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What can the philosophy of mathematics learn from the history of mathematics?

> You ask me about the idiosyncracies of philosophers? ... There is their lack of historical sense, their hatred of even the idea of becoming, their Egyptianism. They think they are doing a thing *honour* when they dehistoricize it, *sub specie aeterni*—when they make a mummy of it.
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> Nietzsche *Twilight of the Idols*

This paper began as a contribution to a workshop called 'Towards a new epistemology of mathematics'.[1] The motivating perception of this workshop was that philosophers of mathematics are no longer content to restrict their enquiries to questions that they can address using formal logic and conceptual analysis. Mathematical research is natural, in the sense that naturally-evolved creatures do it, and it is social, in the sense that the validation of new mathematics is a collective activity. 'Collective validation' does not merely mean that mathematicians check each other's proofs (though this is an important aspect). Mathematicians judge mathematical work for depth, importance, interest, elegance, *etc*. in referees' reports to journals, book reviews, PhD exam reports and other kinds of communication. Thus, evaluation is, in most of the important instances, a group activity. To explore either of these avenues (natural and social), it is not enough to look at mathematics as modelled in meta-mathematics, nor is it enough to study completed mathematics as presented in journals or textbooks; philosophers must look at mathematics in the making, that is, at mathematical practice. This is difficult, but fortunately, there is a professional body of academics already documenting and analysing mathematical activity: historians of mathematics. Of course, historians are not the only students of mathematical practice. Mathematicians[2], philosophers[3] and neurologists[4] have their contributions to make, among others. However, the focus of this paper is the thought that historians may have something to offer.

Perhaps. But philosophers of science called on historians of science for help in the 1960s and 1970s, with mixed results. Ronald Giere, writing in 1973, called the relationship between history of science and philosophy of science a 'marriage of convenience'; John Zammito, thirty years later, declares it a 'failed marriage'.[5] No doubt others will wish to insist that the romance is still alive, but it cannot be denied that this has been a difficult relationship, and less fertile than some hoped for at the time of the wedding. The

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[1] I am grateful to the conveners of this workshop for the opportunity to present these thoughts. Also to Michèle Friend for valuable criticisms and suggestions.
[2] E.g. Davis & Hersh (1995).
[3] E.g. Breger & Grosholz (2000); Corfield (2003).
[4] E.g. Butterworth (1999).
[5] Giere (1973) p. 282; Zammito (2004). Giere's paper is a review of the published proceedings of a conference convened precisely to assess the relationship between history and philosophy of science.

basic difficulty, from the philosophical point of view, is that historiography does not offer unmediated access to the past—nor could it. History is an academic discipline with its own interests and methods, its own *a priori* structure. The two partners see the world differently and have different ambitions. Before the history and philosophy of mathematics rush into wedlock, we would do well to consider some of these differences.[6]

There is a traditional contrast between history and philosophy: history deals with the particular and the temporal, while philosophy deals with what is universal and a-temporal (or in linguistic terms, tense-less). This contrast shows itself in the characteristic professional vices of historians and philosophers respectively. The temptation for historians is antiquarianism, that is, collecting interesting old facts as some people collect silver spoons. In philosophy, the temptation is to buy universality at the price of abstracting up so many levels that we lose touch with the phenomena that we hoped to understand.

However, this simple distinction is itself excessively abstract. The relation between history and philosophy is more interesting, dynamic and purpose-sensitive than this. Historiographic practices are themselves too various to stand in a simple, abstract and uniform relation to philosophy; philosophy is, if anything, yet more heterogeneous. The relationship between these disciplines may depend on who is drawing it. For example, here is Emily Grosholz, comparing Sasaki's book about Descartes's mathematics with Bos's work on the same topic:

> H.J.M. Bos's recent (2001) book on Descartes's mathematics… though written by a historian of mathematics, seems motivated by a thoroughly philosophical interest in how changes in mathematical procedures, representation, and ontology take place. Professor Sasaki's concern, by contrast, is to chart how ideas are transmitted textually from one era or culture to another, and to make precise the chronology of Descartes's acquisition, or relinquishing, of certain ideas.[7]

For Grosholz, Bos's interest in procedures, representation and ontology led him into philosophy. Sasaki, on the other hand, is a purer historian (according to Grosholz) because he investigates textual transmission and chronology. Now, Bos is a historian, not a philosopher. He writes about antique mathematics for its own sake, rather than for any light it might shed on mathematics in general or on our mathematical practices now. A philosopher of mathematics who chose to write about Descartes would have to explain how a four-hundred-year-old example could illustrate anything philosophically interesting about mathematics as it is practiced now (which is not to say that antique mathematics cannot be philosophically illuminating of contemporary mathematics, but the connection would have to be argued). Grosholz's point is not to suggest that Bos is a philosopher in disguise; rather it is that philosophy is part of Bos' historiographic motivation. Bos and Sasaki are both historians, but they stand in different relations to philosophy.

I hope that this example has made it plausible that one can relate history and philosophy in different ways for different purposes. How then should we relate history and philosophy given our purpose of establishing a new epistemology for mathematics? Here is a list of ideas one might have in mind when saying that one wants to bring the history of mathematics into the philosophy of mathematics:

1. The temporal dimension of logic
2. Explanatory Appeal to Context rather than to General Principles
3. Heraclitean Flux

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[6] For the courtship: Aspray & Kitcher (1988).
[7] Grosholz (2005) p. 337.

4. All history is the History of Thought
5. History is Non-Judgmental

This list starts at the least philosophically problematic and ends at a point where we seem to leave philosophy altogether. So then, the question becomes, is it possible to stop part-way down?

1) The temporal dimension of logic

By the phrase 'the temporal dimension of logic', I intend the Popperian thought that we adopt a theory in part because it solves a problem present in the theory it replaced. Most of Popper's immediate[8] predecessors (principally, the Vienna circle) thought that one should accept a scientific theory solely in virtue of its relationship with the empirical evidence. Popper did not deny the importance of empirical evidence, but he thought that its function is to make trouble for theories, not to confirm them. In his view, our current scientific orthodoxies are our least lousy theories so far. We adopt our latest theory, T_n because it solves a problem present in its immediate predecessor T_{n-1} (without reducing content or resorting to *ad hoc* manoeuvres). The details of Popper's philosophy of science no longer command widespread assent, but most philosophers of science accept that theories are tested against each other rather than directly against nature. We accept our current theory because it beat (past tense) the previous champion.

In *Proofs and Refutations*, Lakatos added a temporal dimension to the philosophy of mathematics. Why should we accept one mathematical definition rather than another (think of the various definitions of the continuum, or of integration, or, take one of Lakatos' examples, the definition of measurable set[9])? We accept a refined, technical definition, according to Lakatos, because it allows us to prove a theorem or solve a problem that previously defeated us or left us in some way mathematically unsatisfied. We define continuity, for example, the way we do (ε-δ) because earlier definitions (such as 'function you can plot without taking your pencil off the page') ran into trouble of one kind or another.

The introduction of this temporal dimension does not lead, by itself, to historicism, anti-realism or relativism. However, it does require the abandonment of wholly algorithmic conceptions of rationality. There is no formula for 'improves on' or 'solves a problem for'. Steering scientific research typically requires judgment calls that will not reduce to mechanical rules. For example, under criticism Popper had to temper his emphasis on refutation with the thought that in its early phase a theory cannot survive without the help of some dogmatism on the part of its defenders. Theories are not born fully formed and ready to undertake severe empirical tests. However, there is no hope of calculating the moment when a theory becomes mature enough to stand or fall without such dogmatism. Nor is there any hope of specifying exactly how much dogmatism is appropriate.[10] Moreover, introducing this temporal element changes the unit and standard of appraisal. Instead of asking 'is this theory true?' we have to ask 'is this series of theories (or concepts or programmes or disciplinary matrices) improving?' This is not to say

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[8] Of course, there were historically-minded philosophers of science before Popper, such as Whewell and Duhem. Moreover, French philosophy of science in the inter-war years was heavily historical. For Koyré, the spirit of the time was, "tellement infectée d'historicisme qu'elle ne conçoit pas qu'il puisse y avoir d'elle-même une autre connaissance que la connaissance historique, époque qui n'admet pas qu'elle puisse se comprendre et s'expliquer à elle-même si ce n'est à travers et en fonction de son passé, son histoire." Quoted in Jorland (1981 p. 72). See also Koyré (1973 p. 17) "le style de notre époque, éperdument théorique, éperdument pratique, mais aussi éperdument historique…". However, this tradition does not seem to have influenced Popper.

[9] *Proofs and Refutations* pp. 152-155.

[10] "One must treat budding programmes leniently: programmes may take decades before they get off the ground and become empirically progressive… there is no refutation without a better theory." (Lakatos 1978a p. 6)

that the concept of truth drops out of the picture; without it, activities such as proving, presenting counterexamples, identifying contradictions, *etc*. become unintelligible. These activities play essential roles in mathematical practice, so they had better make sense.[11] Hence, the philosophy of mathematical practice requires some conception of truth, even if it is conditional, attenuated or domain-specific.[12]

The chief point for the present purpose is that introducing this temporal element into logic does not bring philosophy into close collaboration with history. On the contrary, this approach encourages philosophers to write rational reconstructions of selected episodes from the history of science and mathematics. Rational reconstructions are quite different from the accounts that historians give of the same episodes. To see the difference, consider the explication of the term 'rational reconstruction' in Hans Reichenbach's *Experience and Prediction* (1938).[13] The programmatic section of this work is a series of distinctions: between 'external' and 'internal' relations; between psychology and logic; and between the contexts of discovery and justification. The point of these distinctions is to divorce the epistemological question from its historical context, so that the intellectual activity of the actual, human scientist may be replaced with an ideal logical relation between theory and evidence.[14] For historians looking at such treatments of mathematics or science, these distinctions are arbitrary and the rational reconstruction is a fantasy. Historians have no interest in knowing how the history of science would have gone, had the agents been ideal scientists rather than real people. For philosophy, these rational reconstructions are less useful than they first appear. In the rational-reconstruction idiom, the philosopher's conception of rationality shapes his rational reconstructions, and in doing so obliterates the original contours of the episodes reconstructed. Since a typical philosopher (such as Reichenbach) insists on just one conception of rationality, all his rational reconstructions come out the same shape. This philosophical approach thus quickly degenerates into a night in which all cows are black. To put the same point another way, the rational-reconstructionist philosopher is like a mediocre pavement artist whose caricatures all look the same.

That was the experience in the philosophy of natural science. In the philosophy of mathematics, the most common form of rational reconstruction is also the most radical, namely, to identify mathematical proof with the gap-free, fully formal proofs of meta-mathematics. Just as in rational-reconstructionist philosophy of science, the effect is to obscure the specificity of mathematical practices. It is not quite true to say that all proofs look the same in this light—some fully formal proofs require stronger logical resources than do others. Nevertheless, in fully formal proofs, the logical machinery of the system does all the work of inference. Thus, identifying mathematical proof with fully formal proofs tends to wash out the differences between proof-ideas. However, this approach lacks the temporal element that Popper introduced into the logic of science. We are looking for a mathematical parallel with rational-reconstructionist philosophy of science.

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[11] That is why Wittgenstein's suggestion that mathematical statements are expressions of grammar cannot be the whole account. Grammatical expressions (in Wittgenstein's sense) cannot have counterexamples (that is most of what 'grammatical' means here). It is impossible to take seriously the falsehood of a grammatical expression, therefore an attempt to prove one can only be quixotic.

[12] This notion of truth-in-practice is not be confused with the meta-mathematical notion of truth-in-$\mathcal{L}$ (where $\mathcal{L}$ is a fully formalised language with an explicit specification of its well-formed formulae).

[13] Reichenbach credits the term *rationale Nachkonstruktion* to Carnap in *Der logische Aufbau der Welt* (Berlin and Leipzig, 1928).

[14] "Epistemology thus considers a logical substitute rather than real processes. For the logical substitute the term *rational reconstruction* has been introduced…" Reichenbach (1938) p. 5.

The proper parallel is with the rational reconstructions of episodes in the history of mathematics offered by Lakatos and his followers. Here, the temporal element is explicitly present. Lakatos complained in the introduction to *Proofs and Refutations* that "Formalism disconnects the history of mathematics from the philosophy of mathematics"[15] ('formalism' in Lakatos' sense is "the school of mathematical philosophy which tends to identify mathematics with its formal axiomatic abstraction (and the philosophy of mathematics with metamathematics"). A little later, he offers a paraphrase of Kant, "The history of mathematics, lacking the guidance of philosophy, has become *blind*, while the philosophy of mathematics, turning its back on the most intriguing phenomena in the history of mathematics, has become *empty*."[16] He never argued at any length for first half of this formula.[17] Regarding the second half, *Proofs and Refutations* takes instructive details and isolated quotations from historical sources, but it does not offer a history of the Descartes-Euler formula (nor did Lakatos pretend otherwise[18]). He never theorised the relation between history and philosophy of mathematics beyond these few remarks, but he did offer an account of the relation between history and philosophy of science. This, though, returns us to the earlier problem: in spite of his best efforts, Lakatos' philosophy of science suffers from the same difficulty as other rational-reconstructionist accounts. Every episode in the history of science has to have a research programme with a hard core, positive and negative heuristics, *etc.*.[19] Lakatos' rational reconstructions may have been more supple and subtle than those of his rivals, but they still press the endless variety of history into a common frame. To combine a pair of Hegelian metaphors, when Lakatos' methodology of research programmes paints its grey on grey; when dusk falls and the owl of Minerva takes flight, the cows may not all be black, but in the fading light they all start to look rather similar.[20]

Explanatory Appeal to Context rather than to General Principles

It seems, then, that bringing history into epistemology requires more than the merely temporal element present in Popper and Lakatos. Perhaps it requires the thought that historians do not explain events by subsuming them under general schemes (be they causal laws, methodological models or conceptions of rationality), but rather by setting events in their proper historical contexts. This is controversial within the philosophy of history, but it is common to most theorists of historiography that historical explanations do not appeal to general laws in the way that explanations in natural science do. After all, our complaint about rational reconstructions is that they separate scientific thought from its context in order to caricature it. But if context plays a special role in historical explanation, we have to ask, what sort of context? Unqualified, the word 'context' covers a multitude of entanglements. There are three obvious candidates:

- Mathematical Context
- Intellectual Context

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[15] Lakatos 1976 p. 1

[16] *Op. cit.* p. 2.

[17] "I maintain that all historians of science *who hold that the progress of science is progress in objective knowledge*, use, willy-nilly, some rational reconstruction." (Lakatos 1978a p. 192; see also op. cit. pp. 120-121).

[18] "My purpose was to distil a methodological message from the history, rather than to write history itself." (Lakatos 1978a p. 192).

[19] Some authors have tried to carry the methodology of scientific research programmes (or parts of it, with modifications) from natural science into mathematics (see Hallett 1979, Koetsier 1991, Corfield 2003); for criticism of MSRP see Larvor (1998) esp. chapters four and six; for criticism of Methodologies of Mathematical Research Programmes, see *Op. Cit.* and Larvor (1997). Kitcher offered another philosophically motivated history of mathematics in his (1983), in which real characters give way to an ideal mathematical agent, just as Reichenbach recommends.

[20] The cows appear in paragraph sixteen of the preface to the *Phenomenology of Spirit*; the owl of Minerva takes flight in the penultimate paragraph of the preface to the *Philosophy of Right*.

- Institutional/Social Context

No-one, including Reichenbach, would object setting a mathematical idea or theory in its mathematical context. The point of Reichenbach's distinction between internal and external relations is to ensure that epistemologists relate scientific sentences only to other scientific sentences. That is what 'internal' means in Reichenbach's phrase 'internal relations'. However, the development of a mathematical idea may make no sense in isolation from its wider intellectual context. For example, the development of 'mathematical' statistics (as opposed to 'vital' statistics) is related in subtle ways to the change in biology from essentialism to the Darwinian emphasis on gradual variation.[21] Beyond such intellectual connections, there are institutional factors. For example, the structure of incentives and rewards within the professional mathematical community may affect the content of the mathematical work (e.g. 'pure' mathematics is sometimes valued, sometimes despised). The history of statistics serves to illustrate the next point too; the growth of a mathematical idea may depend on the institutional or social context the mathematician finds himself in. For example, governments began to collect census data when the size and health of the population became politically important. Mathematics often develops in response to practical problems, and these practicalities may condition the mathematics for as long as it continues to develop. Thus, a question in the epistemology of mathematics ('what is the justification for using this mathematical concept rather than that?') may require an excursion beyond the bounds of the strictly mathematical, scientific or intellectual. In other words, a properly epistemological question may require us to violate Reichenbach's distinction between internal and external relations.

We saw earlier that the temporal dimension of logic introduced a new question: is this body of thought progressing or degenerating? In other words, after Popper, we should think of a body of scientific thought as something that changes, and ask whether the changes are for the better. Suppose now that we have to consider the wider context of a body of mathematical or scientific thought in order to understand it. There will always be some part of that context undergoing change. If contextualism is true, then change ramifies through all the contextual connections. This introduces the next idea.

Heraclitean Flux

Earlier, we saw Grosholz suggest that philosophical concerns were part of Bos' motivation in his book on Descartes' mathematics. I suggested then that Bos might defend himself against the accusation that he has fallen from history into philosophy, simply by pointing out that he was writing about antique mathematics. A philosopher who wrote about Descartes' mathematics would have to explain what philosophical insight he hopes to gain from a study of antique mathematics. Now, there are answers that such a philosopher could give. The same question arises when philosophers write about antique philosophy; in the final section of this paper, we will look at Bernard Williams' attempt to explain the philosophical benefits for *us, now*, of studying Descartes' philosophy. However, I wish here to consider another reply that Bos might make to Grosholz.

If we wished to defend Bos' status as a historian more vigorously, we could appeal to the view that history is not defined by its subject matter but rather by its interest in change. Certainly (argues the defence), philosophers study mathematical procedures, representation, and ontology, but as a historian, Bos studies *changes* in mathematical procedures, representation, and ontology. On this view, if something does not change, it is of no interest to historians except, perhaps, as a background against which changes might be tracked. Gibbon said that history is "little more than the register of the crimes,

[21] Magnello (2006) p. 220.

follies and misfortunes of mankind".[22] His point is that a historian may spend an entire chapter on a battle, and then allow a century of peace and prosperity to go by in a single paragraph. This does not merely reflect a taste for dramatic material. Significant battles change the course of history in obvious ways, whereas the changes wrought by lasting peace and economic stability are more elusive. Crimes, follies and misfortunes predominate in political and military historiography just because historians study change. The same thought expresses itself in the well-known curse: *may you live in interesting times*.

In order to comprehend and explain changes, historians must first bring them into view. They do this by distinguishing earlier and later versions of whatever it is that undergoes the change in question. Consequently, the historiography tends to break up the unity of the object of enquiry. For example, while the philosopher investigates the epistemology and ontology of *mathematics*, the historian will insist that there is no mathematics as such but rather ancient Greek mathematics, Chinese mathematics, the mathematics of the Italian renaissance, and so on. Naturally, to understand the development of mathematics during the Italian renaissance, we must distinguish the mathematics of the early renaissance from that of the later renaissance. These divisions can continue indefinitely. For his part, the philosopher, faced with the dissolution of his object of enquiry, will insist that despite all the variety and change, these diminishing temporal parts are all mathematics. This could degenerate into a stand-off in which each discipline insists on its own *a priori* stance. However, the question whether mathematics has an essence, or whether its unity is rather that of a narrative, is of central philosophical interest. It requires philosophers of mathematics to take seriously some unfashionable philosophical ideas (such as the dialectic of unity in diversity) and question some philosophical dogmas (such as the intelligibility of appealing to atemporal standards of rationality).

To think of historiography as the explanation of change helps to diagnose what went wrong when philosophers offered general models of rational theory-change, and expected to find them present in the historical record. Philosophers expected to find that the great scientists of the past acted in conformity with a universal 'logic of science' (such as those on offer from the logical positivists, Popper, Lakatos and more recently Bayesianism). Such a logic would have to be an ahistorical essence, that is, something that is active in the history of science but not acted on by it. An appeal to such essences is unhistorical—from a Heraclitean historical perspective, anything that makes a difference in history is changed by its participation in events. On the whole, philosophers of science have learned this lesson; those who develop abstract methodological models do not expect these ahistorical models to explain the successes and failures of real historical scientists.[23] Insofar as philosophers of science see something like a 'scientific method' in the history of science, they recognise it as a temporal achievement: scientists had to learn how to learn.[24]

To say that historiography is about change is not to say that it is exclusively concerned with revolutions and ruptures. Change can be gradual, and it can come about as the result of stability (for example, a long period of peace might deprive a nation of its military edge; a long period of intellectual stagnation might entrench some habits of thought while others fall into oblivion). In the history of mathematics, there are

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[22] Pp. 105-6. This remark is often mistaken as an expression of cynicism about mankind, when in fact it makes a point about historiography. "Antoninus diffused order and tranquillity over the greatest part of the earth. His reign is marked by the rare advantage of furnishing very few materials for history, which is, indeed, little more than the register of the crimes, follies and misfortunes of mankind."

[23] E.g. Laudan, "I believe that the requirement that a methodology or epistemology must exhibit past science as rational is thoroughly wrong-headed." (1996 p. 195-196).

[24] I lay out this argument in more detail in Larvor (2007).

stabilities and continuities that span centuries. For example, Viète, writing at the end of the sixteenth century, uses Eudoxus' theory of proportions (developed in the fourth century BC) to develop his algebra.[25] *Pace* Gibbon, it seems, such survivals and stabilities are part of the historian's field, just as much as revolutions and breaks. However, Eudoxus' theory of proportions is of particular interest to historians of early modern mathematics precisely because it contributed to, and was in turn transformed by, the rapid development of mathematics in the seventeenth century. To understand that, we do not need to know what Eudoxus thought about it; we need to know how Viète and his contemporaries understood the theory of proportions. Now, the theory of proportions as understood by Viète and his contemporaries is a different object from the theory of proportions as understood by Eudoxus. Otherwise, Viète could not have done something new with it. Therefore, where we thought we had an invariant (the theory of proportions), we now have variation (the theory of proportions according to Eudoxus versus the theory of proportions according to Viète). In short, the view that historiography is principally about change depends on another thesis, which we will now consider.

All History is the History of Thought

This slogan, due to R.G. Collingwood[26], summarises his view that historians understand and explain events in the past by reconstructing the thoughts of the people who acted in those events. Collingwood's conception of historiography is not an established orthodoxy among historians and theorists of history. A standard objection is that he puts undue emphasis on the actions of individuals and underplays trans-personal forces. However, Collingwood's slogan can account for a common feature of historiography: it is perceptions that matter. To understand the history of Christianity and Christendom, for example, it is not enough to read the Bible. The contents of the Bible are not directly effective; what matters are people's beliefs about the contents of the Bible. Failure to appreciate this point makes a mystery of the fact that a text containing the words "blessed are the peacemakers" and which distinguishes the kingdom of God from the empire of Caesar, could inspire so much state violence (for example). Turning to the case of mathematics, even if there are mathematical objects or truths laid up in a Platonic heaven, or gapless proofs that an ideal mathematician could give 'in principle', such ideal items are no more historically effective than the contents of scripture. As we just saw, to understand the historical role of the theory of proportions in the emergence of early modern algebra, it is not enough to understand Eudoxus. One must reconstruct Viète's thoughts about the theory of proportions.

At this point, we seem to be teetering on the edge of social-constructivism. However, we need not fall over the edge. Collingwood's claim is just that material (and, presumably, mathematical) facts become historically significant only when mediated by thought.[27] This remark is independent of the philosophical account we give of the word 'fact'. The reason for mentioning this is that historians often write sentences such as, "In the seventeenth century, the universe suddenly got much bigger". Read literally, this is false, but what is meant, of course, is that the universe suddenly seemed much bigger.[28] Historians of science run no danger by using this shorthand because they study changes in thoughts about the universe, and

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[25] I am grateful to an anonymous referee for this example and a number of other well-taken points.

[26] Collingwood (1994 p. 115)

[27] Collingwood's remark needs qualification: a change in the weather can change the outcome of a battle or sink a ship without anyone having to think about it. Nevertheless, his fundamental point survives, because purposes and convictions brought the armies to the field and put the ship to sea, and more thoughts determine the consequences. For example, if the king's eldest son dies in war or at sea, his death will depend for its significance on beliefs about royal succession.

[28] The universe is expanding, so it did get bigger in the seventeenth century as it does every century—but not suddenly.

leave the universe itself to natural scientists. For philosophers, this distinction is a dilemma: is philosophy continuous with science and mathematics, or is it a meta-enquiry? That is, do philosophers of mathematics study mathematical objects and structures in a way continuous with and complementary to the work of mathematicians? Or do philosophers study mathematical thought and practice, in a way continuous with and complementary to the work of historians?

Collingwood sketched the materials for an answer to this question in the final pages of *The Idea of Nature.* There, he argued that natural science is not the sole kind of objective enquiry. It is not even the most fundamental kind of objective enquiry. A scientist works by relating his or her thoughts and experiences to the recorded thoughts and experiences of other scientists. Therefore, the status of natural science as objective enquiry depends on the possibility of relating thoughts to thoughts intelligibly. That is, unless a scientist can *explain to others* the connection between two thoughts, and unless he can do so convincingly (which is a matter of interpretation and judgment), the purported connection will never become part of science. This is the core of what Collingwood understood by 'history'. He concluded, therefore, that, "natural science as a form of thought exists and always has existed in a context of history, and depends on historical thought for its existence."[29] So too with mathematics. Mathematics, like natural science, cannot account for itself as a rational human activity. To take up the dilemma at the end of the previous paragraph: there is no reason why philosophers should not take on questions that arise within mathematics. Take, for example, the discussion of the merits of category theory as a foundational framework for mathematics. Philosophers can and do contribute to this debate. However, it is not long before prior questions arise: what virtues are desirable in a foundational framework? Why should we want a foundational framework at all? These more properly philosophical questions invite prior questions of their own: how can we answer such value-questions without arbitrariness? Pursuing this line, one will eventually run into the following fact: the ideas that we live by (including the ideas that guide our mathematical enquiries) are not fixed points. They are our least bad attempts so far to solve problems that are themselves evolving as a result of our efforts to live by our least bad notions so far. The intelligibility of our mathematical enquiries depends on the intelligibility of this historical process.

Now that we have raised value-questions, we have a new problem. Historiography is not supposed to issue value-judgments. Philosophy, for its part, crucially involves evaluation. So philosophers flirting with history had better understand this value-neutrality, if only to make sure that they are not seduced into relativism.

History is Non-Judgmental

Historians do not make absolute moral judgments about characters living in times different to their own, because it makes no historical sense to do so. We can ask whether Caesar was a good emperor or a good man by Roman standards.[30] To ask whether he was a good ruler or a good man by our standards would require us to pluck Caesar from his historical moment and consider him in isolation. Our earlier arguments show why this would be unhistorical. We understand Caesar by relating him to his circumstances rather than by treating him as an instance of a general type. Deprived of his circumstances, he becomes unintelligible. We can ask without anachronism how his character and conduct compare to those of his contemporaries such as Brutus or Augustus. But this comparison makes sense just because similar circumstances and institutions formed them, so it is reasonable to ascribe differences in conduct to

[29] Collingwood (1960) p. 177.

[30] As we do so we should bear in mind that Roman standards were no more clear-cut than ours are today. Whether Julius Caesar was a good emperor is as complex a question as whether Margaret Thatcher was a good prime minister. There is no ideologically neutral standard against which to measure prime ministers or emperors.

differences in character. A comparison with, for example, Cromwell would have no such rigour. In order to ask how Julius Caesar would have acted in Oliver Cromwell's place, we have to imagine a Caesar who understood early modern English politics and religion, that is, a Caesar partly formed by those institutions, in other words, not Julius Caesar at all. Moreover, as we saw in a previous section, history sees all things in flux, including people's characters. There is, to the historical eye, no fixed essence called 'the character of Caesar' upon which we can conduct thought-experiments. Character and circumstance work on each other, and we cannot know what character Caesar would have had in circumstances dramatically different from those in which he lived. Finally, Collingwood's slogan requires the historian to read Caesar's words and deeds as expressions of his thoughts. That is possible, but only because we know a great deal about the culture to which Caesar belonged, and because he did indeed *belong* to it.

It is a commonplace that the aim of professional historiography is to understand the past, not to judge it. However, the argument just made suggests that historians do not have a choice in the matter if they wish to remain historians in good standing. They do not need to forswear absolute normative judgments of events and people in the past. Rather, it is logically impossible to make absolute evaluative judgments without giving up the historical point of view. At most, historians can offer local, historically relative judgments (e.g. that this Roman pursued Roman ends by Roman means more successfully than that Roman). It seems, then, that the prospects for a fruitful engagement between history and philosophy are unpromising. We began with history and philosophy as opposite poles (one temporal and particular, the other tenseless and universal). Our further analysis has done little to bring them together. We now have the additional problem that the central terms in epistemology ('rational', 'adequate', 'progressive', *etc*.) are normative while absolute normative judgments are, it seems, unhistorical.

The 'History of Philosophy' and the 'History of Ideas'

There is, however, a place where history and philosophy come together, and that is the history of philosophy itself. This has been the subject of some discussion lately, as philosophers in the 'analytic' tradition have addressed the charge that they read antique philosophy as if it had appeared in the most recent edition of *Mind*.[31] I wish in particular to appeal to a distinction that Bernard Williams introduced between 'history of philosophy' and 'history of ideas'.[32] Williams' terminology is confusing, since almost everyone understands by the term 'history of philosophy' precisely what he called 'history of ideas'. There is no established alternative expression in English for what he called 'history of philosophy'. Awkward though his terms are, his distinction is what we need.

What, then, is Williams' distinction between what he calls 'history of philosophy' and 'history of ideas'? For him, history of ideas is historiography properly so called. History of ideas 'looks sideways'[33] to the context in which an idea arose and took root. It studies (among other things) influences, chronology and routes of transmission (recall the quotation from Grosholz, above). It has no philosophical ambition. In contrast, 'history of philosophy' *does* have philosophical ambition. It *is* philosophy. The historian of philosophy (in Williams' sense) uses historiographic techniques to place texts and actors in their proper contexts and understand them in their proper times, but only insofar as this serves the philosophical goals of the enquiry. Nevertheless, professional philosophers have not always managed even that degree of historical sense. In his *Autobiography*, Collingwood complained of philosophers who insisted that Greek ethical theory had the same object as Kantian ethical theory, and proved it by translating a Greek ethical

[31] See Sorell and Rogers (2005)
[32] In *Descartes: The Project of Pure Enquiry*, preface.
[33] Williams (1994) p. 19.

term as 'ought'.[34] This was, said Collingwood, like translating the Greek word for 'trireme' as 'steamer' and then gleefully showing that the ancient Greeks had a very poor understanding of steamers. Such failures of historical sensitivity rob the past of any philosophical interest. That is why philosophers have to learn from historians how to respect the past

Williams argued that philosophy is a 'humanistic' discipline. Part of what he meant is that philosophy should not seek to examine its objects from an absolute standpoint, or to use Nagel's phrase, a view from nowhere. Even if such a perspective is possible, philosophy should not seek it. Most of our philosophical problems derive from the fact that we are finite, fleshy and located in some specific time, place and culture. From an absolute standpoint, few of *our* philosophical problems would arise. In Williams' words:

> Philosophy has to learn the lesson that conceptual description (or, more specifically, analysis) is not self-sufficient; and that such projects as deriving our concepts *a priori* from universal conditions of human life, though they indeed have a place (a greater place in some areas of philosophy than others), are likely to leave unexplained many features that provoke philosophical enquiry.[35]

This should sound familiar to seekers after a new epistemology for mathematics. We need a new epistemology precisely because the old epistemologies try to ground *our* mathematics in allegedly universal requirements of rationality such as formal logic or the transcendental unity of apperception. In doing so, these approaches leave unexplained the evaluative judgments of mathematicians. Why do mathematicians celebrate some theorems as deep and elegant results? Other theorems, proved with no less formal rigour, enjoy no such accolades. If we think that these judgments are not arbitrary, we have to give some account of them, and this account will require resources that we cannot find among the universal conditions of formal rationality. Specifically, it requires some philosophical reflection on mathematical practice. However, *current* mathematical practice is not self-explaining; there is no serious prospect of giving a mathematical account of elegance, depth, explanatory power or any of the other criteria by which mathematicians judge mathematical work. There is, of course, a mathematical account of the validity of gapless proofs expressed in fully formal systems, but since such proofs (or even approximations to them) do not appear in mathematical practice except as objects of study, this does not help us. That is why we epistemologists of mathematics have to consider the history of mathematical practice.

Williams' view requires that we philosophers learn something of the *process* of historical research as well as its products. We should not do this in order to become historians. Rather, we should do this in order *not* to become historians. If we simply copy what historians do, we run the risk of passing out of philosophy and into history.[36] On the other hand, if we try to guard against historicism as Reichenbach did, by drawing distinctions *a priori*, we return to the principal shortcoming of the old epistemologies, that is, both the philosophical problems and the philosophical resources of finite, temporal creatures fall out of sight. The route between these extremes is to understand the process of historical research well

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[34] Collingwood p. 63ff. See Williams (2006) p. 181.

[35] *Philosophy as a Humanistic Discipline* p. 192. Here and elsewhere, Williams seems to take it that the history of natural science vindicates current scientific practice straightforwardly. The discussion here, and the marital difficulties of history and philosophy of science, suggest otherwise. However, in fairness Williams was principally concerned with ethics, in comparison with which natural science must seem unproblematic.

[36] This is what happened to Thomas Kuhn. See Larvor (2003).

enough to borrow its methods and its sensibility just far enough to serve our philosophical ends. That is the point of this brief excursion into the philosophy of history. What remains is to find an informative name for this sort of historically engaged and self-aware philosophy.

References

Aspray, W. and P. Kitcher (1988). *History and philosophy of modern mathematics*. Minneapolis, University of Minnesota Press.

Breger H. & Grosholz, E. (2000). *The growth of mathematical knowledge*. Boston, Kluwer Academic Publishers.

Butterworth, B. (1999) *The Mathematical Brain*. London: Macmillan.

Carnap, R. (1928) *Der logische Aufbau der Welt.* Berlin and Leipzig.

Collingwood, R.G. (1960) *The Idea of Nature.* Oxford: O.U.P. Knox (ed.)

Collingwood, R.G. (1994) *The Idea of History.* Oxford: O.U.P. Revised edition; J. van der Dussen (ed.).

Collingwood, R.G. (2002) *An Autobiography.* Oxford: Clarendon.

Corfield, D. (2003). *Towards a philosophy of real mathematics*. Cambridge, UK ; New York: Cambridge University Press.

Davis, P. J., R. Hersh, et al. (1995). *The mathematical experience*. Boston: Birkhäuser.

Gibbon, E. (1980) *The Decline and Fall of the Roman Empire*. Penguin.

Giere, R.N. (1973) 'History and Philosophy of Science: Intimate Relationship or Marriage of Convenience?' Review of *Historical and Philosophical Perspectives of Science, Minnesota Studies in the Philosophy of Science, 5* by R. H. Stuewer in *The British Journal for the Philosophy of Science*, Vol. 24, No. 3. (Sep., 1973), pp. 282-297.

Grosholz, E. (2005) review of Sasaki's *Descartes's Mathematical Thought* in *Philosophia Mathematica* 13(3) 337-342.

Hallett, Michael (1979) 'Towards a Theory of Mathematical Research Programmes' (in two parts) *British Journal for the Philosophy of Science* 30 pp. 1–25, 135–159.

Jorland, G. (1981) *La science dans la philosophie: Les recherches épistémologiques d'Alexandre Koyré*. Paris: Éditions Gallimard.

Kitcher, Philip (1983) *The Nature of Mathematical Knowledge.* Oxford: Oxford University Press.

Koetsier, Teun (1991) *Lakatos' Philosophy of Mathematics: A Historical Approach* (Studies in the History and Philosophy of Mathematics vol. 3). Amsterdam: North Holland.

Koyré, A. (1946) 'Philosophie de l'histoire' *Europe*, September, 108-117.

Koyré, A. (1973) *Études d'Histoire de la Pensée Philosophique*. Paris: Gallimard. (Original: Presses Universitaires de France, 1966)

Lakatos, Imre (1976) *Proofs and Refutations: the logic of mathematical discovery*. Cambridge: Cambridge University Press.

Lakatos, Imre (1978a and 1978b) *Philosophical Papers* (volumes 1 and 2). Worrall and Currie (eds), Cambridge: Cambridge University Press.

Larvor, B. (1997) 'Lakatos as Historian of Mathematics'. *Philosophia Mathematica*, vol. 5, no. 1, pp. 42-64.

Larvor, B. (1998) *Lakatos, an introduction*. London: Routledge.

Larvor, B. (2003) 'Why did Kuhn's *Structure* cause a fuss?' in *Studies in History and Philosophy of Science*, vol. 34/2 pp. 369 - 390

Larvor, B. (2007) 'Between Logic and History' in Friend, Goethe, Harizanov & Valentina (eds) *Induction, Algorithmic Learning Theory, and Philosophy.* Springer.

Laudan, Larry (1987) 'Progress or Rationality? The prospects for Normative Naturalism'. *American Philosophical Quarterly* 24 (1): 19–31. Reprinted in Papineau (ed.) *The Philosophy of Science*. Oxford: Oxford University Press, 1996.

Magnello, M.E. (2006) 'Victorian vital and mathematical statistics' *Bulletin of the British Society for the History of Mathematics* 21(3) 219-229.

Nietzsche, F. (1888) *Twilight of the Idols the Idols,* trans. *R. J* . Hollingdale (Middlessex, U.K.: Penguin, 1968)

Reichenbach, H. (1938) *Experience and Prediction: an analysis of the foundations and structure of knowledge.* Chicago: University of Chicago Press.

Sorell, T. and Rogers, G. A. J. (2005) *Analytic Philosophy and History of Philosophy*. Oxford: Oxford University Press.

Williams, B.A.O. (1978) *Descartes: The Project of Pure Enquiry*. Harmondsworth: Penguin; Hassocks: Harvester Press.

Williams, B.A.O. (1994) 'Descartes and the Historiography of Philosophy' in *Reason, Will and Sensation: Studies in Descartes' Metaphysics*, ed. John Cottingham (Oxford: Clarendon Press), 19-27; reprinted in *The Sense of the Past: essays in the history of philosophy*, ed. Myles Burnyeat (Princeton: Princeton University Press, 2006).

Williams, B.A.O. (2006) *Philosophy as a Humanistic Discipline*. Princeton and Oxford: Princeton University Press. (A.W. Moore ed.)

Zammito, John H. (2004) *A Nice Derangement of Epistemes: Post-positivism in the Study of Science from Quine to Latour*. The University of Chicago Press.