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Scattering of non-uniform incident fields by long cylinders

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Abstract

We investigate experimentally far-field scattering from cylinders with illumination non-uniform along the axis of the cylinder. Scattered intensity as a function of angle in two orthogonal directions is examined. Variation along the scattering angle is found to be little affected by the illumination profile. However, variation in the transverse direction follows closely the Fourier transform of the illumination pattern and reproduces the angular distribution of the incident wave. These finding apply to circular as well as hexagonal cross-section cylinders.

1 Introduction

There is much interest in scattering measurements involving cylindrical particles. Applications include particle characterization and more fundamental issues, such as testing of new theoretical models of scattering, for example for long hexagonal columns [1]. Plane-wave illumination is typically assumed, either for the sake of simplifying interpretation or because models allowing more general incidence patterns do not yet exist for the required cylinder properties. However, in experimental practice plane wave illumination is not achievable: laser beams with Gaussian irradiance profiles are routinely used, focused to various degrees. Furthermore, producing long cylinders of uniform thickness is not always possible, for example when non-circular cross-sections are required [2]. This gives rise to conflicting demands on the dimensions of the incident beam: on the one hand, a narrow beam is needed so that only a short, relatively uniform section of the cylinder is illuminated; on the other hand, a beam sufficiently broad is required so as to provide uniform illumination across the width of the cylinder, i.e. to ensure that the equivalent incident angular spectrum of plane waves is narrow. A solution to this problem appears to be provided by using a beam with non-circular cross-section (for example elliptical) with the shorter beam dimension along the cylinder axis. However, a question arises whether the narrowness of the beam influences the scattering pattern, most notably as a function of the polar angle θ, defined as the projection of the scattering angle onto a plane perpendicular to the cylinder – see Fig. 1 for the coordinate system adopted here.

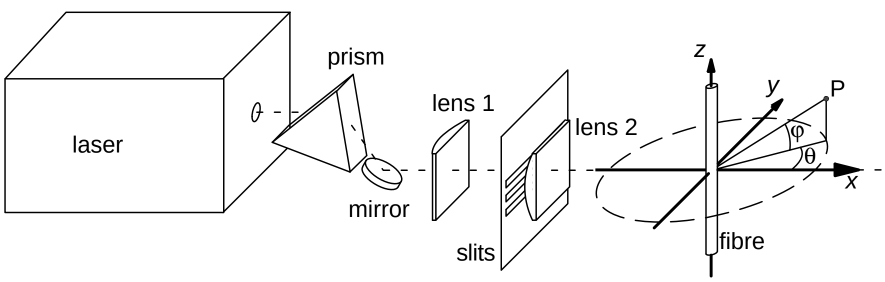

Fig. 1. Schematic experimental arrangement and the coordinate system, showing elevation angle φ and polar angle θ of point P where irradiance is measured.

A rigorous theory of scattering by homogeneous circular cylinders illuminated by a Gaussian beam modelled as an angular spectrum of plane waves was developed by Lock, who then went on to examine the influence of the beam structure on the angular dependence of scattering [3]. In particular, Lock found that scattering only slowly modulated the irradiance in the direction of the elevation angle φ. This variation of irradiance was dominated by Fraunhofer-diffracted amplitude of the focal plane incidence profile. In other words, the amplitude variation along φ was approximately the Fourier transform of the incident (Gaussian) amplitude profile. On this basis Lock conjectured that, in general, "*the scattered intensity at each* [elevation] *coordinate on the viewing screen for arbitrary* [incident beam will] *contain the imprint of a single plane-wave component in the* [angular] *Fourier spectrum of the incident beam*" [3]. This conjecture does not appear to have been previously tested.

We study experimentally the scattering from cylinders with non-uniform illumination. In particular, we use an elliptical Gaussian beam as well as diffraction patterns from single and multiple slits, so as to provide different angular spectra of constituent incident plane waves. The variation of scattering with the elevation angle φ and the polar angle θ under different illumination conditions is examined.

2 Results

If an aperture placed in front of a lens is illuminated by a plane wave, the field $E(z)$ at the focal point of the lens is proportional to the Fourier transform of the field distribution immediately following the aperture, or aperture function $A(z')$ – see e.g. reference [4]. In a one-dimensional case we can write

$$E(z) \propto \int_{-\infty}^{\infty} A(z') \exp\left(\frac{-2\pi i z z'}{f\lambda}\right) dz' \qquad (1)$$

where z' is the spatial coordinate at the aperture, λ is the wavelength of light and f the focal length. Note that $\sin\varphi = z'/f$, allowing us to express the aperture function in terms of the elevation angle φ. Also, in the case of Gaussian illumination f is replaced by the distance from the lens to the beam waist. If a fibre is placed at the focal point then accordingly to Lock's conjecture [3] the φ-dependence of scattering is a Fourier transform of the focal plane amplitude profile. However, since repeated application of the Fourier transform recovers the original function with reversed sign of the argument, i.e. $\mathrm{F}(\mathrm{F}(A(\varphi)) = 2\pi A(-\varphi)$, the φ-direction scattering profile can be expected to be a mirror image of the lens aperture function. We will now proceed to show that this is indeed the case.

Light scattering measurements were done using a fibre-optics diffractometer covering the range of scattering angles from 3 to 177° [5]. A multi-line Ar-ion laser was the light source and a prism was used to select the wavelength, 514.5 nm. The direction of polarization was vertical. Two cylindrical lenses with focal lengths 200 and 50 mm and acting in the horizontal and vertical plane, respectively, were used together with anamorphic magnification from the prism to focus the beam into elliptical cross-section – see Fig. 1. The beam-waist semi-axes were 80 μm horizontally and 13 μm vertically. Diffraction slits aligned horizontally, cut into a 200 μm thick copper sheet, were put just in front of the second lens. Single glass fibres, made as described previously [2], were placed at the beam waist with their axis vertical. Irradiance profiles at the beam waist were obtained by imaging a fibre onto a CCD camera using a 0.16 numerical aperture microscope objective. Scattering profiles as a function of elevation angle and two-dimensional scattering were recorded using projection screens and a CCD camera.

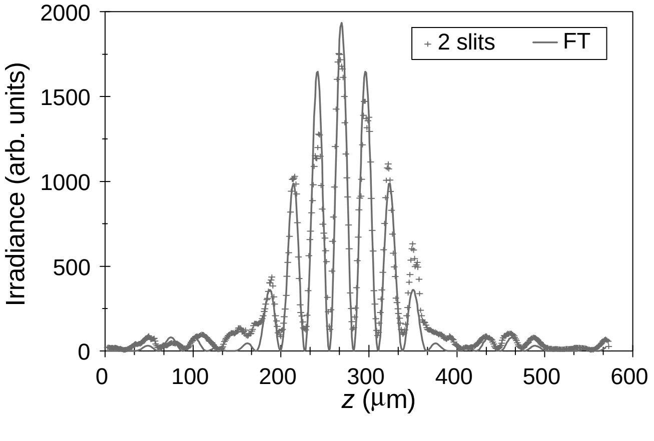

Fig. 2. Irradiance profile at the beam waist measured along the z coordinate, with a double slit placed in front of the second focusing lens. The square of the Fourier transform of the aperture function of the slit is shown for comparison (solid line) assuming uniform (not Gaussian) slit illumination; vertical scale has been adjusted for best fit.

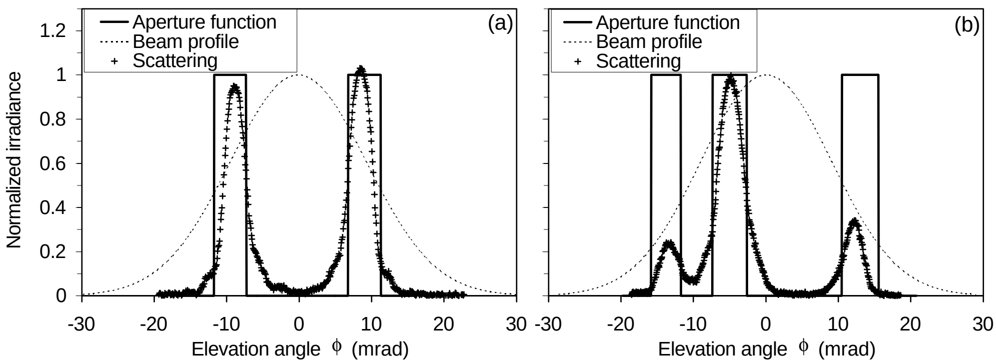

Fig. 3. Measured irradiance profiles as a function of the elevation angle φ: incident beam at the slit (best fit, broken line), slit aperture function for uniform illumination (solid line), and scattering from a 4 μm diameter glass fibre illuminated by a slit diffraction pattern, measured at $\theta = 10°$ (+). Plots for a double slit (a) and an asymmetric triple slit (b) are shown.

Fig. 2 shows a measured profile of irradiance on a fibre illuminated with a focused diffracted wave produced by a double slit. The profile closely follows that calculated from the the Fourier transform of the lens amplitude function - despite a simplifying assumption of uniform slit illumination. Since only the irradiance at the fibre and not the amplitude (and phase) is known, we cannot apply the Fourier transform directly to predict the scattered profile. However, this is not strictly necessary since the scattered profile can be expected to be a mirror image of the aperture function. Therefore, we can simply compare the scattered profile measured along the φ coordinate with the irradiance at the focusing lens expressed as a function of angle, as we show in Fig. 3. For both the double slit and an asymmetric triple slit the profiles can be seen to agree quite well, considering that the slits and the lens were imperfect and the incident illumination was from a Gaussian beam, not a plane wave. Fig. 4 compares scattered patterns as a function of scattering angle for various illumination profiles. There is very little variation between the three cases shown and the patterns closely resemble theoretical prediction calculated for a homogeneous cylinder illuminated by a plane wave [6]. We have also examined scattering from hexagonal glass fibres [2] and observed that in qualitative terms our findings are not limited to circular cross-section cylinders – see Fig. 5. Moreover, the φ-coordinate dependence of scattering appears to be preserved at all scattering angles – see Fig. 6. Consequently, the scattering is concentrated in more or less separate cones with a common vertex at the fibre, each cone corresponding to a dominant frequency in the angular spectrum of incident plane waves, or in our case a slit in the aperture pattern.

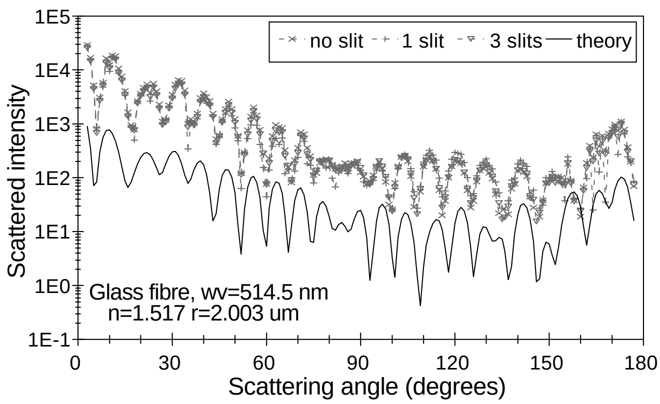

Fig. 4. Scattered intensity as a function of scattering angle for a circular cross-section fibre with a diameter of about 4 μm and refractive index of 1.517, illuminated by an elliptical Gaussian beam (×) and diffraction patterns produced by one (+) or three (∇) slits. Computed best-fit pattern is also shown (solid line, offset vertically) [6].

3 Conclusions

We have investigated scattering from long cylinders illuminated non-uniformly along their axis. The variation of scattering along the direction of scattering angle is little affected by the illumination profile. The scattered irradiance profile along the elevation angle φ is approximately proportional to the square of the Fourier transform of the illumination amplitude. For the case of a fibre at the focal point of a lens, the φ-direction scattering profile closely resembles the mirror image of the lens illumination pattern. Our findings justify the practice of using non-circular beams strongly focused in one direction in scattering experiments on cylindrical scatterers.

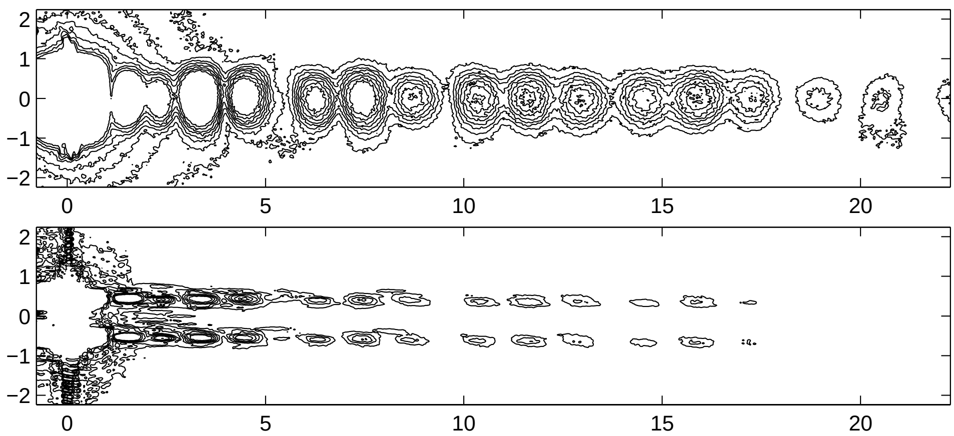

Fig. 5. Contour plots of two-dimensional scattering patterns produced by a hexagonal cross-section fibre about 27 μm in diameter [2]. In the upper panel the incident beam is unobstructed; in the lower one a double slit is in the beam. The horizontal coordinate is θ, the vertical φ, the units are degrees. Low-angle peaks as truncated due to saturation.

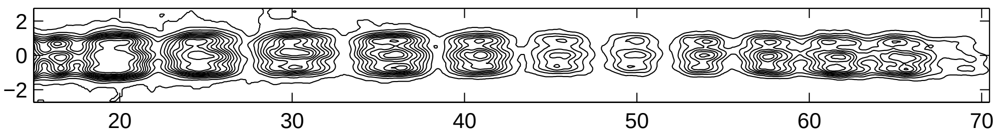

Fig. 6. Contour plot of a two-dimensional scattering pattern produced by a hexagonal cross-section fibre about 8 μm in diameter [2]. The horizontal coordinate is θ, the vertical φ, the units are degrees.

Acknowledgement

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