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Regressive intracohort redistribution in nonfinancial defined contribution pension

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Abstract

Nonfinancial defined contribution (NDC) pension systems have recently become popular because they provide the strong incentives of the private funded systems without requiring a difficult transition period. Using the framework of mechanism design, these systems have theoretically been criticized because they neglect the regressive intracohort redistribution: longer lived workers retire later and are rewarded as if their life expectancies were average. Now we document this by Hungarian data, and giving up the framework of mechanism design, we corroborate our earlier qualitative findings withthe more realistic benefit adjustment function and wage heterogeneity.

Keywords: nonfinancial defined contributions, variable retirement, adverse selection, actuarial fairness

JEL classification: C61, C63, D82, D91, H55

Korosztályon belüli regresszív újraelosztás az eszmei nyugdíjszámlájú nyugdíjrendszerben

András Simonovits

Összefoglaló

Az eszmei nyugdíjszámlájú nyugdíjrendszerek mostanában nagyon népszerűvé váltak, mert a nélkül teremtették meg a tőkésített magánnyugdíj-rendszerek erős ösztönzőit, hogy szükség lett volna a nehéz átmeneti időszakra. A mechanizmustervezés keretét alkalmazva elméletileg bíráltuk ezeket a rendszereket, mert elhanyagolták a korosztályon belüli regresszív újraelosztást: a hosszabb életűek később mennek nyugdíjba és úgy jutalmazzák őket, mintha a várható élettartamuk átlagos lenne. A tanulmányban magyar adatokkal dokumentáljuk a kontraszelekciót és – feladva a mechanizmustervezést – megerősítjük korábbi kvalitatív eredményeinket reálisabb járadékigazítási függvénnyel és heterogén keresetekkel.

Tárgyszavak: eszmei nyugdíjszámla, változó nyugdíjba vonulás, kontraszelekció, biztosításmatematikai méltányosság

JEL kódok: C61, C63, D82, D91, H55

1. Introduction

NDC is an acronym for *nonfinancial (or earlier: notional) defined contribution* and it refers to newly introduced pension systems, where an unfunded system mimics a funded one without suffering from the costly transition period. The basic idea of the NDC scheme is paying a *life annuity*: the annual pension benefit is calculated as the ratio of the stock of accumulated nonfinancial contributions and the remaining life expectancy. This allows for variable (flexible) retirement age: those, who want to retire before/after reaching the full benefit retirement age, receive a properly reduced/increased annual benefit. (This principle of the so-called *actuarially fair adjustment* or *delayed retirement credit* is applied well beyond the NDC scheme, notably in the very progressive US Social Security system. N.B. A progressive scheme redistributes income from the rich to the poor.) If workers only differed in their labor disutilities or in the dates of claiming their benefits, then the NDC scheme would provide perfect incentives, maximize social welfare and ensure the balance between revenues and expenditures (Holzmann and Palmer, 2006 and Holzmann, Palmer and Robalino, 2012). (The impact of *delayed claiming* social security benefits was analyzed theoretically as well as empirically by Coile, Diamond, Gruber and Jousten (2002).)

A lot of researchers have underlined that—apart from differences between males and females—life expectancy is strongly and positively correlated with the lifetime average earnings. Therefore under tight link between lifetime contributions and annual benefits, there is a hidden regressive lifetime redistribution from the poorer to the richer (e.g. Breyer and Hupfeld, 2009). Krémer (2012) observed that the quite unequal distribution of the Hungarian *entry* pensions becomes quite equal in the distribution of *all* pensions partly because pensioners with low benefits die much earlier than the others. (The other part of this discrepancy is caused by the slow phasing-out of earlier granted progressive benefits.) Similarly, Divényi and Kézdi (2012) proved empirically that the poor health status of the Hungarian population above 50 years is one important cause of the low old-age employment and by implication, of low but heterogeneous life expectancies.

The proponents of the NDC scheme have discussed several problems of *intercohort redistribution* of NDC, most notably, how to choose the fictitious interest rate and the remaining life expectancy under rising life expectancy and sinking fertility (for a fresh analysis, see Knell (2012).) Following Diamond and Mirrlees (1978) on disability retirement, another group of analysts have criticized the *regressive intracohort redistribution* in the NDC scheme, or more generally, in actuarial fair adjustment in old-age pensions: Fabel (1994), Eső and Simonovits (2002), Diamond (2003, Chapter 7), Cremer, Lozachmeur and Pestieau (2004), Simonovits (2006), Sheshinski (2008), Bommier, Leroux and Lozachmeur (2011) and Eső, Simonovits and Tóth (2011). Their common starting point has been the obvious observation that actuarially fair adjustment (especially NDC) neglects the regressive redistribution caused by adverse selection: expectedly longer/shorter lived retire later/earlier and rewarded/punished as if the whole cohort had a common stochastic lifespan—regressive redistribution.

For technical reasons, modeling the socially optimal design, most of the above-mentioned critical writings assumed common earnings and our papers used a primitive adjustment formula, neglecting the possibility that early death may precede late retirement. Among authors allowing for that complication, Diamond (2003) restricted his analysis to the choice between minimal and maximal retirement ages, while Sheshinski

(2008) discussed learning the survival curve. Furthermore, there were very few data to substantiate our theoretical findings. For whatever reason, the proponents of the NDC scheme have hardly noticed lest accepted our critique on excessive intracohort redistribution.

In this paper our earlier critique of NDC is revisited and extended to other pension systems. First, nationwide Hungarian data are presented on all those old-age pensioners who deceased in 2004 (unpublished work of Judit Marosi and Rudolf Borlói). Even though there were very weak incentives to postpone retirement and quite strong incentives to early retirement, there was a very strong adverse selection in the period closed in 2004. To give just one data on adverse selection: males aged 65 had a remaining life expectancy 13.1 years, while those who retired at age 65 lived on average a surprisingly long period of 24.3 years. Second, giving up the mathematically sophisticated mechanism design approach, and incorporating Banyár (2011)'s critique on our earlier formulation, now the original primitive NDC adjustment rule is replaced by a realistic one. (I do not follow him, however, in his transformation of the NDC scheme into a minimum flat scheme.) Under plausible assumptions, the NDC scheme is still unbalanced, i.e. the average revenues are much lower than the average expenditures are. Furthermore, the lifetime balances keep decreasing with life expectancies.

Third, the category of *regular* benefit–retirement age schedules is introduced and the inevitability of regressive redistribution is also proved. Artificial numerical examples illustrate the analytical theorems. For example, using a 30 percent contribution rate, and eliminating an aggregate deficit about 2.5-year total wage, our *balanced* NDC scheme punishes the shortest lived (who died at 63) and lowest paid by 5.1-year own wage! On the other hand, the longest lived (who died at 93) and highest paid is rewarded by 9-year own wage! In the *compressed* NDC scheme, the lifetime balances are still large in absolute values and decreasing with the life expectancies but much compressed, starting from 2.6-year surplus and ending with 3.8-year deficit.

It is to be underlined that in the present approach, the mortality rates are given exogeneously and are independent of the retirement age and the benefit. In reality, however, the meaningful work lengthens the lifespan and the increased benefits make room for better health care and longer lives.

The approach also neglects the very important gender differentials, though males earn much more and die much earlier than females do. Copying the practice of public pensions, we use a unisex framework, weakening the positive correlation between lifetime earnings and life expectancies, prevailing for males and females separately.

The structure of the remainder of the paper is as follows. Section 2 displays the Hungarian data on adverse selection. Section 3 analyzes the actuarial pitfalls of the NDC system, while Section 4 proves the inevitability of some regressive redistribution for any regular scheme. Section 5 contains numerical illustrations on the balanced and the compressed NDC system and Section 6 concludes.

2. Data

We present Tables 1 and 2 contrasting the remaining life expectancies with the expected pension span of all Hungarian male and female old-age pensioners who died in 2004.

We shall see the adverse selection mentioned in the Introduction. Rows with erstwhile full benefit retirement age (60 for males and 55 for females) are italicized.

Table 1. Expected lifespan and retirement age, male, years

Life expectancy $L + D_R$	Retirement age $L + R$	Relative frequency $100 f_R$	Expected pension span $D - R$	Remaining life expectancy M_R	Estimation error S_R
69.3	57	7.4%	12.3	18.0	5.7
71.5	58	6.0%	13.5	17.3	3.8
73.2	59	4.6%	14.2	16.7	2.5
77.2	*60*	*60.5%*	*17.2*	*16.1*	*–1.1*
79.1	61	12.7%	18.1	16.4	–1.7
82.9	62	3.9%	20.9	14.9	–6.0
85.4	63	2.1%	22.4	14.3	–8.1
86.4	64	1.6%	23.4	13.7	–9.7
89.3	65	1.4%	24.3	13.1	–11.2

Remarks. $L = 20$, $S_R = D_R - R - M_R$. Number of observations: 28.5 thousands.

Table 2. Expected lifespan and retirement age, female, years

Life expectancy $L + D_R$	Retirement age $L + R$	Relative frequency $100 f_R$	Expected pension span $D - R$	Remaining life expectancy M_R	Estimation error S_R
66.8	52	2.3%	14.8	27.4	12.6
68.8	53	2.0%	15.8	26.6	10.8
73.7	54	2.3%	19.7	25.7	6.0
75.7	*55*	*46.2%*	*20.7*	*24.9*	*4.2*
79.6	56	16.8%	23.6	24.1	0.5
81.3	57	7.9%	24.3	23.3	–1.0
82.7	58	5.9%	24.7	22.5	–2.2
84.4	59	4.0%	25.4	21.7	–3.7
86.7	60	4.3%	26.7	20.9	–5.8
86.6	61	2.6%	25.6	20.1	–5.5
86.5	62	2.0%	24.5	19.3	–5.2
86.5	63	1.7%	23.5	18.5	–5.0
87.0	64	1.0%	23.0	17.7	–5.3
86.5	65	1.0%	21.5	16.9	–4.6

Remarks. $L = 20$, $S_R = D_R - R - M_R$. Number of observations: 30.3 thousands.

Note that our statistical data are quite rudimentary, they do not distinguish between survival profiles of subsequent cohorts. Nevertheless, I am convinced that they correctly reflect the adverse selection. We start the presentation with explaining column 2, classifying the deceased according to their retirement ages between 57 (52) and 65 for males (females). Column 3 contains the relative frequencies of these groups. Column 4 is the most interesting one, depicting the expected retirement span as a function of the retirement age. Adding up the columns 2 and 4 yields the life expectancy, displayed in column 1. In contrast to males, females retiring above age 59 appear to differ only in their labor disutilities but not in their life expectancies. Column 5 contains the usual remaining (average) life expectancies, regardless of the retirement status. Column 6 displays the estimation error of government, being the signed difference of columns 4 and 5. Note that for males, the estimation error drops from 5.7 (at retirement age 57) to –11.2 years (at 65), while for females, the change is from 12.6 (at age 52) to –4.6 years (at age 65).

3. A critique of the NDC scheme

Having presented the data, we work out the model to criticize the NDC scheme, comparing symmetric information and asymmetric information. For convenience, we neglect childhood: $L = 0$, hence the type-specific adult retirement age and the length of employment are equal. We assume that the population is stationary, there is neither inflation nor personal income taxation (for an exception, Cremer et al., 2004). We also assume that the government knows the probability distribution of lifespans. Every worker chooses his retirement age (i.e. when he stops working and claims the benefit) depending on his life expectancy, consumption utilities and labor disutility. In harmony with the practice of public pension systems, there is no sex discrimination.

Symmetric information

Following the proponents of the NDC system, we start our analysis with the assumption of *symmetric information*: the government and the workers have the same stochastic information on lifespans (more realistically, life expectancies).

Let f_i be the probability of death at age i, for $i = \alpha, \ldots, \omega$; $\sum_{i=\alpha}^{\omega} f_i = 1$, α being the earliest age what is relevant for old-age retirement, while ω is the maximal life span. We define the *remaining (average) life expectancy* of aged a, where a is a positive integer:

$$M_a = \frac{\sum_{i=a+1}^{\omega} f_i(i-a)}{\sum_{i=a+1}^{\omega} f_i}, \qquad a = \alpha, \ldots, \omega - 1. \tag{1a}$$

We shall assume that M_a is decreasing but the decrease is limited: $M_a > M_{a+1} \geq M_a - 1$.

We shall need the intermediate values to be defined by linear interpolation. Let A be a positive real number and $a = [A]$ be its integer part, while $\{A\}$ be its fractional part, i.e. $A = a + \{A\}$. Then the generalized remaining life expectancy is

$$M(A) = (1 - \{A\})M_a + \{A\}M_{a+1}. \tag{1b}$$

Note that $M(A)$ is continuous and decreasing.

We assume that a worker earns a positive annual wage w during his active lifetime. Thus a worker retiring at age R has accumulated a nonfinancial pension wealth τRw, yielding an annual NDC benefit

$$b^N(R) = \frac{\tau Rw}{M(R)}. \tag{2}$$

Note that delaying retirement by one year raises the numerator and decreases the denominator, thus doubly raises the annual benefit!

Then, regardless of the value of the retirement age R, his expected lifetime (pension) balance is zero:

$$z^N(R) = \tau Rw - M(R)b^N(R) = 0. \tag{3}$$

We can also consider another symmetric case, when both the government and the workers know the exact life expectancies. Then the full-information benefit rule

$$b^F(R) = \frac{\tau Rw}{D - R} \tag{4}$$

also leads to a zero lifetime balance:

$$z^F(R) = \tau Rw - (D - R)b^F(R) = 0. \tag{5}$$

It is remarkable that assuming common retirement age and using the full-information rule, Breyer–Hupfeld (2009) totally eliminated redistribution.

Asymmetric information

We have already seen in the previous subsection that the assumption of symmetric information contradicts the facts presented in Tables 1 and 2, namely that D_R was a steeply increasing sequence. Therefore we assume now *asymmetric information*: only the workers know exactly their own lifespans (or life expectancies) but the government only knows the stochastic distribution.

For simplicity, now we change the dependence of D_R into R_D etc. Assume that the life expectancies rather than the retirement ages are integers. Assume that there are $n = \omega - \alpha + 1 \geq 2$ types, with integer life expectancy $D = \alpha, \ldots, \omega$, relative frequency f_D, annual wage w_D and retirement age R_D. Introducing the government's *estimation error of pension span*

$$S_D = D - R_D - M(R_D),$$

the type-specific balance is

$$z_D^N = \tau R_D\, w_D - (D - R_D)\frac{\tau R_D\, w_D}{M(R_D)} = (M(R_D) - D + R_D)b^N(R_D) = -S_D\, b^N(R_D). \tag{6}$$

In words, the NDC-lifetime balance is the product of the negative of estimation error and the corresponding benefit.

To obtain analytical results, we must make various assumptions. From now on we number our assumptions as A1, A2 etc.

We assume that the retirement age is a weakly increasing function of the life expectancy:

$$R_D \le R_{D+1}, \qquad D = \alpha, \alpha+1, \dots, \omega-1. \qquad (A1)$$

Additionally we also assume that the annual wage is a weakly increasing function of the life expectancy:

$$w_D \le w_{D+1}, \qquad D = \alpha, \alpha+1, \dots, \omega-1. \qquad (A2)$$

A1–A2 are acceptable from a logical as well as an empirical point of view. A simple consequence of A1–A2, $M(R_D) \ge M(R_{D+1})$ and (2) is that the benefits are increasing with life expectancy: $b_D^N \le b_{D+1}^N$.

In harmony with Tables 1 and 2, we also assume the existence of a positive integer $\tilde{D}$ such that for a life expectancy lower/higher than $\tilde{D}$, the estimation error is negative/positive:

$$S_D < 0 \quad \text{if} \quad D \le \tilde{D} \quad \text{and} \quad S_D \ge 0 \quad \text{if} \quad D > \tilde{D}. \qquad (A3)$$

Due to (6), the lifetime balances of the shorter/longer lived are positive/negative:

$$z_D^N > 0 \quad \text{if} \quad D \le \tilde{D} \quad \text{and} \quad z_D^N \le 0 \quad \text{if} \quad D > \tilde{D}. \qquad (7)$$

Strengthening (A3), we may assume that the estimation error is also an increasing fraction of the life expectancy. This assumption implies that the retirement age stays away from the life expectancy, i.e. $R_\omega \ll \omega$. (For example, in the US, $R_\omega = 70$ years.)

$$S_\alpha < \cdots < S_{\tilde{D}} < 0 \le S_{\tilde{D}+1} < \cdots < S_\omega. \qquad (A3^*)$$

We can now formulate

Theorem 1. *a) Under A1–A3*, the lifetime balance z_D^N of the longer lived is either negative or zero and decreasing with life expectancy D:*

$$0 \ge z_{\tilde{D}+1}^N > \ldots > z_\omega^N. \qquad (8a)$$

b) If, in addition to a), the wages and the retirement ages are both type-invariant, then the lifetime balance z_D^N of the shorter lived is positive and decreasing with life expectancy D:

$$z_\alpha^N > \cdots > z_{\tilde{D}-1}^N > z_{\tilde{D}}^N > 0. \qquad (8b)$$

Proof. a) Considering the longer lived, by A3*, the first factor of z_D^N is negative and decreasing; by A1–A2, the second factor is positive and increasing, therefore their product is negative and decreasing.

b) If w_D and R_D are invariant, then the benefit b_D^N is also invariant, therefore z_D^N is proportional to S_D, hence A3* applies. ∎

Remarks. (i) A3* trivially holds for the primitive approach when $S_D = D - D^*$. (ii) Also, A3* trivially holds if the retirement age is type-invariant: $R_D \equiv R^*$, when

$S_D = D - R^* - M(R^*)$. (iii) Note that without assuming constant wages and retirement ages, (8b) cannot be proved but we will return to it in Theorem 3.

Example 1. As a simple illustration, we consider the continuously uniform lifespan distribution, where the distribution function is

$$F(x) = \frac{x-\alpha}{\omega-\alpha}, \qquad \alpha \le x \le \omega.$$

It is also assumed that the adult retirement age is a homogeneous linear function of the adult life expectancy: $R(D) \equiv \rho D$, where $1/2 < \rho < 1$. Following Banyár's critique, we assume that the earliest exit precedes the latest retirement: $\alpha < \rho\omega$. Then the average life expectancy is

$$D^* = \frac{\alpha+\omega}{2};$$

while the remaining life expectancy is

$$M(A) = D^* - A \quad \text{if} \quad 0 \le A < \alpha \quad \text{and} \quad M(A) = \frac{1}{2}(\omega - A) \quad \text{if} \quad \alpha < A \le \omega.$$

Consequently, introducing the switching point $D_S = \rho^{-1}\alpha$ of $M(A)$, the estimation error is

$$S(D) = D - D^* \quad \text{if} \quad \alpha \le D \le D_S \quad \text{and} \quad S(D) = \frac{1}{2}[(2-\rho)D - \omega] \quad \text{if} \quad D_S < D \le \omega.$$

Note that $\tilde{D} = D^*$ and A3 as well as A3* holds.

We can now turn to the sign of the *average* balance:

$$z^* = \sum_{D=\alpha}^{\omega} f_D\, z_D. \tag{9}$$

To generalize our earlier result on the deficit in the NDC scheme (Theorem 3 of Simonovits, 2003), we must look for a further sufficient condition. We propose the following; the earning-weighted average of the conditional estimation errors is either positive or zero:

$$s = \sum_{D=\alpha}^{\omega} f_D\, S_D\, w_D \ge 0. \tag{A4}$$

Returning to the original model, where $S_D = D - D^*$ and $w_D \equiv 1$, $s = 0$. If the earnings were constant ($A2^o$) and the lifespan distribution were uniform (Example 1), then A4 would not hold: $s < 0$. But we shall see in Table 3 below that a slight rise in the wage–lifespan function would make A4 valid. Anyway, for type-invariant retirement ages, A4 is not only a sufficient but a necessary condition for $z^{*N} \le 0$. In summary, A4 seems to be satisfactory.

Theorem 2. *If A1–A4 hold, then the average NDC-balance is negative:* $z^{*N} < 0$ *except if the retirement age is type-invariant:* $R_D \equiv R^*$ *and if* $s = 0$ *holds, then* $z^{*N} = 0$.

Proof. Inserting z_D^Ns [(6)] into z^* [(9)] and using notation $b_D^N = b^N(R_D) = \beta_D^N w_D$, the NDC balance

$$z^{*N} = -\sum_{D=\alpha}^{\omega} f_D \, S_D \, w_D \, \beta_D^N \tag{10}$$

can be estimated from above. Note that the replacement rates

$$\beta_D^N = \frac{\tau R_D}{M(R_D)}$$

form a weakly increasing sequence. By A3, for $D \le \tilde{D}$, $S_D < 0$ and for $D > \tilde{D}$, $S_D \ge 0$. Cutting z^{*N} into two at $\tilde{D}$, the first subsum contains positive products, while the second does negatives. Inequalities

$$\beta_D^N \le \beta_{\tilde{D}}^N \quad \text{for} \quad D \le \tilde{D} \quad \text{and} \quad \beta_D^N \ge \beta_{\tilde{D}}^N \quad \text{for} \quad D > \tilde{D}$$

and A4 imply

$$z^{*N} \le -\sum_{D=\alpha}^{\omega} f_D \, \beta_{\tilde{D}}^N S_D \, w_D = -\beta_{\tilde{D}}^N \sum_{D=\alpha}^{\omega} f_D \, S_D \, w_D \le 0.$$

If $R_\alpha < R_\omega$ (i.e. $\beta_\alpha^N < \beta_\omega^N$) or $s < 0$, then $z^{*N} < 0$. ∎

Of course, the resulting deficit needs to be eliminated, i.e. by suitably reducing the contribution rate τ to $\hat{\tau}(< \tau)$ in the corresponding *balanced* NDC-formula:

$$\hat{b}^N(R_D) = \frac{\hat{\tau} R_D \, w_D}{M(R_D)}. \tag{11}$$

Note that we neglected the unfavorable reaction of the adjustment on the retirement ages.

At this point we must admit that our modeling of asymmetric information is still inadequate: we assume that the government knows R_D but does not know D. Similarly, the incorporation of wage heterogeneity enables the government to infer D from w_D. The best justification for these inadequacies is as follows: in reality, there are shorter lived with lower labor disutility and longer lived with higher labor disutility who retire at the same age. Similarly, those who die at age D may have earned various wages.

4. Regular benefit–retirement age schedules

We want to show that Theorem 1 can be generalized from constant wages and retirement ages and the NDC scheme to variable wages and retirement ages and some other schedules, called regular. This way we reformulate Eső et al. (2011, Theorem 1) from common to increasing earnings (in A2). Therefore our critique of the NDC system is addressed not against redistribution *per se* but its excessive degree. We leave open whether the system is balanced or not. Before entering the discussion, we present an (irregular) example without any redistribution.

Example 2. The case where there is no redistribution. Let the replacement rate be type-invariant: $b_D = \beta w_D$ ($\beta > 0$) and let the (adult) retirement age be proportional to the (adult) life expectancy: $R_D \equiv \rho D$ ($1/2 < \rho < 1$). For a balanced system (where $\tau\rho = \beta(1-\rho)$), each type's lifetime balance is zero:

$$z_D = \tau w_D \rho D - \beta w_D (1-\rho) D = 0, \qquad D = \alpha, \dots, \omega.$$

This rule, however, eliminates any incentives: it does not charge any deduction for early retirement and does not pay any reward for later retirement.

We define now the concept of *regular* schedule (and drop superscript N) for (b_D) by additional assumptions A1* and A5.

The induced increase in the retirement age R_D is nonnegative and less than or equal to the ratio of the D's benefit b_D to the sum of the contribution τw_D and the same benefit:

$$0 \le \Delta R_D \le \frac{b_D}{\tau w_D + b_D}, \qquad D = \alpha \dots, \omega - 1. \tag{A1*}$$

We also assume that the difference ratio of the benefit to the wage is at least as large as the ratio of the next retirement age to the next actual pension span:

$$\frac{\Delta b_D}{\Delta w_D} \ge \frac{R_{D+1}}{D + 1 - R_{D+1}}, \qquad D = \alpha \dots, \omega - 1. \tag{A5}$$

The upper bound on ΔR_D in A1* has no clear economic content but it is worth substituting the NDC benefits (2) into (A1*). Indeed, (A1*) becomes

$$\Delta R_D^N \le \frac{R_D}{M(R_D) + R_D} < 1. \tag{A1*N}$$

Only its relaxation into $\Delta R_D < 1$ is a natural requirement. Indeed, why should type $D+1$ work at least one year longer than type D just because he is going to live one year longer?

(A5) on the benefit–wage difference ratio is even more difficult to interpret. In Diamond (2003) and Eső et al. (2011), the wage was type-independent, therefore (A5) was automatically satisfied. A5 only means that the type-dependent wage increases much less with the life expectancy than the benefit does. For example, if $R_D \equiv \rho D$, then A5 states that the difference ratio should be at least $\rho/(1-\rho) \ge 1$.

Theorem 3. *a) Under A1*, A2 and A5, for any regular schedule, the lifetime balance is a weakly decreasing sequence of the life expectancy:*

$$z_\alpha \ge \dots \ge z_D \ge z_{D+1} \ge \dots \ge z_\omega. \tag{12}$$

b) If the system is balanced, then there exists an age $\hat{D}$ such that

$$z_{\hat{D}} \ge 0 \ge z_{\hat{D}+1}.$$

Proof. a) Starting from $z_{D+1} = (\tau w_{D+1} + b_{D+1})R_{D+1} - b_{D+1}(D+1)$, we shall arrive to z_D by increases. Introducing $b_{D+1} = b_D + \Delta b_D$ and $w_{D+1} = w_D + \Delta w_D$,

$$z_{D+1} = [\tau(w_D + \Delta w_D) + b_D + \Delta b_D]R_{D+1} - (b_D + \Delta b_D)(D+1)$$
$$= (\tau w_D + b_D)R_{D+1} - b_D(D+1) + [\tau \Delta w_D R_{D+1} - \Delta b_D(D+1-R_{D+1})].$$

By A5, the third term in [ ] is negative or zero, therefore we can drop it. Introducing also $R_{D+1} = R_D + \Delta R_D$, the difference between the first and the second terms can be estimated by (A1*) as

$$z_{D+1} \le (\tau w_D + b_D)R_{D+1} - b_D(D+1) = z_D + (\tau w_D + b_D)\Delta R_D - b_D \le z_D.$$

b) Trivial. ∎

5. Numerical illustrations

Different pension rules imply different retirement ages, and this should be taken into account in the comparisons (cf. Eső et al., 2011). Here we only display two simple numerical examples, illustrating the balanced and the compressed NDC systems, respectively without taking into account their impact on the choice of retirement ages.

We use the simplest analytical lifespan distribution function, namely the uniform one in Example 1. We shall use $\alpha = 42$ and $\omega = 72$ years, omitting the 21 years of childhood. (In other words, people die between ages 63 and 93 years.) Furthermore, we assume a linear retirement age–lifespan function $R \equiv 2D/3$ and a linear wage–lifespan function $w_i = w_\alpha + \delta(i - \alpha)$ such that the expected wage is equal to 1, for $w_\alpha = 0.9$, implying $\delta = 0.0066....$ We apply $\tau = 0.3$.

We provide useful aggregate statistics on the balanced NDC scheme, denoting by $\mathbf{E}$ the expectations: $\mathbf{E}D = 57$ years, $\mathbf{E}w = 1.00$, $\mathbf{E}R = 38$ years, $\mathbf{E}b = 0.558$, $\mathbf{E}z = 0$, $\mathbf{D}z = 4.8$ and $s = -0.007$. (We have chosen the minimal wage w_α so high that $s \le 0$ just holds.) To eliminate the deficit, the benefits were proportionally reduced by 18.7 percent by choosing $\hat{\tau} = 0.244$ in (11).) The size of the corresponding standard deviation can only be judged by comparison to the second simulation.

The first column of Table 3 displays the life expectancies. The next two columns provide the type-specific benefits and balances for the balanced NDC scheme, respectively. Note that the shortest lived's annual benefit is quite low: about 34 percent of the lowest net wage, while the longest lived's annual benefit is quite high: about 140 percent of the highest net wage. The lifetime balances are decreasing, starting from higher than 5.1 years own wage and ending with lower than –9.1 years own wage.

Turning to the compressed NDC scheme, we *dampen* the incentives of original NDC benefit function by taking only its power θ, where $0 < \theta < 1$ and multiply it by an appropriate constant $(b^*)^{1-\theta}$ (Simonovits, 2003):

$$b_D = (b_D^N)^\theta (b^*)^{1-\theta}, \tag{13}$$

where $\theta = 0.5$ and $b^* = 0.527$, making the expected lifetime balance zero. We renounce the discussion of the incentive effect on retirement ages.

The aggregate statistics of the compressed NDC scheme are as follows: $\mathbf{E}R = 38$ years, $\mathbf{E}b = 0.501$, $\mathbf{E}z = 0.05$ and $\mathbf{D}z = 2.166$. The size of the corresponding standard deviation is only 45% of the balanced one, a great improvement.

The last two columns of Table 3 display the type-specific benefits and lifetime balances of the compressed run. Note that the shortest lived's annual benefit is quite low but higher than originally: about 59 percent of the lowest net wage, while the longest lived's annual benefits are quite high but much less than before: about 108 percent of the highest net wage. The lifetime balances are still decreasing but much compressed, starting from 2.6 years surplus and ending with 3.8 years deficit. The lifetime balance again decreases with life expectancy.

Table 3. *Outcomes for original and compressed NDC schemes*

Life expectancy D	Balanced N-benefit $\hat{b}_D^N$	Lifetime N-balance $\hat{z}_D^N$	Compressed M-benefit b_D^M	Lifetime M-balance [year] z_D^M
42	0.212	4.579	0.371	2.371
45	0.250	4.523	0.402	2.250
48	0.295	4.307	0.436	2.046
51	0.348	3.881	0.474	1.740
54	0.412	3.175	0.515	1.307
57	0.490	2.090	0.562	0.716
60	0.588	0.480	0.616	–0.080
63	0.713	–1.878	0.679	–1.145
66	0.816	–3.964	0.726	–1.974
69	0.936	–9.964	0.777	–2.974
72	1.078	–10.032	0.834	–4.177

Remark. $\hat{b}_D^N$ from (11) and b_D^M from (13). Except for column 1, the date are given in terms of the average wage.

At the end we note that using a more realistic distribution function (with hump-shaped rather than constant density function), probably the distortion would be less but not negligible. The replacement of proportional retirement ages by a more concentrated schedule would also reduce the distortion.

6. Conclusions

The NDC scheme is basically a reasonable pension system, achieving an automatic rewarding/punishing of late/early retirement. Nevertheless, it neglects the impact of the life expectancy on the choice of retirement age and the strong positive correlation between life expectancy and (lifetime average) wage. Therefore this system achieves a too strong regressive redistribution from the expectedly shorter lived and worse paid to the expected longer lived and better paid. We showed that qualitatively similar redistribution occurs for all regular (and other) schemes but its size can be made much smaller. Further research with calibrated data is needed to determine the socially optimal modification of the NDC system via mechanism design. Anyway, much caution is needed to use the NDC scheme, even if means-testing or pension credit softens its impact.

References

Banyár, J. (2012): "Proposal for an Optimal Benefit Formula" (in Hungarian), *Szigma, 42*, 105–124.

Bommier, A.; Leroux, M.-L. and Lozachmeur, J.-M. (2011): "Differential Mortality and Social Security", *Canadian Journal of Economics 44*, 273–289.

Breyer, F. and Hupfeld, S. (2009): "Fairness of Public Pensions and Old-Age Poverty", *Finanz Archiv/Public Finance Analysis 65*, 358–380.

Coile, C.; Diamond, P., Gruber, J. and Jousten, A. (2002): "Delays in Claiming Social Security Benefits", *Journal of Public Economics 84*, 357–385.

Cremer, H., Lozachmeur, J-M. and Pestieau, P. (2004): "Social Security, Retirement Age and Optimal Income Taxation", *Journal of Public Economics 88* 2259–2281.

Diamond, P. (2003): *Taxation, Incomplete Markets and Social Security*, Munich Lectures, Cambridge, MA, MIT Press.

Diamond, P. and Mirrlees, J. (1978): "A Model of Social Insurance with Variable Retirement", *Journal of Public Economics 10*, 295–336.

Divényi, J. and Kézdi, G. (2012): "On the Causes of Low Employment of Population above 50 in Hungary. The Role of Incentives, Cognitive Abilities and Health State" (in Hungarian), *Social Report* Budapest, (eds. Kolos, T.–Tóth, I. Gy.) TÁRKI, 190–208.

Eső, P. and Simonovits, A. (2002): "Designing Optimal Benefit Rules for Flexible Retirement", *Discussion Paper CMS-EMS 1353*, Northwestern University, Evanston, IL.

Eső P.; Simonovits, A. and Tóth, J. (2011): "Designing Benefit Rules for Flexible Retirement: Welfare and Redistribution", *Acta Oeconomica 61*, 3–32.

Fabel, O. (1994): *The Economics of Pensions and Variable Retirement Schemes*, New York, Wiley.

Holzmann, R. and Palmer, E., eds. (2006): *Pension Reform through NDCs: Issues and Prospects for Non-Financial Defined Contribution Schemes*, Washington D.C. World Bank.

Holzmann, R.; Palmer, E. and Robalino, D., eds. (2012): *NonFinancial Defined Contribution Pension Schemes in a Changing Pension World*, Washington D.C. World Bank.

Knell, M. (2012): "Increasing Life Expectancy and Pay-As-You-Go Pension Systems", Österreichische Nationalbank, Working Paper 179.

Krémer, B. (2012): Why do Societies are Afraid of Longevity? (in Hungarian), to appear in the (Hungarian) Sociological Review.

Sheshinski, E. (2008): *The Economic Theory of Life Annuities*, Princeton, Princeton University Press.

Simonovits, A. (2003): “Designing Optimal Linear Rules for Flexible Retirement”, *Journal of Pension Economics and Finance 2*, 273–293.
Simonovits, A. (2006): “Optimal Design of Pension Rule with Flexible Retirement: The Two-Type Case”, *Journal of Economics 89*, 198–222.