Teaching Mathematics with Audiograph

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ABSTRACT

Mathematics teachers are charged with the task of teaching children not only how to solve problems, but also how to explain their solutions. There is, however, very little pedagogical support for this enterprise, and very few tools that support the process. We argue that mathematical explanation is essentially a multi-modal activity, requiring the integration of several modes of discourse, and that learners need tools capable of capturing such discourse in ways that can be edited and developed. The Audiograph system is proposed as a possible candidate for this, some experiences of using this with students of mathematics are introduced, and the potential for future work discussed.

Keywords

Mathematical explanation, Audiograph, multimodal interfaces, pedagogical issues, teaching/learning strategies

INTRODUCTION and motivation

The research outlined in this paper is motivated by the observation that students of all ages seem to find great difficulty in producing clear explanations of mathematical activity. In some cases, of course, this can be directly attributed to a basic lack of subject knowledge, but often students who are able to "get the right answer" seem unable to reconstruct their solution processes into a rational mathematical explanation. It might be argued that this is a symptom of the reduction of formality within the mathematics curriculum generally: without proof how can we have explanation, but this is a simplistic view. Many of the more formally presented topics in mathematics (such as Euclidean Geometry and the solution of algebraic equations) are traditionally accompanied by mysterious frames into which the student is required to force the written representation of the solution. Magic incantations such as "Q.E.D." or "We assume that the result holds for x, and show that it holds for x+1" are commonly copied from exemplars and re-used without understanding. These frames, we would argue, are meaningful only to those who already understand the subject. Cobb et al. refer to these as "sociomathematical norms …[which] establish… what counts as an acceptable mathematical explanation and justification" (Cobb et al. 2001, p.126). Once a student has mastered the technique for forcing a solution into the appropriate frame, the teacher usually provides the required "tick", indicating success. If the question required only an answer this seems quite reasonable, but the tick is usually forthcoming even if the question required an "explanation" of the answer. Thus, even if the student has no idea how to produce such an explanation, the frame provided has produced an illusion of understanding. We would argue, therefore, that the presence of formality may obscure the problems a student has in explanation, but it is not sufficient to overcome it.

An important point to note is that mathematical explanation is a discourse *about* mathematics, generally requiring a fair degree of fluency *within* various mathematical domains. Whilst there has been much debate about, and research into, the processes that students use in finding the solutions within domains, there seems to have been very little discussion of the problems associated with the problems of mathematical explanation itself. Indeed, explanation *per se* seems to be a much-neglected topic in both educational research and psychology: as Donaldson notes

"This brings up the question … how and when does the ability to explain develop? Despite the considerable educational relevance of this question, there is a dearth of research which addresses it directly."(Donaldson 86, p1).

Of course, this might not matter: after all mathematics has progressed in spite of this handicap for many centuries, so why address it now? One answer is simply that teachers are now *required* to do so. The U.K. National Curriculum (DFE, 1995) at Key Stage 2 (i.e. children aged 7-11) specified that

"Pupils should be taught to .... explain their reasoning" and this has remained within the revised National Curriculum in Mathematics (DfEE, 1999a) which requires children at Key Stage 2 to be taught to "explain their methods and reasoning". The preliminary report of the Numeracy Task Force (DfEE, 1998) also suggested that "numerate pupils should explain their methods and reasoning" and teachers should "collect information about... the clarity of explanations given in oral and written responses." The OFSTED (1999) review of Primary schools in England (1994-8) also suggests that schools should monitor and review how time is managed in lessons to ensure that pupils have the opportunity to *learn how to explain* their methods and present a reasoned argument. Thus mathematical explanation is no longer a higher-level task, which somehow transcends the substantive curriculum and just "happens", but it needs to be addressed in its own right: it has become a topic, just like algebra or geometry. Moreover, teachers must be able to identify and record progression within this, and keep records to provide evidence of this. Unlike topics like algebra and geometry, however, there is currently a complete lack of support for teachers in this area of the curriculum. The lack of current pedagogical support might suggest that the problem is actually too hard to be tackled, in spite of the National Curriculum requirements, but a classroom-based study carried out by one of the authors suggests that "mathematical explanation" can be taught (Loomes, 1999, 2001). It has become clear, however, that traditional paper-based approaches to recording mathematical artefacts are not sufficient upon which to build a pedagogically sound approach to the teaching of explanation across a wide range of ages and abilities.

A Role for Audiography?

We would argue that teachers need considerable support if a curriculum addressing the teaching of mathematical explanation is to be implemented. One aspect of this support concerns the artefacts of mathematics. Currently teachers use models of the application domain (blocks, shapes, counters etc.), but the actual "mathematics" produced by the students is usually thought of as being a paper-based artefact, typically comprising handwritten text, mathematical notation and diagrams. Whilst there will frequently be spoken discourse, this is usually seen as a means to an end, and is restricted to part of the learning process, rather than being seen as a valid component of the explanation. Paradoxically, a significant part of the explanation *experienced* by the students in the form of teaching will be spoken and informal. It is also likely to be multi-modal, making use of speech, pre-printed materials, handwritten incremental materials (such as the building up of a solution on a board alongside other modes of discourse), gestures relating components, reference to physical models, and anything else that a teacher thinks might help. Thus, at one level, a student sees explanation as an evolving thing, developing in response to feedback from an audience, making use of a wide variety of resources. There will be multiple strands developing, some of which are fragmented as lemmas are developed off-line, several variants of some parts will be provided to meet the needs of different students, and partial results will be stored and retrieved as needed. At another level, however, explanation seems to be reduced to a somewhat pointless template for producing written answers to questions. If we are to bridge the gap between this unstructured, rich environment and the sociomathematical norms of traditional mathematics, we believe it is necessary to allow students better access to the components on route. This problem has been addressed by Cole and Engestrom (1993) in the context of educational research

"Audio and video tape recording, films, and computers have all, in their own way, enabled us to interact with the phenomena of mind in a more sophisticated way. We can now not only talk about the mutual constitution of human activities, but display it in scientifically produced artefacts" (p. 43).

Whereas researchers want to capture snapshots of the world for analysis, teachers and students need systems that are dynamic, so that the artefacts can be viewed statically, for purposes of summative and formative assessment, but also as objects of development and evolution. From this viewpoint, we see mathematical explanation not as pre-conceived frames to be filled, but as narrative structures to be developed.

This is essentially a constructivist view, in Papert's interventionalist sense (Papert, 1991), where we aim to improve our understanding of how knowledge is constructed so that we can structure learning activities to support the development of the individual student. In the words of Bers and Best (1999), "Computational tools become computational construction kits (Resnick et al., 1996) when they support users as designers of their own projects by making both personal and epistemological connections"

If we analyse some of the modes of expression frequently encountered whilst explanations are being developed we can draw up an informal list of requirements for such a system.

- Many problems start their life (as far as the student is concerned) on paper. This typically includes diagrams, typed text and mathematical formulae. Thus the ability to integrate printed materials into the developing discourse seems important as students (and more experienced mathematicians) often annotate the expression of the problem at the early stages of devising a solution (for example, underlining key components, adding angles to diagrams or linking elements in a sequence).

- Students often work together in solving problems, and they do so using speech. Similarly students often work "out loud", talking themselves through problems. Teachers often provide spoken help and verbal feedback during problem solving tasks. Questions may be posed in spoken form (or spoken form may be provided alongside a written question to provide context and hints).

- Drawings, both carefully constructed and jottings, also play an important part in mathematical explanations, not only in the final presentation but also during the development.

- Mathematical notation needs to be written. This may be "correct", conforming to all the sociomathematical norms, but could also be private language, understood only by the writer at the outset, typically translated into more conventional forms when the presentation is tidied up for "publication".

- Rubbing out is important, both completely (where its presence may confuse either the writer or the intended audience later) or semi-transparently (for example, when cancelling fractions, where absence of the initial form may be equally confusing later).

- Gesture is vital. The importance of gesture in scientific explanation has recently been discussed by Roth, who observes "the analysis of an individual's gesture and talk over and about inscriptions shows how deeply integrated these are. Furthermore, the changing relation of gesture and talk over time also suggests that, for the individual, there is a change in the nature of the display" (Roth 2001, p. 55).

This rather simplistic analysis suggests that we need a system capable of integrating speech and pen-based developments, with interleaving of these also allowing gestures (such as highlighting with a pen) and erasures of various types. In essence, we want a flexible, multimodal system that can be adapted by the individual as the construction of an explanation develops. Cohen and Oviatt (1995) & Oviatt and Cohen (1991), have noted that users tend to prefer speech for descriptive purposes, including giving properties of objects, describing what needs to be done and discussing past and future events. This is perhaps not surprising, given that most people can speak clearly faster than they can write legibly, and with less physical effort. Handwritten input, however, is often preferred where the descriptions suggest the use of numbers, iconic representations and diagrams (Oviatt, 1997 & Suhm, 1998). Thus, there is no preferred single mode for all tasks: users like to have the power to select the mode according to details of the task. As Oviatt et al. (2000) have noted:

"Taken together, the speech and pen modes easily can be used to provide flexible descriptions of objects, events, spatial layouts, and their interrelation. This is largely because spoken and pen-based inputs provide complementary capabilities. For example, analysis of the linguistic content of users' integrated pen-voice constructions has revealed that basic subject, verb and object constituents are almost always spoken, whereas those describing locative information invariably are written or gestured (Oviatt, DeAngeli, & Kuhn, 1997). This complementarity of spoken and gestural input also has been identified as a theme during interpersonal communication (McNeill, 1992)." (Oviatt et al. 2000, p.268).

The Audiograph Tool

The Audiograph tool was originally conceived as a means of allowing university lecturers to prepare materials for distribution over the WWW which fitted naturally into their normal modes of working, and required no additional specialist training or skills in the use of technology (Jesshope, Shafarenko and Slusanschi, 1998). The basic model replicates the normal lecture producer-consumer model in part, which may be caricatured as a lecturer preparing slides and a "script" to deliver as an accompaniment, the two being linked in delivery by components such as referential speech ("as you can see at the bottom of the slide…."), gesture (highlighting accompanied by a phrase such as "look here…") and the handwritten annotation of slides. A significant feature of Audiograph is that it is multimodal, supporting background text and graphics, speech, simple gesture, real-time handwriting, and real-time drawing, but at the same time very simple to use. There have recently been several studies of technology to provide such multimodal interfaces. As noted by Oviatt et al. (2000),

"The growing interest in multimodal interface design is inspired largely by the goal of supporting more transparent, flexible, efficient, and powerfully expressive means of human-computer interaction. Multimodal interfaces also are expected to be easier to learn to use, and they are preferred by users for many applications "(p. 265).

It is important to stress, however, that most studies of multimodal interfaces refer to systems with multimodal *control* interfaces. For example, controlling equipment by speech or movement. Audiograph is not multimodal in this sense, but in its data interface. Thus many of the often-cited limitations of multimodal systems are not pertinent here (for example, the need to train the system to recognise particular users, or the high error rates associated with speech recognition). The actual authoring interface is shown in figure 1.

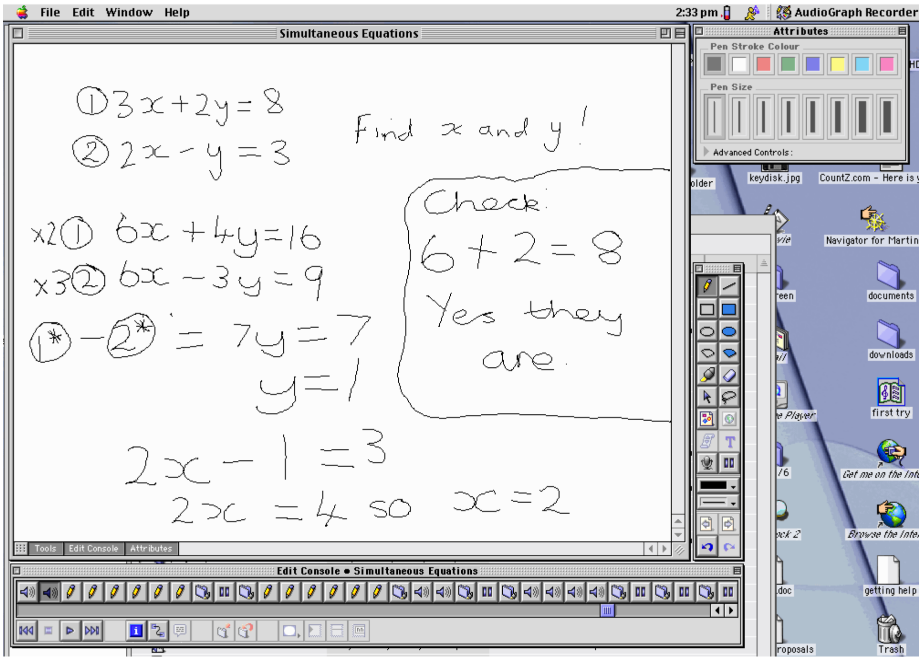

Figure 1: Audiograph in use for authoring.

The main window shows an imported slide presenting a pair of simultaneous algebraic equations handwritten by a teacher (the solution is developed by a student, and is referred to below). The tool palette to the right of the main window provides click-on controls for selecting a pen for handwriting or gesture, an eraser, a highlighting tool, a microphone for speech input and a pause. The other tools need not concern us here. The edit console below allows the sequence of actions comprising pen strokes, speech fragments, erasures, pauses, etc. to be edited. Presentations can also be optimised for delivery over the WWW, but that is not relevant to this paper.

The ease of use, public-domain nature and multi-modal features of the Audiograph make it an excellent tool for exploring the teaching of mathematical explanation. The fact that recording is sequential, with no multiple streams (unlike most complex multimedia authoring packages) is not a problem, indeed it is a virtue. Because the background slide (or worksheet), voice, handwritten text and gestures all result in independent events, it is simple to amend elements of a presentation, or move content from one mode to another. For example, a student who knows what needs saying ("this angle is the same as that angle") but does not know, or has forgotten, the conventional ways of naming angles, can use speech and gesture to achieve an acceptable explanation. With support from peers or a teacher, this can subsequently be amended to include a more formal line of mathematics to replace or supplement the speech fragment. There are also cases where students know exactly what they are doing, and even say the right thing, but the audience is tuned in to hearing something rather different. For example, in explaining an answer to a Key Stage 2 National Curriculum Test question which required pupils to take number patterns presented in a series of triangular shapes and fill in the missing numbers in the final triangle, one child wrote "The numbers are going up in threes ….." which suggests to most teachers of mathematics that somewhere there is a sequence n, n+3, n+6, … In fact, there are three numbers <2,3,4> which go up together to become <4,5,6> and <6,7,8>. Thus, just as children may "go up in twos" to collect their books, so these numbers are "going up in threes". The existence of a soundtrack and gestures, indicating "the numbers" grouped in threes would make this absolutely clear, whereas the purely written form looks "wrong".

Achieving this development of multimodal explanations with more traditional representational devices such as video recording would be very problematic, as a recording of a student standing in front of a whiteboard combines all of these events into a single data stream which can only be untangled by complex and expensive editing tools. For example, editing a video so that a spoken phrase is replaced by something written on a board is non-trivial.

The pen-based interface lends itself readily to mathematics, allowing symbols and simple diagrams to be produced to accompany spoken and written text. Mathematical word processors are not so simple to use, and would constrain the pupil to the mathematical norms as most only allow for well-formed expressions, and so pupils could not create their own notations as intermediate steps or present mistakes indicating they need help.

using audiograph in teaching mathematical explanation

We have explored two possible uses of this with students. First, individuals have used the system in the production of solutions to simple problems, and the work submitted was marked and returned by the teacher using the same technology. Second, groups of students were asked to develop audiograph presentations to be placed on the Web as resources for other pupils to use. Several interesting features emerged from these trials.

First, the learning required to use the technology effectively took just a few minutes. All of the students were familiar with basic computer use, and the only "strange" feature was a graphics tablet. A headset microphone was used, enabling good quality sound to be achieved in a typical classroom setting. Typically, students mastered the technology within a few minutes. The graphics tablet presented no problems, except for the drawing of geometric diagrams, where some students wanted to draw accurately, even when a sketch was clearly sufficient. One solution to this was to use a sheet of paper placed over the tablet, and a pen that actually drew, but then the erasure model of the system is compromised. A second "solution", which unfortunately places the cost of the system well above most educational establishments, is to use an interactive screen which can be drawn upon: experiments with this system are currently underway.

Second, there were interesting features that emerged when the spoken modes were analysed alongside the written ones. Sometimes this discrepancy indicated trivial slips, which could be easily spotted (for example, saying 5 and writing down 6). Very often students said the one thing, but wrote something that meant something different to more experienced mathematicians. Sometimes they said something that was not really correct but, interestingly, wrote down the correct thing (as if the template they already knew took over when they moved to a written form). For example, in the solution presented in Figure 1, there were two such instances. The symbol "=" is frequently overloaded in written mode, denoting concepts such as numerical equality, implication and identity, leading to mathematically incorrect statements. The audio strand associated with this Figure contains "If we take equation two star from one star *we get....*" The written form contains an equal sign to indicate "we get". This particular student is a very competent mathematician, who often uses algebra from choice, and hence tends to use symbols rather than words. There was, however, a lack of knowledge as to how to represent "we get", so the nearest approximation was used. When the two equations were transformed (to give 1* and 2*) the student actually continued to refer to them as 1 and 2, but wrote the correct form. This suggests that the formal need to introduce a correct name was appreciated, but the student realised that no confusion would arise if a short form was used informally. The ability to mark this assignment multimodally meant that the discrepancies between the audio and written presentations could be highlighted as they occurred, simply pointing out what was happening. There was no need to cover the page in red ink to provide feedback (which would have been required to spell out in detail what the overloading had led to, and suggested that the solution was "wrong" whereas in fact it was simply the grasp of as-yet untaught issues that were being addressed), but equally appropriate feedback could be provided to a pupil who was clearly ready to move on. The ability to separate the meta-level commentary on the explanation (the teacher's voice) from the annotations on the pupils work seemed helpful.

A third situation that arose, typically amongst older students, was a desire to reduce the amount of spoken explanation, replacing it with more conventional mathematics. In the extreme, one group of students who were preparing solutions for the WWW actually insisted on removing all speech, claiming that "mathematics can't include talking". It is interesting to note that the English curriculum within the U.K. has moved firmly towards a system where speaking, listening and writing are firmly established with both teaching and assessment. Mathematics has made similar changes to the curriculum, with provision for "oral and mental" activities included in the National Numeracy Strategy (DfEE, 1999b), but this is, as yet, not reflected the assessment mechanisms.

Future work

These explorations are, as yet, embryonic and small scale. They have convinced us, however, that audiography has a valuable role to play in the mathematics curriculum. There are several avenues that we would like to explore, and these are briefly described below.

First, the ability to allow teachers to assess and provide feedback on drafts of explanations (like English teachers do on narratives) seems very valuable. We would like to explore this for particular classes of students, such as those with learning difficulties or problems with motor skills. Our discussions with teachers suggest that the transition from spoken to written forms is often the place were many difficulties seem to reside, and a systematic study of these which teachers can carry out for themselves might prove useful.

Second, we would like to mirror some of the work on narrative group work carried out in the English curriculum within mathematics. For example, the activity of older students producing resources (typically books) for younger ones is well-established, and brings the benefit of making the authors consider and discuss the intended audience, which in turn is reflected in their choice of expressive devices. We believe that similar activities (such as the production of web pages for younger pupils or parents) would bring similar benefits, and audiograph makes this very simple to do. The issue of providing an audience for mathematical explanations is a complex one, and space does not permit a detailed discussion, but evidence suggests that students consider mathematical explanations simply as a device relevant to assessment tasks, rather than communication. This might explain the willingness of more able students to accept the socio-mathematical frames so willingly and uncritically (in contrast, they seem very willing to criticise social norms for narrative in literature).

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