# Essays in Applied Macroeconomic Theory: Volatility, Spreads, and Unconventional Monetary Policy Tools

by

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# Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).



I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party.

I declare that my thesis consists of approximately 34000 words.

# Statement of conjoint work

Chapter 3 of this thesis is based on research that I undertook while working as a Research Specialist at the Central Reserve Bank of Peru. This work was jointly co-authored with Dr. César Carrera and I contributed a minimum of 50% of the work.

# Abstract

This thesis contains three essays that employ macroeconomic theory to study the implications of volatility, financial frictions and reserve requirements.

The first essay uses an imperfect information model where agents solve a signal extraction problem to study the effect of volatility on the economy. A real business cycle model where the agent faces imperfect information regarding productivity is used to address the question. The main finding is that the variance of the productivity process components has a small negative short run impact on the economy's real variables. However, imperfect information dampens the effects of volatility associated to permanent components of productivity and amplifies the effects of volatility associated to transitory components.

The second essay presents a partial equilibrium characterization of the credit market in an economy with partial financial dollarization. Financial frictions (costly state verification and banking regulation restrictions), are introduced and their impact on lending and deposit interest rates denominated in domestic and foreign currency studied. The analysis shows that reserve requirements act as a tax that leads banks to decrease deposit rates, while the wedge between foreign and domestic currency lending rates is decreasing in exchange rate volatility and increasing in the degree of correlation between entrepreneurs' returns and the exchange rate.

The third essay introduces an interbank market with two types of private banks and a central bank into a New-Keynesian DSGE model. The model is used to analyse the general equilibrium effects of changes to reserve requirements, while the central bank follows a Taylor rule to set the policy interest rate. The paper shows that changes to reserve requirements have similar effects to interest rate hikes and that both monetary

policy tools can be used jointly in order to avoid big swings in the policy rate or a zero bound.

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Last but not least, I dedicate this thesis to my family. To my parents, Hugo and Flora, my wife, Arlé, and my daughters, Daira and Bianca.

# Contents

# List of Tables

# List of Figures

# Chapter 1

# Total Factor Productivity and Signal Noise Volatility in an Incomplete Information Setting

## 1.1 Introduction

General equilibrium models of incomplete information are rapidly gaining prominence in the literature. They provide a useful framework to study agent responses in a context of unobservable variables complemented with signals. Thus, they have been used to analyse the impact of permanent and temporary shocks affecting a wide array of exogenous variables on agent decisions when they cannot distinguish perfectly between them. Several studies have reported on the impact of the introduction of this expectation mechanism compared to the usual full information one and, perhaps, the most interesting result of the comparison has been the appearance of a "hump-shaped" consumption impulse response function even with CRRA utility functions (the standard New Keynesian framework requires the assumption of habit formation in order to deliver the same result).

Other studies have used this framework to explore the relative importance of "real" versus "noise" shocks, the latter being considered as measurement error or forecast error amongst other interpretations.

Yet, we are not aware of many attempts to explore the consequences of higher volatility in either real or noise shocks. Intuitively, higher volatility of real shocks (permanent or temporary) implies a higher degree of uncertainty in future outcomes, whereas higher noise shock volatility could be interpreted as a decrease in forecast quality and/or precision.

This paper focuses on the effects of higher real or noise volatility and its impact on both long and short run dynamics. In particular, the effects of higher total factor productivity (TFP) variance will be explored, together with higher noise variance in the signal used to obtain information about the components of TFP.

In order to study the effects of volatility, known time-varying variances are added to a standard signal extraction problem. The agent only observes actual productivity and a signal; he must use this information to extract (i) the permanent component, (ii) the transitory component, and (iii) the noise term in the productivity process. This is a standard problem which has been considered by several authors. The innovation of this paper is that standard deviations of the three stochastic terms are allowed to change over time.

Given that the model used is a standard neoclassical growth model, it is known that an approximation that imposes certainty equivalence gives an accurate solution to the model, which might lead to thinking that this exercise is not interesting. However, certainty equivalence states that a change in the unconditional variance has no effect on the policy rule, it does not say that changes in the conditional variance have no effect. Also, it might be the case that things are different when considering changes in the variances of unobserved components (as is done here) compared to the case when you consider changes in the variance of observed components. This turns out to be the case.

Turning to related literature, the model explored here draws heavily from Blanchard et al. (2009). They present a simple consumption model where the random-walk result holds and then assume imperfect information in the form of unobservable variables coupled with signals delivering information about them. They show the consumer's signal extraction problem, solve it, and then proceed to evaluate the model empirically. Amongst other things, they show that an "econometrician" with no informational advantage to the agents cannot distinguish between news and noise shocks from the estimation of structural VAR's and that noise shocks play an important role in short-run fluctuations.

Blanchard et al. (2009) assume log productivity has a permanent (unit root) and transitory component. Productivity itself is perfectly observable but its components are not. In order to gain some information about the permanent component, a signal based on it is included in the agent's information set. This paper will employ the same set up, expanding the model to include all the pieces of a classic real business cycle, allowing general equilibrium analysis.

The signal extraction literature and the effect of volatility on the signal extraction problem goes all the way back to the seminal contribution of Lucas (1972). In his paper, Lucas proposes a model in which producers cannot distinguish between idiosyncratic shocks to their product's relative demand and aggregate shocks, stemming from monetary policy due to imperfect information. He shows that given that it is optimal for producers to change their production in response to the idiosyncratic shock but not the aggregate one, higher "noise" volatility (in the form of greater variance of aggregate -monetary- shocks) reduces the response of producers to any given shock (idiosyncratic or aggregate), effectively making the aggregate supply more inelastic.

Townsend (1983) proposes a model where firms try to distinguish between aggregate and market specific shocks using a recursive Kalman filter. Given that market specific shocks are independently drawn for each market, Townsend (1983) finds that there is information relevant to market $i$ in the economic variables characterizing market $j$. Thus, market participants not only need to solve a recursive Kalman filtering problem using the variables relevant to their market but must also incorporate the information contained in the forecasts elaborated in other markets. He finds that the equilibrium response to shocks in the model exhibits persistence, cross-correlation between markets and dampening (compared to the full-information equilibrium).

Sargent (1991) carries forward the work done by Townsend (1983), describing a method to solve for the equilibrium of a model in which agents are extracting signals from observations on endogenous variables. He imposes an assumption regarding the formulation of agent's forecasts (restricting them to ARMA processes) which allows him to solve for a symmetric equilibrium.

Crucially, Lucas (1972), Townsend (1983) and Sargent (1991) assume that the signal extraction problem arises from the need to distinguish an idiosyncratic shock from an aggregate shock. Townsend (1983) and Sargent (1991) compound the problem by recognizing that the idiosyncratic shock is drawn multiple times and thus there is information to be glimpsed from other agent's forecasts when performing our own. In our paper, the signal extraction problem will arise because of the impossibility to distinguish between permanent and transitory shocks, both of which are aggregate. Thus, the representative agent framework is preserved and there is no "forecasting the forecasts of others" problem as in Townsend (1983) and Sargent (1991).

Lorenzoni (2009) presents a model of business cycles driven by shocks to consumer expectations regarding aggregate productivity. Agents are hit by heterogeneous productivity shocks, they observe their own productivity and a noisy public signal regarding aggregate productivity. The public signal gives rise to "noise shocks", which have the features of aggregate demand shocks: they increase output, employment and inflation in the short run and have no effects in the long run.

The dynamics of the economy following an aggregate productivity shock are also affected by the presence of imperfect information: after a positive productivity shock output adjusts gradually to its higher long-run level, and there is a temporary negative effect on inflation and employment.

His paper explores the idea of expectation-driven cycles, looking at a model where technology determines equilibrium output in the long run, but consumers only observe noisy signals about technology in the short run. The presence of noisy signals produces expectation errors. The role of these expectation errors in generating volatility at business cycle frequencies constitutes the main result. The author is interested in the interaction of productivity and "noise" shocks in generating the business cycle, he endows the agent with a Kalman filter that is used to "learn" about the nature of the shocks.

Lorenzoni's work differs from Blanchard et al. (2009) in that the former assumes log productivity is the result of a "permanent" (unit root) process plus an i.i.d. component. Thus, his "noise" shock is initially mistaken for a real one, mechanism that drives his result. Blanchard et al. (2009) fail to include the extra i.i.d. component into productivity, essentially allowing some shocks to be perfectly identifiable.

Collard et al. (2009) provide a broad review of the effects of imperfect information on the business cycle. Using a New Keynesian framework, they introduce imperfect information in several different ways, affecting productivity or monetary policy. The objective of their paper is to estimate the impact of the imperfect information assumption in business cycle fluctuations and they conclude that it is *"quantitatively relevant, conceptually satisfactory and empirically plausible"*.

Hassler (1996) studies a model with time-varying uncertainty and irreversible investment. An increase in uncertainty increases the value of waiting, leading the agent to allow his capital to depreciate longer before investing to bring it back to its optimum level. He concludes that fluctuations in uncertainty of moderately high frequency could potentially have substantial effects on the timing and volatility of demand, providing a potential explanation for business cycle fluctuations.

Bloom (2009) shows that several cross-sectional dispersion measures for firms are countercyclical. He proposes a model with non-convex capital and labor adjustment costs where an increase in uncertainty leads to a result similar to Hassler (1996): firms temporarily pause their investment and hiring, leading to a rapid drop and posterior overshoot in aggregate output and employment. Thus, his uncertainty shocks generate short, sharp recessions and posterior recoveries.

Similar to Hassler (1996) and Bloom (2009), this paper finds that volatility shocks have a negative short-run impact on output. Additionally, we attempt to explore how volatility shocks interact with imperfect information: our agents solve a signal extraction problem in order to estimate the permanent and transitory components of TFP. We find that when incorporating imperfect information in the model, higher volatility of transitory shocks has an even larger (negative) effect on output while higher volatility of permanent shocks has a smaller effect. The reason appears to be that higher volatility of transitory shocks, ceteris paribus, makes it harder for the agent to identify the really important shocks (that is, the permanent ones).

Clearly, the vast majority of work done on imperfect information models with signal extraction has not attempted to study the effects of the process or signal noise volatility.

Senhadji (2000) attempts to estimate TFP for several country groups over different time periods. He reports that log productivity volatility differs significantly between country groups and over time.

Evidence of volatility shocks, fluctuations in the standard deviation of TFP over time, is hard to find. Arias et al. (2007) report a decrease in the volatility of TFP shocks post-1983 for the U.S. and show that this is a major factor explaining the reduced cyclical volatility of output and its components that can be observed since 1983. They do not report time series for TFP volatility though.

Given the scarcity of current literature on this topic, we motivate time-series volatility shocks by constructing a measure of TFP volatility. Fernald (2012) describes a methodology to construct a quarterly time series of TFP growth for the U.S. and has made it publicly available through the website of the Center for the Study of Income and Productivity at the Federal Reserve Bank of San Francisco.[1] Figure 1-1 shows the standard deviation (calculated over a window of 12 quarters) of his TFP growth series and its trend (calculated using a Hodrick-Prescott filter) where quarter to quarter growth has been annualized. It is evident in the data that there are significant deviations from trend (Figure 1-2) which we associate with our concept of volatility shocks (note the correspondence is not perfect because our concept of dispersion relates to the cross-section variance rather than its time-series counterpart). It is our claim that there is enough time-series fluctuation in TFP volatility to justify the exploration of its consequences in the short run (business cycle).[2]

In order to address the question, we will construct a simple real business cycle model with a consumer-producer agent who faces imperfect information regarding the components of productivity and deals with the problem by applying a Kalman filter to the information at his disposition ("total" log productivity and a signal regarding its permanent component).

[1] The series can be downloaded from: `http://www.frbsf.org/csip/research/tfp/quarterly_tfp.xls`

[2] Incidentally, the data in Fernald (2012) seems to support the claim made by Arias et al. (2007) that TPF shocks have become less volatile post-1983 in the U.S.

## 1.2 The Model

The model is largely based on a standard real business cycle structure with a representative agent in charge of consumption and production decisions. The agent's signal extraction problem will generate all expectations required to solve the optimization problem he faces. Output is produced according to:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}. \tag{1.1}$$

Capital accumulation follows the usual definition:

$$K_{t+1} = I_t + (1-\delta) K_t. \tag{1.2}$$

We will assume the only asset available in the economy is physical capital implying the following aggregate resource constraint:

$$Y_t = C_t + I_t. \tag{1.3}$$

The agent's objective is to maximize the (subjective) present discounted value of utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U\left(C_t\right) - \frac{L_t^{1+\eta}}{1+\eta} \right\}; \quad U\left(C_t\right) = \frac{C_t^{1-\gamma}}{1-\gamma}. \tag{1.4}$$

Thus, using standard solution methods it can be shown the agent must choose consumption according to the following Euler equation:

$$U'\left(C_t\right) = \beta E_t \left[ U'\left(C_{t+1}\right) \left( \alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + 1 - \delta \right) \right]. \tag{1.5}$$

where $\alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha}$ is next period's marginal product of capital ($MPK_{t+1}$).

Labour supply will be the result of equating marginal disutility of labour to its marginal product expressed in consumption units:

$$L_t^{\eta} = (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha} U'(C_t). \tag{1.6}$$

Following Blanchard et al. (2009), we will assume the log of TFP, $a_t = \ln A_t$, is the sum of two components:

$$a_t = x_t + z_t. \tag{1.7}$$

The permanent component, $x_t$, follows a unit root process of the form:

$$\Delta x_t = \rho_x \Delta x_{t-1} + \epsilon_t. \tag{1.8}$$

thus, positive $\epsilon_t$ shocks gradually push log productivity ($a_t$) towards a new (higher) steady state (see the first panel of Figure 1-3).

The transitory component, $z_t$, follows a stationary AR(1) process of the form:

$$z_t = \rho_z z_{t-1} + \eta_t. \tag{1.9}$$

implying positive $\eta_t$ shocks temporarily increase log productivity and eventually die out (see the first panel of Figure 1-4).

Both parameters $\rho_x$ and $\rho_z$ are in $(0, 1)$, and $\epsilon_t$ and $\eta_t$ are zero-mean shocks with unconditional variance $\sigma_\epsilon^2$ and $\sigma_\eta^2$ respectively.

In order to introduce volatility shocks, the conditional variances of $\epsilon_t$ and $\eta_t$ must vary over time and have some degree of autocorrelation. The GARCH(1,1)[3] framework lends itself naturally to our purposes:

$$\sigma_{j,t}^2 = (1 - \alpha_1 - \beta_1)\sigma_j^2 + \alpha_1 \sigma_{j,t-1}^2 + \beta_1 j_{t-1}^2 \tag{1.10}$$

for $j = \{\epsilon, \eta, \nu\}$.[4] Note that coefficients $\alpha_1$ and $\beta_1$ control the speed of adjustment of

[3] Generalized AutoRegressive Conditional Heteroscedasticity model with one autoregressive (AR) component and one moving average (MA) component.

[4] $\sigma_{\nu,t}^2$ is the conditional variance of the noise shock $\nu_t$ which will be introduced formally later on.

conditional variance $\sigma^2_{j,t}$ to the unconditional variance $\sigma^2_j$.

Given this data-generating process for $\sigma^2_{j,t}$, our volatility "shocks" will be modelled as a one time increase (or decrease) in $j_{t-1}$ that pushes $\sigma^2_{j,t}$ above its steady state value (the unconditional variance $\sigma^2_j$) while ignoring the impact of $j_{t-1}$ on the components of TFP. The rationale behind this strategy is that a shock to $j_t$ would imply additional (first order) effects that we are trying to avoid. An alternative would be to simulate separately two shocks of the same magnitude but opposite sign ($j_t$ and $-j_t$) and define the impulse response to a volatility shock as half the sum of their impulse responses (which should be non-zero after the level effects cancel out). Note though that in this case we would only obtain an impact on $\sigma^2_{j,t+1}$, not $\sigma^2_{j,t}$.

Agents observe productivity $a_t$ but not the individual components. For the sake of analytical convenience, log productivity will be assumed to follow a random walk:

$$a_t = a_{t-1} + u_t, \tag{1.11}$$

with the unconditional variance of $u_t$ equal to $\sigma^2_u$. Thus, certain restrictions on the parameters of its components must be imposed to guarantee consistency with the breakdown postulated earlier (equation (1.7)). In particular,

$$\rho_x = \rho_z = \rho, \tag{1.12}$$

and

$$\sigma^2_\epsilon = (1-\rho)^2 \sigma^2_u, \quad \sigma^2_\eta = \rho\sigma^2_u, \tag{1.13}$$

for some $\rho$ in $(0,1)$ are sufficient conditions to guarantee both (1.7) and (1.11) hold.

## 1.3 Model Solution and Calibration

The key to solving the agent's problem lies in the formulation of:

$$E_t\left[U'\left(C_{t+1}\right)\left(\alpha A_{t+1}K_{t+1}^{\alpha-1}L_{t+1}^{1-\alpha}+1-\delta\right)\right]. \tag{1.14}$$

In the traditional RBC model, a policy function in terms of state variables $K_t$ and $A_t$ can be used to calculate this expectation. In the set up proposed in this paper, this is far from enough.

Aggregate productivity does not provide enough information to solve the consumer's problem. Intuitively, the optimal response to a permanent shock $\epsilon_t$ (that affects permanent process $x_t$) should be different than the response to a transitory shock $\eta_t$[5] (affecting process $z_t$). The problem is that our agent cannot observe the $\epsilon_t$ and $\eta_t$ shocks nor the actual processes $x_t$ and $z_t$. He has to use available information in order to estimate them. We will assume our agent does this using a recursive (time-varying) Kalman Filter.

The agent will have to solve a signal extraction problem by means of the Kalman filter. Following Blanchard et al. (2009) again, each period the agent observes current productivity $a_t$ and receives a signal, $s_t$, which provides noisy information regarding the permanent component of the productivity process:

$$s_t = x_t + \nu_t, \tag{1.15}$$

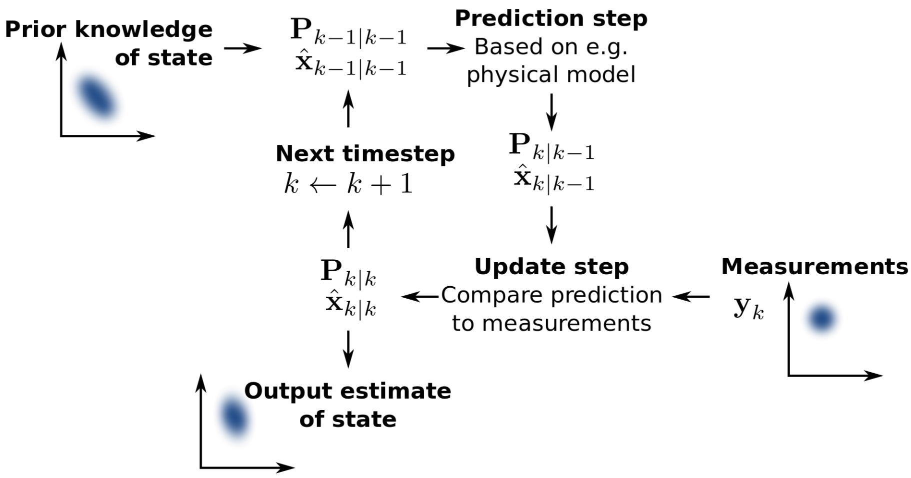
where $\nu_t$, our noise shock, has zero-mean and unconditional variance $\sigma_v^2$ (the conditional variance of this shock follows a GARCH(1,1) process as well).

Note that without the signal it would be impossible for the agent to decompose changes in productivity between permanent and temporary shocks. Furthermore, the agent knows the model behind productivity in detail: the particular functional forms involved and parameter values ($\rho$ and the conditional and unconditional variance of the three shocks).

Thus, the agent enters period $t$ with his knowledge of the model plus beliefs formed last period regarding current productivity $\left(x_{t|t-1}, x_{t-1|t-1}, z_{t|t-1}\right)$, observes productivity and the signal $(a_t, s_t)$ and uses all that information to update his expectations using the Kalman filter. The following figure[6] illustrates the process quite well:

[5] The optimal response to a noise shock $\nu_t$ would be zero since noise has no effect on actual productivity.

[6] Created by Petteri Aimonen, taken from: en.wikipedia.org/wiki/Kalman_filter. The equations used




Note that prior knowledge of the state includes matrix $P$ which captures the degree of precision of the estimates $\left(x_{t|t-1}, x_{t-1|t-1}, z_{t|t-1}\right)$. Formally, $P_{t|t-1}$ is the variance of the prediction error $\left(x_t - x_{t|t-1},\, x_{t-1} - x_{t-1|t-1},\, z_t - z_{t|t-1}\right)$.

In Blanchard et al. (2009), matrix $P$ is time invariant and can be found solving a Ricatti equation implied by the Kalman filter. Then, prediction of the state can be obtained using,

$$\begin{bmatrix} x_{t|t} \\ x_{t-1|t} \\ z_{t|t} \end{bmatrix} = A \begin{bmatrix} x_{t-1|t-1} \\ x_{t-2|t-1} \\ z_{t-1|t-1} \end{bmatrix} + B \begin{bmatrix} a_t \\ s_t \end{bmatrix} \tag{1.16}$$

where the matrices $A$ and $B$ depend on parameters of the model.

In the case being studied here, matrix $P$ changes over time because of changes in volatility. Intuitively speaking, higher shock variance affects the precision of the Kalman filter estimations over time. Thus, matrices $A$ and $B$ above are time dependent and the problem has to be solved using a time-varying Kalman filter. In order to do this, a history of the shocks is simulated (consistent with (1.10)) and fed to the Kalman filter together with the history of conditional variances. The filter then returns a history of estimated productivity components $\left(x_{t|t}, z_{t|t}\right)$ complete with precision estimates $P_{t|t}$.[7]

What variables should be included in the policy function? It is fairly obvious that

to implement the time-varying Kalman filter can be found in the appendix and in several texts including Hamilton (1994).

[7] A seed value is required to start the filter: a null vector is used for the component estimates and the invariant $P$ from the Ricatti equation for the initial precision estimate.

current capital $K_t$ should be there and the estimated productivity components $\left(x_{t|t}, z_{t|t}\right)$ as well. Other candidates include the precision estimates $P_{t|t}$ and the conditional shock volatilities ($\sigma^2_{\epsilon,t}$, $\sigma^2_{\eta,t}$ and $\sigma^2_{\nu,t}$). The problem with the last two options is that they overlap each other to a certain extent in the sense that they contain basically the same information. Thus, given the focus of the paper, we will pick the conditional shock volatilities. Note that including them in the state variable set is akin to the assumption that they are observable by the agent. This assumption is consistent with the fact that the Kalman filter takes the conditional variances as input (implying they are known). Intuitively speaking, our agent is aware of how uncertain productivity is at every point in time but cannot discover the true level of the productivity components.

Once we have selected the variables to include in the policy function we need to actually characterize it. In order to do this, a modified version of the parametrized expectations (PEA) algorithm of den Haan and Marcet (1990) is used.

In den Haan and Marcet (1990), expectation (1.14) is estimated using a history of shocks and an assumption regarding the functional form of the expectation in terms of the state variables. For our purposes, the analogue would be:

$$
\begin{aligned}
&E_t\left[U'\left(C_{t+1}\right)\left(\alpha A_{t+1}K_{t+1}^{\alpha-1}L_{t+1}^{1-\alpha}+1-\delta\right)\right] \\
&\approx \exp\Big(\theta_1+\theta_2\left(\log\left(K_t\right)-\log\left(K_{ss}\right)\right)+\theta_3 x_{t|t}+\theta_4 z_{t|t}+\theta_5 x_{t|t}^2+\theta_6 z_{t|t}^2 \\
&+\theta_7 x_{t|t}z_{t|t}+\theta_8\Big(\left(\log\left(K_t\right)\right)^2-\left(\log\left(K_{ss}\right)\right)^2\Big)+\theta_9\log\left(K_t\right)x_{t|t} \\
&+\theta_{10}\log\left(K_t\right)z_{t|t}+\theta_{11}\left(\sigma^2_{\epsilon,t}-\sigma^2_{\epsilon,ss}\right)+\theta_{12}\left(\sigma^2_{\eta,t}-\sigma^2_{\eta,ss}\right)+\theta_{13}\left(\sigma^2_{\nu,t}-\sigma^2_{\nu,ss}\right) \\
&+\theta_{14}\sigma^2_{\epsilon,t}x_{t|t}+\theta_{15}\sigma^2_{\epsilon,t}z_{t|t}+\theta_{16}\sigma^2_{\eta,t}x_{t|t}+\theta_{17}\sigma^2_{\eta,t}z_{t|t}+\theta_{18}\sigma^2_{\nu,t}x_{t|t} \\
&+\theta_{19}\sigma^2_{\nu,t}z_{t|t}\Big) \qquad (1.17)
\end{aligned}
$$

imposing the restriction

$$
\theta_1=\log\left(U'\left(C_{ss}\right)\left(\alpha A_{ss}K_{ss}^{\alpha-1}L_{ss}^{1-\alpha}+1-\delta\right)\right) \qquad (1.18)
$$

where the subscript "$ss$" refers to the non-stochastic steady state.

Restriction (1.18) is necessary in order to guarantee that all impulse response functions converge to the non-stochastic steady state. This is analogous to imposing that the approximate solution to the model be calculated in a neighbourhood of the non-stochastic steady state. For the same reason, capital is always included as a deviation from its non-stochastic steady state value and unconditional variances are introduced in deviations from their steady state.

An alternative to restriction (1.18) would be to allow the parameterized expectations algorithm to estimate $\theta_1$ and introduce all variables in non-deviation form. In that case, the approximated solution would converge to the stochastic steady state of the parameterized expectation. Given that the latter is notoriously difficult to find, constructing impulse response functions (which are usually presented in terms of deviations from steady state) would imply a serious challenge. Thus, the imposition of restriction (1.18) and the inclusion of the deviations from steady state values is basically a modelling device to enhance analytical tractability. It forces our (approximated) solution to converge to the non-stochastic steady state. Bearing in mind that doing approximations around the non-stochastic steady state is standard practice, we believe this is an acceptable simplification.

The vector of $\theta's$ is estimated by iteration until convergence is achieved.[8] Alternative specifications of the parametrized expectation were explored in order to check for stability of the estimates (adding $P's$ and $\sigma's$, just $P's$, and others).

Equations (1.1) - (1.3) and (1.6) together with the parametrized Euler equation and the Kalman filter results form a system where all variables are driven by productivity, the signal and volatilities (from the agent's point of view) which, in turn, ultimately depend on the history of the three shocks (from the researcher's point of view).

Turning to the model calibration, slightly non-standard values will be used for $\alpha$, $\beta$, $\gamma$, $\delta$ and $\eta$, transformed to their quarterly equivalents as shown in Table 1.1 (quarterly-adjusted values have been rounded). The reason is that for this particular set of parameters, den Haan and Marcet (1990) provide good starting values to initialize the PEA algorithm (reducing the time required to perform calculations).

For the parameter governing the relative importance of permanent versus temporary

[8]With the exception of $\theta_1$ which is always fixed.

Table 1.1: Calibration of RBC parameters

| Parameter | Reference value | Quarterly-adjusted value |
|---|---|---|
| $\alpha$ | 0.33 | 0.33 |
| $\beta$ | 0.96 | 0.99 |
| $\gamma$ | 0.95 | 0.95 |
| $\delta$ | 0.05 | 0.01 |
| $\eta$ | 2 | 2 |

shocks, $\rho$, a value of 0.89 will be taken, using the estimation results of Blanchard et al. (2009). This will imply permanent shocks that build up slowly (see the first panel of Figure 1-3) and temporary shocks that take a long time to decay (see the first panel of Figure 1-4). The parameters governing the evolution of conditional variance $\alpha_1$ and $\beta_1$ are set at 0.7 and 0.2 respectively so that variance shocks decay slowly over time.

The only parameters left are the unconditional variances of the three shocks. These will be the steady state values of the conditional variances and they are calibrated to be consistent with Blanchard et al. (2009) as shown in Table 1.2.

Table 1.2: Variance of shocks

| Parameter | Steady State |
|---|---|
| $\sigma_u$ | 0.67% |
| $\sigma_\epsilon$ | 0.07% |
| $\sigma_\eta$ | 0.63% |
| $\sigma_\nu$ | 0.89% |

Senhadji (2000) provides estimates of the standard deviation of total factor productivity for several country groups over the 1960-1994 period. His average for "Industrial Countries" is fairly close to the value reported by Blanchard et al. (2009) (which is for the US only). Furthermore, he finds that yearly $\sigma_u$ for Middle East and North Africa is roughly 5.7% (the highest volatility he reports); that value will be used approximately for the $\sigma_\epsilon$ and $\sigma_\eta$ shock: for simplicity, we have constructed it as an increase in standard deviation by a factor of 2. Similarly the noise variance shock has been constructed by doubling the magnitude of $\sigma_\nu$. Note that this assumption is consistent with some of the extreme values observed in the time-series standard deviation of quarterly business sector TFP growth reported in Figure 1-1 for the last decade (over the last ten years, the minimum volatility has been 1.56 and the maximum 3.75).

Using all of the above, PEA yields estimates for the coefficients of the parametrized expectation. These are shown in Table 1.3.

Table 1.3: Estimated coefficients for the parametrized expectation

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| $\log(k_t) - \log(k_{ss})$ | -0.3905 | -0.2327 | -0.3305 | -0.4431 | -0.2924 |
| $x_t$ or $x_{t\|t}$ * | -0.8626 | -1.0928 | -0.9445 | -0.7608 | -0.9987 |
| $z_t$ or $z_{t\|t}$ * | -0.3222 | -0.2106 | -0.5155 | -0.2853 | -0.4238 |
| $x_t^2$ or $x_{t\|t}^2$ * | -0.0219 | -0.0594 | -0.0434 | -0.0001 | -0.0482 |
| $z_t^2$ or $z_{t\|t}^2$ * | 0.0428 | 0.0694 | -0.0408 | -0.1355 | -0.1363 |
| $x_t z_t$ or $x_{t\|t} z_{t\|t}$ * | -0.0756 | -0.0340 | -0.1405 | -0.0624 | -0.1037 |
| $(\log(k_t))^2 - (\log(k_{ss}))^2$ | -0.0073 | -0.0251 | -0.0180 | -0.0011 | -0.0205 |
| $\log(k_t)\, x_t$ or $\log(k_t)\, x_{t\|t}$ | 0.0256 | 0.0773 | 0.0557 | 0.0015 | 0.0625 |
| $\log(k_t)\, z_t$ or $\log(k_t)\, z_{t\|t}$ | 0.0466 | 0.0183 | 0.0934 | 0.0373 | 0.0622 |
| $\sigma_{\epsilon,t}^2 - \sigma_{\epsilon,ss}^2$ | | | | -306.33 | -97.449 |
| $\sigma_{\eta,t}^2 - \sigma_{\eta,ss}^2$ | | | | -1.9110 | -3.3789 |
| $\sigma_{\nu,t}^2 - \sigma_{\nu,ss}^2$ | | | | | -0.5049 |
| $\sigma_{\epsilon,t}^2 x_t$ or $\sigma_{\epsilon,t}^2 x_{t\|t}$ * | | | | -24.312 | 62.066 |
| $\sigma_{\epsilon,t}^2 z_t$ or $\sigma_{\epsilon,t}^2 z_{t\|t}$ * | | | | 16858 | -35162 |
| $\sigma_{\eta,t}^2 x_t$ or $\sigma_{\eta,t}^2 x_{t\|t}$ * | | | | 2.0525 | -0.4931 |
| $\sigma_{\eta,t}^2 z_t$ or $\sigma_{\eta,t}^2 z_{t\|t}$ * | | | | 119.11 | 445.24 |
| $\sigma_{\nu,t}^2 x_t$ or $\sigma_{\nu,t}^2 x_{t\|t}$ * | | | | | -1.2809 |
| $\sigma_{\nu,t}^2 z_t$ or $\sigma_{\nu,t}^2 z_{t\|t}$ * | | | | | -233.09 |

* Second term applies to the model of column (5) only.

In order to gain insight into the mechanisms operating in the model, five different versions of it are solved using PEA. The results for each model are shown in a separate column in Table 1.3. The fifth column contains the coefficients of the parametrized expectation defined in equation (1.17). The other four columns show coefficients for full information variants of the model defined as follows:

1. Base specification with constant variances as shown in Table 1.2.
2. High $\sigma_\epsilon^2$ (twice the base value) with constant variances.
3. High $\sigma_\eta^2$ (twice the base value) with constant variances.
4. Time-varying volatilities.[9]

## 1.4 Results

In spite of the slight differences in the coefficients between columns (1) - (3) of Table 1.3, the impulse response functions generated by those three models are virtually identical. This verifies the well known result that changes in the unconditional variance

[9]The terms relating to the noise shock volatility $\sigma_{\nu,t}^2$ are left out because the noise component of the signal (and the signal itself, actually) is irrelevant in a context of full information.

have no quantitatively relevant impact on the policy function that characterizes the solution to the real business cycle model.

However, changes over time in conditional variances do have an impact on the short run dynamics of the model, as shown by the coefficients of columns (4) and (5).

Generally speaking, the agent's response to the permanent component of productivity (actual or estimated through the Kalman filter) is always bigger than the one generated by changes in the transitory component.

The coefficients associated to the conditional variances show that the agent's response to a volatility shock is qualitatively the same no matter which volatility changes: the only differences seem to be of magnitude. In particular, note that the coefficient corresponding to $\sigma_\epsilon^2$ is very high because the magnitude of this variance is much smaller than that of $\sigma_\eta^2$ or $\sigma_\nu^2$.

Figures (1-3) to (1-8) show impulse response functions for the permanent ($\epsilon$), transitory ($\eta$), noise ($\nu$), $\sigma_\epsilon^2$, $\sigma_\eta^2$, and $\sigma_\nu^2$ shocks over 20 quarters for both the full information model in column (4) of Table 1.3 and the imperfect information model in column (5).

When the permanent shock takes place, consumption increases right from the outset. This is basically the agent's response to the wealth component of the shock. Since productivity is factor-neutral, labour and capital increase as well (the latter much more slowly given the necessary build up of investment through time). Thus, there seems to be a substitution of labour for capital in the short run (the agent works more while capital accumulates). In the imperfect information case, not all the observed variation in log productivity is attributed to the permanent component when the shock hits. In fact, the agent believes roughly 60% of the shock is transitory. As time goes by, the agent gradually realizes the true nature of the shock, increasing consumption slowly. This explains the relevant differences in the shape of investment and labour in the imperfect information case: the agent tries to build up as much capital as possible during the (wrongly) perceived transitory shock. Output dynamics basically follow those of productivity.

[Figure 1-3 about here]

The transitory shock tells a different story. With imperfect information, the agent almost identifies the true nature of the shock (roughly 90% of the variation is attributed to the transitory component). Thus, the agent proceeds to smooth out the benefits of the shock (although not as much as in the full information case). Consumption and labour response at time zero is mild, but investment response is relatively high (about 3.8% increase). The idea will be to accumulate capital in order to sustain higher output for as long as possible to allow for a longer-lasting consumption increase.

[Figure 1-4 about here]

The impulse response function derived from a signal noise shock delivers unexpected results.

[Figure 1-5 about here]

Differing from Blanchard et al. (2009), the response of consumption and output is very small. The biggest impulse obtained, that of investment, falls 1% right at the moment the shock hits. The contradiction can be explained by comparing Euler equations. Blanchard et al. (2009) impose an Euler equation consistent with a random walk, $c_t = E_t[c_{t+1}]$ and drastically simplify the supply side to obtain the following consumption function in equilibrium:

$$c_t = \lim_{j \to \infty} E_t \left[ a_{t+j} \right].$$

Note that this implies consumption will depend on the expected limit value of the $a_t$ process alone. Since any transitory shock eventually dies out, that expected value will depend solely on the current expectations of the permanent component of productivity:

$$c_t = x_{t|t} + \frac{\rho}{1-\rho} \left( x_{t|t} - x_{t-1|t} \right).$$

On the other hand, the model presented in this paper results in an Euler equation that predicts consumption depends on the entire expected present and future history of log productivity: the transitory component matters. Iterating on the Euler equation (1.5):

$$U'(C_t) = E_t \left[ \prod_{i=0}^{\infty} \{ \beta ( MPK_{t+1+i} + 1 - \delta ) \} \lim_{\tau \to \infty} U'(C_{t+\tau}) \right]$$

The current marginal utility of consumption depends on i) all present and future values of the marginal product of capital and ii) the long-run marginal utility of consumption. Shocks to the permanent component of productivity have an impact on both factors but transitory shocks will only have an effect on the first factor.

Thus, Blanchard et al. (2009) report consumption impulse responses that depend solely on the evolution of $x_{t|t}$ and $x_{t-1|t}$ (for the permanent shock these estimates change obviously; for the transitory shock, recall that the agent erroneously interprets part of the transitory shock as a permanent one when the shock hits and only "learns" the truth gradually). This has no significant impact on the qualitative aspect of the response to permanent and transitory shocks, but makes a big difference when it comes to signal noise shocks.

When a signal noise shock hits this system, the signal jumps, but observed log productivity does not change. Given that the Kalman filter is always trying to "split" observed changes in these variables (into the permanent and transitory components of productivity), the signal jump of $\sigma_\nu$ magnitude will be interpreted as an increase in the permanent component mirrored by a decrease in the transitory component, both with magnitude equal to roughly a quarter of $\sigma_\nu$. Since Blanchard et al. (2009) have consumption depending on the permanent component only, they obtain a non-trivial impulse response, but that will not be the case for the model developed in this paper: the signal noise shock will generate insignificant responses given that the permanent and transitory components move in opposite directions and the agent learns fairly quickly that the signal jump had no "real" basis. Still, Figure 1-5 shows that the agent's response to the noise shock is reasonable: the initial increase in consumption and decrease in labour would indicate an attempt to spread out the benefits of a non-existent positive shock to the future. Given that productivity has not changed, the increase in consumption has to be financed by cutting down investment. The mistake makes output fall below steady state (as capital and labour fall) and the situation is corrected (with lower consumption and higher investment and labour) fairly quickly.

Turning to the analysis of variance shocks (Figures 1-6, 1-7 and 1-8), it turns out that they all generate qualitatively similar responses. Apparently, the agent's point of view is that an increase in uncertainty, wherever its procedence may be, is always the same. Taking the $\sigma_\eta^2$ shock as an example (Figure 1-7), the increase in variance reduces output by 0.012% as a result of lower labour in the imperfect information case. Investment falls as well (0.3% when the shock hits) and this triggers a fall in capital. Consumption rises mildly (0.04%) and falls back quickly.

The increase in $\sigma_\eta^2$ reduces the agent's expected return from an extra unit of savings, $E_t \left[U'\left(C_{t+1}\right)\left(MPK_{t+1} + 1 - \delta\right)\right]$ (see Table 1.3). Given the Euler equation (1.5), the current marginal utility of consumption, $U'(C_t)$ must fall as well. Given the lower incentives to save, the agent reduces his savings (and therefore investment) and uses the resources freed up to increase consumption.

[Figure 1-7 about here]

Note that the $\sigma_\eta^2$ shock is very similar, qualitatively, to the noise shock ($\nu$). The intuition is fairly straightforward: noise shocks decrease the ability of the agent to estimate the components of productivity correctly (through the Kalman filter). This is exactly the same thing the variance shock does.

Another important observation is the fact that imperfect information seems to dampen the $\sigma_\epsilon^2$ shock while amplifying the $\sigma_\eta^2$ shock. With imperfect information, the agent can no longer observe the components of TFP. Greater $\sigma_\eta^2$ will imply that in the short run, a higher proportion of the (unobserved) future shocks will be temporary. On the other hand, greater $\sigma_\epsilon^2$ will imply that a higher proportion of future shocks will be permanent. Our agent has a negative response to uncertainty in general (thus the fall in investment) but he has a preference regarding the kind of uncertainty he dislikes the most when in an imperfect information setting: that of temporary shocks. We claim the reason behind this is that higher volatility of temporary shocks reduces his ability to identify the important (permanent) ones in the signal extraction problem.

Why is the impact of a variance shock so small? Generally speaking, the magnitude of the change in $\sigma_u^2$ we use is too small to generate a significant response in output. Apparently, our TFP volatility shocks are too small but, given the estimates reported in Senhadji (2000) it might be the case that these shocks are larger in other countries with higher average TFP volatility compared to the U.S.

Still, the model presented seems to deliver a strong conclusion regarding the effects of TFP volatility: variance shocks seem to be relevant for the short run behaviour of certain macroeconomic aggregates (such as investment). The model used might seem to indicate they are quantitatively small but this statement must be qualified by the fact we restrict the analysis to a real business cycle framework. Incorporating frictions and other bells and whistles to the model could potentially amplify the effects.

## 1.5 Conclusion

An imperfect information model with an agent facing a signal extraction problem has been developed, solved and calibrated. Then, changes in process and signal noise volatility of productivity are introduced and their impact studied together with more traditional permanent, transitory and noise shocks to productivity.

Permanent and transitory level shocks have similar effects to those shown in Blanchard et al. (2009). Noise shocks are much smaller given that the agent responds to the (negative) estimated transitory component that results coupled with the (positive) estimated permanent component.

The model clearly shows that process variance of all types has a small short run impact on the economy's real variables even though we isolate it from the (usually) associated level shocks. Given that both positive and negative level shocks can potentially be coupled with higher volatility, this would be a source of asymmetry in the business cycle.

The method used to solve the model under imperfect information (a time-varying Kalman filter coupled with the parametrized expectations algorithm of den Haan and Marcet (1990)) is new to the best of the author's knowledge and should be easily extendable to more realistic models capable of amplifying the responses reported here.

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# A. Appendix

## A.1 Additional Figures

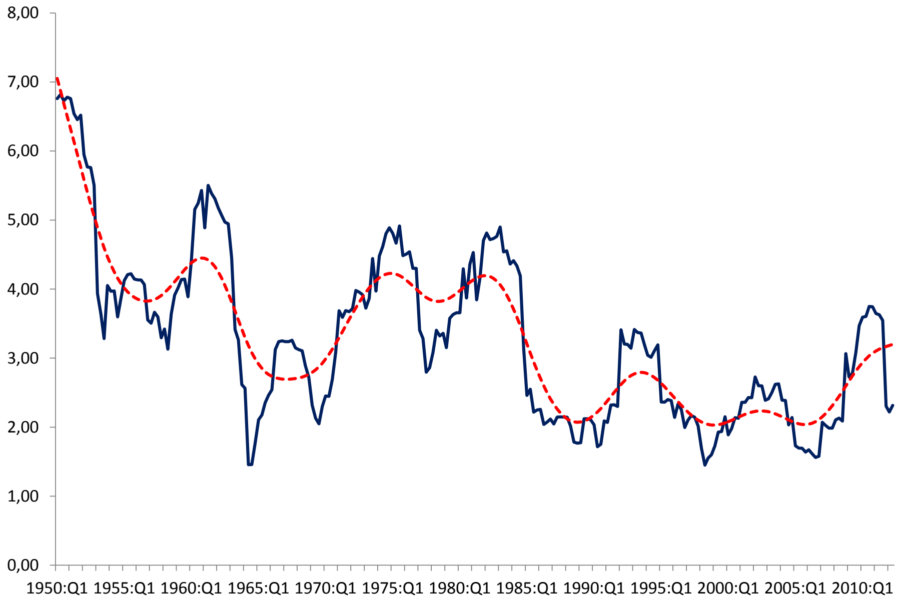


Figure 1-1: Standard deviation of quarterly business sector TFP growth (solid) and its HP-trend (dashed, $\lambda = 1600$).

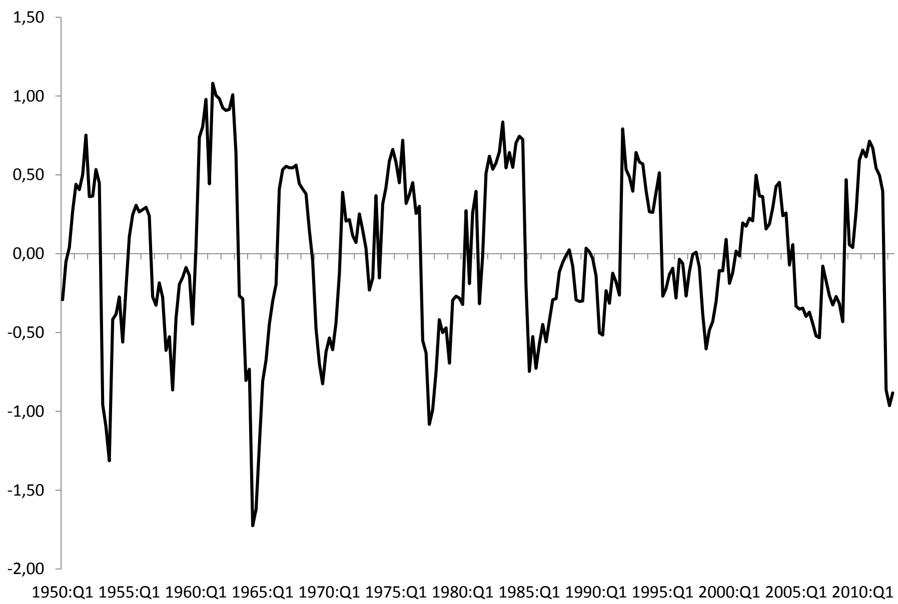


Figure 1-2: Cycle component of the standard deviation of quarterly business sector TFP growth

## A.2 A time-varying Kalman filter for unobservable permanent and transitory shocks

Consider an I(1) unobserved process given by

$$x_t = (1+\rho)\, x_{t-1} - \rho x_{t-2} + \epsilon_t,$$

and an I(0) process

$$z_t = \rho z_{t-1} + \eta_t.$$

We assume $x_t$ and $z_t$ are unobservable but their sum,

$$a_t = x_t + z_t,$$

which we call "log productivity" is observable.

Even though he cannot observe the components directly, the agent receives a signal

$$s_t = x_t + \nu_t,$$

containing information regarding the "permanent" process $x_t$.

Following standard notation in the filtering literature, this set up can be rewritten as follows:

*State process*

$$\xi_t = F\xi_{t-1} + w_t$$

$$w_t \sim N\left(0, Q_t\right)$$

where

$$\xi_t \equiv \begin{bmatrix} x_t \\ x_{t-1} \\ z_t \end{bmatrix}, F \equiv \begin{bmatrix} 1+\rho & -\rho & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho \end{bmatrix},$$

$$w_t \equiv \begin{bmatrix} \epsilon_t \\ 0 \\ \eta_t \end{bmatrix}, Q_t \equiv \begin{bmatrix} \sigma^2_{\epsilon,t} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\eta,t} \end{bmatrix}$$

and

*Measurement process*

$$m_t = H\xi_t + u_t$$

$$u_t \sim N\left(0, R_t\right)$$

where

$$m_t \equiv \begin{bmatrix} a_t \\ s_t \end{bmatrix}, H \equiv \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, u_t \equiv \begin{bmatrix} 0 \\ \nu_t \end{bmatrix}, R_t \equiv \begin{bmatrix} 0 & 0 \\ 0 & \sigma^2_{\nu,t} \end{bmatrix}.$$

The Kalman filter algorithm starts by predicting the current state $\widehat{\xi}_{t|t-1}$ and estimate precision $P_{t|t-1}$ before observing the measurement:

$$\widehat{\xi}_{t|t-1} = F\widehat{\xi}_{t-1|t-1}$$

$$P_{t|t-1} = FP_{t-1|t-1}F^T + Q_t$$

Then, information from the measurement is incorporated into the estimate.

Measurement residual:

$$\widetilde{y}_t = m_t - H\widehat{\xi}_{t|t-1}$$

Residual covariance:

$$S_t = HP_{t|t-1}H^T + R_t$$

Optimal Kalman gain:

$$K_t = P_{t|t-1}H^T S_t^{-1}$$

Updated (after measurement) state estimate:

$$\widehat{\xi}_{t|t} = \widehat{\xi}_{t|t-1} + K_t\widetilde{y}_t$$

Updated estimate precision:

$$P_{t|t} = (I - K_t H)\, P_{t|t-1}.$$

Given initial values $\widehat{\xi}_{0|0}$ and $P_{0|0}$, simulated series ($x_t$, $z_t$, $a_t$, $s_t$, $Q_t$ and $R_t$) can be used to generate the full history of estimates $\widehat{\xi}_{t|t}$ and $P_{t|t}$. In the model presented in the main body of this paper, the agent's policy function is parametrized using known capital $K_t$, estimates $\widehat{\xi}_{t|t}$ and variances $Q_t$ and $R_t$.

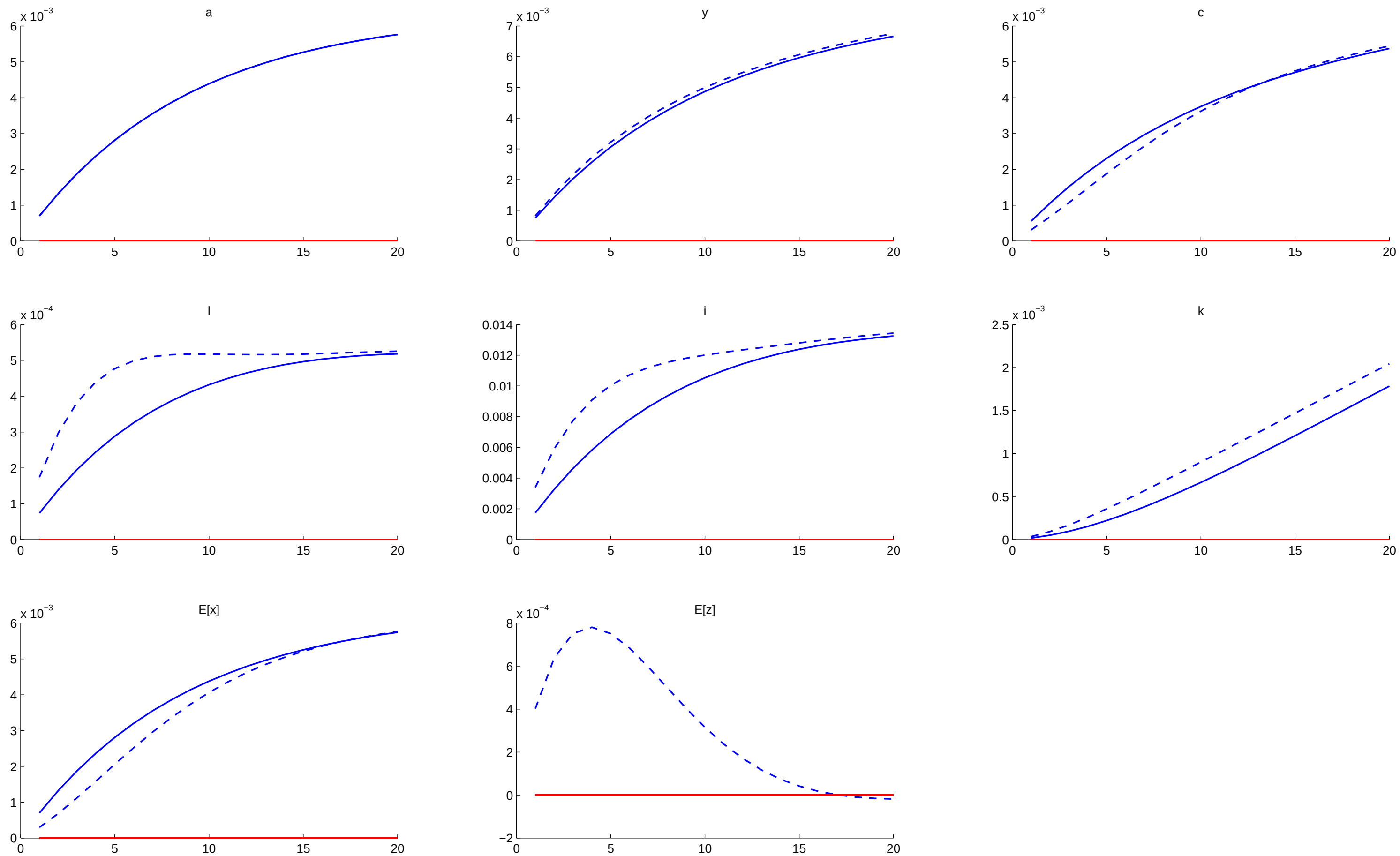


Figure 1-3: Impulse response functions for a permanent shock of one s.d. (0.07%) Full information: solid, Imperfect information: dashed

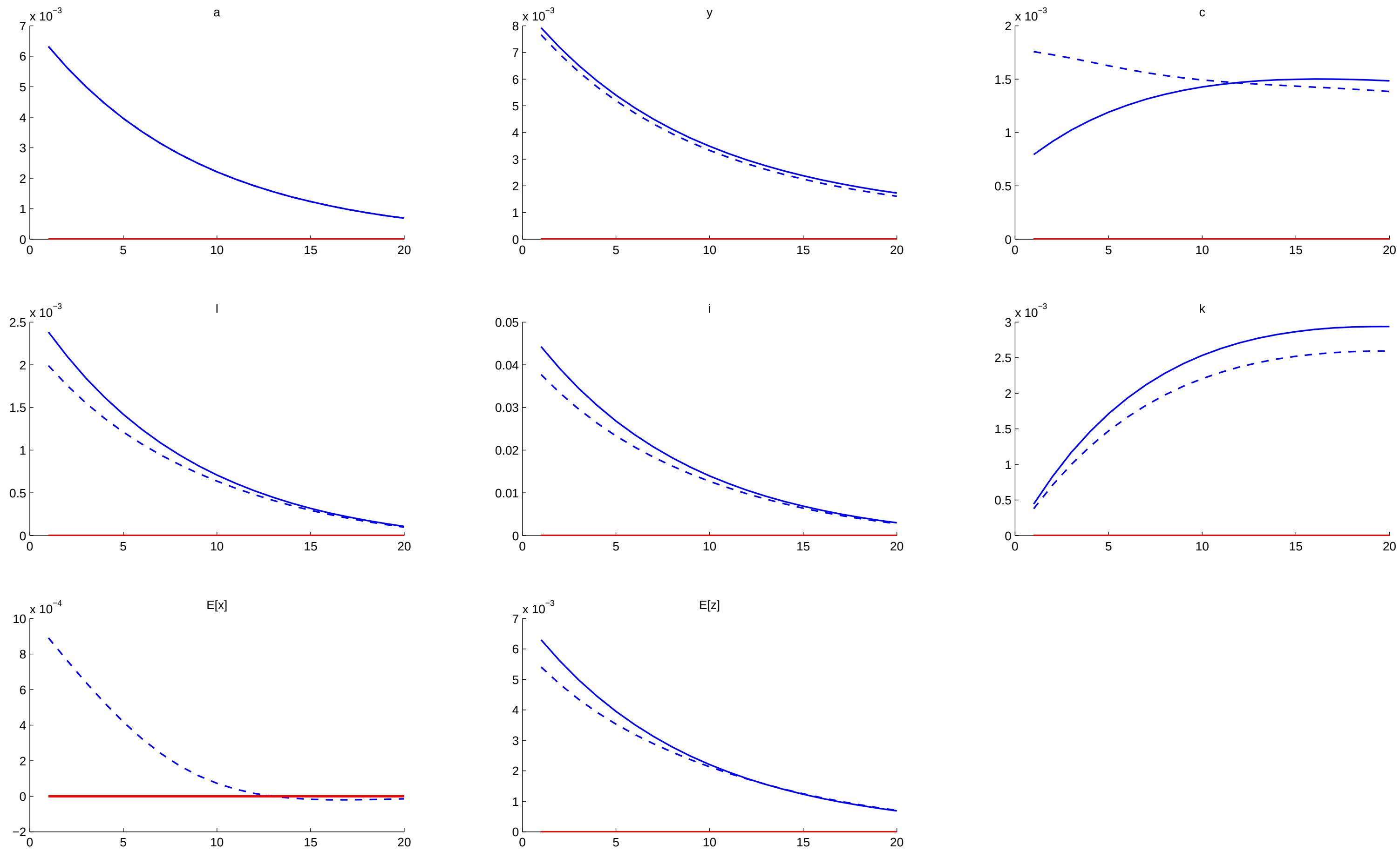


Figure 1-4: Impulse response functions for a transitory shock of one s.d. (0.63%) Full information: solid, Imperfect information: dashed

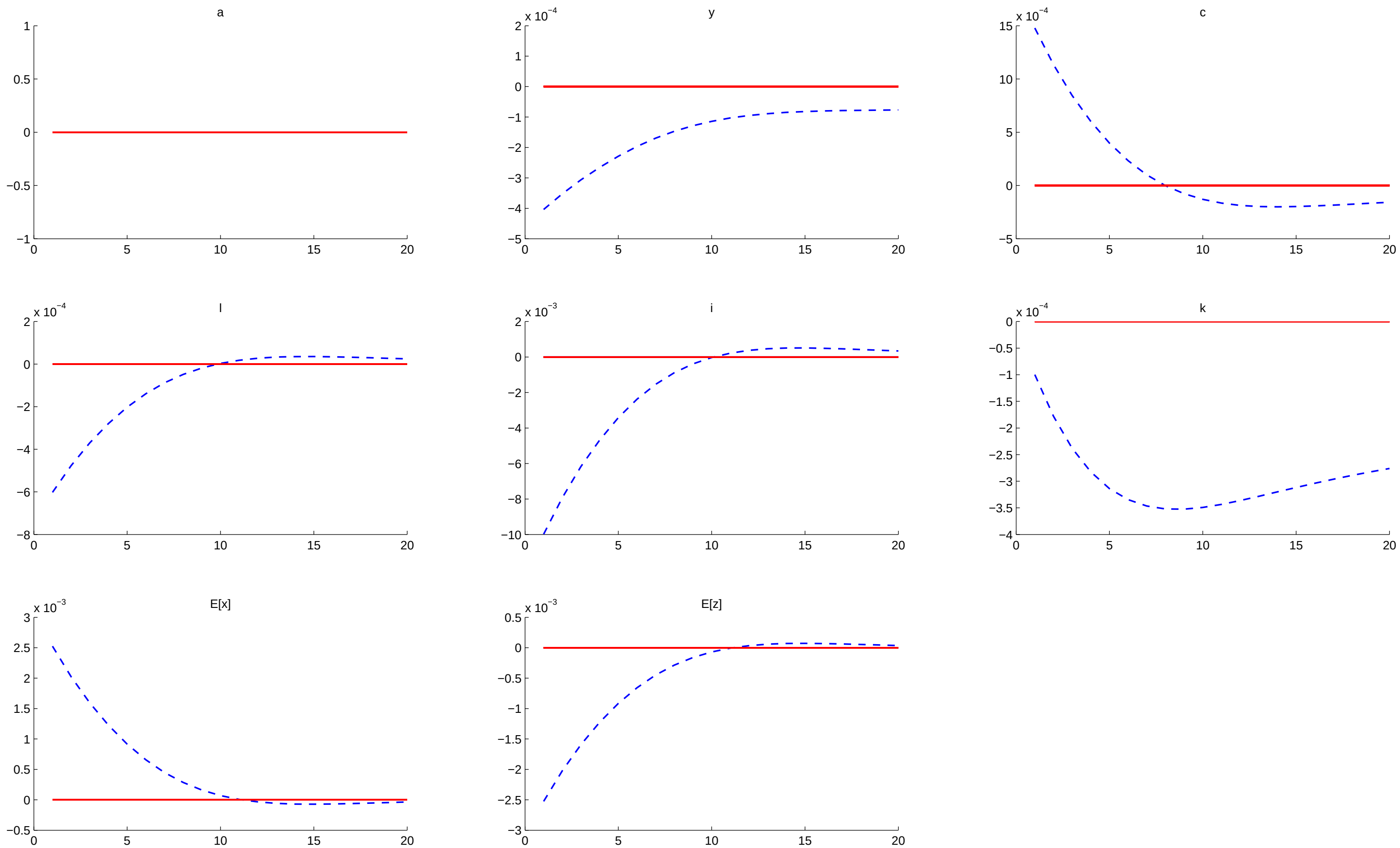


Figure 1-5: Impulse response functions for a noise shock of one s.d. (0.89%) Full information: solid, Imperfect information: dashed

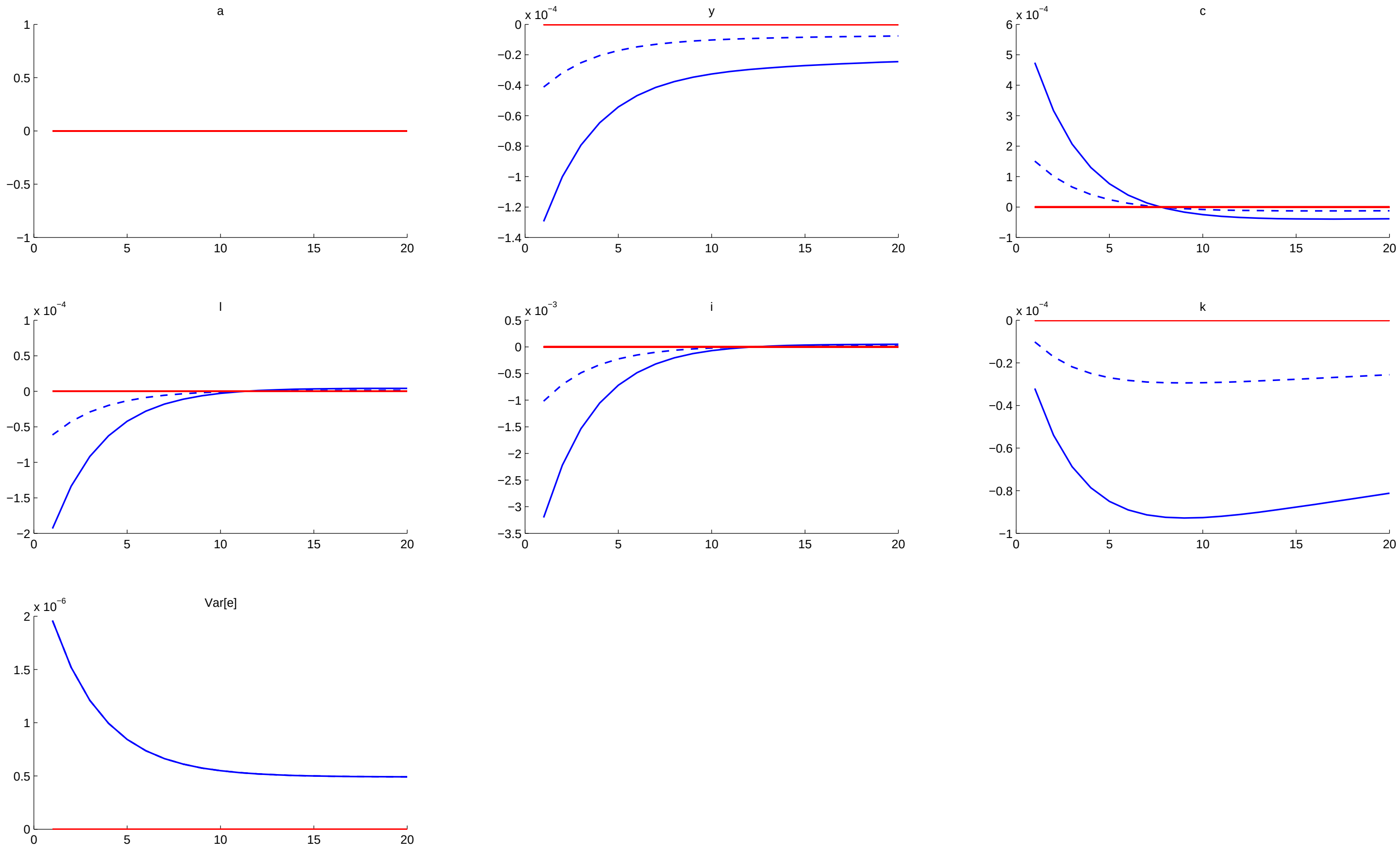


Figure 1-6: Impulse response functions for a $\sigma_\epsilon$ shock (0.07%) Full information: solid, Imperfect information: dashed

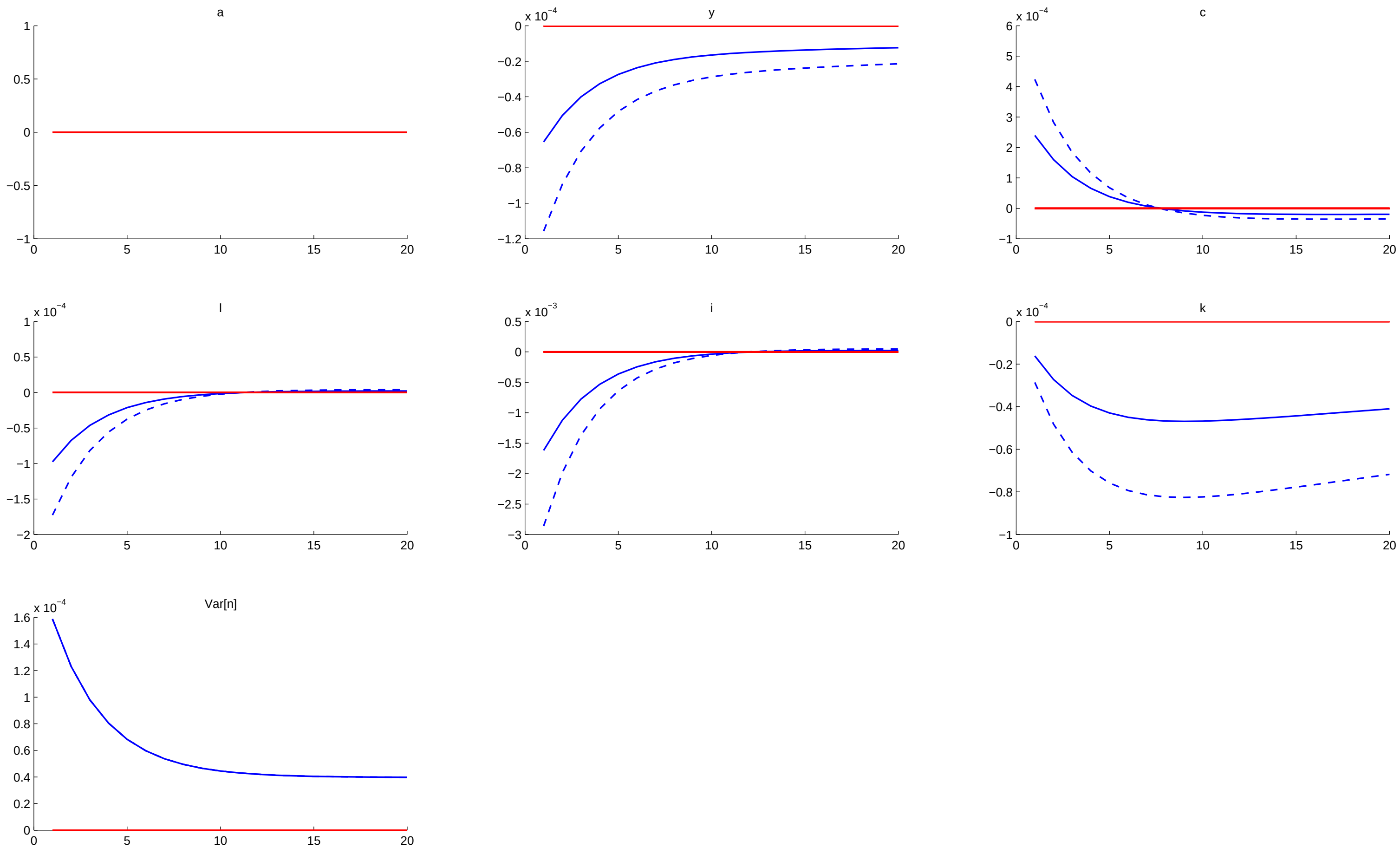


Figure 1-7: Impulse response functions for a $\sigma_\eta$ shock (0.63%) Full information: solid, Imperfect information: dashed

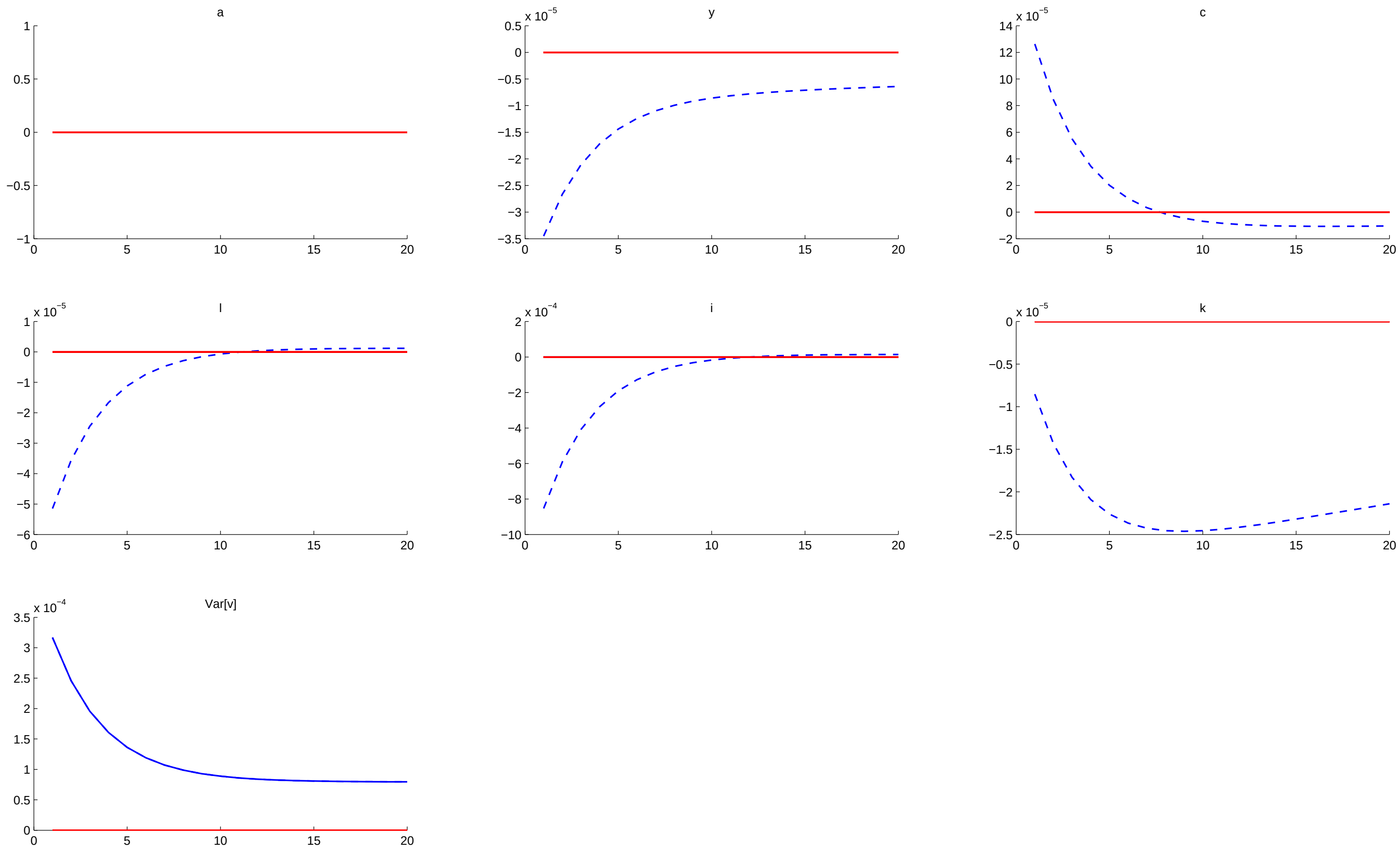


Figure 1-8: Impulse response functions for a $\sigma_\nu$ shock (0.89%) Full information: solid, Imperfect information: dashed

# Chapter 2

# Financial Frictions and the Interest-Rate Differential in a Dollarized Economy

## 2.1 Introduction

The objective of this paper is to study the impact of financial frictions on lending and deposit interest rate differentials in an economy characterized by partial financial dollarization. In order to do this, I extend the Bernanke et al. (1999) financial accelerator mechanism to incorporate financial dollarization and banking regulation restrictions in a partial equilibrium setting.

Two types of financial frictions are incorporated in the model: first, lending banks face monitoring costs when foreclosing entrepreneurs defaulting on their loans. This is the standard costly state verification (CSV) mechanism of Townsend (1979) that was introduced in a DSGE framework by Carlstrom and Fuerst (1997) and Bernanke et al. (1999). The second type of financial frictions are reserve requirements imposed on banks. The banking set up is a modification of the one used in Cohen-Cole and Martínez-García (2010).

Financial dollarization is present because banks offer loans to entrepreneurs which are denominated in domestic and foreign currency. The reason behind this is that banks themselves accept deposits denominated in domestic and foreign currency from households. The deposit market is assumed to be competitive.

A secondary objective of the paper is to develop a model that incorporates financial frictions in order to explore the effects of monetary policy over interest rate differentials. This is important because the Peruvian economy is partially dollarized and allows for deposits and loans in both domestic and foreign currency, a feature that may distort the conventional transmission mechanism.

Céspedes et al. (2004) propose a small open economy model with financial dollarization where all loans are extended in foreign currency. They use it to explore balance sheet effects: when their small open economy experiences a depreciation as a result of a negative external shock, entrepreneurial debt increases, reducing entrepreneurial net worth and thus contracting investment due to financial frictions. Castillo et al. (2009) study a medium-sized small open economy DSGE with partial financial dollarization tailored to the Peruvian economy in order to analyse the impact of the degree of financial dollarization on the monetary policy transmission mechanism. However, they fix the degree of loan dollarization exogenously and characterize the relationship between the lending rates in domestic and foreign currency with an uncovered interest rate parity condition. Gondo and Orrego (2011) use a small open economy model similar to Céspedes et al. (2004) incorporating exogenous partial dollarization in order to evaluate the impact of de-dollarization on the response of GDP to a negative external shock. These authors assume an uncovered interest rate parity condition governs the relationship between interest rates in domestic and foreign currency as well.

All of these approaches assume some exogenous degree of financial dollarization and a given wedge between the lending rates in domestic and foreign currency. This paper attempts to relax those assumptions, characterizing the interaction between an endogenous degree of loan dollarization and an endogenous wedge between domestic and foreign currency lending rates. In order to do so, the assumption of arbitrage between internal (domestic) and external (foreign) borrowing is removed and replaced by a domestic bank which offers loans in both domestic and foreign currency. Crucially, this bank will set the interest rate charged on both loans.

The analysis carried out in this paper yields several insights. First, reserve requirements on domestic or foreign currency deposits act as a tax that leads banks to decrease deposit rates. This seems to explain the very low foreign currency deposit rates observed in the Peruvian economy (which suffers from partial financial dollarization). More interestingly though, arbitrage between banks' funding sources implies that manipulating interbank rates through monetary policy can have an impact on foreign currency deposit rates. This is a direct result of the assumption that a foreign currency deposit in a domestic bank cannot be perfectly substituted by a foreign currency deposit in a foreign bank.

Second, the wedge between domestic and foreign currency lending rates is increasing in exchange rate volatility and decreasing in the degree of correlation between entrepreneurs' returns and the exchange rate. That exchange rate volatility pushes down foreign currency lending rates should be no surprise: ceteris paribus, higher volatility implies the entrepreneur is undertaking more exchange rate risk associated to foreign currency loans and this must be compensated (in equilibrium) with lower interest rates being charged on those loans. However, the analysis presented in this paper indicates that exchange rate volatility is not the whole story: the degree of correlation between borrowers' returns on capital and the exchange rate matters as well.

In a competitive setting with no frictions, banks can avoid exchange rate risk by matching their assets and liabilities per currency, borrowers in Peru usually cannot. Thus, banks can offer loans in different currencies charging the same interest rates (provided expected depreciation is zero). A borrower facing those interest rates would have no incentive to take a loan in foreign currency, exposing himself to exchange rate risk, unless his return on assets has some degree of correlation with the exchange rate. This is the main insight the analysis performed in this paper yields: higher correlation between borrowers' returns on capital and the exchange rate implies a higher disposition to take on foreign currency loans because they provide partial insurance against exchange rate fluctuations (that would impact the entrepreneur's return on capital through the correlation). This benefit implies entrepreneurs are willing to accept higher lending rates on foreign currency loans when the correlation is higher.

Thus, the model predicts that sectors with higher correlation between their return on assets and the exchange rate (e.g.: exporters) face lower lending rate spreads (defined as domestic minus foreign) while sectors with low correlation are offered higher spreads.

In order to provide some intuition regarding the behaviour of lending and deposit interest rates in a dollarized economy, section 2.2 presents historical data showing the behaviour of interest rates and reserve requirements in Peru during the last decade. Section 2.3 provides the set up for the model, section 2.4 shows the conditions of the optimal loan contract and section 2.5 incorporates the solution procedure for the model. Section 2.6 discusses the results and section 2.7 concludes the paper.

## 2.2 Data

For the Peruvian economy, there are four reference interest rates: TAMN, TIPMN, TAMEX, and TIPMEX. They are defined as follows:

- TAMN is the weighted-average lending rate in domestic currency.
- TAMEX is the weighted-average lending rate in foreign currency.
- TIPMN is the weighted-average deposit rate in domestic currency.
- TIPMEX is the weighted-average deposit rate in foreign currency.

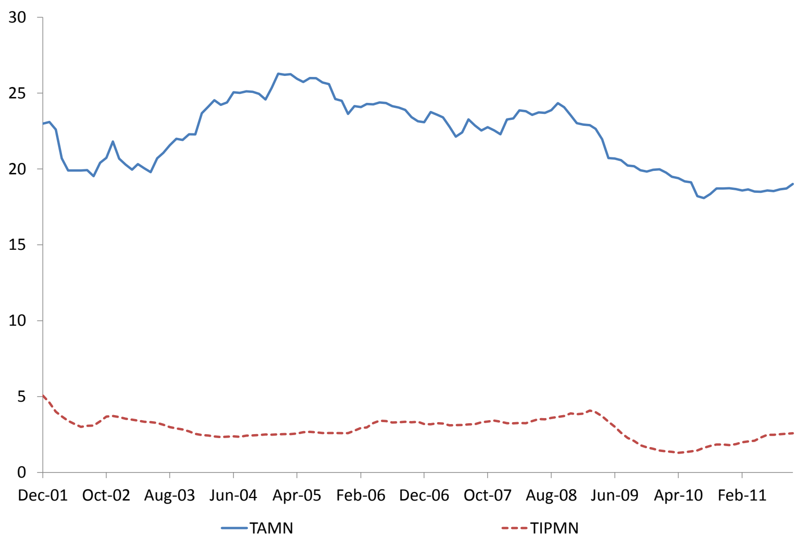


Figure 2-1: Interest rates on loans and deposits in domestic currency

These rates are calculated on a daily basis. The data is public and available from the Central Bank of Peru's website. Figures 2-1 and 2-2 show the monthly average of each rate for the period 2001 - 2010. The difference between domestic currency lending

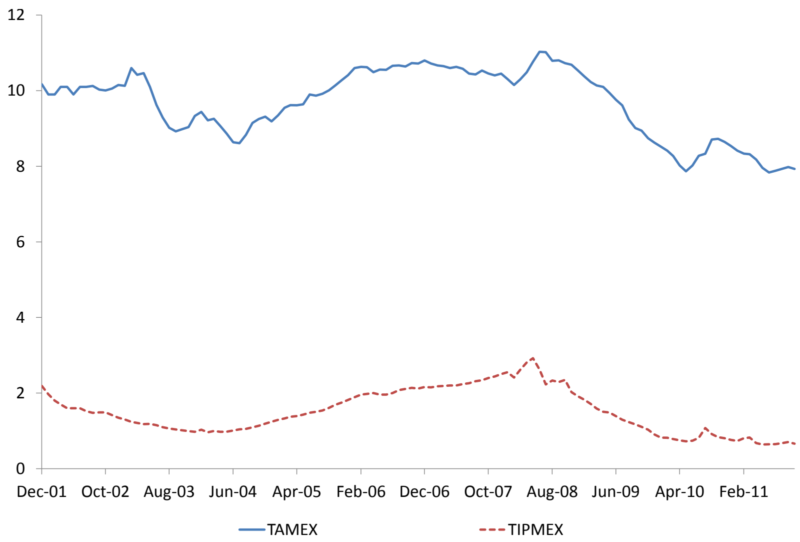


Figure 2-2: Interest rates on loans and deposits in foreign currency

and deposits rates is 19 percentage points compared to 8 percentage points for foreign currency. As expected, lending rates are higher than deposit rates. In order to gain further insight on the factors behind this significant difference in spreads the same rates will be presented below, grouped by instrument (loans and deposits).

Figure 2-3 shows lending rates for both currencies. On average, the difference between the lending rate in domestic versus foreign currency is 13 percentage points. This fact explains most of the difference in spreads observed in Figures 2-1 and 2-2 and may suggest a higher external premium required for loans in domestic currency which in turn may be consistent with risk averse firms. In Figure 2-4, the average difference between domestic and foreign deposit rates is shown to be of just 2 percentage points. Given that households are probably risk averse as well, why isn't the interest rate on foreign currency deposits (much) higher than its domestic currency counterpart? Since most household expenditures are denominated in domestic currency (some prices in Peru are set in foreign currency, rent for example), the household incurs in exchange rate risk when it deposits its savings in foreign currency and should be compensated for that risk in equilibrium with a higher interest rate.

It is our claim that the reason why the data contradicts this intuition are Peru's higher reserve requirements on foreign deposits which make this funding alternative costly for banks, pushing down the interest rate they're willing to offer on these deposits. There are some alternative explanations, of course. Expected depreciation of domestic currency could push up the domestic currency deposit rate. Note though that it would be hard to justify a persistent expected depreciation over the span of 10 years (see Figure 2-4) and the monthly depreciation data for Peru (Figure 2-10 in the appendix) is inconsistent with sustained expected depreciation.

Domestic currency deposit rates being higher than their foreign currency counterparts could also be explained by a "peso effect": the idea that households attach a positive probability to a disastrous event (a hyperinflation, for example) and foreign currency deposits protect their wealth in this doomsday scenario (acting effectively as insurance). Note though that Peruvian households can switch their savings between deposits in domestic and foreign currency with minimal transaction costs (these days it can be done over the Internet, provided the household has an open account in both currencies) and a hyperinflation does not occur overnight.

Yet another possible explanation could be constructed by invoking the possibility of differences in the bank's creditworthiness in one currency versus another. Given the liquid nature of the foreign exchange market, such differences can be ruled out: if the bank cannot fulfil his obligations in a particular currency then all it needs to do is buy said currency in the market using the other. If the bank has liquidity issues in both currencies then both deposit rates should be affected in the same way and this would have no impact on the wedge between them.

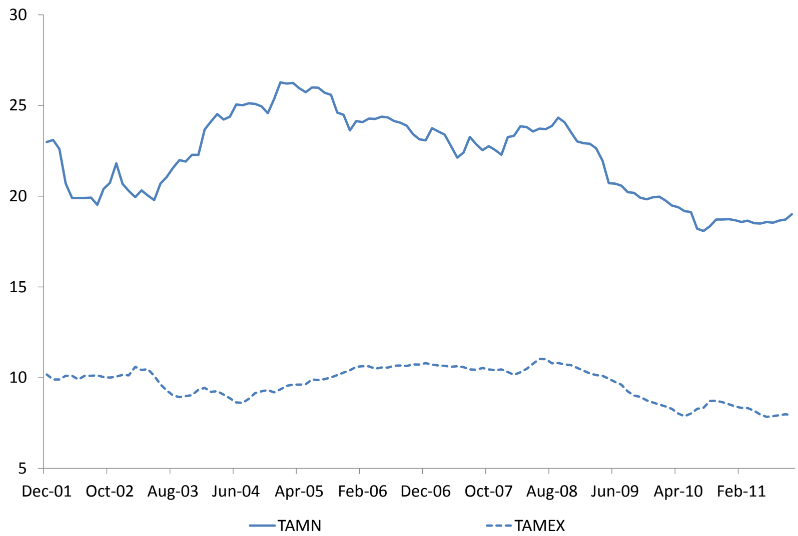


Figure 2-3: Domestic and foreign currency lending rates

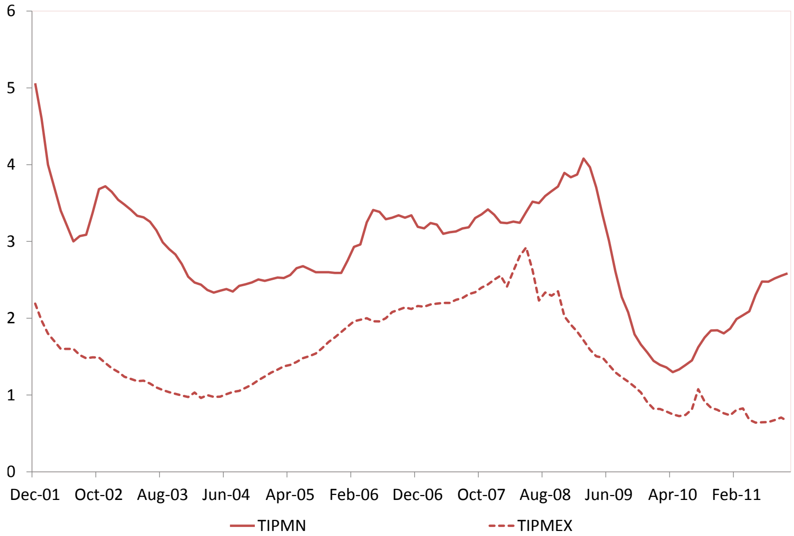


Figure 2-4: Domestic and foreign currency deposit rates

Regarding reserve requirements, the Central Reserve Bank of Peru enforces a higher reserve requirement for deposits and obligations in foreign currency compared to domestic currency as mentioned before. Figure 2-5 shows the evolution of the rate of reserves that banks effectively hold in order to comply with reserve requirements by type of currency (these are slightly above the required reserves imposed by the central bank). Reserve requirements as a monetary policy instrument became more relevant in the past three years (as a result of the crisis) and they are now actively used as a complement to the policy rate (the reference interest rate). Figure 2-6 illustrates this phenomenon: banks are being forced to hold more reserves when the reference interest rate increases and vice versa.

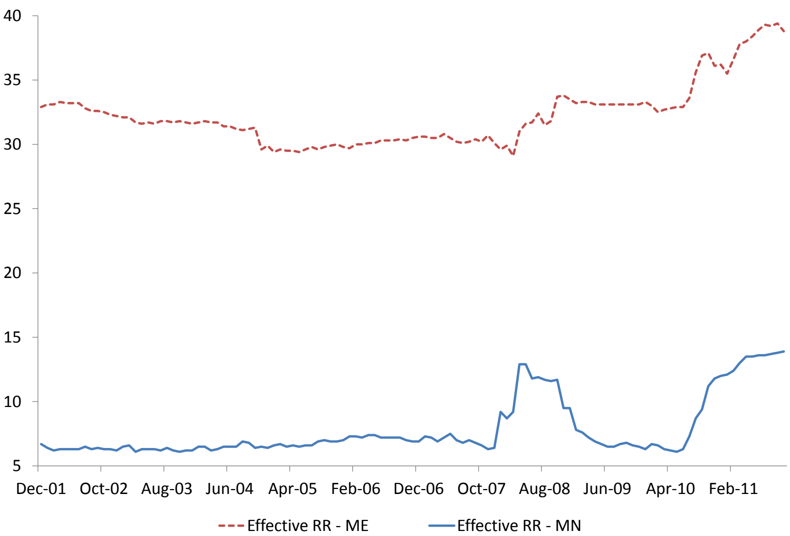


Figure 2-5: Effective reserves on deposits and obligations in foreign (ME) and domestic (MN) currency

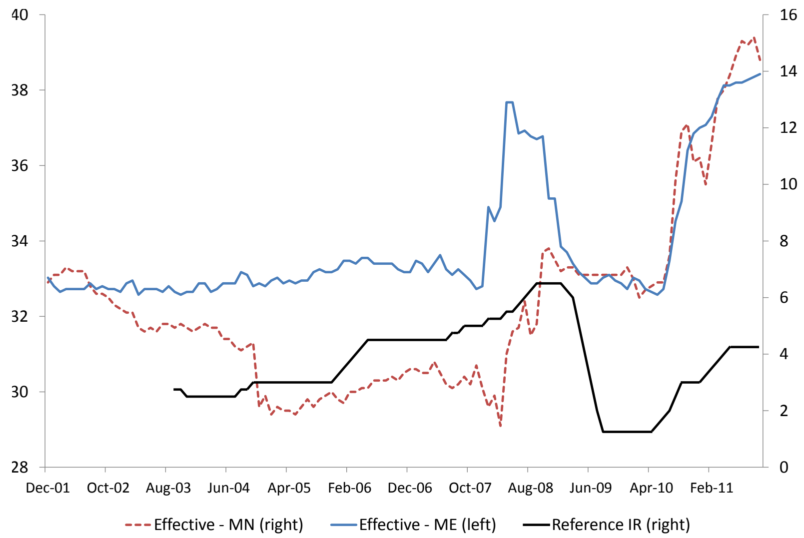


Figure 2-6: Effective reserves in foreign (ME) and domestic (MN) currency and reference interest rate

Besides the effect these reserve requirements have on the wedge between deposit rates, it is important to note that there are hints of a negative relationship between deposit rates and reserve requirements, particularly in the last three years.

Given the information extracted from the data presented, a good characterization of interest rate differentials in a dollarized economy such as the Peruvian one should address:

1. The wedge between domestic and foreign currency lending rates.

2. The relationship between domestic and foreign currency deposit rates and their interaction with reserve requirements.

Next section provides a highly stylized model which strives to provide a framework to study these issues.

## 2.3 The Model

This section analyses a partial equilibrium model where an entrepreneur interacts with a bank. The entrepreneur demands loans denominated in domestic and foreign currency from the bank. The bank funds itself by taking deposits in domestic and foreign currency from households.

Given the partial equilibrium set up, some characteristics of the entrepreneur are considered exogenous. Particularly, his average return on capital and how correlated it is with the nominal exchange rate. In the bank's case, it is assumed the deposit market is competitive and thus, deposit rates are taken as given.[1]

It is assumed entrepreneurs take real loans denominated in domestic ($L$) and foreign ($L^*$) currency in order to finance the acquisition of physical capital $K$. The market price of physical capital is fixed at unity and the bank charges gross nominal interest rate $R^L$ on loans denominated in domestic currency and gross nominal interest rate $R^{*L}$ on loans denominated in foreign currency.

The assumption that entrepreneurs take both types of loans is somewhat strong. For example, it could be argued that a firm producing non-traded goods would be better off without any foreign currency debt (unless it is making an exchange rate bet but we will rule those out later when expected depreciation is set to zero). Note though that even if the firm produces non-traded goods only, some of its customers might perceive income in foreign currency (implying a small but positive correlation between its return on capital and the nominal exchange rate) or the non-traded sector might be subject to positive spillovers from the traded sector (a depreciation which benefits the traded sector could produce higher demand for non-traded goods from households because of complementarity and strong wealth effects).

Following Bernanke et al. (1999), the entrepreneur faces an idiosyncratic shock $\omega$ to his (stochastic) nominal return $R^E$ over assets $K$. On top of this, the entrepreneur also faces uncertainty with respect to next period's nominal exchange rate $S'$ (defined as the price in domestic currency of one unit of foreign currency). Given that interest factors are fixed before the realization of any shocks, if $\omega$ turns out to be too small or $S'$ too high, the entrepreneur cannot repay his debt (in domestic and foreign currency) and goes bankrupt. In this scenario, the bank pays a monitoring cost to recoup what is left of the entrepreneur's assets.

Thus, the bankruptcy space will be:

$$\omega R^E K < R^L L + R^{*L} \frac{S'}{S} L^*. \tag{2.1}$$

[1]Actually, the credit market is competitive as well, but lending rates are *not* considered given.

Note this implies there is no difference in the default rates on domestic and foreign currency loans: an entrepreneur that goes bankrupt defaults on both loans.

We can define the cut-off $\varpi$ as the particular value for the idiosyncratic shock that allows the entrepreneur to pay his debt without any excess profit:

$$\varpi R^E K = R^L L + R^{*L} \frac{S'}{S} L^*. \tag{2.2}$$

where $S$ denotes the current nominal exchange rate.

Note that uncertainty with respect to next period's nominal exchange rate (from here on denoted simply as "exchange rate" since the real exchange rate does not play a part in this model) and the entrepreneur's return (which could depend on the exchange rate as well), implies the cut-off is stochastic. This is the first major departure from Bernanke et al. (1999): when entrepreneurs' income and liabilities are denominated in the same currency, the cut-off $\varpi$ is fixed. Here, the exchange rate makes one of the liabilities stochastic, implying there will be a different cut-off for every possible realization of next period's exchange rate.

The exchange rate risk implied by equation (2.2) could be avoided with some form of forward contract priced from interest rate differentials. This possibility is ruled out by assumption. The justification stems from the observation that in the Peruvian economy (which provides our motivation), financial derivatives are subject to non-trivial transaction costs. Furthermore, the forward market for foreign currency suffers from liquidity issues and the minimum transaction size is big enough to act as an entry barrier for a large number of firms.

It is assumed that entrepreneurs possess some net worth, $N$, which is required as collateral in order to obtain loans from banks. The entrepreneur's balance sheet links the entrepreneur's net worth and outstanding loans to capital:

$$K = L + L^* + N. \tag{2.3}$$

Note that entrepreneurs always allocate all resources available to capital acquisition. The reason behind this is that the expected return on capital is always greater than the return of alternative assets (such as bank deposits).

Banks will lend to entrepreneurs and take deposits in both currencies. They also participate in an interbank market where they can get additional funding $IB$. Even though the inclusion of an interbank market is unnecessary to answer this paper's questions (the determinants of lending and deposit interest rate spreads), it provides an intuitive way to analyse the effects of monetary policy on the different interest rates, something that will be done later on.

The bank's balance sheet equates the loans made to entrepreneurs to bank's liabilities:[2]

$$L + L^* = (1 - \varphi^*) D^* + (1 - \varphi) D + IB. \tag{2.4}$$

Here, $\varphi$ and $\varphi^*$ stand for the fractions of domestic and foreign currency deposits required as reserves by the banking regulatory agency. Note that we assume all of the bank's financial assets and liabilities ($L$, $L^*$, $D$, $D^*$, $IB$) are one period and thus abstract from any maturity risk by assumption. This is a common modelling device in the macroeconomic literature and it has the implication that our model is silent regarding the nature of potential interactions between exchange rate and maturity risk.

In this set-up, banks will never have an incentive to hold excess reserves because interest paid on them is very low, as will be discussed later on. Note also that the bank's balance sheet does not rule out the possibility of currency mismatches: $L^*$ does not necessarily equal $(1 - \varphi^*) D^*$. Thus, it is implicitly assumed that the bank has access to a liquid and competitive foreign currency spot market where it can, for example, exchange foreign currency obtained via deposits ($D^*$) for domestic currency required to issue domestic currency loans ($L$).

The entrepreneur's expected benefit after loan repayment is,

$$\Pi^E \equiv E\left[\int_{\varpi}^{\infty} \left(\omega R^E K - R^L L - R^{*L} \frac{S'}{S} L^*\right) dF(\omega)\right], \tag{2.5}$$

[2]Actually, Peru's banking system also funds its operations with foreign credit lines obtained from foreign banks and/or investment firms. Even though this type of funding is empirically relevant, this set up abstracts from it given that it would unnecessarily complicate the exposition without adding significant results.

where the integral comprises the stochastic return on assets and the other terms are loan repayments to the bank. Following Bernanke et al. (1999), it is assumed that the idiosyncratic shock's probability density function, $F'(\omega)$, corresponds to that of a log-normal distribution with $E[\omega] = 1$ and $Var[\log(\omega)] = \sigma^2$ the latter being exogenous to the model.[3] Using (2.2), the objective function can be re-written as,

$$\begin{aligned} \Pi^E &\equiv E\left[\int_{\overline{\omega}}^{\infty} (\omega - \overline{\omega})\, dF(\omega)\, R^E K\right] \\ &= E\left[f(\overline{\omega})\, R^E K\right]. \end{aligned} \tag{2.6}$$

Function $f(\overline{\omega})$ has been analysed extensively since Carlstrom and Fuerst (1997): it is the entrepreneur's share of investment returns $R^E K$. The marginal effect of $\overline{\omega}$ on the entrepreneur's share is negative and increasing.

Bank loans are paid back whenever $\omega > \overline{\omega}$. Otherwise, the bank forecloses the entrepreneur and pays a fraction $\mu$ of his remaining assets in order to cover monitoring costs. The central bank offers some remuneration on reserves to compensate for the fact that they cannot be lent.

Besides maximizing profit, we will assume the bank is interested in minimizing his long position in foreign currency. There are several justifications for this: the authority in charge of regulation finds banking balance sheets matched by currency desirable,[4] or the bank could dislike the exchange rate risk incurred whenever it finds itself in a short or long position. Thus, the bank's objective function becomes,

[3] It follows that $E[\log(\omega)] = -\sigma^2/2$.

[4] This is actually the case in Peru: banks face restrictions on how long or short they can go with respect to foreign currency in their net asset positions.

$$\Pi^B \equiv E\left[\begin{array}{c} \int_{\overline{\omega}}^{\infty}\left(R^L L + R^{*L}\frac{S'}{S}L^*\right)dF(\omega) + (1-\mu)\int_0^{\overline{\omega}}\omega R^E K dF(\omega) \\ +R^R\varphi D - R^D D + R^{*R}\frac{S'}{S}\varphi^* D^* - R^{*D}\frac{S'}{S}D^* \\ -R^{IB}IB - \frac{\chi}{2}\left(\frac{L^*}{L^*+L} - \frac{(1-\varphi^*)D^*}{L^*+L}\right)^2 \end{array}\right]$$

$$= E\left[\begin{array}{c} \left[\int_{\overline{\omega}}^{\infty}\overline{\omega}dF(\omega) + (1-\mu)\int_0^{\overline{\omega}}\omega dF(\omega)\right]R^E K \\ -\left(R^D - \varphi R^R\right)D - \left(R^{*D} - \varphi^* R^{*R}\right)\frac{S'}{S}D^* \\ -R^{IB}IB - \frac{\chi}{2}\left(d^L - \frac{(1-\varphi^*)D^*}{L^*+L}\right)^2 \end{array}\right]$$

$$= E\left[\begin{array}{c} g(\overline{\omega})R^E QK - \left(1 - \varphi\frac{R^R}{R^D}\right)R^D D \\ -\left(1 - \varphi^*\frac{R^{*R}}{R^{*D}}\right)R^{*D}\frac{S'}{S}D^* - R^{IB}IB - \frac{\chi}{2}\left(d^L - \frac{(1-\varphi^*)D^*}{L^*+L}\right)^2 \end{array}\right] \tag{2.7}$$

where $R^D$ and $R^{*D}$ are the nominal interest factors paid on deposits in domestic and foreign currency respectively, $R^R$ and $R^{*R}$ are the nominal interest factors the central bank pays on reserves in domestic and foreign currency, and $R^{IB}$ is the nominal interest factor being charged on interbank loans. All these interest rates will be taken by the bank as given and, furthermore, $R^R$, $R^{*R}$ and $R^{IB}$ will be considered exogenous in the model.

Function $g(\overline{\omega})$ represents the bank's share of investment returns, the marginal effect of $\overline{\omega}$ on it is positive and decreasing.

The last term in the bank's objective function acts as an adjustment cost whenever the bank's long position in foreign currency deviates from zero. The long position is calculated by subtracting liabilities denominated in foreign currency $(D^*)$ from assets in foreign currency $(L^* + \varphi^* D^*)$ and it is expressed as fraction of total assets $(L + L^*)$. After some algebra, an expression for loan dollarization ($d^L = \frac{L^*}{L^*+L}$) shows up in that last term.

Following Bernanke et al. (1999), we define the entrepreneur's asset to net worth ratio (leverage) $p$, as

$$p = \frac{K}{N} \tag{2.8}$$

Thus, using the entrepreneur's balance sheet and this definitions we can re-write the cut-off as

$$\overline{\omega} = \frac{1}{R^E}\frac{p-1}{p}\left(R^L\left(1-d^L\right)+R^{*L}\frac{S'}{S}d^L\right). \tag{2.9}$$

and the bank's balance sheet as

$$p-1 = \left(1-\varphi^*\right)\frac{D^*}{N}+\left(1-\varphi\right)\frac{D}{N}+\frac{IB}{N} \tag{2.10}$$

Similarly, the entrepreneur's and bank's objectives require re-writing,

$$\Pi^E \equiv E\left[f\left(\overline{\omega}\right)R^e\right]pN, \tag{2.11}$$

$$\Pi^B \equiv E\left[\begin{array}{c} g\left(\overline{\omega}\right)R^E p - \left(1-\varphi\theta^R\right)R^D\frac{D}{N} \\ -\left(1-\varphi^*\theta^{*R}\right)R^{*D}\frac{S'}{S}\frac{D^*}{N} - R^{IB}\frac{IB}{N} - \frac{\chi}{2}\left(d^L - \frac{\left(1-\varphi^*\right)\frac{D^*}{N}}{p-1}\right)^2 \end{array}\right]N, \tag{2.12}$$

where it is assumed that reserve remunerations are a fraction of their respective deposit rates ($R^R = \theta^R R^D$ and $R^{*R} = \theta^{*R}R^{*D}$).

## 2.4 Optimal Loan Contract

In Bernanke et al. (1999), lenders (ultimately the household sector) are risk averse while entrepreneurs are risk neutral. Thus, they find that the optimal contract involves a state-contingent loan rate, insuring the household against any aggregate risk (diversification over a large number of entrepreneurs takes care of idiosyncratic risk) and ensuring the lender's zero-profit condition holds in all states of the world. Even though this is formally correct, state-contingent loan rates are hardly realistic. There are non-trivial transaction costs involved in formulating these kind of contracts and enforcing them. Furthermore, proper evaluation of a state-contingent interest rate offer from the borrower's point of view would require him to incur in additional costs necessary to obtain information regarding the potential states of the world and their associated probabilities.

All of these costs are obviously not incorporated in the Bernanke et al. (1999) model, resulting in the optimality of state-contingent loan rates. This paper does not model them either, but we decide to strive for realism ruling out state-contingent loan rates. Thus, the contract we will construct can be interpreted as a constrained optimum where we restrict our domain to the set of loan contracts involving fixed loan rates. As a consequence, our bank's profit will be zero only in expectation and generally not zero after the realization of aggregate uncertainty.

The (constrained) optimal contract can be obtained by maximizing (2.11) subject to (2.12) being equal to zero and that the balance sheet identity (2.10) holds. The implicit assumption is that banks are competitive and they offer the best possible contract to the entrepreneur, at the cost of driving down their profits all the way to zero. Replacing the threshold with the expression shown in (2.9). The variables of the problem are $p$, $d^L$, $\frac{D}{N}$, $\frac{D^*}{N}$, $\frac{IB}{N}$ and the lending interest rates.

The first order conditions for an interior solution of the problem are:

$$\begin{aligned} p : 0 = E & \left[ \left( f\left(\overline{\omega}\right) + f'\left(\overline{\omega}\right) p \frac{\partial \overline{\omega}}{\partial p} \right) R^E \right] \\ & + \lambda E \left[ \left( g\left(\overline{\omega}\right) + g'\left(\overline{\omega}\right) p \frac{\partial \overline{\omega}}{\partial p} \right) R^E \right] \\ & - \mu - \lambda \chi \left( d^L - \frac{\left(1-\varphi^*\right) \frac{D^*}{N}}{p-1} \right) \frac{\left(1-\varphi^*\right) \frac{D^*}{N}}{\left(p-1\right)^2}, \end{aligned} \tag{2.13}$$

$$d^L : 0 = E\left[ f'\left(\overline{\omega}\right) \frac{\partial \overline{\omega}}{\partial d^L} R^E \right] + \lambda E \left[ g'\left(\overline{\omega}\right) \frac{\partial \overline{\omega}}{\partial d^L} R^E \right] - \lambda \chi \left( d^L - \frac{\left(1-\varphi^F\right) \frac{D^F}{N}}{p-1} \right), \tag{2.14}$$

$$\frac{D}{N} : 0 = -\lambda \left(1 - \varphi \theta^R\right) R^D + \mu \left(1 - \varphi\right), \tag{2.15}$$

$$
\begin{aligned}
\frac{D^*}{N} : 0 = &-\lambda \left(1 - \varphi^* \theta^{*R}\right) R^{*D} E \left[\frac{S'}{S}\right] + \mu \left(1 - \varphi^*\right) \\
&+ \lambda \chi \left(d^L - \frac{\left(1 - \varphi^*\right) \frac{D^*}{N}}{p - 1}\right) \frac{\left(1 - \varphi^*\right)}{p - 1},
\end{aligned}
\tag{2.16}
$$

$$
\frac{IB}{N} : 0 = -\lambda R^{IB} + \mu, \tag{2.17}
$$

$$
R^L : 0 = E\left[f'\left(\overline{\omega}\right) \frac{\partial \overline{\omega}}{\partial R^L} R^E\right] + \lambda E\left[g'\left(\overline{\omega}\right) \frac{\partial \overline{\omega}}{\partial R^L} R^E\right], \tag{2.18}
$$

$$
R^{*L} : 0 = E\left[f'\left(\overline{\omega}\right) \frac{\partial \overline{\omega}}{\partial R^{*L}} R^E\right] + \lambda E\left[g'\left(\overline{\omega}\right) \frac{\partial \overline{\omega}}{\partial R^{*L}} R^E\right]. \tag{2.19}
$$

where $\lambda$ is the Lagrangian multiplier associated to the bank's zero profit condition and $\mu$ is the Lagrangian multiplier associated to the bank's balance sheet.

Condition (2.18) is standard in the financial accelerator literature and simplifies to

$$
E\left[f'\left(\overline{\omega}\right)\right] + \lambda E\left[g'\left(\overline{\omega}\right)\right] = 0, \tag{2.20}
$$

which implies the Lagrangian multiplier $\lambda$ is constant. Condition (2.19) is new and simplifies to

$$
E\left[f'\left(\overline{\omega}\right) \frac{S'}{S}\right] + \lambda E\left[g'\left(\overline{\omega}\right) \frac{S'}{S}\right] = 0. \tag{2.21}
$$

This condition will complement (2.20) to define the relationship between interest charged in domestic and foreign currency. Given that the threshold is a function of next period's exchange rate $S'$, the exchange rate cannot be eliminated from the expression.

Expressions (2.20) and (2.21) can be combined to gain some intuition about their implications:

$$\frac{E\left[f'\left(\overline{\omega}\right)\frac{S'}{S}\right]}{E\left[f'\left(\overline{\omega}\right)\right]}=\frac{E\left[g'\left(\overline{\omega}\right)\frac{S'}{S}\right]}{E\left[g'\left(\overline{\omega}\right)\right]} \tag{2.22}$$

The LHS of expression (2.22) can be interpreted as the marginal rate of substitution between $R^L$ and $R^{*L}$ along the entrepreneur's indifference curve (it is $\frac{\partial\Pi^E/\partial R^{*L}}{\partial\Pi^E/\partial R^L}$). The RHS is the slope of the restriction imposed by the bank's zero-profit condition ($\frac{\partial\Pi^B/\partial R^{*L}}{\partial\Pi^B/\partial R^L}$) which acts as a marginal rate of transformation. Note that in order for this condition to pin down the ratio $\frac{R^L}{R^{*L}}$ the potential for some degree of correlation between $\overline{\omega}$ and $\frac{S'}{S}$ is required. This is the case in our model because the threshold, $\overline{\omega}$, is stochastic and depends on $\frac{S'}{S}$ (see equation (2.9)).

The first order condition with respect to the loan "dollarization" variable, $d^L$, simplifies to,

$$\begin{aligned}&\left(E\left[f'\left(\overline{\omega}\right)\right]+\lambda E\left[g'\left(\overline{\omega}\right)\right]\right)R^L\left(p-1\right)+\lambda\chi\left(d^L-\frac{\left(1-\varphi^*\right)\frac{D^*}{N}}{p-1}\right)\\&=\left(E\left[f'\left(\overline{\omega}\right)\frac{S'}{S}\right]+\lambda E\left[g'\left(\overline{\omega}\right)\frac{S'}{S}\right]\right)R^{*L}\left(p-1\right).\end{aligned} \tag{2.23}$$

It is easy to note that if the first order conditions with respect to $R^L$ (2.20) and $R^{*L}$ (2.21) hold, then (2.23) pins down loan dollarization:

$$d^L=\frac{\left(1-\varphi^*\right)\frac{D^*}{N}}{p-1} \tag{2.24}$$

which, after some rearranging, implies

$$d^L=\frac{\left(1-\varphi^*\right)d^D}{\left(1-\varphi^*\right)d^D+\left(1-\varphi\right)\left(1-d^D\right)+\frac{IB}{D^*+D}} \tag{2.25}$$

where $d^D$ stands for deposit dollarization ($\frac{D^*}{D^*+D}$).

The degree of deposit dollarization will be a household decision in the end. Appendix A.2 shows how it can be determined in the context of a household that can save in domestic and foreign currency deposits. The market equilibrium ratio of interbank loans to total deposits ($\frac{IB}{D^*+D}$) is not determined in the model either. Note though that aggregate interbank loans must equal the net position of the central bank in the interbank market and are thus exogenous.[5]

Further simplification of expression (2.25) yields:

$$L^* = (1 - \varphi^*)\, D^* \tag{2.26}$$

implying the pseudo adjustment cost introduced in the bank's objective function results in no currency mismatches on the bank's balance sheet in an interior equilibrium. The bank's foreign currency assets ($L^* + \varphi^* D^*$) equal its foreign currency liabilities ($D^*$) implying its domestic currency assets equal its domestic currency liabilities as well (to see this, replace equation (2.26) in the bank's balance sheet, equation (2.4)). Thus, the bank never faces exchange rate risk: it does not require a premium to extend foreign currency loans. Figure 2-11 in Appendix A.1 shows that Peruvian banks' long position on foreign currency basically follows the nominal exchange rate. When that effect is removed, the long position would become practically flat somewhere close to zero.[6]

Conditions (2.15), (2.16) and (2.17) can be combined to show that:

$$\frac{(1-\varphi\theta^R)}{(1-\varphi)} R^D = \frac{(1-\varphi^*\theta^{*R})}{(1-\varphi^*)} R^{*D} E\left[\frac{S'}{S}\right] = R^{IB} \tag{2.27}$$

which means that in order for both types of deposit (in domestic and foreign currency) and interbank loans to coexist, the net cost of funding from each source must be equal. The bank's demand for every type of funding is completely elastic as long as condition (2.27) holds (if one of the terms in the equality was smaller than the others, all banks would prefer that particular type of funding). Since $\varphi$, $\varphi^*$, $\theta^R$, $\theta^{*R}$, $R^{IB}$ and expected depreciation $E\left[\frac{S'}{S}\right]$ are considered exogenous in the model, this expression determines the equilibrium deposit rates.

Note that (2.27) resembles a modified uncovered interest rate parity (UIP) rela-

[5] Indeed, without a central bank interbank loans must cancel out in the aggregate: every loan taken by a bank must be given by another. Since all banks are identical, this would imply $IB = 0$.

[6] Peruvian banking regulation plays a role in this matter as well: banks are not allowed to take very long or short positions in foreign currency on their balance sheet. The "adjustment cost" introduced in the bank's benefits captures this nicely.

tionship but the intuition behind it is very different. The theory behind a standard UIP relationship is based on the assumption of perfect capital mobility allowing arbitrage opportunities that will result in identical returns on assets in different currencies. Whenever the return on an asset denominated in currency A is bigger than the return of a similar asset in currency B, investors will sell asset B and buy currency A in order to buy its asset. The result is an appreciation of currency A. Thus, the UIP relationship determines the spot exchange rate, not the interest rates.

In the case of equation (2.27), the deposit rates are being determined because perfect capital mobility does not hold. Foreign currency deposits are not an asset subject to arbitrage from the household's point of view. Households cannot obtain a foreign asset with the same characteristics as the foreign currency deposit being offered by the domestic bank. In particular, foreign assets cannot be made liquid through an ATM or on demand[7] and they are subject to indivisibilities, maturity and legal restrictions.

Figure 2-4 provides further support to the argument. During the last decade, average foreign currency denominated deposit rates have fluctuated between 1 and 2 percent in Peru. Given that those are nominal rates and U.S. inflation has fluctuated between 2 and 4 percent for the same period, real rates being paid on foreign currency deposits have been negative. If it existed, arbitrage would not allow that.

Thus, equilibrium in the deposit market would require that $R^D$ and $R^{*D}$ adjust in the long run to guarantee (2.27) holds. A competitive deposit market with households offering both deposits in domestic and foreign currency coupled with a competitive interbank market are enough to support the result. If, for example, $R^D$ was too low for condition (2.27) to hold, all banks would be demanding domestic currency deposits from households and none would demand foreign currency deposits. Market equilibrium would push up the domestic currency deposit rate in this scenario given that households generally prefer a diversified portfolio.[8]

Note that a central bank can have an impact on either $R^D$ or $R^{*D}$ by changing its associated reserve requirement or the interest rate paid on reserves. Thus, a central bank

[7] This is the case in Peru. Households are free to open deposit accounts in foreign currency and these operate exactly like their domestic currency counterparts. This institutional development was a result of the hyperinflation episode the economy suffered during the late 80s and very early 90s.

[8] Appendix A.2 presents an example of such a household.

striving for a particular equilibrium funding mix (say, with low deposit dollarization) would adjust the relative reserve requirements and relative interest paid on reserves accordingly. In Peru's case, given the central bank's objective of de-dollarization, a high reserve requirement imposed on foreign currency deposits (high $\varphi^*$) coupled with low interest paid on foreign currency reserves held at the central bank (low $\theta^{*R}$) would push foreign currency deposit rates down, inducing households to offer banks less foreign currency funding and more domestic currency.[9]

It is assumed that the aggregate supply of interbank funds is controlled by the central bank. Given that condition (2.27) defines a perfectly elastic demand for interbank funds, the central bank determines quantity. This allows a degree of control over bank's funding rates in general: given a finite level of aggregate net worth at any point in time, total loans issued to entrepreneurs will be constrained (there is an optimal entrepreneurial leverage determined by the optimal loan contract), implying a finite total funding required by banks. If the central bank offers more funds (at a lower interbank rate $R^{IB}$), this will displace deposits, decreasing both deposit rates.

Thus, we find that it is theoretically possible for the central bank to have an impact on foreign currency deposit rates. This depends crucially on our assumption of no arbitrage with alternative deposits opportunities in foreign countries, which we justified by pointing out that these foreign instruments are not proper substitutes for a foreign currency deposit in a domestic bank. This result is extreme and reality is probably half way: there must be some degree of substitution between foreign currency deposits in domestic and foreign banks but as long as it is imperfect, we claim the intuition being presented here still comes through: the central bank should be able to have some impact on the foreign currency deposit rate when manipulating the interbank rate. This intuition is empirically testable but we have no knowledge of any study attempting to do so using data from a country with partial financial dollarization. Most interest rate pass-through studies done for these economies tend to assume implicitly that UIP holds (see for example Lahura (2006)).

The remaining first order condition, with respect to $p$, is standard as well and

[9] In Peru, reserves are not remunerated up to a legal minimum. Given that the reserve requirement set by the central bank is higher than that minimum, additional required reserves must be remunerated. Currently, the interest paid on additional foreign currency reserves equals the 1-month LIBOR rate divided by 4.

simplifies to:

$$E\left[f\left(\overline{\omega}\right)\frac{R^{E}}{R^{IB}}\right]+\lambda\left(E\left[g\left(\overline{\omega}\right)\frac{R^{E}}{R^{IB}}\right]-1\right)=0, \tag{2.28}$$

note that the equalization of net funding costs implies $R^{IB}$ is also the bank's weighted cost of funding.

The last condition required to characterize the (partial) equilibrium of this problem is that bank's expected profits must be zero, i.e.: there is perfect competition in banking. Simplification of that condition yields,

$$E\left[g\left(\overline{\omega}\right)\frac{R^{E}}{R^{IB}}\right]p-(p-1)=0. \tag{2.29}$$

This condition also resembles one found in Bernanke et al. (1999).

## 2.5 Solution

Equations (2.20), (2.21), (2.28) and (2.29) form a system in four unknowns: $R^{L}$, $R^{*L}$, $p$ and $\lambda$. All traces of the cut-off $\overline{\omega}$ can be eliminated from them using (2.9).

Given the functional forms and moments present in the four-equation system, in order to proceed, second order approximations will be used. In particular, a second-order approximation of (2.20) and (2.21) calculated in the vicinity of $E\left[\overline{\omega}\right]$ and $E\left[\frac{S'}{S}\right]$ will be used to pin down the relationship between the lending interest rates.

Combining the approximate versions of (2.20) and (2.21) it can be demonstrated that,

$$\begin{aligned}&\frac{f'\left(E\left[\overline{\omega}\right]\right)+\lambda g'\left(E\left[\overline{\omega}\right]\right)}{f''\left(E\left[\overline{\omega}\right]\right)+\lambda g''\left(E\left[\overline{\omega}\right]\right)}\left(E\left[\frac{S'}{S}\right]-1\right)\\&=E\left[\overline{\omega}\right]E\left[\frac{S'}{S}\right]-E\left[\overline{\omega}\frac{S'}{S}\right].\end{aligned} \tag{2.30}$$

This condition states that, barring expected appreciation or depreciation of the exchange rate, the cut-off and ex-ante depreciation of the exchange rate must be independent.

In order to proceed, we will assume that the nominal exchange rate follows a random walk process:

$$E\left[\frac{S'}{S}\right] = 1. \tag{2.31}$$

There is a large literature in the field of international macroeconomics that justifies this assumption.[10] Perhaps the most important contribution can be found in Meese and Rogoff (1983) where the key result is that no structural model of the nominal exchange rate can out-predict the naive random walk model at short to medium term horizons (less than year). Peruvian data seems to conform to this result: average monthly $S_{t+1}/S_t$ equals 0.998 in a sample dating from January 2005 to March 2011.[11]

Using the latter assumption and the definition of the cut-off on (2.30) and simplifying implies:

$$R^L E\left[\frac{1}{R^E}\left(\frac{S'}{S} - 1\right)\right](1 - d^L) + R^{*L} E\left[\frac{1}{R^E}\frac{S'}{S}\left(\frac{S'}{S} - 1\right)\right] d^L = 0. \tag{2.32}$$

This expression determines the relationship between both lending interest rates. Two particular cases are worth noting in order to gain some intuition on what (2.32) implies. If we assume $R^E$ is non-stochastic (fixed) and equal to $\overline{R}^E$, then (2.32) implies a corner solution where $d^L$ must be zero. The optimal contract involves no debt denominated in foreign currency and first order conditions (2.14) and (2.19) vanish. On the other hand, if $R^E$ equals $\frac{S'}{S}$ (which implies perfect correlation with the exchange rate) then (2.32) results in $d^L$ being equal to one, the entrepreneur is not offered any debt in domestic currency and first order conditions (2.14) and (2.18) vanish.

In principle, in both of these cases a modified version of equation (2.27) appears. The bank will keep a currency-matched balance sheet if the net cost of its funding

[10] See Frankel and Rose (1995) for a survey.
[11] See Figure 2-10 in the Appendix.

alternatives are all the same (implying zero deposit dollarization in the first case and full deposit dollarization in the second one in order to minimize the bank's long position in foreign currency). If the market deposit rates do not imply equal funding costs across currencies, i.e. equation (2.27) does not hold, the bank will set deposit dollarization to some fraction if the deposit rate on the currency he does not use for lending is low enough. Thus, he will balance the drive to avoid currency mismatches against the desire to benefit from cheap funding in the currency he does not use for lending. Note that in these cases the model cannot pin down the relationship between deposit rates in equilibrium.

This reasoning leads us to the conclusion that the correlation between $R^E$ and $\frac{S'}{S}$ is key. This makes sense: if a well defined demand for loans is to exist in both currencies, those denominated in foreign currency must provide some additional benefit to the entrepreneur given that they expose him to exchange rate risk; when $R^E$ and $\frac{S'}{S}$ have some degree of correlation, foreign currency liabilities act as a form of insurance. The implication is that the Bernanke et al. (1999) set up implies some degree of risk aversion, which can be traced to the presence of the cut off on the probability of default.

Even though the objective functions depend on expected payoffs only, they are not risk-neutral in the classic sense. We can define risk-neutrality broadly as the case in which the objective function is a linear function of the (random) payoffs. Taking the entrepreneur's objective function (2.5) as an example, the entrepreneur's payoff is $\omega R^E K - R^L L - R^{*L}\frac{S'}{S}L^*$ if $\omega > \overline{\omega}$ and zero otherwise. Classic risk-neutrality would require that the stochastic properties of the payoffs have no effect on $\Pr(\omega > \overline{\omega})$ which is clearly not the case here (in particular, $R^E$ and $\frac{S'}{S}$ have an impact on $\overline{\omega}$). Thus, the entrepreneur's objective function is not a linear function of the payoffs. Note that the same reasoning applies to the bank's objective function.

In order to simplify the relationship between the lending interest rates some more, an assumption regarding the stochastic process of $R^E$ is made:

$$\frac{1}{R^E} = \gamma\frac{1}{\overline{R}^E} + (1-\gamma)\,\frac{S}{S'}. \tag{2.33}$$

Note this assumption embodies the particular cases mentioned above when $\gamma = 1$ (entrepreneurial return is non-stochastic) and $\gamma = 0$ (entrepreneurial return is perfectly correlated with the exchange rate). In general, $0 < \gamma < 1$ implies there is some degree of correlation between the stochastic entrepreneurial return and the exchange rate, a condition necessary for the lending interest rates to be well defined. Using (2.33) on (2.32):

$$R^{*L} = \overline{R}^{E} \frac{(1-\gamma)}{\gamma} \frac{\left(E\left[\frac{S}{S'}\right] - 1\right)}{Var\left[\frac{S'}{S}\right]} \frac{\left(1-d^{L}\right)}{d^{L}} R^{L}. \tag{2.34}$$

Thus, there is a positive relationship between both lending interest rates.[12] Furthermore, the interest rate charged on foreign currency denominated loans is decreasing on the variance of the exchange rate and the degree of dollarization of credit; it is increasing on the degree of correlation between the entrepreneur's returns and the exchange rate. These results are all fairly intuitive: higher exchange rate variance implies higher exchange rate risk being taken on by the entrepreneur on foreign currency loans (given that his return on assets is imperfectly correlated with the exchange rate) and he will have to be offered a lower interest rate on them in compensation.

A higher degree of loan dollarization implies higher exposure to exchange rate risk as well. This will be compensated with a lower interest rate on foreign currency loans. If the degree of correlation between the entrepreneur's returns and the exchange rate is higher then foreign currency denominated loans become better insurance against exchange rate risk (liabilities would increase when the return on asset increases and vice versa) and the entrepreneur will be willing to pay a higher interest rate on loans which provide said insurance.

This result also highlights the fact that exchange rate risk faced by the entrepreneur taking foreign currency denominated credit can be compensated through a lower foreign currency lending interest rate, $R^{*L}$, or a lower credit dollarization ratio, $d^{L}$. Since both options are perfect substitutes (in the sense that both can perfectly compensate the entrepreneur for exchange rate risk) then the credit dollarization ratio is tied to deposit dollarization in (2.25) and the entrepreneur is compensated exclusively through $R^{*L}$. Credit dollarization being tied to deposit dollarization is a necessary simplification that arises from the fact that we assume all entrepreneurs taking loans from the bank share

[12] Jensen's Inequality states that given a convex function $g$, $E[g(x)] > g(E[x])$. Thus, $E[1/x] > 1/E[x]$ implying $E[S/S'] > 1/E[S'/S] = 1$.

the same $\gamma$. A more realistic set up would involve entrepreneurs with different values of $\gamma$ taking loans from the same bank. Given that the bank would have to offer them the same (or at least, fairly similar) interest rates, an endogenous credit dollarization ratio dependent on the distribution of $\gamma$ should be obtainable. Such an exercise is explored in Appendix A.3.

Thus, an interesting extension to the model would imply exploring the possibility of a continuum of entrepreneurs with different $\gamma^i$ (see Figure 2-7). Two extreme possibilities would arise then: the bank could offer them all the same ratio of loan dollarization and compensate them through different foreign currency lending rates or offer one unique foreign currency lending rate and allow each entrepreneur to pick his own dollarization ratio. Reality, as usual, is probably half way between these two extremes. Empirical evidence shows there is a distribution of loan dollarization in firm data (Figure 2-7) and there are several foreign currency lending rates (see Figures 2-8 and 2-9 for example). Given the added complication that $\gamma^i$ is probably not perfectly observable by the bank, it is possible that an optimal strategy involves offering a menu of interest rates associated to typical loan categories (micro, small, medium, corporate loans) and allowing firms to pick their loan dollarization ratio subject to the interest rates operating in the specific category in which the bank classifies them. Note though that there must be some degree of arbitrage between the interest rates in different categories so the bank must take that into account when offering its menu. The characterization of such a model goes beyond the scope of this paper and is left for further research.

The remaining equations, (2.20), (2.28) and (2.29) form a system with three unknowns very similar to the one studied in Bernanke et al. (1999). In order to ease the comparison, the system analysed by Bernanke et al. (1999) is reproduced here:

$$f'\left(\overline{\omega}\right) + \lambda g'\left(\overline{\omega}\right) = 0 \tag{2.35}$$

$$f\left(\overline{\omega}\right)\frac{R^E}{R} + \lambda\left(g\left(\overline{\omega}\right)\frac{R^E}{R} - 1\right) = 0 \tag{2.36}$$

$$g(\overline{\omega})\frac{R^E}{R}p-(p-1)=0 \tag{2.37}$$

All variables used have similar interpretations to the ones introduced previously and $R$ is the risk free rate: Bernanke et al. (1999) assumed entrepreneurs obtained loans from a financial intermediary that funded itself from households at rate $R$. Given that the formulation studied by Bernanke et al. (1999) has a non-stochastic threshold, the solution of the system is a fixed vector $(\lambda, \overline{\omega}, p)$ satisfying (2.35), (2.36) and (2.37) with $\frac{R^E}{R}$, the external finance premium, being the only "exogenous" variable, coupled with functional forms $f$ and $g$.

Given the non linearity of the system, the strategy in Bernanke et al. (1999) is to begin using (2.35) to show that $\partial\lambda/\partial\overline{\omega}$ is positive. This result coupled with a total derivative of (2.36) results in $\partial\left(\frac{R^E}{R}\right)/\partial\overline{\omega}$ being positive as well. Finally, a similar procedure on (2.37) is used to demonstrate that $\partial p/\partial\overline{\omega}$ must be positive too. Thus, it must be the case that $\partial\left(\frac{R^E}{R}\right)/\partial p$ is positive, i.e.: the external finance premium is increasing in the entrepreneur's leverage (the asset to net worth ratio).

The strategy employed by Bernanke et al. (1999) has to be slightly modified in order to apply it to this problem. First, the stochastic nature of the cut-off makes taking derivatives with respect to it slightly awkward. Second, the entrepreneurial return $R^E$ is stochastic as well, so $\partial\left(\frac{R^E}{R^{IB}}\right)/\partial\overline{\omega}$ is probably not well defined.

Thus, instead of using the cut-off as the "link" variable between the external finance premium and $p$, the lending interest rate on loans denominated in domestic currency, $R^L$, will take this role.

As a first step, note that (2.9) implies

$$\frac{\partial\overline{\omega}}{\partial R^L}=\frac{1}{R^E}\frac{p-1}{p}\left(1-d^L\right)>0 \tag{2.38}$$

a result we will be relying on.

In order to show that $\partial\lambda/\partial R^L$ is positive, note that

$$\lambda^{-1} = -\frac{E\left[g'\left(\overline{\omega}\right)\right]}{E\left[f'\left(\overline{\omega}\right)\right]} \tag{2.39}$$

from (2.20). Furthermore,

$$\frac{\partial \lambda^{-1}}{\partial R^{L}} = -\frac{1}{\lambda^{2}} \frac{\partial \lambda}{\partial R^{L}} \tag{2.40}$$

thus, proving that $\partial\lambda^{-1}/\partial R^{L}$ is negative is sufficient for our purposes. Given the (implicit) definitions of $f$ and $g$ given in (2.6) and (2.7), (2.39) can be expressed as:

$$\lambda^{-1} = 1 - \mu \frac{E\left[\overline{\omega} F'\left(\overline{\omega}\right)\right]}{E\left[1 - F\left(\overline{\omega}\right)\right]}. \tag{2.41}$$

In Bernanke et al. (1999), the expression $\overline{\omega} F'\left(\overline{\omega}\right) / \left(1 - F\left(\overline{\omega}\right)\right)$ is assumed to be increasing in $\overline{\omega}$. An analogous assumption would be sufficient for our purpose but given the nuances of the problem, further detail will be provided. In particular,

$$\frac{\partial \lambda^{-1}}{\partial R^{L}} = -\mu \left( \frac{E\left[\left(F'\left(\overline{\omega}\right) + \overline{\omega} F''\left(\overline{\omega}\right)\right) \frac{\partial \overline{\omega}}{\partial R^{L}}\right]}{E\left[1 - F\left(\overline{\omega}\right)\right]} + \frac{E\left[\overline{\omega} F'\left(\overline{\omega}\right)\right] E\left[F'\left(\overline{\omega}\right) \frac{\partial \overline{\omega}}{\partial R^{L}}\right]}{E\left[1 - F\left(\overline{\omega}\right)\right]^{2}} \right), \tag{2.42}$$

implying we require

$$E\left[\left(F'\left(\overline{\omega}\right) + \overline{\omega} F''\left(\overline{\omega}\right)\right) \frac{\partial \overline{\omega}}{\partial R^{L}}\right] + \frac{E\left[\overline{\omega} F'\left(\overline{\omega}\right)\right] E\left[F'\left(\overline{\omega}\right) \frac{\partial \overline{\omega}}{\partial R^{L}}\right]}{E\left[1 - F\left(\overline{\omega}\right)\right]} > 0 \tag{2.43}$$

to guarantee our result. Given our assumption that $\omega$ follows a log-normal distribution, it can be shown that

$$F''\left(\overline{\omega}\right) = -\frac{1}{\overline{\omega}} \left(1 + \frac{\log\left(\overline{\omega}\right) + \frac{\sigma^{2}}{2}}{\sigma^{2}}\right) F'\left(\overline{\omega}\right) \tag{2.44}$$

which allows us to rewrite (2.43):

$$E\left[\left(\frac{E\left[\overline{\omega} F'\left(\overline{\omega}\right)\right]}{E\left[1 - F\left(\overline{\omega}\right)\right]} - \frac{\log\left(\overline{\omega}\right) + \frac{\sigma^{2}}{2}}{\sigma^{2}}\right) F'\left(\overline{\omega}\right) \frac{\partial \overline{\omega}}{\partial R^{L}}\right] > 0. \tag{2.45}$$

Note there are three terms being multiplied in (2.45). Given that the last two terms, $F'(\overline{\omega})$ and $\partial\overline{\omega}/\partial R^L$, are positive, in order for the condition to hold, the first one must be positive too. A low average cut-off is enough to guarantee that will be the case.

The next step requires demonstrating $\partial\left(\frac{R^E}{R^{IB}}\right)/\partial R^L$ is positive. The added difficulty comes from the fact that $R^E$ is stochastic. Total differentiation of expression (2.28) with respect to $R^L$ yields:

$$\begin{aligned} & E\left[f(\overline{\omega})\frac{\partial\left(\frac{R^E}{R^{IB}}\right)}{\partial R^L} + f'(\overline{\omega})\frac{R^E}{R^{IB}}\frac{\partial\overline{\omega}}{\partial R^L}\right] \\ & + \lambda E\left[g(\overline{\omega})\frac{\partial\left(\frac{R^E}{R^{IB}}\right)}{\partial R^L} + g'(\overline{\omega})\frac{R^E}{R^{IB}}\frac{\partial\overline{\omega}}{\partial R^L}\right] \\ & = E\left[1 - g(\overline{\omega})\frac{R^E}{R^{IB}}\right]\frac{\partial\lambda}{\partial R^L} \end{aligned} \tag{2.46}$$

Using (2.20) and (2.38) on this expression and simplifying results in:

$$E\left[\left(f(\overline{\omega}) + \lambda g(\overline{\omega})\right)\frac{\partial\left(\frac{R^E}{R^{IB}}\right)}{\partial R^L}\right] = \frac{\partial\lambda}{\partial R^L}\frac{1}{p}. \tag{2.47}$$

This expression will hold if $\partial\left(\frac{R^E}{R^{IB}}\right)/\partial R^L$ is positive given that $f(\overline{\omega}) + \lambda g(\overline{\omega})$ is positive for every possible realization of $\overline{\omega}$.

Finally, applying a similar procedure to (2.29) yields:

$$\frac{\partial p}{\partial R^L} = E\left[g(\overline{\omega})\frac{\partial\left(\frac{R^E}{R^{IB}}\right)}{\partial R^L}\right]p^2 + E\left[g'(\overline{\omega})\right]\frac{p(p-1)\left(1-d^L\right)}{R^{IB}} \tag{2.48}$$

where previous results guarantee the right hand side must be positive. Thus, the conclusion is analogous to Bernanke et al. (1999):

$$\frac{\partial\left(\frac{R^E}{R^{IB}}\right)}{\partial p} > 0 \tag{2.49}$$

the external finance premium must be increasing in the firm's leverage.

## 2.6 Discussion

Results (2.27) and (2.34) are the main contributions of the model developed. The model predicts a modified interest rate parity condition to govern deposit rates and several factors affecting the wedge between domestic and foreign currency lending rates.

Result (2.27) requires some elaboration. The optimal contract involves not one but two dollarization decisions: that of loans and deposits. The deposit dollarization decision results in (2.27). It follows that the reason behind this result is the fact that banks are basically risk neutral when it comes to the funding decision (the bank's objective function is always linear in both types of deposit).

Turning to the implications of (2.27), the inverse relation between a deposit interest rate and its corresponding reserve requirement is evident. If we move one step further and consider $R^{IB}$, the domestic currency interbank rate, to be the monetary policy instrument, then it is clear that monetary policy can influence the domestic and foreign currency deposit rates directly or through the use of reserve requirements.

There is another implication behind (2.27). If we abandon the (long run) assumption of $E[S'] = S$, then short run fluctuations in the exchange rate imply short run movements in the bank's deposit rates.

Deposit dollarization seems to be left hanging in the air. Even though this is true in the set up, adding the household's saving decision along the lines of Devereux and Sutherland (2007) pins down this variable, without modifying the results presented above. This is shown in Appendix A.2.

Moving on to result (2.34), it provides plenty of mileage to explain the wedge between domestic and foreign currency lending rates. The first suspect is the variance of the exchange rate. Noting that $R^L$ and $R^{*L}$ are both interest factors, a small variance can help a great deal towards explaining the big difference between lending rates observed in the data. Furthermore, a small correlation between entrepreneur's returns and the exchange rate (high $\gamma$) can also help explain the difference. Empirical analysis would be required to assess these claims, but that is left for further research.

The wedge between domestic and foreign currency lending rates could be explained by other factors. Some alternative theories are:

1. Firms taking foreign currency loans are "better" (they are more productive, bigger, less risky, etc.) than firms taking domestic currency loans.

2. There is a lower inflation risk premium on foreign currency debt.

3. Firms taking loans in foreign currency have access to foreign credit markets where they can obtain cheap loans. Domestic banks are forced to offer them low interest loans while enjoying a degree of monopoly power on firms that can only borrow domestically (the latter end up paying high interest on domestic currency loans).

The first theory does not hold up because there are all types of firms taking loans both in domestic and foreign currency. Figure 2-7 shows loan dollarization ratios for a random sample of 2300 Peruvian firms.[13] It would be a mistake to think that firms that take mostly foreign currency loans are "better" than those taking domestic currency loans. Digging into the data shows that firms taking foreign currency loans tend to be associated with the export sector while those taking domestic currency loans do business domestically but there are big firms on both extremes. To illustrate this, it can be pointed out that loan dollarization of micro-credit in the sample is close to 30% while the same figure for large and medium firms is 83%. There is a significant proportion of both large and medium firms taking domestic currency loans and micro firms taking foreign currency loans.

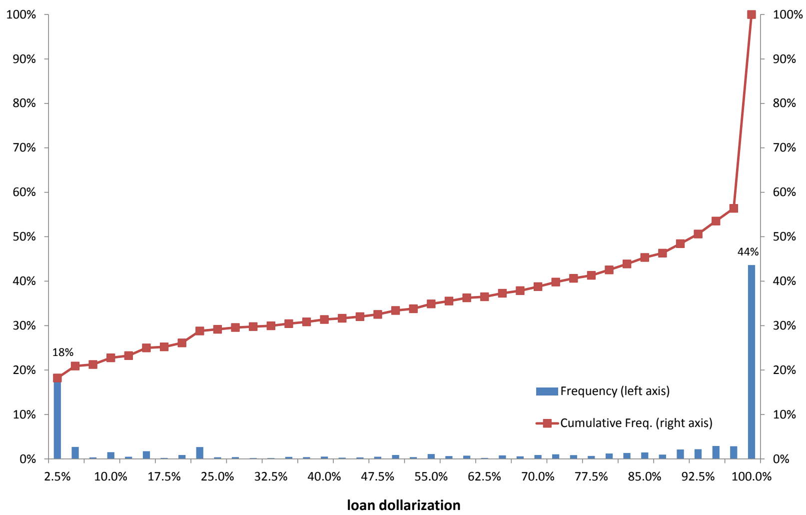


Figure 2-7: Loan dollarization on a random sample of 2300 firms (December 2010)

[13]The sample data used to elaborate Figure 2-7 was obtained from Peru's banking supervision agency.

It could be argued that these differences in composition between domestic and foreign currency loans are large enough to explain the spread between domestic and foreign currency lending rates provided large firms are somehow better than micro firms. We address that argument next.

Comparing lending rate spreads between micro-credit and commercial credit in Peru (Figures 2-8 and 2-9)[14] yields a significant difference. If firms taking commercial credit[15] are better than micro firms, how can the spreads be different? Firm quality should affect the lending rate level on both currencies in the same way.

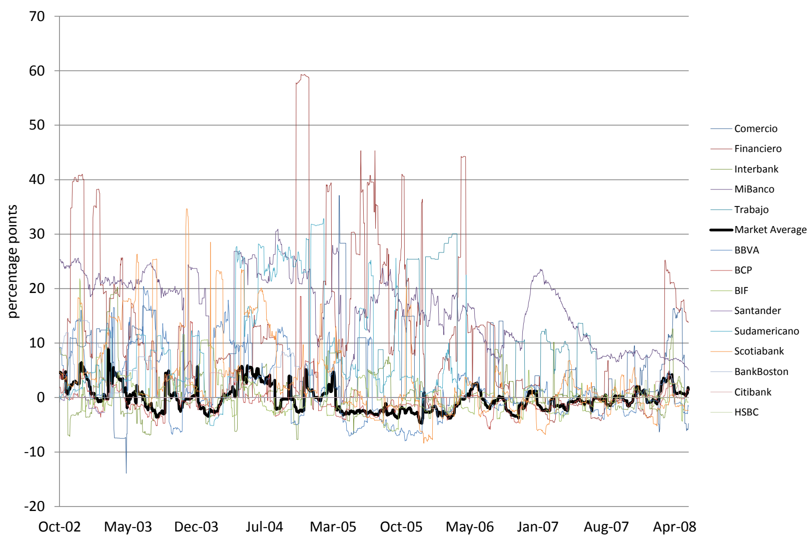


Figure 2-8: Lending rate spread (domestic minus foreign) on commercial loans

The second theory can be rejected using two arguments: first, inflation in Peru has been low during the period shown in these graphs (fluctuating around 3%). Thus, inflation in Peru and USA has been similar and the associated risk premia should not differ. Second, different inflation risk premia would yield a unique spread between domestic and foreign currency lending rates, it cannot explain the difference in spreads shown in Figures 2-8 and 2-9.

The third theory fails on similar grounds: if micro firms have no access to foreign credit markets, why would banks offer them cheaper loans on foreign currency? Foreign currency loans to micro firms should be priced with the same mark-up as domestic

[14] Data used to elaborate Figures 2-8 and 2-9 was obtained from the website of the Peruvian banking supervision agency: www.sbs.gob.pe

[15] This type of credit is generally offered to large, medium and small firms in Peru but not to micro firms.

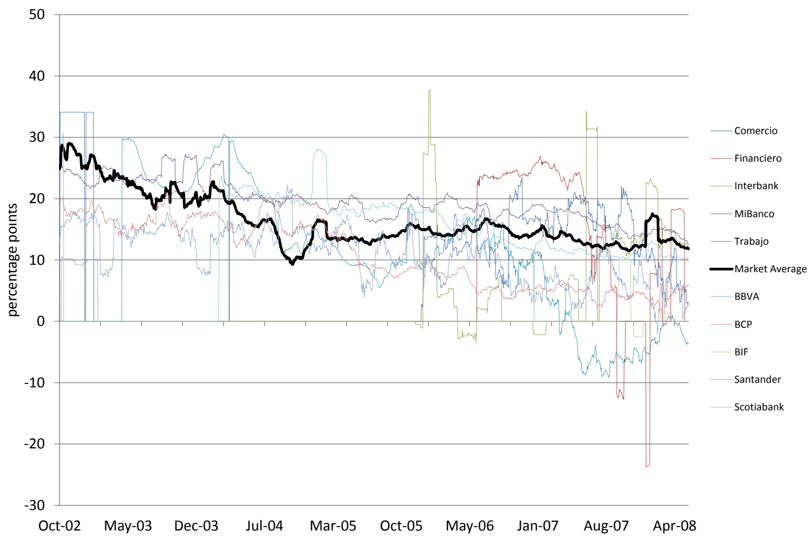


Figure 2-9: Lending rate spread (domestic minus foreign) on loans to micro firms

currency loans.

The explanation proposed in this paper is that firms taking commercial loans face a high degree of correlation between their return on assets and the nominal exchange rate (high $\gamma$). Thus, banks have no need to push down the foreign currency lending rate to entice these firms to take foreign currency loans. On the other hand, micro firms face very low correlation between their return on assets and the nominal exchange rate (low $\gamma$). Banks have to offer them substantially cheaper foreign currency loans (compared to domestic currency ones) to make them attractive. Crucially, we base our argument on the belief that neither group of firms has an extreme value of $\gamma$: we abstract from the possibility of $\gamma$ being exactly equal to zero or one (which would yield corner solutions in our model).

A more realistic characterization of the degree of credit dollarization, $d^L$, proves troublesome. Equation (2.34) shows that exchange rate risk can be compensated through the lending rate differential or by changing the degree of credit dollarization. In a more realistic context, it would be expected that the bank has to lend to a variety of entrepreneurs with different $\gamma$. Given that the bank should offer a similar interest rate differential to all of them, it follows that the degree of credit dollarization should depend on the distribution of $\gamma$ (see Appendix A.3).

Monetary policy can influence the variance of the exchange rate. In the extreme, a fixed exchange rate scheme should wipe the distinction between domestic and foreign currency lending rates according to (2.34). Regarding the correlation between entrepreneur's returns and the exchange rate, it is unclear whether monetary policy could affect the wedge between interest rates through this channel. General equilibrium analysis would probably be required to answer that question.

### 2.6.1 Thought experiment: a monetary policy shock

What are the implications of a monetary policy shock in this (partial equilibrium) model? If we take the domestic currency interbank rate, $R^{IB}$, to be the monetary policy instrument, then the analysis developed in this paper would suggest:

*Foreign and domestic currency deposit rates should increase.* This follows from (2.27). The wedge between domestic and foreign currency deposit rates remains invariant.

*Domestic currency lending rates should increase.* An increase in the domestic interbank rate implies higher bank marginal funding cost. This will drive up the equilibrium cut-off $\overline{\omega}$ (this can be shown with a total derivative of (2.28) with respect to $R^{IB}$). Given the positive relationship between the cut-off and the domestic currency lending rate, the latter must increase as well.

*Foreign currency lending rates should increase.* This follows from (2.34) and the fact that domestic currency lending rates should increase. Again, the wedge between domestic and foreign currency lending rates should remain the same.

## 2.7 Conclusion

The model presented provides some interesting insights into the relationship between the different interest rates that arise in an economy with partial financial dollarization such as Peru. Still, there is quite a lot of work pending in order to gain more insight into these relationships.

The first point that must be made is the need for quantitative analysis. Empirical test of the propositions made and evaluation of the magnitudes involved is crucial to continue progress in this area.

Another important issue is the fact that partial equilibrium does not allow a complete analysis of the implications of this mechanism for monetary policy. Incorporating this set up into a dynamic stochastic general equilibrium model should be fairly straightforward with the added benefit of being able to quantify some of the predictions through proper calibration.

Turning to the model's relevance, it is important to point out that even though the set up is motivated by financial dollarization in general and Peru's characteristics in particular, the mechanism developed has other applications. The first, and most obvious one, that comes to mind are international banks operating in several countries. These institutions "lend" to financial intermediaries worldwide in several different currencies. These local financial intermediaries can be interpreted as our "entrepreneurs" with international banks incurring in country risk wherever they lend. Obviously, a similar parallel can be made for global investment funds and other financial institutions operating worldwide. Indeed, the international macrofinance literature has recently taken great interest in this topic, encouraged by the global financial crisis.

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# A. Appendix

## A.1 Additional figures

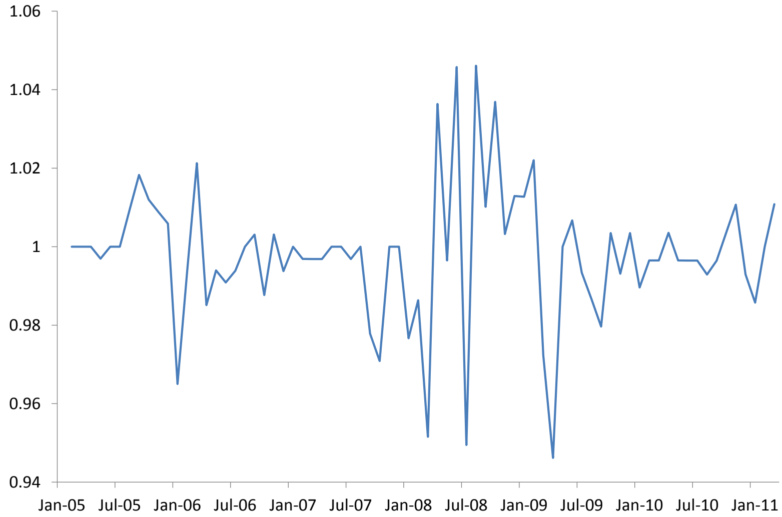



Figure 2-10: Monthly depreciation in Peru ($S_{t+1}/S_t$)

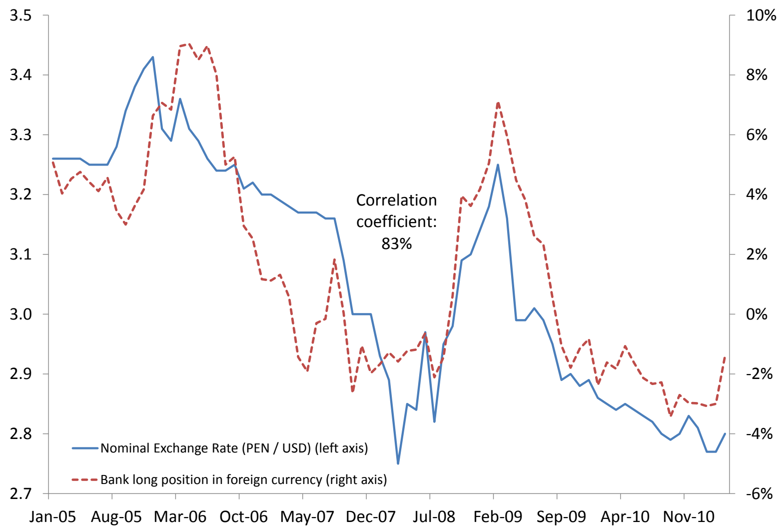



Figure 2-11: Long Position of Peru's Banking System (Net dollar assets as a percentage of total assets) and Nominal Exchange Rate

## A.2 Household portfolio decision in a partially dollarized context

This section borrows heavily from Devereux and Sutherland (2007).

Assume households have the objective:

$$\max \sum_{t=0}^{\infty} \beta^t E_t \left[ \frac{{C_t}^{1-\sigma^{-1}}}{1-\sigma^{-1}} \right]$$

subject to a budget constraint:

$$P_t C_t + D_t + D_t^* = Y_t + R_{t-1}^D D_{t-1} + R_{t-1}^{D*} \frac{S_t}{S_{t-1}} D_{t-1}^*$$

where income ($Y_t$) is exogenous, stochastic, and possibly correlated with the nominal exchange rate ($S_t$). The household must divide savings between two deposit accounts, one denominated in domestic currency ($D_t$) and the other in foreign currency ($D_t^*$). Note that savings allocated to the foreign currency deposit account must be converted to foreign currency in order to earn interest $R_t^{D*}$.

In order to set up the problem properly, we introduce two transformations. Total savings ($W_t$) are defined as:

$$D_t + D_t^* = W_t$$

Then, deposit dollarization ($\alpha_t$) is defined as:

$$\alpha_t = \frac{D_t^*}{W_t}$$

Thus, the budget constraint becomes,

$$W_t = \alpha_{t-1} \left( R_{t-1}^{D*} \frac{S_t}{S_{t-1}} - R_{t-1}^D \right) W_{t-1} + R_{t-1}^D W_{t-1} + Y_t - P_t C_t$$

The Lagrangian of the problem would be:

$$\max_{C_t, W_t, \alpha_t} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t E_t \left[ \frac{{C_t}^{1-\sigma^{-1}}}{1-\sigma^{-1}} + \lambda_t \left( \begin{array}{c} -W_t + \alpha_{t-1} \left( R_{t-1}^{D*} \frac{S_t}{S_{t-1}} - R_{t-1}^D \right) W_{t-1} \\ +R_{t-1}^D W_{t-1} + Y_t - P_t C_t \end{array} \right) \right]$$

Taking first order conditions, we obtain the following expressions:

$$C_t : \left(C_t^{-\frac{1}{\sigma}} - \lambda_t P_t\right) = 0$$

$$W_t : E_t\left[-\beta^t \lambda_t + \beta^{t+1}\lambda_{t+1}\left(\alpha_t\left(R_t^{D*}\frac{S_{t+1}}{S_t} - R_t^D\right) + R_t^D\right)\right] = 0$$

$$\alpha_t : E_t\left[\lambda_{t+1}\left(R_t^{D*}\frac{S_{t+1}}{S_t} - R_t^D\right) W_t\right] = 0$$

Combining the first and last expressions,

$$E_t\left[R_t^{D*}\frac{P_t}{P_{t+1}}\frac{S_{t+1}}{S_t}C_{t+1}^{-\frac{1}{\sigma}}\right] - E_t\left[R_t^D\frac{P_t}{P_{t+1}}C_{t+1}^{-\frac{1}{\sigma}}\right] = 0$$

Devereux and Sutherland (2007) take a second-order approximation of an analogous expression in order to figure out portfolio composition in their country portfolio set up. The main issue discussed by them is the particular point around which the approximation is done. They show that using the non-stochastic equilibrium as the reference point yields correct solutions for the equilibrium portfolio composition. Furthermore, only one second-order approximation needs to be done: the budget constraint is required in order to solve for $\alpha$ but a first-order approximation of it is sufficient.

A second-order approximation of the last expression yields:

$$\left(r_t^{D*} - r_t^D\right)\left(1 - \frac{1}{\sigma}E_t\left[c_{t+1}\right] - E_t\left[\pi_{t+1}\right]\right) - \frac{1}{\sigma}E_t\left[c_{t+1}\frac{S_{t+1}}{S_t}\right]$$

$$+\frac{1}{\sigma}E_t\left[c_{t+1}\right] - E_t\left[\frac{P_{t+1}}{P_t}\frac{S_{t+1}}{S_t}\right] + 1 + E_t\left[\pi_{t+1}\right] \approx 0$$

In steady state, all deviations are zero ($r_t^{D*}, r_t^D, c_{t+1}, \pi_{t+1}$), but the expectations on products are not. Consumption might have some covariance with the exchange rate and prices as well (particularly if the household consumption basket includes goods priced in foreign currency).

In order to solve for the equilibrium portfolio, a first-order approximation of the budget constraint is required:

$$c_{t+1} + R^D \frac{W}{PC}\left(\frac{P_{t+1}}{P_t} - 1\right) = \alpha \frac{W}{PC} R^{D*} r_t^{D*} + (1-\alpha)\frac{W}{PC} R^D r_t^D$$

$$+\alpha R^{D*}\frac{W}{PC}\left(\frac{S_{t+1}}{S_t} - 1\right) + \frac{R^D}{C}\left(\frac{W_t}{P_t} - \frac{W}{P}\right) + \frac{1}{C}\left(\frac{Y_{t+1}}{P_{t+1}} - \frac{Y}{P}\right) - \frac{1}{C}\left(\frac{W_{t+1}}{P_{t+1}} - \frac{W}{P}\right)$$

From this approximation, $E_t\left[c_{t+1}\frac{S_{t+1}}{S_t}\right]$ can be constructed and replaced in the previous expression. Further simplification and application of steady state values will yield:

$$\alpha \approx \frac{\left(R^D \frac{W}{P} - \sigma C\right) Cov\left[\frac{P'}{P}, \frac{S'}{S}\right] - Cov\left[\frac{Y'}{P'}, \frac{S'}{S}\right] + Cov\left[\frac{W'}{P'}, \frac{S'}{S}\right]}{R^D \frac{W}{P} Var\left[\frac{S'}{S}\right]}$$

Which is the result we require. Deposit dollarization is increasing in the covariance between prices and the nominal exchange rate: if prices increase when the exchange rate depreciates, foreign currency deposits hedge this risk. On the other hand, higher covariance between real income $(\frac{Y}{P})$ and the nominal exchange rate discourages deposit dollarization since income itself would be the hedge consumers require against exchange rate risk. Higher exchange rate variance discourages deposit dollarization as well.

### A.3 Heterogeneous entrepreneurs

Consider the case where there are three types of entrepreneurs, one demanding loans in foreign currency only (e.g. an exporter), one demanding loans in domestic currency only (e.g. sells only to the domestic market) and one demanding a mix of foreign and domestic currency loans (as in the model presented in Section 2.2). The objective of the following exercise is to hint at the determination of an endogenous credit dollarization ratio without a convex "cost" associated to the bank's long position in foreign currency.

Let superscript $E$ stand for our exporter entrepreneur that takes loans in foreign currency only. Using notation similar to that employed in the main body of the paper his balance sheet would be

$$K^{E}=L^{E,F}+N^{E},$$

where $L^{E,F}$ refers to real loans taken by entrepreneur $E$ on foreign currency $F$. This implies leverage,

$$p^{E}\equiv\frac{K^{E}}{N^{E}}=\frac{L^{E,F}}{N^{E}}+1.$$

We need to define a cut-off:

$$\overline{\omega}^{E}R^{e,E}K^{E}=R^{L,F}\frac{S'}{S}L^{E,F}$$

where $R^{e,E}$ is the return on capital of entrepreneur $E$ and $R^{L,F}$ is the lending interest factor on foreign currency.

The benefit of entrepreneur $E$ then can be shown to equal

$$\Pi^{E}=E\left[\int_{\overline{\omega}^{E}}^{\infty}\left(\omega^{E}-\overline{\omega}^{E}\right)dF\left(\omega^{E}\right)R^{e,E}\right]K^{E}$$

$$\Pi^{E}=E\left[f^{E}\left(\overline{\omega}^{E}\right)R^{e,E}\right]p^{E}\frac{N^{E}}{N}N$$

where $N$ is aggregate net worth over all 3 types of entrepreneurs.

Similar equations can be derived for the “domestic” (with superscript $N$) and “mixed” entrepreneurs (superscript $M$):

Domestic:

$$K^{N}=L^{N,D}+N^{N}$$

$$p^{N}\equiv\frac{K^{N}}{N^{N}}=\frac{L^{N,D}}{N^{N}}+1$$

$$\overline{\omega}^N R^{e,N} K^N = R^{L,D} L^{N,D}$$

(here $L^{N,D}$ refers to real loans taken by entrepreneur $N$ in domestic currency $D$, $R^{e,N}$ is his return on capital and $R^{L,D}$ is the lending interest factor on domestic currency)

$$\Pi^N = E\left[\int_{\overline{\omega}^N}^{\infty} \left(\omega^N - \overline{\omega}^N\right) dF\left(\omega^N\right) R^{e,N}\right] K^N$$

$$\Pi^N = E\left[f^N\left(\overline{\omega}^N\right) R^{e,N}\right] p^N \frac{N^N}{N} N$$

Mixed:

$$K^M = L^{M,D} + L^{M,F} + N^M$$

$$p^M \equiv \frac{K^M}{N^M} = \frac{L^{M,D}}{N^M} + \frac{L^{M,F}}{N^M} + 1$$

$$\overline{\omega}^M R^{e,M} K^M = R^{L,D} L^{M,D} + R^{L,F} \frac{S'}{S} L^{M,F}$$

$$\Pi^M = E\left[\int_{\overline{\omega}^M}^{\infty} \left(\omega^M - \overline{\omega}^M\right) dF\left(\omega^M\right) R^{e,M}\right] K^M$$

$$\Pi^M = E\left[f^M\left(\overline{\omega}^M\right) R^{e,M}\right] p^M \frac{N^M}{N} N$$

The new variables have similar interpretations to those presented before.

The bank's balance sheet needs to be modified to account for the different loans being given out:

$$L^{E,F} + L^{M,F} + L^{N,D} + L^{M,D} = \left(1 - \varphi^F\right) D^F + \left(1 - \varphi^N\right) D^N + IB$$

which can be rewritten as:

$$p^E \frac{N^E}{N} + p^N \frac{N^N}{N} + p^M \frac{N^M}{N} - 1 = \left(1-\varphi^F\right)\frac{D^F}{N} + \left(1-\varphi^N\right)\frac{D^N}{N} + \frac{IB}{N}$$

where, formally, $N = N^E + N^N + N^M$.

Thus, bank profit can be defined as,

$$\Pi^B = \left(\begin{array}{c} \int_{\overline{\omega}^E}^{\infty} R^{L,F}\frac{S'}{S}L^{E,F}dF\left(\omega^E\right) + (1-\mu)\int_0^{\overline{\omega}^E}\omega^E R^{e,E}K^E dF\left(\omega^E\right) \\ +\int_{\overline{\omega}^N}^{\infty} R^{L,D}L^{N,D}dF\left(\omega^N\right) + (1-\mu)\int_0^{\overline{\omega}^N}\omega^N R^{e,N}K^N dF\left(\omega^N\right) \\ +\int_{\overline{\omega}^M}^{\infty}\left(R^{L,D}L^{M,D} + R^{L,F}\frac{S'}{S}L^{M,F}\right)dF\left(\omega^M\right) \\ +(1-\mu)\int_0^{\overline{\omega}^M}\omega^M R^{e,M}K^M dF\left(\omega^M\right) \\ +R^{R,N}\varphi^N D^N - R^{D,N}D^N + R^{R,F}\frac{S'}{S}\varphi^F D^F - R^{D,F}\frac{S'}{S}D^F - R^{IB}IB \end{array}\right)$$

which can be rewritten in terms of cut-off and leverage ratios to yield:

$$\Pi^B = \left(\begin{array}{c} g^E\left(\overline{\omega}^E\right)R^{e,E}p^E\frac{N^E}{N} + g^N\left(\overline{\omega}^N\right)R^{e,N}p^N\frac{N^N}{N} + g^M\left(\overline{\omega}^M\right)R^{e,M}p^M\frac{N^M}{N} \\ -\left(1-\varphi^N\theta^{R,N}\right)R^{D,N}\frac{D^N}{N} - \left(1-\varphi^F\theta^{R,F}\right)R^{D,F}\frac{S_{+1}}{S}\frac{D^F}{N} - R^{IB}\frac{IB}{N} \end{array}\right)N$$

Thus, the bank's problem consists in maximizing $\Pi^E+\Pi^N+\Pi^M$ subject to $\Pi^B=0$ and the modified bank balance sheet holding. Note that there are implicit weights in the definition of entrepreneurial benefits and they depend on the relative size of entrepreneurs' net worth.

The solution to this problem is similar to the one found in the main body with respect to the funding side ($R^{D,N}$, $R^{D,F}$ and $R^{IB}$) but a bit different when it comes to first order conditions coming from lending interest factors, leverage and credit dollarization:

$$
\begin{aligned}
R^{L,D} : 0 = {} & E\left[f_1^N\left(\overline{\omega}^N\right) + \lambda g_1^N\left(\overline{\omega}^N\right)\right]\left(p^N - 1\right)\frac{N^N}{N} \\
& + E\left[f_1^M\left(\overline{\omega}^M\right) + \lambda g_1^M\left(\overline{\omega}^M\right)\right]\left(p^M - 1\right)\left(1 - d^M\right)\frac{N^M}{N}
\end{aligned}
$$

$$
\begin{aligned}
R^{L,F} : 0 = {} & E\left[\left(f_1^E\left(\overline{\omega}^E\right) + \lambda g_1^E\left(\overline{\omega}^E\right)\right)\frac{S'}{S}\right]\left(p^E - 1\right)\frac{N^E}{N} \\
& + E\left[\left(f_1^M\left(\overline{\omega}^M\right) + \lambda g_1^M\left(\overline{\omega}^M\right)\right)\frac{S'}{S}\right]\left(p^M - 1\right)d^M\frac{N^M}{N}
\end{aligned}
$$

$$
\begin{aligned}
p^E : 0 = {} & E\left[f^E\left(\overline{\omega}^E\right)R^{e,E}\right]p^E + E\left[f_1^E\left(\overline{\omega}^E\right)\frac{S'}{S}\right]R^{L,F} \\
& + \lambda\left(E\left[g^E\left(\overline{\omega}^E\right)R^{e,E}\right]p^E + E\left[g_1^E\left(\overline{\omega}^E\right)\frac{S'}{S}\right]R^{L,F} - R^{IB}p^E\right)
\end{aligned}
$$

$$
\begin{aligned}
p^N : 0 = {} & E\left[f^N\left(\overline{\omega}^N\right)R^{e,N}\right]p^N + E\left[f_1^N\left(\overline{\omega}^N\right)\right]R^{L,D} \\
& + \lambda\left(E\left[g^N\left(\overline{\omega}^N\right)R^{e,N}\right]p^N + E\left[g_1^N\left(\overline{\omega}^N\right)\right]R^{L,D} - R^{IB}p^N\right)
\end{aligned}
$$

$$
\begin{aligned}
p^M : 0 = {} & E\left[f^M\left(\overline{\omega}^M\right)R^{e,M}\right]p^M + E\left[f_1^M\left(\overline{\omega}^M\right)\right]R^{L,D} \\
& + \lambda\left(E\left[g^M\left(\overline{\omega}^M\right)R^{e,M}\right]p^M + E\left[g_1^M\left(\overline{\omega}^M\right)\right]R^{L,D} - R^{IB}p^M\right)
\end{aligned}
$$

$$
\begin{aligned}
d^L : & \left(E\left[f_1^M\left(\overline{\omega}^M\right)\frac{S'}{S}\right] + \lambda E\left[g_1^M\left(\overline{\omega}^M\right)\frac{S'}{S}\right]\right)R^{L,F} \\
& = E\left[f_1^M\left(\overline{\omega}^M\right) + \lambda g_1^M\left(\overline{\omega}^M\right)\right]R^{L,D}
\end{aligned}
$$

$$\lambda : 0 = E\left[g^{E}\left(\overline{\omega}^{E}\right)R^{e,E}\right]p^{E}\frac{N^{E}}{N} + E\left[g^{N}\left(\overline{\omega}^{N}\right)R^{e,N}\right]p^{N}\frac{N^{N}}{N}$$
$$+ E\left[g^{M}\left(\overline{\omega}^{M}\right)R^{e,M}\right]p^{M}\frac{N^{M}}{N} - R^{IB}\left(p^{E}\frac{N^{E}}{N} + p^{N}\frac{N^{N}}{N} + p^{M}\frac{N^{M}}{N} - 1\right)$$

where $f_1$ and $g_1$ refer to the derivatives of functions $f$ and $g$ respectively.

Note that in this system, the relative weights of the entrepreneurs ($\frac{N^E}{N}$, $\frac{N^N}{N}$, and $\frac{N^M}{N}$) matter. Finding a solution to the problem would require numerical methods but such a solution should pin down the dollarization ratio in terms of the relative weights and the particular distributions of the idiosyncratic shock for each type (if the distributions were the same, functions $f$ and $g$ would be unique).

# Chapter 3

# Interbank Market and Macroprudential Tools in a DSGE Model

## 3.1 Introduction

In this paper, we study the effects of transitory changes in reserve requirements in a general equilibrium context. In particular, we are interested in the short-run effects of transitory shocks to reserve requirements on real and financial variables and the transmission mechanism behind those effects. We also explore the interaction of reserve requirement shocks with traditional, interest-rate based, monetary policy shocks.

We find that reserve requirement shocks are qualitatively similar to traditional monetary policy shocks. They generate a short-run fall in inflation, output and asset prices while pushing up lending and deposit rates. However, reserve requirement shocks differ from monetary policy shocks in that they expand interbank lending and contract households' deposits in the model. Additionally, we show that changes in reserve requirements can complement traditional monetary policy actions such as a hike in interest rates. Thus, our policy-maker can obtain the same desired impact on real aggregates with a smaller change in the interest rate, provided he is willing to complement his actions with a change to reserve requirements.

In our model, an increase in reserve requirements reduces the loanable funds of financial intermediaries. These institutions will react demanding more interbank lending, pushing up the interest rate charged on those operations due to higher monitoring costs (a financial friction). Thus, banks' average cost of funding will increase. This cost hike will be transmitted to lending rates and deposits rates, resulting in a slowdown of economic activity.

When used together, reserve requirements can partly substitute interest rate hikes. The reason behind this is that both variables affect banks' average cost of funding. This is particularly relevant when the policy-maker faces a situation where the desired response would be a very big shift in the policy rate,[1] or, even worse, the required policy action entails getting uncomfortably close to the zero lower bound.

Standard New-Keynesian models cannot accommodate both a reserve requirement shock and monetary policy formulated by a Taylor Rule. The reason is that changes to reserve requirements alter the monetary base but the latter becomes endogenous when the monetary policy interest rate is governed by a Taylor Rule.[2] Any reserve requirement "shocks" become undone immediately by endogenous changes to the policy rate.

In spite of the theoretical conundrum exposed above, central banks in Latin America have been using reserve requirements as a policy tool recently, usually in conjunction with traditional, interest-rate based, policy actions. Furthermore, Tovar et al. (2012) provide empirical and anecdotal evidence that *"monetary and macroprudential instruments, including reserve requirements, appear to have complemented each other in recent episodes"*.

Reserve requirements have also been used as a macroprudential tool in Latin America for the past decade. As Tovar et al. (2012) report,

> *"...policy makers in Latin America have adopted a number of macroprudential instruments to manage the procyclicality of bank credit dynamics to the private sector and contain systemic risk. Reserve requirements, in particular, have been actively employed."*

[1] Historically, policy makers tend to be reluctant to do this, possibly as an endogenous response to uncertainty.

[2] This is the case in the models developed in Clarida et al. (1999) and Bernanke et al. (1999), for example.

In order to have changes in reserve requirements interacting with traditional monetary policy (i.e.: a Taylor Rule), we propose a New-Keynesian model incorporating a financial system with frictions, particularly in the interbank market. It turns out that very little work has been done in this area: studying interbank markets and the frictions associated with them in a general equilibrium context is a relatively new subject. Thus, our work also contributes to the literature by providing a fresh take on how to model the agents that participate in this market and their interactions.

Interbank markets play an important role in the transmission process from monetary policy to economic activity because they help allocate resources between financial institutions. Financial frictions (usually involving credit) and regulation (hair-cuts, reserve requirements, and collateral constraints) constitute important features of interbank markets that have an impact on their effectiveness in amplifying or dampening the real effects of monetary policy.

The rest of the paper is organized as follows. Section 3.2 is devoted to short review of related literature, section 3.3 describes our model with an interbank market. Section 3.4 details the calibration procedure. Section 3.5 presents our results. Finally, section 3.6 concludes.

## 3.2 A (Short) Literature Review on Reserve Requirements and Interbank Markets

### 3.2.1 Reserve Requirements

The literature on reserve requirement shocks in general equilibrium is very scarce. Our model bears some resemblance to Edwards and Vegh (1997) and, more recently, Prada (2008). The work of Edwards and Vegh (1997) shows how foreign business cycles and shocks to the banking system affect output and employment through fluctuations in bank credit. In this context, they explore the countercyclical use of reserve requirements and find they can be used to insulate the economy from the world business cycle. In order to obtain this result, Edwards and Vegh (1997) assume the production of banking services is costly.

Costly banking services are present in Prada (2008) as well. This author elaborates on the work done by Edwards and Vegh (1997) adding New-Keynesian rigidities to the open economy (Calvo (1983) pricing, investment adjustment costs in the spirit of Christiano et al. (2005), etc.). He finds that reserve requirements do not have quantitatively significant effects.

Our model bears some resemblance to Edwards and Vegh (1997) because the financial friction we impose on the interbank market (a monitoring cost) is akin to their "costly banking services". Yet, neither Edwards and Vegh (1997) nor Prada (2008) include an interbank market in their model. More importantly, their findings with respect to reserve requirements are different from ours. Edwards and Vegh (1997) does not study short-run changes in reserve requirements and their effect on real aggregates nor how they interact with policy rates. Prada (2008) dismisses reserve requirements because his quantitative results are not significant while we find that reserve requirements impact the economy in a similar manner than policy rates and, more importantly, they can be used to complement policy rate hikes.

There is a broader literature on reserves and how monetary policy can be carried out by altering them. This literature has become particularly relevant given the use of quantitative easing (QE) by major economies in the aftermath of the financial crisis of 2008. QE can be interpreted as an expansion of the central bank's balance sheet, via creation of reserves (printing money) or taking additional liquid fiscal liabilities. The extra reserves are then used to purchase assets from the financial or private sector. Thus, this increase in reserves is expansionary because it provides liquidity to financial institutions (and the private sector) in exchange for their illiquid, risky assets. In our paper, an increase in reserves is obtained imposing higher reserve requirements on retail banks. Thus, our expansion of reserves involves reducing retail bank's liquidity, not expanding it. Given that liquidity becomes more scarce, interest rates increase and the economy slows down.

Chadha and Corrado (2012) study a model in which banks choose their optimal asset mix between loans and reserves (required and voluntary). They find justification for Basel III type policies that aim at providing incentives for banks to maintain greater reserves. However, they claim that fomenting cyclical variations in reserve holdings (by, for example, setting the interest paid on reserves as a function of the policy rate) could help limit the procyclicality of private sector lending and increase the efficacy of monetary policy. Chadha and Corrado (2012) assume the existence of an exogenous target for reserves which loosely corresponds to our reserve requirement. However, Chadha and Corrado (2012) allow for endogenous deviations from the target and suggest the possibility of actively changing the interest paid on reserves to help stabilize the economy. In our set up, the (target) reserve requirement must be met exactly. Banks have no incentive to accumulate reserves beyond the requirement (they pay little interest) and are not allowed by regulation to let reserves fall below the requirement. Thus, our requirement is more strict than Chadha and Corrado (2012) where reserves falling below target impose a (marginally increasing) penalty on the bank. Furthermore, we study the effects of shocks to required reserves (as a fraction of deposits) and assume reserve remuneration is fixed. Thus, Chadha and Corrado (2012) study how incorporating endogenous reserves (compared to fixed) changes the economy's responses to various shocks. Our concern is how shocks to required reserves (set exogenously by regulation) impact the economy.

Chadha et al. (2012) extends Chadha and Corrado (2012) to allow for the possibility of QE type policies where the central bank expands its balance sheet via reserve creation or higher fiscal liabilities. They show that QE can help stabilise the economy in the event of a negative shock by easing financial conditions when the interest rule is constrained. QE in their model operates by improving banks' asset mix in favour of more liquidity, which reduces the impact of the negative shock on the external finance premium. Note though that the increase in reserve provision on the part of the central bank is achieved by creating reserves or expanding fiscal liabilities. The central bank is bringing new liquidity into the financial system. In our set up, an expansion is achieved with a contraction of required reserves because this liberates liquidity already present in the system, giving banks more resources to lend.

### 3.2.2 Interbank Markets

The financial accelerator of Bernanke et al. (1999) usually amplifies, spreads, and gives more persistence to different types of shocks in the economy, particularly shocks that directly affect financial intermediaries. After the financial crisis of 2007 - 2009, several economists use Bernanke et al. (1999) as a stepping stone for valid extensions of the original model. One of those extensions is the inclusion of an interbank market. As Walsh (2010) points out, imperfect credit markets make the policy interest rate insufficient to characterize the monetary policy stance. Moreover, credit effects may arise when frictions are present in these financial markets. Thus, one source of motivation for recent research is the nature of the transmission of monetary policy through more than one interest rate (interest rate pass-through) and the conditions of such transmission (the nature of credit markets).

The inclusion of credit markets and, more importantly, their imperfections in general equilibrium analysis has yielded novel implications for the formulation of monetary policy. Curdia and Woodford (2010) introduce an endogenous spread between lending and deposit rates justified by monitoring costs. They found that the optimal Taylor rule in this setup calls for a response to credit spreads. Christiano et al. (2013) introduce exogenous shocks to the cross-section variance of entrepreneurial returns in the costly state verification framework. Their paper implies that optimal monetary policy could require a response to a risk premium (the difference between the lending rate offered to an entrepreneur undertaking a risky project and the risk-free rate). Monacelli (2008) proposes a model where borrowing households face a collateral constraint on nominal borrowing. He finds that the Ramsey monetary policy in the presence of this imperfection requires some smoothing of durable prices which affect the value of assets that act as collateral.

The recent literature reviews of Carrera (2012) and Roger and Vlcek (2012), highlight the lack of models featuring an interbank market. In that regard, the work of Gerali et al. (2010), Curdia and Woodford (2010), Dib (2010), and Hilberg and Hollmayr (2011) are among the first on this arena.

The banking sector in Gerali et al. (2010) encompasses many banks each composed of two "retail" branches and one "wholesale" unit. The first retail branch is responsible for giving out differentiated loans to households and entrepreneurs; the second for raising deposits. The wholesale unit manages the capital position of the group. In Curdia and Woodford (2010), the frictions associated with financial intermediation (intermediation requires real resources and bank lending activities create opportunities for borrowers to take out loans without being made to repay) determine both the spread between borrowing and lending rates and the resources consumed by the intermediary sector. Dib (2010) introduces the distinction between banks that only raise deposits and banks that only give out credit, and sets them up in an interbank market in which the first group of banks borrows from the second group.

Hilberg and Hollmayr (2011) take a different approach and separate the interbank market in two types of banks: commercial banks and investment banks. Hilberg and Hollmayr notice that only a few banks actually interact with the central bank, and then fund the rest of the banking system. While the capital of the banks plays an important role in Gerali et al. (2010) and Dib (2010), for Hilberg and Hollmayr (2011) it is the structure of the market and collateral that matters the most.

We partially follow on the structure of Hilberg and Hollmayr (2011) (see Figure 3-1). The hierarchical interbank market is a good representation of the structure in the U.S. (only Primary Dealers deal with the central bank whereas a vast group of commercial banks is not allowed to deal directly with the monetary authority) and in Europe (only 6 out of 2500 banks are allowed to participate in the bidding process in main refinancing operations of the ECB and other banks rely on interbank funding).[3]

We depart from Hilberg and Hollmayr (2011) in four dimensions: (i) retail banks are subject to required reserves,[4] (ii) narrow banks incur in monitoring costs, (iii) retail banks obtain funding from households and narrow banks, not the central bank and (iv) the bond market is used by the central bank to implement monetary policy in the form of open market operations.

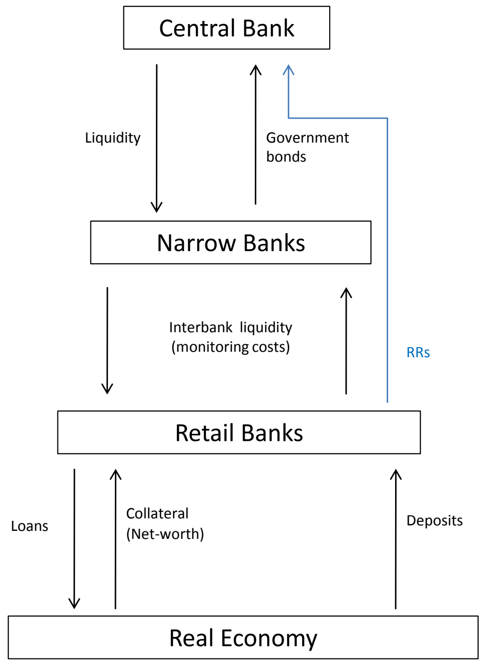
The interbank market is key for central banks because it is the place where financial institutions are allowed to trade liquidity and a monetary model that carefully considers the central bank's transmission mechanism should closely study its structure and main characteristics.

It is a fact that there is asymmetric information in any interbank market. Incomplete information in the form of banks' risk exposures characterizes this market (see, for

[3] See Chapter 11 in Walsh (2010) for a description of the FED's operating procedures, and http://www.ny.frb.org/markets/primarydealers.html for more information on the FED's Primary Dealers.

[4] These can also be interpreted as liquidity requirements in line with Basel III proposals.

Figure 3-1: Interbank market structure




example, Pritsker (2013) and Freixas and Jorge (2008)). On the other hand, Dinger and Hagen (2009) point out that banks are particularly good at identifying the risk of other banks and present evidence of the importance of interbank transactions. King (2008) shows that high risk banks pay more than safe banks for interbank loans.

Thus, our hierarchical modelling strategy of the financial system is justified by the need to incorporate the monitoring costs that arise when asymmetric information is present. Our point is that there are interesting results that can be derived from the observed structure of the interbank market. We argue that our work complements those in which the characterization of the interbank market includes a financial friction, ours taking the form of a monitoring cost.

We incorporate monitoring costs in the same fashion as Curdia and Woodford (2010). In doing this, we find that reserve requirements can actually complement the effects of the interest rate, a result that helps understand the importance of this macro-prudential tool.

## 3.3 The Model

Our model exhibits a fairly standard real sector coupled with the financial accelerator mechanism of Bernanke et al. (1999) (taking some additional elements of Cohen-Cole and Martínez-García (2010)). On top of this, we add an interbank market structure along the lines of Hilberg and Hollmayr (2011), with bank monitoring costs in Curdia and Woodford (2010) fashion.

Even though this model does not justify the existence of banks (or why they should be regulated), it is still flexible enough to capture the transmission of monetary policy with an interbank market operating. In that sense, banks are assumed to be essential because they provide households with the only risk-free asset in the economy (deposits) and entrepreneurs can only attract external finance from banks.[5]

There are two financial frictions in the model: one on the liability side of retail banks and one on the asset side of narrow banks. Our first friction takes the form of an adjustment cost on deposit rates (given imperfect competition in the banking sector, a la Gerali et al. (2010)). Our second friction arises from convex monitoring costs (a la Curdia and Woodford (2010)) originated by interbank loans from narrow banks to retail banks.

Finally, our model has one-period nominal loan contracts. Contracts are nominal by assumption but we consider the feature to be realistic and it has the added benefit of allowing us to introduce (minor) Fisherian debt deflation effects in the monetary policy transmission mechanism.

### 3.3.1 Households

We assume a continuum of households that have an identical utility function. The utility function of each household is additively separable in consumption, $(C_t)$, real cash holdings, $(CHS_t/P_t)$, and labor $(H_t)$. Thus, the household's objective is to maximize:

[5]The model abstracts from bank moral hazard, bank runs, etc. (as in, for example, Dib (2010)) in order to stress the role of the interbank market and its frictions.

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \frac{(C_s - bC_{s-1})^{1-\sigma^{-1}}}{1-\sigma^{-1}} + \chi_M \frac{\left(\frac{CSH_s}{P_s}\right)^{1-\sigma_M^{-1}}}{1-\sigma_M^{-1}} - \chi_H \frac{H_s^{1+\varphi^{-1}}}{1+\varphi^{-1}} \right\} \tag{3.1}$$

where $0 < \beta < 1$ is the subjective intertemporal discount factor, $b$ is the habit parameter in household consumption, $\sigma > 0$ and $\sigma_M > 0$ are the elasticities of intertemporal substitution of consumption and real cash holdings respectively, and $\varphi > 0$ is the Frisch elasticity of labor supply.

We include real cash holdings in the household's utility function to generate a money demand. This "money in the utility function" (MIU) approach has been studied extensively in the literature (see, for example, Walsh (2010)). It is usually rationalized arguing that money holdings provide transaction services, facilitating the acquisition of consumption goods by, for example, reducing the time needed to purchase them. It should be noted that without this assumption households would never hold cash: any asset offering a positive return (e.g.: deposits) would be a superior substitute.

Household income is derived from renting labor to wholesale producers at competitive nominal wages ($W_t$). Given that households own the retailers and capital goods producers, they receive their total real profits ($\Pi_t^R$ and $\Pi_t^K$ respectively). The unanticipated profits of retail banks are also fully rebated to households in each period ($\Pi_t^{RB}$). Turning to assets, households demand one period deposits which pay a fixed nominal interest, invest in shares which entitle them to a proportional fraction of the narrow banks' dividends ($DIV_t^{NB}/S_{t-1}$) and hold cash balances between periods. Available income is used to finance aggregate consumption ($C_t$), open new deposits ($D_t$), invest in shares ($P_t^S S_t$), hold cash ($CSH_t/P_t$), and pay the real (lump-sum) tax bill ($T_t^S + T_t^B$). Therefore, the households' budget constraint is defined as:

$$
\begin{aligned}
C_t + T_t^S + T_t^B + D_t + P_t^S S_t + \frac{CSH_t}{P_t} &= \frac{W_t}{P_t} H_t + R_{t-1}^D D_{t-1} \frac{P_{t-1}}{P_t} \\
&+ \left( \frac{DIV_t^{NB} + P_t^S S_{t-1}}{P_{t-1}^S S_{t-1}} \right) P_{t-1}^S S_{t-1} + \frac{CSH_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} \\
&+ \Pi_t^R + \Pi_t^K + \Pi_t^{RB}
\end{aligned} \tag{3.2}
$$

where $R_t^D$ is the nominal one-period interest rate offered to depositors by retail banks, $P_t^S$ is the narrow bank's relative price per share, and $P_t$ is the consumer price index (CPI, defined later). As a convention, $D_t$ denotes real deposits from time $t$ to $t+1$. Therefore, the interest rate $R_t^D$ paid at $t+1$ is known and determined at time $t$.

From the household's first order conditions we obtain,

$$
(C_t - bC_{t-1})^{-\frac{1}{\sigma}} - \lambda_t = \beta b E_t \left[ (C_{t+1} - bC_t)^{-\frac{1}{\sigma}} \right] \tag{3.3}
$$

$$
\chi_H H_t^{\frac{1}{\phi}} = \lambda_t \frac{W_t}{P_t} \tag{3.4}
$$

$$
\lambda_t = \beta E_t \left[ \lambda_{t+1} \frac{P_t}{P_{t+1}} \right] R_t^D \tag{3.5}
$$

$$
E_t \left[ \lambda_{t+1} \frac{P_t}{P_{t+1}} \right] R_t^D = E_t \left[ \lambda_{t+1} \left( \frac{DIV_{t+1}^{NB} + P_{t+1}^S S_t}{P_t^S S_t} \right) \right] \tag{3.6}
$$

$$
\chi_M \left( \frac{CSH_t}{P_t} \right)^{-\frac{1}{\sigma_M}} = \lambda_t \left( \frac{R_t^D - 1}{R_t^D} \right) \tag{3.7}
$$

thus, (3.3) implies the Lagrange multiplier associated to the household's budget constraint, $\lambda_t = (C_t - bC_{t-1})^{-\frac{1}{\sigma}} - \beta b E_t \left[ (C_{t+1} - bC_t)^{-\frac{1}{\sigma}} \right]$, is the marginal utility of consumption.

Condition (3.5) is the Euler equation that links consumption to the deposit rate and past consumption (given habit formation). Condition (3.4) is the household's labour supply.

Money demand is characterized by equation (3.7) and equation (3.6) captures the demand for shares: the household will demand them until the real return on shares equals the real return on the alternative financial instrument, deposits.

### 3.3.2 Wholesale Producers

We assume the existence of a representative wholesale producer operating under perfect competition. Our wholesale producer employs entrepreneurial ($H_t^E$) and household ($H_t$) labor combined with rented capital goods ($K_t$) in order to produce homogeneous wholesale goods ($Y_t^W$). The technology involved is Cobb-Douglas:

$$Y_t^W = e^{a_t}(K_t)^{1-\psi-\varrho}(H_t)^{\psi}(H_t^E)^{\varrho} \tag{3.8}$$

where $a_t$ is a productivity shock.

In this constant returns-to-scale technology, the non-managerial and managerial labor shares in the production function are determined by the coefficients $0 < \varrho < 1$ and $0 < \psi < 1$. As in Bernanke et al. (1999), the managerial share ($\varrho$) is assumed to be very small. The productivity shock follows an AR(1) process of the following form:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{3.9}$$

where $\varepsilon_t^a$ is normal i.i.d. (with zero mean and $\sigma_a^2$ variance) and $\rho_a$ captures the degree of persistence of the shock.

Wholesale producers seek to maximize their nominal profits:

$$P_t\Pi_t^W = P_t^W Y_t^W - R_t^W K_t - W_t H_t - W_t^E H_t^E \tag{3.10}$$

where $\Pi_t^W$ is the real profit of the wholesale producer, $P_t^W$ is the nominal price of the wholesale good, $R_t^W$ is the nominal rent paid per unit of capital to entrepreneurs, and $W_t$ and $W_t^E$ are the nominal wages of household and entrepreneurial labor respectively.

The first order conditions for this problem result in the usual demands for labor (household and entrepreneurial) and capital,

$$R_t^W = (1 - \psi - \varrho)\frac{P_t^W Y_t^W}{K_t} \tag{3.11}$$

$$W_t = \psi \frac{P_t^W Y_t^W}{H_t} \tag{3.12}$$

$$W_t^E = \varrho \frac{P_t^W Y_t^W}{H_t^E} \tag{3.13}$$

Wholesale producers make zero profits. Households, who own these firms, do not receive any dividends. Entrepreneurs receive income from their supply of managerial labor and rented capital to wholesalers. Wholesale producers rent capital from the entrepreneurs and return the depreciated capital after production has taken place.

### 3.3.3 Capital Goods Producers

We assume a continuum of competitive capital goods producers who at time $t$ purchase a bundle of retail goods that will be used as "investment" ($X_t$) and depreciated capital ($(1-\delta)K_t$) to manufacture new capital goods ($K_{t+1}$). The production of new capital is limited by technological constraints. We assume that the aggregate stock of new capital considers investment adjustment costs and evolves following the law of motion:

$$K_{t+1} = (1-\delta)K_t + \Phi\left(\frac{X_t}{X_{t-1}}\right) X_t \tag{3.14}$$

where $\Phi(\cdot)$ is an investment adjustment cost function. We follow Christiano et al. (2005) and describe the technology available to the capital good producer as:

$$\Phi\left(\frac{X_t}{X_{t-1}}\right) = \left[1 - 0.5\kappa \frac{\left(\frac{X_t}{X_{t-1}} - 1\right)^2}{\frac{X_t}{X_{t-1}}}\right] \tag{3.15}$$

where $\frac{X_t}{X_{t-1}}$ is the investment growth rate and $\kappa > 0$ regulates the degree of concavity of the technological constraint. Note that given our functional choice, $\Phi(1) = 1$ and $\Phi'(1) = 0$ implying constant returns to scale in steady state only.[6]

A representative capital goods producer chooses his investment ($X_t$) and depreciated capital ($(1-\delta)K_t$) demand to maximize the expected discounted value of his profits, solving the following problem:

$$E_t \sum_{s=t}^{\infty} M_{s-t}^{H} \left\{ Q_s K_{s+1} - (1-\delta) Q_s K_s - X_s \right\} \tag{3.16}$$

where $M_{s-t}^{H}$ is a stochastic discount factor and $Q_t$ is the price of new capital for entrepreneurs which determines the relative cost of investment in units of consumption (Tobin's Q).

Since households own the capital goods producers, the latter compute present value using the household's stochastic discount factor defined as:

$$\begin{aligned} M_{\tau}^{H} &= \beta^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \frac{P_t}{P_{t+\tau}} \\ &= \begin{cases} 1 & \tau = 0 \\ \prod_{i=0}^{\tau-1} \frac{1}{R_{t+i}^{D}} & \tau > 0 \end{cases} \end{aligned} \tag{3.17}$$

where the second equality can be obtained using the household's Euler equation.

Given the capital goods producer's production function, the marginal rate of transformation of depreciated capital to new capital is unity. Thus, it must be the case that, in equilibrium, depreciated and new capital share the same price $Q_t$. Furthermore, any quantity of depreciated capital is profit-maximizing as long as its price is the same as that of new capital: the capital good producer's demand for depreciated capital is perfectly elastic at price $Q_t$. Since entrepreneurs (discussed later) will supply depreciated capital inelastically, market clearing guarantees capital goods producers acquire all the depreciated capital stock from entrepreneurs at price $Q_t$. Thus, we take a short-cut to

[6] This follows from the realization that the marginal product of investment in the production of new capital is $\Phi'\left(\frac{X_t}{X_{t-1}}\right)\frac{X_t}{X_{t-1}} + \Phi\left(\frac{X_t}{X_{t-1}}\right)$, an expression that equals unity only in steady state where $\frac{X_t}{X_{t-1}} = 1$.

this result incorporating $(1-\delta)Q_tK_t$ directly in the capital goods producer objective function.

The first order conditions derived from the optimization process of the capital goods producers yield a standard link between our Tobin's Q analogue ($Q_t$) and investment ($X_t$):

$$Q_t\left[\Phi\left(\frac{X_t}{X_{t-1}}\right)+\Phi'\left(\frac{X_t}{X_{t-1}}\right)\frac{X_t}{X_{t-1}}\right]=1+\frac{1}{R_t^D}E_t\left[Q_{t+1}\Phi'\left(\frac{X_{t+1}}{X_t}\right)\left(\frac{X_{t+1}}{X_t}\right)^2\right] \quad (3.18)$$

Aggregate profits for the capital goods producers are defined as:

$$\Pi_t^K = Q_tK_{t+1}-(1-\delta)Q_tK_t - X_t \quad (3.19)$$

Given perfect competition, there are no profits in steady state: if $X_t = X$ for all $t$, equation (3.14) yields $K_{t+1}-(1-\delta)K_t - X_t = 0$ and equation (3.18) yields $Q_t = 1$. Together, these results guarantee profits are null in steady state. However, during the transition towards steady state, capital goods producers can generate short-term profits (or losses) because their production function no longer has constant returns to scale in the inputs $(K_t, X_t)$ when $X_{t-1}$ differs from $X_t$: the marginal product of investment, $\Phi'\left(\frac{X_t}{X_{t-1}}\right)\frac{X_t}{X_{t-1}}+\Phi\left(\frac{X_t}{X_{t-1}}\right)$, deviates from unity. When transitioning from one steady state equilibrium to another, the adjustment cost function deviates from its optimal level (unity), changing the marginal product of investment.

### 3.3.4 Retailers

There is a continuum of retailers indexed by $z \in [0,1]$ that purchase the homogeneous good ($Y_t^W$) from wholesalers and differentiate it costlessly in order to sell it to households, entrepreneurs, and capital goods producers (for consumption or investment). These customers love variety and demand a CES bundle ($Y_t$) composed by the differentiated varieties ($Y_t(z)$) offered by retailers, aggregated with elasticity of substitution $\theta > 1$:

$$Y_t = \left[ \int_0^1 Y_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{3.20}$$

Standard optimization of a CES utility defined over the retail good varieties yields a relative demand for each variety:

$$Y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\theta} Y_t \tag{3.21}$$

where $P_t(z)$ is the price of variety $z$ being charged at time $t$ and $P_t$ is a price index given by:

$$P_t = \left[ \int_0^1 P_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}} \tag{3.22}$$

Given the opportunity, a retailer will set its price $(P_t(z))$ to maximize the expected discounted value of its profit stream. Because of the market structure in which these firms operate (monopolistic competition), they have the power to charge a retail mark-up over the price of the homogeneous good. However, their re-optimizing processes are constrained by nominal rigidities as in Calvo (1983). At time $t$, an individual retailer maintains its price fixed from the previous period with probability $0 < \alpha < 1$. Thus, it is allowed to optimally reset its price with probability $(1 - \alpha)$.

Our paper concentrates on the effects of reserve requirements and the incorporation of an interbank market into a relatively standard New - Keynesian DSGE framework, without delving into welfare implications or optimal policy analysis. We choose to eliminate the distortion introduced by retailers' mark-up pricing in order to simplify the algebra, ensuring all prices in the economy are equal in steady state. This has no effect on the dynamic properties of the model. In order to remove the mark-up, we will assume the government subsidizes a fraction $(\tau^R)$ of retailers' input costs. Therefore, the retailer pays only $\left(1 - \tau^R\right) P_t^W$ per unit of wholesale good acquired.[7]

A retailer $z$ that is allowed to change its price at time $t$ will choose it to maximize:

[7]In this set up, it can be shown that $\tau^R = \frac{1}{\theta}$ is required to guarantee $P_t = P_t^W$ in equilibrium.

$$E_t \sum_{s=t}^{\infty} M_{s-t}^{H} \alpha^{s-t} \left\{ \left( \tilde{P}_t(z) - (1-\tau^R) P_s^W \right) \tilde{Y}_{s,t}(z) \right\} \tag{3.23}$$

where $\tilde{P}_t(z)$ is the optimal price chosen at time $t$ and $\tilde{Y}_{s,t}(z) = \left( \frac{\tilde{P}_t(z)}{P_s} \right)^{-\theta} Y_s$ is the relative demand of good $z$ at time $s$ given that its price remains fixed at $\tilde{P}_t(z)$.

The first order condition for this problem is:

$$E_t \sum_{s=t}^{\infty} M_{s-t}^{H} \alpha^{s-t} \left\{ \left( \tilde{P}_t(z) - \frac{\theta}{\theta - 1} (1-\tau^R) P_s^W \right) \tilde{Y}_{s,t}(z) \right\} = 0 \tag{3.24}$$

where $\frac{\theta}{\theta-1}$ would be the retail mark-up without the government's subsidy.

Since all re-optimizing retailers face a symmetric problem, the aggregate CPI ($P_t$) can be expressed as a weighted geometric average of "old" and "new" prices:

$$P_t = \left[ \alpha P_{t-1}^{1-\theta} + (1-\alpha) \tilde{P}_t^{\,1-\theta} \right]^{\frac{1}{1-\theta}} \tag{3.25}$$

where $\tilde{P}_t = \tilde{P}_t(z)$ is the symmetric optimal price.

Wholesale goods market clearing requires that aggregate retailer demand equal the total output of wholesale producers:

$$\int_0^1 Y_t(z) dz = Y_t^W \tag{3.26}$$

Introducing the individual retailer relative demands into this expression will result in:[8]

$$Y_t = \left( \frac{P_t^*}{P_t} \right)^{\theta} Y_t^W \tag{3.27}$$

where

$$P_t^* = \left[ \int_0^1 P_t(z)^{-\theta} dz \right]^{-\frac{1}{\theta}} = \left[ \alpha \left( P_{t-1}^* \right)^{-\theta} + (1-\alpha) \tilde{P}_t^{\,-\theta} \right]^{-\frac{1}{\theta}} \tag{3.28}$$

[8] Recall that $Y_t$ is a CES bundle of the individual $Y_t(z)$ and generally not equal to $\int_0^1 Y_t(z) dz$.

is an alternative price index introduced to ease notation and highlight the efficiency distortion due to sticky prices.[9]

Aggregate nominal profits transferred to the households are:

$$P_t \Pi_t^R = \int_0^1 \left( P_t(z) - \left(1 - \tau^R\right) P_t^W \right) Y_t(z) dz \tag{3.29}$$

which implies,

$$\Pi_t^R = Y_t - (1 - \tau^R) \frac{P_t^W}{P_t} Y_t^W. \tag{3.30}$$

### 3.3.5 Entrepreneurs and Retail Banks

The following description of the interaction between entrepreneurs and retail banks draws heavily from Bernanke et al. (1999), Christiano et al. (2010) and Gerali et al. (2010). Entrepreneurs supply one unit of managerial labor ($H_t^E = 1$) to wholesale producers inelastically. They accumulate real net worth ($N_t$) and take real loans ($L_t$) in order to buy new capital ($K_{t+1}$) from capital goods producers at relative price $Q_t$. Thus, an entrepreneur's balance sheet can be described as:

$$Q_t K_{t+1} = L_t + N_t \tag{3.31}$$

There is risk involved in the entrepreneurial activity: after the acquisition of new capital, entrepreneurs experience a private idiosyncratic shock $\omega$ which transforms the capital they acquired, $K_{t+1}$, into $\omega K_{t+1}$. In the literature incorporating the financial accelerator mechanism described in Bernanke et al. (1999), $\omega$ is usually assumed to be log-normally distributed with parameters $\mu_\omega$ and $\sigma_\omega$. These parameters are then picked to be consistent with $E[\omega] = 1$ and a particular steady state default rate on the loans. We follow this convention.

At the end of period $t$, an individual entrepreneur receives a nominal wage, $W_t^E$, and earns income from capital rented to the producers of wholesale goods, $R_t^W \omega K_t$,

[9] Actually, $P_t$ and $P_t^*$ are identical in a first-order approximation.

plus the resale value of the depreciated capital which is sold back to the capital goods producers ($(1-\delta)P_tQ_t\omega K_t$). Therefore, we can express the individual entrepreneur's nominal return on capital as the ratio between income received from renting capital and selling it after depreciation divided by its nominal cost:

$$\omega R_t^E = \omega \frac{R_t^W K_t + (1-\delta)P_tQ_tK_t}{P_{t-1}Q_{t-1}K_t} \tag{3.32}$$

where $R_t^E$ is defined implicitly as the gross nominal return on capital of the average entrepreneur.

At $t$, our representative entrepreneur signs a loan contract with a retail bank specifying a loan amount ($L_t$) and a nominal lending rate ($R_t^L$). Both the entrepreneur and the retail bank understand the loan is destined to finance part of the acquisition of new capital, $K_{t+1}$. The debt has to be repaid at time $t+1$. In case of default, retail banks can only appropriate the gross capital return of the entrepreneur at that time, i.e. $\omega R_{t+1}^E P_tQ_tK_{t+1}$.

Thus, we can define the cut-off $\overline{\omega}_{t+1}$ as the particular value of the idiosyncratic shock $\omega$ that allows the entrepreneur to honor his debt next period, leaving him with zero net income (individual entrepreneurs experiencing an idiosyncratic shock $\omega < \overline{\omega}_{t+1}$ default on their loans):

$$\overline{\omega}_{t+1} R_{t+1}^E P_tQ_tK_{t+1} = R_t^L P_t L_t$$

$$\overline{\omega}_{t+1} = \frac{R_t^L P_t L_t}{R_{t+1}^E P_tQ_tK_{t+1}} \tag{3.33}$$

where $\overline{\omega}_{t+1}R_{t+1}^E$ is the minimum return that entrepreneurs require in order to pay back to the bank, and $R_t^L P_t L_t$ is the payment amount agreed with the bank at time $t$. Note that the cut-off $\overline{\omega}_{t+1}$ depends positively on the lending rate ($R_t^L$) and negatively on the entrepreneur's leverage ($Q_tK_{t+1}/N_t$).

The loan market is competitive. There is a continuum of retail banks that offer contracts with lending rate $R_t^L$, obtain deposits at rate $R_t^D(j)$ in a market characterized by monopolistic competition, and take the interest rate on the (competitive) interbank market $R_t^{IB}$ as given. On the liability side, the representative retail bank has deposits $(D_t(j))$ and interbank funds $(IB_t(j))$ that are obtained from households and narrow banks, respectively. These funds are allocated by the retail bank into loans $(L_t(j))$ to entrepreneurs and reserves at the central bank $(RR_tD_t(j))$, constituting the asset side of the retail bank's balance sheet. Reserves are compulsory due to regulation: a fraction $RR_t$ of every unit of deposits received by the retail bank must be deposited at the central bank.

Table 3.1: Balance Sheet of Retail Banks

| Assets | Liabilities |
|---|---|
| Loans $(L_t)$ | Deposits $(D_t)$ |
| Reserves $(RR_tD_t)$ | Interbank loans $(IB_t)$ |

The balance sheet identity of the retail bank is:

$$L_t = (1 - RR_t)D_t + IB_t \tag{3.34}$$

When the entrepreneur's idiosyncratic shock is below the cut-off, $\omega < \overline{\omega}_{t+1}$, the bank forecloses the entrepreneur. Given the private nature of the idiosyncratic shock, the retail bank must pay a monitoring cost in order to observe $\omega$ and absorb the entrepreneur's gross capital return. Following convention in the costly state verification literature dating back to Townsend (1979), we assume that in this scenario, the retail bank keeps a fraction $(1 - \mu)$ of the entrepreneur's gross capital return after paying for monitoring costs and the entrepreneur walks out empty handed.

The retail bank's expected real profits next period are:

$$\begin{aligned} E_t\left[\Pi_{t+1}^{RB}\right] = E_t\Bigg[\Bigg(&\int_{\overline{\omega}_{t+1}}^{\infty} R_t^L L_t dF(\omega) + (1-\mu)\int_0^{\overline{\omega}_{t+1}} \omega R_{t+1}^E Q_t K_{t+1} dF(\omega) + \\ &R_t^{RR} RR_t D_t(i) - R_t^D(i) D_t(i) - \frac{\kappa^D}{2}\left(\frac{R_t^D(i)}{R_{t-1}^D(i)} - 1\right)^2 R_t^D D_t - \\ &R_t^{IB} IB_t\Bigg)\frac{P_t}{P_{t+1}}\Bigg] \end{aligned} \tag{3.35}$$

where $F(\omega)$ is the cumulative density of the idiosyncratic shock and $R_t^{RR}$ is the gross interest the central bank pays on reserves.

The retail bank has three income sources: loan repayments from entrepreneurs that performed well (those with $\omega > \overline{\omega}_{t+1}$), the gross capital return of defaulting entrepreneurs (with $\omega < \overline{\omega}_{t+1}$) net of monitoring costs and interest on reserves deposited at the central bank. On the other hand, retail bank expenses include deposit repayment with interest ($R_t^D(i)D_t(i)$), interbank loan repayment with interest ($R_t^{IB}IB_t$) and an adjustment cost on the deposit rate. The inclusion of an adjustment cost on the deposit rate can be justified theoretically using the classic menu costs argument of the price rigidity literature: retail banks incur in costs to market their deposit "product" in the form of advertising material. These costs increase whenever the deposit rate changes.

Departing from Gerali et al. (2010), we do not include adjustment costs related to the lending or interbank rates. The rationale behind this decision is that lending rates usually vary on a client to client basis (thus, there is no unique number to publicize) and the interbank market rate is determined on a day to day basis in a perfectly competitive market with almost perfect information. From an empirical perspective, the deposit rate adjustment cost is meant to capture the stylized fact that the pass-through from interbank rates to deposit rates is small and slow while the pass-through to lending rates is much bigger and faster (see Craig and Dinger (2010), Fuentes and Berstein (2004) and Sorensen and Werner (2006) for micro evidence on this). Note that this mechanism will also imply that consumption (which responds to the deposit rate) will have a smoother response to monetary policy shocks than investment (which will be associated with the lending rate).

Recall that the loan market (where retail banks and entrepreneurs interact) and the interbank market (where narrow banks and retail banks meet) are competitive but the deposit market is not. Following Gerali et al. (2010), monopolistic competition in the deposit market implies that every retail bank faces a particular demand for its slightly differentiated deposit ($D_t(i)$). We assume the consumer loves variety and demands a bundle of deposits ($D_t$) constructed as a CES aggregate of the individual deposits with elasticity of substitution $\epsilon$. Thus, the retail bank sets its deposit rate $R_t^D(i)$ taking into account the consumer's relative demand for its particular deposit given by:

$$D_t(i) = \left(\frac{R_t^D(i)}{R_t^D}\right)^{\epsilon} D_t \tag{3.36}$$

where $R_t^D$ is a CES index of the deposit rates $R_t^D(i)$.

The retail bank's expected real profit next period can be simplified using its balance sheet and the cut-off definition in order to substitute away $L_t$ and $R_t^L$ which yields:

$$\begin{aligned} E_t\left[\Pi_{t+1}^{RB}\right] = E_t\Bigg\{ \Bigg[ & g(\overline{\omega}_{t+1}) R_{t+1}^E p_t - \left( \frac{(R_t^D(i) - R_t^{RR} RR_t)}{1 - RR_t} + \right. \\ & \left. \left(1 - \frac{(1 - RR_t)}{p_t - 1}\left(\frac{R_t^D(i)}{R_t^D}\right)^{\epsilon} \frac{D_t}{N_t}\right)\left(R_t^{IB} - \frac{\left(R_t^D(i) - R_t^{RR} RR_t\right)}{1 - RR_t}\right)\right) \\ & (p_t - 1) - \frac{\kappa^D}{2}\left(\frac{R_t^D(i)}{R_{t-1}^D(i)} - 1\right)^2 R_t^D \frac{D_t}{N_t} \Bigg] N_t \left(\frac{P_t}{P_{t+1}}\right) \Bigg\} \end{aligned} \tag{3.37}$$

where $p_t \equiv \frac{Q_t K_{t+1}}{N_t}$ is the entrepreneur's leverage and

$$g\left(\overline{\omega}_{t+1}\right) \equiv \left[\overline{\omega}_{t+1} \Pr\left(\omega > \overline{\omega}_{t+1}\right) + (1 - \mu) E\left(\omega \mid \omega < \overline{\omega}_{t+1}\right) \Pr\left(\omega < \overline{\omega}_{t+1}\right)\right] \tag{3.38}$$

is the fraction of the gross nominal return on capital of the average entrepreneur $(R_{t+1}^E)$ that is given to the retail bank in compensation for the loan it provided at time $t$. $g\left(\overline{\omega}_{t+1}\right)$ is increasing in $\overline{\omega}_{t+1}$ given a reasonably small steady state default rate and some restrictions on the parameters of $F(\omega)$ imposed by Bernanke et al. (1999).

Turning back to entrepreneurs, their aggregate profit next period would be:

$$\left(\int_{\overline{\omega}_{t+1}}^{\infty} \omega R_{t+1}^E Q_t K_{t+1} dF(\omega) - R_t^L L_t\right)\left(\frac{P_t}{P_{t+1}}\right) \tag{3.39}$$

Note that only entrepreneurs that manage to repay their loans $(\omega > \overline{\omega}_{t+1})$ make a profit. Using the cut-off $(\overline{\omega}_{t+1})$ and leverage $(p_t)$ definitions to substitute away $R_t^L$ and $L_t$ again, results in a new expression for entrepreneurs' profit:

$$[E(\omega \mid \omega > \overline{\omega}_{t+1}) \Pr(\omega > \overline{\omega}_{t+1}) - \overline{\omega}_{t+1} \Pr(\omega > \overline{\omega}_{t+1})] R^E_{t+1} p_t N_t \left(\frac{P_t}{P_{t+1}}\right) \tag{3.40}$$

Given a well behaved distribution of $\omega$ (e.g.: log-normal), it can be demonstrated that $f(\overline{\omega}_{t+1}) \equiv E(\omega \mid \omega > \overline{\omega}_{t+1}) \Pr(\omega > \overline{\omega}_{t+1}) - \overline{\omega}_{t+1} \Pr(\omega > \overline{\omega}_{t+1})$, the average entrepreneur's share of $R^E_{t+1}$, is decreasing in $\overline{\omega}_{t+1}$.[10]

Retail banks need to offer entrepreneurs a loan contract specifying $R^L_t$ and $L_t$. However, we follow common practice in the literature redefining the problem in terms of $\overline{\omega}_{t+1}$ and $p_t$ to facilitate exposition. Intuitively speaking, a higher lending rate is equivalent to a higher cut-off and a bigger loan can be interpreted as higher leverage. Given the competitive environment in the loans market, retail banks will offer entrepreneurs the most desirable contract possible, driving down the present discounted value of their profits to zero.

Thus, the optimal contract is determined by the retail bank choosing $\overline{\omega}_{t+1}$, $p_t$ and $R^D_t(i)$ to maximize the expected present discounted value of the entrepreneurs' aggregate profit subject to the restriction that the expected present discounted value of their own profits is non-negative.[11]

The Lagrangian of the problem we just described would be:

$$\begin{aligned}
\max_{\overline{\omega}_{t+1}, p_t, R^D_t(i)} \mathcal{L} = E_t \Bigg[ & \sum_{s=t}^{\infty} M^H_{s-t} f(\overline{\omega}_{t+1}) R^E_{s+1} p_s N_s \frac{P_s}{P_{s+1}} \\
& + \lambda \sum_{s=t}^{\infty} M^H_{s-t} \Bigg\{ g(\overline{\omega}_{t+1}) R^E_{s+1} p_s - R^{IB}_s (p_s - 1) \\
& + \left(\frac{R^D_s(i)}{R^D_s}\right)^{\epsilon} \frac{D_s}{N_s} \left(R^{IB}_s (1 - RR_s) - R^D_s(i) + R^{RR}_s RR_s\right) \\
& - \frac{\kappa^D}{2} \left(\frac{R^D_s(i)}{R^D_{s-1}(i)} - 1\right)^2 R^D_s \frac{D_s}{N_s} \Bigg\} N_s \frac{P_s}{P_{s+1}} \Bigg]
\end{aligned} \tag{3.41}$$

[10] It should be noted that the retail bank's share and the entrepreneur's share do not add up to unity: $g(\overline{\omega}_{t+1}) + f(\overline{\omega}_{t+1}) < 1$ because part of $R^E_{t+1}$ is lost due to monitoring costs.

[11] Actually, the present discounted value of retail banks' profits will have to be zero at the optimum, otherwise they could obtain a better outcome cutting the lending rate marginally to offer entrepreneurs a more desirable contract.

where $\lambda$ is a (constant) lagrangian multiplier and $M^H_{s-t}$ is the households' stochastic discount factor, previously defined.

The solution to this problem yields the financial accelerator of Bernanke et al. (1999): a positive relationship between the external finance premium $(E_t\left[R^E_{t+1}\right]/R^{IB}_t)$ and entrepreneurial leverage, defined as the ratio of assets to net worth $(Q_t K_{t+1}/N_t)$. The particular functional form of the relationship depends on $f(\cdot)$, $g(\cdot)$ and their derivatives. We follow common practice and approximate it by,

$$\frac{E_t\left[R^E_{t+1}\right]}{R^{IB}_t} = \left[\frac{Q_t K_{t+1}}{N_t}\right]^{\upsilon} \tag{3.42}$$

where $\upsilon$ is the (positive) elasticity of the external finance premium with respect to leverage.

This relationship constitutes the entrepreneur's demand for new capital (recall that the retail bank is maximizing entrepreneurial profit): it is intuitive that demand for $K_{t+1}$ should be decreasing in $Q_t$ and increasing in $E_t\left[R^E_{t+1}\right]$ and $N_t$. That demand for new capital should be decreasing in $R^{IB}_t$ is not very intuitive but we can remedy this by pointing out that higher $R^{IB}_t$ must translate into higher $R^L_t$ in order to comply with the participation constraint of the retail bank (zero expected discounted present value of profits).

It is important to note that the costly-state verification framework implies that external funding is more expensive for the entrepreneur than internal funding always. Thus, the entrepreneur always uses all available net worth $N_t$ plus some loans to fund the acquisition of new capital.

Given our assumption of monopolistic competition in the market for deposits (a la Gerali et al. (2010)), the retail bank does not find it optimal to perfectly arbitrage between its sources of funding when transitioning from one steady state to another. Adjustment costs and monopolistic competition imply the following relationship between the deposit rate $(R^D_t)$ and the net cost of funding obtained from narrow banks in the interbank market (after imposing the condition that $R^D_t(i) = R^D_t$ for all $i$ by symmetry).

$$\kappa^D \left( \frac{R_t^D}{R_{t-1}^D} - 1 \right) \frac{R_t^D}{R_{t-1}^D} E_t \left[ \frac{P_t}{P_{t+1}} \right] =$$
$$\left( -1 - \epsilon + \epsilon \frac{R_t^{IB}(1 - RR_t) + R_t^{RR} RR_t}{R_t^D} \right) E_t \left[ \frac{P_t}{P_{t+1}} \right] +$$
$$\frac{\kappa^D}{R_t^D} E_t \left[ \left( \frac{D_{t+1}}{D_t} \right) \left( \frac{R_{t+1}^D}{R_t^D} - 1 \right) \left( \frac{R_{t+1}^D}{R_t^D} \right)^2 \frac{P_{t+1}}{P_{t+2}} \right] \tag{3.43}$$

Once the optimal contract between entrepreneur and retail bank has been defined, all that remains is to characterize entrepreneurial net worth and entrepreneurial consumption. We assume that each period, after settling with the retail banks, entrepreneurs make a decision whether to stay in business or "retire". In order to avoid unnecessary complications, we follow the literature and assign the value $\gamma$ to the probability that a particular entrepreneur will remain in business. Entrepreneurs choosing not to retire use all their gross return on capital plus labor income to accumulate net worth for the next period:

$$N_t = \gamma f(\overline{\omega}_t) R_t^E Q_{t-1} K_t \left( \frac{P_{t-1}}{P_t} \right) + \frac{W_t^E}{P_t} \tag{3.44}$$

This expression provides insight on the necessity of entrepreneurial labor. In our set up, an individual entrepreneur that incurs in default might not choose to "retire". Given that the retail bank has appropriated all his assets, he needs some net worth in order to participate in the loan market next period (retail banks do not lend to entrepreneurs with zero net worth). Entrepreneurial labor provides the net worth "seed" required to start over. We implicitly assume that credit history is erased every period and the entrepreneur's past credit performance does not affect his ability to obtain a loan from a retail bank in period $t$ in any way.

Entrepreneurs that exit the market ("retire") consume their entire gross return on capital in period $t$:

$$C_t^E = (1 - \gamma) f(\overline{\omega}_t) R_t^E Q_{t-1} K_t \left( \frac{P_{t-1}}{P_t} \right) \tag{3.45}$$

### 3.3.6 Narrow banks

Our set up assumes the existence of a handful of competitive narrow banks. These financial institutions are key in our model. Just like retail banks, narrow banks require funding in period $t$ in order to make financial investments that pay off in $(t+1)$.[12] There is a crucial difference though: retail banks promise a fixed return in exchange for funding (in the form of deposits and interbank loans) whereas narrow banks offer a variable return on their shares.

Funds obtained by issuing shares $(P_t^S S_t)$ are used by narrow banks to invest in government bonds $(B_t^{NB})$, purchased in the open market at relative price $P_t^B$, and offer interbank loans $(IB_t)$ competitively to retail banks.

Table 3.2: Balance Sheet of Narrow Banks

| Asset | Liabilities |
|---|---|
| Government bonds $(P_t^B B_t^{NB})$ | Equity $(P_t^S S_t)$ |
| Interbank Loans $(IB_t)$ | |

Note that narrow bank liabilities consist only of equity obtained by issuing shares $(P_t^S S_t)$, these represent household investment (in a financial sense) in the narrow bank. Thus, the balance sheet of a representative narrow bank is the following:

$$P_t^B B_t^{NB} + IB_t = P_t^S S_t \tag{3.46}$$

Participation in the open market is restricted to narrow banks only. The reason is that our stylized open market is meant to resemble a secondary bond market where the central bank will carry out open market operations (repos) buying and selling its own holdings of government bonds. Thus, participation in the market is limited due to regulatory restrictions.[13]

Narrow banks choose optimally their supply of interbank lending $(IB_t)$ and demand of government bond holdings $(P_t^B B_t^{NB})$ obtained through open market opera-

[12]Note that real sector firms such as retail and capital goods producers do all their business in $t$ and thus require no funding in a financial sense.

[13]Hilberg and Hollmayr (2011) argue that a hierarchical interbank market is justified by the structure found in the U.S. where only Primary Dealers deal with the central bank whereas a vast group of commercial banks is not allowed to directly deal with the monetary authority. In Europe, only 6 out of 2500 banks are allowed to participate in the bidding process in main refinancing operations of the ECB and other banks rely on interbank funding.

tions. The interest rate on interbank loans ($R_t^{IB}$) is the competitive outcome of the profit-maximizing behaviour of both bank types. However, the equilibrium price of government bonds ($P_t^B$) will be chosen by the central bank ensuring consistency with its monetary policy objectives.

It is important to note that aggregate interbank liquidity always runs from narrow to retail banks in the model. There is no intra-sectoral borrowing/lending in equilibrium given that all retail banks are identical (narrow banks too). In order to produce intra-sectoral borrowing and lending in equilibrium we would need to introduce some sort of heterogeneity between retail and/or narrow banks but, provided said heterogeneity does not entail a financial imperfection, we claim it would not alter our results. We do use a financial imperfection (monitoring costs, introduced below) to characterize the relationship between narrow and retail banks: this modelling device should yield similar results to a setup where we define a "bank" as an agent that combines the roles of a retail and narrow bank into a single entity and introduce heterogeneity between these new banks (idiosyncratic shocks to their deposit supply, $D_t(i)$, for example).

In order to simplify exposition and highlight the interaction between narrow banks and the central bank, we will assume that government bonds are consols issued at some undisclosed point in the past at relative price $P^B$ (without a subscript) and that they pay a fixed nominal interest ($R^B$) perpetually. Thus, buying a government bond unit at time $t$ makes the holder eligible to receive a fixed coupon payment of $\left(R^B - 1\right) P^B$ at time $(t + 1)$.[14] Furthermore, government bond supply is fixed for the duration of our shock experiments.

We use consols as a modelling device in order to avoid the unnecessary complication of having the goverment re-issue its bond stock every period where the price and interest paid on bonds would have to respond to the economy's state. An alternative would be to assume a fixed supply of very long-term bonds but note that would introduce time-to-maturity as another factor affecting bond price besides its demand as a financial asset.

Narrow bank's dividends are defined as:

[14] Multiplying the interest rate $\left(R^B - 1\right)$ by $P^B$ is necessary to ensure $R^B$ is an interest factor expressed in nominal good units and not bond units, making it comparable to other interest rates introduced before. In order to normalize $P^B$ to unity, $R^B$ must be equal to the steady state deposit rate.

$$DIV_t^{NB} = R_{t-1}^{IB} IB_{t-1}\left(\frac{P_{t-1}}{P_t}\right) + \left(\frac{\left(R^B - 1\right) P^B \frac{P_{t-1}}{P_t} + P_t^B}{P_{t-1}^B}\right) P_{t-1}^B B_{t-1}^{NB}$$
$$- \Xi\left(IB_{t-1}\right)\left(\frac{P_{t-1}}{P_t}\right) - P_t^S S_{t-1} \tag{3.47}$$

where $\Xi\left(IB_{t-1}\right)$ is a convex monitoring cost incurred by the narrow bank when lending to a retail bank.

The monitoring cost captures all expenses incurred by the narrow bank during the evaluation process, follow-up and monitoring that takes place for the duration of its credit relationship with a retail bank. We argue that the central bank is ill-equipped to perform the monitoring task and, therefore, does not offer loans directly to retail banks. This monitoring cost constitutes an important friction in our interbank market set up.[15] It will ensure there is an endogenous spread between the interbank rate and the return on government bonds, introducing a financial accelerator effect: the intebank rate responds not only to opportunity cost (the return on government bonds) but also to the volume of lending to retail banks, $IB_t$. This allows us to model traditional monetary policy (the central bank sets the opportunity cost of funds by manipulating the return on government bonds) coexisting with reserve requirements shocks because the latter will have an effect on the retail banks' demand for interbank funds, $IB_t$. Without the monitoring cost, reserve requirement shocks would be perfectly offset by the central bank's Taylor rule and have no effect on output.

It could be argued that it would make sense to incorporate monitoring costs when modelling retail banks as well. Note though that retail banks have a financial accelerator already, incorporating monitoring costs in their problem would complicate the model unnecessarily.

The narrow bank maximizes shareholder return:

[15] See Curdia and Woodford (2010) for more details on this particular formulation of monitoring costs in a context of households borrowing from financial intermediaries.

$$\max_{IB_t, B_t^{NB}} E_t\left[\frac{DIV_{t+1}^{NB} + P_{t+1}^{S} S_t}{P_t^S S_t}\right] = E_t\left[\frac{R_t^{IB} IB_t\left(\frac{P_t}{P_{t+1}}\right) - \Xi(IB_t)\left(\frac{P_t}{P_{t+1}}\right)}{P_t^B B_t^{NB} + IB_t} + \frac{\left(\frac{(R^B-1)P^B\left(\frac{P_{t-1}}{P_t}\right) + P_{t+1}^B}{P_t^B}\right) P_t^B B_t^{NB}}{P_t^B B_t^{NB} + IB_t}\right] \tag{3.48}$$

First order conditions for this problem are:

$$E_t\left[R_t^{IB}\left(\frac{P_t}{P_{t+1}}\right) - \Xi'(IB_t)\left(\frac{P_t}{P_{t+1}}\right)\right] = E_t\left[\frac{DIV_{t+1}^{NB} + P_{t+1}^{S} S_t}{P_t^S S_t}\right] \tag{3.49}$$

and

$$E_t\left[\frac{\left(R^B - 1\right) P^B\left(\frac{P_t}{P_{t+1}}\right) + P_{t+1}^B}{P_t^B}\right] = E_t\left[\frac{DIV_{t+1}^{NB} + P_{t+1}^{S} S_t}{P_t^S S_t}\right] \tag{3.50}$$

implying the expected return on interbank loans and government bond investments must equate to shareholder return. Eliminating shareholder return from the first order conditions yields:

$$E_t\left[R_t^{IB}\left(\frac{P_t}{P_{t+1}}\right) - \Xi'(IB_t)\left(\frac{P_t}{P_{t+1}}\right)\right] = E_t\left[\frac{\left(R^B - 1\right) P^B\left(\frac{P_t}{P_{t+1}}\right) + P_{t+1}^B}{P_t^B}\right] \tag{3.51}$$

Thus, the interest rate being charged to retail banks ($R_t^{IB}$) depends positively on the volume of interbank lending ($IB_t$) and negatively on the price of bonds ($P_t^B$). The central bank will exploit this relationship when pursuing monetary policy, effectively turning the narrow bank's return on government bonds into its monetary policy instrument. In order to do this, the central bank will supply(demand) government bonds in the open market whenever it wants to contract(expand) money supply, effectively setting $P_t^B$.

### 3.3.7 Central bank

The central bank's liabilities correspond to the components of the monetary base: retail bank reserves ($RR_tD_t$) and household cash holdings ($CSH_t/P_t$). On the asset side, the central bank holds the remaining government bonds ($P_t^B B_t^{CB}$). Thus, the total government bond supply ($P_t^B B_t$) must equate to the joint demand from narrow banks and the central bank ($P_t^B B_t^{CB} + P_t^B B_t^{NB}$).

Table 3.3: Balance Sheet of the Central Bank

| Asset | Liabilities |
|---|---|
| Government bonds ($P_t^B B_t^{CB}$) | Reserves ($RR_tD_t$) |
| | Cash holdings ($\frac{CSH_t}{P_t}$) |

The balance sheet of the central bank is as follows:

$$P_t^B B_t^{CB} = RR_tD_t + \frac{CSH_t}{P_t} \tag{3.52}$$

The central bank obtains interest and capital gains from its bond holdings.[16] These funds are used to pay some interest on reserves ($R_t^{RR}$) but, given the fact that part of the central bank's funding has zero cost (cash holdings), the central bank makes profits in steady state. These profits ($\Pi_t^{CB}$) are transferred to the government.

$$\begin{aligned}\Pi_t^{CB} = &\left(\frac{\left(R^B - 1\right) P^B \left(\frac{P_{t-1}}{P_t}\right) + P_t^B}{P_{t-1}^B}\right) P_{t-1}^B B_{t-1}^{CB} - R_{t-1}^{RR} RR_{t-1} D_{t-1} \left(\frac{P_{t-1}}{P_t}\right) \\ &- \frac{CSH_{t-1}}{P_{t-1}} \left(\frac{P_{t-1}}{P_t}\right) \end{aligned} \tag{3.53}$$

The central bank in this model controls liquidity by conducting open market operations, buying or selling bonds to the narrow bank. We assume the central bank's interventions are guided by a pseudo Taylor rule: if contemporaneous inflation is above its target, the central bank sells government bonds in the secondary open market to the narrow banks, pushing down their price. The result is a higher return on government

[16] We could assume the central bank does not receive interest on its bond holdings and our results would not be affected. This is because the central bank will transfer its profits to the government anyway. We choose to include interest payments to the central bank from the government in order to avoid confusion and ease exposition.

bonds for narrow banks and a decrease of the central bank's monetary base. Additionally, the central bank also reacts to deviations of output from its long run trend in a similar fashion. It is useful to introduce an auxiliary variable, $R_t^P$ to help characterize traditional monetary policy:

$$\left(\frac{R_t^P}{\bar{R}^P}\right) = \left(\frac{R_{t-1}^P}{\bar{R}^P}\right)^{\rho_R} \left[\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}\right]^{1-\rho_R} \exp\left(\varepsilon_t^R\right) \tag{3.54}$$

where $\rho_R$ captures interest rate rigidity, $\phi_\pi$ is the weight of inflation in the Taylor rule, $\phi_y$ is the weight of the output-gap, and $\varepsilon_t^R$ is our i.i.d. monetary policy shock.

Auxiliary variable $R_t^P$ is useful because it characterizes clearly the central bank's monetary policy stance. Given our assumption that the central bank's actual monetary policy instrument is the narrow bank's return on government bonds, our auxiliary "policy rate" must be "translated" into a bond price:

$$\frac{\left(R^B - 1\right) P^B P_t + E_t\left[P_{t+1} P_{t+1}^B\right]}{P_t P_t^B} = R_t^P \tag{3.55}$$

this expression characterizes the central bank's "demand" for government bonds. The central bank will adjust its bond holdings ($B_t^{CB}$) until bond price ($P_t^B$) is consistent with (3.55).

Note that equation (3.55) suggests an alternative modelling assumption: if goverment debt consisted of one-period, zero-coupon bonds, then monetary policy could target the market discount rate of those bonds. Our assumption of a fixed bond supply would then translate to the government issuing the same volume of debt every period.

In our set up, the central bank has a second, albeit unconventional, monetary policy tool: the reserve requirement rate ($RR_t$). Given our intent of studying the pure effects of this instrument on the short-run evolution of the model economy, we do not tie it to a particular rule:

$$\left(\frac{1 + RR_t}{1 + \overline{RR}}\right) = \left(\frac{1 + RR_{t-1}}{1 + \overline{RR}}\right)^{\rho_{RR}} \exp\left(\varepsilon_t^{RR}\right) \tag{3.56}$$

where $\rho_{RR}$ captures reserve requirement rigidity and $\varepsilon_t^{RR}$ is an i.i.d. shock.

### 3.3.8 Government

We rule out government demand for retail goods. Given that our model focuses on monetary policy and the interactions taking place in the interbank market, we try to minimize the government's role in our model economy. The government's intertemporal budget constraint is:

$$\Pi_t^{CB} + P_t^B B_t + T_t^S + T_t^B = \tau^R \frac{P_t^W}{P_t} Y_t^W + (R^B - 1) P^B B_{t-1} \frac{P_{t-1}}{P_t} + P_t^B B_{t-1} \qquad (3.57)$$

where $T_t^B$ is a (small) lump-sum tax required to finance part of the interest payments on the stock of government bonds $B_{t-1}$.

We make a number of simplifying assumptions to characterize government behaviour. First, we impose that lump-sum tax $T_t^S$ be destined to finance the retailer subsidy exclusively:

$$T_t^S = \tau^R \frac{P_t^W}{P_t} Y_t^W \qquad (3.58)$$

Second, as explained before, we assume government bond supply to be fixed ($B_t = B_{t-1} = B$). Introducing these assumptions allows us to rewrite the government's budget constraint:

$$\Pi_t^{CB} + T_t^B = (R^B - 1) P^B B \frac{P_{t-1}}{P_t} \qquad (3.59)$$

Note then that fluctuations in central bank profit ($\Pi_t^{BC}$) or inflation ($P_{t-1}/P_t$) must be compensated by altering the government's lump-sum "bond tax" ($T_t^B$) paid by households. Interest paid on central bank reserves ($R_{t-1}^{RR} RR_{t-1} D_{t-1} \left(\frac{P_{t-1}}{P_t}\right)$) will have an impact on the government's budget constraint through its effect on the central bank's bottom line, $\Pi_t^{CB}$.

### 3.3.9 Goods and bonds market equilibrium

All that is left to tie up our model is to define the resource constraint. Production of the (final) retail good is allocated to private consumption (by households and entrepreneurs), investment (by capital goods producers), deposit rate adjustment costs, and to cover monitoring costs incurred by retail banks (costly state verification) and narrow banks. The resource constraint takes the following form:

$$\begin{aligned} Y_t = C_t + C_t^E + X_t + \frac{\kappa^D}{2}\left(\frac{R_t^D}{R_{t-1}^D} - 1\right)^2 R_{t-1}^D D_{t-1}\left(\frac{P_{t-1}}{P_t}\right) \\ + \left(1 - f(\overline{\omega}) - g(\overline{\omega})\right) R^E Q_{t-1} K_t \left(\frac{P_{t-1}}{P_t}\right) + \Xi(IB_{t-1})\left(\frac{P_{t-1}}{P_t}\right) \end{aligned} \tag{3.60}$$

Finally, bonds market equilibrium requires:

$$B = B_t^{CB} + B_t^{NB} \tag{3.61}$$

Table 3.4 summarizes the model.

Table 3.4: Flow of Resources

| Agent | Inflow | Outflow |
|---|---|---|
| consumer | $\frac{W}{P}H + R^{D}_{-1}D_{-1}(\frac{P_{-1}}{P}) + \Pi^{R} + \Pi^{K} + \Pi^{RB} + \left(\frac{DIV^{NB}+P^{S}S_{-1}}{P^{S}_{-1}S_{-1}}\right) P^{S}_{-1}S_{-1} + \frac{CSH_{-1}}{P_{-1}}(\frac{P_{-1}}{P})$ | $C + T^{S} + T^{B} + D + P^{S}S + \frac{CSH}{P}$ |
| retailer | $Y + \tau^{R}\frac{P^{w}}{P}Y^{w}$ | $\Pi^{R} + \frac{P^{w}}{P}Y^{w}$ |
| capital good producer | $QK_{+1}$ | $\Pi^{K} + (1-\delta)QK + X$ |
| wholesale producer | $\frac{P^{w}}{P}Y^{w}$ | $\frac{W}{P}H + \frac{R^{W}}{P}K + \frac{W^{E}}{P}$ |
| entrepreneur BS | $L + N$ | $QK_{+1}$ |
| entrepreneur return | $\frac{R^{W}}{P}K + (1-\delta)QK$ | $R^{E}Q_{-1}K\left(\frac{P_{-1}}{P}\right)$ |
| entrepreneur BC | $f(\overline{\omega})\, R^{E}Q_{-1}K\left(\frac{P_{-1}}{P}\right) + \frac{W^{E}}{P}$ | $C^{E} + N$ |
| retail bank | $g(\overline{\omega})\, R^{E}Q_{-1}K(\frac{P_{-1}}{P}) + R^{RR}_{-1}RR_{-1}D_{-1}(\frac{P_{-1}}{P})$ | $\Pi^{RB} + R^{D}_{-1}D_{-1}(\frac{P_{-1}}{P}) + R^{IB}_{-1}IB_{-1}(\frac{P_{-1}}{P})$ |
| retail bank BS | $D + IB$ | $L + RRD$ |
| narrow bank | $R^{IB}_{-1}IB_{-1}\left(\frac{P_{-1}}{P}\right) + \left(\frac{(R^{B}-1)P^{B}\left(\frac{P_{-1}}{P}\right)+P^{B}}{P^{B}_{-1}}\right) P^{B}_{-1}B^{NB}_{-1}$ | $DIV^{NB} + \Xi\,(IB_{-1})\left(\frac{P_{-1}}{P}\right) + P^{S}S_{-1}$ |
| narrow bank BS | $P^{S}S$ | $P^{B}B^{NB} + IB$ |
| government | $T^{S}$ | $\tau^{R}\frac{P^{w}}{P}Y^{w}$ |
| more government | $T^{B} + \Pi^{CB} + P^{B}B$ | $(R^{B}-1)P^{B}B_{-1}\left(\frac{P_{-1}}{P}\right) + P^{B}B_{-1}$ |
| central bank | $\left(\frac{(R^{B}-1)P^{B}\left(\frac{P_{-1}}{P}\right)+P^{B}}{P^{B}_{-1}}\right) P^{B}_{-1}B^{CB}_{-1}$ | $\Pi^{CB} + R^{RR}_{-1}RR_{-1}D_{-1}(\frac{P_{-1}}{P}) + \frac{CSH_{-1}}{P_{-1}}(\frac{P_{-1}}{P})$ |
| central bank BS | $RRD + \frac{CSH}{P}$ | $P^{B}B^{CB}$ |
| resource constraint | $C + C^{E} + X + (1 - f(\overline{\omega}) - g(\overline{\omega}))\, R^{E}Q_{-1}K\left(\frac{P_{-1}}{P}\right) + \Xi\,(IB_{-1})\left(\frac{P_{-1}}{P}\right)$ | $Y$ |

Notes: Bond market requires $B = B^{NB} + B^{CB}$, adjustment costs and $t$ subscripts have been omitted to improve presentation.

## 3.4 Calibration

Our calibration of the model's parameters captures the key features of the U.S. economy. In Table 3.5 and 3.6 we report the calibration values and steady state values and ratios.

Regarding the households, the steady-state gross domestic inflation rate $(P_t/P_{t-1})$ is set equal to 1.00. The discount factor, $(\beta)$ is set to 0.99 to match the historical averages of nominal deposit and risk-free interest rates, $R_t^D$ and $R_t^P$. The intertemporal substitution parameter in workers' utility functions ($\sigma$ and $\sigma_M$) is set to 1 following Bernanke et al. (1999). Assuming that workers allocate one third of their time to market activities, we set the parameter determining the weight of leisure in utility $(\chi_H)$ and the inverse of the elasticity of intertemporal substitution of labor $(\varphi)$ to 1.0 and 0.33, respectively. The habit formation parameter, $(b)$, is set to 0.75, as estimated in Christiano et al. (2010).

The capital share in aggregate output production $(1 - \psi - \varrho)$ and the capital depreciation rate $(\delta)$ are set to 0.33 and 0.025, respectively. The parameter measuring the degree of monopoly power in the retail-goods market $(\theta)$ is set to 6, which would have implied a 20 per cent mark-up without the government subsidy.

The nominal price rigidity parameter $(\alpha)$ in the Calvo set up is assumed to be 0.75, implying that the average price remains unchanged for four quarters.

The probability that an entrepreneur will stay in the market the next period is 0.97. In the same line, the probability that an entrepreneur does not meet the required income to avoid default $(\Pr(\omega < \bar{\omega}))$ is 0.0075.

The adjustment cost parameter on the retail deposit rate, $\kappa^D$ is set to 3.5, as in Gerali et al. (2010). They actually estimate $\kappa^D$ using Euro data but we are not aware of estimates done for this parameter with U.S. data. Furthermore, we have performed some sensitivity analysis and the model's impulse responses change very little with $\kappa^D$.[17]

The elasticity of substitution $\epsilon$ between deposits at different retail banks is set to 237.5 in order to ensure consistency with the steady state interest rates reported in Table

[17]Increasing it by a factor of 3 has virtually no distinguishable effect on the impulse responses to shocks.

3.6.[18]

Turning to the narrow banks, monitoring costs are captured using the functional form $\Xi(IB_t) = \Xi_0(IB_t)^\eta$, as in Curdia and Woodford (2010). The parameters $\Xi_0$ and $\eta$ are set to 0.000726 and 10 respectively, to be consistent with the spread between $R^{IB}$ and $R^P$ (which equals $\Xi'(IB)$ in steady state) and minimize the magnitude of $\Xi(IB)$ (which ends up being less than 0.02% of GDP in our calibration implying its impact on output dynamics is virtually null).

Monetary policy parameter $\phi_\pi$ is set to 1.5 while $\phi_Y$ is set to zero (as in Bernanke et al. (1999)). These values satisfy the Taylor principle (see Taylor (1993)).

Following Bernanke et al. (1999), the steady-state leverage ratio of entrepreneurs $(1 - N/K)$, is set to 0.5, matching the historical average. The steady-state elasticity of the external finance premium ($\upsilon$) is set to 0.05, the value that is used by Bernanke et al. (1999).

Table 3.5: Parameter Calibration

| | | | |
|---|---|---|---|
| Preferences | | | |
| $\beta = 0.99$ | $b = 0.75$ | $\sigma = 1$ | $\chi_M = 0.008$ |
| $\sigma_M = 1$ | $\chi_H = 1$ | $\varphi = 0.333$ | $\theta = 6$ |
| Technologies | | | |
| $\delta = 0.025$ | $\psi = 0.66$ | $\varrho = 0.01$ | $\kappa = 8$ |
| Nominal rigidities | | | |
| $\alpha = 0.75$ | | | |
| Financial sector | | | |
| $\mu = 0.12$ | $\kappa^D = 3.5$ | $\epsilon = 237.5$ | $\upsilon = 0.0506$ |
| $\gamma = 0.9728$ | $\Xi_0 = 0.000726$ | $\eta = 10$ | |
| Monetary policy | | | |
| $\phi_\pi = 1.5$ | $\rho_R = 0.7$ | | |
| Government | | | |
| $\tau^R = 0.166$ | | | |
| Exogeneous processes | | | |
| $\rho_a = 0.95$ | $\rho_{RR} = 0.9$ | | |

## 3.5 Results

We solve the model taking a first order linear approximation using Dynare.[19] Figure 3-2 shows the model's impulse responses to a one percent productivity shock. Most

[18] In steady state, $((1 + 1/\epsilon)R^D - RR * R^{RR})/(1 - RR) = R^{IB}$

[19] www.dynare.org

Table 3.6: Steady-State Values and Ratios

| Variables | Definitions | Values |
|---|---|---|
| $\pi$ | inflation | 1.0000 |
| $R^{IB}$ | interbank rate | 1.0143 |
| $R^{D}$ | deposit rate | 1.0097 |
| $RR$ | reserve requirements | 0.06 |
| $R^{RR}$ | reserve requirements' remuneration rate | 1.0092 |
| $C/Y$ | household's consumption to output | 0.681 |
| $C^{E}/Y$ | entrepreneur's consumption to output | 0.143 |
| $I/Y$ | investment to output | 0.177 |
| $K/Y$ | capital stock to output | 7.069 |
| $L/Y$ | lending to output | 1.961 |
| $D/Y$ | deposit to output | 1.699 |
| $IB/Y$ | interbank funding to output | 0.363 |
| $CSH/Y$ | cash holding to output | 0.551 |
| $P^{S}S/Y$ | shares to output | 0.472 |

variables exhibit fairly standard behaviour. Output returns slowly to steady state thanks to habit formation, adjustment costs associated to investment, and the shock's own persistence. In addition, the expansion of supply induces a fall in inflation and the central bank responds by cutting the policy rate which in turn pushes down the interbank rate.

Given the assumption of competitive pricing at the wholesale level, wholesale prices ($P^{W}$) drop much more in response to the supply (productivity) shock than retail prices ($P$). As a result, the rental rate on capital, $R^{W}$, plummets. This explains the fall in the entrepreneur's return in spite of the increase in the price of capital that results from higher productivity. The initial positive effect on net worth is the result of a higher real valuation of capital acquired last period given the surprise deflation. This effect is short-lived though because eventually the entrepreneur's negative return prevails and pushes net worth down.

Given the fact that capital is increasing and its price suffers a positive shock, decreasing net worth results in higher leverage being adopted by entrepreneurs, expanding their demand for loans. This higher leverage explains the increase in the lending rate over the first 10 quarters (the initial drop in the lending rate is a consequence of the monetary policy pass-through: from the policy rate to the interbank rate and then to the lending and deposit rates).

Households experience a positive wealth effect which explains the behaviour of consumption (combined with habit formation). The same wealth effect interacts with the intertemporal substitution effects induced by the response of deposit rates to yield the path of deposits shown in Figure 3-2. The wealth effect is responsible for the overall increase in deposits while the short term evolution of the deposit rates (which move in a similar fashion to the policy rate) explains the dip observed in the first 5 quarters.

From the point of view of retail banks, the sustained increase in loans coupled with the short term dip in deposits of the first 5 quarters results in a need to rely on interbank lending for a few quarters. Narrow banks provide the funding, reducing their holdings of government bonds. The central bank is thus required to hold more bonds and increases the monetary base (cash) in order to accommodate this.

Given that output takes a very long time to converge back to equilibrium in the model, inflation stays below steady state for a long time. This explains why the policy and interbank rates remain below steady state even after 20 quarters. They take longer to converge.

[Figure 3-2 about here]

The model's impulse responses to a (negative) monetary policy shock of 50 basis points are shown in Figure 3-3. Output decreases and returns slowly to steady state. The demand contraction has a negative impact on inflation, which leads the monetary authority to decrease the policy rate quickly after the shock.

The demand for capital collapses. On one hand, wholesale prices decrease sharply (they are not subject to rigidity) and output drops as well, decreasing the rental rate of capital that wholesalers pay to entrepreneurs. Furthermore, the opportunity cost of funds has increased (the higher policy interest rate pushes up the interbank, lending and deposit rates) and this leads banks to charge more for the loans being offered to entrepreneurs. Both effects contract entrepreneur's demand for capital, resulting in a sharp drop of its price.

The sharp drop in entrepreneur's return (explained mostly by the fall in the rental price of capital, $R^W$) coupled with the fall in the price of capital explain the evolution of entrepreneur's net worth. Note though that the fall in net worth is much more persistent than the causes. The reason is that the financial accelerator mechanism introduces endogenous persistence: it is hard for the entrepreneur to recover from a fall in net worth because the lending he receives is a function of net worth itself. This mechanism operates in similar fashion to that described in Kiyotaki and Moore (1997) where credit constraints interact with asset prices to generate persistence. The financial accelerator acts as a "soft" credit constraint.

The fall in the price of capital induces capital good producers to reduce investment, generating a gradual fall in capital given the adjustment costs involved.

The external finance premium (EFP) increases given the paths of entrepreneurial returns and the interbank rate. This prompts higher leverage offered to entrepreneurs, which explains the increase in loans. Given that the EFP takes a long time to converge to equilibrium (because of the interbank rate), loans stay above trend for a long time.[20] This persistent higher leverage explains why the lending rate behaves differently than the interbank, policy and deposit rates (which are basically following the monetary policy rule). The effect of the endogenous EFP on the lending rate overcomes that of the interbank rate.

There is a strong increase in deposits given the initial higher deposit rates paid on them, prompting a decrease in retail banks' demand for interbank funds which translates into a short lived increase in narrow banks' demand for government bonds. The central bank adjusts money supply to accommodate the needs of narrow banks.

Given the negative monetary conditions that prevail eventually (the interest rate drops below steady state because inflation is very persistent) household funding offered to the financial sector should eventually drop below steady state. Bear in mind though, that households allocate savings between deposits and shares. Even though deposits never fall below steady state, funds being allocated to narrow banks in the form of equity investment do (equity must equal interbank loans plus government bond holdings, the latter stay below steady state a long time). Thus, total household saving eventually falls (monetary conditions are restrictive) and this prompts a mild increase in consumption

[20] The increase in loans does not match empirical evidence. This is a common problem in the literature associated with the financial accelerator of Bernanke et al. (1999): the increase in the EFP as a result of monetary tightening expands leverage, resulting in higher lending. A countercyclical EFP leads to countercyclical leverage (see equation (3.42)).

(this is a typical RBC effect: given that output does not drop much, lower savings must expand consumption).

There could be concern about output's convergence to steady state given the impulse response shown in Figure 3-3. Initially, output falls because of investment and entrepreneur's consumption (which is basically a small fraction of net worth). As investment and entrepreneur's consumption start to recover, consumption begins to fall (habit formation delays the fall in consumption). The effect of consumption initially makes output fall again but eventually investment and entrepreneur's consumption increasing reverse that trend and output converges. In a sense, the very long persistence of output is a direct result of consumption (by households and entrepreneurs) being perhaps too persistent.

[Figure 3-3 about here]

The reserve requirement shock depicted in Figure 3-4 corresponds to an increase in reserve requirements from 6% to 9%. The resulting decrease in aggregate demand pushes down output and inflation.

The reserve requirement shock bears important similarities with the monetary policy shock of Figure 3-3. Tight monetary conditions explain the drop in the price of capital which prompts the fall in investment. Deflation has a negative effect on the wholesale price and this decreases the entrepreneur's returns. Net worth falls and recuperates slowly, the EFP increases (in a countercyclical fashion) and this raises leverage, pushing loans and the lending rate up.

The key difference with the monetary policy shock lies in the monitoring cost. Higher reserve requirements prompt retail banks to switch funding from deposits to interbank loans. This increases narrow banks' monitoring costs. The higher monitoring costs prompt the latter to push up the interbank rate (which eventually has an impact on the deposit rate as well given the pass-through). Thus, for a few quarters, the interbank rate and the policy rate move in opposite directions: the central bank pushes the policy rate down to fight deflation and the fall in output, monitoring costs push the interbank rate up. Monitoring costs effectively prevent the central bank's Taylor rule from perfectly offsetting the reserve requirement shock.

Narrow banks find themselves with more funding available (households are switching their savings from deposits to equity) and demand government bonds from the central bank which reduces money supply accordingly. Thus, besides the composition effects, both shocks boil down to a contraction of the monetary base (understood as cash plus reserves).

Additionally, this shock produces a negative effect on the government's net revenue for two reasons: first, higher reserve requirements imply higher interest payments done by the central bank, which reduces its profits; second, higher government bond holdings by narrow banks imply higher interest payments done by the government (because interest payments on central bank government bond holdings revert back to the government when the central bank transfers its profits). The government will be forced to increase the lump-sum tax ($T_t^B$) to balance its budget.

[Figure 3-4 about here]

Reserve requirement shocks could have an even bigger impact on our model economy. To justify this claim, note that traditional monetary policy (i.e. the Taylor rule) is trying to undo the effects of the shock right from the outset which is counterintuitive. Thus, in Figure 3-5 we show the combined effects of a monetary policy (50bps) and reserve requirement (3 per cent) shock. Reserve requirements can help obtain a bigger reaction of output and inflation for a given interest rate policy shock. This is because the interbank rate is pushed up by a higher policy rate and monitoring costs (resulting from the retail banks' need for more interbank lending) at the same time. The result suggests that lower movements in interest rate can achieve the same desired inflation and output, if used together with a consistent reserve requirement policy.

[Figure 3-5 about here]

Furthermore, it is worth mentioning that reserve requirements could stabilize this economy on their own. Monetary policy at its core is all about manipulating the monetary base which is the sum of reserves and cash holdings in our model. Still, we do not propose reserve requirements as a substitute of traditional monetary policy: they are essentially a tax and thus entail deadweight loss and inefficient allocations. An evaluation of the welfare implications of using reserve requirements as the main stabilization instrument compared to traditional (interest rate based) monetary policy is beyond the scope of this paper.

## 3.6 Conclusions

When the central bank regulates the interbank market using reserve requirements, the monetary authority also affects liquidity in the banking sector, first, and the economy, later. This way of influencing bank funding, without any use of the policy interest rate, is a macroprudential tool.

In terms of modelling, the introduction of an interbank market allows a better identification of the final effects of different type of shocks in the economy. Important conclusions such as the complementarity of a central bank's tools can be potentially answered in a model with this additional feature.

The properties of macroprudential tools, developed in this model, are combined with the traditional effect of an interest rate policy shock. The complementarity of these two tools is one of our results. Reserve requirements act as a tax to financial intermediation, increasing the cost of funding economic activity through deposits and ultimately affecting output and inflation. Thus, a central bank can achieve a similar reaction on inflation and output with a lower increase of the policy interest rate if reserve requirements are increased at the same time. This is particularly relevant when the required policy rate cut is very big and could bring the monetary authority close to the zero lower bound, a problem faced by several countries in the wake of the Lehman bankruptcy.

Our results are in line with those of Carrera (2012) and Whitesell (2006). In his review of the relevant literature, Carrera (2012) finds that complementarity of these policy tools is normally achieved using different modelling strategies, however there is room for more research to explore the mechanism by which these and other related tools operate (e.g. collaterals). In the same line, Whitesell (2006) shows that combined policies of interest rate and reserve requirements result in lower volatility of the policy interest rate.

While the research conclusion for this paper is clear enough, this model can be extended to consider the possibility of collateral from retail banks to either narrow banks or a shadow banking system. The flexibility of our model allows for questions that are directly related with the liquidity of the financial system, and that is part of our research agenda.

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# A. Appendix: The Model - Log linear equations

## A.1 Households

All uppercase variables (except first line of each mini section) represent steady state values. Lowercase variables are deviations from steady state. No subscript implies variable is in current period.

The first derivative of the instantaneous utility of consumption

$$UC*(C-hab*C_{-1})^{(\frac{1}{\sigma})}=1$$

$$uc+\left(\frac{1}{\sigma}\right)\left(\frac{1}{1-hab}c-\frac{hab}{1-hab}c_{-1}\right)=0$$

-

Marginal utility of consumption

$$\lambda=UC-\beta*hab*UC_{+1}$$

$$\lambda=\left(\frac{1}{\sigma}\right)(\frac{\beta*hab}{1-\beta*hab}\left(\frac{1}{1-hab}c_{+1}-\frac{hab}{1-hab}c\right)-\frac{1}{1-\beta*hab}\left(\frac{1}{1-hab}c-\frac{hab}{1-hab}c_{-1}\right))$$

-

Household's budget constraint

$$C_t+T_t^S+T_t^B+D_t+P_t^S S_t+\frac{CSH_t}{P_t}=\frac{W_t}{P_t}H_t+R_{t-1}^D D_{t-1}\frac{P_{t-1}}{P_t}+DIV_t^{NB}+P_t^S S_{t-1}+\frac{CSH_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}+\Pi_t^R+\Pi_t^K+\Pi_t^{RB}$$

$$\frac{C}{Y}c+\frac{T^B}{Y}t^B+\frac{D}{Y}d+\frac{P^S S}{Y}\left(p^S+s\right)+\frac{CSH}{Y}\left(csh-p\right)=\psi y+\left(1-\psi\right)\left(p-p^w\right)+\delta\frac{K}{Y}q+\frac{P^B B^{NB}}{Y}\left(p^B-R^B_{SS}p^B_{-1}\right)+R^B_{SS}\frac{P^B B^{NB}}{Y}\left(p^B_{-1}+b^{NB}_{-1}\right)$$
$$-\frac{\Xi\left(IB\right)}{Y}\left(\eta ib_{-1}-\pi\right)+\frac{CSH}{Y}\left(csh_{-1}-p_{-1}-\pi\right)+\left(\left(R^D-R^{RR}RR\right)\frac{D}{Y}+R^{IB}\frac{IB}{Y}\right)\left(\frac{g'\left(\overline{\omega}\right)\overline{\omega}}{g\left(\overline{\omega}\right)}\left(r^L_{-1}+l_{-1}-r^E-q_{-1}-k\right)+r^E-\pi+q_{-1}+k\right)$$
$$+R^{RR}RR\frac{D}{Y}\left(r^P_{-1}-\pi-rr_{-1}-d_{-1}\right)$$

-

Household's Euler equation

$\frac{\lambda}{P}=\beta R^d\frac{\lambda_{+1}}{P_{+1}}$

$\lambda=r^d+\lambda_{+1}-\pi_{+1}$

-

Labor supply

$W*\lambda/P=\chi_h*H^{(\frac{1}{\phi})};$

$p^w-p+y+\lambda=(\frac{\phi+1}{\phi})h$

but $y^w=a+(1-\psi-\varphi)k_{-1}+\psi h$ then

$p^w-p+\lambda=\frac{\phi(1-\psi)+1}{\phi\psi}y-\frac{\phi+1}{\phi\psi}((1-\psi-\varphi)k_{-1}+a)$

-

Demand for shares

$$E_t\left[\lambda_{t+1}\frac{P_t}{P_{t+1}}\right]R_t^D = E_t\left[\lambda_{t+1}\left(\frac{DIV_{t+1}^{NB}+P_{t+1}^S S_t}{P_t^S S_t}\right)\right]$$

$$r_t^D - \pi_{t+1} + p_t^S + s_t = \frac{DIV^{NB}}{DIV^{NB}+P^S S} div_{t+1}^{NB} + \frac{P^S S}{DIV^{NB}+P^S S}\left(p_{t+1}^S + s_t\right)$$

-

Money demand

$$\chi_M\left(\frac{CSH_t}{P_t}\right)^{-\frac{1}{\sigma_M}} = \lambda_t\left(\frac{R_t^D-1}{R_t^D}\right)$$

$$-\frac{1}{\sigma_M}(csh-p) = \lambda + \left(\frac{1}{R^D-1}\right) r^D$$



## A.2 Retailer supply and aggregation

Retailer profit

$$\Pi^R = Y - (1-\tau^R) * \frac{P^w}{P} * Y^w$$

$$\pi^R = y - \frac{(1-\tau^r)}{\tau^r}(p^w - p)$$

-

retailer demand = wholesaler supply

$(P^{\epsilon}) * Y = (P^{*})^{\epsilon} * Y^{w}$

$y = y^{w}$

-

domestic price evolution, alternative cpi weights

$P^{*} = (\alpha {P^{*}_{-1}}^{(-\theta)} + (1-\alpha) P^{z(-\theta)})^{(-1/\theta)}$

$p^{*} = \alpha p^{*}_{-1} + (1-\alpha) p^{z}$

-

additional variables required to characterize price setting

$MUCPVN = MUCP * VN$

$mucpvn = mucp + vn$

-

ídem

$MUCPVD = MUCP * VD$

$mucpvd = mucp + vd$

-

domestic price evolution

$$P = (\alpha P_{-1}{}^{(1-\theta)} + (1-\alpha)P^{z(1-\theta)})^{(\frac{1}{1-\theta})}$$

$$p = \alpha p_{-1} + (1-\alpha)p^{z}$$

thus, $p = p*$

-

optimal retail price derivation

$$P^{z}(\theta - 1)VD = \theta(1-\tau^{R})VD$$

$$p^{z} + vd = vn$$

-

$$VN * MUCP = Y * (P)^{\theta} * P^{w} * MUCP + \alpha * \beta * MUCPVN_{+1}$$

$$vn = \frac{Y}{Y+\alpha\beta VN}(y + \theta p + p^{w}) + \frac{\alpha\beta VN}{Y+\alpha\beta VN}(vn_{+1} - r^{D})$$

but $VN = Y/(1-\alpha\beta)$ then

$vn = (1-\alpha\beta)(y + \theta p + p^{w}) + \alpha\beta(vn_{+1} - r^{d})$ -

$$VD * MUCP = Y * (P^{\epsilon}) * MUCP + \alpha * \beta * MUCPVD_{+1}$$

$$vd = \frac{Y}{Y+\alpha\beta VD}(y + \epsilon p) + \frac{\alpha\beta VD}{Y+\alpha\beta VD}(vd_{+1} - r^{d})$$

but $VD = Y/(1-\alpha\beta)$ then

$$vd = (1-\alpha\beta)(y+\theta p) + \alpha\beta(vd_{+1} - r^d)$$

then, given that $p^z = vn - vd$

$$vn - vd = (1-\alpha\beta)p^w + \alpha\beta(vn_{+1} - vd_{+1})$$

$$p^z = (1-\alpha\beta)p^w + \alpha\beta p^z_{+1}$$

but $p^z = \frac{1}{1-\alpha}p - \frac{\alpha}{1-\alpha}p_{-1}$

$$\frac{1}{1-\alpha}p - \frac{\alpha}{1-\alpha}p_{-1} = (1-\alpha\beta)p^w + \alpha\beta(\frac{1}{1-\alpha}p_{+1} - \frac{\alpha}{1-\alpha}p)$$

going for Phillips curve:

$$-\alpha\beta p_{+1} + (1+\alpha^2\beta)p - \alpha p_{-1} - (1-\alpha)(1-\alpha\beta)p = (1-\alpha)(1-\alpha\beta)p^w - (1-\alpha)(1-\alpha\beta)p$$

$$-\alpha\beta(p_{+1} - p) + \alpha(p - p_{-1}) = (1-\alpha)(1-\alpha\beta)(p^w - p)$$

$$-\alpha\beta\pi_{+1} + \alpha\pi = (1-\alpha)(1-\alpha\beta)(p^w - p)$$

Phillips curve:

$$\pi = \beta\pi_{+1} + \frac{(1-\alpha)}{\alpha}(1-\alpha\beta)(p^w - p)$$

-

## A.3 Capital Goods Producers

Capital accumulation

$K = (1 - \delta) * K_{-1} + CPHI * X$

$k = (1 - \delta)k_{-1} + \delta x$

-

Tobin's Q

$Q(1 + \kappa - \kappa(\frac{X}{X_{-1}})) = 1 - \frac{0.5\kappa\beta}{\lambda}(\frac{\lambda_{+1}Q_{+1}X_{+1}^2}{X^2} - \lambda_{+1}Q_{+1})$

$q - \kappa(x - x_{-1}) = -\kappa\beta(x_{+1} - x)$

-

## A.4 Wholesale Producer

Production function

$Y^w = \exp(a) * K_{-1}{}^{(1-\psi-\varphi)} * H^\psi * (H^e)^\varphi$

$y^w = a + (1 - \psi - \varphi)k_{-1} + \psi h$

-

Productivity shock

$a = \rho^a * a_{-1} + \varepsilon^a$

-

Capital demand

$R^w * K_{-1} = (1 - \psi - \varphi) * P^w * Y^w$

$r^w + k_{-1} = p^w + y^w$

-

Household labor demand

$W * H = \psi * P^w * Y^w$

$w + h = p^w + y^w$

-

Entrepreneurial labor demand

$W^e * H^e = \varphi * P^w * Y^w$

$w^e = p^w + y^w$ -

## A.5 Entrepreneurs

Net return on capital (definition)

$$R^e * P_{-1} * Q_{-1} = R^w + (1-\delta) * P * Q$$

$$r^e - \pi + q_{-1} = \frac{R^e-(1-\delta)}{R^e}(p^w - p + y - k_{-1}) + \frac{(1-\delta)}{R^e} q$$

-

Entrepreneur's balance sheet

$$Q * K = B + N$$

$$q + k = \left(\frac{B}{K}\right) b + \left(\frac{N}{K}\right) n$$

-

Entrepreneur's net worth evolution

$$N_t = \gamma f\left(\overline{\omega}_{t-1}\right) R_t^e Q_{t-1} K_{t-1} \left(\frac{P_{t-1}}{P_t}\right) + \frac{W_t^e}{P_t}$$

$$n = \gamma f\left(\overline{\omega}\right) R^e \frac{K}{N}\left[\frac{f'(\overline{\omega})\overline{\omega}}{f(\overline{\omega})}\left(r_{-1}^L + l_{-1} - r^E - q_{-1} - k_{-1}\right) + r^e + q_{-1} + k_{-1} - \pi\right] + \varrho \frac{Y}{K}\frac{K}{N}\left(p^w - p + y\right)$$

-

Entrepreneur's consumption

$$C_t^E = (1-\gamma)\, f\left(\overline{\omega}_{t-1}\right) R_t^e Q_{t-1} K_{t-1} \left(\frac{P_{t-1}}{P_t}\right)$$

$$\frac{C^E}{Y} c^E = (1-\gamma)\, f\left(\overline{\omega}\right) R^e \frac{K}{Y} \left[\frac{f'(\overline{\omega})\overline{\omega}}{f(\overline{\omega})} \left(r^L_{-1} + l_{-1} - r^E - q_{-1} - k_{-1}\right) + r^e + q_{-1} + k_{-1} - \pi\right]$$

-

## A.6 Retail Bank

Retail bank balance sheet

$$L = (1-RR)D + IB$$

$$l = (1-RR)\frac{D}{L}d - RR\frac{D}{L}rr + \frac{IB}{L}ib$$

-

Retail bank profit

$$\Pi_{t+1}^{RB} = \{g(\overline{\omega})R_{t+1}^{e} p_{t+1} - \left[\frac{\left(R_{t+1}^{D}(i) - R_{t+1}^{RR} RR_{t+1}\right)}{1-RR_{t+1}} + \left(1 - \frac{(1-RR_{t+1})}{p_{t+1}-1}\left(\frac{R_{t+1}^{D}(i)}{R_{t+1}^{D}}\right)^{\epsilon} \frac{D_{t+1}}{N_{t+1}}\right)\left(R_{t+1}^{ib} - \frac{\left(R_{t+1}^{D}(i) - R_{t+1}^{RR} RR_{t+1}\right)}{1-RR_{t+1}}\right)\right]$$
$$(p_{t+1}-1) - \frac{\kappa^d}{2}\left(\frac{R_{t+1}^{D}(i)}{R_t^{D}(i)} - 1\right)^2 R_{t+1}^{D} \frac{D_{t+1}}{N_{t+1}}\} N_{t+1} \left(\frac{P_t}{P_{t+1}}\right)$$

-

Determination of deposit rates (as in Gerali et al. (2010))

$$\left(-1-\epsilon+\epsilon\frac{R^{ib}_{t+1}(1-RR_{t+1})+R^{RR}_{t+1}RR_{t+1}}{R^{D}_{t+1}}\right)\frac{P_t}{P_{t+1}}-\kappa^d\left(\frac{R^D_{t+1}}{R^D_t}-1\right)\frac{R^D_{t+1}}{R^D_t}\frac{P_t}{P_{t+1}}+SDF\frac{D_{t+2}}{D_{t+1}}\kappa^d\left(\frac{R^D_{t+2}}{R^D_{t+1}}-1\right)\left(\frac{R^D_{t+2}}{R^D_{t+1}}\right)^2\frac{P_{t+1}}{P_{t+2}}=0$$

$$r^D=\frac{\kappa^D}{1+\epsilon+(1+\beta)\kappa^D}r^D_{-1}+\frac{\beta\kappa^D}{1+\epsilon+(1+\beta)\kappa^D}E\left[r^D_{+1}\right]+\frac{R^{IB}}{R^D}\frac{\epsilon(1-RR)}{1+\epsilon+(1+\beta)\kappa^D}r^{IB}-\frac{R^{IB}-R^{RR}}{R^D}\frac{\epsilon RR}{1+\epsilon+(1+\beta)\kappa^D}rr+\frac{R^{RR}}{R^D}\frac{\epsilon RR}{1+\epsilon+(1+\beta)\kappa^D}r^P$$

-

Threshold

$$\overline{\omega}R^E_{+1}QK_{+1}=R^L_{+1}L_{+1}$$

$$\frac{d\overline{\omega}}{\overline{\omega}}+r^E_{+1}+q+k=r^L+l$$

-

External finance premium

$$\frac{R^E_{+1}}{R^{IB}}=\rho\left(\overline{\omega}\right)$$

$$r^E_{+1}-r^{IB}=\frac{\rho'(\overline{\omega})\overline{\omega}}{\rho(\overline{\omega})}\left(r^L+l-r^E_{+1}-q-k\right)$$

## A.7 Narrow Bank

Balance sheet

$$P^BB^{NB}+IB=P^SS$$

$$\frac{P^B B^{NB}}{Y}\left(p^B + b^{NB}\right) + \frac{IB}{Y} ib = \left(\frac{P^B B^{NB}}{Y} + \frac{IB}{Y}\right)\left(p^S + s\right)$$

-

Narrow bank benefit/dividend

$$R_{-1}^{IB} IB_{-1}\left(\frac{P_{-1}}{P}\right) + \left(\frac{\left(R^B-1\right)\left(\frac{P_{-1}}{P}\right)+P^B}{P_{-1}^B}\right) P_{-1}^B B_{-1}^{NB} = DIV^{NB} + \Xi\left(IB_{-1}\right)\left(\frac{P_{-1}}{P}\right) + P^S S_{-1}$$

$$\frac{DIV^{NB}}{Y} div^{NB} = R^{IB}\frac{IB}{Y}\left(r_{-1}^{IB} + ib_{-1} - \pi\right) + \frac{P^B B^{NB}}{Y}\left(p^B - R^B p_{-1}^B - \left(R^B-1\right)\pi\right) + R^B \frac{P^B B^{NB}}{Y}\left(p_{-1}^B + b_{-1}^{NB}\right) - \frac{\Xi(IB)}{Y}\left(\eta ib_{-1} - \pi\right) - \frac{P^S S}{Y}\left(p^S + s\right)$$

-

Interbank supply of funds

$$R^{IB}\left(\frac{P}{P_{+1}}\right) - \Xi'\left(IB\right)\left(\frac{P}{P_{+1}}\right) = \left(\frac{DIV_{+1}^{NB}+P_{+1}^S S}{P^S S}\right)$$

-

Government bonds' demand

$$\left(\frac{\left(R^B-1\right)\left(\frac{P}{P_{+1}}\right)+P_{+1}^B}{P^B}\right) = \left(\frac{DIV_{+1}^{NB}+P_{+1}^S S}{P^S S}\right)$$

$$\frac{P^S S}{Y}(p_{+1}^B - R^B p^B - (R^B - 1)\pi_{+1}) = \{R^{IB}\frac{IB}{Y}\left(r^{IB} + ib - \pi_{+1}\right) + \frac{P^B B^{NB}}{Y}\left(p_{+1}^B - R^B p^B\right) + R^B\frac{P^B B^{NB}}{Y}\left(p^B + b^{NB}\right) - \frac{\Xi\left(IB\right)}{Y}\left(\eta ib - \pi_{+1}\right)$$
$$- \frac{P^S S}{Y}\left(p_{+1}^S + s_{+1}\right) + \frac{P^S S}{Y}p_{+1}^S - \left(\frac{DIV^{NB}}{Y} + \frac{P^S S}{Y}\right)p^S\}$$

-

In equilibrium,

$$R^{IB} - \Xi'\left(IB\right) = \left(\frac{\left(R^B - 1\right)\left(\frac{P}{P_{+1}}\right) + P^B_{+1}}{P^B}\right)\left(\frac{P_{+1}}{P}\right)$$

$$\frac{R^{IB}\frac{IB}{Y}}{R^{IB}\frac{IB}{Y} - \eta\frac{\Xi(IB)}{Y}} r^{IB} - \frac{\eta\frac{\Xi(IB)}{Y}}{R^{IB}\frac{IB}{Y} - \eta\frac{\Xi(IB)}{Y}} \frac{(\eta - 1)}{\widetilde{\Xi} R^{IB}} \frac{\Xi(IB)}{Y}\left(\frac{IB}{Y}\right)^{-1} ib = -\frac{1}{R^B}\pi_{+1}\frac{1}{R^B}p^B_{+1} - p^B + \pi_{+1}$$

## A.8 Central Bank & Government

Central bank's balance sheet



$$P^B B^{CB} = RRD + \frac{CSH}{P}$$

$$\left(RR\frac{D}{Y} + \frac{\frac{CSH}{P}}{Y}\right)\left(p^B + b^{CB}\right) = \frac{RRD}{Y}\left(rr + d\right) + \frac{\frac{CSH}{P}}{Y}\left(csh - p\right)$$

-

Central bank's profits

$$\Pi^{CB} = \left(\frac{\left(R^B - 1\right)\left(\frac{P_{-1}}{P}\right) + P^B}{P^B_{-1}}\right) P^B_{-1} B^{CB}_{-1} - R^{RR}_{-1} RR_{-1} D_{-1}\left(\frac{P_{-1}}{P}\right) - \frac{CSH_{-1}}{P_{-1}}\left(\frac{P_{-1}}{P}\right)$$

$$\left(R^B\frac{B^{CB}}{Y} - R^{RR}RR\frac{D}{Y} - \frac{\frac{CSH}{P}}{Y}\right)\pi^{CB} = P^B\frac{B^{CB}}{Y}\left(p^B + R^B b_{-1}^{CB} - \left(R^B - 1\right)\pi\right) - R^{RR}RR\frac{D}{Y}\left(r_{-1}^{RR} + rr_{-1} + d_{-1} - \pi\right)$$
$$- \frac{\frac{CSH}{P}}{Y}\left(csh_{-1} - p_{-1} - \pi\right)$$

-

Taylor rule

$\frac{R}{R^{SS}} = (\frac{R_{-1}}{R^{SS}})^{\rho^r} * (\Pi^{\phi_\pi} * (\frac{Y}{Y^{SS}})^{\phi_y})^{(1-\rho_r)} * \exp(\varepsilon^r)$

$r = \rho_r r_{-1} + (1 - \rho_r)(\phi_\pi \pi + \phi_y y) + \varepsilon^r$

-

Open market operations

$\left(\frac{\left(R^B-1\right)(\frac{P}{P_{+1}})+P_{+1}^B}{P^B}\right)\frac{P_{+1}}{P} = R$

$-\frac{1}{R^B}\pi_{+1} + \frac{1}{R^B}p_{+1}^B - p^B + \pi_{+1} = r$

-

Reserve requirement shock

$rr = \rho^{RR} rr_{-1} + \epsilon^{RR}$

-

Remuneration to reserve requirements should be a fraction of the policy rate:

$R^{RR} = \theta^{RR} R^{P}$ with $\theta^{RR} < 1$

$r^{RR} = r^{P}$

-

Tax to finance subsidy to retailers

$T^{S} = \tau^{r} \frac{P^{w}}{P} Y^{w}$

-

Government's budget constraint

$\Pi^{CB} + P^{B} B + T^{B} - G = (R^{B} - 1) B_{-1} + P^{B} B_{-1}$

$P^{B} B^{CB} \left(- \left(R^{B} - 1\right) \pi + p^{B}\right) + R^{B} P^{B} B^{CB} b_{-1}^{CB} - R^{RR} RRD \left(r_{-1}^{RR} + rr_{-1} + d_{-1} - \pi\right) - CSH \left(csh_{-1} - p_{-1} - \pi\right) + T^{B} t^{b} - Gg = (R^{B} - 1) B \left(b - \pi\right)$

## A.9 Resource Constraint and Bond Market

Resource constraint

$Y = C + C^{e} + X + (1 - f\left(\overline{\omega}\right) - g\left(\overline{\omega}\right)) R^{e} Q_{-1} K (\frac{P_{-1}}{P}) + \Xi \left(IB_{-1}\right) (\frac{P_{-1}}{P}) + \frac{\kappa^{d}}{2} \left(\frac{R^{D}}{R_{-1}^{D}} - 1\right)^{2} R_{-1}^{D} D_{-1} (\frac{P_{-1}}{P})$

$$y = \frac{C}{Y}c + \frac{C^e}{Y}c^e + \frac{X}{Y}x - R^e\frac{K}{Y}(f'\left(\overline{\omega}\right) + g'\left(\overline{\omega}\right))d\overline{\omega} + (1 - f\left(\overline{\omega}\right) - g\left(\overline{\omega}\right))R^e\frac{K}{Y}\left(r^e - \pi + q_{-1} + k\right) + \frac{\Xi(IB)}{Y}\left(\eta i b_{-1} - \pi\right)$$

-

Bond market

$$B = B^{CB} + B^{NB}$$

$$\frac{P^B B}{Y}b = \left(\frac{P^B B}{Y} - \frac{P^B B^{NB}}{Y}\right)b^{CB} + \frac{P^B B^{NB}}{Y}b^{NB}$$

Inflation definition

$$\pi = p - p_{-1}$$

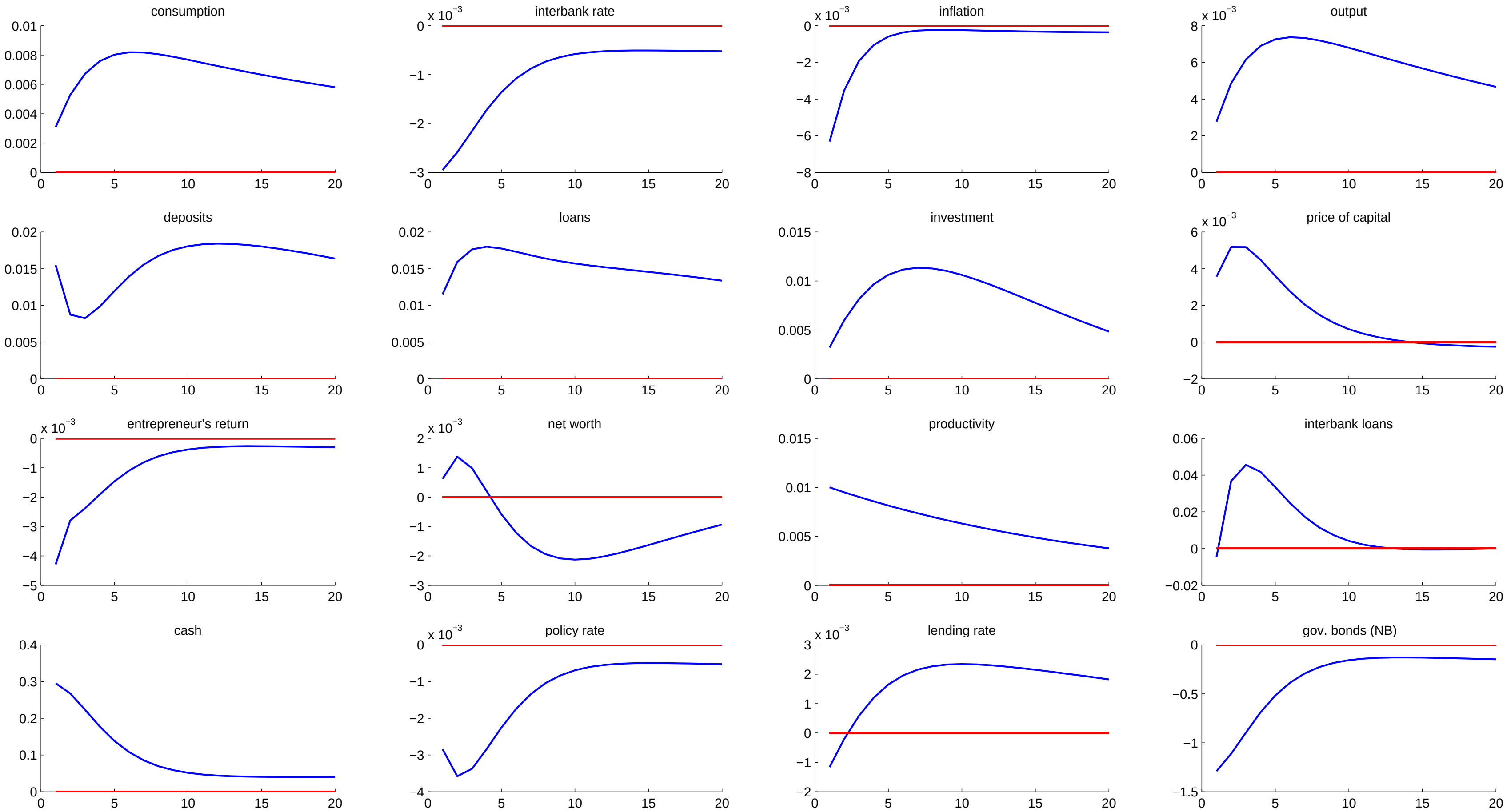


Figure 3-2: Productivity shock

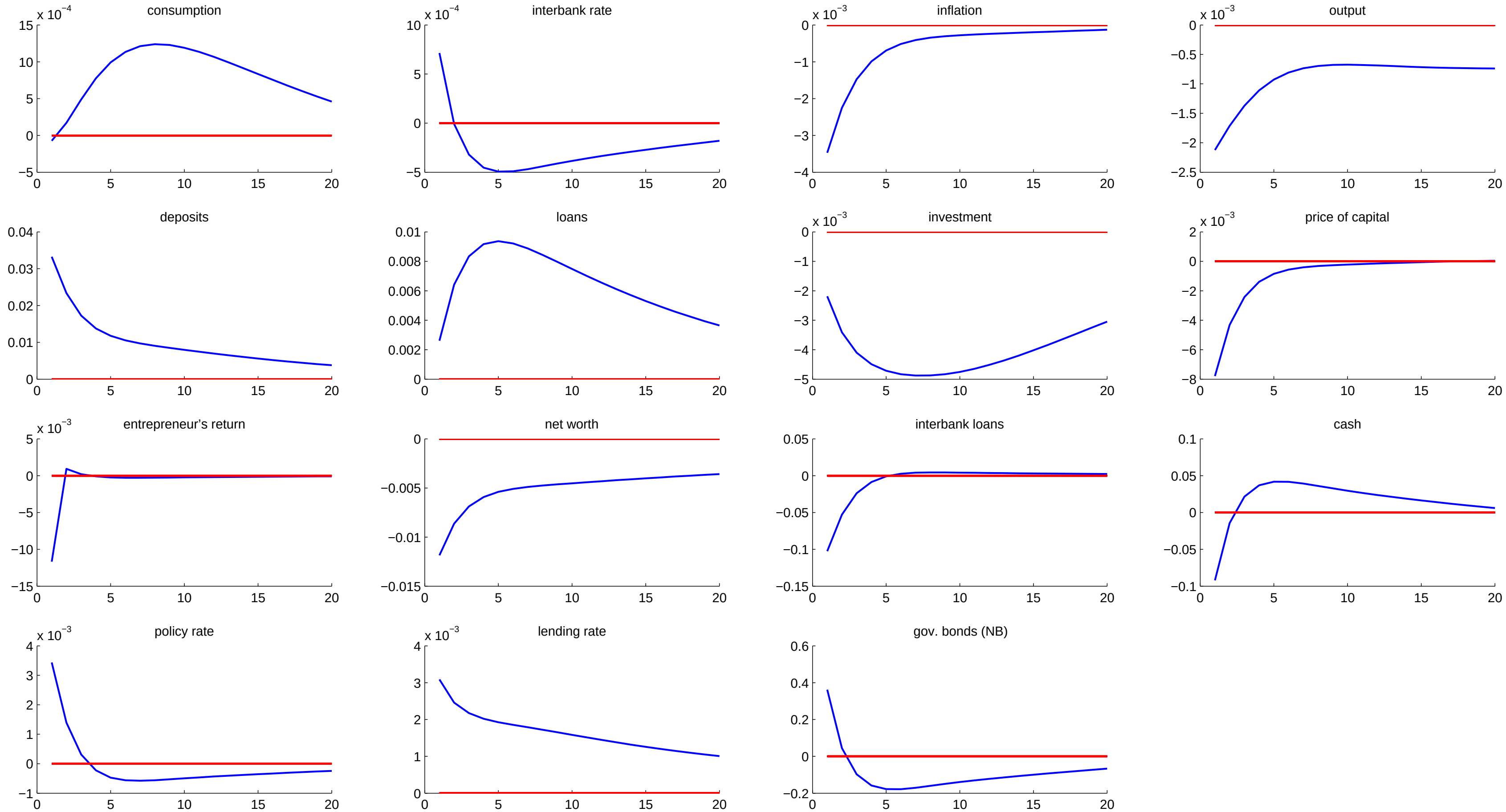


Figure 3-3: Monetary policy (MP) shock

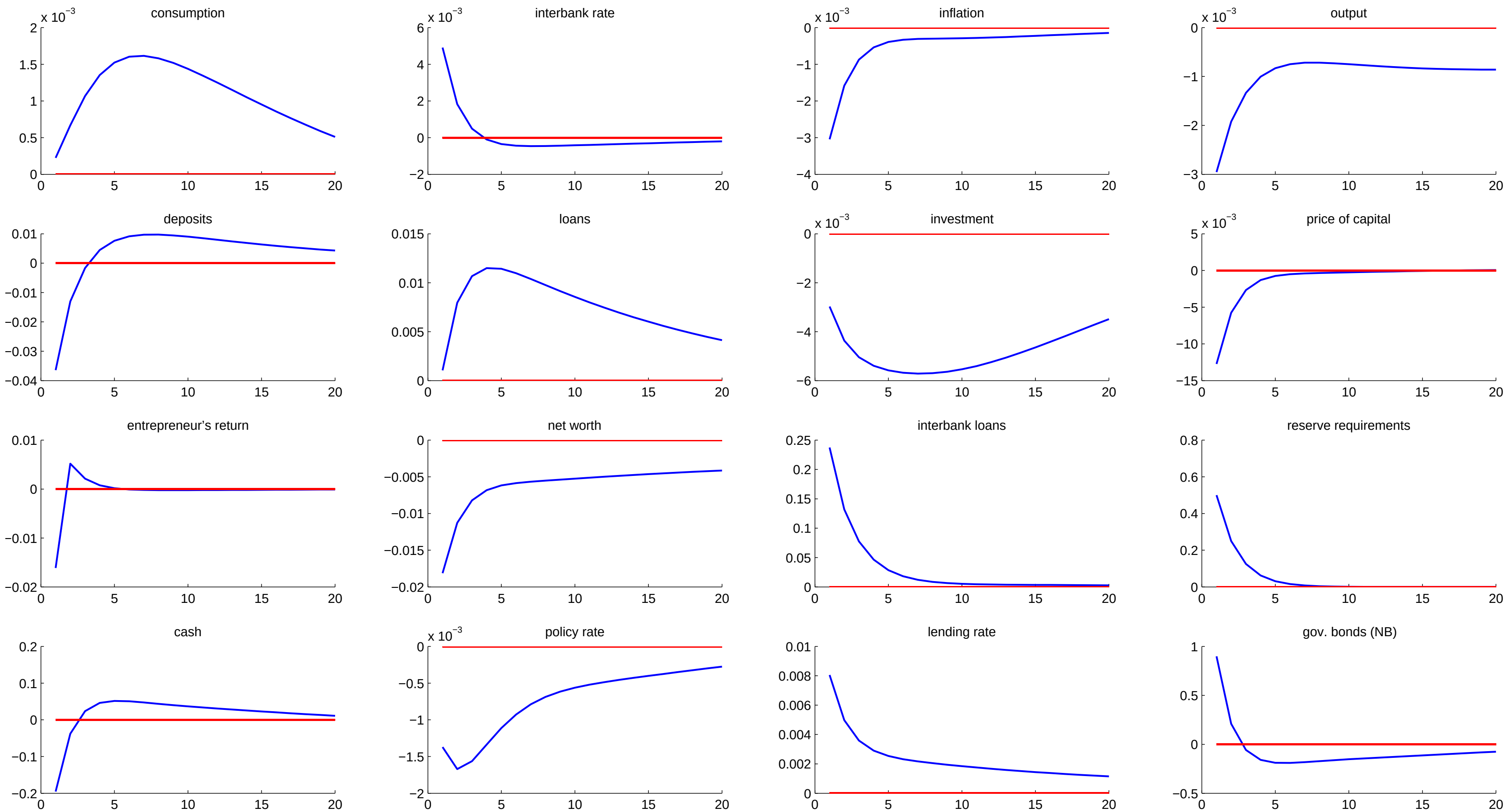


Figure 3-4: Reserve requirement (RR) shock

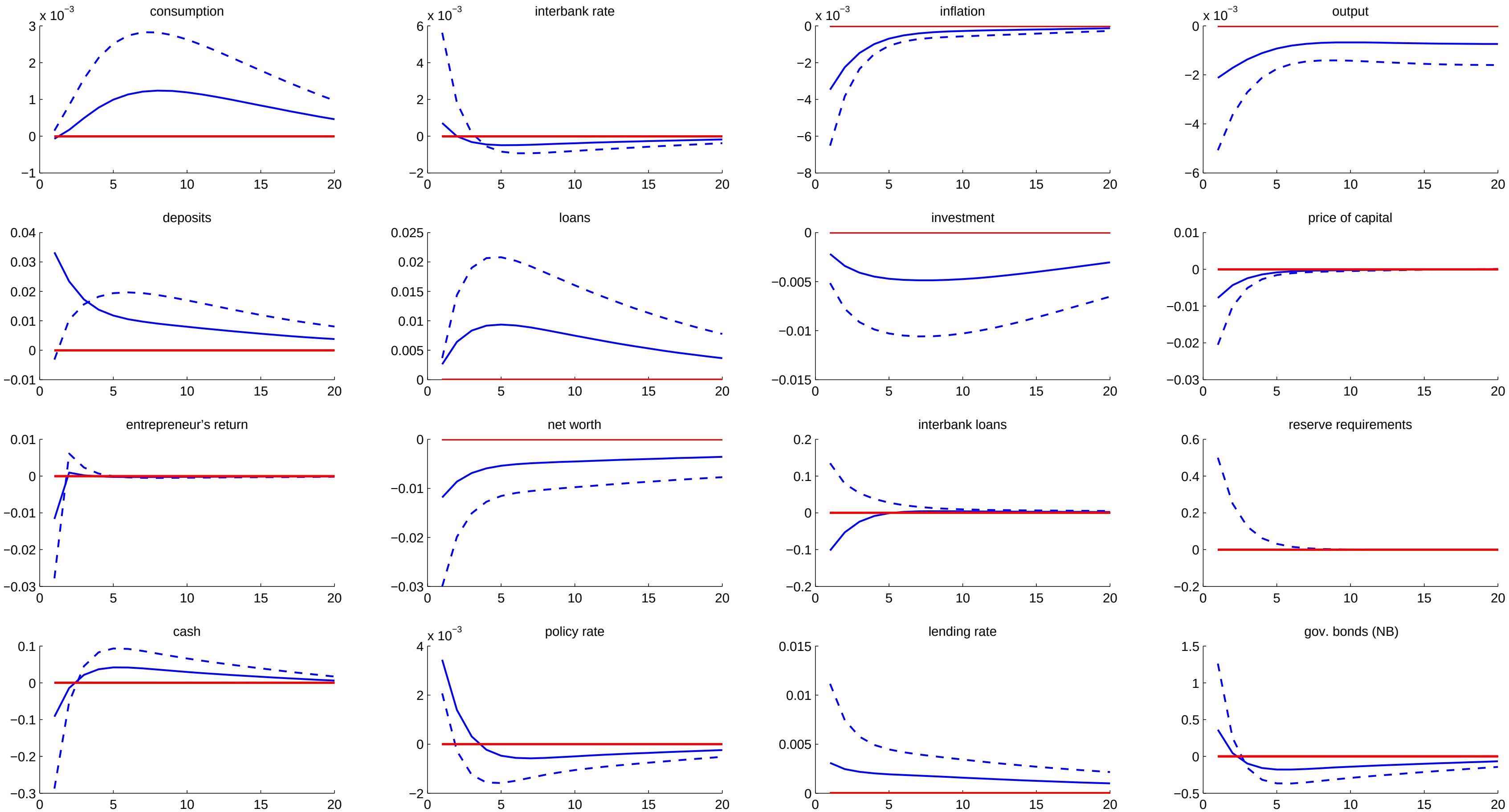


Figure 3-5: Combined MP and RR shock (dashed) versus MP shock only (solid)