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Zooming In and Out from the Mental Number Line: Evidence for a Number Range Effect

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Authors Note

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Abstract

The representation of numbers is commonly viewed as an ordered continuum of magnitudes, referred to as the mental number line. Previous work has repeatedly shown that number representations evoked by a given task can be easily altered, yielding an ongoing discussion about the basic properties of the mental number line and how malleable they are. Here we studied whether the resolution of the mental number line is fixed or depends on the relative magnitudes that are being processed. In two experiments, participants compared the same number pairs under two conditions that differed in terms of the overall range of numbers present. We report a novel number range effect, such that comparisons of the same number pairs were responded to faster under the smaller vs. larger number range. This finding is consistent with the idea that the resolution of the mental number line can be adjusted, as if a unit difference is perceived as larger in smaller ranges.

Key words: mental number line, context effects, mental resolution, size effect, distance effect.

Numbers are abstract entities, representing numerical magnitudes, which we can combine using specific rules. Regardless of the physical way in which these mental symbols are presented (Arabic numerals, number words, dot arrays, etc.) or processed (e.g., in a visual or auditory modality), their semantic meaning is thought to be captured by an analogue magnitude representation associated with the intraparietal sulcus (for a review see Feigenson, Dehaene, & Spelke, 2004), commonly referred to as the mental number line (Restle, 1970). Extensive evidence shows how number representations can be changed by certain contexts or task requirements. It is worth noting that the insight of a malleable mental number line sharply contrasts with a prior intuition that numbers would be represented in a purely propositional way.

Researchers have identified some key characteristics for the mental representation of numbers, such as condensed scaling and left-to-right spatial orientation. The relevant empirical evidence concerns three effects: the distance effect (i.e., better discrimination of numbers that are further apart than when they are close), the size effect (i.e., better performance for small than large numbers; Moyer & Landauer, 1967) and the SNARC (Spatial-Numerical Association of Response Codes) effect (i.e., faster responding to large/small numbers with the right/left hand, respectively; Dehaene, Bossini, & Giraux, 1993). Both the distance and size effects are generally robust and rarely affected by training (e.g., Dehaene, 1997; Zbrodoff & Logan, 2005; but see Verguts & Van Opstal, 2005).

In contrast, the flexibility of the SNARC effect has been repeatedly observed. This effect seems to depend on relative, and not absolute, numerical magnitude, such that the same numbers (e.g., 4 and 5) can be associated with left-hand responses, when used as the smallest numbers in the presented range (e.g., 4-9), and with the right-hand responses, when used as the largest numbers in the presented range (e.g., 0-5; Dehaene et al., 1993; Fias, Brysbaert, Geypens, & d’Ydewalle, 1996). Moreover, the SNARC effect can be easily reversed (or diluted) by manipulating the task instructions (e.g., Bächtold, Baumüller, & Brugger, 1998), incompatible spatial mapping/positioning (e.g., Fischer, Mills, & Shaki, 2010; Fischer, Shaki, & Cruise, 2009; Notebaert, Gevers, Verguts, & Fias, 2006; Shaki & Fischer, 2008) and memory requirements or load (e.g., Lindemann, Abolafia, Pratt, & Bekkering, 2008; van Dijck & Fias; 2011; van Dijck, Gevers, & Fias, 2009).

In line with these findings, it seems that short-term numerical representations are constructed online, during task execution, in accordance with the specific characteristics of the task at hand. Therefore, the same numbers can be considered small or large and evoke contrasting directions in space, depending on the putative mental number line, which is generated by the task. Such “working representations” appear to be particularly affected by task demands when people are intentionally asked to process numerical magnitudes, compared to situations in which the numerical magnitudes are processed automatically (i.e., not as part of the task requirements, e.g., Tzelgov, 1997; Tzelgov & Ganor-Stern, 2005). However, even in the latter case, there seems to be a context-dependent ranking of numerical magnitudes, at least in the sense of which number is automatically perceived to be the smallest (Pinhas & Tzelgov, 2012). Consistently with these ideas, a recent model by Cohen Kadosh and Walsh (2009) proposed that numerical processing starts with non-abstract numerical representations in the parietal cortex. These representations are assumed to be automatic in the sense that they are not affected by task demands. At a later stage, abstract representations may emerge in accordance with the task requirements in the prefrontal cortex, which is associated with training effects, working memory and strategy application (see also Gilbert & Burgess 2008; Tudusciuc & Nieder, 2007).

The objective of the present study was to explore whether the granularity or resolution of the evoked mental number line can also be changed by context. Consider a numerical comparison task, in which participants are presented with pairs of numbers (one pair at a time) and are asked to select the larger number in each pair. In one condition participants see the pairs 10-11, 11-12 ... 18-19, while in the second condition, they see the pairs 10-11, 11-12 ... 38-39. The two conditions can be called ‘small number range’ and ‘large number range’, to reflect the corresponding differences in the number range in the two conditions. The numbers in all pairs differ by 1 unit. However, in the small number range, for example, 11 and 12 differ by 1 unit in a range of 10 units, while in the large number range the difference is 1 unit in a range of 40 units. According to the well-known ‘distance effect’, there is faster responding for pairs of numbers which are further apart. What is not known is whether differences in perceived distance are fixed (i.e., depend on absolute numerical magnitude) or variable, depending on which other numbers are present. If the latter hypothesis is true, we expect faster reaction times (RTs) when comparing the numbers 11 vs. 12 in the small than in the large number range. Accordingly, a ‘range effect’ (to mean: a difference in RT for responding to the same pair of stimuli, depending on whether the pair appears in the small or large number range condition), would be consistent with an interpretation that the evoked mental number line keeps getting adjusted, as additional number pairs are observed, and its resolution depends on the overall range of the processed numbers.

Experiment 1

Method

Participants and Design

Participants were 62 mostly undergraduate students (mean age 23 years, 33 female, 54 right-handed), who were compensated with course credit. The experiment was designed to manipulate two within-participant factors: range (small vs. large range) and pair (the particular pairs of numbers which were common in the small and large ranges; these were 11-12, 12-13, 13-14, 14-15, 15-16, 16-17, 17-18, 18-19).

Procedure and Materials

The task was computer-based. E-Prime software controlled the presentation of the stimuli (Schneider, Eschman, & Zuccolotto, 2002). Participants were asked to perform a numerical comparison task, identifying the larger number, between two numbers concurrently presented on a screen. When the number presented on the left/right of the screen was larger, they were asked to press the left (A)/right (L) response key, respectively. The trial structure involved the presentation of a fixation cross for 500 ms, followed by a pair of numbers, which remained visible until a response was made. Subsequently, a blank screen was shown for 500 ms. Numbers appeared in 40 point size Arial font, in white, against a black background. Each number within each pair was presented equally often on the left and right side of the screen.

All number pairs consisted of adjacent numbers. Each participant was presented with two blocks of trials (block order was counterbalanced across participants), corresponding to the two range conditions. In the small number range block, there were 80 trials in total (each pair between 11-19 was presented 10 times) and in the large number range block 176 trials (each pair between 11-19 was presented 10 times and pairs between 21 and 39 were presented 6 times each; pairs involving the numbers 20 and 30 were excluded). Order of trials in each block was randomized. There was a self-paced break between the two blocks and halfway the large range block.

Analysis

In this and the following experiment significance level was defined as $p < .05$. We report partial eta squared (η^2_p) values calculated as SSeffect / (SSeffect + SSerror). RTs shorter than 200 ms or longer than 2,000 ms were excluded from all analyses (less than 1% of the data). Error rates were low (average = 4%, SD = 3%) and, after checking that there were no speed-accuracy trade-offs, were not analyzed further. Finally, one participant was excluded from the analysis due to a high error rate (24%).

Results and Discussion

We focus on the number pairs common to the small and large number range conditions. Figure 1 shows that RTs for correct responses were faster in the small range (651 ms), as opposed to the large range (679 ms), for exactly the same pairs of numbers. We explored this difference with a 2 × 2 × 8 mixed-design ANOVA, with order of blocks (small first, large first) as a between-participant variable and number range (small, large) and pair (11-12, 12-13, 13-14, 14-15, 15-16, 16-17, 17-18, 18-19) as within-participant variables. There were significant main effects for both number range [$F_{(1,59)} = 18.11$,MSE = 11,024, $\eta^2_p = .23$] and pair [$F_{(7,413)} = 14.09$, MSE = 4,452, $\eta^2_p = .19$], but no interaction between the two factors [$F_{(7,413)} = 1.84$, MSE = 2,593, n.s., $\eta^2_p = .03$,]. Next, we evaluated the presence of the size effect, according to which pairs of smaller numbers are responded faster than those of larger ones. As expected, the linear component of the pair effect was significant [$F_{(1,59)} = 17.16$, MSE = 5,225, $\eta^2_p = .23$]. Finally, we examined the size effect for all the pairs in the large number range condition [$F_{(1,60)} = 154.14$, MSE = 3,983, $\eta^2_p = .72$].[1] Note that the size effect was more pronounced when considering the greater set of numbers in the large number range condition, probably due to the bigger change in the overall magnitude of the pairs (which included a change in decades). Last, there was no reliable main effect for the order of blocks or interactions involving this factor (all ps > .12, all $\eta^2_p < .04$).

The results of this experiment indicate that the same pair of numbers will be responded to differently, depending on whether they appeared together with numbers across a smaller or larger number range. We predicted that the same number difference would be psychologically less salient when the overall range was greater. This prediction, labeled the number range effect, was confirmed. Note that the information about number range was entirely driven by the trials participants went through: there was no other aspect of the procedure which informed participants regarding range differences. Moreover, we tested and broadly confirmed the presence of the standard size effect.

Even though the frequency of the critical pairs in Experiment 1 was matched across the two number ranges, the large range condition included several filler trials (to indicate the greater range) and there were no comparable filler trials in the small range condition. Thus, the total number of trials in the large range was about twice as large, and so there is a possibility that the number range effect was due to the lower probability of a critical trial in the large range condition. This possibility was tested (and ruled out) in Experiment 2, in which we matched the number of trials across each range condition. Accordingly, each condition had an equal number of critical (i.e., common) pairs and an equal number of fillers. Furthermore, the intra-pair distance was manipulated in Experiment 2, which allowed a test of the distance effect.

Experiment 2

Method

Participants and Design

Participants were 70 undergraduate students (mean age 21 years, 50 female, 66 right-handed), who were compensated with course credit. This experiment was designed to test the number range effect as well as the distance and size effects. As in Experiment 1, there were two within-participant factors: number range (small vs. large) and a pair (the common pairs for both the range conditions were 12-13, 17-18, 13-17, 12-17 and 13-18).

Procedure and Materials

The trial structure and task administration were identical to those in Experiment 1.While in Experiment 1, number pairs consisted of adjacent numbers, in this experiment the numerical distance between numbers varied to match the intra-pair distances of 1, 4 and 5. Moreover, there was the same number of critical number pairs in the small and large number range conditions, as well as the same number of filler number pairs. Specifically, the critical pairs were 12-13, 17-18 (distance 1), 13-17 (distance 4), and 12-17, 13-18 (distance 5); each critical pair had a frequency of 10. In the small range condition, there were 6 filler number pairs (numbers between 11 and 19), each with a frequency of 6. In the large range condition, there were 18 filler number pairs (numbers between 11 and 39), each with a frequency of 2. Thus, in both conditions, there were overall 86 trials.

Analysis

We excluded RTs shorter than 200 ms or longer than 2,000 ms (less than 1% of the data). Error rates were low (average = 3%, SD = 2%). They were only analyzed to rule out speed-accuracy trade-offs and will not be discussed further.

Results and Discussion

As before, we examined the mean RTs of correct responses for the number pairs which were common between the small and large number range conditions (Figure 2). We conducted a 2 × 2 × 5 mixed-design ANOVA, with order of blocks (small first, large first) as a between-participant variable and number range (small, large) and pair (12-13, 17-18, 13-17, 12-17, 13-18) as within-participant variables. As before, we observed significant main effects of both number range [$F_{(1,68)} = 17.44$, MSE = 6,802, $\eta^2_p = .20$] and pair [$F_{(4,272)} = 48.93$,MSE = 2,644, $\eta^2_p = .42$], but no interaction between the two [$F_{(4,272)} = 1.45$, MSE = 1,876, $\eta^2_p = .02$]. Next, we computed the linear trend for the effect of pair. This was significant and indicates the presence of a distance effect [$F_{(1,68)} = 157.11$, MSE = 3,135, $\eta^2_p = .70$].[1] As can be seen in Figure 2, responses were faster for pairs 12-13 and 17-18 (distance 1) than for pair 13-17 (distance 4), than for pairs 12-17 and 13-18 (distance 5). No other effects in the analysis were significant (all ps > .12, all $\eta^2_p < .03$). Last, we evaluated the size effect, which was less straightforward to explore in the current design, given the use of variable intra-pair distances. Therefore, we simply averaged the RTs for all adjacent pairs of numbers in the large number block within the first (11-19), second (21-29), and third (31-39) decades and compared the means differences, with a repeated measures ANOVA. Mean RTs for the first, second and third decades were 638 ms, 684 ms and 700 ms, respectively. There was a main effect for decade [$F_{(2,138)} = 19.02$, MSE = 3,790, $\eta^2_p = .22$], as well as a significant linear component [$F_{(1,69)} = 45.87$, MSE = 2,911, $\eta^2_p = .40$], providing evidence for a reliable size effect.

General Discussion

The mental number line metaphor is extensively used to describe the representation of numbers (e.g. Dehaene, 1997; Restle, 1970), despite an ongoing challenge of fully characterizing its properties. The current study tested whether the mental number line induced by a given task has a fixed resolution or whether the resolution depends on the overall set of numbers which are being processed. Our results clearly showed that the same two numbers are responded to faster when presented in a smaller vs. larger number range. We suggest that such a finding can be interpreted in terms of the perception of the distance between two numbers being affected by the overall range of the other numbers present in the task. Our reasoning for favoring such an interpretation is simple: we know from the distance effect that more distant number pairs are responded to faster. In our task, the same pair of numbers was responded to faster in the smaller vs. the larger range condition. Since these two conditions were matched in other relevant respects, the decrease in response time can be plausibly interpreted as an increase in perceived distance. Given the above, our findings are consistent with the hypothesis that the resolution of the mental number line changes, in a manner that is similar to zooming in and out from the relevant numbers which are being processed. That is, the subjective impression of number differences seems to be a function not of absolute numeric magnitudes, but rather of numeric magnitudes adjusted by the overall range.

The current results extend previous findings showing that the association between numbers and space can change depending on the task at hand. For example, the same numbers were found to be associated with a right or left spatial direction, and accordingly perceived as large or small, depending on their rank against the other numbers employed in the task (Dehaene et al., 1993; Fias et al., 1996). In another recent study, participants were asked to compare the physical size of single-digit numbers while ignoring their numerical magnitudes. It was concluded that number range can be processed automatically, at least in the sense of which number was perceived as the smallest (Pinhas & Tzelgov, 2012). Here, we provide evidence that the used number range, which varied between blocks, presumably affects the discriminability between numbers. Future studies can test whether such effects can be found at the level of a single trial (cf. Fischer et al., 2009) or whether they are constrained to manipulations made at the block level. Additional work can focus on the question of exactly how malleable the properties of the mental number line can be, depending on the distributional properties of the numbers processed for a particular task.

It should be noted that the number range effect reported here was found for within-decade comparisons of two-digit numbers. Such comparisons can be resolved based on the unit digits alone, since the decade digits are always identical in a comparison pair. Yet it was evident from the findings for the number range effect and the size effect that the decade digits were not ignored, even though their processing was not relevant for the task. Processing of both the unit and the decade digits of two-digit numbers usually takes place under conditions of simultaneous presentation, irrespective of task requirements (e.g., Ganor-Stern, Pinhas, & Tzelgov, 2009; Nuerk, Weger, & Willmes, 2001).

Finally, the present research regarding flexibility in the resolution of the mental number line was partly motivated by findings in similarity and categorization that indicate possibly analogous effects of learning on the resolution of psychological space (e.g., Nosofsky, 1988; Tversky, 1977). Thus, the tantalizing possibility presents itself, that perhaps such similarities might point to the existence of shared principles guiding working memory, when generating context-dependent representations (regardless of whether these representations are for numbers, or perceptual stimuli in a categorization task, or more abstract conceptual stimuli). While it is currently, premature to speculate about the nature of such shared principles, this is an exciting direction for future work.

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Footnotes

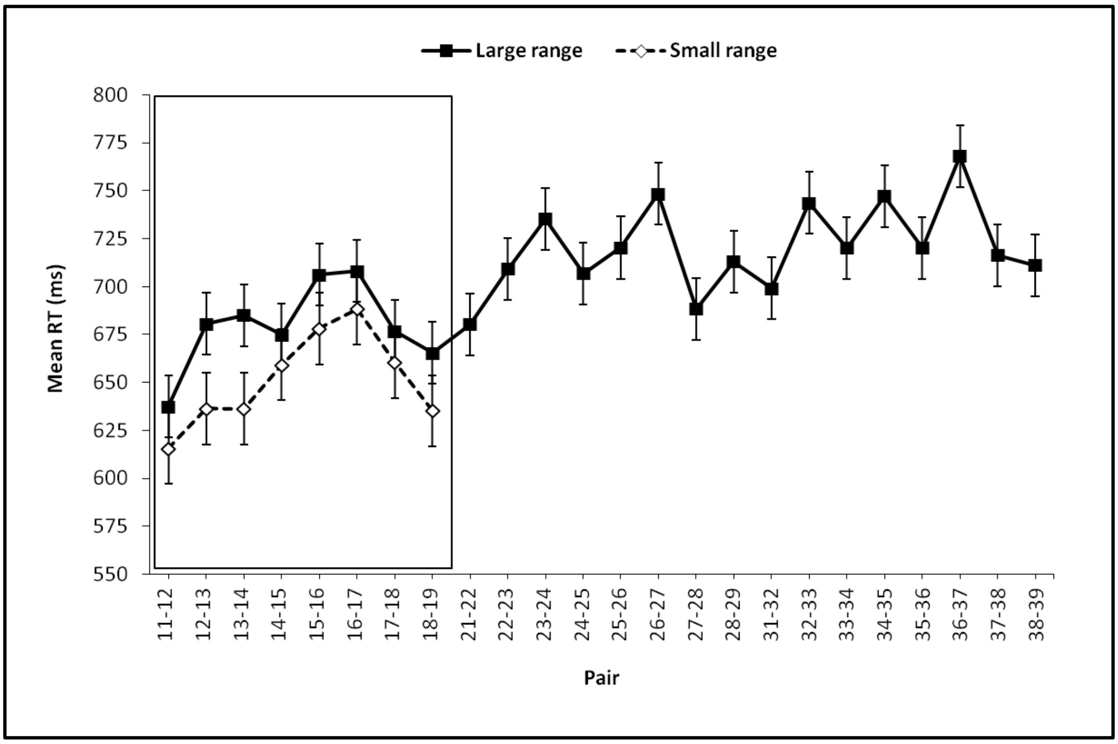
[1] Given the unequal spacing between the independent variable levels, compatible linear trend coefficients were computed according to Robson (1959).

Figure Captions

Figure 1. Mean RT as a function of number range and pair in Experiment 1. Common pairs for both range conditions are framed. Vertical bars denote 0.95 confidence intervals.

Figure 2. Mean RT as a function of number range and pair in Experiment 2. The intra-pair distances of the pairs 12-13, 17-18, 13-17, 12-17 and 13-18 are 1, 1, 4, 5, and 5, respectively. Vertical bars denote 0.95 confidence intervals.

Figure 1


Large range
Small range
Mean RT (ms)
800
775
750
725
700
675
650
625
600
575
550
11-12
12-13
13-14
14-15
15-16
16-17
17-18
18-19
21-22
22-23
23-24
24-25
25-26
26-27
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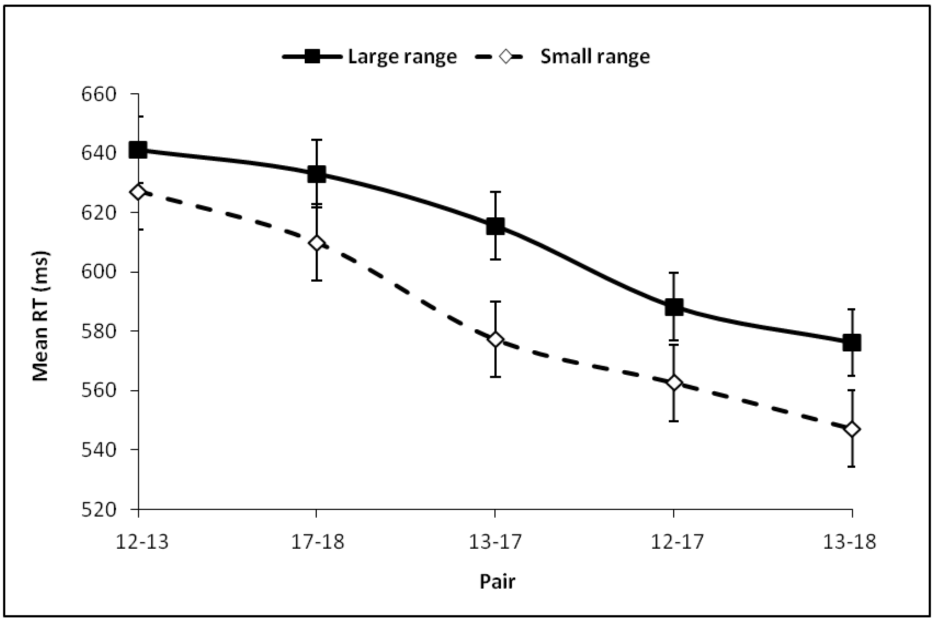
32-33
33-34
34-35
35-36
36-37
37-38
38-39
Pair

Figure 2


Large range
Small range
660
640
620
600
580
560
540
520
Mean RT (ms)
12-13
17-18
13-17
12-17
13-18
Pair