dependence is not (Jarrett, 1984; Shimony, 1984, 1990; Arntzenius, 1994). The true theory about the quantum world, they maintain, very likely is 'outcome dependent and parameter *in*dependent', just as quantum theory is. Furthermore, Jarrett's probabilistic dependences have been given a metaphysical interpretation. While parameter dependence is mostly seen as constituting a causal relation, outcome dependence is interpreted as a non-causal influence ('passion at-a-distance': Shimony, 1984; Redhead, 1986, 1987, 1989) or a kind of non-separability / physical holism (Howard, 1985, 1989; Teller, 1986, 1989; Jarrett, 1989, 2009; Healey, 1991, 1994; Fogel, 2007). It should also be noted that these arguments which rely on the distinction between outcome dependence and parameter dependence are not uncontroversial. They have been criticized mainly for the fact that regardless of whether parameter dependence or outcome dependence holds, there must be a causal relation between the two wings (Butterfield, 1992; Jones and Clifton, 1993; Maudlin, 2011; Berkovitz, 1998a,b).

> one. While it is possible that outcome information can be revealed from shared hidden variables, [...] the setting information must be non-locally transferred.

Abstracting from the causal talk that is implicit in both quotations ('influence', 'transferred'), we can sum up that the information theoretic approach suggests a different picture than Jarrett's probabilistic analysis. While Jarrett's argument seems to say that either there is a probabilistic dependence on the distant outcome *or* on the distant setting, the informational approach makes clear that one of the outcomes must depend on information of both the distant outcome *and* the distant setting.

It might be hoped that the tension between the two approaches can be explained away by pointing to the fact that they use different concepts of dependence, viz. probabilistic dependence as opposed to informational dependence. This, however, is not the case: the concept of (Shannon mutual) information between two variables $\boldsymbol{x}$ and $\boldsymbol{y}$, which, basically, is at the core of both Maudlin's and Pawlowski's argument, is just a measure for the strength of the correlation between $\boldsymbol{x}$ and $\boldsymbol{y}$. Hence, informational dependence implies probabilistic dependence, i.e. Maudlin's result implies that there must be a probabilistic dependence between an outcome and its distant setting. The result of Pawlowski et al. adds that in principle also a dependence between the outcomes is required, which however might be screened-off by shared hidden variables. Since it is common to allow for such hidden variables, Pawlowski's analysis does not add a necessary requirement in terms of probabilistic dependences. Maudlin's condition that there must be a probabilistic dependence on the distant setting is the crucial insight of the information theoretic approach. This result seems to contradict Jarrett's analysis which *suggests* that one can avoid a dependence between an outcome and its distant setting if one accepts a dependence between the outcomes. So the information theoretic result is clearly stronger than the probabilistic one and if it is correct, Jarrett's standard analysis feigns an option that is not really available.

Which of the analyses is correct? Surprisingly, there is no clear answer available. Both analyses seem to be established by good and valid arguments and both coexist in the discussion. There exists no convincing explanation how the two analyses both of which seem to be correct can have seemingly different results. In this paper we shall present an analysis which fills this gap: we shall improve on the Bell-Jarrett argument such that it becomes explicit *which probabilistic dependences exactly follow from the violation of Bell inequalities.* The result will approve Maudlin's analysis in probabilistic terms: the violation of Bell inequalities implies that there must be a dependence between an outcome and its distant parameter of some kind.

The phrase 'of some kind' will turn out to be crucial. Our result will not mean that parameter dependence in the usual sense necessarily holds. Particularly, quantum mechanics is not ruled out by our new analysis. Parameter dependence in the normal sense is just one kind of a probabilistic dependence between an outcome and its distant parameter, which is characterised by a very specific set of conditional variables. But there are other kinds of parameter dependences (conditional on other variables) which will turn out to be important. Thus, our argument will also make precise what Maudlin's result leaves open, namely which kind of parameter dependence *exactly* is required.

Distinguishing different kinds of parameter dependences will furthermore have the consequence that Jarrett's result, 'outcome dependence or parameter dependence', is not wrong. Understood in a literal sense, it is perfectly correct. It is 'just' highly misleading, especially in this catchy short form, because it suggests and has been (falsely!) read to mean that one can avoid *any* probabilistic dependence of an outcome on its distant parameter if one accepts that the outcomes depend on each other. In its correct literal sense, however, it only says that one can avoid a *certain* dependence of an outcome on its distant parameter, namely parameter dependence, if one accepts a *certain* dependence between the outcomes. The latter statement does not logically contradict the fact that one must have *some* kind of parameter dependence. This will become clearer in the course of this paper, when we will have explicitly defined the corresponding mathematical expressions.

Our argument follows the Bell-Jarrett approach to quantum non-locality and shows how it can be improved towards a stronger result, which recovers the information theoretic implications. The procedure involves two steps. First, the conclusion of the Bell argument provides the probabilistic characterisation of quantum non-locality, which, second, is analysed in terms of pairwise probabilistic dependences. Accordingly, in a first part, we shall be concerned with strengthening the Bell argument such that it gives a stronger conclusion than the failure of local factorisation (section 2). The basic idea will be that the violation of Bell inequalities excludes even more than just the local theories, because certain kinds of *non*-local theories turn out to imply Bell inequalities as well. This makes explicit that the violation of Bell inequalities has considerably stronger and more informative consequences on a probabilistic level than believed so far. Jarrett's analysandum—the failure of local factorisation, which follows from the standard Bell argument—is weaker than it could and should be.

In a second part, we analyse the new conclusion in terms of pairwise independences, in a similar manner as Jarrett analysed the failure of local factorisation (section 3). The result of this analysis will be that, regardless of whether the outcomes depend on another or not, there must be some kind of dependence between at least one of the outcomes and both its local and its distant parameter.

A final note before we start. When we talk about probabilistic dependences in the following, for instance about a dependence between an outcome and its distant parameter, one should resist the temptation to interpret them causally. If two variables $\boldsymbol{x}$ and $\boldsymbol{y}$ are dependent conditional on another variable $\boldsymbol{z}$, there might be a causal relation from $\boldsymbol{x}$ to $\boldsymbol{y}$, but there does not have to. Alternatively, there might be a causal relation in the reverse direction or there might be a common cause or $\boldsymbol{z}$ might just be a common effect of $\boldsymbol{x}$ and $\boldsymbol{y}$. Correlation is not causation. The inference from probabilistic facts to causal facts is subtle and here we shall not be concerned with this question. We are just trying to get the probabilistic facts right. Causal inferences are left for future work, but we believe that a clear probabilistic account is a suitable basis for doing that inference.

2. Two concepts of quantum non-locality

2.1. EPR/B experiments and correlations

Many arguments for a quantum non-locality consider an EPR/B setup with polarisation measurements of photons (fig. 1; Einstein, Podolsky, and Rosen 1935; Bohm 1951; Clauser and Horne 1974). One run of the experiment goes as follows: a suitable source C (e.g. a calcium atom) is excited and emits a pair of photons whose quantum mechanical polarisation state $\boldsymbol{\psi}$ is entangled. Possible hidden variables of this state are called $\boldsymbol{\lambda}$, so that the *complete* state of the particle at the source is $(\boldsymbol{\psi}, \boldsymbol{\lambda})$. Since the preparation procedure is usually the same in all runs, the quantum mechanical state $\boldsymbol{\psi}$ is the same in all runs and will not explicitly be noted in the following. (One may think of any probability being conditional on one fixed state $\boldsymbol{\psi} = \psi_0$.) After the emission, the photons move in opposite direction towards two polarisation measurement devices A and B, whose measurement directions $\boldsymbol{a}$ and $\boldsymbol{b}$ are randomly chosen among two of three possible settings ($\boldsymbol{a} = 1, 2$; $\boldsymbol{b} = 2, 3$) while the photons are on their flight. A photon either passes the polariser (and is detected) or is absorbed by it (and is not detected), so that at each measuring device there are two possible measurement outcomes $\boldsymbol{\alpha} = \pm$ and $\boldsymbol{\beta} = \pm$.

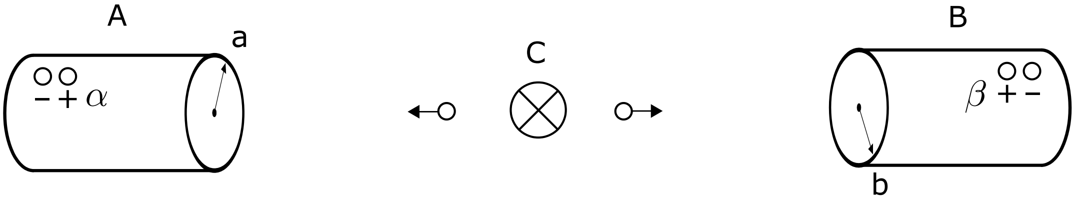

Fig. 1: EPR/B setup

On a probabilistic level, the experiment is described by the joint probability distribution $P(\alpha\beta ab\lambda) := P(\boldsymbol{\alpha} = \alpha, \boldsymbol{\beta} = \beta, \boldsymbol{a} = a, \boldsymbol{b} = b, \boldsymbol{\lambda} = \lambda)$ of the five random variables just defined.[3] We shall consistently use bold symbols $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{a}, \dots)$ for random variables and normal font symbols $(\alpha, \beta, a, \dots)$ for the corresponding values of these variables. We use indices to refer to *specific* values of variables, e.g. $\alpha_- = -$ or $a_1 = 1$, which provides useful shorthands, e.g. $P(\alpha_-\beta_+a_1b_2\lambda) := P(\boldsymbol{\alpha} = -, \boldsymbol{\beta} = +, \boldsymbol{a} = 1, \boldsymbol{b} = 2, \boldsymbol{\lambda} = \lambda)$. Expressions including probabilities with non-specific values of variables, e.g. $P(\alpha|a) = P(\alpha)$, are meant to hold for all values of these variables (if not otherwise stated).

Containing the hidden states $\boldsymbol{\lambda}$, which are by definition not measurable, the total distribution is empirically not accessible ('hidden level'), i.e. purely theoretical. Only the marginal distribution which does not involve $\boldsymbol{\lambda}$, $P(\alpha\beta ab)$, *is* empirically accessible and

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[3] While the outcomes and settings are discrete variables, the hidden state may be continuous or discrete. In the following I assume λ to be discrete, but all considerations can be generalized to the continuous case.

is determined by the results of actual measurements in EPR/B experiments (‘observable level’).[4]

Although the EPR/B setup is constructed in order to weaken and minimize correlations between the involved variables,[5] a statistical evaluation of a series of many runs with similar preparation procedures yields that there *are* observable correlations: the outcomes are correlated given the parameters,[6]

$$P(\alpha\beta|ab) = P(\alpha|\beta ab)P(\beta) = \begin{cases} \cos^2\phi_{ab}\cdot\frac{1}{2} & \text{if } \alpha=\beta \\ \sin^2\phi_{ab}\cdot\frac{1}{2} & \text{if } \alpha\neq\beta \end{cases} \tag{Corr}$$

(where ϕ_{ab} is the angle between the measurement directions a and b). These famous EPR/B correlations between space-like separated measurement outcomes have first been measured by Aspect et al. (1982) and are correctly predicted by quantum mechanics.

2.2. The standard Bell argument for quantum non-locality

Since according to (Corr), one outcome depends on both the other space-like separated outcome as well as on the distant (and local) parameter, the observable part of the probability distribution is clearly non-local. Bell (1964) could show that EPR/B correlations are so extraordinary that even if one allows for hidden states $\boldsymbol{\lambda}$ one cannot restore locality: given EPR/B correlations the *theoretical* probability distribution (including possible hidden states) must be non-local as well. Hence, *any possible* probability distribution which might correctly describe the experiment must be non-local.

This ‘Bell argument for quantum non-locality’, as I shall call it, runs as follows. Bell realised that EPR/B correlations have the remarkable feature to violate Bell inequalities. Since Bell then did not know that suitable measurements indeed yield the correlations, the violation was merely hypothetical, but today the violation of Bell inequalities is an empirically confirmed fact. It follows that at least one of the assumptions in the derivation of the inequalities must be false. Indeterministic generalizations (Bell, 1971; Clauser and Horne, 1974; Bell, 1975) of Bell’s original deterministic derivation (1964)

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[4] The (theoretical) transition from the total probability distribution to the observable marginal distribution is given by a marginalisation over λ, $P(\alpha\beta ab) = \sum_\lambda P(\alpha\beta ab\lambda)$. In order to be empirically adequate, any theoretical distribution must in this way yield the distribution which describes the statistics of EPR/B measurements.

[5] First, the settings are random and statistically independent. Second, the parameters are set *after* the emission, so that the setting may not influence the state of the particles at the emission. And, finally, but most importantly, the wings of the experiment are space-like separated, so that according to the First Signal Principle of relativity there cannot be any influence from one outcome to the other or from one setting to the outcome on the other wing.

[6] A correlation of the outcomes means that the joint probability $P(\alpha\beta|ab)$ is in general not equal to the product $P(\alpha|ab)P(\beta|ab) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$.

employ two probabilistic assumptions, 'local factorisation'[7]

$$P(\alpha\beta|ab\lambda) = P(\alpha|a\lambda)P(\beta|b\lambda) \tag{ℓF}$$

and 'autonomy'

$$P(\lambda|ab) = P(\lambda). \tag{A}$$

Another type of derivation (Wigner, 1970; van Fraassen, 1989; Graßhoff et al., 2005) additionally requires the empirical fact that there are perfect correlations (PCorr) between the outcomes if the measurement settings are equal. For both types of derivation we have the dilemma that any empirically correct probability distribution of the quantum realm must either violate autonomy or local factorisation (or both). Since giving up autonomy seems to be ad hoc and implausible ('cosmic conspiracy'), most philosophers conclude that the empirical violation of Bell inequalities implies that local factorisation fails.[8] And since local factorisation states the factorisation of the hidden joint probability distribution into *local* terms, the *failure* of local factorisation indicates a certain kind of *non*-locality, which is specific to the quantum realm—hence 'quantum non-locality'.

In order to make the logical structure clear let me note the Bell argument in an explicit logical form (where (I1), (I2), ... indicate intermediate conclusions). Here and in the following I shall use the Wigner-type derivation of Bell inequalities because, as we will see, it is the most powerful one allowing to derive Bell inequalities from the widest range of probability distributions:

(P1) There are EPR/B correlations: (Corr)

(P2) EPR/B correlations violate Bell inequalities: (Corr) $\rightarrow \neg$(BI)

  (I1) Bell inequalities are violated: $\neg$(BI) (from P1 & P2, MP)

(P3) EPR/B correlations include perfect correlations: (Corr) $\rightarrow$ (PCorr)

  (I2) There are perfect correlations: (PCorr) (from P1 & P3, MP)

(P4) Bell inequalities can be derived from autonomy, perfect correlations and local factorisation: (A) $\wedge$ (PCorr) $\wedge$ (ℓF) $\rightarrow$ (BI)

  (I3) Autonomy or local factorisation has to fail: $\neg$(A) $\vee$ $\neg$(ℓF) (from I1 & I2 & P4, MT)

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[7] 'Local factorisation' is my term. Bell calls (ℓF) 'local causality', some call it 'Bell-locality', but most often it is simply called 'factorisation' or 'factorisability'. Bell's terminology already suggests a metaphysical interpretation, which I would like to avoid in this paper, and the latter two names are too general since, as I shall show, there are other forms of the hidden joint probability which can be said to factorise; hence '*local* factorisation'.

[8] We should note that, though not a majority view, there are suggestions to explain the violation of Bell inequalities by a violation of autonomy (e.g. Costa de Beauregard, 1979; Sutherland, 1983; Price, 1994; Szabó, 2000). Our analysis in this paper does not apply to these cases. We shall consistently assume that autonomy holds.

(P5) Autonomy holds: (A)

(C1) Local factorisation fails: $\neg(\ell\mathrm{F})$ (from I3 & P5)

(P6) Quantum non-locality is the failure of local factorisation:
$(\mathrm{QNL}) :\leftrightarrow \neg(\ell\mathrm{F})$ (definition)

It is obvious that the argument from (P1)–(P5) to (C1) is valid. It shows that if autonomy holds, EPR/B correlations mathematically imply a non-locality which is called quantum non-locality, (P6). (P6) is *not* a premise of the Bell argument but labels its result with an appropriate name; it determines what *quantum non-locality according to the standard view* amounts to an a probabilistic level.[9] It is clear that if the Bell argument could be modified to have a stronger conclusion, the definition (P6) would have to be adapted. What we call 'quantum non-locality' depends on the result of the Bell argument. In this sense the *analysis* of quantum non-locality, in which (P6) functions as a premise (it determines the analysandum, see section 3), is based on the Bell argument. Note that defining quantum non-locality as the conclusion of the Bell argument, the logical structure of the argument is such that quantum non-locality only provides *necessary conditions* for EPR/B correlations, i.e. for being empirically adequate. So we have to keep in mind that the analysis of quantum non-locality is *not* an analysis of EPR/B correlations but of a necessary condition for them.

The core idea of my critique concerning the standard view of quantum non-locality is that the result of the Bell argument is weaker as it could be. I do not say that the argument is invalid nor do I say that one of its premises is not sound, I just say that the argument can be made considerably stronger and that the stronger conclusion will allow us to define a tighter, more informative concept of quantum non-locality: one can be much more precise about what EPR/B correlations imply (if we assume that autonomy holds) than just saying that local factorisation has to fail. I shall show that besides the local classes EPR/B correlations also exclude certain non-local classes. Given this new result, the standard definition of quantum non-locality (P6) will turn out to be inappropriately weak, because it includes those non-local classes which can be shown to be forbidden.

Specifically, I shall show that it is premise (P4) which can be made stronger (while leaving the other premises basically at work). Being an implication from autonomy, perfect correlations and local factorisation to Bell inequalities, it is clear that we can make (P4) the stronger the weaker we can formulate the antecedens, i.e. the assumptions to derive the inequalities. This idea is not essentially a new one. Since Bell's original proof

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[9] We should note that the term 'quantum non-locality' is not unambiguous. According to the definition we use here, (P6), it refers to probabilistic facts—not to causal or metaphysical ones. Moreover, we do not mean the *specific* non-locality inherent in quantum mechanics (or the true theory of the quantum world, whatever it may be). It is clear that one could be more precise about this latter non-locality than saying that it violates local factorisation. Rather, by 'quantum non-locality' here and in the following we denote the general probabilistic fact, which follows from the violation of Bell inequalities and which characterises *any* viable theory of the quantum realm. Given the standard Bell argument this is just the failure of local factorisation.

(1964) considerable efforts have been made to find derivations with weaker and weaker assumptions. For example, one of the milestones was to show that one can derive Bell inequalities without the original assumption of determinism. Currently, autonomy and local factorisation seem to constitute the weakest set of probabilistic assumptions which allow a derivation. What will be new about my approach is to try to find *alternatives to local factorisation*, which (given autonomy and perfect correlations) also imply that Bell inequalities hold. Since local factorization is the weakest possible form of local distributions, it is clear that such alternatives have to involve a kind of *non-locality*, i.e. what I am trying to show in the following is that we can derive Bell inequalities from certain non-local probability distributions.

2.3. Classes of probability distributions

We can find potential alternatives to local factorisation if we consider what it is: a particular feature of the *hidden joint probability*, as I shall call $P(\alpha\beta|ab\lambda)$. According to the product rule of probability theory, for *any* of the possible hidden probability distributions the joint probability of the outcomes (given the other variables) can be written as a product,

$$\begin{aligned} P(\alpha\beta|ab\lambda) &= P(\alpha|\beta ba\lambda)P(\beta|ab\lambda) && (1)\\ &= P(\beta|\alpha ab\lambda)P(\alpha|ba\lambda). && (2)\end{aligned}$$

Since there are two product forms, one whose first factor is a conditional probability of $\boldsymbol{\alpha}$ and one whose first factor is a conditional probability of $\boldsymbol{\beta}$, for the time being, let us restrict our considerations to the product form (1), until at the end of this section I shall generalize the results to the other form (2).

The product form (1) of the hidden joint probability holds in general, i.e. for all probability distributions. According to probability distributions with appropriate independences, however, the factors on the right-hand side of the equation reduce in that certain variables in the conditionals can be left out. If, for instance, outcome independence holds, $\boldsymbol{\beta}$ can disappear from the first factor, and the joint probability is said to 'factorise'. *Local* factorisation further requires that the distant parameters in both factors disappear, i.e. that parameter independence holds. Prima facie, any combination of variables in the two conditionals in (1) seems to constitute a distinct product form of the hidden joint probability. Restricting ourselves to *irreducibly hidden* joint probabilities, i.e. requiring $\boldsymbol{\lambda}$ to appear in both factors, there are $2^5 = 32$ *combinatorially* possible forms (for any of the three variables in the first conditional and any of the two variables in the second conditional *besides* $\boldsymbol{\lambda}$ can or cannot appear). Table 1 shows these conceivable forms which I label by (H_1^α) to (H_{32}^α) (the superscript α is due to the fact that we have used (1) instead of (2)).

The specific product form of the hidden joint probability is *the essential feature* of the probability distributions of EPR/B experiments. For, as we shall see, it not only determines whether a probability distribution can violate Bell inequalites but also carries unambiguous information about which variables of the experiment are probabilistically

Table 1: Classes of probability distributions

	I	II	III	IV		V	VI		VII	VIII	IX
	(H_i^α):	$P(\alpha\beta\vert ab\lambda) = \ldots$									
	i	$P(\alpha\vert$ β	b	a	$\lambda)\ \cdot P(\beta\vert$	a	b	$\lambda)$	□(BI)	[Group]	Notes
strong non-locality$^\alpha$	1	1	1	1		1	1		0	iv	
	2	1	1	1		1	0		0	iv	
	3	1	1	1		0	1		0	iv	$\mathrm{QM}_{\mathrm{PE}}$
	4	1	1	0		1	1		0	iv	
	5	1	0	1		1	1		0	iv	
	6	0	1	1		1	1		0	iv	Bohm
	7	1	1	1		0	0		0	iv	$\mathrm{QM}_{\mathrm{ME}}$
	8	0	1	1		1	0		0	iv	
	9	0	1	1		0	1		0	iv	
	10	1	0	0		1	1		0	iv	
	11	0	1	0		1	1		0	iv	
	12	0	0	1		1	1		0	iv	
	13	0	1	1		0	0		0	iv	
	14	0	0	0		1	1		0	iv	
weak non-locality$^\alpha$	15	1	1	0		1	0		1	iii	
	16	1	0	1		0	1		1	iii	
	17	1	0	1		1	0		1	i	
	18	1	1	0		0	1		1	i	
	19	1	1	0		0	0		1	i	
	20	1	0	1		0	0		1	i	
	21	1	0	0		1	0		1	i	
	22	0	1	0		1	0		1	ii	
	23	0	0	1		1	0		1	i	
	24	1	0	0		0	1		1	i	
	25	0	1	0		0	1		1	i	
	26	1	0	0		0	0		1	i	
	27	0	1	0		0	0		1	i	
	28	0	0	0		1	0		1	i	
locality$^\alpha$	29	0	0	1		0	1		1	ii	local fact.
	30	0	0	1		0	0		1	i	
	31	0	0	0		0	1		1	i	
	32	0	0	0		0	0		1	i	
Analysis:		$\neg(\mathrm{OI}_1)$	$\neg(\mathrm{PI}_1^\alpha)$	$\neg(\ell\mathrm{PI}_1^\alpha)$		$\neg(\mathrm{PI}_2^\beta)$	$\neg(\ell\mathrm{PI}_2^\beta)$				

independent of another. Virtually any interesting philosophical question involving probabilistic facts of EPR/B experiments depends on the specific product form of the hidden joint probability. Hence, it is natural to use the product form of the hidden joint probability in order to classify the probability distributions. We can say that each product form of the hidden joint probability constitutes a *class of probability distributions* in the sense that probability distributions with the same form (but different numerical weights of the factors) belong to the same class. In order to make the assignment of probability distributions to classes unambiguous let us require that each probability distribution belongs only to that class which corresponds to its *simplest* product form, i.e. to the form with the minimal number of variables appearing in the conditionals (according to the distribution in question).

This scheme of classes is comprehensive: Any probability distribution of the EPR/B experiment must belong to one of these 32 classes. In this systematic overview, the class constituted by local factorisation is (H^{α}_{29}) (see table 1, column IX), and if we allow that there might be no hidden states $\boldsymbol{\lambda}$, we can assign the quantum mechanical distribution to class (H^{α}_{7}).[10] The de-Broglie-Bohm theory falls under class (H^{α}_{6}), and similarly any other theory of the quantum realm has its unique place in one of the classes. The advantage of this classification is that it simplifies matters insofar we can now derive features of *classes* of probability distributions and can be sure that these features hold for all members of a class, i.e. for all theories whose probability distributions fall under the class in question.

The feature that we are most interested in is, of course, which of these classes (given autonomy) imply that Bell inequalities hold. We provide the answer by the following theorem:

> **Theorem 1**: Given autonomy, perfect correlations and perfect anti-correlations, a hidden joint probability implies Bell inequalities if each of the two factors in its product form involves at most one parameter.

Before we shall comment on the theorem, let us say that its proof can be found in the mathematical appendix. Although the proof is the core of our argument in this paper, nothing in what follows depends on understanding its details. We should remark that the perfect (anti-)correlations are required for letting a maximum of product forms imply Bell inequalities.

The result of theorem 1 is remarkable. So far it has been believed that *local* product forms imply Bell inequalities, but the theorem does not refer to this characteristic at all. It just requires that an outcome may not depend on both parameters. What does this mean? Which classes imply Bell inequalities according to the theorem? In order

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[10] The quantum mechanical distribution belongs to this class if the quantum state is *maximally* entangled, which is the typical case in EPR/B experiments (e.g. $|\psi\rangle = \sqrt{p}|+\rangle|+\rangle + \sqrt{1-p}|-\rangle|-\rangle$ with $p = \frac{1}{2}$). The slightest deviation from maximal entanglement ($p \neq \frac{1}{2}$), however, breaks the symmetry of the state. The probability distribution of such *partially* entangled states shows a dependence on the local setting in the second factor; they fall in class (H^{α}_{3}). We denote the quantum mechanical distribution for maximally entangled states by $\mathrm{QM}_{\mathrm{ME}}$ and that for partially entangled ones by $\mathrm{QM}_{\mathrm{PE}}$.

to make this clear let us partition the classes into three groups, depending on which variables appear in their constituting product forms:[11]

Local$^\alpha$ classes: (H^α_{29})–(H^α_{32}) (imply Bell inequalities)
each factor only contains time-like (or light-like) separated variables

Weakly non-local$^\alpha$ classes: (H^α_{15})–(H^α_{28}) (imply Bell inequalities)
at least one of the factors involves space-like separated variables, but none of the factors involves both parameters

Strongly non-local$^\alpha$ classes: (H^α_{1})–(H^α_{14}) (do not imply Bell inequalities)
at least one of the factors involves both parameters

(Note the superscript α, which indicates that we refer to classes deriving from (1) instead of from (2)).

Local classes involve only time-like (or light-like) separated variables in the factors of their hidden joint probability. When a product form is local, none of the outcomes depends on its distant parameter, so none of the outcomes depends on both parameters—hence, according to theorem 1, Bell inequalities follow. This is a well-known fact.

The surprising consequence of theorem 1 is that *even certain non-local classes imply Bell inequalities*. We have called those non-local classes, which imply Bell inequalities, '*weakly* non-local$^\alpha$' (in contrast to *strongly* non-local$^\alpha$ ones, which do not). According to weakly non-local$^\alpha$ classes, at least one of the factors in the product form involves space-like separated variables, but none of the factors involves both parameters. So weakly non-local$^\alpha$ classes do fulfil the criterion of theorem 1, which means that they imply Bell inequalities, although they involve non-local dependences. For instance, the outcomes may depend on their distant parameters if they do not depend on their local one,

$$P(\alpha\beta|ab\lambda) = P(\alpha|b\lambda)P(\beta|a\lambda). \tag{H^α_{22}}$$

Or an outcome may depend on the other outcome,

$$P(\alpha\beta|ab\lambda) = P(\alpha|\beta a\lambda)P(\beta|b\lambda), \tag{H^α_{16}}$$

or both of these non-localities may occur,

$$P(\alpha\beta|ab\lambda) = P(\alpha|\beta b\lambda)P(\beta|a\lambda). \tag{H^α_{15}}$$

Our proof of theorem 1 shows that all these product forms imply Bell inequalities, because none of its factors involves both parameters. From the perspective of the standard view this result is surprising because here we have cases where (as we shall show in the second part) parameter dependence or outcome dependence or both hold, and still Bell inequalities are implied. We emphasise that the proof of theorem 1, which shows that

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[11] Column VIII in table 1 introduces an even finer partition of the classes, which, however, is only relevant for the proof of theorem 1. We shall not comment on it in the main text.

besides local theories also weakly non-local$^\alpha$ ones imply Bell inequalities, is the source of all new consequences we shall derive in this paper.

As soon as there is a dependence on *both* parameters in at least one of the factors of the product form, which is how we have defined strongly non-local$^\alpha$ classes, one can*not* derive Bell inequalities from the product form any more. For each of these product forms one can easily find examples of probability distributions which do *violate* Bell inequalities. Thus, strongly non-local$^\alpha$ classes do *not* imply Bell inequalities, because *some* distributions in each of the classes violate them. This is, however, not to say that probability distributions in those classes necessarily violate Bell inequalities. On the contrary, one can as well find probability distributions of that form, which *obey* Bell inequalities. So depending on both parameters in one of the factors is only a *necessary* condition for violating Bell inequalities; it is not a sufficient one. Sufficient criteria to violate Bell inequalities would have to involve conditions for the *strength* of the correlations. As we have said in the introduction, mutual information between two variables is a measure for how strong the correlation between them is, so the information theoretic works which derive numerical values for how much mutual information has to be given in order to violate Bell inequalities, provide an answer to that question (see Maudlin 1994, ch. 6 and Pawlowski et al. 2010).

To sum it up: as opposed to what the standard discussion suggests, it is *not* true that local factorisation (and the other local product forms) are the *only* product forms which allow deriving Bell inequalities. Rather, we have found that 18 of the 32 logically possible classes imply Bell inequalities if autonomy (and perfect (anti-)correlations) hold, among them 14 *non-local* classes. Column VII of table 1 indicates which classes imply Bell inequalities and which do not ('□(BI)' means *necessarily, Bell inequalities hold*).

2.4. A stronger Bell argument for quantum non-locality

This consequence from theorem 1, that one can derive Bell inequalities also from certain non-local product forms, enables us to strengthen premise (P4) in the Bell argument. We can now write:

(P4′) Bell inequalities can be derived from autonomy, perfect correlations, perfect anti-correlations and any local$^\alpha$ or weakly non-local$^\alpha$ class of probability distributions:

$$\Big[(\mathrm{A}) \wedge (\mathrm{PCorr}) \wedge (\mathrm{PACorr}) \wedge \Big(\bigvee_{i=15}^{32} (\mathrm{H}_i^\alpha)\Big)\Big] \rightarrow (\mathrm{BI})$$

Compared to (P4), we have made two changes. First, we have replaced local factorisation in the antecedent by the *disjunction* of the local$^\alpha$ and weakly non-local$^\alpha$ classes (including local factorisation (H_{29}^α)). This makes the antecedent of (P4′) weaker than that in (P4) and, hence, the argument stronger. Second, we have added the condition that there are perfect anti-correlations (PACorr), since certain weakly non-local$^\alpha$ classes require them for the derivation. This additional assumption however does not weaken the argument since the perfect anti-correlations follow from the EPR/B correlations (as the perfect correlations do). We just have to modify premise (P3) to

(P3′) EPR/B correlations include perfect correlations and perfect anti-correlations: (Corr) → (PCorr) ∧ (PACorr)

Changing these two premises has a considerable effect on the Bell argument. Instead of the standard conclusion (C1), that the violation implies the failure of local factorisation, by the modified argument from (P1), (P2), (P3′), (P4′) and (P5), we arrive at the essentially stronger conclusion:

(C1′) Both local$^\alpha$ and weakly non-local$^\alpha$ classes fail:

$$\Big(\bigwedge_{i=15}^{32} \neg(\mathrm{H}_i^\alpha)\Big)$$

While the original result, the failure of local factorisation, implied that all local$^\alpha$ classes fail (because the other local classes are specializations of local factorisation), the new result additionally excludes all weakly non-local$^\alpha$ classes.

Our considerations leading to this new result of the Bell argument rest on the fact that we have found alternatives to local factorisation from writing the hidden joint probability according to the product rule (1) and conceiving different possible product forms (table 1). However, we can as well write the hidden joint probability according to the second product rule (2), and similar arguments as above lead us to a similar table as table 1, whose classes, (H_1^β)–(H_{32}^β), differ to those in table 1 in that the outcomes and the parameters are swapped. For instance, class (H_{16}^β) is defined by the product form $P(\alpha\beta|ab\lambda) = P(\beta|\alpha b\lambda)P(\alpha|a\lambda)$ in contrast to (H_{16}^α), which is constituted by $P(\alpha\beta|ab\lambda) = P(\alpha|\beta a\lambda)P(\beta|b\lambda)$. Note that this new classification is a *different partition* of the possible probability distributions. Any probability distribution must fall in exactly one of the classes (H_1^α)–(H_{32}^α) *and* in exactly one of the classes (H_1^β)–(H_{32}^β). Analogously to theorem 1 one can proof for the new partition that the local$^\beta$ and weakly non-local$^\beta$ classes imply Bell inequalities as well, so that we can reformulate (P4′) as:

(P4″) Bell inequalities can be derived from autonomy, perfect correlations, perfect anti-correlations and any local$^\alpha$, weakly non-local$^\alpha$, local$^\beta$ or weakly non-local$^\beta$ class of probability distributions:

$$\Big[(\mathrm{A}) \wedge (\mathrm{PCorr}) \wedge (\mathrm{PACorr}) \wedge \Big(\bigvee_{i=15}^{32} (\mathrm{H}_i^\alpha) \vee \bigvee_{i=15}^{32} (\mathrm{H}_i^\beta)\Big)\Big] \rightarrow (\mathrm{BI})$$

Then we can formulate an even stronger Bell argument from (P1), (P2), (P3′), (P4″) and (P5) to

(C1″) All local$^\alpha$, weakly non-local$^\alpha$, local$^\beta$ and weakly non-local$^\beta$ classes fail:

$$\Big(\bigwedge_{i=15}^{32}\neg(\mathrm{H}_i^{\alpha})\wedge\bigwedge_{i=15}^{32}\neg(\mathrm{H}_i^{\beta})\Big)$$

Since in section 2.2 we have seen that the result of the original Bell argument defines what we call quantum non-locality, it is clear that given this new result we have to adapt the definition appropriately. It simply becomes implausible to stick to the old, looser definition *including* weakly non-local classes, given that we now know that we have to *exclude* them. With the new result of the Bell argument we can be much more precise about what quantum non-locality amounts to on a probabilistic level and re-define it as:

(P6′) Quantum non-locality is the failure of the disjunction of all local$^{\alpha}$, weakly non-local$^{\alpha}$, local$^{\beta}$ and weakly non-local$^{\beta}$ classes (i.e. it is not the case that both factors in each product form involve at most one parameter):

$$\text{(QNL')} :\leftrightarrow \left(\bigwedge_{i=15}^{32}\neg(\mathrm{H}_i^{\alpha})\wedge\bigwedge_{i=15}^{32}\neg(\mathrm{H}_i^{\beta})\right) \qquad \text{(definition)}$$

This is the first main result of my investigation. (P6′) takes the notion of quantum non-locality from *any* kind of non-locality (the mere failure of local factorisation) to a more *specific* one (namely exclusive the weakly non-local$^{\alpha}$ and weakly non-local$^{\beta}$ classes). Since our scheme of logically possible classes is comprehensive, the failure of all local$^{\alpha}$ and weakly non-local$^{\alpha}$ classes is equivalent to the fact that one of the strongly non-local$^{\alpha}$ classes, (H_1^{α})–$(\mathrm{H}_{14}^{\alpha})$, holds. Analogously, if a probability distribution is neither local$^{\beta}$ nor weakly non-local$^{\beta}$ it must be strongly non-local$^{\beta}$, i.e. belong to one of the classes (H_1^{β})–$(\mathrm{H}_{14}^{\beta})$. Therefore, equivalently to (P6′) we can say:

(P6″) Quantum non-locality is strong non-locality$^{\alpha}$ and strong non-locality$^{\beta}$ (i.e. at least one of the factors in each product form must involve both parameters):

$$\text{(QNL')} \leftrightarrow \left(\bigvee_{i=1}^{14}(\mathrm{H}_i^{\alpha})\wedge\bigvee_{i=1}^{14}(\mathrm{H}_i^{\beta})\right)$$

2.5. Discussion I

According to the logic of the Bell argument, we have noted in section 2.2, quantum non-locality is a *necessary* condition for EPR/B correlations and their violation of Bell inequalities. This still holds for the new result and definition (since the logical structure of the argument has not essentially changed). However, we now know that the new concept of quantum non-locality is *not sufficient* for EPR/B correlations because we have seen that there are strongly non-local distributions which do *not* violate Bell inequalities. On the other hand, since we have also found that all strongly non-local classes include distributions which violate Bell inequalities and reproduce EPR/B correlations, we can say that on a qualitative level, which only considers the product forms of the hidden joint probability, i.e. probabilistic dependences and independences, we cannot improve the argument any more. It is impossible to reach a stronger conclusion than (C1$''$) by showing that we can derive Bell inequalities from still further classes. For if my argument and counterexamples are correct, there are no classes left which in general may imply Bell inequalities. Any future characterisation of quantum non-locality which is more detailed must involve reference to the *strengths* of the correlations in the strongly non-local classes. In this sense, my new definition of quantum non-locality, although not being sufficient for EPR/B correlations, captures their strongest possible consequences on a qualitative probabilistic level.

Why has this stronger consequence of the Bell argument, that we have derived, been overlooked so far? Obviously, it has wrongly been assumed that local factorisation is the *only* basis to derive Bell inequalities, and the main reason for neglecting other product forms of hidden joint probabilities might have been the fact that, originally, Bell inequalities were derived to capture consequences of a *local* worldview. The main result of former considerations was that locality has consequences which are in conflict with the quantum mechanical distribution. Given this historical background, the idea to derive Bell inequalities from non-local assumptions maybe was beyond interest because the conflict with locality was considered to be the crucial point; or maybe it was neglected because Bell inequalities were so tightly associated with locality that a derivation from non-locality sounded totally implausible. Systematically, however, since it is now clear that the quantum mechanical distribution is empirically correct and Bell inequalities are violated, it is desirable to draw as strong consequences as possible, which requires to check without prejudice whether some non-local classes allow a derivation of Bell inequalities as well—and this is what we have done here.

Before we go on with the argument let me shortly digress on how my argument resolves two common misunderstandings in the debate about quantum non-locality. First, in the discussion Bell inequalities are so closely linked to locality that one could have the impression that Bell inequalities are locality conditions in the sense that, if a probability distribution obeys a Bell inequality, it must be local. Of course, Bell's argument never really justified that view, for the logic of the standard Bell argument is that local factorisation (given autonomy and perfect (anti-)correlations) is merely *sufficient* (and not necessary) for Bell inequalities. Maybe the association between Bell inequalities and locality might have arisen from the the fact that up to now local factorisation has been the *only* product form which was shown to imply Bell inequalities. Given only this information, it was at least possible (though unproven) that the holding of Bell inequalities implies locality. However, since we have shown that weakly non-local classes in general imply Bell inequalities and since the simulations show that even some strongly non-local distributions can conform to Bell inequalities, it has become explicit that this is not true. *Not all* probability distributions obeying Bell inequalities are *local.* So if a probability distribution obeys a Bell inequality it does not have to be local.

Second, sometimes it has been said that the violation of Bell inequalities by EPR/B correlations implies that there *cannot* be a screener-off for the correlations (e.g. van Fraassen, 1989). Table 1 shows that this is not true either. Among the strongly non-local classes there are classes according to which $\boldsymbol{\alpha}$ is screened-off from $\boldsymbol{\beta}$ (namely all classes with the value 0 in column II). Consider for example class (H_6^α) for which the product form reads $P(\alpha\beta|ab\lambda) = P(\alpha|ab\lambda)P(\beta|ab\lambda)$. Here we have it that the conjunction of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{\lambda}$ screens the correlation off. Including the distant parameter for both outcomes, this is of course *not* a *local* screener-off. But that there can be a screener-off for the correlations, although non-local, shows that it is not true saying that EPR/B correlations disprove Reichenbachs common cause principle. This claim would only be true if you exclude non-local screener-offs by adding the premise of locality, which, however, would be odd, because the violation of Bell inequalities shows that we cannot avoid a non-locality anyway. Of course, the true theory of the quantum realm might *in fact* have a distribution which does not screen-off, but to argue for that one needs further assumptions; this claim is not in general implied by the violation of Bell inequalities.

Finally, and maybe most importantly, my argument points out that the discussion so far has been based on an inappropriate concept of quantum non-locality. Capturing all non-local classes the standard concept of quantum non-locality (the failure of local factorisation (P6)) includes classes which we have found to be compatible with Bell inequalities (weakly non-local classes). In this sense *the standard concept is inappropriately weak*, i.e. weaker as it should be. Therefore, if we analyse this concept, as Jarrett did, and take it as informing us about the consequences of EPR/B correlations violating Bell inequalities, we must expect to be misled, either in that some of the options we arrive at are not really available or that the analysantia do not cut the problem at its natural joints (in section 3.5 we will see that the latter is the case). This is the core of my critique concerning the standard view.

Excluding local *and* weakly non-local classes, my new concept (P6′) picks out a proper subset of the classes which are included in the standard concept, and comprises only those classes that do *not* imply Bell inequalities (strongly non-local classes): it is considerably stronger and much more informative than the standard concept. Moreover, I have just argued that it cannot be made stronger on a qualitative probabilistic level, because it comprehensively describes the qualitative probabilistic consequences of EPR/B correlations violating Bell inequalities. In this sense, my new concept is an appropriate basis for an analysis (while the standard concept is not). Analysing the new stronger concept will be the subject of the following section, and it will turn out that the result of *this* analysis differs significantly from Jarrett's.

3. Analysing quantum non-locality

The aim of this section is to provide an analysis of the *new* concept of quantum non-locality (P6′), which was the result of the modified Bell argument in the last section. Jarrett proved that the standard concept of quantum non-locality (P6) is equivalent to the disjunction of outcome dependence and parameter dependence. The *idea* of Jarrett's analysis is that a specific product form of the hidden joint probability (such as local factorisation), which is a *complex* independence condition, can be analysed by *pairwise* independences (such as outcome independence or parameter independence). Our new concept of quantum non-locality (P6′) is a conjunction of two disjunctions of several product forms and, hence, a complex independence condition as well. So we can apply Jarrett's idea to our new case and understand 'analysis' as providing an expression in terms of pairwise probabilistic independences which is equivalent to the new concept. I first recall shortly Jarrett's analysis and introduce an appropriate set of independences, which will serve as analysantia. Then I shall develop an analysis for each of the classes (H_i^α) and subsequently of the disjunction $\bigvee_{i=15}^{32}(\mathrm{H}_i^\alpha)$ (the first part of the disjunction of classes which imply Bell inequalities). Finally, we can transfer our results from the first to the second part $\bigvee_{i=15}^{32}(\mathrm{H}_i^\beta)$, and the negation of the disjunction of the two parts will yield the analysis of quantum non-locality (P6′).

3.1. Jarrett's analysis of quantum non-locality

Jarrett (1984) had the idea that one can be more explicit about the probabilistic nature of quantum non-locality (P6) by analysing the probabilistic statement local factorisation (ℓF) in terms of pairwise conditional probabilistic independences. By a 'pairwise conditional probabilistic independence' I mean the fact that a random variable $\boldsymbol{x}$ is independent of another $\boldsymbol{y}$ given a conjunction of further variables $\boldsymbol{z}$. This is said to be true iff for all values of the variables the joint probability over the variables makes the following equation true:

$$P(x|yz) = P(x|z) \tag{3}$$

The independence is noted as $I(\boldsymbol{x}, \boldsymbol{y}|\boldsymbol{z})$. If, however, there is at least one set of values for which (3) does not hold, the variables $\boldsymbol{x}$ and $\boldsymbol{y}$ are called dependent given $\boldsymbol{z}$, and this probabilistic dependence is noted as $\neg I(\boldsymbol{x}, \boldsymbol{y}|\boldsymbol{z})$.

Jarrett uses three pairwise independences: 'outcome independence' is defined as $I(\boldsymbol{\alpha}, \boldsymbol{\beta}|\boldsymbol{ab\lambda})$ and 'parameter independence' as a conjunction of two independences, $I(\boldsymbol{\alpha}, \boldsymbol{b}|\boldsymbol{a\lambda}) \wedge I(\boldsymbol{\beta}, \boldsymbol{a}|\boldsymbol{b\lambda})$. (Originally, Jarrett denotes these independences as 'completeness' and 'locality' respectively, but we shall use the now established names.) Jarrett proved mathematically that

(P7) Local factorisation is equivalent to the conjunction of outcome independence and parameter independence:

$$(\ell\mathrm{F}) \leftrightarrow I(\boldsymbol{\alpha}, \boldsymbol{\beta}|\boldsymbol{ab\lambda}) \wedge I(\boldsymbol{\alpha}, \boldsymbol{b}|\boldsymbol{a\lambda}) \wedge I(\boldsymbol{\beta}, \boldsymbol{a}|\boldsymbol{b\lambda}) \tag{4}$$

From (P6) and (P7) he concluded that

(C2) Quantum non-locality is equivalent to the disjunction of outcome dependence or parameter dependence:

$$(\mathrm{QNL}) \leftrightarrow \neg I(\boldsymbol{\alpha}, \boldsymbol{\beta}|\boldsymbol{ab\lambda}) \vee \neg I(\boldsymbol{\alpha}, \boldsymbol{b}|\boldsymbol{a\lambda}) \vee \neg I(\boldsymbol{\beta}, \boldsymbol{a}|\boldsymbol{b\lambda}) \tag{5}$$

which is the *probabilistic analysis of quantum non-locality according to the standard view* ('Jarrett's analysis'). The analysis is correct, but since (as we have seen) the analysandum (P6) is inappropriately weak, its result is not as informative as it could be and, as we will see, in fact misleading about the nature of quantum non-locality. Therefore, what we aim to do is to analyse the new stronger concept of quantum non-locality (P6′) in terms of pairwise conditional independences, which will give us a clearer picture of what quantum non-locality amounts to.

3.2. Pairwise independences

The first step towards an analysis is to get an overview which concepts can play the role of the analysantia. In table 2 I introduce those nine pairwise independences which will be relevant. Among the relevant independences we find the normal outcome independence, $I(\boldsymbol{\alpha}, \boldsymbol{\beta}|\boldsymbol{ab\lambda})$, as well as $I(\boldsymbol{\alpha}, \boldsymbol{b}|\boldsymbol{a\lambda})$, one independence of the conjunction which is usually called 'parameter independence'. Here we see a first problem with the standard names: how shall we call the latter if its conjunction with $I(\boldsymbol{\beta}, \boldsymbol{a}|\boldsymbol{b\lambda})$ is called 'parameter independence'? I have tried to stay as close to the standard names as possible, but obviously further qualifications are needed. My suggestion is to continue to use the name 'parameter independence' for all independences between an outcome and its distant parameter, but to add the outcome in question, namely 'α-parameter independence' or 'β-parameter independence' respectively. Further differentiation in the nomenclature is required by the fact that there is another α-parameter independence in the table, $I(\boldsymbol{\alpha}, \boldsymbol{b}|\boldsymbol{\beta a\lambda})$, which differs from the one already mentioned in the conditional variables (it additionally includes the outcome $\boldsymbol{\beta}$). Such independences of the same type but with different conditional variables are different independences and are in general logically independent of another: one can hold or not irrespective of whether the other does or does not. (One can show that only if one involves more than two independences logical restrictions appear.) I discern them by indices, e.g. the former is called 'α-parameter independence$_2$', the latter 'α-parameter independence$_1$'. Of course, there are further α-parameter independences (namely those conditional on $\boldsymbol{\beta\lambda}$ and $\boldsymbol{\lambda}$) which, however, do not play any role for the analysis here.

Similarly to the parameter independences I define local parameter independences (see table 2), which instead of the independence of an outcome on its distant parameter $(\boldsymbol{\alpha}, \boldsymbol{b})$ claim the independence of an outcome on its *local* parameter $(\boldsymbol{\alpha}, \boldsymbol{a})$. Besides these new names I have also introduced short labels for each independence, which we will mainly use in the following.

Given these new concepts we are now in a position to clearly see one of the sources of confusion in the standard discussion. 'Outcome dependence or parameter dependence' does *not necessarily* mean that if you accept outcome dependence you can avoid parameter dependence in the sense of *any kind* of dependence of an outcome on its distant parameter (conditional on whatever variables). The slogan just says that in this case you can avoid parameter dependence in the usual sense of $\neg(\mathrm{PI}_2^\alpha) \vee \neg(\mathrm{PI}_2^\beta)$, while other kinds of parameter dependences like $\neg(\mathrm{PI}_1^\alpha)$ might still hold! And indeed the analysis of the new concept will yield that at least one of the two parameter dependences $\neg(\mathrm{PI}_1^\alpha)$ and $\neg(\mathrm{PI}_2^\beta)$ *must* hold. Parameter dependence in this broader sense cannot be avoided but will turn out to be a necessary condition for violating the Bell inequalities.

Table 2: Definition of conditional independences

independence	standard name	new name	label
$I(\boldsymbol{\alpha}, \boldsymbol{\beta} \vert \boldsymbol{a}\boldsymbol{b}\boldsymbol{\lambda})$	outcome independence	outcome independence$_1$	(OI$_1$)
$I(\boldsymbol{\alpha}, \boldsymbol{b} \vert \boldsymbol{\beta}\boldsymbol{a}\boldsymbol{\lambda})$	–	α-parameter independence$_1$	(PI$_1^\alpha$)
$I(\boldsymbol{\alpha}, \boldsymbol{b} \vert \boldsymbol{a}\boldsymbol{\lambda})$	[part of] parameter ind.	α-parameter independence$_2$	(PI$_2^\alpha$)
$I(\boldsymbol{\beta}, \boldsymbol{a} \vert \boldsymbol{\alpha}\boldsymbol{b}\boldsymbol{\lambda})$	–	β-parameter independence$_1$	(PI$_1^\beta$)
$I(\boldsymbol{\beta}, \boldsymbol{a} \vert \boldsymbol{b}\boldsymbol{\lambda})$	[part of] parameter ind.	β-parameter independence$_2$	(PI$_2^\beta$)
$I(\boldsymbol{\alpha}, \boldsymbol{a} \vert \boldsymbol{\beta}\boldsymbol{b}\boldsymbol{\lambda})$	–	α-local parameter independence$_1$	(ℓPI$_1^\alpha$)
$I(\boldsymbol{\alpha}, \boldsymbol{a} \vert \boldsymbol{b}\boldsymbol{\lambda})$	–	α-local parameter independence$_2$	(ℓPI$_2^\alpha$)
$I(\boldsymbol{\beta}, \boldsymbol{b} \vert \boldsymbol{\alpha}\boldsymbol{a}\boldsymbol{\lambda})$	–	β-local parameter independence$_1$	(ℓPI$_1^\beta$)
$I(\boldsymbol{\beta}, \boldsymbol{b} \vert \boldsymbol{a}\boldsymbol{\lambda})$	–	β-local parameter independence$_2$	(ℓPI$_2^\beta$)

3.3. Analysing the classes

With these pairwise independences we can now attempt to analyse each class of probability distributions. For the analysis of the classes (H$_i^\alpha$) in table 1 we shall need the first five independences in table 2 (the other four independences plus outcome independence$_1$ are only used for the analysis of the classes (H$_1^\beta$); see below). We have noted the corresponding dependences in the bottom line of table 1 such that each dependence is associated with one of the columns II–VI. The idea is that the dependence holds in a class if the column of that class contains a '1'. Otherwise, i.e. if it contains a '0', the corresponding independence holds. The result of this analysis is stated by the following theorem:

> **Theorem 2**: Each class in table 1 is equivalent to the conjunction of the specific pattern of independences (see the bottom line) indicated by 0's in columns II–VI.

The proof of theorem 2 can be found in the mathematical appendix.

The theorem means that each pattern of independences corresponds to exactly one of the classes, e.g.

$$(\mathrm{H}_7^\alpha) \leftrightarrow (\mathrm{PI}_2^\alpha) \wedge (\ell\mathrm{PI}_2^\alpha). \tag{6}$$

One can see from the table that *each of the five independences corresponds to exactly one of the five variables in the conditionals of the factors*: if a certain independence *holds*, the corresponding variable does *not* appear (and vice versa), and if a certain independence *fails*, the corresponding variable does *appear* (and vice versa). Specifically, if (OI$_1$) holds, the first factor of the hidden joint probability does not involve the other outcome $\boldsymbol{\beta}$ (and vice versa), and if it does not, the first factor includes it (and vice versa). Similarly,

(PI_1^α) and $(\ell\mathrm{PI}_1^\alpha)$ correspond to the distant and the local parameter in the *first* factor respectively, while (PI_2^α) and $(\ell\mathrm{PI}_2^\alpha)$ are linked to the distant and the local parameter in the *second* factor respectively. So the holding or failure of each of the five independences has a very well defined impact on the product form of the hidden joint probability (and vice versa), and the conjunction of *all* independences which hold according to a certain probability distribution determines its product form, i.e. its class (and vice versa).[12]

3.4. Quantum non-locality as double parameter dependence

Finally, with the analysis of the single classes we shall now formulate the second main result of this paper, the analysis of the new, stronger concept of quantum non-locality (P6′). We had found that quantum non-locality is the failure of all local$^\alpha$, weakly non-local$^\alpha$, local$^\beta$ and weakly non-local$^\beta$ classes and that these classes are characterized by the fact that their constituting product forms involve *at most one parameter in each of its factors*. Let us first give an analysis of the local$^\alpha$ and weakly non-local$^\alpha$ classes. Our analysis of the single classes (H_1^α)–(H_{32}^α) has revealed that each variable in the conditionals of the factors corresponds to exactly one of the five independences in table 1. The distant parameter in the first factor corresponds to α-parameter independence$_1$, (PI_1^α), and the local parameter to α-local parameter independence$_1$, $(\ell\mathrm{PI}_1^\alpha)$. So the first factor involves at most one parameter if and only if at least one of these independences holds, $(\mathrm{PI}_1^\alpha) \vee (\ell\mathrm{PI}_1^\alpha)$. Similarly, at most one parameter appears in the second factor iff β-parameter independence$_2$ or β-local parameter indepence$_2$ hold, $(\mathrm{PI}^\beta) \vee (\ell\mathrm{PI}^\beta)$. So we have found the following equivalence:

(P7′a) The disjunction of local$^\alpha$ and weakly non-local$^\alpha$ classes is equivalent to the fact that $\boldsymbol{\alpha}$ is independent$_1$ of at least one parameter and $\boldsymbol{\beta}$ is independent$_2$ of at least one parameter:

$$\left(\bigvee_{i=15}^{32}(\mathrm{H}_i^\alpha)\right) \leftrightarrow \left[\Big((\mathrm{PI}_1^\alpha) \vee (\ell\mathrm{PI}_1^\alpha)\Big) \wedge \Big((\mathrm{PI}_2^\beta) \vee (\ell\mathrm{PI}_2^\beta)\Big)\right]$$

In a very similar way as we have proceeded for the classes (H_1^α)–(H_{32}^α) one can find an analysis for the classes (H_1^β)–(H_{32}^β) (remember the table which is symmetric to table 1 in swapping the outcomes and the parameters and apply all considerations mutatis mutandis):

(P7′b) The disjunction of local$^\beta$ and weakly non-local$^\beta$ classes is equivalent to the fact that $\boldsymbol{\beta}$ is independent$_1$ of at least one parameter and $\boldsymbol{\alpha}$ is independent$_2$ of at least one parameter:

$$\left(\bigvee_{i=15}^{32}(\mathrm{H}_i^\beta)\right) \leftrightarrow \left[\Big((\mathrm{PI}_1^\beta) \vee (\ell\mathrm{PI}_1^\beta)\Big) \wedge \Big((\mathrm{PI}_2^\alpha) \vee (\ell\mathrm{PI}_2^\alpha)\Big)\right]$$

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[12] Note that according to table 1 local factorisation is analysed as $(\mathrm{H}_{29}^\alpha) \leftrightarrow (\mathrm{OI}_1) \wedge (\mathrm{PI}_1^\alpha) \wedge (\mathrm{PI}_2^\beta)$, while according to Jarrett it is $(\mathrm{H}_{29}^\alpha) \leftrightarrow (\mathrm{OI}_1) \wedge (\mathrm{PI}_2^\alpha) \wedge (\mathrm{PI}_2^\beta)$, i.e. in Jarrett's claim (PI_1^α) is replaced by (PI_2^α). Given that (OI_1) holds, the replacement is logically correct, because one can show that $(\mathrm{OI}_1) \wedge (\mathrm{PI}_1^\alpha) \leftrightarrow (\mathrm{OI}_1) \wedge (\mathrm{PI}_2^\alpha)$. So the two analyses of (H_{29}^α) are equivalent.

Since (P6′) defined quantum non-locality as the *failure* of the disjunction of all local$^\alpha$, weakly non-local$^\alpha$, local$^\beta$ and weakly non-local$^\beta$ classes, the negation of the disjunction of (P7'a) and (P7'b) finally yields the analysis of quantum non-locality:

(C2)′ Quantum non-locality is equivalent to the fact that $\boldsymbol{\alpha}$ depends$_1$ on both parameters or $\boldsymbol{\beta}$ depends$_2$ on both parameters and $\boldsymbol{\beta}$ depends$_1$ on both parameters or $\boldsymbol{\alpha}$ depends$_2$ on both parameters:

$$\begin{aligned}(\mathrm{QNL}') \quad \leftrightarrow \quad & \Bigg\{ \bigg[\Big(\neg(\mathrm{PI}_1^\alpha) \wedge \neg(\ell\mathrm{PI}_1^\alpha) \Big) \vee \Big(\neg(\mathrm{PI}_2^\beta) \wedge \neg(\ell\mathrm{PI}_2^\beta) \Big) \bigg] \wedge \\ & \wedge \quad \bigg[\Big(\neg(\mathrm{PI}_1^\beta) \wedge \neg(\ell\mathrm{PI}_1^\beta) \Big) \vee \Big(\neg(\mathrm{PI}_2^\alpha) \wedge \neg(\ell\mathrm{PI}_2^\alpha) \Big) \bigg] \Bigg\}\end{aligned}$$

While the definition of quantum non-locality in (P6′) was in terms of product forms, here we have the equivalent expression, the analysis, in terms of pairwise independences. It is a rather complex logical expression whose meaning and implications are not at all easy to grasp. A first understanding might be attained by making explicit how this analysis of quantum non-locality is also an analysis of the conjunction of strongly non-local$^\alpha$ and strongly non-local$^\beta$ classes (which is necessarily so, see (P6″)). These classes were characterized by the fact that at least one of the factors in each product form must involve both parameters and this is exactly what (C2′) says: The first term in the first disjunction, $\neg(\mathrm{PI}_1^\alpha) \wedge \neg(\ell\mathrm{PI}_1^\alpha)$ ('α-double parameter dependence$_1$'), guarantees a dependence on both parameters in the first factor of the product forms (H_i^α), the second term in the first disjunction, $\neg(\mathrm{PI}_2^\beta) \wedge \neg(\ell\mathrm{PI}_2^\beta)$ ('β-double parameter dependence$_2$'), implies a similar fact for the second factor of these forms, and analogously, the second disjunction entails a dependence on both parameters in at least one of the factors of the product forms (H_i^β) (and vice versa).

So the analysis involves double parameter dependences for each outcome in two different forms, either conditional on all other variables (double parameter dependence$_1$) or conditional on all other variables excluding the other outcome (double parameter dependence$_2$). The logic of the expression has it that these can hold in different combinations, but whichever combination does, there is one thing that necessarily follows if (C2′) is true:

(C3) Double parameter dependence: at least one of the outcomes depends probabilistically on both parameters (in at least one of the forms double parameter dependence$_1$ or double parameter dependence$_2$).

For one *can* avoid that *one* of the outcomes is double parameter dependent$_1$ and double parameter dependent$_2$, but then it follows that the respective other outcome must be double parameter dependent$_1$ as well as double parameter dependent$_2$. Of course, you can also have mixed cases in which both outcomes are double parameter dependent (in one or both of the two forms), but in any case you have double parameter dependence of at least one of the outcomes.

So we have found two results: the precise probabilistic analysis of quantum non-locality (C2′) and a general feature of and deriving from that analysis (C3), that at least one of the outcomes must be double parameter dependent. Since quantum non-locality is a necessary condition for EPR/B correlations (if autonomy holds) double parameter dependence of at least one of the outcomes, which is implied by quantum non-locality, is a *necessary* condition for EPR/B correlations as well: whenever we find that EPR/B correlations hold, double parameter dependence (C3) must hold as well. So given that measurement results in our world yield EPR/B correlations (and assuming autonomy), we can be sure that at least one of the outcomes depends both on the local as well as on the distant parameter. Note, however, that we have *not* shown that quantum non-locality, and hence neither its analysis (C2′) nor double parameter dependence (C3), is *sufficient* for the violation of Bell inequalities. If an outcome depends on both parameters in the sense of (C2′) the correlations between the two wings *might* be sophisticated enough to violate Bell inequalities—but they need not be. We have noted above that the product form and, hence, the dependences alone cannot guarantee a violation because, additionally, the correlations must have a certain strength.

3.5. Discussion II

We shall now compare our new results (C2′) and (C3) with that of the two existing analyses, the probabilistic one by Jarrett and the information theoretic one by Maudlin (and Pawlowski et al.).

(1) The main message of my result is that given EPR/B correlations and autonomy *one cannot avoid some kind of dependence between at least one of the outcomes and both parameters (C3).* This is the unambiguous probabilistic requirement of quantum non-locality according to my new analysis. *It confirms the information theoretic result in qualitative probabilistic terms.* The information theoretic approach has yielded that at least one of the outcomes must depend on information about the distant (and the local) parameter. Since information implies correlation,[13] the two results agree. This makes explicit that even by a probabilistic analysis using the Bell-Jarrett methodology one can recover the strong results of the information theoretic approach.

On the other hand, Jarrett's analysis does not bring out the essential feature of quantum non-locality on a probabilistic level (C3): from his result 'outcome dependence or parameter dependence' one just cannot see that, necessarily, there must be some kind of double parameter dependence. This difference between Jarrett's and Maudlin's result is the tension we have noted in the introduction. Here we see that my new analysis supports Maudlin's claim, and there are two good reasons to regard this as definitely resolving the tension in favour of Maudlin: First, my argument is very much like Jarrett's except that it analyses the new stronger notion of quantum non-locality (which, contrary to Jarrett's, picks out only strongly non-local classes; cf. P6 and P6′). Thus, my analysis supersedes Jarrett's because its *analysandum is much more informative.* Second, the

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[13] Remember that we noted in the introduction that Shannon mutual information, which is the concept that Maudlin's and Pawlowski et al.'s considerations essentially are based on, is a measure for the strength of a correlation.

fact that *the results from two independent and reliable approaches*, my probabilistic analysis and Maudlin's information theoretic argument, *converge* is strong evidence for their correctness. We conclude that our argument on the probabilistic level resolves the tension between Maudlin's and Jarrett's analysis in favour of the former. Something must be wrong or at least inappropriate about Jarrett's result and we shall shortly show what exactly it is.

(2) Before that we should note that our result is in one sense weaker and in one sense stronger than the information theoretic one. It is weaker because it is purely qualitative: it just says which probabilistic dependences are required, but it is tacit about how strong the correlations have to be in order to violate Bell inequalities. In section 2.3 I have argued that such qualitative results can only be necessary conditions for a violation. Having the right dependences for violating Bell inequalities does not mean that the inequalities are in fact violated. Sufficient criteria must involve conditions on the strength of the correlations. This is what the information theoretic approach derives in terms of quantities of bits.

In another sense, however, our result is also stronger than the information theoretic one. Our detailed result (C2′) makes explicit which probabilistic dependences *exactly* are required given that Bell inequalities are violated. Maudlin's analysis just implies that we need *some* dependence of an outcome on its distant parameter. But which precisely? We have seen that there are different kinds of parameter dependences, which differ in the conditional variables. For instance, it cannot be an unconditional parameter dependence because that would contradict the empirical distribution. So which are the ones that are required? My detailed result (C2′) makes this explicit. Either one of the outcomes depends on its distant parameter given the local setting and the hidden variables or it depends on the distant parameter given the local setting, the hidden variables *and* the other outcome. (The condition is rather complex and requires certain other dependences if the one or the other holds, but one cannot avoid that one of the mentioned parameter dependences holds.) This detailed characterisation cannot be obtained from the present information theoretic results. It might be important for the discussion of quantum non-locality on the other levels. Causal inference (cf. Pearl, 2000; Spirtes et al., 2000), for instance, is very sensitive to the exact kind of dependences and independences of a given situation.

(3) What is wrong about Jarrett's analysis? We now know (by C3) that one cannot avoid a certain dependence of the outcomes on their distant settings. We have also observed that Jarrett's result 'outcome dependence or parameter dependence' does not bring out this necessary requirement. Even worse, it stands in a certain tension to this result because it seems to suggest the contrary: that one can avoid *any* dependence of the outcomes on their distant settings if the outcomes depend on another. If this suggestive interpretation were the correct reading of Jarrett's result, it would be plainly wrong. However, this is not what it literally says. 'Parameter dependence' here does not mean *any* kind of parameter dependence but a very specific kind, namely parameter dependence$_2$. Saying that one can avoid this specific kind does not mean that there is no dependence of the outcomes on their distant parameters at all. Our presentation of different kinds of parameter dependences (see table 2) has made explicit that parameter

dependence$_2$ is only one among several kinds, all of which might hold if parameter dependence$_2$ fails. So Jarrett's result is compatible with our result (C2$'$) that one can avoid parameter dependence$_2$ only if parameter dependence$_1$ holds.

This reasoning shows that, though in a literal sense not logically contradicting our findings, Jarrett's result is *liable to be misunderstood to its non-literal false sense*, that one can avoid *any* kind of parameter dependence if outcome dependence holds. In this sense, Jarrett's result is misleading. In fact, it seems that Jarrett's result has to a large extent received this unfortunate interpretation. There is a bunch of literature about quantum non-locality (on any level, whether causal, spatio-temporal or metaphysical) which is based on Jarrett's distinction, and which discusses in detail what outcome dependence or parameter dependence would amount to, the preferred solution being outcome dependence without parameter dependence. But in most cases this makes only sense, if one believes that by neglecting parameter dependence one can avoid *any* kind of parameter dependence! If the authors in that debate would have known that one cannot avoid some kind of parameter dependence anyway, they would surely not have spent so much time on finding arguments why outcome dependence rather than parameter dependence holds. Much of the debate based on Jarrett's analysis seems to adhere to the wrong non-literal reading of Jarrett's result.

(4) We are now in a position to discard the typical prejudices about quantum mechanics which stem from such a false non-literal reading of Jarrett's result. Is quantum mechanics not a counterexample to my result? Is it not a theory which violates Bell inequalities although there is no dependence on the distant parameter? It is true, quantum mechanics is well known to be 'outcome dependent and parameter independent', but again this is not to be understood that according to quantum mechanics there is no probabilistic dependence of an outcome on the distant parameter at all. In fact, it is easy to check, which independences hold according to quantum mechanics: one can calculate all relevant conditional probabilities from the quantum mechanical probability distribution for the EPR/B experiment (Corr). A simple comparison of these probabilities then shows which of the independences hold and which do not, and it turns out that quantum mechanics is parameter dependent$_1$, $\neg(\mathrm{PI}_1^\alpha)$ and $\neg(\mathrm{PI}_1^\beta)$, so according to quantum mechanics each outcome *does* depend on its distant parameter! This parameter dependence in quantum mechanics is not as surprising as it may seem since, according to the formalism, the measurement direction at A determines the possible collapsed states at B and the actual outcome at A only determines in which of the (two) possible states the photon state at B collapses. So contrary to what the standard talk suggests, quantum mechanics is parameter dependent, and it is important to see that it is as well local parameter dependent$_1$, $\neg(\ell\mathrm{PI}_1^\alpha)$ and $\neg(\ell\mathrm{PI}_1^\beta)$ (while it is local parameter independent$_2$, $(\ell\mathrm{PI}_2^\alpha)$ and $(\ell\mathrm{PI}_2^\beta)$), because then, the quantum mechanical distribution fulfills the requirement of a quantum non-locality by rendering the first terms of the two disjunctions in (C2$'$) true. If my argument in this paper is true, it cannot be otherwise. For if quantum mechanics were not parameter dependent in this double sense, it could not (as it does) violate Bell inequalities.

(5) Finally, we shall now show that even in a correct literal reading, Jarrett's result

is problematic: understood as providing insight about quantum non-locality (on a probabilistic level), it is highly deceptive because it rests on inappropriate categories. This point becomes clear, if one investigates how Jarrett's analysantia outcome dependence$_1$, α-parameter dependence$_2$ and β-parameter dependence$_2$ relate to the new concept of quantum non-locality. Here is, first, how they do *not* relate: Jarrett's analysis of the weaker concept is a disjunction of these three dependences and it could have been that the analysis of the stronger concept just cancels one or two of the elements in the disjunction, revealing them as options which are not really available. For instance it might have been that the new analysis yields just $\neg(\mathrm{PI}_2^\alpha) \vee \neg(\mathrm{PI}_2^\beta)$, cancelling $\neg(\mathrm{OI}_1)$. However, it turns out that this is not the case. The logical structure of the new analysis is not just a simplification of the former, but, in fact, is much more complicated involving new concepts (parameter dependence$_1$, local parameter dependence$_1$ and local parameter dependence$_2$) and not involving others (outcome dependence$_1$). This suggests that *Jarrett's categories outcome dependence*$_1$ *and parameter dependence*$_2$ *cannot capture the new concept of quantum non-locality.*

Table 3: Jarrett's classes of possible probability distributions

Label	$\neg(\mathrm{OI}_1)$	$\neg(\mathrm{PI}_2^\alpha) \vee \neg(\mathrm{PI}_2^\beta)$	Notes	
(J_1)	1	1		quantum non-locality
(J_2)	0	1	Bohm	
(J_3)	1	0	QM	
(J_4)	0	0		locality

To make this explicit, consider the partition of the probability distributions according to the dependences in Jarrett's analysis (table 3). There are four classes, which I call 'Jarrett's classes' and label as (J_1)–(J_4). Any of the 32 possible classes from table 1 must fall into one of Jarrett's coarse-grained classes. While the local classes belong to (J4), any of the classes (J1)–(J3) includes both weakly and strongly non-local classes. So Jarrett's non-local classes, which are assumed to be able to violate Bell inequalities, *mix probability distributions which can with such which cannot* (see fig. 2). *They do not cut the probability distributions at their natural joints!* This means that neither outcome dependence$_1$ nor parameter dependence$_2$ are necessary or (contrary to Jarrett's analysis) sufficient for the new concept of quantum non-locality. Providing, for instance, the information that a certain probability distribution is outcome dependent$_1$ does not tell you whether it can violate Bell inequalities or not. The crucial fact is whether double parameter dependence of a certain kind holds, and α-parameter dependence$_2$ and β-parameter dependence$_2$ at least play a *certain* role in this complex condition. Outcome dependence$_1$, however, does *not* play any *essential* role for the general concept of quantum non-locality. Not being able to capture the new concept, we conclude that the partition according to Jarrett's categories outcome dependence$_1$ and parameter dependence$_2$ is inappropriate or unnatural for the analysis of quantum non-locality.

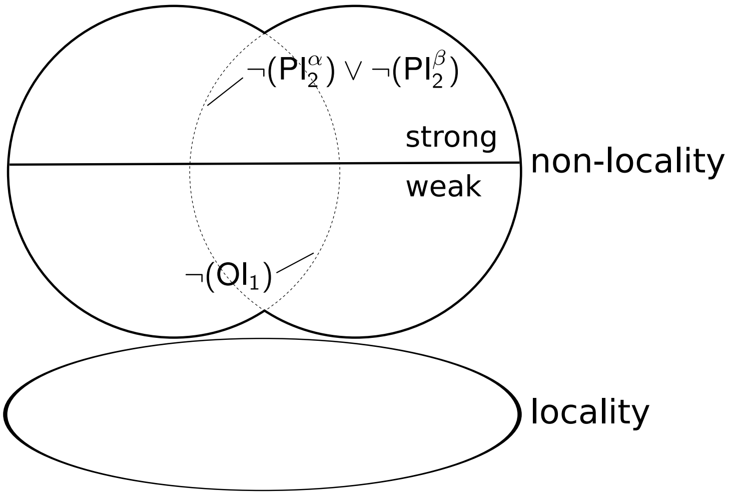

Fig. 2: Outcome dependence$_1$ and parameter dependence$_2$ vs. weak and strong non-locality. 'Strong non-locality' means *strong*$^\alpha$ and *strong*$^\beta$ *non-locality* (i.e. those distributions which can violate Bell inequalities), while 'weak non-locality' means *weak*$^\alpha$ or *weak*$^\beta$ *non-locality* (i.e. those non-local distributions which imply Bell inequalities).

So it seems that a significant amount of the debate after Jarrett's paper which has focused on the question of the formal, physical and metaphysical differences between outcome dependence$_1$ and parameter dependence$_2$, in order to decide which of the two does hold, is misguided. *'Outcome dependence or parameter dependence?' is just the wrong question if one wants to explore deeper into the nature of quantum non-locality*, because each of the two options subsumes probability distributions which can and such which cannot violate Bell inequalities. Making this question a guide to quantum non-locality is like asking whether those humans which can get pregnant have dark or fair hair. Rather, the natural question, the new analysis shows, is which of the outcomes is double parameter dependent and whether it is double parameter dependent$_1$ or double parameter dependent$_2$.

4. Conclusion

In this paper we have investigated what EPR/B correlations, which violate Bell inequalities, imply on a qualitative probabilistic level. We started our considerations by giving a comprehensive overview of the possible types of probability distributions, which can describe EPR/B experiments (table 1). The overview has revealed that one can derive Bell inequalities not only from local theories but also from a large range of non-local ones, which we have called weakly non-local. This has enabled us to formulate a stronger Bell argument than usual to exclude local *and* weakly non-local theories of the quantum world. Since the result of the Bell argument defines what we appropriately call quantum non-locality, this new result yielded a tighter, more informative concept of quantum non-locality, which describes more precisely what the violation of Bell inequalities implies on a probabilistic level. In fact, my new concept of quantum non-locality, although not being sufficient for EPR/B correlations, captures the strongest possible consequences of EPR/B correlations on a *qualitative* probabilistic level. In this sense, my new concept is appropriate, while the standard concept is too weak. Furthermore, the argument shows that Bell inequalities are *not* locality conditions (because weakly non-local theories obey them) and that their violation does *not* necessarily imply that there is no screener-off for the correlations.

In a second part, we have given an analysis of the new concept of quantum non-locality, similar to how Jarrett analysed the failure of local factorisation. We have provided an exact logical expression in terms of pairwise independences, which is equivalent to the new concept. (It includes two types of parameter dependences with respect to each outcome, namely parameter dependence$_2$, which is the usual parameter dependence, and parameter dependence$_1$, which differs from the usual parameter dependence in that the conditional variables include the other outcome respectively.) A general feature of the result is that at least one of the outcomes must depend on both parameters, either including parameter dependence$_1$ or parameter dependence$_2$ (while outcome dependence is neither necessary nor sufficient for the new concept quantum non-locality). Confirming the information theoretic considerations by Maudlin (and Pawlowski et al.) on a probabilistic level, this result resolves the tension between Maudlin's and Jarrett's analysis in favour of the former. Jarrett's result does not bring out this necessary requirement of EPR/B correlations. Rather, it is liable to be misunderstood to the contrary meaning that one can avoid any dependence between the outcomes and their distant parameters if one accepts a dependence between the outcomes. (This understanding, however, is not what it literally says.) A closer examination revealed that even in a literal reading the result of Jarrett's analysis, 'outcome dependence or parameter dependence', is deceptive: it does not cut the probability distributions at their natural joints (it mixes theories which can violate Bell inequalities with such which cannot). So it turns out that asking whether outcome dependence or parameter dependence holds is a deeply misleading question if one aspires to understand quantum non-locality.

These deficiencies of Jarrett's analysis require that the debate based on the inappropriate disjunction 'outcome dependence or parameter dependence' needs a fundamental revision. There are two main issues. First, regarding the question whether EPR/B correlations are compatible with relativity, many have argued that parameter dependent$_2$ theories are forbidden by relativity, while outcome dependent (and parameter independent$_2$) theories might peacefully coexist with relativity. However, we now know that theories in this latter class which correctly describe the EPR/B correlations, e.g. quantum mechanics, must be parameter dependent$_1$. As we cannot avoid parameter dependence in some sense, we have to check the alleged inconsistency of parameter dependence and relativity again. Is the so far widely neglected parameter dependence$_1$ as problematic for relativity as parameter dependence$_2$ has been said to be? Do the arguments which have been adduced against parameter dependence$_2$ apply to parameter dependence$_1$ as well? Either at least one of them can live in harmony with Lorentz invariance or no theory correctly reproducing the EPR/B correlations by a quantum non-locality can be compatible with relativity on a fundamental level.

Second, concerning the metaphysical implications of quantum non-locality it has been argued that while parameter dependence$_2$ requires a causal relation (action at-a-distance), outcome dependence is best understood as a non-causal connection (non-separability / holism). Since one cannot take refuge in outcome dependence any more: does that mean that we necessarily have to accept action at-a-distance? If yes, between which variables? Or can the idea of a non-separability be made intelligible even for parameter dependent theories? These and similar questions have to be addressed. Some old arguments might be transferred to the new situation, some might not. In any case, the debate will need a fresh look. It seems that we are far from being finished with our enquiries into the nature of quantum non-locality, since it has just received new probabilistic foundations.

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A. Mathematical Appendix

A.1. Proof of theorem 1

In order to prove theorem 1 it will provide useful to partition the classes into four groups depending on which variables appear in their constituting product forms (see table 1, column VIII):

(i) At least one of the parameters does not appear at all: (H_{17}^{α})–(H_{21}^{α}), (H_{23}^{α})–(H_{28}^{α}), (H_{30}^{α})–(H_{32}^{α})

(ii) Both parameters appear separately, one in each factor: (H_{22}^{α}), (H_{29}^{α})

(iii) As (ii) but the first factor additionally involves the outcome $\boldsymbol{\beta}$: (H_{15}^{α}), (H_{16}^{α})

(iv) Both parameters appear together in at least one of the factors: (H_{1}^{α})–(H_{14}^{α})

While group (iv) is identical to the group of strongly non-local$^{\alpha}$ classes, the groups (i)–(iii) provide a finer partition of the union of the local$^{\alpha}$ and weakly non-local$^{\alpha}$ classes. In terms of the partition (i)–(iv), confirming theorem 1 means to prove that *given autonomy and perfect (anti-)correlations, the classes belonging to groups (i), (ii) and (iii) imply Bell inequalities.* Additionally, we shall show that classes in group (iv) in general do *not* imply Bell inequalities.

A.1.1. Group (i)

Consider one version of the Wigner-Bell inequality (Wigner, 1970; van Fraassen, 1989),

$$P(\alpha_-\beta_+|a_1b_3) \leq P(\alpha_-\beta_+|a_1b_2) + P(\alpha_-\beta_+|a_2b_3). \tag{7}$$

We can write the probabilities in terms of the hidden probability distribution if we sum over λ,

$$P(\alpha_-\beta_+|ab) = \sum_{\lambda} P(\alpha_-\beta_+|ab\lambda)P(\lambda|ab), \tag{8}$$

and assuming autonomy (A), we can further rewrite it as

$$P(\alpha_-\beta_+|ab) = \sum_{\lambda} P(\alpha_-\beta_+|ab\lambda)P(\lambda). \tag{9}$$

It is obvious that in this form the empirical joint probability $P(\alpha_-\beta_+|ab)$ depends on the parameters only via the hidden joint probability $P(\alpha_-\beta_+|ab\lambda)$. Hence, if a certain parameter does not appear in a specific product form of the hidden joint probability (group (i)), the empirical joint probability becomes independent of this parameter. Consider, for instance, how class (H_{17}^{α}), the product form of which does not involve the parameter $\boldsymbol{b}$,

$$P(\alpha\beta|ab\lambda) = P(\alpha|\beta a\lambda)P(\beta|a\lambda), \tag{10}$$

makes the empirical joint probability independent of $\boldsymbol{b}$:

$$P(\alpha_-\beta_+|ab) = \sum_\lambda P(\alpha_-|\beta_+a\lambda)P(\beta_+|a\lambda)P(\lambda) = P(\alpha_-\beta_+|a). \tag{11}$$

Inserting this empirical joint probability, which does not depend on $\boldsymbol{b}$, into the Bell-Wigner inequality, reveals that in this case the inequality holds trivially, just because it has lost its functional dependence on $\boldsymbol{b}$:[14]

$$P(\alpha_-\beta_+|a_1) \leq P(\alpha_-\beta_+|a_1) + P(\alpha_-\beta_+|a_2) \tag{12}$$

(H^α_{17}) implying that Bell inequalities hold is surprising because its constituting product form is both *non-local* and *non-factorising*: $\boldsymbol{\beta}$ depends on the distant parameter $\boldsymbol{a}$ in the second factor, $P(\beta|a\lambda)$, and $\boldsymbol{\alpha}$ depends on $\boldsymbol{\beta}$ in the first, $P(\alpha|\beta a\lambda)$, i.e. $\boldsymbol{\lambda}$ and $\boldsymbol{a}$ do *not* screen-off the outcomes from another. However, very similarly, we can show that all other classes in group (i) meet the requirements of Bell inequalities: no matter what kind of non-localities they involve, if at least one of the parameters does not appear in the product form, Bell inequalities hold trivially. Hence, we can conclude that if autonomy holds (which we have used to simplify the expectation value in (9)) distributions in group (i) imply that Bell inequalities hold:[15]

$$\left[(\mathrm{A}) \wedge \Bigg(\bigvee_{\substack{i=17-21\\23-28\\30-32}} (\mathrm{H}^\alpha_i)\Bigg)\right] \rightarrow (\mathrm{BI}) \tag{13}$$

A.1.2. Group (ii)

Let us now turn to distributions in group (ii). Since according to this group both parameters appear in the product form (one in each factor), it is clear that, contrary to group (i), here Bell inequalities do not hold just because of the functional dependences. However, local factorisation (H^α_{29}) belongs to this group and we know how we can derive Bell inequalities with this product form of the hidden joint probability. Since the derivation from the other class in this group, (H^α_{22}), is very similar, let me first sketch a derivation with local factorisation, which is based on the ideas of Wigner (1970) and van Fraassen (1989).

We proceed from the empirical fact that there are perfect correlations between the measurement outcomes if the settings equal another:

$$P(\alpha_\pm\beta_\mp|a_ib_i) = 0 \tag{14}$$

Similarly to (9), using autonomy and local factorisation we can rewrite the empirical

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[14] Note that (11) even *directly* contradicts the empirical distribution (not only indirectly by making Bell inequalities true), because it states that the empirical joint probability does not depend on one of the parameters, which is wrong.

[15] The sign '$\bigvee$' denotes a multiple disjunction, e.g. $\bigvee_{i=1..n}(\mathrm{H}^\alpha_i) := (\mathrm{H}^\alpha_1) \vee (\mathrm{H}^\alpha_2) \vee \ldots \vee (\mathrm{H}^\alpha_n)$.

joint probability in terms of the hidden joint probability,

$$P(\alpha_\pm\beta_\mp|a_ib_i) = 0 = \sum_\lambda P(\lambda)P(\alpha_\pm|a_i\lambda)P(\beta_\mp|b_i\lambda). \tag{15}$$

Since probabilities are non-negative (and we assume $P(\lambda) > 0$ for all λ), at least one of the two remaining factors in each summand must be zero, i.e. for all values of i and λ we must have:

$$\begin{aligned} &\big[P(\alpha_+|a_i\lambda) = 0 \quad \vee \quad P(\beta_-|b_i\lambda) = 0\big] && (16)\\ \wedge \quad &\big[P(\alpha_-|a_i\lambda) = 0 \quad \vee \quad P(\beta_+|b_i\lambda) = 0\big] && (17)\end{aligned}$$

There are two cases. Suppose first that $P(\alpha_+|a_i\lambda) = 0$. From there all other probabilities follow as either 0 or 1:

$$\overset{\text{(CE)}}{\Rightarrow} P(\alpha_-|a_i\lambda) = 1 \qquad \overset{(17)}{\Rightarrow} P(\beta_+|b_i\lambda) = 0 \qquad \overset{\text{(CE)}}{\Rightarrow} P(\beta_-|b_i\lambda) = 1$$

Here, '(CE)' stands for 'complementary event' and refers to a theorem of probability theory that the sum of the probability of an event A and of its complementary event $\overline{A}$ is 1, e.g. $P(\alpha_+|a_i\lambda) + P(\alpha_-|a_i\lambda) = 1$.

Assume, second, that $P(\beta_-|b_i, \lambda) = 0$. Again all other probabilities are determined to be either 0 or 1:

$$\overset{\text{(CE)}}{\Rightarrow} P(\beta_+|b_i\lambda) = 1 \qquad \overset{(17)}{\Rightarrow} P(\alpha_-|a_i\lambda) = 0 \qquad \overset{\text{(CE)}}{\Rightarrow} P(\alpha_+|a_i\lambda) = 1$$

In order to avoid contradiction the two cases have to be disjunct. So given a certain measurement direction i, the two cases define a partition of the values of $\boldsymbol{\lambda}$: all values of $\boldsymbol{\lambda}$ for which $P(\alpha_+|a_i\lambda) = 0$ belong to the set $\Lambda(i)$, while all other values, for which $P(\alpha_-|a_i\lambda) = 0$, belong to $\overline{\Lambda(i)}$. Note that each value of i defines a different partition.

We can use the fact that the $\boldsymbol{\lambda}$-partitions depend on just *one* parameter i to calculate the hidden joint probability $P(\alpha\beta|ab\lambda)$ for *any* choice of measurement directions a_ib_j by forming intersections of partitions for different parameters (see table 4). Since all values are either 0 or 1 we have shown that determinism holds on the hidden level.

Given table 4, i.e. determinism and the composability of the $\boldsymbol{\lambda}$-partitions (each of which depends on just one parameter), it is easy to show that Wigner-Bell inequalities must hold. Consider the inequality

$$P(X \cap \overline{Z}) \leq P(X \cap \overline{Y}) + P(Y \cap \overline{Z}), \tag{18}$$

which in general holds for any events X, Y, Z of a measurable space (the validity of the inequality is obvious if one draws a Venn diagram, see (Neapolitan and Jiang, 2006)). Assuming $X = \Lambda(1)$, $Y = \Lambda(2)$ and $Z = \Lambda(3)$ gives the inequality

$$P(\Lambda(1) \cap \overline{\Lambda(3)}) \leq P(\Lambda(1) \cap \overline{\Lambda(2)}) + P(\Lambda(2) \cap \overline{\Lambda(3)}). \tag{19}$$

We can now express the probabilities in the inequality by the empirical probability

Table 4: Values of the hidden joint probability

	$\lambda \in$			
	$\Lambda(i) \cap \Lambda(j)$	$\Lambda(i) \cap \overline{\Lambda(j)}$	$\overline{\Lambda(i)} \cap \Lambda(j)$	$\overline{\Lambda(i)} \cap \overline{\Lambda(j)}$
$P(\alpha_+\beta_+ \vert a_i b_j \lambda) =$	0	0	0	1
$P(\alpha_+\beta_- \vert a_i b_j \lambda) =$	0	0	1	0
$P(\alpha_-\beta_+ \vert a_i b_j \lambda) =$	0	1	0	0
$P(\alpha_-\beta_- \vert a_i b_j \lambda) =$	1	0	0	0

distribution if we use the hidden joint probability from table 4, e.g.:

$$\begin{aligned} P(\Lambda(1) \cap \overline{\Lambda(2)}) &\overset{(\sigma\text{-additivity})}{=} \sum_{\lambda \in \Lambda(1) \cap \overline{\Lambda(2)}} P(\lambda) = \\ &\overset{(\text{table } 4)}{=} \sum_{\lambda} P(\lambda) P(\alpha_-\beta_+ | a_1 b_2 \lambda) = \\ &\overset{(\text{A})}{=} P(\alpha_-\beta_+ | a_1 b_2) \end{aligned} \tag{20}$$

The resulting inequality is the Wigner-Bell inequality (7).

This derivation reminds us how local factorisation together with autonomy and perfect correlations implies Bell inequalities. The other class in group (ii), (H^{α}_{22}), differs from local factorisation in that the parameters are swapped: instead of a dependence of each outcome on the local parameters it involves a dependence on the *distant* ones. Regardless of the implicit non-locality it can be used to derive a Bell inequality in a very similar way: given (H^{α}_{22}), instead of (15) we have

$$P(\alpha_\pm \beta_\mp | a_i b_i) = 0 = \sum_{\lambda} P(\lambda) P(\alpha_\pm | b_i \lambda) P(\beta_\mp | a_i \lambda). \tag{21}$$

and by very similar arguments we arrive at a similar partition of the values of $\boldsymbol{\lambda}$: $\Lambda(i)$ denotes all values of lambda for which $P(\alpha_+ | b_i \lambda) = 0$, while the complementary set $\overline{\Lambda(i)}$ is defined by $P(\alpha_- | b_i \lambda) = 0$. If we then calculate the values of the hidden joint probability we arrive at the very same result as in table 4—and the rest of the derivation runs identically up to the Bell-Wigner inequality (7). Hereby we have found another *non-local* hidden joint probability which implies Bell inequalities and the result for group (ii) reads

$$\left[(\text{A}) \wedge (\text{PCorr}) \wedge \Bigl(\bigvee_{i=22,29} (\text{H}^{\alpha}_{i}) \Bigr) \right] \rightarrow (\text{BI}). \tag{22}$$

A.1.3. Group (iii)

Up to this point one might have been surprised about how easy one can derive Bell inequalities from product forms other than local factorisation, but that one can do it even from classes in group (iii) is my strongest claim. These classes include both parameters, one in each factor, so they do not trivially imply Bell inequalities as classes in group (i). Neither, it seems, can they imply Bell inequalities in the same way as classes in group (ii) because they additionally involve $\boldsymbol{\beta}$ in the first factor. However, they do imply Bell inequalities, and they do it in a very similar (yet slightly more complicated) way than classes in group (ii), if besides perfect correlations for equal settings (14) we also assume perfect anti-correlations (PACorr) for perpendicular settings $(a_i b_{i_\perp})$:

$$P(\alpha_\pm, \beta_\pm | a_i, b_{i_\perp}) = 0 \tag{23}$$

Let me sketch the proof for class $(\mathrm{H}_{16}^{\alpha})$ which follows along the lines of that for local factorisation. By autonomy and the product form of $(\mathrm{H}_{16}^{\alpha})$ we rewrite (14) and (23) as

$$P(\alpha_\pm \beta_\mp | a_i b_i) = 0 = \sum_\lambda P(\lambda) P(\alpha_\pm | \beta_\mp a_i \lambda) P(\beta_\mp | b_i \lambda) \tag{24}$$

$$P(\alpha_\pm \beta_\pm | a_i b_{i_\perp}) = 0 = \sum_\lambda P(\lambda) P(\alpha_\pm | \beta_\pm a_i \lambda) P(\beta_\pm | b_{i_\perp} \lambda), \tag{25}$$

and again, at least one of the factors in each summand must vanish, i.e. for all values of i and $\boldsymbol{\lambda}$ (assuming $P(\lambda) > 0$) we must have:

$$\begin{aligned}
&\left[P(\alpha_+|\beta_- a_i\lambda) = 0 \quad \vee \quad P(\beta_-|b_i\lambda) = 0\right] && (26)\\
\wedge\ &\left[P(\alpha_-|\beta_+ a_i\lambda) = 0 \quad \vee \quad P(\beta_+|b_i\lambda) = 0\right] && (27)\\
\wedge\ &\left[P(\alpha_+|\beta_+ a_i\lambda) = 0 \quad \vee \quad P(\beta_+|b_{i_\perp}\lambda) = 0\right] && (28)\\
\wedge\ &\left[P(\alpha_-|\beta_- a_i\lambda) = 0 \quad \vee \quad P(\beta_-|b_{i_\perp}\lambda) = 0\right] && (29)
\end{aligned}$$

As above, from these conditions we can infer that all involved probabilities must be 0 or 1, depending on which of the following two cases holds.

If $P(\alpha_+|\beta_- a_i\lambda) = 0$:

$$\begin{aligned}
&\overset{\text{(CE)}}{\Rightarrow} P(\alpha_-|\beta_- a_i\lambda) = 1 \qquad && \overset{(29)}{\Rightarrow} P(\beta_-|b_{i_\perp}\lambda) = 0\\
&\overset{\text{(CE)}}{\Rightarrow} P(\beta_+|b_{i_\perp}\lambda) = 1 && \overset{(28)}{\Rightarrow} P(\alpha_+|\beta_+ a_i\lambda) = 0\\
&\overset{\text{(CE)}}{\Rightarrow} P(\alpha_-|\beta_+ a_i\lambda) = 1 && \overset{(27)}{\Rightarrow} P(\beta_+|b_i\lambda) = 0\\
&\overset{\text{(CE)}}{\Rightarrow} P(\beta_-|b_i\lambda) = 1 &&
\end{aligned}$$

If $P(\beta_-|b_i\lambda) = 0$:

$$\begin{array}{llll}
\overset{\text{(CE)}}{\Rightarrow} & P(\beta_+|b_i\lambda) = 1 & \overset{(27)}{\Rightarrow} & P(\alpha_-|\beta_+a_i\lambda) = 0 \\
\overset{\text{(CE)}}{\Rightarrow} & P(\alpha_+|\beta_+a_i\lambda) = 1 & \overset{(28)}{\Rightarrow} & P(\beta_+|b_{i_\perp}\lambda) = 0 \\
\overset{\text{(CE)}}{\Rightarrow} & P(\beta_-|b_{i_\perp}\lambda) = 1 & \overset{(29)}{\Rightarrow} & P(\alpha_-|\beta_-a_i\lambda) = 0 \\
\overset{\text{(CE)}}{\Rightarrow} & P(\alpha_+|\beta_-a_i\lambda) = 1 & &
\end{array}$$

The cases are disjunct and, hence, define a partition for the values of $\boldsymbol{\lambda}$ for each measurement direction i: $\Lambda(i)$ includes all values of $\boldsymbol{\lambda}$ for which $P(\beta_+|b_i\lambda) = 0$, while $\overline{\Lambda(i)}$ includes the complementary values, which make $P(\beta_-|b_i\lambda) = 0$. Calculating the hidden joint probability $P(\alpha\beta|ab\lambda)$ for an arbitrary choice of measurement directions a_ib_j gives us the very same result as in table 4—and again a Wigner Bell inequality follows.

Since the derivation for class (H_{15}) runs mutatis mutandis, we have shown:

$$\left[(\mathrm{A}) \wedge (\mathrm{PCorr}) \wedge (\mathrm{PACorr}) \wedge \left(\bigvee_{i=15,16} (\mathrm{H}_i^\alpha)\right)\right] \rightarrow (\mathrm{BI}) \tag{30}$$

A.1.4. Group (iv)

Finally, classes of group (iv) do *not* imply Bell inequalities. Involving both parameters in at least one of the factors, they neither fulfill Bell inequalities by their functional dependences nor do they admit of deriving a Bell inequality in the manner of classes in group (ii) or (iii). In order to rule out that there are other kinds of derivations one has to find explicit examples of probability distributions for each class in the group which *violate* Bell inequalities. Requiring just *any* example we can assume a toy model with only two possible hidden states ($\boldsymbol{\lambda} = 1, 2$). Then the probability distribution $P(\alpha\beta ab\lambda)$ is determined by assigning a value to each of the $2^5 = 32$ probabilities which conform to the laws of probability theory (each value lies in the interval $[0, 1]$ and all values sum to 1). Furthermore, the values have to be chosen such that autonomy and the specific product form of the class in question hold and that Bell inequalities are violated. I have found appropriate distributions for each class (H_1^α)–(H_{14}^α) by solving numerically a corresponding set of equations. This fact, that some probability distributions of these classes violate Bell inequalities, means that none of these classes implies Bell inequalities in general, i.e. by its constituting product form. Of course, this does *not* mean that *all* probability distributions in these classes *violate* Bell inequalities: in fact one can as well find examples of probability distributions in each class (H_1^α)–(H_{14}^α) which *fulfill* Bell inequalities. This means that for these classes the product form alone does not determine whether Bell inequalities hold or fail; whether they do depends on the numerical values of the specific distribution. On the general level of the classes we can only say that classes in group (iv) neither imply Bell inequalities nor do they imply their failure. Probability distributions in those classes *can* violate Bell inequalities.

A.2. Proof of theorem 2

The proof of the equivalences of product forms and conjunctions of independences involves only some basic laws of probability theory. Providing an analysis for *each* product form of the hidden joint probability, this claim is an extension of Jarrett's analysis of local factorisation (P7). One can demonstrate the equivalence for each hidden joint probability separately (analogous to how Jarrett derived (P7)), but the following constructive method is more elegant: in the case that the hidden joint probability factorises according to the product rule, (H_1^α), none of the relevant independences holds (and vice versa). Then we consider the five cases in which exactly *one* independence holds (H_2^α)–(H_6^α). Here is the proof of $(\mathrm{H}_2^\alpha) \leftrightarrow (\ell\mathrm{PI}_2^\beta)$:

$$\boxed{\leftarrow} \quad P(\alpha\beta|ab\lambda) = P(\alpha|\beta ba\lambda)P(\beta|ab\lambda) \overset{(\ell\mathrm{PI}_2^\beta)}{=} P(\alpha|\beta ba\lambda)P(\beta|a\lambda) \tag{31}$$

$$\boxed{\rightarrow} \quad P(\beta|ab\lambda) = \sum_\alpha P(\alpha\beta|ab\lambda) \overset{(\mathrm{H}_2^\alpha)}{=} P(\beta|a\lambda)\sum_\alpha P(\alpha|\beta ba\lambda) = P(\beta|a\lambda) \tag{32}$$

The equivalence $(\mathrm{H}_3^\alpha) \leftrightarrow (\mathrm{PI}_2^\beta)$ can be shown mutatis mutandis (just swap the local with the distant parameter). $(\mathrm{H}_4^\alpha) \leftrightarrow (\ell\mathrm{PI}_1^\alpha)$ can be derived as follows:

$$\boxed{\leftarrow} \quad P(\alpha\beta|ab\lambda) = P(\alpha|\beta ba\lambda)P(\beta|ab\lambda) \overset{(\ell\mathrm{PI}_1^\alpha)}{=} P(\alpha|b\beta\lambda)P(\beta|ab\lambda) \tag{33}$$

$$\boxed{\rightarrow} \quad P(\alpha|\beta ba\lambda) = \frac{P(\alpha\beta|ab\lambda)}{P(\beta|ab\lambda)} \overset{(\mathrm{H}_4^\alpha)}{=} \frac{P(\alpha|\beta b\lambda)\cancel{P(\beta|ab\lambda)}}{\cancel{P(\beta|ab\lambda)}} = P(\alpha|\beta b\lambda) \tag{34}$$

The equivalences $(\mathrm{H}_5^\alpha) \leftrightarrow (\mathrm{PI}_1^\alpha)$ and $(\mathrm{H}_6) \leftrightarrow (\mathrm{OI}_1)$ are proved similarly. Then, by pairs of these five equivalences involving one independence, we prove equivalences with two independences, and subsequently, equivalences with three independences, and so on. Here is an example how to derive an equivalence with two independences, $(\mathrm{H}_7^\alpha) \leftrightarrow (\mathrm{PI}_2^\beta) \wedge (\ell\mathrm{PI}_2^\beta)$, on the basis of the corresponding equivalences with one independence respectively:

$$\boxed{\leftarrow} \quad (\mathrm{PI}_2^\beta) \wedge (\ell\mathrm{PI}_2^\beta) \xleftrightarrow{(31),\,(32)} (\mathrm{PI}_2^\beta) \wedge (\mathrm{H}_2^\alpha) \overset{(35)}{\rightarrow} (\mathrm{H}_7^\alpha)$$

$$P(\alpha\beta|ab\lambda) \overset{(\mathrm{H}_2^\alpha)}{=} P(\alpha|\beta ba\lambda)P(\beta|ab'\lambda) \overset{(\mathrm{PI}_2^\beta)}{=} P(\alpha|\beta ba\lambda)P(\beta|a'b'\lambda) \tag{35}$$

$$\boxed{\rightarrow} \quad (\mathrm{H}_7^\alpha) \overset{(*)}{\rightarrow} (\mathrm{H}_3^\alpha) \leftrightarrow (\mathrm{PI}_2^\beta); \quad (\mathrm{H}_7^\alpha) \overset{(*)}{\rightarrow} (\mathrm{H}_2^\alpha) \leftrightarrow (\ell\mathrm{PI}_2^\beta)$$

$(*)$: (H_7^α) is a common special case of (H_2^α) and (H_3^α); if (H_7^α) holds, then a forteriori (H_2^α) and (H_3^α):

$$\begin{aligned} \forall a, a', b, b' :\quad & P(\alpha\beta|ab\lambda) = P(\alpha|\beta ba\lambda)P(\beta|a'b'\lambda) && (\mathrm{H}_7^\alpha) \\ \forall a = a', b, b' :\quad & P(\alpha\beta|ab\lambda) = P(\alpha|\beta ba\lambda)P(\beta|a'b'\lambda) && (\mathrm{H}_2^\alpha) \\ \forall a, a', b = b' :\quad & P(\alpha\beta|ab\lambda) = P(\alpha|\beta ba\lambda)P(\beta|a'b'\lambda) && (\mathrm{H}_3^\alpha) \end{aligned}$$

As one can see by this constructive method the proofs remain basic and short, even for the more complex equivalences. Similarly, with some patience, we can derive step by step the other equivalences between product forms and independences in table 1.

References

Arntzenius, F. (1994). Space-like connections. *British Journal for the Philosophy of Science 45*(1), 201–217.

Aspect, A., J. Dalibard, and G. Roger (1982). Experimental test of Bell's inequalities using time-varying analyzers. *Physical Review Letters 49*, 1804–1807.

Bell, J. S. (1964). On the Einstein-Podolsky-Rosen paradox. *Physics 1*(3), 195–200.

Bell, J. S. (1971). Introduction to the hidden-variable question. In B. d'Espagnat (Ed.), *Foundations of quantum mechanics: Proceedings of the International School of Physics 'Enrico Fermi', Course XLIX*, pp. 171–181. New York: Academic.

Bell, J. S. (1975). The theory of local beables. *TH-2053-CERN*.

Berkovitz, J. (1998a). Aspects of quantum non-locality i: Superluminal signalling, action-at-a-distance, non-separability and holism. *Studies in History and Philosophy of Modern Physics 29*(2), 183–222.

Berkovitz, J. (1998b). Aspects of quantum non-locality II: Superluminal causation and relativity. *Studies in History and Philosophy of Modern Physics 29*(4), 509–545.

Bohm, D. (1951). *Quantum Theory.* Englewood Cliffs: Prentice-Hall.

Butterfield, J. N. (1992). David Lewis meets John Bell. *Philosophy of Science 59*, 26–43.

Clauser, J. F. and M. A. Horne (1974). Experimental consequences of objective local theories. *Physical Review D 10*(2), 526–535.

Costa de Beauregard, O. (1979). Time symmetry and the Einstein paradox - II. *Il Nuovo Cimento 51B*, 267–279.

Einstein, A., B. Podolsky, and N. Rosen (1935). Can quantum mechanical description of physical reality be considered complete? *Physical Review 47*, 777–780.

Fogel, B. (2007). Formalizing the separability condition in bell's theorem. *Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics 38*(4), 920 – 937.

Graßhoff, G., S. Portmann, and A. Wüthrich (2005). Minimal assumption derivation of a Bell-type inequality. *British Journal for the Philosophy of Science 56*(4), 663–680.

Healey, R. A. (1991). Holism and nonseparability. *Journal of Philosophy 88*, 393–421.

Healey, R. A. (1994). Nonseparable processes and causal explanation. *Studies in History and Philosophy of Science 25*, 337–374.

Howard, D. (1985). Einstein on locality and separability. *Studies in History and Philosophy of Science 16*, 171–201.

Howard, D. (1989). Holism, separability, and the metaphysical implications of the Bell experiments. In J. T. Cushing and E. McMullin (Eds.), *Philosophical Consequences of Quantum Theory: Reflections on Bell's Theorem*, pp. 224–253. Notre Dame: University of Notre Dame Press.

Jarrett, J. P. (1984). On the physical signifance of the locality conditions in the Bell arguments. *Nous 18*, 569–590.

Jarrett, J. P. (1989). Bell's theorem: A guide to the implications. In J. T. Cushing and E. McMullin (Eds.), *Philosophical Consequences of Quantum Theory: Reflections on Bell's Theorem*, pp. 60–79. Notre Dame: University of Notre Dame Press.

Jarrett, J. P. (2009). On the separability of physical systems. In W. C. Myrvold and J. Christian (Eds.), *Quantum Reality, Relativistic Causality, and Closing the Epistemic Circle*, pp. 105–124. Department of Philosophy, University of Illinois at Chicago, IL USA: Springer Netherlands.

Jones, M. R. and R. K. Clifton (1993). Against experimental metaphysics. In P. A. French, T. E. Uehling, and H. K. Wettstein (Eds.), *Philosophy of science*, pp. 295–316. Notre Dame, Ind: University of Notre Dame Press.

Maudlin, T. (1994). *Quantum Non-locality and Relativity : Metaphysical Intimations of Modern Physics.* Oxford: Blackwell.

Maudlin, T. (2011). *Quantum non-locality and relativity : metaphysical intimations of modern physics.* Oxford: Wiley-Blackwell.

Neapolitan, R. E. and X. Jiang (2006). A tutorial on learning causal influence. In D. E. Holmes and L. C. Jain (Eds.), *Innovations in machine learning : theory and applications*, pp. 29–72. Berlin ; New York: Springer.

Norsen, T. (2009). Local causality and completeness: Bell vs. jarrett. *Foundations of Physics 39*(3), 273–294.

Pawlowski, M., J. Kofler, T. Paterek, M. Seevinck, and C. Brukner (2010). Non-local setting and outcome information for violation of Bell's inequality. *New Journal of Physics 12*(8), 083051.

Pearl, J. (2000). *Causality: Models, Reasoning, and Inference.* Cambridge, UK: Cambridge University Press.

Price, H. (1994). A neglected route to realism about quantum mechanics. *Mind 103*, 303–336.

Redhead, M. L. G. (1986). Relativity and quantum mechanics—conflict or peaceful coexistence? *Annals of the New York Academy of Sciences 480*(1), 14–20.

Redhead, M. L. G. (1987). *Incompleteness, Nonlocality, and Realism : A Prolegomenon to the Philosophy of Quantum Mechanics.* Oxford New York: Clarendon Press Oxford University Press.

Redhead, M. L. G. (1989). Nonfactorizability, stochastic causality, and passion-at-a distance. In J. T. Cushing and E. McMullin (Eds.), *Philosophical Consequences of Quantum Theory: Reflections on Bell's Theorem*, pp. 145–153. Notre Dame: University of Notre Dame Press.

Shimony, A. (1984). Controllable and uncontrollable non-locality. In S. Kamefuchi (Ed.), *Foundations of Quantum Mechanics in the Light of New Technology*, pp. 225–230. Tokyo: The Physical Society of Japan.

Shimony, A. (1990). An exposition of Bell's theorem. In A. I. Miller (Ed.), *Sixty-two Years of Uncertainty : Historical, Philosophical, and Physical Inquiries Into the Foundations of Quantum Mechanics*, pp. 33–43. New York ; London: Plenum.

Spirtes, P., C. Glymour, and R. Scheines (2000). *Causation, Prediction, and Search.* Cambridge, MA: The MIT Press.

Sutherland, R. I. (1983). Bell's theorem and backwards-in-time causality. *International Journal of Theoretcial Physics 22*(4), 377–384.

Szabó, L. E. (2000). Attempt to resolve the EPR-Bell paradox via Reichenbach's concept of common cause. *International Journal of Theoretcial Physics 39*, 901–911.

Teller, P. (1986). Relational holism and quantum mechanics. *Brit. J. Phil. Sci. 37*, 71–81.

Teller, P. (1989). Relativity, relational holism and the Bell inequalities. In J. T. Cushing and E. McMullin (Eds.), *Philosophical consequences of quantum theory. Reflections on Bell's theorem*, pp. 208–223. Notre Dame: University of Notre Dame Press.

van Fraassen, B. C. (1989). The charybdis of realism: Epistemological implications of Bell's inequality. In J. T. Cushing and E. McMullin (Eds.), *Philosophical Consequences of Quantum Theory: Reflections on Bell's Theorem*, pp. 97–113. Notre Dame: University of Notre Dame Press.

Wigner, E. P. (1970). On hidden variables and quantum mechanical probabilities. *American Journal of Physics 38*, 1005–1009.