HELSINKI INSTITUTE OF PHYSICS INTERNAL REPORT SERIES

HIP-2013-01

Aspects of symmetry breaking in supersymmetric models

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ACADEMIC DISSERTATION

To be presented, with the permission of the Faculty of Science of the University of Helsinki, for public criticism in the Main Auditorium (D101) at Physicum, Gustaf Hällströmin katu 2A, Helsinki, on the 11th of October 2013 at 12 o'clock.

Helsinki 2013

ISBN 978-952-10-8110-1 (print)
ISBN 978-952-10-8111-8 (pdf)
ISSN 1455-0563
http://ethesis.helsinki.fi
Helsinki University Print
Helsinki 2013

High up in the North in the land called Svithjod, there stands a rock. It is a hundred miles high and a hundred miles wide. Once every thousand years a little bird comes to this rock to sharpen its beak. When the rock has thus been worn away, then a single day of eternity will have gone by.

–Hendrik Willem Van Loon

T. Rüppell: Aspects of symmetry breaking in supersymmetric models,
University of Helsinki, 2013, 87 pages,
Helsinki Institute of Physics, Internal Report Series, HIP-2013-01,
ISBN 978-952-10-8110-1,
ISSN 1455-0563.

Abstract

Supersymmetry is a widely used extension of the Standard Model of particle physics. It extends the Standard Model by adding a symmetry between bosonic and fermionic particles and introduces superpartners – particles with similar quantum numbers but opposite spin statistics – for each of the Standard Model fields. The scalar partners of Standard Model particles allow for the construction of Lorentz and gauge invariant terms in the Lagrangian that break symmetries (or near symmetries) of the Standard Model, such as CP, flavor, baryon number, and lepton number. This presents both a challenge in explaining the absence of large symmetry breaking effects, and an opportunity for indirect discovery of Supersymmetry in precision experiments.

In the Standard Model the mentioned symmetries are accidental and unrelated. In a supersymmetric model they must be specifically reinstated and dynamically broken, or be otherwise constrained. As a consequence, these symmetries can become linked via the specifics of their breaking mechanisms. The subject of this thesis is the construction of models where this happens and the consequent relationships between symmetry breaking observables as well as the effects on other phenomenologically interesting quantities, such as Dark Matter and neutrino masses. The first part of the thesis is an introduction to the Standard Model, and the second part details the specifics of supersymmetric model building. The third part introduces the subject matter of the papers included in this thesis and presents some of the papers' key findings.

Acknowledgments

The research for this thesis was carried out with the support of the Helsinki Institute of Physics (HIP) and the Department of Physics of the University of Helsinki. I also gratefully acknowledge the numerous travel grants I have received from the Magnus Ehrnrooth foundation.

This thesis would not exist if it were not for the support and guidance of my advisor Katri Huitu. It has been a privilege working with her and I am thankful for the many insightful discussions we have had about physics, research and and life in general. I also wish to thank all of my collaborators, especially Prof. Dr. Gudrun Hiller for inspiring me and Jari Laamanen for pushing me when needed. I wish to thank Prof. Jukka Maalampi and Prof. Sourov Roy for refereeing my thesis. Their thorough and insightful comments elevated the quality of my writing significantly. In particular I want to thank David Weir who graciously proofread the entire manuscript.

The collegial atmosphere at HIP and the Departments of Physics and the many friendships I have gained while studying and working there are an invaluable part of my life; Janne, Mikko, Julius, Antti, Samuli, Janne (again), Vesa, Antti (again), Ville, Janne (once more), Jari, Lasse, and David, to name but a few. My fellow academicians from scouting, Ansku and Karhu, and from the extended family of a jazz orchestra, Maija, Liisa and Simo – many heartfelt thanks for your support and (occasional) commiseration. To all of you, and to those many friends whom I fail to mention here (though not for lack of fondness): Thank you for your excellent services in defeat of tedium.

I am very fortunate to have a loving and supportive family; my parents and sister and her family, and a wonderful flush of in-laws and associated out-laws. Thank you all for being there and knowing when not to ask about my thesis. Finally, my greatest thanks go to the love of my life, Elli, for believing in me and encouraging me, and to our son, Otto, for filling our days with wonder.

Helsinki, September 2013
Timo Rüppell

List of Included Papers

The three articles [1–3] included in this thesis are:

1. M. Frank, K. Huitu, and T. Rüppell,
Higgs and neutrino sector, EDM and ϵ_K in a spontaneously CP and R-parity breaking supersymmetric model,
Eur. Phys. J. **C52**, 413, (2007),

2. G. Hiller, K. Huitu, T. Rüppell, J. Laamanen,
A Large Muon Electric Dipole Moment from Flavor?,
Phys. Rev. D **82**, 093015, (2010),

3. K. Huitu, J. Laamanen, L. Leinonen, S. Rai, and T. Rüppell,
Comparison of neutralino and sneutrino dark matter in a model with spontaneous CP violation,
JHEP, **1211**, 129, (2012),

The articles will be referred to as Paper 1-3 throughout this thesis.

Author's Contribution

Paper 1: The paper examines the low energy parameter space in an extended NMSSM model that minimally produces tree level spontaneous R-parity and CP violation.

I wrote a Mathematica software package to calculate various observables from a given Superpotential and parameter input. I also carried out all numerical computations and the systematic exploration of the parameter space of the model. The procedural part of the paper, sections 2-5, were written by me with minor corrections and comments from the other authors.

Paper 2: The paper examines the interplay of two probes of CP and flavor violation: the muon electric dipole moment and the flavor violating rare decay $\tau \to \mu\gamma$.

I applied the Mathematica packages developed by me for Paper 1 and expanded the statistical analysis to include a novel way to quantify fine-tuning. I wrote section III and the Appendix, and performed all the numerical calculations in section III.

Paper 3: The paper examines the effect of spontaneous CP violation on the properties of dark matter candidates in an extended NMSSM model with a right handed neutrino.

I further developed the Mathematica package used in Paper 1 and Paper 2 to export FORTRAN code for mass matrix diagonalizations to be used in conjunction with the microMEGAs program. I implemented our model for microMEGAs using lanHEP as well as Perl and Mathematica based optimization scripts I wrote specifically for this task. I carried out all of the numerical calculations and the analysis of the results in section 4 and performed the selection of the benchmark points used in section 6. I wrote sections 2, 4 and most of section 3.

Contents

Chapter 1

Introduction

1.1 Historical Background

Modern particle physics uses the concepts of fields and gauge symmetries to describe the fundamental interactions of matter and the forces of nature. Particles are seen as spatially localized excitations of a field and during the 1920s the relativistic treatment of these wavefunctions led to the formulation of quantum field theory (QFT). Specifically, in 1927 Paul Dirac presented a theory for the simplest case, the interaction of the electromagnetic field and charged matter [4, 5]. The importance of this first QFT was its ability to model processes with changing particle numbers, such as an electron radiating a photon, which were not present in the earlier quantum mechanics of Werner Heisenberg and Erwin Schrödinger. The process of turning classical field variables into quantum operators, which create or annihilate particles when operating on a quantum state, is called canonical quantization. Since in this approach the wave functions of the quantum mechanical representations of particles are themselves field variables, this process is also called second quantization.

The next great advance was made in the late 1930s by Hermann Weyl, who introduced the concept of locally changing symmetries. Starting from an attempt to allow a locally varying scale (gauge) factor in Einstein's theory of relativity, when the scale factor was made complex the change in scale became a change in phase; scale invariance turned into a U(1) symmetry. Making a free field QFT invariant under this symmetry will produce the interaction terms of the Dirac theory.

The electromagnetic field of Dirac's formalism was replaced by the U(1) gauge factor, the interpretation being that the photon is an excitation of a U(1) gauge field, a gauge boson. This first gauge theory was the theory of quantum electrodynamics (QED) and became widely used during the 1940s. QED is – to this date – the best tested theory in physics, but more importantly it was a crucial step on the way to our understanding of the forces of nature in terms of local gauge symmetries of a quantum field theory.

Richard Feynman introduced the path integral formulation of quantum mechanics in 1948 [6–10]. It was based on the action principle and variational calculus, well understood methods from classical physics and the works of Joseph-Luis Lagrange and William Hamilton [11–13]. The method of path integral quantization is manifestly Lorentz invariant in every step, an improvement on the earlier work of Wolfgang Pauli and Werner Heisenberg [14, 15]. This was especially important since even very simple quantum field theory calculations yielded infinite results. Dealing with these infinities systematically, a process called renormalization, requires a Lorentz invariant framework. In renormalization, infinities that arise are absorbed by physical quantities such as charge and mass. For his work on the renormalization of QED, Feynman shared a Nobel Prize in Physics with Julian Schwinger and Sin-Itiro Tomonaga who had concurrently developed an alternate, operator based, method [16–20]. Freeman Dyson later showed the methods to be equivalent and systematized the renormalization of QED [21]. The property of renormalizability – that there exist only a finite number of infinities – is now considered a prerequisite to the acceptance of a QFT.

In 1954, Chen-Ning Yang and Robert Mills, generalized the U(1) gauge symmetry method to the non-abelian group SU(2) [22]. Their idea was to model the strong interactions of the proton and neutron by gauging the isospin symmetry. The main problem of this approach was that gauge bosons remain massless as long as the symmetry is preserved and thus would produce observable long-range effects. In the absence of such effects, non-abelian groups were unsuccessful until the early 1960's when work by Jeffrey Goldstone [23] and Yoichiro Nambu [24–26] on spontaneous symmetry breaking showed how to generate masses for gauge bosons. On the basis of those works, Peter Higgs, François Englert, and Robert Brout – among others[1] – formulated what is now called the Higgs mechanism in 1964 [31–34]. This mechanism predicts the existence of a particle called the Higgs boson, whose interaction terms give rise to the masses of all other particles via spontaneous symmetry breaking. Together with Sheldon Glashow's work on uniting the electromagnetic and weak force into the electroweak force in the framework of an SU(2)$\times$U(1) gauge theory [35], this set the stage for the Standard Model (SM) of particle physics.

The Standard Model was formulated during the late 1960s around work by Abdus Salam and John Clive Ward [36, 37], and Steven Weinberg [38] who proposed including the Higgs mechanism to spontaneously break Glashow's electroweak symmetry. Historically Glashow, Salam and Weinberg are credited with the creation of the Standard Model, and they received the Nobel Prize in 1979 for their work in uniting the electroweak force. In 1972 Gerardus 't Hooft and Martinus Veltman proved that the theory was mathematically consistent, i.e. renormalizable [39]. In 1973 the discovery of the weak neutral current at CERN [40–42], as predicted by the Standard Model, led to widespread acceptance of the theory. Around the same time, developments in the study of Yang-Mills theories led to the discovery of asymptotic freedom by David Gross and Frank Wilczek [43–45] and independently by David Politzer [46]. Asymptotic freedom is a key ingredient in the description of strong interactions and the resulting SU(3)$_C$ color gauge theory of quantum chromodynamics (QCD) is canonically included in the Standard Model. Thus formed, the Standard Model describes consistently the strong, weak and electromagnetic interactions between the fundamental constituents of matter: quarks and leptons.

[1] Due to the recent discovery of the Higgs boson it has become an exercise of academic rigor to identify the correct origin of the Higgs mechanism. Other contributors include Guralnik, Hagen, and Kibble [27], Anderson [28], Klein and Lee [29], and Gilbert [30].

	Quarks									Leptons					
	$\binom{u_L}{d_L}$	$\binom{c_L}{s_L}$	$\binom{t_L}{b_L}$	u_R	c_R	t_R	d_R	s_R	b_R	$\binom{\nu_e}{e_L}$	$\binom{\nu_\mu}{\mu_L}$	$\binom{\nu_\tau}{\tau_L}$	e_R	μ_R	τ_R
SU(3)$_C$	**3**									**1**					
SU(2)$_L$	**2**			**1**						**2**			**1**		
U(1)$_Y$	$\frac{1}{3}$			$\frac{4}{3}$			$-\frac{2}{3}$			-1			-2		

Table 1.1: The Standard Model matter content including the representations and charges under the SM gauge groups.

1.2 The Standard Model

The gauge group of the Standard Model is SU(3)$_C\times$SU(2)$_L\times$U(1)$_Y$ corresponding to the strong (QCD), weak left handed and weak hypercharge interactions, respectively. The constituents of matter, quarks and leptons, are spin-$\frac{1}{2}$ fermions and come in three families. They are charged under one or more of the gauge symmetries and are grouped into representations of the gauge symmetries accordingly. Quarks are the only particles charged under SU(3)$_C$ and form color triplets. All left-handed particles form SU(2)$_L$ doublets, L^i for the leptons and Q^i for the quarks with $i = 1, 2, 3$ indicating the family. Table 1.1 shows the charges and representations of these particles. The Standard Model does not contain a completely neutral matter particle – such a particle is often called a singlet since it falls in the singlet representation of each gauge group. The right-handed neutrino is an example of such a singlet, and is often introduced in extensions of the Standard Model.

1.2.1 Theoretical Formulation

The free field Lagrangian for a fermion ψ with mass m is [47, 48]

$$\mathscr{L} = \bar{\psi}(i\partial_\mu\gamma^\mu - m)\psi \tag{1.1}$$

with γ_μ the Dirac gamma matrices and $\bar{\psi} = \psi^\dagger\gamma^0$. The gauge principle requires invariance of the Lagrangian under a local symmetry transformation, $\psi \to e^{ig\epsilon^a(x)t_a}\psi$, where t_a are the symmetry generators of the gauge group, $\epsilon^a(x)$ the coordinate dependent gauge transformation and g the universal gauge coupling of the symmetry group. The variation of the derivative in the Lagrangian,

$$\delta(\partial_\mu\psi) = ig\epsilon^a t_a\left(\partial_\mu\psi\right) + ig\left(\partial_\mu\epsilon^a\right)t_a\psi, \tag{1.2}$$

now contains an additional $\partial_\mu\epsilon^a$ term. This term can be removed by adding a gauge field A^a_μ which transforms as $\delta A^a_\mu = \partial_\mu\epsilon^a + f^{abc}\epsilon^b A^c_\mu$, where f^{abc} are the gauge group's structure constants[2]. Now the derivative in the Lagrangian (1.1) can be replaced by a covariant derivative $\mathcal{D}_\mu = \partial_\mu - igA^a t_a$, after which the Lagrangian remains invariant under local gauge transformations. The covariant

[2] In the U(1) case the transformation is simply $\delta A_\mu = \partial_\mu\epsilon + \epsilon A_\mu$.

derivative of the SM includes the gauge fields G^a, W^i and B,

$$\mathcal{D}_\mu = \partial_\mu - i\frac{g_3}{2}\boldsymbol{\lambda}\cdot\mathbf{G}_\mu - i\frac{g_2}{2}\boldsymbol{\sigma}\cdot\mathbf{W}_\mu - i\frac{g_1}{2}B_\mu Y, \tag{1.3}$$

where g_1, g_2 and g_3 are the gauge couplings of U(1)$_Y$, SU(2)$_L$ and SU(3)$_C$ respectively, Y is the hypercharge, σ^i are the Pauli spin matrices (A.2) and λ^a are the Gell-Mann matrices (A.6). It is easy to see how gauging a free field theory introduces interaction terms between a pair of fermions and a gauge boson $\bar{\psi}\psi A$. Also, note that in the case of the U(1) group the gauge transformation of A has a nice connection to classical field theory where electromagnetic field configurations remain invariant under the addition of a total derivative of a scalar field to the vector potential, represented here by $\epsilon(x)$.

In order to write a kinetic term for the gauge fields introduced this way, it is necessary to define the gauge field strength

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu, \tag{1.4}$$

with g the appropriate gauge coupling. In the case of SU(2), f^{abc} is the completely antisymmetric tensor ϵ^{abc}; for U(1) it is zero. Using the field strength we can write gauge invariant kinetic terms for the gauge fields as

$$\mathscr{L}_{gk} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} - \frac{1}{4}W^i_{\mu\nu}W^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}. \tag{1.5}$$

Again, it is easily seen how this kinetic term gives rise to gauge boson self-interactions in the non-abelian case. It is possible to add a term $\theta\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$ to the Lagrangian. This term violates CP, and its possible presence in the otherwise CP conserving strong interactions is often called the strong CP problem [49, 50]. Finally, note that the fermion mass term in the Lagrangian (1.1) is generally not gauge invariant, similarly a mass term for the weak gauge bosons is not possible without breaking gauge invariance.

The Higgs mechanism [31, 32, 34] solves the problem of gauge boson masses by introducing the Higgs field, a spin-0 scalar doublet ϕ with hypercharge $Y = 1$ and suitable self interactions, into the theory. The Lagrangian for this field is

$$\mathscr{L}_\phi = (\mathcal{D}_\mu\phi)^\dagger(\mathcal{D}^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2, \tag{1.6}$$

where the covariant derivative leads again to interactions between the Higgs and the gauge bosons. If the parameters of the potential are such that $\mu^2 < 0$ and $\lambda > 0$, the minimum of the potential lies at a non-zero field value $|\phi|^2 = -\mu^2/(2\lambda) \equiv v^2$. This is the vacuum expectation value (VEV) of the Higgs field, $v = \langle\phi\rangle$. Choosing an appropriate minimum field configuration, ϕ_0, breaks the SU(2)$_L\times$U(1)$_Y$ symmetry and generates mass terms for specific combinations of the W^i and B gauge fields,

$$|\mathcal{D}_\mu\phi_0|^2 = \left|\left(-i\frac{g_2}{2}\sigma\cdot W_\mu - i\frac{g_1}{2}B_\mu\right)\begin{pmatrix}0\\ v\end{pmatrix}\right|^2 = m_W^2 W^-_\mu W^{+\mu} + m_Z^2 Z_\mu Z^\mu. \tag{1.7}$$

Now $W^\pm$ and Z are the physical W and Z bosons with masses $m_W^2 = g_2^2v^2/2$ and $m_Z^2 = (g_1^2 + g_2^2)v^2/2$. The field combination of W^3_μ and B_μ orthogonal to Z_μ is written as A_μ. A_μ remains massless since there is an unbroken symmetry left in the system corresponding to the combination $Q = T^3 + Y/2$ of hypercharge and isospin. This is the generator of electric charge; A_μ is identified as the photon – the gauge boson of the electromagnetic symmetry U(1)$_{EM}$. Contrary to adding mass terms for the W and Z bosons by hand, this process of spontaneous symmetry breaking protects the renormalizability of the theory as it is not the symmetry of the Lagrangian but that of the vacuum of the theory which is broken when μ^2 becomes negative. In 1973, Coleman and Weinberg showed that this is possible to achieve via radiative corrections to the scalar potential [51].

In addition to providing masses for the weak gauge bosons, the Higgs field generates the masses of fermions via the Yukawa interaction

$$\mathscr{L}_y = -y_e^{ij}\bar{L}^i\phi\, l_R^j - y_d^{ij}\bar{Q}^i\phi\, d_R^j - y_u^{ij}\bar{Q}^i\phi^c u_R^j + \text{h.c.}, \tag{1.8}$$

where $\phi^c = -i\sigma^2\phi^*$ and y are the 3×3 Yukawa matrices over family indices i, j. Once ϕ acquires a VEV the fermion mass term in the Lagrangian (1.1) is generated as $m = yv$. Although this mechanism introduces masses in a gauge invariant way, the Yukawa matrices remain arbitrary and thus offer no explanation for the structure of the fermion mass spectrum.

The Yukawa matrix of the charged leptons y_e can be made diagonal by two chiral rotations on L and l_R; this leaves the theory invariant and fixes the gauge eigenstates to be equal to the mass eigenstates. In the quark sector, rotating Q and u_R similarly diagonalizes the Yukawa matrix of the up type quarks y_u. Since Q is now fixed, the left-handed down type quarks d_L cannot absorb the rotation that diagonalizes the Yukawa matrix of the down type quarks y_d. This rotation between the gauge eigenstates d_L and mass eigenstates d'_L remains physical and is the origin of the Cabibbo–Kobayashi–Maskawa (CKM) matrix and the source of flavor changing charged current interactions in the Standard Model. The CKM matrix also contains one phase that cannot be absorbed by redefinitions of the phases of the quark fields. This phase remains physical and is the source of CP violation in the Standard Model. The CKM matrix is commonly parametrized in terms of a small expansion parameter λ [52]:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta}) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \tag{1.9}$$

Due to unitarity, the elements of the CKM matrix can be used to construct so-called unitary triangles in the complex plane. From the parametrization (1.9) it is easy to see that the commonly used unitarity constraint $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ results in a triangle with sides of roughly equal size. The area of these triangles is constant and functions as a measure of CP violation called the Jarlskog invariant [53],

$$J_{CP} = \mathrm{Im}[V_{ij}V_{kl}V_{il}^*V_{kj}^*], \quad i \neq j, k \neq l. \tag{1.10}$$

1.2.2 Experimental Measurement

The Standard Model has been verified by numerous precision experiments over the years, the most significant confirmations being the discovery of the weak currents in 1973 followed in 1983 by the

Leptons		Quarks	
ν_e	$<$ 225 eV ‡	u	2.3 $^{+0.7}_{-0.5}$ MeV
ν_μ	$<$ 0.19 MeV ‡	d	4.8 $^{+0.7}_{-0.3}$ MeV
ν_τ	$<$ 18.2 MeV ‡	s	95 $\pm$ 5 MeV
e	0.511 MeV	c	1.275 $\pm$ 0.025 GeV
μ	105.7 MeV	b	4.18 $\pm$ 0.03 GeV
τ	1.777 GeV	t	173.07 $\pm$ 0.52 $\pm$ 0.72 GeV
Gauge Bosons			
$W^\pm$	80.4 GeV	Z	91.2 GeV
G^a	0	A	0
Higgs Boson			
H		125.9 $\pm$ 0.4 GeV	

Table 1.2: The masses of elementary particles. The experimental errors for the electron and muon are six and eight orders of magnitude smaller than the most significant digit, respectively. For the tau and the weak gauge bosons the error is about four orders of magnitude lower than the most significant digit.
‡ In light of the large mixing angles present in the neutrino sector, flavor based limits are not optimal and a much better limit can be achieved for a combined neutrino mass $m_{tot}^2 < (0.5\ \text{eV})^2$ [68].

discovery of the W and Z – the gauge bosons of the weak force – at CERN [54–56], for which Carlo Rubbia and Simon van der Meer received the Nobel Prize in 1984.

Around the time of the formation of the Standard Model only the up, down, and strange quarks were known. All hadronic bound states known at the time could be understood as bound states of these quarks falling nicely into representations of an SU(3) flavor symmetry. The discovery of deep inelastic scattering in the late 1960s [57–59] solidified the quark model as a description of the strong interactions attributing the global flavor symmetry to the existence of constituent quarks and earned Jerome Friedman, Henry Kendall and Richard Taylor the Nobel Prize in 1990. In 1974 the charm quark was discovered at SLAC and BNL [60, 61], followed by the bottom quark at Fermilab in 1977 [62, 63]. Due to the heavy mass of the top quark it took until 1995 for it to be seen at Fermilab [64, 65]. Finally, announcements made by the LHC's CMS and ATLAS collaborations at CERN on July 4th 2012 regarding the discovery of a new boson with an approximate mass of 125 GeV strongly suggest that the Higgs boson has been discovered at last[3] [66, 67]. Table 1.2 shows the current state for measured values of the masses of these particles.

The non-perturbative nature of the strong force at low energies makes it very challenging to measure the quark masses and the elements of the CKM matrix accurately. The intricacies of the

[3]The discovered boson may yet be shown to be a composite particle, but in this thesis it will be assumed to be an elementary particle.

$\lvert V_{ud}\rvert = 0.97425$	$\lvert V_{us}\rvert = 0.2252$	$\lvert V_{ub}\rvert = 4.15\times10^{-3}$
$\lvert V_{cd}\rvert = 0.230$	$\lvert V_{cs}\rvert = 1.006$	$\lvert V_{cb}\rvert = 40.9\times10^{-3}$
$\lvert V_{td}\rvert = 8.4\times10^{-3}$	$\lvert V_{ts}\rvert = 42.9\times10^{-3}$	$\lvert V_{tb}\rvert = 0.89$

Table 1.3: The CKM matrix elements. Blue indicates the order of magnitude of the experimental error.

theoretical and experimental difficulties are beyond the scope of this work, and I refer the interested reader to the Review of Particle Physics [68]. Table 1.3 summarizes the current measurements of the CKM matrix elements. Extensions of the Standard Model often introduce new sources of CP violation and it is important to precisely determine the CP phase of the CKM matrix as well as over constrain the unitarity triangle. Recently, it has been proposed to disentangle SM and possible new physics contributions [69, 70]. This is particularly interesting for models of spontaneous CP violation which presume a CP-conserving real CKM matrix at tree level. Figure 1.1 shows the current bounds on the unitarity triangle.

The discovery of atmospheric neutrino oscillation in 1998 [71] made it necessary to incorporate a mechanism for neutrino masses into the Standard Model. The standard Yukawa mechanism requires the addition of one or more right-handed neutrinos to the model. This addition incidentally creates a mixing matrix, analogous to the CKM matrix, for the left-handed neutrinos – the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [72, 73]. A right-handed neutrino also allows for the seesaw mechanism for generating small neutrino masses [74–77]. On its own, the left-handed neutrino can have a mass if it is a Majorana particle[4], and discovery of neutrinoless double beta ($0\nu2\beta$) decay [79–82] would confirm this. However, the $0\nu2\beta$ process is a fourth order weak interaction and difficult to observe. A Heidelberg and Moscow collaboration has claimed observation [83] but issues with the statistical analysis, nuclear matrix elements, and interpretation of the results have brought the discovery into question [84]. The most recent results on double beta decay from the LNGF experiment at Gran Sasso National Laboratories fail to confirm the discovery, but note that several experiments are currently underway to test the claim [85].

The challenges of direct measurements in the neutrino sector are on par with those in the quark sector, albeit for different reasons. The weak nature of their interactions and the sizable background from cosmic radiation requires large detectors situated underground. Measurements of the flavor oscillation of accelerator, atmospheric or solar neutrinos are mapped into two independent mass squared differences and the three rotation angles of the PMNS matrix [86, 87]. Table 1.4 summarizes these results.

[4] Majorana particles are their own anti-particle making it possible to write a gauge invariant mass term without the need for a charge conjugate field. The existence of such particles was proposed by Ettore Majorana in 1937 [78].

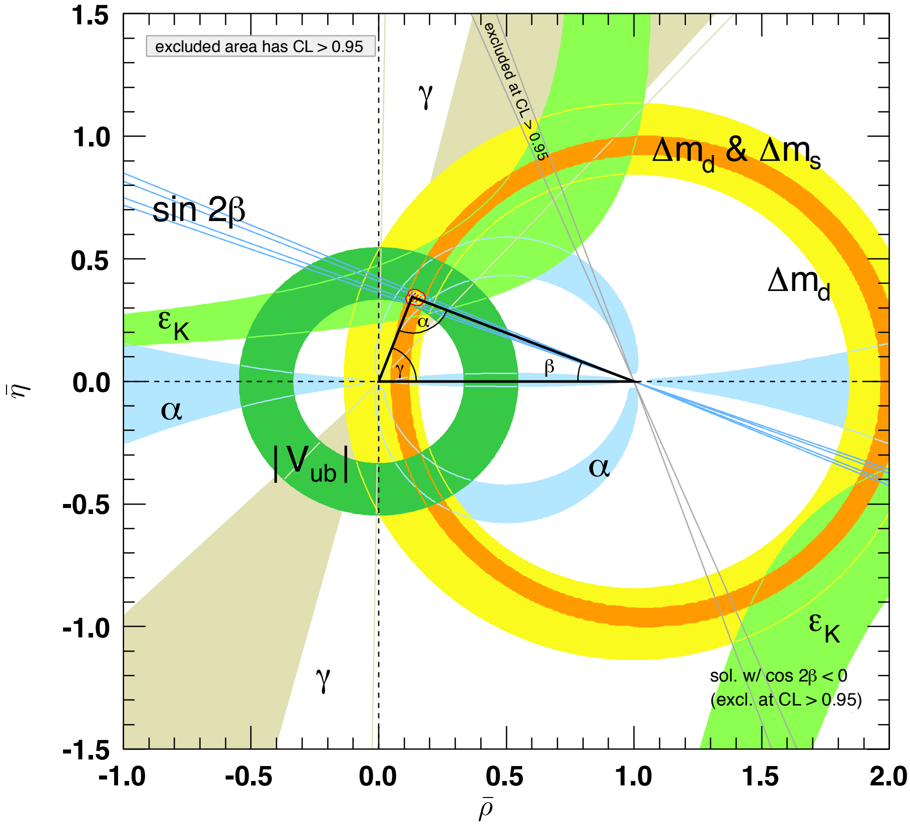


Figure 1.1: The unitarity triangle. Various measurements constrain the upper corner, $(\bar{\rho}, \bar{\eta})$, of the triangle. The measurements correspond to $J_{CP} = 2.96^{+0.20}_{-0.16} \times 10^{-5}$. Taken from the Review of Particle Physics [68].

$\Delta m^2_{\odot}$	$(7.58\ ^{+0.22}_{-0.26}) \times 10^{-5}$ eV^2
$\lvert\Delta m^2_A\rvert$	$(2.35\ ^{+0.12}_{-0.09}) \times 10^{-3}$ eV^2
$\sin^2(\theta_{12})$	$0.306\ ^{+0.018}_{-0.015}$
$\sin^2(\theta_{23})$	$0.42\ ^{+0.08}_{-0.03}$
$\sin^2(2\theta_{13})$	0.096 ± 0.013

Table 1.4: The neutrino oscillation data.

1.2.3 Higgs mass

The masses of the weak gauge bosons [68], $m_W \simeq$ 80.4 GeV and $m_Z \simeq$ 91.2 GeV, together with measurements of the gauge couplings fix the Higgs VEV v at 175 GeV. The mass of the Higgs, $m_h^2 = 2\lambda v^2$, on the other hand, is a free parameter in the Standard Model and can only be measured directly from observation of the Higgs boson. There are, however, several ways for indirectly arriving at bounds for the Higgs mass. Unitarity of the scattering amplitude in gauge boson interactions is perhaps the simplest of these. The WW scattering amplitude [88–90]

$$\mathcal{A}(W_L^+ W_L^- \to W_L^+ W_L^-) = -\sqrt{2} G_F m_\phi^2 \left[\frac{s}{s - m_\phi^2} + \frac{t}{t - m_\phi^2} \right] \tag{1.11}$$

grows with s when $m_\phi \to \infty$. Even for a finite m_ϕ, partial wave unitarity can be used to extract an upper bound on the mass,

$$m_\phi^2 \leq \frac{4\pi\sqrt{2}}{3G_F} \simeq (700 \text{ GeV})^2. \tag{1.12}$$

Including one loop corrections lowers this upper limit to approximately 350 GeV [91].

The unitarity limit is not a particularly low and better limit can be arrived at by considering the running of the Higgs quartic coupling [90]. Starting with the pure scalar model of (1.6) without gauge interactions, the renormalization group equations (A.19) simplify to

$$\frac{d}{dt}\lambda = \frac{3\lambda^2}{4\pi^2}. \tag{1.13}$$

Solving this leads to an extrapolation of the coupling at an arbitrary scale, $\lambda(Q)$, when the coupling is known at some reference scale, $\lambda(Q_0)$.

$$\lambda(Q) = \lambda(Q_0) \left[1 - \frac{3\lambda(Q_0)}{4\pi^2} \log\left(\frac{Q^2}{Q_0^2} \right) \right]^{-1}. \tag{1.14}$$

Regardless of how small $\lambda(Q_0)$ is initially, $\lambda(Q)$ becomes infinite at some large value of Q – this is known as the Landau pole. Conversely, $\lambda(Q) \to 0$ as $Q \to 0$, which results in a trivial free field theory without interactions. Requiring λ to be finite, that is for the theory to remain perturbative, at some high scale Λ gives us a limit on the Higgs mass at the electroweak scale v:

$$m_\phi^2 < \frac{8\pi^2 v^2}{3\log(\Lambda^2/v^2)}. \tag{1.15}$$

The scale Λ denotes the scale at which new physics enters and changes the theory. If this is, say, the gauge unification scale 10^{16} GeV, then the limit on the Higgs mass is $m_\phi <$ 170 GeV. Expecting new physics at lower scales weakens this limit; $\Lambda =$ 2 TeV yields $m_\phi <$ 620 GeV. This simple analysis breaks down when approaching the limit as λ becomes large and higher order corrections to (1.13) should be taken into account. Similarly one should include corrections from fermion – in particular top quark – interactions. A thorough analysis leads to an upper limit $m_\phi <$ 170 GeV [92, 93].

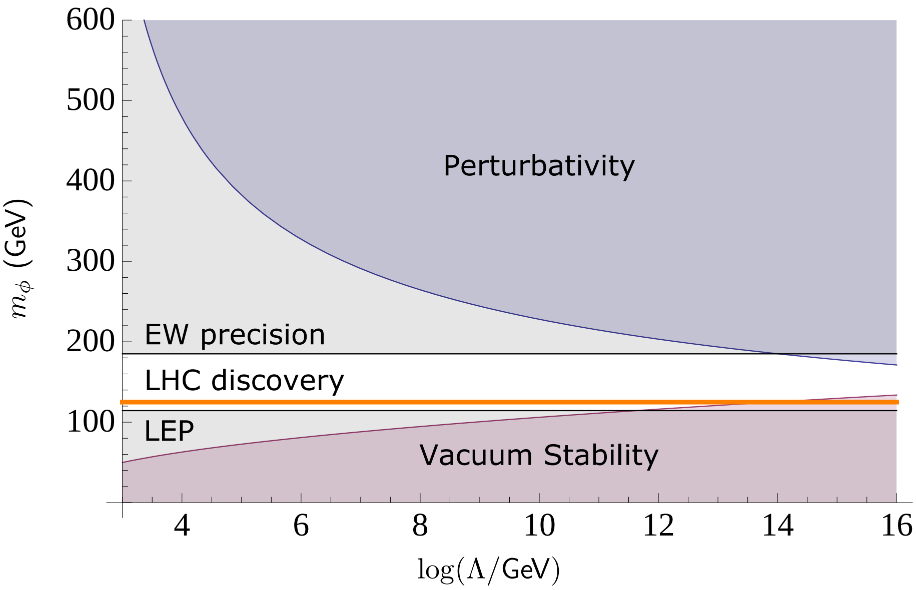


Figure 1.2: The Standard Model limits on the Higgs mass.

A lower limit on the Higgs mass can be arrived at by demanding that λ remains positive up to the scale Λ. A naive first order approach solves the RGE for small λ yielding

$$m_\phi^2 > \frac{3}{16\pi^2 v^2}\left[2m_W^4 + m_Z^4 - 4m_t^4\right] \overset{m_t=0}{\simeq} (7\ \text{GeV})^2, \tag{1.16}$$

which is not very useful. A more thorough approach that includes second order corrections gives a limit of $m_\phi \gtrsim 130$ GeV for $\Lambda = 10^{16}$ GeV [94–96]. Since the Higgs boson was discovered with a mass of 125 GeV the vacuum stability bound has received a lot of attention; more recent studies including 3-loop beta functions of the Higgs boson self coupling λ give $m_\phi > 129 \pm 3$ GeV [97, 98]. The result is highly dependent on both the top mass and the Higgs boson mass and suggests the possibility of a metastable vacuum.

Prior to the discovery of the Higgs boson at the LHC, LEP had put a lower bound on the mass of the Higgs boson of 114.4 GeV at a 95% confidence level [99]. Also, since the Higgs boson is present in loop level calculations of the Standard Model[5], precision measurement of any parameter of the Standard Model can be used to put a limit on its mass. The combined upper limit from such measurements at Fermilab and CERN was 185 GeV at a 95% confidence level if the direct search bound was taken into account [100]. The limits on the Higgs mass discussed in this section are shown in Figure 1.2.

[5] In general these corrections have leading logarithmic dependence with increasing powers of the gauge couplings for higher orders. This effect of the gauge couplings is commonly called screening.

1.3 Beyond The Standard Model

The importance of the Standard Model can be seen in the number of Nobel laureates that are associated with the model. In addition to the laureates already mentioned in Section 1.1, Gross, Wilczek, Politzer, Nambu, 't Hooft and Veltman received Nobel Prizes for their work with the Standard Model. In addition to these, Nobel Prizes were also awarded for the discovery of the weak gauge bosons, discovery of the τ-lepton and the electron neutrino, discovery of the muon neutrino, and discovery of CP-violation in the kaon system. The main success of the Standard Model, however, is the wide variety of tests it has passed since its formulation. The quintessential example being the electrons $g-2$ measurement, which differs from the theoretical expectation at a level of one part in a billion [101].

Despite this success of the Standard Model, it is commonly accepted to be only a low energy approximation, albeit a good one, of a more fundamental theory. For example, the observed baryon asymmetry of the universe requires more CP violation than there is present in the Standard Model [102–105]. The standard model also does not include gravity, indeed the interactions of a spin-2 graviton are non-renormalizable in the framework of quantum field theory[6] [108, 109]. The existence of cold dark matter, first discovered in 1933 by Fritz Zwicky [110], is well established and the Standard Model does not provide an elementary particle candidate for it. More recently, the discovery of neutrino oscillation is a further hint at physics beyond the Standard Model. Lastly, the apparent unification of gauge couplings at 10^{16} GeV, only three orders of magnitude below the Planck scale, suggests the existence of a grand unified theory (GUT) of the strong and electroweak interactions.

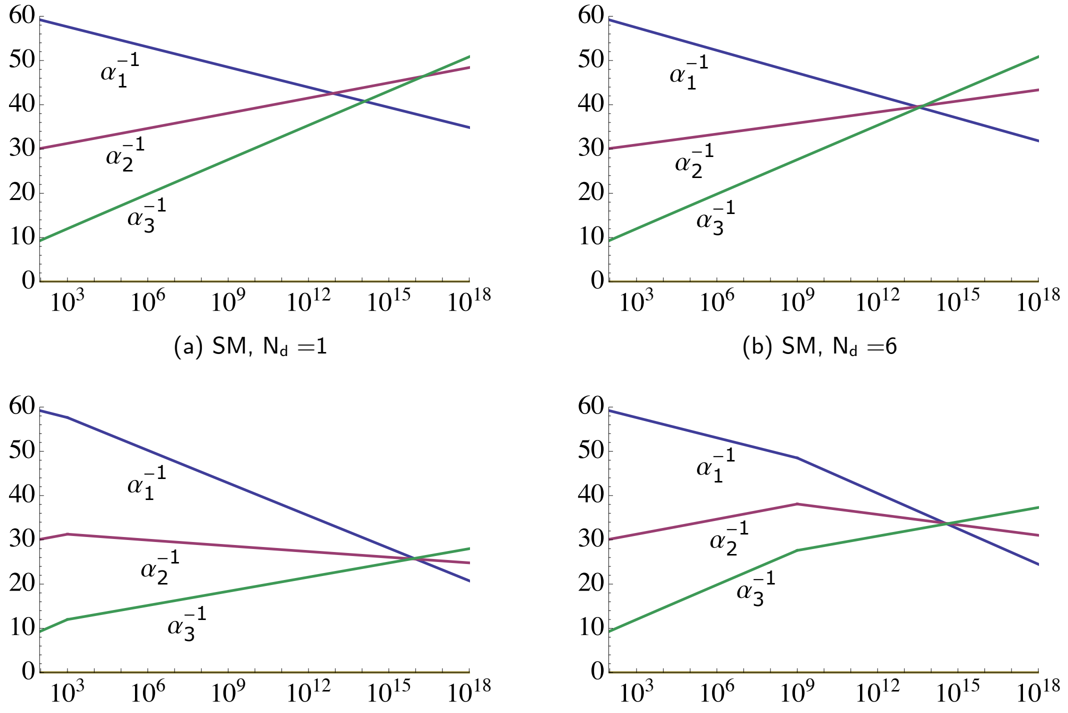
1.3.1 Gauge Coupling Unification

In the early 1970s, scaling in deep inelastic scattering hinted at the asymptotic freedom of strong interactions. This led to the hypothesis that the strong and electroweak interactions could have the same strength at some high scale and that consequently there might be a unified description of these forces using some unifying gauge group [111–113].

The radiative corrections to the Standard Model gauge couplings are scale dependent. Requiring physics to be unchanged by the choice of this arbitrary renormalization scale μ leads to differential equations called the renormalization group equations (RGE), which relate the parameters at different energy scales. The RGEs for the three gauge couplings of the Standard Model are

$$\mu\frac{\partial}{\partial\mu}\alpha_i = \frac{b_i}{2\pi}\alpha_i^2 + \frac{b_{ij}}{8\pi^2}\alpha_i^2\alpha_j, \qquad \begin{cases} \alpha_1 = \frac{5}{3}g'^2/4\pi \\ \alpha_2 = g_2^2/4\pi \\ \alpha_3 = g_3^2/4\pi \end{cases} \tag{1.17}$$

[6] An intuitive explanation for this is given by Shomer [106], noting that renormalizability of a theory implies the high energy limit to be a conformal field theory (CFT). Gravity's asymptotic density of states, however, is dominated by black holes and thus radically different from a CFT. Consequently, gravity as a low energy effective field theory models something other than a quantum field theory. Currently the most promising candidate for a proper quantum theory of gravity is superstring theory [107].



(a) SM, $N_d = 1$

(b) SM, $N_d = 6$

(c) SUSY, $M_{SUSY} = 10^3$ GeV, $N_d = 2$

(d) SUSY, $M_{SUSY} = 10^9$ GeV, $N_d = 4$

Figure 1.3: The unification of gauge couplings. The inverse couplings α_i^{-1} as functions of the energy μ[GeV] in the SM and SUSY for differing numbers of Higgs doublets N_d and SUSY scale M_{SUSY}.

where the constants b_i and b_{ij} are given in the Appendix A.4. Knowing the values of the parameters at a low energy scale it is possible to extrapolate their value at higher energies. More precise measurements soon showed that the three gauge couplings do not in fact meet at a single point[7]. Since the RGE constants b_i, b_{ij} depend on the number of fermion families and Higgs doublets in the theory, there is some room for adjustment. Specifically, extending the Higgs sector is required as the number of families affects all couplings equally to the first order. The problem of such extensions is that they tend to decrease the scale of unification and are incompatible with limits on the proton lifetime.

Supersymmetry (SUSY) provides an alternative way of modifying the theory's particle content. In 1981 it was shown by Dimopoulos and others that gauge coupling unification worked in supersymmetric models [115–118]. In fits of gauge coupling unification for supersymmetric models there is an additional parameter, which is the scale at which the RGEs of the Standard Model switch to those of the supersymmetric model. In the simplest case unification requires $M_{SUSY} \sim 10^3$ GeV [114]. Similar to the Standard Model case, one can extend the Higgs sector to adjust the scale of supersymmetry. Figure 1.3 shows the gauge coupling unification in a variety of scenarios.

[7] By the early 1990s the gauge couplings were found to miss a common intersection by more than seven standard deviations [114].

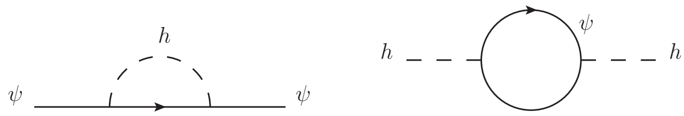


Figure 1.4: One loop contribution to the fermion and scalar mass.

1.3.2 Hierarchy Problem

The most compelling argument – from a theoretical perspective – against the Standard Model is the so-called hierarchy problem [119–121]. It becomes apparent when examining the loop order correction to the Higgs boson mass. Consider a simple model of a Higgs scalar ϕ and a fermion ψ and a Yukawa interaction [122],

$$\mathscr{L}_\phi = \bar{\psi}(i\partial_\mu\gamma^\mu)\psi + |\partial_\mu\phi|^2 - m_s^2|\phi|^2 - \left(\frac{\lambda_f}{2}\bar{\psi}\psi\phi + \text{h.c.}\right). \tag{1.18}$$

For simplicity we assume that spontaneous symmetry breaking occurs and expand around the vacuum expectation value $\phi = h + v$, with h now the physical scalar field. The fermion field gets a mass term $m_f\bar{\psi}\psi$ with $m_f = \lambda_f v$. One can now calculate the corrections to m_f and m_s due to the loops shown in Figure 1.4,

$$\delta m_f = -\frac{3\lambda_f^2 m_f}{32\pi^2}\log\left(\frac{\Lambda^2}{m_f^2}\right) \tag{1.19a}$$

$$\delta m_s^2 = -\frac{4\lambda_f^2}{16\pi^2}\left[\Lambda^2 + \left(m_s^2 - 6m_f^2\right)\log\left(\frac{\Lambda}{m_f}\right) + \left(2m_f^2 - \frac{m_s^2}{2}\right)\left(1 + I_1\left(\frac{m_s^2}{m_f^2}\right)\right)\right], \tag{1.19b}$$

where Λ is the regularization cut off scale, $I_1(x)$ is the integral

$$I_1(a) = \int_0^1 \mathrm{d}x \log(1 - ax(1-x)) = 2\sqrt{\frac{4-a}{a}}\arctan\left(\sqrt{\frac{a}{4-a}}\right) - 2, \quad 0 < a < 4, \tag{1.20}$$

and terms which vanish as $\Lambda \to \infty$ are omitted. Looking at δm_f, we see that the correction is not too severe at the GUT scale $\Lambda \sim 10^{16}$ GeV or even at the Planck scale $\Lambda \sim 10^{19}$ GeV. The proportionality of δm_f to m_f is due to the fact that the Lagrangian has a chiral symmetry

$$\psi_L \to e^{i\theta_L}\psi_L \qquad \psi_R \to e^{i\theta_R}\psi_R, \tag{1.21}$$

which is broken by the fermion mass term. When intact, the symmetry will protect the mass ($\delta m_f \to 0$ as $m_f \to 0$) to all loop orders; when broken, the corrections to the mass will be proportional to the symmetry breaking [25, 26]. Conversely the Higgs scalar mass receives corrections that diverge quadratically as $\Lambda \to \infty$ and setting $m_s = 0$ is of no help. Indeed, no symmetry protects a scalar mass term – leaving the Higgs boson mass sensitive to any high scale that exists in the theory. This problem persists even if the scalar is not coupled to these matter fields directly. If the fermions and the Higgs boson are both charged under some gauge group G, then the contributions of the loops

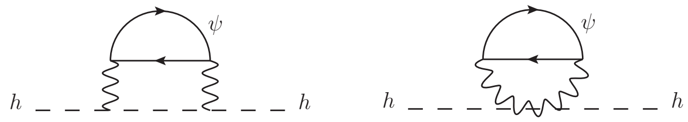
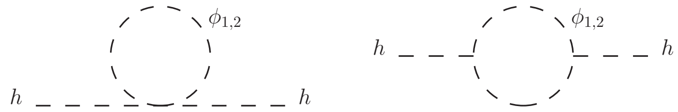


Figure 1.5: Two loop contributions to the scalar mass from fermions coupled to the scalar via gauge interactions.



(a) Quartic interaction (b) Cubic interaction

Figure 1.6: Loop contributions to the scalar mass from bosonic fields.

shown in Figure 1.5 yield (up to terms that vanish as $\Lambda \to \infty$) [123]

$$\delta m_s^2 = C_h \cdot D_\psi \left(\frac{g^2}{16\pi^2} \right) \left[a\Lambda^2 + 48 m_f^2 \log\left(\frac{\Lambda}{m_f} \right) \right], \tag{1.22}$$

where C_h and D_ψ are the Casimir operator and Dynkin index, respectively, and a depends on the particulars of the momentum cut off.

Taking the Higgs boson mass to be around 125 GeV and assuming Λ to be the Planck scale, we have to conclude that cancellations in the perturbation series have to be fine tuned to thirty-two orders of magnitude. This is called the hierarchy problem.

Solutions to the Hierarchy Problem

Let us now introduce two additional scalars $\phi_{1,2}$ to the theory:

$$\mathscr{L} = |\partial_\mu \phi_1|^2 - m_1^2 |\phi_1|^2 + |\partial_\mu \phi_2|^2 - m_2^2 |\phi_2|^2 + \lambda_s |\phi|^2 (|\phi_1|^2 + |\phi_2|^2) + \mathscr{L}_\phi. \tag{1.23}$$

The interactions give rise to the loops in Figure 1.6. The contributions to m_s are (up to terms that vanish when $\Lambda \to \infty$)

$$(\delta m_s^2)_q = -\frac{4\lambda_s}{16\pi^2} \left[\Lambda^2 - m_1^2 \log\left(\frac{\Lambda}{m_1} \right) - m_2^2 \log\left(\frac{\Lambda}{m_2} \right) \right], \tag{1.24a}$$

$$(\delta m_s^2)_c = \frac{4 m_f^2}{16\pi^2} \frac{\lambda_s^2}{\lambda_f^2} \left[2 - 2\log\left(\frac{\Lambda}{m_1} \right) - 2\log\left(\frac{\Lambda}{m_2} \right) + I_1\left(\frac{m_s^2}{m_1^2} \right) + I_1\left(\frac{m_s^2}{m_2^2} \right) \right]. \tag{1.24b}$$

The contribution from the loop with cubic (c) interactions is only logarithmically divergent but the quartic (q) interactions lead to a quadratic divergence similar to the one from the fermion loop. This divergence is exactly cancelled by the quadratic divergence in Equation (1.19b) if $\lambda_f^2 = -\lambda_s$. The logarithmic divergencies proportional to $m_{1,2}^2$ and m_f^2 can all be canceled by setting the fermion and scalar masses equal, $m_f = m_{1,2} = m$. We are then left with,

$$\delta m_s^2 = -\frac{4\lambda_f^2}{16\pi^2} m_s^2 \log\left(\frac{\Lambda}{m}\right) + \text{finite term}, \tag{1.25}$$

where the remaining divergent term is due to evaluating the corrections at the physical pole mass of the external particle, $p^2 = m_s^2$. If the fermion and scalar masses are only approximately equal, there will be a divergent term proportional to $\delta m^2 = m_f^2 - m_{1,2}^2$, which leads to an upper bound for the mass splitting between the fermions and their scalar "partners".

Let us briefly assume that Λ is not a physical quantity since it arises from our choice of regularization. One can alternatively use Pauli-Villars regularization [124] or dimensional regularization [125]. Using the latter, a more interesting conclusion about the mass relation can be drawn from

$$\begin{aligned}(\delta m_s^2)_{d-reg} \quad = \quad & -\frac{4\lambda_f^2}{16\pi^2}\left[\left(m_s^2 - (2m_f^2 - m_1^2 - m_2^2)\right)\left(\frac{1}{\epsilon} - \gamma + \mathcal{O}(\epsilon)\right)\right. \\ & \left. -2m_f^2 I_1\left(\frac{m_s^2}{m_1^2}\right) - 2m_f^2 I_1\left(\frac{m_s^2}{m_2^2}\right)\right], \end{aligned} \tag{1.26}$$

where $\epsilon = 2 - d/2$, d is the dimension and γ is the Euler number. Now the masses of fermions and scalars do not have to be equal but rather $2m_f^2 = m_1^2 + m_2^2$. This implies that one of the scalars must be lighter than the fermion, which is contradicted by experimental searches. Additionally it should be noted that without a symmetry governing the addition of these new scalars, the mass relation breaks down at higher loop orders. Supersymmetry was eventually found to correctly perform the role of the needed symmetry [126].

Several other solutions to the hierarchy problem have been attempted. As pointed out above, the scalar nature of the Higgs boson leaves it unprotected against large radiative corrections. Technicolor models circumvent this by replacing the Higgs boson with a composite particle; a fermion condensate [127–129]. Many of these theories have significant problems with flavor changing neutral currents as well as with the correct generation of the masses of the heavier fermions [130]. More recently it was realized that the problem of quadratic divergences can be avoided if the Planck scale were much lower, say, at the TeV scale. Models of extra dimensions [131–134] achieve this by diluting the gravitational interaction into more than four spacetime dimensions.

Chapter 2

Supersymmetry

2.1 Historical Background

Supersymmetry extends quantum field theory to include operators that change the spin of a field thus relating fermions and bosons.

In 1967 Coleman and Mandula proved that there exists no symmetry to relate particles of different spin[1] [135]. Their no-go theorem, however, does not apply if the Poincaré algebra is extended to a graded Lie algebra including anti-commuting symmetry generators [136]. More importantly, the theorem requires Lorentz invariance, making it irrelevant to the study of non-relativistic hadron spectroscopy where for example SU(6) was used to relate meson and baryon multiplets of different spins.

It is in this context – a search for a model of the string interactions – that supersymmetry was discovered in 1971 by Ramond [137]. This was quickly followed by the first formulation of a supersymmetric string theory of bosons and fermions by Neveu and Schwarz [138] as well as Gervais and Sakita [139]. In 1974 Haag, Łopuszański and Sohnius extended Coleman and Mandula's no-go theorem to include this new supersymmetry [140]. At the same time Volkov and Akulov [141–143], and independently Wess and Zumino [144–147], formulated the first supersymmetric quantum field theories. Shortly thereafter Freedman, Ferrara, and van Nieuwenhuizen localized supersymmetry using the Noether procedure and showed that it leads to the inclusion of the graviton and its supersymmetric partner the gravitino – which can be thought of as the gauge field of local SUSY transformations – in a theory called supergravity (SUGRA) [148–150].

The first supersymmetric extension of the SM, which solved the problem discussed in Section 1.3.2 of the origin of the scalar partners, was formulated by Fayet and Farrar [151–154]. The first realistic minimal supersymmetric extension of the Standard Model was presented by Dimopoulos and Georgi in 1981 [115]. Today, SUSY is perhaps the most broadly adopted extension of the SM [123, 155, 156]. A large part of the impetus for building the LHC was the machine's potential to discover SUSY, and while constrained models of minimal SUSY are starting to be ruled out, it remains a remarkably

[1] The Coleman–Mandula theorem states that, for a Lie group composed of an internal symmetry group G and the Poincaré group P, the generators of G must commute with the generators of P.

resilient theory if only due to its complexity.

2.2 Basic Formulation

2.2.1 Algebra

A graded Lie algebra is defined by the graded commutation relation

$$[t_a, t_b\} \equiv t_a t_b - (-1)^{\eta_a \eta_b} t_b t_a = i f^c_{ab} t_c \tag{2.1}$$

where η_a is the grading of the generator t_a, and is by convention 1 for fermionic generators and 0 for bosonic ones. The f^c_{ab} are the structure constants of the algebra, satisfying $f^c_{ba} = -(-1)^{\eta_a \eta_b} f^c_{ab}$. This so-called superalgebra also leads to the super Jacobi identity

$$(-1)^{\eta_a \eta_b} \left[[t_a, t_b\}, t_c\right\} (-1)^{\eta_c \eta_a} \left[[t_c, t_a\}, t_b\right\} (-1)^{\eta_b \eta_c} \left[[t_b, t_c\}, t_a\right\} = 0. \tag{2.2}$$

We now introduce fermionic generators that transform under an internal symmetry G to the Poincaré group as two component Weyl spinors[2] Q. Lorentz invariance and the superalgebra can be used to derive the (anti)commutation relations

$$\left[P^\mu, Q_\alpha{}^A\right] = \left[P^\mu, \bar{Q}^{\dot{\alpha}}{}_A\right] = 0 \tag{2.3a}$$

$$\left[M^{\mu\nu}, Q_\alpha{}^A\right] = -i(\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta{}^A \tag{2.3b}$$

$$\left[M^{\mu\nu}, \bar{Q}^{\dot{\alpha}}{}_A\right] = -i(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}} \bar{Q}^{\dot{\beta}}{}_A \tag{2.3c}$$

$$\left\{Q_\alpha{}^A, \bar{Q}_{\dot{\beta}B}\right\} = 2\delta^A{}_B (\sigma^\mu)_{\alpha\dot{\beta}} P_\mu \tag{2.3d}$$

$$\left\{Q_\alpha{}^A, Q_\beta{}^B\right\} \equiv \epsilon_{\alpha\beta} Z^{AB}, \tag{2.3e}$$

where Z^{AB} is the central charge[3] of G and commutes with all of G. P_μ and $M_{\mu\nu}$ are the usual generators of translations and Lorentz transformations of the Poincaré group. If all the central charges are 0, the internal symmetry group G is U($\mathcal{N}$) and one uses the term $\mathcal{N}$ = 1, 2, ... supersymmetry. An important property of supersymmetric theories can be shown using Equation (2.3d),

$$\langle H \rangle = \langle 0|P_0|0\rangle = \frac{1}{4N}\langle 0| \sum (QQ^* + Q^*Q)|0\rangle = \frac{1}{4N}\sum \left(|Q|0\rangle|^2 + |\bar{Q}|0\rangle|^2\right) \geq 0, \tag{2.4}$$

meaning the vacuum energy is positive definite and that, for a supersymmetric vacuum, where $Q|0\rangle = \bar{Q}|0\rangle = 0$, the vacuum energy is zero. If supersymmetry were a manifest symmetry of

[2] For the definitions and relations of Dirac, Majorana and Weyl spinors, see the Appendix A.2.

[3] Similarly, the anticommutator of $\bar{Q}$ defines a central charge Z^*. Z and Z^* are linear combinations of the generators of G and belong to the abelian invariant subalgebra of G. Commutation relations for Q, $\bar{Q}$ and the generators of G can be written but are not pertinent to this discussion.

nature, this would neatly solve the cosmological constant problem[4]. Even with supersymmetry broken in nature, negative radiative contributions to the vacuum energy can appear in the framework of supergravity and cancel the positive contributions [162, 163]. However, it is still necessary to fine-tune these cancelations [164].

Defining a spin vector $W^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}P_\nu M_{\rho\sigma}$, one can show that the generators $Q/\bar{Q}$ raise or lower the spin of a state by $\frac{1}{2}$. Successive operations on a given spin state produce a SUSY representation or supermultiplet. The commutator (2.3a) can be used to easily show that all states in a supermultiplet have equal mass and that the fermionic degrees of freedom match the bosonic ones. The only case that gives rise to chiral supermultiplets is the simplest formulation, $\mathcal{N}=1$ SUSY. For larger numbers of supersymmetry generators, a multiplet containing a chiral fermion ψ_L would either contain a spin 1 vector field, A^μ, or a fermion with opposite chirality ψ_R. Fields in the same supermultiplet transform the same way under the gauge group so – in the first case – we would end up with fermions in the adjoint representation, which is contrary to the experimentally observed behavior. The second case is excluded by the need to treat chiral representations differently under SU$(2)_L$, which is also experimentally well established.

While low energy models of supersymmetry assume $\mathcal{N}=1$, this is not to say that extended supersymmetries with $\mathcal{N}\geq 2$ are useless. For example, dimensional reduction uses an equivalence of $\mathcal{N}=2$ supersymmetry in three dimensions and $\mathcal{N}=1$ supersymmetry in 4 dimensions [165], $\mathcal{N}=4$ supersymmetric Yang-Mills theory is used in the AdS–CFT duality conjecture by Maldacena [166] and $\mathcal{N}=8$ supersymmetry is employed in linking supergravity and string theory [167, 168]. The $\mathcal{N}=8$ theory is said to be the maximally extended supersymmetry. This is due to problems arising from spin > 2 fields; in particular it becomes difficult to eliminate the unphysical degrees of freedom which grow with the spin of a particle. In the case of the the spin 2 graviton, local supersymmetry as well as its associated gauge field, the spin $\frac{3}{2}$ gravitino, arise as constraints that eliminate all unphysical degrees of freedom [107]. The consistency of a model with such a gravity multiplet was shown by Deser and Zumino in 1976 [169]. It is not known how to couple fields with spins greater than two to gravity – or fields of any other spin – in a consistent manner [170].

2.2.2 Superfields

In the approach of Wess and Zumino, the supersymmetric Lagrangian is formulated by considering infinitesimal supersymmetry transformations on the free field Lagrangian of a massless scalar and spinor. Requiring closure of the superalgebra leads to the introduction of auxiliary fields into the supermultiplet. For constructing the Lagrangian a more powerful method introducing the concept of superspace and superfields was proposed by Salam and Strathdee [171–174]. In direct analogy to spacetime coordinates x^μ parametrizing translations generated by P_μ, there are four fermionic

[4]The non-vanishing energy content of the vacuum in a quantum theory was first pointed out by Nernst in 1916 [157] and the connection to the cosmological constant was first pointed out by Lemaître in 1934 [158]. Zel'dovich was the first to correctly calculate the contribution to the cosmological constant from quantum fluctuations in 1967 [159, 160], and currently the estimate of the discrepancy between the observed value and theoretical prediction is $\sim 10^{123}$ [161].

coordinates θ_α that parametrize supersymmetry translations generated by Q and $\bar{Q}$. These Grassmann variables transform as two component Weyl spinors θ and $\bar{\theta}$. Together with x^μ they form a superspace[5] and a we can define an element of the super Lie algebra as

$$G(x,\theta,\bar{\theta}) = e^{-i(x^\mu P_\mu - \theta Q - \bar{\theta}\bar{Q})}. \tag{2.5}$$

Left and right multiplication on a superfield $S(x,\theta,\bar{\theta})$ by an infinitesimal superspace translation lets us extract the operators Q and $\bar{Q}$ as well as the two new operators D and $\bar{D}$:

$$\begin{aligned} Q_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, & \bar{Q}_{\dot{\alpha}} &= -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu, \\ D_\alpha &= \frac{\partial}{\partial\theta^\alpha} + i(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, & \bar{D}_{\dot{\alpha}} &= -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu. \end{aligned} \tag{2.6}$$

The D and $\bar{D}$ operators anticommute with Q and $\bar{Q}$ and thus commute with $\delta_\xi = \xi Q + \bar{\xi}\bar{Q}$, the supersymmetry generator[6], when operating on a superfield. This is a useful property as it allows the construction of covariant constraints, which are needed to reduce the degrees of freedom in a general superfield

$$\begin{aligned} S(x,\theta,\bar{\theta}) &= \phi(x) + \theta\xi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta m(x) + \bar{\theta}\bar{\theta}n(x) \\ &\quad + \theta\sigma^\mu\bar{\theta}V_\mu(x) + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\psi(x) + \theta\theta\bar{\theta}\bar{\theta}d(x), \end{aligned} \tag{2.7}$$

which has four scalars, four spinors and one vector component field. The covariant constraint $\bar{D}_{\dot{\alpha}}S = 0$ defines a left[7] chiral superfield Φ. Noting that any function of the coordinates θ and $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ satisfies this constraint, we can write

$$\begin{aligned} \Phi(y,\theta) &= \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \\ &= \phi(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial^2\phi(x) \\ &\quad + \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x), \end{aligned} \tag{2.8}$$

where F is a purely auxiliary degree of freedom that can be eliminated via the field equations. The coefficient of $\theta\theta$ in a left chiral superfield is called the F-term and will be useful in constructing the Lagrangian since it transforms under supersymmetry as a total derivative. A right chiral superfield is similarly defined by the constraint $D_\alpha\Phi^\dagger(y^\dagger,\bar{\theta}) = 0$.

Another covariant constraint[8] we can apply is $S^\dagger = S$, which defines a vector superfield V. Using the definition (2.7), one immediately gets a set of constraints on the component fields. To further

[5]Supercoordinates are formally the extension of real numbers to nilpotent dual numbers, $\theta_\alpha^2 = 0$. Derivatives are easily defined in the usual way but it is slightly more involved to show that integration over superspace coordinates is equivalent to derivation [175, 176].

[6]Here ξ and $\bar{\xi}$ are Grassmann numbers that function as the infinitesimal parameters of the SUSY transformation.

[7]The "left" and "right" naming convention is chosen such that the Weyl spinor component ψ of the chiral superfield is left- or right-handed respectively.

[8]Additional constraints using the covariant derivatives tend to restrict the x dependence of the field components via differential equations on x-space. For example, $D\Phi = 0$ yelds $\Phi =$ a constant and $DD\Phi = 0$ implies a massless supermultiplet.

eliminate unphysical degrees of freedom, we identify the coefficient of $\theta\sigma^\mu\bar\theta$ as a gauge boson. This lets us define a super gauge transformation

$$\mathrm{V} \to \mathrm{V}' = \mathrm{V} + (\Phi + \Phi^\dagger) \quad \Rightarrow \quad V_\mu \to V_\mu + i\partial_\mu(\phi - \phi^*). \tag{2.9}$$

Comparing V and V', we find that only two of the component fields are invariant under this gauge transformation. Using this transformation to gauge away most of the other component fields leads to the so-called Wess–Zumino (WZ) gauge in which

$$\mathrm{V}(x,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta V_\mu(x) + \theta\theta\bar\theta\bar\lambda(x) - \bar\theta\bar\theta\theta\lambda(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(x), \tag{2.10}$$

where we identify λ and $\bar\lambda$ as the spin-$\frac{1}{2}$ partners of the gauge boson V_μ and D is an auxiliary degree of freedom that transforms as a total derivative under supersymmetry. Generally, the coefficient of $\theta\theta\bar\theta\bar\theta$ in a vector superfield is called the D-term.

In the WZ gauge, powers greater than two of the vector superfield are zero and V^2 leads to a mass term which is not gauge invariant. This indicates that the component fields hidden by the WZ gauge fixing are not unphysical. Analogously to the Standard Model case, in order to write down gauge invariant interactions we have to construct a supersymmetric field strength,

$$\begin{aligned} W_\alpha &= \frac{1}{2g}\bar D\bar D e^{-\mathsf{V}}(D_\alpha e^{\mathsf{V}}) \\ &\downarrow \text{WZ} \\ W^a_\alpha &= \bar D\bar D D_\alpha \mathrm{V}^a + igf^{abc}\bar D\bar D(D_\alpha \mathrm{V}^b)\mathrm{V}^c \end{aligned} \tag{2.11}$$

where $\mathsf{V} \equiv 2gt_a\mathrm{V}^a$. The supersymmetric field strength W^a_α in (2.11) is a chiral superfield and in component form, using the WZ gauge and coordinates y as previously defined,

$$W^a_\alpha(y,\theta) = 4i\lambda^a_\alpha + 4\theta_\alpha D^a(y) + 2i(\sigma^\mu\bar\sigma^\nu)_\alpha^{\ \beta} V^a_{\mu\nu}(y)\theta_\beta + 4\theta\theta(\sigma^\mu)_{\alpha\dot\alpha}\mathcal{D}_\mu\bar\lambda^{a\dot\alpha}, \tag{2.12}$$

where $\mathcal{D}_\mu$ is the gauge covariant derivative and $V^a_{\mu\nu}$ is the non-abelian field strength.

The Gravity multiplet

There is one more relevant supermultiplet that can be constructed in $\mathcal{N}$=1 supersymmetry, the gravity multiplet consisting of the spin 2 graviton and the spin $\frac{3}{2}$ gravitino, $({e_m}^\mu, \psi_\mu)$. One can write a globally supersymmetric Lagrangian for this multiplet by combining the Rarita–Schwinger Lagrangian for a spin $\frac{3}{2}$ fermion [177] with the Einstein Lagrangian [178],

$$\mathscr{L} = -\frac{\mathbf{e}}{2\kappa^2}{R_{\mu\nu}}^{mn}{e_m}^\mu{e_n}^\nu - \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar\psi_\mu\gamma_5\gamma_\nu\mathcal{D}_\rho\psi_\sigma, \tag{2.13}$$

where $\mathbf{e}$ is the determinant of the vierbein ${e_m}^\mu$, ${R_{\mu\nu}}^{mn}$ the Riemann tensor, κ the gravitational coupling constant which has dimension of inverse mass, and the Lorentz covariant derivative $\mathcal{D}_\rho = \partial_\rho - \frac{\mathrm{i}}{4}{\omega_\rho}^{mn}\sigma_{mn}$ now contains the so-called spin connection ${\omega_\rho}^{mn}$ of a local Lorentz transformation[9].

[9] The term corresponding to a global Lorentz transformation is absent from the covariant derivative since the connection $\Gamma^\tau_{\rho\sigma}$ is symmetric in its lower indices and cancels out against $\epsilon^{\mu\nu\rho\sigma}$.

The SUSY algebra closes on-shell[10] under local SUSY transformations, indeed local supersymmetry implies supergravity. Closure of the Lagrangian off-shell introduces auxiliary fields

$$\mathscr{L} \ni -\frac{\mathbf{e}}{3}\left(S^2 + P^2 - A_m A^m\right) \tag{2.14}$$

which, when eliminated, introduce an important negative contribution to the scalar potential [179].

2.2.3 Lagrangian

In constructing the Lagrangian we want to include only terms that transform as a total derivative under supersymmetry transformations. As noted above, these are the F-terms of chiral superfields and the D-terms of vector superfields. It is easy to show that products of chiral superfields are chiral superfields and that the product $\Phi^\dagger\Phi$ is a vector superfield. Further, from (2.9) and (2.8) we see that the $\theta\theta\bar{\theta}\bar{\theta}$ component of a gauge transformation of an abelian vector superfield, V^A, is also a total derivative[11]. Thus the general gauge invariant supersymmetric Lagrangian is of the form

$$\begin{aligned}\mathscr{L} &= \frac{1}{64}\left(W^\alpha W_{\alpha|\theta\theta} + \bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}{}_{|\bar{\theta}\bar{\theta}}\right) + \Phi_i^\dagger e^{\mathsf{V}}\Phi_{i|\theta\theta\bar{\theta}\bar{\theta}} + 2\xi^A \mathrm{V}^A{}_{|\theta\theta\bar{\theta}\bar{\theta}} \\ &\quad + \left[\left(b_i\Phi_i + \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}y_{ijk}\Phi_i\Phi_j\Phi_k\right)_{|\theta\theta} + \text{h.c.}\right],\end{aligned} \tag{2.15}$$

where ξ^A is a constant coefficient with dimension mass squared. The exponent in the $\Phi^\dagger e^{\mathsf{V}}\Phi$ term of the Lagrangian (2.15) ensures gauge invariance under the non-abelian supergauge transformation

$$\Phi \;\to e^{-i\Lambda}\Phi \qquad \Phi^\dagger \to \Phi^\dagger e^{i\Lambda^\dagger} \qquad e^{\mathsf{V}} \to e^{-i\Lambda^\dagger}e^{\mathsf{V}}e^{i\Lambda}, \tag{2.16}$$

where $\Lambda \equiv 2gt_a\Lambda^a$ is a chiral superfield. The second term in the Lagrangian (2.15) is sometimes called the Kähler potential and the polynomial of chiral fields

$$W = b_i\Phi_i + \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}y_{ijk}\Phi_i\Phi_j\Phi_k \tag{2.17}$$

is called the superpotential. In the superpotential, m_{ij} and y_{ijk} are antisymmetric and non-zero only for gauge invariant combinations of fields. Clearly $b_i = 0$, unless there is a gauge singlet in the theory. For a renormalizable globally supersymmetric theory, the Kähler potential is uniquely specified and the superpotential terminates at trilinear terms. It is now straightforward to solve for the auxiliary fields F and D. We have

$$F_i^\dagger \;=\; -\frac{\partial W(\phi)}{\partial\phi_i} \;=\; -b_i - m_{ij}\phi_j - y_{ijk}\phi_j\phi_k \tag{2.18}$$

$$D^a \;=\; -g\phi_i^\dagger t^a\phi_i - \xi, \tag{2.19}$$

[10] Closure on-shell or off-shell refers to whether the Lagrangian is invariant under SUSY transformations with or without resorting to the equations of motion of the fields.

[11] This applies also to the case where a gauge group G has one or more abelian U(1) factors. The terms added this way to the Lagrangian are called Fayet–Iliopoulos terms. Note that the index A does not denote a non-abelian group but enumerates the U(1) groups with Fayet–Iliopoulos terms.

where the constant $\xi \neq 0$ whenever D^a corresponds to a U(1) group with a Fayet–Iliopoulos term in the Lagrangian (2.15). The auxiliary fields make it very easy to write down the part of the Lagrangian that only depends on scalar component fields and not their derivatives, in other words the scalar potential

$$V = F_i^\dagger F^i + \frac{1}{2} D^a D^a. \tag{2.20}$$

2.3 Supersymmetry Breaking

Supersymmetry is not evident in nature; experiments have failed to find supersymmetric scalar partners with the charges and masses of the SM fermions. It may be that supersymmetry nevertheless is a symmetry of nature but it is broken by the vacuum of the theory at low energy scales. This is analogous to how gauge symmetry is broken by the vacuum in the Higgs mechanism. In contrast to the Higgs mechanism it is not possible to break supersymmetry via radiative corrections [180, 181]. This is a result of a more general property of supersymmetric theories called non-renormalization [182–184].

2.3.1 F- and D-term breaking

Lorentz invariance of the vacuum requires that only scalar fields may develop a vacuum expectation value; perturbation theory demands that the Euclidean action is in a stable minimum configuration which is equivalent to the scalar potential being at a minimum[12]. From Equation (2.4) we know that if the minimum of the scalar potential (2.20) is zero, then supersymmetry is necessarily conserved. It follows that the equations

$$F_i = 0 \qquad D^a = 0 \tag{2.21}$$

must not have a solution in order for supersymmetry to be broken. In the absence of U(1) groups, or if all ξ^A in the Lagrangian (2.15) are zero, there is a solution with $\phi_i = 0$ provided that there is no linear term in the superpotential. The case where b_i is non-zero in the superpotential (2.17) is called O'Raifeartaigh or F-term breaking[13] [185]. If there is at least one U(1) group it is possible to have so-called Fayet–Iliopoulos or D-term breaking [186]. However the equation in question,

$$D = -g \sum_i q_i |\phi_i|^2 - \xi = 0, \tag{2.22}$$

always has a solution if the U(1) is anomaly free. It is thus necessary to have additional terms in the superpotential to break supersymmetry[14].

[12] If this is not a global minimum, it is called a *false vacuum* and there will be a non-zero probability for quantum tunneling into a lower minimum. This is not necessarily a problem if the tunneling probability is low enough.

[13] A suitably simple example is the potential $W = Y(b - X^2) + mZX + f(X)$ which has no solutions for $F_X = F_Y = F_Z = 0$.

[14] If Φ_1 and Φ_2 are the only two fields charged under the U(1) and have opposite charges, then the superpotential term $m\Phi_1\Phi_2$ makes it impossible to have $F_1 = F_2 = D = 0$.

Supersymmetry breaking via these mechanisms is not feasible in the minimal supersymmetric extension of the Standard Model; there is no suitable singlet field for F-term breaking and using the $\mathrm{U}(1)_Y$ group for D-term breaking leads to problems with charged VEVs. Using some thus far undiscovered U(1) leads to difficulties generating suitable masses for all superpartner particles [123].

Supertrace

A more general problem with spontaneous supersymmetry breaking arises from a mass sum rule. Adopting the notation

$$D_i^a = \frac{\partial D^a}{\partial \phi_i} \qquad F_{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}, \tag{2.23}$$

the traces of the mass matrices for the component fields of spin 1, $\frac{1}{2}$, and 0 are, respectively,

$$\mathrm{Tr}\,\mathsf{M}_1^2 = 2|\langle D_i^a \rangle|^2, \tag{2.24a}$$

$$\mathrm{Tr}\,|\mathsf{M}_{\frac{1}{2}}|^2 = |\langle F_{ij} \rangle|^2 + 4|\langle D_i^a \rangle|^2, \tag{2.24b}$$

$$\mathrm{Tr}\,\mathsf{M}_0^2 = 2|\langle F_{ij} \rangle|^2 + 2|\langle D_i^a \rangle|^2 + 2g\langle D^a \rangle \mathrm{Tr}\,t^a, \tag{2.24c}$$

which, when weighted appropriately for degrees of freedom and with different signs for bosons and fermions, give the so-called supertrace mass sum rule

$$\mathrm{STr}\,M^2 = \sum_J (-1)^{2J}(2J+1)\,\mathrm{Tr}\,\mathsf{M}_J^2 = 2g\langle D^a \rangle \mathrm{Tr}\,t^a = 0, \tag{2.25}$$

where the last equality hold when the U(1) groups are non-anomalous. This sum rule can be shown to hold for each supersymmetry representation separately [187], and leads to serious phenomenological problems with the particle spectrum, even if supersymmetry is broken.

The Goldstino

In addition to the practical problems mentioned above with breaking supersymmetry via the F- or D-term, the breaking of global supersymmetry would also introduce a massless Goldstone fermion, the Goldstino. This can be seen by combining the minimization condition of the scalar potential

$$\frac{\partial V}{\partial \phi_i} = F_j \frac{\partial^2 W(\phi)}{\partial \phi_i \partial \phi_j} - g D^a \phi_i^\dagger t^a = 0 \tag{2.26}$$

with the gauge invariance of the superpotential in the vacuum

$$\delta^a W = \frac{\partial W}{\partial \phi_i} \delta^a \phi_i = F_i^\dagger t^a \phi_i = 0, \tag{2.27}$$

into a matrix equation for the vacuum expectation values of the F and D fields,

$$\mathsf{M} \begin{pmatrix} \langle F_j \rangle \\ \langle D^a \rangle \end{pmatrix} = 0 \qquad \text{with} \qquad \mathsf{M} \equiv \begin{pmatrix} \left.\frac{\partial^2 W(\phi)}{\partial \phi_i \partial \phi_j}\right|_{\langle \phi \rangle} & -g\langle \phi_i^\dagger \rangle t^a \\ -g\langle \phi_j^\dagger \rangle t^b & 0 \end{pmatrix}. \tag{2.28}$$

If supersymmetry is broken, then M has to be singular; there is a zero eigenvalue. On the other hand, it can be shown by expanding the Lagrangian that M is also the mass matrix for the fermion component fields ψ and λ of the chiral and vector superfields, respectively.

2.3.2 Local SUSY

Extending supersymmetry to a local symmetry solves several of the problems with SUSY breaking discussed above. In a manner analogous to local gauge theories, the infinitesimal SUSY transformation parameter ξ is allowed to vary locally; the usual Noether procedure then results in a locally invariant Lagrangian. Remarkably, the procedure automatically gives rise to a spin $\frac{3}{2}$ field and a spin 2 field that couple to the energy momentum tensor. These fields are consequently identified as the gravitino and the graviton and the resulting Lagrangian is called supergravity. This procedure was demonstrated for $\mathcal{N}$=1 SUSY by Freedman, van Nieuwenhuizen, and Ferrara in 1976[15] [149].

The Lagrangian (2.13) already hints at the fact that the theory becomes non-renormalizable once gravity becomes involved and it is usual to reformulate the chiral Lagrangian (2.15) as

$$\mathscr{L} = K(\Phi^\dagger e^V, \Phi)_{|\theta\theta\bar{\theta}\bar{\theta}} + \left[f_{AB}(\Phi) W_A^\alpha W_{B\alpha|\theta\theta} + W(\Phi)_{|\theta\theta} + \text{h.c.} \right], \tag{2.29}$$

where the Kähler potential K, gauge kinetic function f_{AB}, and superpotential W can now be general functions of the chiral superfields including non-renormalizeable terms. Closure of the SUSY algebra requires f_{AB} and W to be holomorphic and gauge invariance restricts f_{AB} to transform as a symmetric product of two adjoint representations of the gauge group. Terms of dimension higher than four in the Lagrangian (2.29) are suppressed by powers of the reduced Plank mass M_P, and in the limit $M_P \to \infty$ the Lagrangian (2.15) is recovered. Unless it is useful to the discussion we adopt the notation $M_P = 1$. An interesting feature of supergravity is that the Lagrangian (2.29) actually only depends on two independent dimensionless functions, the gauge kinetic function and the combination, $G = K + \log |W|^2$, called the Kähler function. Derivatives of the Kähler function are denoted by upper and lower indices,

$$G^i = \partial_{\Phi_i} G_{|\Phi \to \phi} \quad \text{and} \quad G_j = \partial_{\Phi_j^\dagger} G_{|\Phi \to \phi}. \tag{2.30}$$

In minimal supergravity models it is usual to make a choice for the so-called Kähler metric

$$G^i_j = K^i_j = \delta^i_j. \tag{2.31}$$

A similarly minimal choice for the gauge kinetic function, $f_{AB} = \delta_{AB}/g_A^2$, leaves the gauginos massless at tree level and it is common to include at least one higher order term

$$f_{AB} = \delta_{AB} \left[1/g_A^2 + f_A^i \Phi_i / M_P + \cdots \right]. \tag{2.32}$$

The coefficients f_A^i in (2.32) effectively determine the structure of the gaugino masses, as will be shortly discussed in the next section.

[15] A more elegant method using tensor calculus and superspace formalism was used by Nath, Arnowitt, and Zumino in 1975 [188, 189] and further developed by Ferrara and van Nieuwenhuizen, as well as Stelle and West in 1978 [190, 191]. Extensive papers on the details of SUGRA and dynamical SUSY breaking were written in 1981 by Witten [192] and van Nieuwenhuizen [179], and a first realistic model presented by Chamseddine, Arnowitt, and Nath in 1982 [193].

Dynamical SUSY breaking

It is instructive to look at a few terms of the complete SUGRA Lagrangian [194],

$$\frac{1}{\mathbf{e}}\mathscr{L} = -\frac{1}{2}R + G^i_j\partial_\mu\phi_i\partial^\mu\phi^*_j - e^G\left[G_i(G^{-1})^i_jG^j - 3\right] + e^{G/2}\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu + \ldots, \tag{2.33}$$

where the gravitational coupling constant κ is set to 1. The second term demonstrates how G^i_j acts as a metric on the space of scalar fields, hence its name. The third term is a part of the scalar potential[16] and notably contains a negative term making it possible for the vacuum energy to be zero even with broken supersymmetry. Finally the factor $e^{G/2}$ in the last term comes from a Weyl rescaling of the vierbein $e_m{}^\mu$ which is necessary for the first term to have the canonical -$\frac{1}{2}R$ form as in Equation (2.13). Crucially, if SUSY is spontaneously broken and G develops a VEV, this rescaling provides a mass for the gravitino $m^2_{3/2} = e^{\langle G\rangle}M^2_P$. If one assumes a vanishing vacuum energy and minimal Kähler metric (2.31) then

$$m^2_{3/2} = \frac{1}{3}\frac{\langle F_iF^{i*}\rangle}{M^2_P} = \frac{1}{3}\frac{M^4_S}{M^2_P}, \tag{2.34}$$

where the SUSY breaking scale M_S is set by the VEV of the auxiliary field $F_i = -e^{G/2}(G^{-1})^j_iG_j$. Cremmer showed that in analogy to the Higgs mechanism it is indeed the Goldstino associated with F- or D-term SUSY breaking that provides the mass to the gravitino; this is called the super-higgs mechanism [195]. The SUGRA Lagrangian also provides a mass $m_{1/2}$ for the gauginos with the term

$$\frac{1}{\mathbf{e}}\mathscr{L} \ni e^{G/2}\frac{1}{4}\frac{\partial f^*_{AB}}{\partial\phi^*_i}(G^{-1})^i_jG^j\bar{\lambda}_A\lambda_B \quad\rightarrow\quad m_{1/2}\sim m_{3/2}, \tag{2.35}$$

if the gauge kinetic function is not minimal[17]. In fact the prevalence of the gravitino mass is nicely illustrated in the modified version of the tree level supertrace formula

$$\mathrm{STr}\,M^2 = 2g\langle D^a\rangle\mathrm{Tr}\,t^a + (N-1)\left(2m^2_{3/2} - \frac{\langle D^a\rangle\langle D^a\rangle}{M^2_P}\right), \tag{2.36}$$

with N the number of chiral supermultiplets.

Gravity-, Anomaly-, and Gauge-mediated SUSY breaking

The minimal field content of a supersymmetric extension of the Standard Model are not suitable for SUSY breaking and it is assumed that an additional hidden sector at a scale M_S contains fields that are singlets under the SM gauge groups and develop a non-zero F-term. This SUSY breaking is then communicated to the visible sector. The gravitational interactions in the SUGRA Lagrangian (2.33) lead to the mass of the gravitino determining the overall scale of any terms in the scalar potential. This is called gravity mediation, and it can in fact be shown that the scale of the low energy effective

[16] The usual F-term has been replaced with $F_i = -e^{G/2}(G^{-1})^j_iG_j$.

[17] Notably, the gaugino masses can be non-universal in models of SUSY breaking if f_{AB} is something other than the singlet component of the $(Adj \bigotimes Adj)_{\text{SYM}}$ product of the GUT group.

potential depends solely on $m_{3/2}$ [196]. Requiring the effective potential – and by proxy the gravitino mass – to be at the EW scale gives a SUSY breaking scale $M_S = \mathcal{O}(10^{10}$ GeV).

The Weyl scaling invariance of the supergravity Lagrangian suffers from a quantum anomaly which introduces a one-loop contribution to all SUSY breaking parameters of the effective potential [197]. If gravity mediation is suppressed, this anomaly mediation can dominate and hence determine the structure of the SUSY breaking terms. The scale of the effective potential is given by $m_{3/2}/16\pi^2$ leading to a heavier gravitino mass and a higher SUSY breaking scale M_S than in the case of gravity mediation.

A third commonly used method for mediating SUSY breaking uses an additional messenger sector at a scale $M_X \lesssim M_S$ whose fields feel the SUSY breaking effect of the hidden sector, develop a SUSY breaking VEV $\langle F_X \rangle$, and communicate its effect to the observable sector via SM gauge interactions [198, 199]. The SUSY breaking scalar masses $m_i \sim g_i^2 \langle F_X \rangle / 16\pi^2 M_X$ are non-universal and generated at loop level, which evades the tree level supertrace constraint (2.36). The gravitino mass (2.34) is determined by the hidden sector scale and can be very light if the messenger scale – and consequently the hidden sector scale – is much smaller than the Planck scale.

Soft SUSY breaking terms

The exact mechanism of SUSY breaking in the hidden sector has little effect on the structure of the SUSY breaking terms. Since it is also unknown which mediation method – or combination of methods – dictates the form of the effective potential, it is common to parameterize all renormalizable low energy SUSY breaking terms that can arise from supergravity. These terms are called soft SUSY breaking terms as it can be shown that they do not introduce quadratic divergences to the theory [200],

$$\begin{aligned}\mathscr{L}_{\text{soft}} = &\Big[C_i\phi_i + B_{ij}\phi_i\phi_j + A_{ijk}\phi_i\phi_j\phi_k + \text{h.c.}\Big] + M_{ij}^2\phi_i\phi_j^* + M_A\bar{\lambda}_A\lambda_A + M_A'\bar{\lambda}_A\gamma_5\lambda_A \\ &+ \Big[D_{jk}^i\phi_i^*\phi_j\phi_k + M_F^{ij}\psi_i\psi_j + M_{iA}\psi_i\lambda_A + \text{h.c.}\Big]\,. \end{aligned} \tag{2.37}$$

Here D_{jk}^i may lead to quadratic divergencies if there are SM gauge singlet superfields, M_F^{ij} is redundant as it can be absorbed in a redefinition of the superpotential and M_{iA} requires a chiral supermultiplet in an adjoint representation of the gauge group. Hence these coefficients – as well as the CP-odd gaugino mass M_A' – are commonly assumed to be zero.

2.4 The Minimal Supersymmetric Standard Model

2.4.1 Formulation

The minimal supersymmetric standard model (MSSM) is constructed by describing the Standard Model fields as component of their respective superfields, writing the most general gauge invariant supersymmetric Lagrangian possible following the prescriptions of Section 2.2 and adding to it the appropriate soft supersymmetry breaking terms (2.37). The only necessary addition to the field

$\Phi^{\mathsf{SU_3SU_2U_1}}$	spin 0	spin $\frac{1}{2}$
$\hat{Q}_i^{\mathbf{3}\ \mathbf{2}\ \frac{1}{3}}$	$\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}_i$	$\begin{pmatrix}u_L\\ d_L\end{pmatrix}_i$
$\hat{U}_i^{\bar{\mathbf{3}}\ \mathbf{1}\ -\frac{4}{3}}$	$\tilde{u}^*_{R,i}$	$u^\dagger_{R,i}$
$\hat{D}_i^{\bar{\mathbf{3}}\ \mathbf{1}\ \frac{2}{3}}$	$\tilde{d}^*_{R,i}$	$d^\dagger_{R,i}$

$\Phi^{\mathsf{SU_3SU_2U_1}}$	spin 0	spin $\frac{1}{2}$
$\hat{L}_i^{\mathbf{1}\ \mathbf{2}\ -1}$	$\begin{pmatrix}\tilde{\nu}_L\\ \tilde{l}_L\end{pmatrix}_i$	$\begin{pmatrix}\nu_L\\ l_L\end{pmatrix}_i$
$\hat{E}_i^{\mathbf{1}\ \mathbf{1}\ 2}$	$\tilde{l}^*_{R,i}$	$l^\dagger_{R,i}$

$\Phi^{\mathsf{SU_3SU_2U_1}}$	spin 0	spin $\frac{1}{2}$
$\hat{H}_1^{\mathbf{1}\ \mathbf{2}\ -1}$	$\begin{pmatrix}H_1^0\\ H_1^-\end{pmatrix}$	$\begin{pmatrix}\tilde{H}_1^0\\ \tilde{H}_1^-\end{pmatrix}$
$\hat{H}_2^{\mathbf{1}\ \mathbf{2}\ 1}$	$\begin{pmatrix}H_2^+\\ H_2^0\end{pmatrix}$	$\begin{pmatrix}\tilde{H}_2^+\\ \tilde{H}_2^0\end{pmatrix}$

Table 2.1: The chiral supermultiplets of the MSSM.

content of the SM is a second Higgs doublet with opposite hypercharge than that of the SM Higgs doublet. This is to have holomorphic Yukawa terms for both up-type and down-type fermions and to preserve the Standard Model anomaly cancellation [156]. The chiral supermultiplets making up the MSSM are shown in Table 2.1.

The superpotential and soft SUSY breaking terms

For a given left handed chiral supermultiplet $\hat{X}$, X denotes the SM component field, and $\tilde{X}$ the supersymmetric partner of X. Defining the antisymmetric tensor $\epsilon_{\alpha\beta}$ for $SU(2)$ contraction with $\epsilon_{12} = 1$ the most general renormalizeable gauge invariant superpotential is

$$W_{\mathsf{MSSM}} = \epsilon_{\alpha\beta}\left(y_E^{ij}\hat{H}_1^\alpha\hat{L}_i^\beta\hat{E}_j + y_D^{ij}\hat{H}_1^\alpha\hat{Q}_i^\beta\hat{D}_j + y_U^{ij}\hat{Q}_i^\alpha\hat{H}_2^\beta\hat{U}_j + \mu\hat{H}_2^\alpha\hat{H}_1^\beta\right) + W_{\not{R}_\mathrm{p}}, \tag{2.38}$$

where $W_{\not{R}_\mathrm{p}}$ contains R-parity violating terms which will be discussed in detail in Section 3.2. For now they are assumed to be zero. The soft SUSY breaking terms and scalar mass terms are

$$\begin{aligned} V_{\mathsf{soft}} = {} & \epsilon_{\alpha\beta}\left(A_E^{ij}y_E^{ij}H_1^\alpha\tilde{L}_i^\beta\tilde{E}_j + A_D^{ij}y_D^{ij}H_1^\alpha\tilde{Q}_i^\beta\tilde{D}_j + A_U^{ij}y_U^{ij}\tilde{Q}_i^\alpha H_2^\beta\tilde{U}_j + B\mu H_2^\alpha H_1^\beta + \text{h.c.}\right) \\ & + M^2_{\Psi,ij}\Psi_i^\dagger\Psi_j + M^2_{\Theta,ij}\Theta_i\Theta_j^* + m^2_{H_1}H_1^\dagger H_1 + m^2_{H_2}H_2^\dagger H_2 + \frac{1}{2}\left(\sum_i M_{\lambda_a}\lambda_a\lambda_a + \text{h.c.}\right) \end{aligned} \tag{2.39}$$

where $\Psi = \{\tilde{L}, \tilde{Q}\}$, $\Theta = \{\tilde{E}, \tilde{U}, \tilde{D}\}$ and λ_a are the gaugino superpartners of the gauge bosons. It is usually assumed, although not necessary, that the A-terms A_X^{ij} have the same flavor structure as the corresponding Yukawa terms y_X^{ij}. Indeed, it is common to assume that all these parameters are flavor diagonal and real, and to only consider third generation Yukawa and A-terms.

The scalar potential

The scalar potential $V = V_{\mathsf{soft}} + V_{\mathsf{D}} + V_{\mathsf{F}}$ is of particular interest because it yields very important constraints. In order for any particular SUSY model to be viable, the scalar potential has to be in a stable minimum configuration. The extremum and minimum conditions

$$\left.\frac{\partial V}{\partial\phi_i}\right|_{\langle\phi\rangle} = 0 \qquad \text{and} \qquad \lambda\left[M^2 = \left.\frac{\partial^2 V}{\partial\phi_i\partial\phi_j^*}\right|_{\langle\phi\rangle}\right] \geq 0, \tag{2.40}$$

where $\lambda[M^2]$ denotes the eigenvalues of the scalar mass matrix M^2, must be satisfied. The field values for which this is so are the fields' vacuum expectation values $\langle\phi\rangle$. Gauge invariance of the scalar potential ensures that setting all charged field VEV's to zero trivially solves the extremum constraint. Neutral fields may develop a non-zero VEV and indeed the Higgs field is expected to do so,

$$\langle H_1^0\rangle = v_1, \quad \langle H_2^0\rangle = v_2, \quad \tan\beta = \frac{v_2}{v_1}. \tag{2.41}$$

It is common to solve the extremum condition for the soft mass parameters of the scalar fields. The minimum condition does not lend itself to the analytic elimination of parameters in any but the most trivial cases and is usually verified numerically when scanning the parameter space of a model.

Depending on the field content of the model the scalar potential may also exhibit so-called flat directions in field space in which the F-term and D-term contributions vanish. In the MSSM the flat directions are mostly lifted by soft scalar masses or non-renormalizeable terms [201]. Conversely, soft SUSY breaking terms can induce deep charge or color breaking (CCB) minima or make the potential unbounded from below (UFB) in a flat direction which leads to constraints for soft SUSY breaking parameters[18] [123, 203].

Higgs boson masses

The scalar mass matrix in Equation (2.40) separates into four parts: one each for the neutral and charged Higgses and the neutral and charged sleptons[19]. In the absence of CP violation the neutral matrices further separate into CP-even and CP-odd parts. In the MSSM the eigenvalues of the 2×2 Higgs matrices are easily solved and give the well-known tree level results [123]

$$m_A^2 = B\mu(\cot\beta + \tan\beta), \tag{2.42a}$$

$$m_{h,H}^2 = \frac{1}{2}\left[(m_A^2 + M_Z^2) \mp \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta}\right], \tag{2.42b}$$

$$m_{H^\pm}^2 = m_A^2 + M_W^2, \tag{2.42c}$$

for the pseudoscalar, scalar, and charged Higgses, respectively. The ensuing upper bound on the lightest Higgs, $m_h^2 \leq M_Z^2$, illustrates the importance of loop corrections. The simplest way to include such corrections is to add field dependent loop order effective corrections to the scalar potential [204]

$$V_{\text{eff}} = V + \frac{1}{16\pi^2}V^{(1)} + \frac{1}{(16\pi^2)^2}V^{(2)} + \ldots, \tag{2.43}$$

where

$$V^{(1)} = \frac{1}{4}\sum_i (-1)^{2s_i}(2s_i + 1)m_i^4\left[\log\frac{m_i^2}{Q^2} - \frac{3}{2}\right], \tag{2.44}$$

with i running over fields that interact with the Higgs and m_i being field-dependent eigenvalues of the corresponding fields' mass matrix. At one loop the upper limit on the lightest Higgs is approximately

[18] The off-diagonal entries in A_X can receive tighter constraints from CCB and UFB considerations than those coming from flavor changing neutral currents [202].

[19] If R-parity is violated sleptons and Higgses mix so there are only two matrices, one neutral and one charged.

130 GeV for $M_{\text{SUSY}} = 1$ TeV. The general form of $V^{(2)}$ is much more complicated, but should be included in any numerical precision calculations since it can shift the mass of the lightest Higgs by a few GeV [205].

Gaugino masses

The SU(3) gauginos do not mix with other particles in the MSSM and thus form gluino mass eigenstates, $\tilde{g}$, with mass M_3. The SU(2) and U(1) gauginos mix with the higgsinos forming neutralinos, χ_i^0, and charginos, $\chi_i^\pm$. The mixing of the initial $(\tilde{\lambda}_0, \tilde{\lambda}_3, \tilde{H}_1^0, \tilde{H}_2^0)$ and $(\tilde{\lambda}^\pm, \tilde{H}^\pm)$ fields are described by the mass matrices

$$M_{\chi^0} = \begin{pmatrix} M_1 & 0 & -\frac{g_1 v_1}{\sqrt{2}} & \frac{g_1 v_2}{\sqrt{2}} \\ 0 & M_2 & \frac{g_2 v_1}{\sqrt{2}} & -\frac{g_2 v_2}{\sqrt{2}} \\ -\frac{g_1 v_1}{\sqrt{2}} & \frac{g_2 v_1}{\sqrt{2}} & 0 & -\mu \\ \frac{g_1 v_2}{\sqrt{2}} & -\frac{g_2 v_2}{\sqrt{2}} & -\mu & 0 \end{pmatrix} \quad \text{and} \quad M_{\chi^\pm} = \begin{pmatrix} M_2 & g_2 v_2 \\ g_2 v_1 & \mu \end{pmatrix}, \tag{2.45}$$

from which it is easy to extract the following tree level sum rules for squared masses

$$m^2_{\chi_1^\pm} + m^2_{\chi_2^\pm} - 2M_W^2 = \mu^2 + M_2^2 \tag{2.46a}$$

$$m^2_{\chi_1^0} + m^2_{\chi_2^0} + m^2_{\chi_3^0} + m^2_{\chi_4^0} - 2M_Z^2 = 2\mu^2 + M_1^2 + M_2^2. \tag{2.46b}$$

These sum rules are quite useful for *ad hoc* phenomenological analysis since neutralino and chargino masses do not get large loop corrections. Typically, the lightest mass eigenstate will receive corrections at the level of 3–8% and the heavier states at the level of 1% [206].

2.4.2 Limitations of the MSSM

While the MSSM solves some of the Standard Model's problems, such as gauge unification and the hierarchy problem, some issues remain and several new problems are introduced. Many of the new parameters that the MSSM introduces violate flavor, CP, or R-parity. The natural scale of the μ term in the superpotential is either zero or the Planck scale, neither of which is a viable choice. None of the newly introduced superpartners is a suitable candidate for the right-handed neutrino and the SM problem of the neutrino masses remains.

Some of the problems of the MSSM can be addressed by suitable assumptions. For example the constrained MSSM (cMSSM) [207] – motivated by gauge unification at some high GUT scale and SUSY breaking – only has four "and a half" parameters, $(m_0, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu))$. Despite being remarkably predictive, this model has yet to be completely excluded by experimental data[20]. Since there are no measurements yet of any of the SUSY parameters, the choice and details of any specific SUSY breaking mediation mechanism are largely arbitrary. Such models can even be overly restrictive to the point of missing phenomenologically interesting predictions [208]. An effective low

[20] It is arguable whether one can completely exclude supersymmetry in any case since it simply decouples from the Standard Model when the SUSY scale is raised sufficiently.

energy approach to SUSY phenomenology is thus well motivated and we take this approach in the papers included with this thesis.

Extending the MSSM is another way of addressing the model's issues. This is commonly done in two ways; by additions to the field content or changes to the symmetries of the model[21]. These extensions can go hand in hand, for example spontaneous R-parity breaking can be introduced via the VEV of a right-handed neutrino, and additional gauge symmetries naturally include new gauge bosons. Conversely, extending the field content of a model can introduce new accidental symmetries which, if broken spontaneously, typically lead to problematic Goldstone states or cosmological domain wall problems. These issues routinely occur in model building and are usually dealt with by explicitly breaking the offending symmetry or, if possible, turning it into a local symmetry. In the works included in this thesis we focus specifically on the CP, R-parity, and flavor symmetries and Chapter 3 details how these symmetries may be broken, how they can be linked with each other, and what some of the phenomenological implications are.

[21] Other notable extensions that do not fall into these two categories are SplitSUSY [209–211] and the concept of extra dimensions [212–216]

Chapter 3

Beyond the MSSM

From a model-building perspective it is appealing to start with a model that is maximally symmetric and see whether it is possible to arrive at a realistic model by dynamically breaking the symmetries we know – or wish – to be broken in nature. In Paper 1 we show how to construct a minimal model that solves many of the MSSM's problems in this way at tree level. In particular, we address the observed smallness of CP violation and the apparent absence of proton decay by imposing conservation of CP and R-parity, and breaking these symmetries by allowing certain fields to develop a VEV. Either of these cases requires extending the MSSM with a singlet field leading to a class of models called next to minimal supersymmetric Standard Models (NMSSM).

3.1 The NMSSM

3.1.1 Formulation

The canonical NMSSM is obtained by adding a singlet field to the Higgs sector of the MSSM. A superpotential term $\lambda\hat{S}\hat{H}_2\hat{H}_1$ replaces the bilinear term $\mu\hat{H}_2\hat{H}_1$ and the corresponding soft SUSY breaking term is similarly replaced. If the scalar component of $\hat{S}$ acquires a VEV, this produces an effective μ term $\mu_{\text{eff}}\hat{H}_2\hat{H}_1$ (with $\mu_{\text{eff}} = \lambda\langle S\rangle$) at the EW scale, solving the μ problem[1] [49]. On its own, this addition introduces a U(1) Peccei-Quinn (PQ) symmetry[2] [218, 219], which, when broken, leads to a massless Goldstone state called the axion. Cosmological and astrophysical constraints on the axion mass and decay constant set stringent limits on the coupling constant, $\lambda < 10^{-8}$, and the VEV of the singlet, $\langle S\rangle > 10^{10}$ GeV [220, 221]. The problem of naturalness therefore reappears. This issue can be easily sidestepped by the addition of a linear, bilinear, or trilinear singlet term – any of which explicitly breaks the PQ symmetry. Since the impetus for adding the singlet is the elimination of dimensionful parameters in the superpotential, it is usual to only add a trilinear term $\frac{1}{3}\kappa\hat{S}^3$ and the corresponding soft SUSY breaking term. The most commonly used superpotential for

[1] Alternatively the μ problem can be solved by a specific choice of the Kähler potential in SUGRA [217].

[2] The Peccei-Quinn symmetry was originally introduced as part of a proposed solution to the so-called strong CP problem.

the NMSSM is

$$W_{\text{NMSSM}} = W_{\text{MSSM}} + \lambda \hat{S}\hat{H}_2\hat{H}_1 + \frac{\kappa}{3}\hat{S}^3, \tag{3.1}$$

where W_{MSSM} does not contain the $\mu\hat{H}_2\hat{H}_1$ term. Formally the undesirable Higgs bilinear term – as well as the otherwise allowed linear and bilinear singlet terms – can be avoided by simply imposing a $\mathbb{Z}_3$ symmetry. Such a symmetry unfortunately leads to cosmological problems with domain walls once the symmetry is broken during the electroweak phase transition. Dissipation of the domain walls requires a $\mathbb{Z}_3$ breaking term in the scalar potential, and it can be shown that imposing a $\mathbb{Z}_2$ R-symmetry on all superpotential terms – including non-renormalizable terms – can produce the requisite $\xi^3 S$ term in the scalar potential with ξ naturally at the EW scale [222–224]. The additional soft SUSY breaking terms are

$$V_{\text{soft}}^{\text{NMSSM}} \ni \left(A_\lambda \lambda S H_2 H_1 + A_\kappa \kappa S^3 + \xi^3 S + \text{h.c.}\right) + m_S^2 |S|^2. \tag{3.2}$$

3.1.2 Phenomenology

A Higgs mass of 125 GeV is problematic in the context of the MSSM, which gives the tree level upper limit for the mass of the lightest Higgs as $m_h^2 \leq m_Z^2 \cos^2 2\beta$. At one loop, assuming small squark mixing and a universal SUSY mass scale M_S, the dominant contribution to the limit is [205]

$$\delta(m_h^2) = \frac{3m_t^4}{16\pi^2 v^2}\left[\log\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right], \tag{3.3}$$

with m_t the top mass and $X_t = A_t - \mu \cot\beta$ the stop mixing parameter. This correction needs to be maximal (corresponding to $X_t = M_S\sqrt{6}$) in order to reach a Higgs mass of 125 GeV with typical SUSY masses at 1 TeV [205]. In the NMSSM the tree level limit receives an additional contribution [225]

$$m_h^2 = m_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta \tag{3.4}$$

which, in the regime of large λ and small $\tan\beta$, reduces the need for large loop corrections[3]. Recent work suggests that when commonly used experimental limits are applied in the context of a constrained version of the NMSSM[4] (cNMSSM), achieving a correct Higgs mass severely limits the parameter space and prefers small values of λ [231]. This effectively decouples the singlet and the resulting phenomenology is similar to the cMSSM. Adopting non-universal Higgs masses and trilinear terms allows for larger values of λ, which leads to interesting phenomenology in the Higgs sector, efficient decay of a SM-like Higgs into into light pseudoscalar singlets or the possibility of a second – as yet undiscovered – Higgs boson near 125 GeV [232, 233].

Indeed, we have shown in Paper 1 that even in a model with 17 Higgs bosons – a consequence of R-parity violation and added right-handed sneutrinos – it is likely that a SM-like Higgs in the

[3] In the extreme limit this leads to models called λSUSY [226] which have $\lambda \leq 2$ and $\tan\beta \simeq 1$. The divergence of the coupling constant λ below the GUT scale is not necessarily a problem [227] and can also be avoided by introducing a suitably low scale at which the underlying model is assumed to change [228, 229].

[4] Similar to the cMSSM with universal masses at the GUT scale and parameters m_0, $m_{1/2}$, A_0, λ, $\tan\beta$, and $\text{sign}(\mu_{\text{eff}})$ [230].

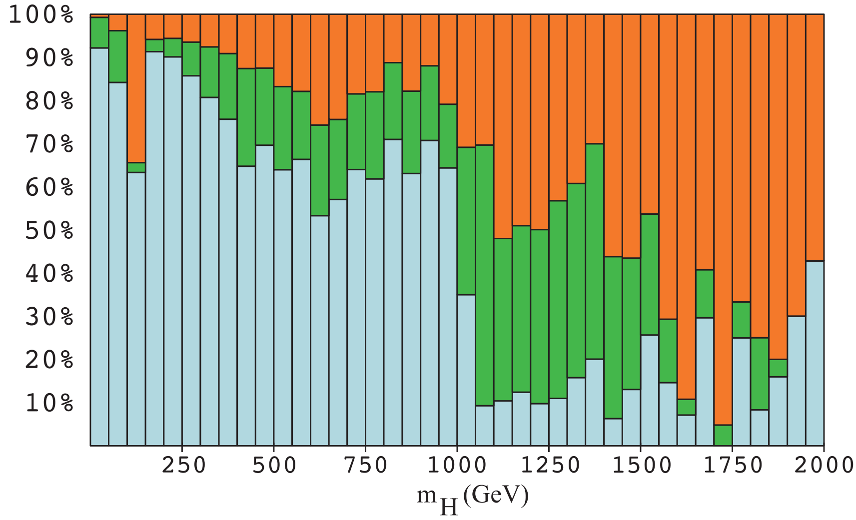


Figure 3.1: Average composition of neutral Higgs particles from Paper 1. Blue correspond to sneutrino admixture, green to singlet admixture, and orange to doublet Higgs admixture.

mass range of 100–150 GeV would be found and that any other Higgs bosons present at low masses will likely have high singlet and sneutrino admixtures making their detection challenging. Figure 3.1 illustrates this effect.

The fermionic partner of the singlet mixes with the higgsinos and gauginos making it possible for the neutralino LSP to be dominantly singlino. This is an interesting alternative dark matter candidate and we investigated this possibility in Paper 3. Some of our results (as well as a discussion of dark matter in general) are presented in Section 3.5.

3.2 R-parity

3.2.1 Formulation

In the context of the Standard Model, the baryon and lepton number B and L are well defined for Dirac fermions. SUSY formally uses Majorana fermions and introduces scalar partners, yet it is not clear how B or L apply in either of those cases. In particular, scalars carrying baryon or lepton number can spoil the conservation of the respective quantum numbers via boson exchange – similar to the effects arising in typical Grand Unification schemes – and lead to a very short proton lifetime.

In an attempt to solve this problem one can construct a new additive quantum number R associated with a continuous U(1)$_R$ symmetry under which also the anticommuting coordinates θ and $\bar{\theta}$ transform with charge -1 and $+1$, respectively [234],

$$S(x_\mu, \theta, \bar{\theta}) \to e^{in_S\alpha} S(x_\mu, e^{-i\alpha}\theta, e^{i\alpha}\bar{\theta}). \tag{3.5}$$

This is called R-symmetry[5] and n_S is the R-character of the superfield field S. From the form of the

[5]R-symmetry is an improved version of Q-invariance [235], and an early attempt at matching bosons and fermions.

chiral superfield (2.8) we can see how the bosonic and fermionic components transform differently under this symmetry,

$$\phi \to e^{in_S\alpha}\phi, \qquad \psi \to e^{i(n_S-1)\alpha}\psi, \qquad F \to e^{i(n_S-2)\alpha}F. \tag{3.6}$$

The superpotential is itself a chiral superfield and must transform under this symmetry with weight $n_W = 2$. Consequently a continuous R-symmetry forbids a $\mu\hat{H}_2\hat{H}_1$ term in the superpotential[6]. Vector superfields are real and thus have R-character zero. The transformation properties of the component fields can be easily seen from the WZ gauge form (2.10),

$$V^\mu \to V^\mu, \qquad \lambda \to e^{i\alpha}\lambda, \qquad \bar{\lambda} \to e^{-i\alpha}\bar{\lambda}, \qquad D \to D, \tag{3.7}$$

which show that a gaugino mass term $M_\lambda\lambda\lambda$ is forbidden by a continuous R-symmetry. For the chargino this leads to one of the eigenvalues being lighter than the W boson,

$$M_{\chi^\pm} = \begin{pmatrix} M_2 = 0 & M_W\sqrt{2}\sin\beta \\ M_W\sqrt{2}\cos\beta & \mu = 0 \end{pmatrix} \quad \to \quad m_{\chi^\pm} < M_W, \tag{3.8}$$

which is excluded by current limits [68]. Similarly, massless gluinos would form light R-hadrons [154, 236] which are excluded by experiment [237]. It is thus necessary to abandon the continuous $\mathrm{U}(1)_R$ in favor of the discrete Z_2 subgroup of R-symmetry. This R-parity can arise naturally in the framework of local SUSY when the gravitino acquires a mass [238]. The R-parity of a given field with spin S, baryon number B and lepton number L is

$$R_p = (-1)^{2S}(-1)^{3B+L} = (-1)^{2S}(-1)^{3(B-L)}, \tag{3.9}$$

where the latter formulation explicitly shows that R_p is conserved with B and/or L broken – as long as the difference $(B-L)$ is conserved modulo two. Standard Model particles have R-parity $+1$ and their superpartners have R-parity -1. Conservation of R_p means that superpartners are only created through pair production, and that any decay chain of such a sparticle will end with the lightest supersymmetric particle (LSP) which is necessarily stable.

3.2.2 R-parity violation

There is no *a priori* reason for R-parity to be conserved, and one can introduce R-parity violation in the MSSM via the superpotential terms

$$W_{\not{R}_p} = \mu_i\hat{H}_2\hat{L}_i + \lambda_{ijk}\hat{L}_i\hat{L}_j\hat{E}_k + \lambda'_{ijk}\hat{L}_i\hat{Q}_j\hat{D}_k + \lambda''_{ijk}\hat{U}_i\hat{D}_j\hat{D}_k, \tag{3.10}$$

where μ_i, λ and λ' violate lepton number and λ'' violates baryon number. Including the corresponding soft SUSY breaking terms, R-parity violation adds 96 parameters to the MSSM. The phenomenologically interesting or dangerous consequences of R-parity violation depend on the details of how lepton number and baryon number are broken – fast proton decay puts extremely stringent limits on

[6] The Higgs superfields must have R-character zero to protect the $\mathrm{U}(1)_R$ symmetry during EW symmetry breaking.

both L and B being violated at the same time. Proton decay is usually avoided by allowing either only B violating or only L violating terms, but any particular choice of non-zero R-parity violating parameters has to be checked for self-consistency against radiative corrections[7] [239].

Individual bounds from collider experiments on λ, λ', and λ'' are only of the order of 10^{-2} to 10^{-3} [240]. Cosmological considerations provide much more stringent bounds of the order of 10^{-6} to 10^{-7} [241]. These bounds arise because of the sphaleron process [242, 243] which keeps baryons and (anti-)leptons in equilibrium in the early universe, and will result in a washout of both baryon and lepton number if either is violated[8].

R-parity can be broken spontaneously if a field with non-zero lepton number acquires a VEV. The only candidate for this in the MSSM is the left-handed sneutrino. The value of its VEV is very constrained, $\langle \nu_L \rangle \ll m_Z$ [245, 246] – much below the natural scale of the sneutrino potential [123]. Extending the field content to include a right-handed neutrino N – adding the Yukawa term $y_N \hat{L}\hat{H}_2\hat{N}$ to the superpotential – provides a more suitable candidate. In either case, the resulting spontaneous breaking of lepton number introduces a massless Goldstone boson called the Majoron [247–249] which is experimentally ruled out by measurements of the invisible Z boson decay width [250, 251]. The appearance of the Majoron can be avoided by also adding terms that explicitly break lepton number but conserve R-parity, such as a bilinear term N^2. In models where the Higgs sector has been expanded by a singlet field S a term $\lambda_N S N^2$ produces the required bilinear N^2 term once the singlet acquires a VEV [252].

In the model we investigate in Paper 1, such a trilinear term also provides a crucial tadpole term for the singlet field S when the right-handed sneutrino acquires a VEV. Such a term is necessary for spontaneous CP violation, as will be discussed in Section 3.3. In addition to CP violation, the coupling λ_N of the trilinear term also controls the mass of the lightest Higgs, since in the limit of the coupling going to zero lepton number is restored and a Goldstone state reappears. The coupling also appears in the neutrino sector, constraining it still further. Figure 3.2 illustrates these limits for a typical data point. Despite this connection between the Higgs sector, neutrino sector, R-parity, and CP violation, we find that satisfying experimental constraints is possible in a low energy SUSY model. In part this is due to compartmentalization of the parameter space, meaning that some subset of parameters can be found to exclusively affect a given experimental quantity – for example in Paper 1 the set of soft SUSY breaking parameters A_U^{22} ,M_Q^{22} and M_U^{22} only show up in the calculations of ϵ_K. Another reason is that R-parity violation imposes few constraints on the model; proton decay is safe by construction and possible limits on the size of the effective R-parity violating parameter $\mu_i \equiv y_{N_i}\langle N_i \rangle$ are taken care of by the smallness of the neutrino Yukawa coupling y_{N_i}.

The general phenomenological consequences of R-parity violation are that the LSP is no longer stable, that superpartners need not be pair produced, and that SM and SUSY particles will mix. In

[7] For example the bilinear term in $W_{\not{R}_p}$ (3.10) can be rotated away in the MSSM due to an accidental SU(4) symmetry between L_i and H_d. However, even if one assumes alignment between the superpotential and soft SUSY breaking parameters, the corresponding bilinear soft term will be regenerated via loop effects.

[8] Complete washout can be avoided if at least one lepton number is conserved [244], but this solution does not work in the presence of lepton flavor violation [241].

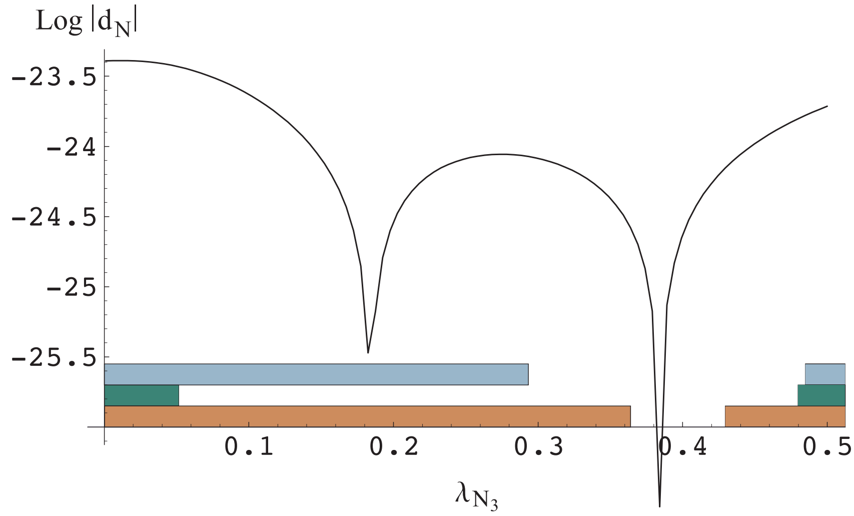


Figure 3.2: The neutron EDM d_N as a function of the coupling λ_{N_3} of the trilinear superpotential term $\lambda_{N_i} S N_i^2$. Exclusion ranges shown are due to vacuum stability (blue), charged Higgs mass limits (green), and neutrino sector constraints (orange).

particular, the mixing of neutrinos and neutralinos introduces the seesaw mechanism for generating small neutrino masses.

3.3 CP and Flavor violation

3.3.1 Background

In the Standard Model it is an accident of gauge and global symmetries that all but one complex phase can be rotated away. Although this process can be repeated in a supersymmetric model, there are a large number of new complex phases in the soft SUSY breaking terms which cannot be absorbed. In the same manner, flavor diagonal neutral couplings – an accident of the field content and symmetries in the SM – cannot be automatically assumed in the MSSM. Experimental constraints put very tight bounds on both flavor changing neutral currents (FCNC) and CP violation, and all soft SUSY breaking terms are usually chosen flavor diagonal and real in the MSSM. On the other hand, it is known that baryogenesis in the early universe requires more CP violation than is present in the SM [102–105] and SUSY is an excellent source for such CP violation. This tension can be addressed by having CP be a good symmetry of the Lagrangian but broken in the vacuum of the theory. One way to do this is to introduce CP violation via radiative corrections [253, 254], but this usually leads to an unacceptable light Goldstone state in the spectrum [255, 256]. An alternative solution is spontaneous CP violation (SCPV) via a complex valued VEV of a scalar field [257–260]. However, the structure of the Higgs sector in the MSSM is such that SCPV is not possible. Indeed it can be shown that one has to add at least one scalar singlet[9] to the model for SCPV to work [263, 264].

An attractive feature of SCPV is that it provides a solution to the strong CP problem[10] [267, 268], but more importantly it introduced CP violation without increasing the parameter count of the underlying model. For each of the additional phases, a corresponding vacuum extremum condition (2.40) is present. These conditions are usually solved for the soft A-terms of the Higgs sector, leaving the phases as free parameters. The individual complex phases in SCPV can be large as long as the combinations that make up physical CP violating phases stay within experimental limits. Indeed, in Paper 3 we show that if the VEVs' phases are constrained to be small, the situation is analogous to radiative CP violation and a light scalar appears in the spectrum. In contrast to radiative CP violation in the MSSM, where the light state is predominantly composed of the pseudoscalar component of the Higgs doublets, here it is generally singlet dominated. The appearance of a light Goldstone state illustrates how the limit of phases going to zero in a SCPV model does not necessarily connect with a corresponding CP-conserving model.

SCPV can be cumbersome in certain situations. For example, in Paper 2 we wanted to investigate the connection of flavor off-diagonal physics with CP violation and SCPV would have been poorly suited to this task. In the case of explicit CP or flavor violation, it is possible to impose structure by relating the CP and flavor violating effects to the structure of the Yukawa interactions. This is generally called minimal flavor violation (MFV) [269]. In Paper 2 we specifically assume a hybrid gauge–gravity mechanism of SUSY breaking with the formation of flavor structure governed by Froggatt–Nielsen type symmetries [270].

3.3.2 Phenomenology

Figure 3.3 shows typical one-loop SUSY contributions to the K^0–$\bar{K}^0$ mixing amplitude $\mathcal{M}_{K\bar{K}}$ which relates to the measured value of the CP violating parameter ϵ_K [68]

$$\epsilon_K = \mathrm{Im}\left[\frac{\mathcal{M}_{K\bar{K}}}{\Delta m_K}\right] = 161.1 \pm 0.5 \times 10^{-5}, \tag{3.11}$$

with Δm_K the mass difference of the neutral kaons. The SUSY contributions have to be small enough to fit into the error of this measurement, or – in the case of spontaneous CP violation – reproduce this value on their own. Much stronger experimental constraints for SUSY CP phases tend to come from the electron electric dipole moment (EDM) due to the very low experimental limit [68]

$$d_e < 0.105 \times 10^{-26}\ \text{ecm}, \tag{3.12}$$

which is roughly the same order of magnitude as typical SUSY contributions to the electron EDM[11]. These contributions come from diagrams shown in Figure 3.4 which make up the amplitude

[9] Two scalar singlets are necessary if one wishes to avoid the need for fine tuning or dimensionful parameters in the Higgs sector of the superpotential [261, 262].

[10] A CP violating parameter θ arises in QCD from considerations involving the chiral anomaly and degenerate QCD vacuum configurations. Experimental limits [265] give $\theta \lesssim 10^{-10}$ implying a high degree of cancellation between unrelated effects in the strong sector [266]. A more detailed discussion is presented in Appendix A.7.

[11] In contrast, the SM contribution to d_e is of the order of 10^{-38} ecm [271].

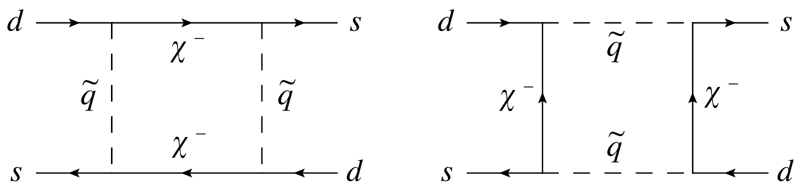


Figure 3.3: Supersymmetric contributions to ϵ_K.

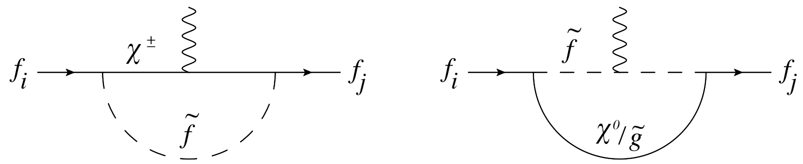


Figure 3.4: Supersymmetric contributions to the electric dipole moment, anomalous magnetic moment, and flavor violating decay of a fermion.

$$\mathcal{M}_{ij} = ie\epsilon^{\mu *} \frac{q^\nu}{2m_{f_j}} \bar{f}_j \, \sigma^{\nu\mu} \left(a_{ij}^L P_L + a_{ij}^R P_R \right) f_i, \tag{3.13}$$

where q^ν is the momentum of the radiated photon, m_{f_j} the mass of the outgoing fermion, $P_{L/R}$ the usual left- and right-handed projection operators, and the coefficients $a_{ij}^{L/R}$ are complicated functions of the couplings, mixings matrices and loop momenta. A remarkable result is that the EDM, anomalous magnetic moment, and rare decay branching ratios of the fermion f_i can all be given in terms of the coefficients $a_{ij}^{L/R}$ [272]

$$d_i = \frac{e}{4m_{f_i}} \mathrm{Im}\left[-a_{ii}^L + a_{ii}^R \right] \tag{3.14a}$$

$$\Delta a_i = \frac{1}{2} \mathrm{Re}\left[a_{ii}^L + a_{ii}^R \right] \tag{3.14b}$$

$$\Gamma(f_i \to f_j \gamma) = \frac{\alpha m_{f_j}}{16} \left(|a_{ij}^L|^2 + |a_{ij}^R|^2 \right). \tag{3.14c}$$

In models where SUSY contributions are the only source of CP and flavor violation, these relations intimately link those two phenomena together and the respective experimental limits can become complementary.

It is worth pointing out that when the fermion f_i in Figure 3.4 is a quark – such as in the case of the flavor violating decay $b \to s\gamma$ – matters are more complicated than presented here due to the hadronic nature of the initial and final states. As we point out in Paper 1, the calculations of ϵ_K are similarly complicated by the hadronic nature of the initial and final states and it is necessary to apply the vacuum insertion approximation (VIA) [273]. It is further usual to average the various SUSY particles in the loop by using the mass insertion approximation (MIA) [274, 275], however in our analysis we used the complete spectrum of the model.

In Paper 2 we investigated the connection between the rare lepton flavor violating decay $\tau \to \mu\gamma$ and the EDM of the muon. In order to separate out any irrelevant CP violation contributions we

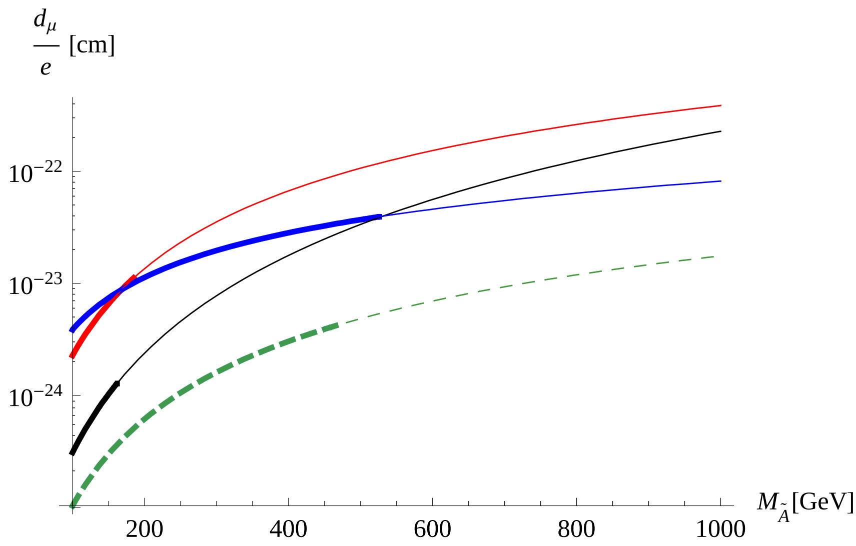


Figure 3.5: The maximal value of the muon EDM as a function of the average slepton mass $M_{\tilde{A}}$. Curves are shown for $\delta_{33}^{L_L E} = 10^{-3}$ (blue), 10^{-4} (red), and 10^{-2} (black) with Br($\tau \to \mu\gamma$) $<$ $4.4 \cdot 10^{-8}$, and for $\delta_{33}^{L_L E} = 10^{-3}$ with Br($\tau \to \mu\gamma$) $< 2 \cdot 10^{-9}$ (green). The thick sections of the curves indicate the region where the mass insertions are valid.

manually added CP phases to only the μ–τ soft SUSY breaking elements. Approximating the flavor changes in Figure 3.4 by mass insertions δ_{ij} [275], one can solve Equations (3.14a) and (3.14b). Given the experimental limit on the rare decay $\tau \to \mu\gamma$, this yields a maximum allowed value for the muon EDM as a function of the average slepton mass as shown in Figure 3.5. We also performed an analysis using exact calculations of the EDM and rare decay branching ratio. In particular we found that for typical ranges of the parameter space the current experimental limits posed no problem but also that it was possible to achieve values that could be detected at current and future experiments. The method we used was to evolve the data set by using a simplified Markov chain Monte Carlo (MCMC) [276]. We found that experimentally interesting ranges for the muon EDM and the $\tau \to \mu\gamma$ rare decay branching ratio are readily accessible. This is demonstrated in Figure 3.6. The main concern in using such a method is that of fine tuning and we formulate an appropriate relative measure of fine tuning based on the preferred ranges of parameters[12]. We found no large fine tuning effect and that even a relatively heavy, $M_{SUSY} > 3$ TeV, scenario can produce interesting values of the muon EDM while avoiding the current limits on the $\tau \to \mu\gamma$ branching ratio.

[12] A commonly used measure of fine tuning is the sensitivity of low energy observables to variations in the high energy parameters of the theory [277, 278]. Since we focused on low energy SUSY this was not a viable choice.

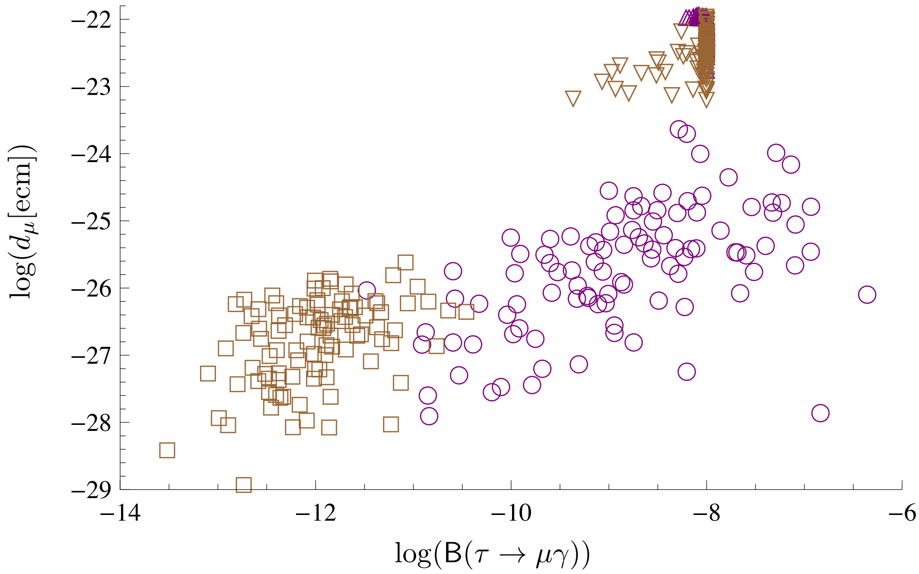


Figure 3.6: The muon EDM d_μ vs. the branching ratio B($\tau \to \mu\gamma$). Distributions before (circles and squares) and after (up and down triangles) the MCMC process are shown for a light, $M_{SUSY} < 1$ TeV, scenario (purple) and a heavy, $M_{SUSY} > 3$ TeV, scenario (brown).

3.4 Neutrinos

3.4.1 Background

In 1957 Bruno Pontecorvo suggested that neutrinos may exhibit similar oscillations as seen in the neutral K-meson system [72]. Five years later, this idea was further developed into the current formalism of flavor oscillations by Ziro Maki, Masami Nakagawa, and Shoichi Sakata [73]. The first experimental indication for neutrino oscillations came in 1968 from the Homestake experiment led by Raymond Davis and John Bahcall, which saw a significant deficit in the expected solar neutrino flux [279]. The discovery of atmospheric neutrino oscillations by the Super-Kamiokande experiment in 1998 [71] established as experimental fact that neutrinos oscillate between flavor eigenstates, and the SNO collaboration proved in 2001 that the solar neutrino deficit, first observed by Davies and Bahcall, is indeed due to neutrino oscillation [280]. Currently – with very few exceptions – all experiments are in agreement with a model parametrized by two mass-squared differences and three mixing angles, and the existence of three flavor eigenstates mixing into three mass eigenstates [68]. Cosmological measurements give a limit[13] on the combined neutrino mass, $m_{\rm tot} < 0.23$ eV, as well as the number of light neutrinos, $N_{\rm eff} = 3.2 \pm 0.2$ [281].

[13]This limit is significantly smaller than laboratory based experimental limits for individual flavor eigenstates shown in Table 1.2.

3.4.2 Masses and Mixing

Giving neutrinos a small mass via a Majorana-type mass term would add a new scale to the Standard Model. A natural way of avoiding this is possible with the so-called seesaw mechanism [74–77]. Introducing a right-handed neutrino with a Majorana mass M_R and a Yukawa term $y_N L H_2 N$ gives a Dirac type neutrino mass $m_D = y_N \langle h_2 \rangle$. The effective mass matrix m_ν for the left-handed neutrinos is approximately given by a block diagonalization of the neutrino mass matrix

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \quad \to \quad m_\nu \simeq -m_D M_R^{-1} m_D^T. \tag{3.15}$$

Thus, with m_D similar to the electron Dirac mass, a right-handed Majorana type mass $M_R \sim 1\text{TeV}$ yields mass scale around the upper limit from Cosmology for neutrino masses. The seesaw mechanism does not strictly need a right-handed neutrino. In models with R-parity violation, m_D can originate from an R-parity breaking bilinear $\mu_i H_2 L_i$ term, or from gauge kinetic terms when the left-handed sneutrino has a non-zero VEV. In these cases M_R in Equation (3.15) is effectively μ or the gaugino mass, respectively. Depending on the form of m_D, all three light neutrinos can get a mass. In the simplest models with only one right-handed neutrino, only one light neutrino gets a mass. It is necessary to invoke radiative corrections to give masses to at least one more light neutrino to be able to fit the two experimentally observed mass-squared differences. In Paper 1, we include three right-handed neutrinos specifically to give a tree level mass to all three neutrinos.

The matrix that diagonalizes M_ν in (3.15) has an approximate block form [282], and the elements of the 3×3 submatrix V_ν, which effectively diagonalizes m_ν, can be used to extract the neutrino mixing parameters listed in Table 1.4. Denoting $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$, the mixing matrix can be written as [68]

$$V_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \tag{3.16}$$

The three mixing angles can be extracted from elements of the matrix (3.16) in the following manner.

$$\sin\theta_{13} = \left|V_\nu^{13}\right|, \qquad \tan\theta_{12} = \left|\frac{V_\nu^{12}}{V_\nu^{11}}\right|, \qquad \tan\theta_{23} = \left|\frac{V_\nu^{23}}{V_\nu^{33}}\right|. \tag{3.17}$$

Of specific interest is the extraction of the CP violating Dirac phase δ,

$$|\delta| = \sin^{-1}\left(\left|\frac{8\,\mathrm{Im}(V_\nu^{21}V_\nu^{*22}V_\nu^{12}V_\nu^{*11})}{\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}}\right|\right), \tag{3.18}$$

which has the Jarlskog invariant J_{CP} (1.10) of the neutrino sector in the numerator. In the quark sector J_{CP} is of the order of 10^{-5}, but in the neutrino sector it can be much larger. Figure 3.7 from Paper 1 shows J_{CP} for a sample of our data set with typical values of the order of 10^{-1}. Recent analysis of neutrino oscillation data suggests that the CP phase of the neutrino sector may indeed be large, $\delta \sim \pi$ [283].

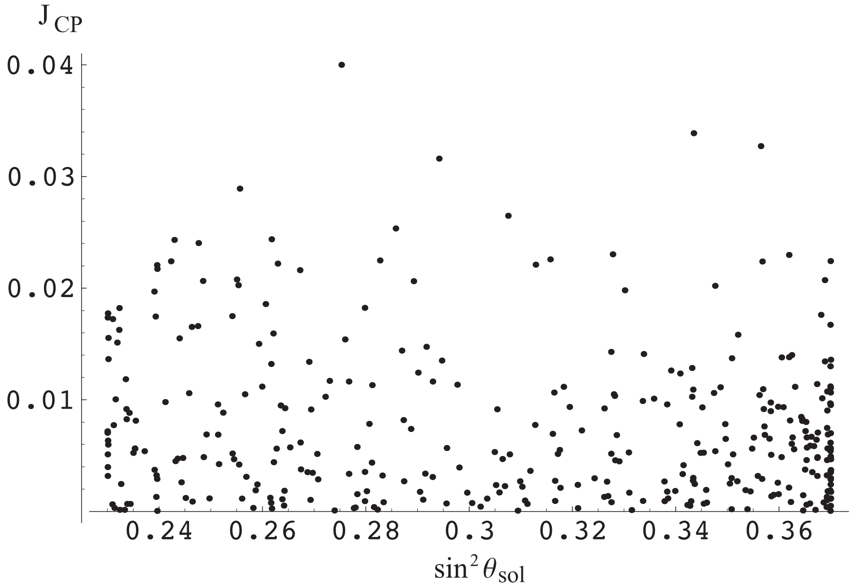


Figure 3.7: The Jarlskog invariant J_{CP} vs. the solar neutrino mixing angle.

3.5 Dark Matter

3.5.1 Cosmology

Alexander Friedman, Georges Lemaître, Howard Robertson, and Arthur Walker proved the uniqueness of a solution – the FLRW metric – to Einstein's field equations in the 1920s and 1930s [284–287]. Using this metric Friedman derived the equation governing the evolution of the universe's scale factor a dependent on the components of the energy density of the universe as

$$\left(\frac{H^2}{H_0^2}\right) = \Omega_R a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda, \tag{3.19}$$

where $H = \dot{a}/a$ is the Hubble parameter and H_0 is its value today. $\Omega_{R/M/k/\Lambda}$ are the radiation, matter, curvature, and vacuum energy density parameters, respectively. Dismissing the currently insignificant radiation density and accounting for the observation of a currently nearly flat universe leaves only the matter and energy partial densities. Measurements of the matter density show it to be roughly six times larger than the amount of luminous matter we can observe and thus the matter density is split into the baryon density and the so-called dark matter (DM) density described by Ω_b and Ω_c, respectively. Denoting by h the reduced Hubble constant (with $H_0 = 100h$ km/s/Mpc), $\Omega_b h^2$, $\Omega_c h^2$, and Ω_Λ make up three out of the six free parameters of the standard model of cosmology [288]. The most recent measurements of the Cosmic Microwave Background (CMB) give the makeup of the universe as 68.3% dark energy, 4.9% ordinary matter, and 26.8% dark matter[14] [281, 293]. This

[14]Other evidence for dark matter comes from galactic rotation curves [110, 289], weak lensing analysis of the so-called Bullet Cluster [290], and measurements of the positron fraction of cosmic rays [291, 292].

corresponds to a physical dark matter density of

$$\Omega_c h^2 = 0.1187 \pm 0.0017. \tag{3.20}$$

Unlike the connection of particle physics to the dark energy density which is – naively – off by approximately 120 orders of magnitude [161], the situation with the dark matter relic density is much better. A weakly interacting stable particle with a correct relic abundance in the universe today can account for the missing portion of the matter density.

The Boltzman equation for the evolution of the number density of a weakly interacting massive particle (WIMP) in an expanding universe is [288]

$$\dot{n} + 3Hn = -\langle \sigma v_M \rangle \left(n^2 - n_{eq}^2 \right), \tag{3.21}$$

where n is the number density, n_{eq} is its value in thermal equilibrium, and the thermally averaged cross section [294, 295] is

$$\langle \sigma v_M \rangle = \frac{1}{8m^4 T K_2^2(m/T)} \int_{4m^2}^{\infty} \sigma(s - 4m^2)\sqrt{s} K_1(\sqrt{s}/T) \mathrm{d}s, \tag{3.22}$$

where T is the temperature, m is the mass of the WIMP, K_i are modified Bessel functions, σ the annihilation cross section of the WIMP into lighter states, and s the usual Mandelstam variable.

In the very early universe T is much larger than m and the thermal equilibrium value of the number density n_{eq} goes as T^3. At the same time, H goes as T^2 so right hand side of the Boltzmann equation (3.21) dominates the evolution of the number density, which consequently tracks the thermal equilibrium value. As the universe cools down and T becomes much smaller than m, n_{eq} falls exponentially with temperature and at some point the annihilation rate drops below the expansion rate of the universe, $\langle \sigma v_M \rangle n < H$, and interactions cease to influence the evolution of the number density. This is called freeze-out, and after it has happened the comoving number density becomes constant. The larger the thermally averaged cross section (3.22) is, the later in the expansion of the universe the freeze-out occurs and the smaller the WIMP relic density becomes.

An estimate of the WIMP relic density can be obtained from Equation (3.21) [296],

$$\Omega_c h^2 = 3 \times 10^{-27} cm^3 s^{-1} \langle \sigma v_M \rangle^{-1} \simeq 0.1, \tag{3.23}$$

if one assumes $\langle \sigma v_M \rangle$ to be an energy independent weak interaction between $\mathcal{O}$(100 GeV) particles. A more thorough treatment of the problem does not significantly change this outcome [297, 298]. The fact that the weak interaction strength and the electroweak mass scale provide a near perfect fit for the completely unrelated measurement of the dark matter relic density is called the WIMP miracle.

3.5.2 Detection

The three main methods for detecting WIMP dark matter are schematically connected via the annihilation cross section of the dark matter candidate as shown in Figure 3.8. The actual relationships are

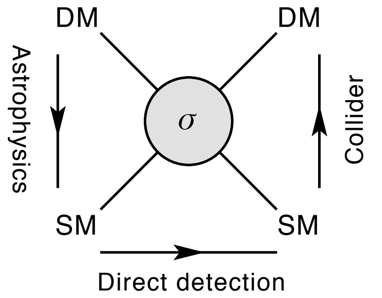


Figure 3.8: The schematic relationship between astrophysical, collider based, and direct detection based WIMP searches.

complicated by the fact that different sets of sub-amplitudes contribute to the individual processes; the thermally averaged cross section (3.22) includes sub-amplitudes for all kinematically possible final states, only a few of which are initial states in collider phenomenology. The case of direct detection is even more complex, since the cross section for elastic scattering involves hadronic bound states and has to be normalized for the particular nucleus used in the experiment [299]:

$$\sigma_{\mathsf{SI}} = \frac{4\mu^2}{\pi}\left(\frac{Z\sqrt{\sigma_p}+(A-Z)\sqrt{\sigma_n}}{A}\right)^2 , \tag{3.24}$$

where μ is the WIMP–nucleon reduced mass, Z and A are the atomic and mass number of the target element, and $\sigma_{n,p}$ the spin-independent cross sections of the dark matter scattering of a neutron and proton, respectively. The matter can further be complicated by a possible isospin dependent contribution

$$\sigma_{\mathsf{SD}} = \frac{32\mu^2 G_F^2}{2J+1}\left(a_p^2 S_{pp} + a_p a_n S_{pn} + a_n^2 S_{nn}\right) , \tag{3.25}$$

with J the WIMP spin, G_F the Fermi coupling constant, $a_{p,n}$ the axial four fermion WIMP–nucleon couplings and S_{xx} structure functions specific to a given nucleus. Astrophysical dark matter searches suffer from uncertainties including a poorly known galactic dark matter distribution, final state dependent boost factors [300], and possible non-perturbative Sommerfeld enhancements of the annihilation cross section [301–303].

The current direct detection limits on the WIMP–nucleon scattering cross section are shown in Figure 3.9. The only experiment claiming a discovery signal is the crystal scintillator based DAMA experiment which reports a 8.2σ effect in annual modulation of the event rate [304]. More recently two cryogenic experiments, CoGeNT and CRESST, have published weaker findings of a 2.8σ annual modulation [305] and a 4σ excess [306], respectively. The most stringent null result comes from the XENON noble liquid experiment [307]; the exclusion of the DAMA result seen in Figure 3.9 is at the level of 3σ. However, if one includes the possibility for spin-dependent effects this discrepancy can be reduced [308, 309]. Interestingly, the XENON bound is starting to push up against the cMSSM preferred region, indeed a recent analysis suggests that the direct detection limit is complementary to the LHC discovery limit by ruling out high mass regions which are currently unreachable by the LHC [310].

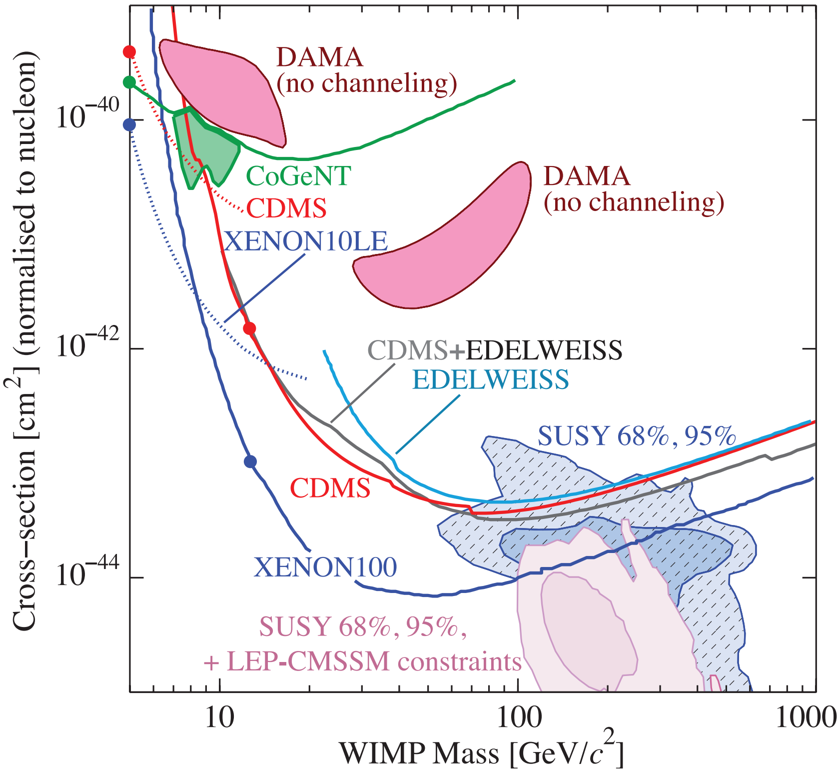


Figure 3.9: Experimental limits on the WIMP–nucleon cross section. Taken from the Review of Particle Physics [68].

Astrophysical detection experiments look for signatures of WIMP annihilation in the cosmic ray spectrum. In particular, observations of the positron fraction of cosmic rays show an increase as a function of energy, which could be explained by WIMP annihilation in the galactic halo[15]. This effect was first observed by the PAMELA satellite experiment [291] and recently confirmed by the AMS experiment attached to the International Space Station [292]. The ground-based Fermi Large Area Telescope (LAT) has also been able to measure this effect [312]. Another channel for indirect detection is the possible observation of a sharp peak in the cosmic ray spectrum due to possible WIMP annihilation into two photons. Recently, the Fermi LAT experiment reported such a photon line in the spectrum at 130 GeV with 3.2σ significance [313]. Finally, WIMPs passing through the Earth or Sun can produce energetic neutrinos from the direction of the Sun or center of the Earth. High-energy neutrino telescopes such as ANTARES [314], IceCUBE [315], and AMANDA [316] are looking for such signals.

Collider based searches for SUSY dark matter reduce to searches for the LSP. These searches are model dependent and possibly complicated by long decay chains with multiple hadronic jets, missing

[15]It is worth noting that the positron excess observation has large uncertainties from modeling and background estimates, and that specific models trying to fit the data are not minimal [311].

energy and momentum components, and large SM backgrounds.

3.5.3 SUSY Dark Matter Candidates

Supersymmetry offers several possible candidates for a dark matter particle[16]. The neutralino, sneutrino, and gravitino all fit the requirements for a weakly interacting neutral particle[17]. If R-parity is conserved and one of these is the LSP then it may have a suitable relic abundance to account for the dark matter relic density. Extensive treatments on SUSY dark matter are given by Ellis et al. [326] and Jungman et al. [296] and more recent overviews by Bertone et al. [327], Kamionkowski [328] and Bergstrom [300].

Since cosmological observations provide a definite value of the dark matter relic density with a relatively small error, this can place rather tight restrictions on the LSP in constrained SUSY models such as the c(N)MSSM [231, 232]. This "WMAP window" should, however, only be considered as an upper bound since it is possible that other sources besides a SUSY WIMP contribute substantially to $\Omega_c h^2$. It is in principle also possible to reduce an *a priori* too high relic density by invoking a very light gravitino LSP making the offending relic density of the now next to LSP a moot point. This is not without its problems, as is briefly discussed in the section on gravitino LSPs below.

Neutralino

The neutralino is the most widely studied SUSY dark matter candidate. In the MSSM it is an admixture of the bino, wino, and higgsino. However, it is a generic feature of many SUSY models that in the parameter space not excluded by LEP, the LSP is bino dominated and the resulting relic density is too large due to highly suppressed couplings in most of the annihilation channels [123]. If this is the case, it is possible to have regions in parameter space where specific mechanisms bring the relic density down. Commonly these are called the co-annihilation, A-funnel, and focus point regions corresponding to specific mechanisms that increase the annihilation cross section (3.22). In the first of these, the next to lightest supersymmetric particle (NLSP) is close in mass to the LSP. This means that during the LSP's freeze-out the NLSP is still abundantly present and the LSP annihilation cross section gets additional contributions from LSP–NLSP co-annihilation processes. The A-funnel refers to the region in parameter space where the LSP annihilation has a resonant decay mode via one of the Higgs bosons – usually the pseudoscalar, hence the name. Finally, the focus point region refers to a region where the composition of the LSP has significant wino and/or higgsino admixtures which

[16] The only SM candidate for a dark matter particle is the neutrino. In order to explain the dark matter relic density it would have to have a mass of 5 eV which is excluded by current limits [293]. As yet undiscovered neutrinos are ruled out at masses below $m_Z/2$ by LEP and at higher masses by direct detection experiments [317–319], and limits on sterile neutrinos can be derived from the Pauli exclusion principle and phase space considerations [320].

[17] SUSY models containing an axion can have its fermionic superpartner, the axino, as an LSP and consequently as a dark matter candidate [321, 322]. Axino dark matter has some of the same characteristics as the gravitino – light and very weakly interacting – but a thorough discussion of axino dark matter is outside the scope of this thesis. Comprehensive reviews are given by Covi et al. [323, 324] and very recently by Choi et al. [325].

increases the efficiency of many of the LSP's annihilation channels[18].

In the NMSSM there is the additional possibility of the neutralino being singlino dominated, which in general also leads to an excessively large relic density. In Paper 3 we study an extended NMSSM model and show how the inclusion of spontaneous CP violation influences the relic density. In particular, we show that the light scalar introduced by SCPV in this model significantly increases the number of annihilation channels. This effect is shown in Figure 3.10. We also show that a naïve

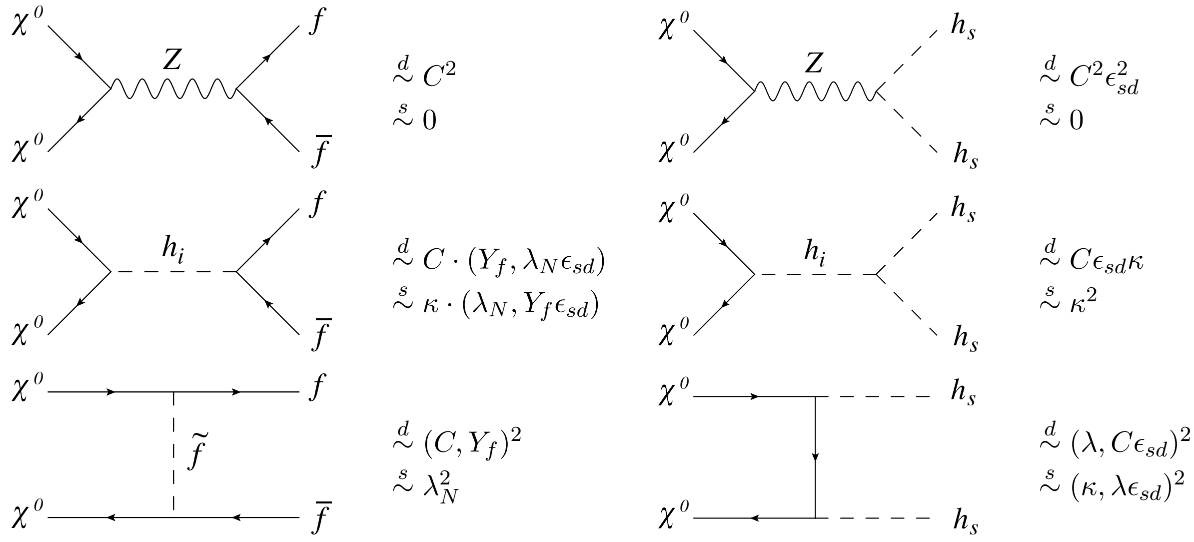


Figure 3.10: Tree level pair annihilation channels for the neutralino LSP into $f\bar{f}$ and $h_S h_S$ final states. Leading coefficients for the higgsino/gaugino (d) and singlet (s) component of the neutralino are given. The electroweak coupling $C \equiv \frac{e}{2 s_W c_W} \simeq 0.37$ and ϵ_{sd} is the doublet admixture of the light singlet state h_S.

analysis of the leading coefficients of the decay channels gives a very good explanation of the behavior of this model. Specifically, we can deduce the overall reduction in the relic density compared to a model without SCPV and the high dependence on the parameter κ which incidentally also dictates the singlino admixture $\epsilon_{\chi s}$ of the LSP. Figure 3.11 shows the clear dependency of the relic density on κ when the LSP has a singlino admixture of more than 0.1 and Figure 3.12 shows both the effect of SCPV for even very small singlino admixtures of the LSP and the increase in relic density as the LSP becomes singlino dominated.

Sneutrino

The left-handed sneutrino is excluded by current experimental constraints [68]. We show in Paper 3 that the inability of the left-handed sneutrino to evade constraints can be seen as a consequence

[18] The focus point region of cMSSM/mSUGRA models is a subsection of a more general class of focus point SUSY models designed to leave the Z-boson mass insensitive to variations on the GUT scale parameters of the theory [329–331].

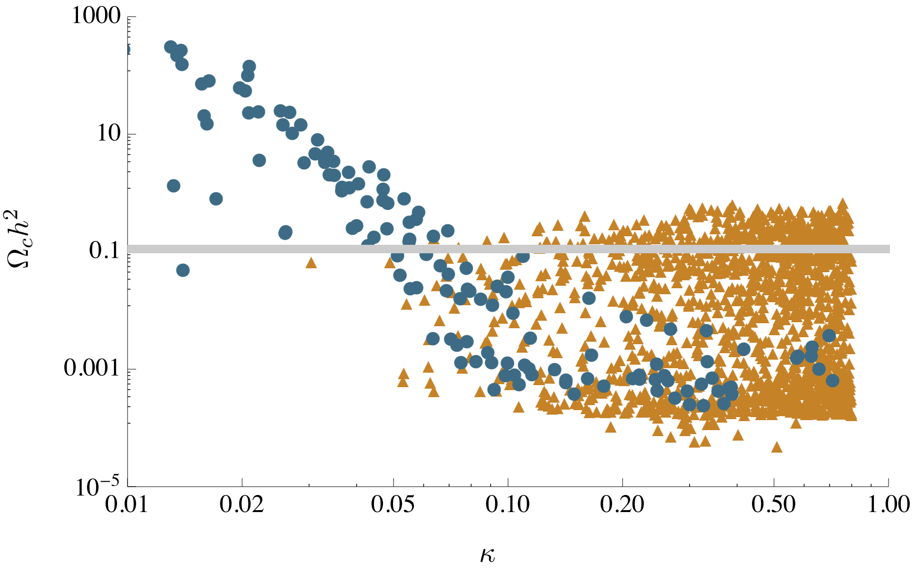


Figure 3.11: The relic density against the trilinear coupling κ for neutralino LSPs. Points with (blue) and without (orange) significant singlino component. The grey band indicates the current WMAP limits on the relic density.

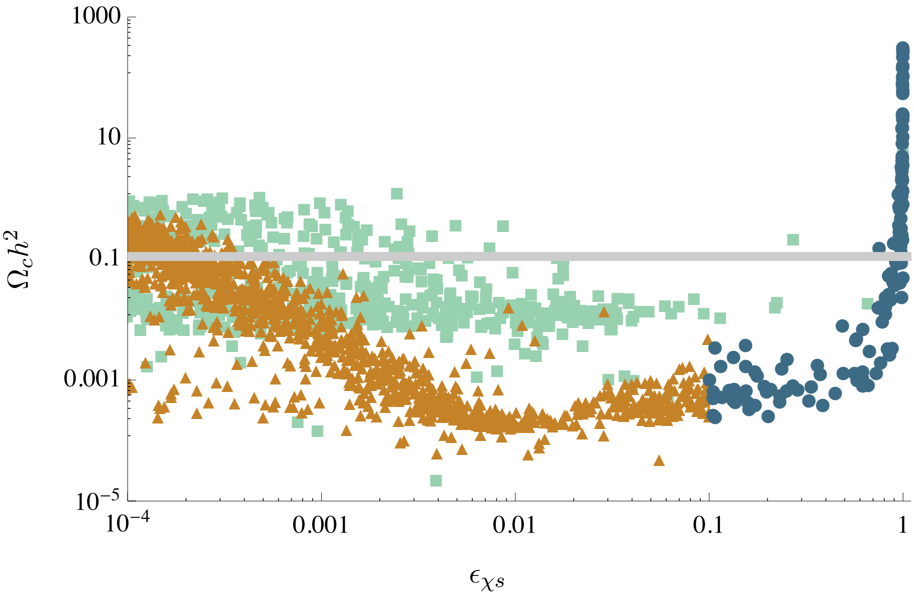


Figure 3.12: The relic density against the singlino component of the neutralino LSP. Points with (blue) and without (orange) significant singlino component. CP-conserving points (green) are shown for comparison. The grey band indicates the current WMAP limits on the relic density.

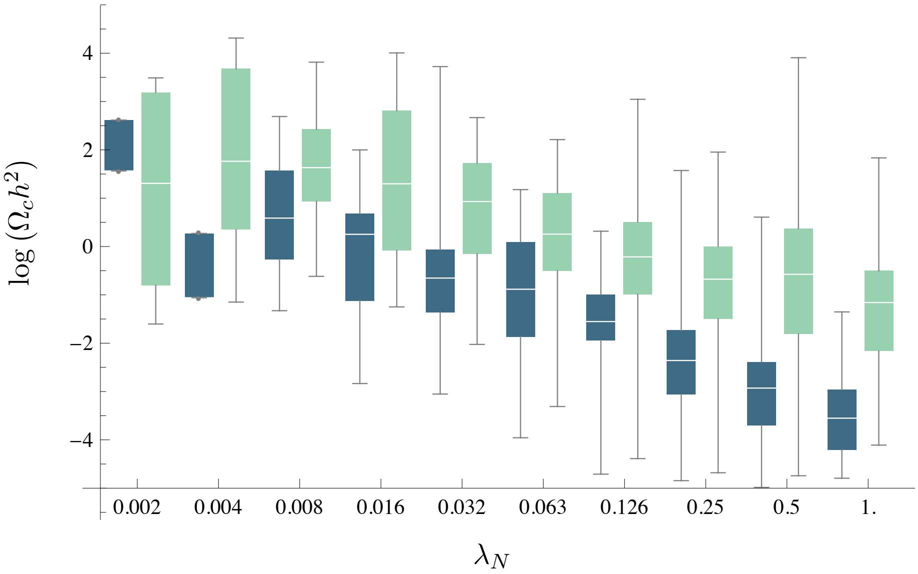


Figure 3.13: A box-and-whisker diagram of the relic density against the trilinear coupling λ_N comparing the distribution of right-handed sneutrino LSPs in the CP-conserving (green) and CP-violating (blue) cases. Boxes are the 25–75 percentile range and whiskers denote the complete range of the data.

of the very rigid annihilation channels, which are dominated by SM gauge interactions and have no dependence on SUSY parameters (other than the left-handed sneutrino's mass). In extended models of SUSY – such as the model we investigated – the right-handed sneutrino is a possible DM candidate. Note that one has to provide a plausible way to thermally produce the right-handed sneutrino in the early universe. In the NMSSM this is usually effected by a $\lambda_N \hat{S}\hat{N}\hat{N}$ term in the superpotential which couples the singlet to the right-handed sneutrino.

We found that the light singlet scalar introduced by SCPV adds annihilation channels in a manner similar to the neutralino LSP case discussed above. Again, we can identify a dominant parametric dependence, this time on the trilinear coupling λ_N. Figure 3.13 shows both the dependency on λ_N as well as the reduction in average relic density between the CP-conserving and CP-violating models.

Gravitino

The gravitino is closely connected to SUSY breaking and difficult to accommodate as a dark matter candidate. In gravity mediated SUSY breaking scenarios it has a mass of 100 GeV – 1 TeV and a lifetime [332]

$$\tau \sim 10^6 \left(\frac{\text{TeV}}{m_{3/2}}\right) \text{s}, \tag{3.26}$$

meaning it decays after nucleosynthesis which ends when the universe is around 10^3 seconds old. The decay products of the gravitino destroy nearly all nuclei formed during nucleosynthesis and upset the observed relative abundances of the light elements [333]. Avoiding this scenario sets a stringent limit on the reheating temperature [334]

$$T_{\text{max}} < 10 \text{ TeV} \times h^2 \left(\frac{m_{3/2}}{100 \text{ keV}}\right) \times \left(\frac{\text{TeV}}{M_3}\right) \simeq 10^{10} \text{ GeV}, \tag{3.27}$$

which is – for typical values of the gluino masses M_3 – too low for regular mechanisms of baryogenesis.

In gauge mediated SUSY breaking scenarios the gravitino can be very light and stable, avoiding problems with nucleosynthesis and overclosing of the universe. Typically

$$\Omega_{3/2}h^2 \simeq \left(\frac{m_{3/2}}{\text{keV}}\right) \times \left(\frac{100}{g^*(T_f)}\right), \tag{3.28}$$

with $g^*(T_f) \sim 100$–200 the number of effective degrees of freedom at freeze-out. The problem in this case arises from the NLSP which will decay to the LSP via gravitational interactions, with a half life much longer than the end of nucleosynthesis. This can lead to similar problems to those encountered in the case of a heavy gravitino with specific limits depending on the decay products of the NLSP [332]. If R-parity is not conserved these limits can be avoided as the NLSP will decay much faster.

3.6 Conclusions and Outlook

The LHC started up in the fall of 2008 and has delivered a vast amount of data – close to 30 fb^{-1} of integrated luminosity. More importantly, the LHC has led to the discovery of the last missing SM particle, the Higgs boson, in July 2012. In this respect the LHC has already fulfilled one of its key purposes. As a discovery machine for SUSY the LHC has so far not delivered[19]. While future runs of the LHC will see the machine operating at significantly higher energies and plans exist for high energy and high luminosity upgrades down the road allowing for studies of a wider mass range, it is becoming increasingly unlikely that a simple "vanilla" version of the MSSM such as the cMSSM or mSUGRA is the correct model of particle physics. Indeed, complementary searches such as the XENON experiment already exclude regions in parameter space that would be otherwise favored by naturalness arguments. Consequently, in the papers included in this thesis we emphasized the study of SUSY as an unconstrained model with low energy parameters as well as extending the model beyond the MSSM. Abandoning the GUT-scale constraints is becoming more common in the literature and models such as the parametric MSSM (pMSSM) [208] attempt to standardize the investigation of the low energy parameter space. In our case the approach let us focus on specific mechanisms – in particular the use of spontaneously broken symmetries – to solve some of the problems of SUSY without missing possibly interesting phenomena.

[19] It can be argued that the discovery of a possibly fundamental scalar particle is in itself a significant, albeit indirect, proof of concept for SUSY – a theory proposing the addition of a multitude of scalars.

One of the most interesting aspects of SUSY is CP and flavor violation and the possibility of the two being connected. Current searches for rare decays and electric dipole moments are not far from typical SUSY contributions and future experiments are set to close the remaining gap [335, 336]. Indeed, we show in Paper 2 that these experiments could discover new physics effects even for a SUSY spectrum that is unreachable at the LHC in the near future.

Further insight into the mechanism of CP violation can be obtained from detailed studies of the Higgs sector. In Paper 3 we showed the interesting effect that SCPV has on the dark matter relic density. In particular, we show the effect of the light singlet state that appears in the spectrum similarly to the axion in radiatively broken CP [255]. In contrast with the axion, which has been largely excluded [68], a singlet dominated light scalar could have evaded discovery thus far. Now that the Higgs boson has been found it may be possible to measure or limit its decay into two singlets, and constrain models with SCPV. Since models such as the NMSSM have generically large couplings between the Higgs bosons and the singlet it may even be possible to exclude the existence of a singlet with mass below $m_h/2$, in which case a majority of SCPV models would become obsolete.

The exploration of SUSY models beyond the MSSM currently faces two technical hurdles. The first is the expanded model space which adds a significant number of parameters especially when these are treated as independent at the weak scale. Novel methods such as Markov chain Monte Carlos that help with the increasing number of parameters can become prohibitively expensive as measured in CPU time if one wishes to include more rigorous treatment of the models (for example by solving RGEs or including detector simulations). The second issue is that, even though in most cases the fundamental work of calculating the RGEs and various other loop corrections has been done for these models, there are few implementations beyond explicit symmetry breaking and very minimal field content extensions in any of the standard software packages. The progress of computing power will solve the first problem and one hopes that continued interest in SUSY will solve the second.

Appendix A

A.1 General

Metric convention

$$\eta^{\mu} = \mathrm{diag}(1,-1,-1,-1) \tag{A.1}$$

Pauli matrices

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.2}$$

$$\sigma^{\mu} = (\sigma^0, \sigma^1, \sigma^2, \sigma^3) \tag{A.3}$$

$$\overline{\sigma}^{\mu} = \sigma^{\nu}\eta_{\mu\nu} \tag{A.4}$$

Gamma matrices and projection operators (Dirac representation)

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu} \\ \overline{\sigma}^{\mu} & 0 \end{pmatrix} \quad \gamma^5 = \begin{pmatrix} -\mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix} \quad P_L = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & 0 \end{pmatrix} \quad P_R = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{1} \end{pmatrix} \tag{A.5}$$

Gell–Mann matrices

$$\begin{gathered}
\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\
\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}
\end{gathered} \tag{A.6}$$

Poincaré algebra

$$[P^{\mu}, P^{\nu}] = 0 \tag{A.7}$$

$$[M^{\mu\nu}, P^{\rho}] = i\left(\eta^{\nu\rho}P^{\mu} - \eta^{\mu\rho}P^{\nu}\right) \tag{A.8}$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = i\left(\eta^{\mu\sigma}M^{\nu\rho} + \eta^{\nu\rho}M^{\mu\sigma} - \eta^{\mu\rho}M^{\nu\sigma} - \eta^{\nu\sigma}M^{\mu\rho}\right) \tag{A.9}$$

A.2 Spinors

Dirac spinors Ψ_D are four-component anti-commuting objects which in the Dirac representation can be written using two component Weyl spinors ξ and χ

$$\Psi_D = \begin{pmatrix} \xi \\ \chi^\dagger \end{pmatrix}. \tag{A.10}$$

The projection operators (A.5) can be used to extract the right- or left-handed components $\chi^\dagger$ or ξ, respectively. Since components have different electroweak interactions it is useful to simply write everything using χ and ξ; this is called the Weyl notation. Majorana spinors Ψ_M are four component Dirac spinors with the added constraint $\chi = \xi$

$$\Psi_M = \begin{pmatrix} \xi \\ \xi^\dagger \end{pmatrix} \tag{A.11}$$

A.3 Grassmann variables

The superspace formulation of SUSY requires the introduction of anti-commutating coordinates θ. Integration over these Grassmann variables is consistently defined by a linear functional called the Berezin integral [176]

$$\int \theta \mathrm{d}\theta = 1, \quad \int \mathrm{d}\theta = 0. \tag{A.12}$$

A.4 Renormalization Group Equations (RGE's)

The two loop renormalization group equations for the gauge couplings are

$$\mu \frac{d}{d\mu}\alpha_i = \frac{1}{16\pi^2}\Big[8\pi b_i + 2b_{ij}\alpha_j\Big]\alpha_i^2 \qquad \begin{cases} \alpha_1 = \frac{5}{3}{g'}^2/4\pi \\ \alpha_2 = g_2^2/4\pi \\ \alpha_3 = g_3^2/4\pi \end{cases} \tag{A.13}$$

with the constants b_i and b_{ij} in the SM and the MSSM

$$b_i^{\mathsf{SM}} = -\begin{pmatrix} 0 \\ \frac{22}{3} \\ 11 \end{pmatrix} + N_f \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + N_d \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix} \tag{A.14}$$

$$b_i^{\mathsf{MSSM}} = -\begin{pmatrix} 0 \\ 6 \\ 9 \end{pmatrix} + N_f \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_d \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix} \tag{A.15}$$

$$b_{ij}^{\mathsf{SM}} = -\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{136}{3} & 0 \\ 0 & 0 & 102 \end{pmatrix} + N_f \begin{pmatrix} \frac{19}{15} & \frac{3}{5} & \frac{44}{15} \\ \frac{1}{5} & \frac{49}{3} & 4 \\ \frac{11}{30} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} + N_d \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{A.16}$$

$$b_{ij}^{\mathsf{MSSM}} = -\begin{pmatrix} 0 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 54 \end{pmatrix} + N_f \begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\ \frac{2}{5} & 14 & 8 \\ \frac{11}{15} & 3 & \frac{68}{3} \end{pmatrix} + N_d \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{2}{7} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{A.17}$$

Here $t = \log(\mu^2/\mu^2_{\mathsf{EW}})$, N_f is the number of families and N_d is the number of complex scalar doublets. The one loop RGE's for the top Yukawa coupling and for the quartic Higgs coupling in the SM are

$$\frac{d}{dt}y_t = \frac{1}{16\pi^2}\left[\frac{9}{2}y_t^3 - 4g_3^2 y_t - \frac{9}{8}g_2^2 y_t - \frac{17}{24}g_1^2 y_t\right], \tag{A.18}$$

$$\frac{d}{dt}\lambda = \frac{1}{16\pi^2}\left[12\lambda^2 + 12\lambda y_t^2 - 12y_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]. \tag{A.19}$$

Full three loop results have been calculated by, inter alia, Chetyrkin and Zoller [337]. The two and three loop corrections to λ are at the level of 1% and -0.04% respectively. In the case of the top Yukawa coupling the corrections are more substantial at 16.6% and 0.7% respectively [97].

A.5 Slepton and squark mass matrices

The slepton and squark mass matrices are derived from the scalar potential

$$\left.\begin{matrix} M^2_{\tilde{l},ij} \\ M^2_{\tilde{\nu},ij} \\ M^2_{\tilde{q},ij} \end{matrix}\right\} = \frac{\partial^2 V_{\mathrm{S}}}{\partial\varphi_i \partial\varphi_j^*} \qquad \left\{\begin{matrix} \varphi = \{\tilde{e}_{L,1}, \tilde{e}_{L,2}, \tilde{e}_{L,3}, \tilde{e}_{R,1}, \tilde{e}_{R,2}, \tilde{e}_{R,3}\} \\ \varphi = \{\tilde{\nu}_{L,1}, \tilde{\nu}_{L,2}, \tilde{\nu}_{L,3}\} \\ \varphi = \{\tilde{q}_{L,1}, \tilde{q}_{L,2}, \tilde{q}_{L,3}, \tilde{q}_{R,1}, \tilde{q}_{R,2}, \tilde{q}_{R,3}\} \end{matrix}\right. \tag{A.20}$$

The charged sleptons form a 6×6 mass matrix

$$M^2_{\tilde{l}} = \begin{pmatrix} M^2_{LL} & M^2_{LR} \\ M^{2\dagger}_{LR} & M^2_{RR} \end{pmatrix}, \tag{A.21}$$

where the sub-matrices are

$$M^2_{LL,ij} = M^2_{L,ij} + v_1^2 y_E^{ik} y_E^{kj*} + \frac{1}{4}(g_1 - g_2)(v_1^2 - v_2^2)\delta_{ij} \tag{A.22}$$

$$M^2_{LR,ij} = v_1 A_{E,ij} - \mu^* v_2 y_E^{ij} \tag{A.23}$$

$$M^2_{RR,ij} = M^2_{E,ij} + v_1^2 y_E^{ik} y_E^{kj*} - \frac{1}{2}g_1^2(v_1^2 - v_2^2)\delta_{ij}. \tag{A.24}$$

The sneutrinos form a 3×3 mass matrix

$$M^2_{\tilde{\nu},ij} = M^2_{L,ij} + \frac{1}{4}(g_1 + g_2)(v_1^2 - v_2^2)\delta_{ij}. \tag{A.25}$$

The squarks form a 6×6 mass matrix analogous to that of the charged sleptons (A.21), but it is common to assume flavor diagonal A-terms and that only the third generation squarks have appreciable mixing,

$$M^2_{\tilde{t}} = \begin{pmatrix} M^2_{Q,33} + m_t^2 + \Delta_{\tilde{u}_L} & v_1 A_{U,33} - \mu^* v_2 y_t \\ v_1 A^*_{U,33} - \mu v_2 y_t & M^2_{U,33} + m_t^2 + \Delta_{\tilde{u}_R} \end{pmatrix} \tag{A.26}$$

$$M^2_{\tilde{b}} = \begin{pmatrix} M^2_{Q,33} + m_b^2 + \Delta_{\tilde{d}_L} & v_2 A_{D,33} - \mu^* v_1 y_b \\ v_2 A^*_{D,33} - \mu v_1 y_b & M^2_{U,33} + m_b^2 + \Delta_{\tilde{d}_R} \end{pmatrix} \tag{A.27}$$

where the D-term contribution is

$$\Delta_\phi = \frac{1}{2}\left(T_{3\phi}g_2^2 - \frac{Y_\phi}{2}g_1^2\right)\left(v_1^2 - v_2^2\right), \tag{A.28}$$

with Y_ϕ the hypercharge and $T_{3\phi}$ the third Isospin component of ϕ.

A.6 Mass matrix diagonalization

The various mass matrices of a SUSY model can be complex valued and either symmetric or arbitrary. Only the neutral scalar matrices can always be presented in a real symmetric form. Consequently, when talking about mass eigenvalues of the fields it must be emphasized that only in the case of the neutral scalars do these always coincide with the eigenvalue decomposition of the corresponding mass matrix. For all other fields the physical masses are the singular values of the singular value decomposition of the corresponding mass matrix

$$U^* M_{\chi^\pm} V^{-1} = \mathrm{diag}(m_{\chi^\pm}) \tag{A.29}$$

$$N^* M_{\chi^0} N^{-1} = \mathrm{diag}(m_{\chi^0}) \tag{A.30}$$

$$R M_\phi^2 R^{-1} = \mathrm{diag}(m_\phi^2), \tag{A.31}$$

where the unitary transformation matrices U, V and N are usually determined by solving the eigenvalue decomposition of $M_X M_X^\dagger$ and $M_X^\dagger M_X$. Likewise the singular values are, up to a sign, the square roots of eigenvalues of $M_X M_X^\dagger$ (or $M_X^\dagger M_X$). The sign has to be recovered by performing the singular value decomposition.

A.7 Strong CP problem

The axial anomaly [338, 339] in QCD gives (when m_q=0)

$$\partial_\mu J_5^\mu = -N_f \frac{g^2}{8\pi^2} \mathrm{Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}], \quad \tilde{G}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}, \quad G_{\mu\nu} \equiv \frac{1}{2}\boldsymbol{\lambda}\cdot\mathbf{G}_{\mu\nu}. \tag{A.32}$$

Since the trace can be written as a total derivative, $\partial_\mu K^\mu$, axial current conservation is restored for a new current $\hat{J}_5^\mu = J_5^\mu + K^\mu$ where

$$K^\mu \equiv \frac{N_f g^2}{4\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left[G_\nu\partial_\rho G_\sigma + ig\frac{2}{3}G_\nu G_\rho G_\sigma\right]. \tag{A.33}$$

The conservation of this current implies an axial U(1) symmetry which is broken by the quark masses. The associated Goldstone boson should have a mass $\lesssim \sqrt{3}m_\pi$; this is experimentally excluded and termed the U(1) problem [340]. However, axial charge conservation for this symmetry is problematic as the surface integral over $\mathrm{d}S_3$ in

$$Q_A = \int \mathrm{d}^3x \hat{J}_5^0 = -\oint \mathrm{d}S_3 \partial \hat{J}_5^i \tag{A.34}$$

can only be discarded if G can be gauged away at $r\to\infty$, i.e. G is a pure gauge, $G \to -ig^{-1}U^{-1}\partial U$. This can always be done locally, e.g. at the infinity, but in general such a transformation may not be able to gauge away G throughout S_3 [266]. This is due to the possibility of gauge configurations with differing topological charges. The topological charge describes how many times U wraps around the gauge group as we move through S_3, and is calculated as

$$\begin{aligned} n &= -\frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\int \mathrm{d}S_\mu \mathrm{Tr}\left[(U^{-1}\partial_\nu U)(U^{-1}\partial_\rho U)(U^{-1}\partial_\sigma U)\right] \\ &= -\frac{g^2}{16\pi^2}\int \mathrm{d}^4x \mathrm{Tr}\left[G_{\mu\nu}\tilde{G}^{\mu\nu}\right], \end{aligned} \tag{A.35}$$

where the second equality follows from inserting the pure gauge into (A.33). These vacuum configurations with different topological charges are degenerate and the amplitude for vacuum tunneling between two configurations with topological charges $n_{1,2}$ is

$$\mathcal{M} \propto e^{-S} = e^{-\frac{1}{2}\int \mathrm{d}^4x \mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}]} \tag{A.36}$$

$$|\mathcal{M}|^2 \propto e^{-\frac{16\pi^2}{g^2}|n_1-n_2|}. \tag{A.37}$$

The integral in the Euclidian action (A.36) is minimized by self-dual or anti-self-dual gauge field configurations, $G_{\mu\nu} = \pm\widetilde{G}_{\mu\nu}$ [341]. These solutions are called pseudoparticles or instantons [342]. It is clear that the vacuum of the theory is infinitely degenerate between states of different topological charge $|n\rangle$. The true vacuum is a linear combination

$$|\theta\rangle = \sum_n e^{in\theta}|n\rangle. \tag{A.38}$$

This is the θ-vacuum and it is invariant under the tunneling discussed above. Assuming we are in a specific vacuum state, $|n\rangle$, necessitates the inclusion of

$$-n\theta = \theta\frac{g^2}{16\pi^2}\mathrm{Tr}\left[G_{\mu\nu}\tilde{G}^{\mu\nu}\right] \tag{A.39}$$

into the Lagrangian to account for the exponent in (A.38). This solution to the U(1) problem was proposed by 't Hooft in 1986 [343]. Unfortunately it also proves that θ is not a spurious parameter of the Standard Model, and introduces CP violation in the strong interactions. Experimental constraints on the neutron EDM give $\theta \lesssim 10^{-10}$. This is called the strong CP problem.

Solutions to the Strong CP Problem

The easiest solution to the strong CP problem is to have one of the quarks, usually the up quark, be massless. In that case a global chiral transformation

$$q \to e^{i\beta\gamma_5}q \tag{A.40}$$

leaves the QCD Lagrangian unchanged and variational analysis shows that (A.39) is cancelled if $2\beta = \theta$. A more realistic approach was proposed by Peccei and Quinn [218, 219, 344], by including the Higgs boson in the chiral transformations. This restores the U(1) broken by the Yukawa terms in the Lagrangian. In turn, this requires the addition of a second Higgs doublet and during electroweak symmetry breaking a new Goldstone particle called the axion appears. The standard axion of Peccei and Quinn was quickly ruled out [345] and currently the two favored tactics which evade experimental bounds are using heavy quarks [346, 347] or a weakly coupled light axion [221, 348].

Spontaneous CP violation is an elegant solution to the strong CP problem. It postulates the conservation of CP in the Lagrangian (setting $\theta = 0$) and introduces the CP violation necessary for processes such as baryogenesis via a complex vacuum expectation value of the Higgs or other scalar. This is particularly attractive in SUSY, where numerous arbitrary CP phases are present at tree level. A rigorous treatment should take into account that radiative corrections will regenerate θ below the scale at which SCPV happens [349–351].

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