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Properties of Bars and Bulges in the Hubble Sequence

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ABSTRACT

Properties of bars and bulges in the Hubble sequence are discussed, based on the analysis of 216 disk galaxies (S0s and spirals from NIRS0S and OSUBGS surveys, respectively). For that purpose we have collected together, and completed when necessary, the various analysis we have previously made separately for early and late types. We find strong photometric and kinematic evidence of pseudobulges in the S0-S0/a galaxies: their bulges are on average fairly exponential, inner disks are common (in 56%), and in many of the galaxies the bulges are rotationally supported. This would be difficult to understand in such gas poor galaxies as in S0s, if these pseudobulge candidates were formed by star formation in the disk in a a similar manner as in spirals. A more likely explanation is that pseudobulges in the early-type galaxies are bar-related structures, connected to the evolution of bars, which interpretation is supported by our Fourier analysis and structural decompositions. Bars in the early-type galaxies are found to have many characteristics of evolved systems: (1) they have flat-top/double peaked Fourier amplitude profiles, (2) bars have typically sharp outer cut-offs, (3) the higher Fourier modes appear in the amplitude profiles, and (4) many bars have also ansae-type morphologies. We show the distributions of bar strength in different Hubble type bins using four bar strength indicators, Q_g, A_2, f_{bar} and the bar length, which are expected to give important clues for understanding the mechanism of how bars evolve.

Key words: galaxies: elliptical and lenticular - galaxies: evolution galaxies: structure

1 INTRODUCTION

One central goal of extragalactic research is to understand what factors drive the morphology of galaxies and how important these factors are over long term evolution. Our understanding of galaxy formation and evolution is still largely dominated by the early scenarios of dissipative monolithic collapse (Eggen, Lynden-Bell & Sandage 1962), and the hierarchical clustering of galaxies (Toomre & Toomre 1972; Toomre 1977; Navarro & Steinmetz 2000). For instance, the bulges in disk galaxies are assumed to have formed either in a single episode of gravitational collapse, or more gradually, via mergers of disk galaxies in the distant past. This results in a classical velocity supported bulge. Hierarchical clustering of the Universe, dominated by cold dark matter, seems to account remarkably well for the large scale structure of galaxies, but it is faced with problems while trying to explain the structural components of individual galaxies. For example, galaxies are not found to have cuspy halos (see Bosma 2003), nor is the observed amount of mass in the bulge component as large as predicted by these models. The discovery that a large majority of the bulges in late-type spiral galaxies are actually pseudobulges formed by secular evolutionary processes has also been difficult to explain in the hierarchical picture of galaxy formation (see the review by Kormendy & Kennicutt 2004; hereafter KK2004).

The monolithic collapse scenario of galaxy formation is also faced with problems when compared with the recent cosmological models and observations. It is particularly difficult to explain the so called “downsizing” effect (Cowie, Songalia & Hu 1996), which on the other hand can be understood naturally in the hierarchical picture of galaxy formation. There is evidence that the most massive galaxies were formed in the early phases of the Universe, having only modest star formation in the disk after the principal starburst event, whereas dimmer galaxies, in which most of the gas was not washed out in the main starburst event, continue actively forming stars until redshift zero (Cowie et al. 1996; Ueda et al. 2003; Steffan et al. 2003). Continued star formation in the central regions of dimmer galaxies might

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lead to the gradual formation of pseudobulges with young stellar populations. The fact that pseudobulges have been found repeatedly in late-type spirals (Andreakis, Peletier & Balcells 1995; Carollo, Stiavelli & Mark 1998), a class dominated by low luminosity spirals, fits to this picture. As shown by Laurikainen, Salo & Buta (2005), however, massive early-type disk galaxies might also have disk-like bulges fairly frequently. Disk-like rotationally supported bulges have been recently detected also kinematically in S0 galaxies by Cappellari et al. (2005). As it is highly improbable that these disk-like bulges were formed by star formation in the disk, some other mechanism is needed to explain them. Although not yet included explicitly in cosmological models, the intrinsic evolution of bars is among the most promising alternatives to explain pseudobulges in the early-type disk galaxies.

Theories of bar formation form an active field of investigation and many interesting results have been obtained. One of the most important discoveries has been that bars evolve due to angular momentum transfer between bars and dark matter halos, so that bars become longer and more massive when evolved over time (Athanassoula 2003). In the early phases of evolution a bar develops a centrally concentrated inner part and a boxy/peanut-shaped structure, components which in dynamical models are generally interpreted as bulges. However, these models have not yet been fully tested with observations. For example, it is unclear to what extent the bulges in galaxies are classical bulges or pseudobulges, or what is the exact role of bars in the formation of pseudobulges? Another open question is what are the details of the angular momentum transfer mechanisms between bars and halos. This is because the dark matter halos and the bar pattern speeds are not well known in galaxies, and also because no systematic comparison of the observed and model-predicted properties of bars and bulges, covering all Hubble type bins, has been done to date. The observed properties of bars in edge-on spirals have been compared with the dynamical models by Lütticke, Dettmar & Pohlen (2000) and Bureau et al. (2006), but a similar comparison has not been made for more face-on galaxies nor for S0s.

In this study the properties of bars, bulges and ovals in the Hubble sequence are studied and compared with various dynamical models in the literature. In order to characterize the nature of bulges we use a photometric approach by applying a 2D multicomponent decomposition method (Laurikainen, Salo & Buta 2005), and for a subsample of galaxies stellar kinematic observations are also collected from the literature. The properties of bars and ovals in the same galaxies are derived by Fourier techniques. The decompositions and most of the Fourier analysis for the individual galaxies have been published in a series of papers by us (Laurikainen et al. 2004, 2006a; Buta, Salo & Laurikainen 2004; Laurikainen, Salo & Buta 2004, 2005; Buta et al. 2005, 2006). Our aim in this paper is to combine all the analysis for the early- and late-type galaxies, and bring them together for a more general discussion in the Hubble sequence. Also, if any of the discussed parameters were not derived earlier for the whole sample of 216 galaxies, the analysis is completed in this study. Such parameters are the ellipticities and the lengths of the bars for the early-type galaxies, and the Fourier amplitude profiles and bar morphologies for the late-type galaxies. Some preliminary results of this study have been reported in the IAU conference proceeding by Laurikainen et al. (2006).

2 SAMPLE

The sample used in this study consists of 216 galaxies, based on the Ohio State University Bright Galaxy Survey (OSUBGS; Eskridge et al. 2002) for spirals, and on the Near InfraRed S0 galaxy Survey (NIRS0S; Laurikainen, Salo & Buta 2005, Buta et al. 2006) for S0-S0/a galaxies. Both are magnitude limited samples ($B_T < 12.5$) avoiding high inclination galaxies ($INC < 65°$). The NIRS0S sample has been observed only partly so far, but when combined with the OSUBGS, a reasonable sized sample is obtained to study the properties of bars and bulges in the Hubble sequence. Figure 1 (middle panel) shows the mean galaxy brightness in each Hubble type index bin. The magnitudes are total K_s-band magnitudes to the limiting surface brightness of 20 $mag\ arcsec^{-2}$, corrected to Galactic extinction. The magnitudes are from Two-Micron All Sky Survey (2MASS) [1] and the extinction values are from the NASA/IPAC Extragalactic Database (NED) [2]. The mean absolute brightnesses of the galaxies in our sample are very similar for the different Hubble types, except for galaxies of types Sc and later, which are dominated by dwarf galaxies. A large majority of the galaxies in our sample are bright: they are slightly brighter than the characteristic brightness $K_s = 23.1$ in Schechter's luminosity function (Gardner et al. 1997). The analysis discussed in this paper is based primarily on H-band (the OSUBGS sample) and K_s-band (NIRS0S sample) images.

3 OBSERVATIONAL EVIDENCE OF PSEUDOBULGES IN THE HUBBLE SEQUENCE

The idea that some bulges might form by slow secular evolutionary processes after the initial rapid phase of galaxy formation was suggested already in 1982 by Kormendy. These so called pseudobulges have more exponential surface brightness profiles than the classical bulges, whose surface brightness profiles closely resemble those found in elliptical galaxies. The concept of a pseudobulge comes in two different flavors: Kormendy (1982; see also KK2004) has suggested that they are disk-like structures formed by star formation in the disk, whereas Athanassoula (2005) has suggested two different types of pseudobulges, *disk-like* and *boxy/peanut* bulges. In her characterization boxy/peanut bulges are thick inner components of bars seen in edge-on galaxies, which structures are related to the characteristic orbital families of bars. Generally we accept the wider concept of a pseudobulge. However, the way how boxy/peanut structures manifest in less inclined galaxies has not yet been studied in detail by

[1] Two Micron All Sky Survey, which is a joint project of the University of Massachusetts and the Infrared Processing and Analysis Center/California Institute of Technology, funded by the National Aeronautics and Space Administration and the National Science Foundation

[2] NASA/IPAC Extragalactic Database (NED), operated by the Jet Propulsion Laboratory in Caltech.

simulations. Therefore we adopt a term "bar-related" for all those pseudobulge candidates that seem unlikely to result from star formation, but which might be related to the orbital structure in bars. In the following potential pseudobulges are identified using the photometric and kinematic approaches, in a similar manner as suggested by KK2004 and Kormendy & Fisher (2005). The main idea is that if the bulges were formed by secular processes in galactic disks, they should remember their disky origin, which is expected to be visible in their surface brightness profiles, stellar populations, and stellar kinematics.

3.1 Photometric approach

In order to identify potential pseudobulges we use the following criteria: (1) the surface brightness profiles are nearly exponential (the shape parameter of the bulge $n < 2$), (2) the galaxies have nuclear bars, rings, or ovals in the disk inside the region of the bulge. In galaxies with pseudobulges the bulge-to-total flux ratio (B/T) is generally small (< 0.5), although this is not a sufficient criterion to confirm the disk-like nature of the bulge. In our analysis we have used a 2D multicomponent decomposition algorithm (Laurikainen, Salo & Buta 2005) which takes into account, not only the bulges and disks, but also multiple bars or ovals. For the bulges a generalized Sersic function is used, and bars and ovals are typically fitted with a Ferrers function. The decompositions for the individual galaxies discussed in this paper have been presented previously by Laurikainen, Salo & Buta (2004, 2005) and by Laurikainen et al. (2004, 2006).

3.1.1 Innermost structures of the disks

In extensive studies, Carollo and her collaborators (Carollo, Stiavelli & Zeeuw 1997; Carollo, Stiavelli & Mark 1998; Carollo et al. 2002) have shown that late-type spirals (Sb-Sbc) frequently have star forming nuclear rings, inner spiral arms and inner disks. In many cases the inner disks are the only bulge-like structures in these galaxies, and some galaxies have no bulge at all. A pseudobulge has been recently found also in one early-type spiral galaxy, NGC 7690, by Kormendy et al. (2006). For some spiral galaxies there is also kinematic evidence confirming the disk-like nature of the bulge (Fálcon-Barroso et al. 2003). However, star formation related inner structures are expected to be less common in S0 galaxies, which are defficient of interstellar matter. In order to check the frequency of the inner structures we combined the data presented by Laurikainen, Salo & Buta (2005) and Laurikainen et al. (2006) for S0-S0/a galaxies. We confirm the earlier result by Laurikainen et al. (2006) showing that even 56 % of S0-S0/a galaxies have either nuclear bars, nuclear disks or nuclear rings inside the bulge.

3.1.2 B/T flux ratios

One of the main results of our decomposition studies is that the typical B/T flux ratio is considerably smaller than generally assumed, particularly for the early-type galaxies. In Figure 1 (upper panel) this is shown, for the first time, for all Hubble types. For comparison, the mean B/T flux-ratios derived by Simien & de Vaucouleurs (1986) in the B-band are also shown. The difference to our result is very large: for example, while for S0-S0/a galaxies Simien & de Vaucouleurs found $< B/T >_B = 0.57$, we find $< B/T >_K = 0.25 \pm 0.10$. One may ask whether this is a wavelength effect, related to the mass-to-luminosity (M/L) ratio, or due to the different decomposition methods used? If it is a wavelength effect we should conclude that recent star formation (visible in the B-band) in the bulge, relative to that in the disk, should be particularly significant in S0 galaxies, which is highly improbable. That the difference is not a wavelength effect becomes clear also by comparing our result with the decompositions made by de Souza, Gadotti & dos Anjos (2004) in the K-band. Using their measurements (their table 1) we find $< B/T >_K = 0.64$ for S0s, which is very similar to that obtained by Simien & de Vaucouleurs in the B-band. A correction related to different star formation time scales at different wavelengths (taken from Schulz et al. 2003) is fairly small, changing the $(B/T)_K = 0.64$ to $(B/T)_B = 0.54$ (see Laurikainen, Salo & Buta 2005 for more details). Obviously wavelength is not capable of explaining the large difference in B/T between this study and that by Simien & de Vaucouleurs. A more likely explanation is that the more sophisticated multicomponent decomposition approach used in our study is capable of accounting for the effects of strong bars and ovals, structures which in the more simple 1D (used by Simien & de Vaucouleurs) or 2D bulge/disk decompositions (used by de Souza, Gadotti & dos Anjos) are easily mixed with the bulges. In fact, it was shown by Laurikainen et al. (2006) that, when applied to the same barred galaxies, both 1D and 2D bulge/disk decompositions give fairly similar high B/T ratios, whereas 2D bulge/disk/bar decompositions give significantly lower B/T. Laurikainen, Salo & Buta (2005) used also synthetic data to demonstrate that most of the bar flux is erroneously assigned to the bulge, if a simple bulge/disk decomposition algorithm is applied to a system with a prominent bar. The $(B/T)_H$ flux ratios we find for spiral galaxies are also smaller than those obtained by Simien & de Vaucouleurs, but the differences are much smaller, presumably because bars in these galaxies are smaller and thus affect less the decompositions.

The mean B/T flux ratios we find are smaller than typically found for classical bulges (see KK2004). If we assume that the mass-to-luminosity (M/L) ratio is constant in these galaxies, the derived flux ratios are also approximations of the relative masses of bulges. In that case our result contradicts the earlier understanding according to which bulges and disks represent approximately equal amounts of mass in galaxies (Schechter & Dressler 1987; Benson, Frenk & Sharples 2002). Nevertheless, a constant M/L-ratio might be valid for most part of the disks, but not necessarily in the central regions of the galaxies. If the bulges were redder than the disk, higher M/L-ratios should be used for bulges (Bell & de Jong 2001). The conversion of flux ratios to mass ratios will be made in a forthcoming paper, where colour index maps will be studied.

3.1.3 Shape and scale parameters of the bulges

Another important finding in this study is that the shape parameter of the bulge, n, is on average smaller than or near 2 for all morphological types (Fig. 1, lower panel). In the generalized Sersic's function the value $n = 1$ corresponds to an exponential disk and $n = 4$ corresponds to the de Vaucouleurs type surface brightness profile. If the concept of a disk-like pseudobulge is accepted this result seems to contradict with the view where only bulges in spiral galaxies later than Sb have small enough n-values to be interpreted as pseudobulges (Carollo, Stiavelli & Mark 1998; see also KK2004). However, pseudobulges in early and late-type galaxies might be different.

It has been shown both for spirals (Andreakis & Sanders 1994) and for elliptical galaxies (Caon, Cappaccioli & D'Onofrio 1993), that for the spheroidal component the generalized Sersic's function (used also in this study) gives a better fit than the $R^{1/4}$-law. Using a Sersic's function for the bulge both Andreakis, Peletier & Balcells (1995) and de Souza, Gadotti & dos Anjos (2004) found large n-values particularly for S0 galaxies ($< n >$=3.7 and 4.1, respectively), whereas Graham (2001) found $< n >$=2 for the galaxies with the same morphological types. The n-values given by Graham are also similar to those obtained by us. How can we understand these disagreeing results? One of the key issues is that the sample by Graham did not include barred galaxies, thus any problems related to the degeneracy of bars and bulges were avoided. Most probably the relative masses of bulges were overestimated in the decompositions by Andreakis, Peletier & Balcells thus leading also for an overestimate of the n-values. Balcells et al. (2003) have shown the importance of high image resolution when deriving the n-values in decompositions. They showed that the decompositions where ground-based images are combined with high resolution Hubble Space Telescope (HST) images, give considerably lower n-values than the decompositions where only ground-based images are used. The small B/T flux ratios and small n-values for S0-S0/a galaxies have been previously reported by Laurikainen, Salo & Buta (2005) and Laurikainen et al. (2006).

We confirm the earlier result by Balcells, Graham & Peletier (2004) showing that galaxies are not scale free, the scale parameter being the relative mass of the bulge (see Fig. 2): both the shape parameter n and the effective radius of the bulge normalized to the scale length of the disk, r_{eff}/h_R, correlate with B/T. A critical B/T value seems to be 0.1, below which the bulge profiles are close to pure exponentials having $n \sim 1$.

The K-band luminosities of these galaxies provide an interesting additional piece of information. For the late-type spirals the B/T flux ratio increases nearly linearly towards brighter galaxies, whereas for the more early-type systems (S0-Sab) B/T seems to be independent of galaxy luminosity (Fig. 2, lower panel). If the bulges in the late-type spirals are largely pseudobulges formed by star formation in the disk, the found correlation can be understood assuming that the bulges are getting more massive when evolved over time, which is also the generally accepted view (KK2004). This is also consistent with our finding that most bulges among the late-type spirals have nearly exponential surface brightness profiles with n = 1-1.5.

On the other hand, bulges in early-type disk galaxies are generally believed to have non-exponential surface brightness profiles, interpreted as evidence of their merger origin. However, as a large fraction of bulges also in S0 galaxies are fairly exponential we should ask how were these bulges formed? The fact that there is no correlation between B/T flux ratio and the galaxy luminosity is consistent with the picture that they have not enough gas that could accounted for the mass of the bulge via star formation. On the other hand, if these nearly exponential bulges were part of the bar one would expect some differences in the B/T flux ratios and the n-values between the barred and the non-barred galaxies. Figure 3 shows that such a difference seems in fact be present. The barred classification here is based on the de Vaucouleurs's Atlas of Galaxies by Buta, Corwin & Odewahn (2007). If the galaxy did not appear in the Atlas, the classification was taken from The Third Reference Catalog of Bright Galaxies (de Vaucouleurs et al. 1991; hereafter RC3). We also tested whether the differences in the distributions of n and B/T-values between the barred and nonbarred S0's are statistically significant. For that purpose the non-paramateric Kolmogorov-Smirnov test was used, and all morphological types with $T \leqslant 2$ were grouped together. According to this test the differences between barred and nonbarred galaxies are real: the probability that their n-parameters are from the same distribution is only 0.02. The same result applies to the B/T ratios. A similar test for later type galaxies ($T > 2$) indicated no difference between barred and nonbarred galaxies, as anticipated from Fig. 3.

As any decomposition, also our multicomponent approach has its limitations. A critical question is whether the uncertainties of the algorithm itself are so large that they alone could produce the found differences between the barred and non-barred galaxies? One way of evaluating this is to look at how much massive nuclear bars or inner ovals can affect the B/T flux ratio in the decompositions. This evaluation was done by Laurikainen, Salo & Buta (2005) who showed that the inclusion of prominent inner structures to the fit do not affect the n-value at all, compared to simple bulge/disk/bar decompositions, but can decrease the mean B/T flux ratio by 0.05, which is slightly smaller than the difference in B/T between the barred and non-barred galaxies.

3.2 Kinematic approach

Another discriminator between the pseudobulges (whether disk-like or bar-related) and the classical bulges is based on the kinematic approach first suggested by Illingworth (1977): classical bulges are supported by random motions of stars (Illingworth 1981), whereas pseudobulges are supported by rotational velocities (Kormendy 1981). A useful discriminator is the parameter V_{max}/σ versus ϵ (Illingworth 1977; Kormendy 1982), where V_{max} is the maximum rotational line of sight velocity of the bulge measured from the absorption lines, σ is the mean stellar line of sight velocity dispersion of the bulge just outside the nucleus, and ϵ is the characteristic ellipticity inside the radius of V_{max}. Therefore V_{max}/σ measures the importance of rotation in supporting the galaxy against its self-gravity.

In Table 1 we have collected the stellar kinematic observations from the literature for all those galaxies in our sample that have measurements along the major axis of the disk. We use those major-axis position angles and inclinations that come out from our ellipse fitting to deep optical or near-infrared images using the ellipse routine in IRAF (PA/INC shown in the table). The bulge-dominated regions of the disks were evaluated, based on our structural decompositions (the radius given in the Table). V_{max} was then taken to be the maximum line of sight rotational velocity inside the bulge-dominated region. The ellipticities inside the bulge dominated region were taken from the radial ϵ-profiles. The last column in Table 1 shows the references for the kinematic data.

The data are shown in Figure 4, where the diagonal curve represents an isotropic oblate-spheroid model taken from Binney (1978). In the early diagrams by Illingworth (1981), Kormendy (1982), and Davies et al. (1983) the elliptical galaxies and bulges of disk galaxies supported by random velocities lie below the oblate line, whereas fast rotating disk-like bulges are expected to appear above that line. The reliability of this approach has been recently verified theoretically by Binney (2005). As the apparent ellipticity and the maximum rotational velocity of the bulge depend also on galaxy inclination, models with different inclinations were calculated by Binney (2005), who showed that the inclination would shift the data points in the V_{max}/σ versus ϵ diagram almost along the isotropic oblate rotator line. We find that the S0 galaxies in our sample appear both above and below the line. Of these galaxies five appear above the line: one of these galaxies, having a high flattening is a non-barred galaxy, whereas the other four galaxies have primary bars and some of them also a nuclear ring. Of the S0 galaxies below the line five are barred and two are non-barred galaxies, one galaxy has a clear secondary bar, and one galaxy has inner spiral arms.

Our emphasis here is not to do any detailed interpretation of the kinematic data as the study for early-type galaxies by Cappellari et al. (2005), based on modeling of 2-dimensional kinematic observations. However, the kinematic data for the galaxies in our sample indicates that many of the early-type galaxies are likely to be rotationally supported. This is also consistent with the results obtained by Cappellari et al. (2005).

4 BARS AND OVALS IN THE HUBBLE SEQUENCE

4.1 How prominent are bars?

Bars are generally characterized as "strong" when they are *long* and *massive* (have large Fourier amplitudes), and also when they *have large ellipticities* or the *tangential forces induced by bars (bar torques) are large.* In de Vaucouleurs' family classification (de Vaucouleurs et al. 1991) bars are divided into strong (B) and weak (AB) bars, based mainly on visual inspection of how prominent the bar looks in the optical image, although the morphology of the bar also comes into the classification in the sense that weak bars typically have more oval-like morphologies. As the orbital families of bars strongly depend on the underlying gravitational potential, one would expect a clear correlation between the ellipticity of a bar and the bar torque, which is indeed found to be the case (Laurikainen, Salo & Rautiainen 2002; Buta, Laurikainen & Salo 2004).

There are many optical (Elmegreen & Elmegreen 1985; Martin 1995; Erwin 2005) and infrared (Regan & Elmegreen 1997; Laurikainen, Salo & Rautiainen 2002; Laurikainen, Salo & Buta 2004) studies showing that bars in early-type spirals are longer than bars in late-type spirals. The estimation of bar length is not trivial (see Athanassoula 2005), but the different bar length estimates (visual inspection, maximum in the ellipticity profile, Fourier phase angle, or the maximum in the force profile) seem to lead to a similar conclusion. Bars in early-type spirals are also found to have smaller ellipticities (Martin 1995; Shlosman, Peletier & Knapen 2000; Whyte et al. 2002; Laurikainen et al. 2002, 2004) and smaller bar torques (Laurikainen, Salo & Rautiainen 2002; Buta, Laurikainen & Salo 2004) than bars in late-type spirals.

In Figure 5 we show four different bar strength indicators calculated for our sample of 216 galaxies: (1) Q_g (bar torque), which is the maximum of relative tangential force in the bar region, normalized to the underlying mean axisymmetric force field, (2) the ellipticity of a bar using the f_{bar} index by Whyte et al. (2002), (3) the relative mass of a bar, as estimated from the $m = 2$ (A_2) and $m = 4$ (A_4) Fourier amplitudes in the bar region, and (4) the length of a bar, as estimated from the phases of the A_2 amplitudes, normalized to the radial scale length of the disk. For Q_g, A_2 and A_4, the values are collected from Laurikainen et al. (2004, 2006), and Laurikainen, Salo & Buta (2005). The values of f_{bar} for the OSUBGS sample are those derived by Whyte et al. (2002, see also Buta, Laurikainen & Salo 2004), whereas for the galaxies in the NIRS0S sample f_{bar} was calculated in this study using the formula given by Whyte et al. (see Table 2):

$$f_{bar} = 2/\pi \; [\arctan(b/a)^{-1/2} - \arctan(b/a)^{+1/2}],$$

where b/a is the minor-to-major axis ratio of a bar. We used b/a derived from the maximum ellipticities in the bar region, obtained from our radial ϵ-profiles, and corrected for the inclination of the disk (see Abraham et al. 1999). For completeness, the lengths of the bars for some of the galaxies in the NIRS0S sample were also estimated in this study based on the phases of the Fourier amplitudes, in a similar manner as estimated previously by us for the other galaxies in our sample. The strength of this comparison is that all bar strength indicators are calculated for the same galaxies using the same homogeneous database. The advantage of using IR-images is also well known: we are not missing bars that might be obscured by dust in the optical region, or where morphology might be masqueraded by pockets of star formation.

In Figure 5 we can look at separately the tendencies for the early (shaded region) and late-type (non-shaded region) galaxies. Characteristic for the spiral galaxies is that bars become *stronger* towards the later Hubble types when the bar torques are concerned, and *weaker* when the lengths or the relative masses of bars are concerned. These tendencies for spiral galaxies have been previously shown by Laurikainen, Salo & Buta (2004), and for bar torques also by Buta, Laurikainen & Salo (2004). Buta et al. (2005) also showed that in some cases spiral arms might affect the bar torques (in which case Q_b was used). Figure 6 we shows that any superposition of spiral arms with bars does not significantly affect the found tendency for the bar torques in the Hubble sequence. For f_{bar} there might be a similar increase toward later types as found for bar torques, but it is not as clear: actually any possible variations in the ellipticity in the

Hubble sequence are within the error bars. This is consistent with the result obtained by Marinova & Jogee (2006) who found that the ellipticity of a bar is practically independent of the Hubble type. In any case we find that bar torque, Q_g, is well correlated with f_{bar} for all morphological types (Fig. 7). This correlation exists also if spiral-corrected Q_b is used, but in that case the dispersion is somewhat larger. The apparent inconsistency between the bar length and the relative mass of a bar in one hand, and the bar torque on the other hand, has been discussed by Laurikainen, Salo & Buta (2004) for the spiral galaxies. They showed that although bars in the early-type spirals are longer and more massive, the bar-induced tangential forces are weaker, because they are diluted by the more massive bulges in these systems.

On the other hand, Figure 5 also shows that the trends we find for spiral galaxies do not extend to S0s, which might be an important clue while evaluating the evolutionary history of galaxies in the Hubble sequence. Bars in the *early-type S0 galaxies* are clearly shorter (see also Erwin 2005) and less massive than bars in the *later-type S0s or in S0/a galaxies*. For bar torques and ellipticities the opposite might be true, but the number of galaxies in our sample is too small to confirm that. In the bar torque approach the largest uncertainty is related to the assumed vertical thickness of the bar (Laurikainen & Salo 2002). We used a vertical thickness based on the empirical relation between the morphological type and the vertical thickness of the disk, as estimated by de Grijs (1998). Nevertheless, this uncertainty (see Fig. 6, in Laurikainen et al. 2004) is clearly smaller than the trend we find for bar torques in the Hubble sequence.

4.2 Fourier amplitude profiles of bars

4.2.1 Early-type galaxies

The Fourier amplitude profiles of 26 S0-S0/a galaxies in our sample have been previously studied by Buta et al. (2006). They showed that the A_2 amplitude profiles of bars can be fitted by symmetric Gaussian functions, using either single (SG), double (DG), or multiple (MG) Gaussian functions. In Buta et al. bars fitted by a double Gaussian function were found to have, on average, larger bar torques than bars fitted by a single Gaussian function. In the following we discuss the other bar strength indicators and the properties of bulges in the same galaxies. As the MG-type profiles are generally very complex we concentrate only on the SG and DG-type bars in the following. The mean parameters of bars and bulges for these galaxies are collected to Table 3. We find that DG-type bars not only have stronger bar torques, but that they are also more elliptical, longer and more massive than SG-type bars. There is also a small difference in the properties of bulges between these two groups of galaxies in the sense that the galaxies with DG-type bars have slightly less massive and more exponential bulges than the galaxies with SG-type bars. Both groups of galaxies have similar K_s-band luminosities, which eliminates the possibility that the found differences were due to a magnitude bias.

In order to understand better the SG and DG nature of bars, we picked up four characteristic examples of early-type galaxies and inspected their properties in more detail (see Fig. 8). NGC 3941 is a typical example of a galaxy with an SG-type bar, whereas NGC 4245, NGC 2859 and NGC 1452 have DG-type bars. For NGC 3941 the maxima in the A_2, A_4 and A_6 amplitude profiles appear at the same radial distance, a behavior which is characteristic for SG-type bars. However, in some other SG-type bars like NGC 1440, the lower modes are shifted towards slightly smaller radial distances. NGC 3941 has two amplitude maxima in the A_2-profile, the main maximum belonging to the primary bar, and the lower maximum (at r<5") to an inner disk. The radial Q_T-profile follows the A_2 amplitude profile so that the two Q_T-maxima appear at the same radial distances as the two amplitude maxima. The surface brightness profile of this galaxy is well fitted when including a bulge, a disk and a bar in the fit. SG-type profiles were found to appear both among the primary and the secondary bars, but all clearly identified secondary bars have SG-type amplitude profiles.

By definition in Buta et al. (2006), DG-type bars have a broad A_2 amplitude profile, of which category the galaxies NGC 4245, NGC 2859 and NGC 1452 are representative examples. NGC 4245 has two density peaks both in the lower (A_2) and in the higher Fourier modes (A_4 and A_6). Again, the radial Q_T-profile follows the A_2 amplitude profile: one can imagine that the broad maximum is actually a superposition of two partially overlapping Q_T-peaks. In the structural decomposition these two components were fitted by two oval/bar components. The interpretation is that the bar has two components, namely a short inner bar, and a longer outer bar. It is highly improbable that the inner component is an oval, because the higher Fourier modes are also visible. In order to identify better the components both in the amplitude profile and in the decomposition plot, in Figure 8 the peaks from the Gaussian fitting are also shown. The parameters for calculating these peaks were taken from Table 3 in Buta et al. (2006).

NGC 1452 has a broad double peaked A_2 amplitude profile, but in this case the higher Fourier modes (A_4, A_6 and A_8) appear clearly only at r $\sim$ 35", which is the more distant component of the A_2 double peak. Also, the Q_T-profile does not completely follow the A_2 amplitude profile: the peak in the Q_T-profile is fairly sharp and coincides with the radial distance where the higher Fourier modes are also significant. This maximum evidently corresponds to a bar, a component which appears also in the surface brightness profile at a similar radial distance. The strongest A_2 maximum appears at r $\sim$ 20", where the higher Fourier modes are weak or absent. At this radial distance there is only a weak shoulder in the Q_T-profile. Most probably this A_2 maximum corresponds to a bright oval, identified also in the structural decomposition. This example shows that although an oval might have a fairly high relative mass, it does not necessarily induce strong tangential forces. It also shows the power of the higher Fourier modes in discriminating ovals from thick inner components of bars.

NGC 2859 has a very broad A_2 amplitude profile, but in this case the higher Fourier modes appear only at fairly small radial distances (r < 40"). The Q_T-profile is extremely shallow showing only a modest peak at the radial distance where the higher Fourier modes are present (r $\sim$ 40"). Our conclusion is that the A_2-profile is broad because the galaxy has a weak bar and a prominent oval extending outside the bar, a conclusion made also by Buta et al. (2006) for this galaxy. The small peak in the A_2 amplitude profile at r <

10", visible also in the surface brightness profile and in the Q_T-profile, corresponds to a secondary bar.

In conclusion, the broad or double peaked A_2 amplitude profiles of bars in the early-type galaxies correspond either to a two-component bar with a thick inner part and a thin outer part, or a superposition of a bar and an oval (small or extended). Many DG-type bars in these galaxies are two-component bars. We find that the DG-type bars are on average stronger than the more simple SG-type bars using all four bar strength indicators. Bars and ovals can be distinguished from each other by inspecting also the higher Fourier modes, which are significant in bars, but not in ovals (due to their smaller ellipticities). Strong bars are also found to have fairly sharp outer cut-offs in the A_2-profiles, in agreement with the finding by Ohta (2002).

4.2.2 Late-type galaxies

We inspected the Fourier amplitude profiles also for all galaxies in the OSUBGS sample as well, although due to the prominence of spiral arms the interpretation of these profiles is not always straightforward. For this reason, any quantitative comparison of SG/DG type bars in different Hubble type bins was not possible. However, similar density profiles as found for the early-type galaxies were found also for the spirals. Examples of characteristic SG and DG-type profiles of bars in spiral galaxies are collected in Figure 9. As in Fig. 8, the radial Q_T-profiles and Fourier amplitude profiles are shown.

'The bar in NGC 4321 manifests itself with a rather weak outer part, affected in morphology by the spiral arms, and an aligned inner part which is well-defined in the near-IR (Knapen et al. 1995). This secondary bar, with a length of some 10 arcsec, has a clear SG-type morphology. The morphological type of this galaxy is SAB(s)bc, but the amplitude profile of the bar is similar as in any other morphological type: the density maximum is sharp and in addition to the A_2 component the higher Fourier modes, A_4 and A_6 (even A_8 is present), are also prominent. All modes appear nearly at the same radial distance, which is also the location of the Q_T-maximum.

The five remaining galaxies in Figure 9 are candidates of DG-type bar profiles, similar to those discussed among the early-type galaxies above. For example, NGC 3583 clearly has a two-component maximum at r < 40", both in the lower and in the higher Fourier modes, indicating that the bar has both an inner and an outer component. The Q_T-peak is also very broad. However the phase of the A_2 component is maintained nearly constant only to $r \sim 25$", which suggests that the outer part of the bar has some spiral-like characteristics, as is visible also in the direct image of this galaxy. The galaxies NGC 4394 and NGC 7479 are more clear examples of two-component bars. Bars in these galaxies have qualitatively similar double-peaked amplitude profiles with prominent higher Fourier modes. The bar in NGC 4593 is qualitatively similar to those in NGC 4394 and NGC 7479. The large A_2 maximum at $r \sim 70$" is caused by the prominent spiral arms starting at the two ends of the bar.

Ohta, Hawabe & Wakamasu (1990) were the first to discover that the higher Fourier modes of bars are characteristic for early-type galaxies, like S0s. They also argued that these modes are absent for the bars in spiral galaxies. However, our analysis indicates that the higher Fourier modes are characteristic for *all* strong bars, independent of the morphological type. For example, the amplitudes of the higher modes are extremely strong in the Sc-type spiral galaxy NGC 7479, having Q_g=0.7, and weaker for NGC 6221 (Q_g =0.44). According to Ohta (1996) the early-type galaxies also have sharp cut-offs in the amplitude profiles at the ends of the bar, but such cut-offs were argued to be missing in bars of late-type galaxies. Due to the strong spiral arms, the shapes of the amplitude profiles in the spiral galaxies are difficult to evaluate. However, NGC 7479 is an example showing that strong bars can have fairly sharp outer cut-offs in their A_2-profiles also in the late-type spirals. Most probably, this property is also related to the strength of the bar rather than to the morphological type. As an example of an early-type bar Ohta used NGC 4643, a galaxy which is also in our sample, and shows a fairly strong bar (Q_g=0.3).

5 DISCUSSION

5.1 Nature of bulges in the Hubble sequence

In the hierarchical clustering model of the Universe dominated by cold dark matter (Toomre & Toomre 1972; White & Rees 1978; Steinmetz & Navarro 2002) the dominant mechanism for producing bulges is by mergers of disk galaxies. Mergers of equal mass galaxies also yield remnants with properties similar to those found in the elliptical galaxies (Barnes 1988; Hernquist 1993; Lima-Neto & Combes 2005; Balcells & Gonzalez 1998). However, a problem with these models is that the properties of bulges in spiral galaxies and, as discussed in this study, even in early-type disk galaxies, do not resemble those of the elliptical galaxies. Nevertheless, it is not clear how strict the criterion for the merger origin is: there is recent evidence based on the cosmological dynamical models by Springer & Hernquist (2005) showing that if sufficient gas remains following a major merger, cooling can quickly reform the disk. This yields remnants that are closer to spiral galaxies, both structurally and kinematically. If this is correct, bulges might still be remnants of hierarchical clustering, particularly in the non-barred early-type galaxies. However, detailed comparison between observations and model predictions are not yet possible.

The fact that the bulges in the spiral galaxies are nearly exponential is consistent with the picture according to which the bulges are largely part of the disk formed by star formation in the disk. The B/T-ratios are also small and B/T flux ratio increases with galaxy luminosity. This can be understood if gas from the outer disk is accreted to the inner parts during the galaxy evolution. Galaxies with higher luminosities have more gas in the disk that can accumulate into starbursting rings and eventually lead to increased bulge masses. Support for this scenario comes from the recent Spitzer Space Telescope observations by Fisher (2006) who showed that galaxies which are structurally identified as having pseudobulges, also have higher central star formation rates than those having classical bulges. Bars may help in transferring the gas towards the central regions of the galaxies, but they are not a necessary requirement for the gas inflow (Sakamoto et al. 1999; Sheth et al. 2005).

The predominance of old stellar populations and the de Vaucouleurs type surface brightness profile are generally used to claim that bulges in S0-S0/a galaxies are merger-built structures (Schweizer 2005). Also, they do not seem to have sufficient gas for producing star formation at a level that could account for the masses of typical pseudobulges (see KK2004). However, as discussed in this study, bulges in the early-type galaxies have many signatures of pseudobulges. For example, they have fairly exponential bulges, 56% of them have inner structures like nuclear rings or nuclear bars, and some of them also have kinematic evidence of rotationally supported bulges. There is also a large kinematic study of E/S0 galaxies by Emsellen et al. (2005, and references there) using high resolution integral field spectroscopy for deriving the kinematic parameters. They found that the bulges in most S0s galaxies are fast rotating systems having also large anisotropies of the bulge. These kinematic properties were suggested to indicate either secular evolution, or heating of the disk due to minor mergers. Our interpretation in this study is that pseudobulges in the early-type disk galaxies are largely bar-related, connected to the evolution of bars. This could also naturally explain why the bulges are non-classical even in such gas poor galaxies as S0s.

5.2 Angular momentum transfer and the evolution of bars in the Hubble sequence

In modern dynamical models, bars are expected to play an important role in galaxy evolution (Athanassaoula 2003). The main idea is that bars evolve due to angular momentum transfer between the bar and the halo, which occurs near the resonances: in particular disk material at the Inner Lindblad Resonance (ILR) will lose angular momentum, while halo material near corotation (CR) and near the outer Lindblad Resonance (OLR) will absorb it. How strong this angular momentum exchange is depends critically on the mass of the halo and its central concentration, the amount of mass in the resonances, and how dynamically cool or hot are the disk and the halo. Athanassoula (2003) reports three different ways of how bars can lose angular momentum: (1) by trapping particles outside the bar into elongated orbits of the bar, a process in which angular momentum is lost from the inner parts of the disk, while the bar becomes *longer*, (2) part or all of the orbits trapped in the bar become more elongated and the bar becomes *thinner*, and (3) bars lose rotational energy leading to a *slow bar*. These three processes are expected to be linked, so that due to strong angular momentum transfer bars simultaneously become longer, thinner and more slowly rotating. Athanassoula's models (2003) have also shown that while bars become longer, their relative masses increase. These trends have been verified later by other self-consistent 3D simulation studies like those made by Martinez-Valpuesta, Shlosman & Heller (2006). The first indication that bars grow when they evolved over time comes already from dynamical models by Sellwood (1980).

Our observations show that bars become longer and more massive from the late-type spirals towards the early-type spirals. These observations are consistent with the models by Athanassoula (2003) if either the dark matter halos or the bulges in the early-type galaxies are more massive or more centrally concentrated than the halos and bulges in late-type galaxies. Our observations can be understood by the same models also if the bars in the early-type galaxies are very old structures, in which case even smaller halos might be sufficient to cause the angular momentum transfer. In that case we might also be witnessing slow evolution of bars in galaxies that gradually lose their gas and change their morphological type in the Hubble sequence. A possible candidate of such evolution is the non-barred galaxy NGC 1411, which is classified as an S0 galaxy, but has a B/T flux ratio which is as small as typically found in Sc-type spirals (see Laurikainen et al. 2006).

On the other hand, and at odds with the model predictions, our analysis also shows that bars do not have larger bar torques nore become more elliptical, towards the early-type spirals. This behavior of the bar torque (Q_g) and the ellipticity of the bar (f_{bar}) however, should not necessarily be taken as a counterargument to the evolutionary models by Athanassoula. This is because Q_g is diluted by the underlying axisymmetric component, generally the bulge, which is more massive in the early-type galaxies, and which is generally not taken into account in the simulations. It would be interesting if the simulations could confirm whether the opposite trends found in the Hubble sequence among the different bar strength indicators can be completely explained by the dilution effect due to more massive bulges in the early-type galaxies as suggested by Laurikainen, Salo & Buta (2004), or whether this result is also related to the different mechanisms of how bars lose their angular momentum.

An additional verification for the hypothesis that bars evolve because they lose angular momentum would be to show that long, evolved bars are more slowly rotating systems than shorter and less evolved bars. However, this has been difficult to prove, mainly because of the difficulties to measure the bar pattern speed, Ω_p. So far, Ω_p has been measured directly using the Tremaine-Weinberg method for some S0 galaxies (Merrifield & Kuijken 1995: Gerssen, Kuijken & Merrifield 2003; Aguerri, Debattista & Corsini 2003; Corsini, Debattista & Aguerri 2003; Rand & Wallin 2004; Debattista & Williams 2004), and only for two spiral galaxies (Gerssen, Kuijken & Merrifiled 2003). These direct measurements of Ω_p have repeatedly pointed to fast bars in the early-type galaxies, contrary to what one would expect for evolved bars. The largest collection of Ω_p measurements for spiral galaxies comes from the 2D sticky particle simulation (Salo et al. 1999) models for 38 spiral galaxies in the OSUBGS sample by Rautiainen, Salo & Laurikainen (2005) and Salo et al. (2006), who showed that the bars in late-type spirals are actually slower rotators than those in early-type spirals. In principle, this could be related to the dark matter halos, because according to Persic, Salucci & Stel (1996) the relative halo mass depends on galaxy luminosity so that galaxies with lower luminosities also have more massive dark matter haloes. However, in the OSUBGS sample the luminosities of Sb and Sbc galaxies, for which lower Ω_p-values were found, have on average the same luminosities as the early-type spirals. Therefore, mass of the dark matter halo alone is not sufficient to explain the slowdown rate of the bar in the late-type spirals. More measurements of Ω_p and of the rotation curves are evidently needed to clarify this issue.

A completely different view of the evolution of bars in spiral galaxies has been presented by Bournaud & Combes (2002), who suggest that in the presence of continuous accretion of external gas bars become fairly short-lived systems, so that bars are recurrently formed and destroyed. In principle this is possible, but our observations do not shed any new light on this issue. The recent simulations by Debattista et al. (2006) suggest that bars are actually fairly robust systems so that high gas masses are required to destroy the bars. Somewhat smaller gas masses to destroy bars are suggested in the simulation models by Hozumi & Hernquist (2005), Bounaud, Combes & Semelin (2005), and Athassoula, Lambert & Dehnen (2005).

Bar strength measurements alone are not sufficient to distinguish whether bars are strong mainly because they have massive dark matter halos, or because the strong bars are very old, formed in the epoch when they still had a large amount of gas. Whatever the case, something is different in galaxies with S0/a type morphologies. For very early types the bars start to decrease in length and lose their mass, probably accompanied by smaller bulge components. An intriguing possibility is that the internal evolution of bars plays an important role in producing these characteristics.

5.3 Internal evolution of bars

The evolution of stellar bars is affected by dynamical instabilities leading to long-term changes in their morphologies. A well known is the so called buckling instability (Combes et al. 1990; Pfenniger & Friedli 1991; Raha et al. 1991; Berentzen et al. 1998; Athanassoula & Misioritis 2002; Athanassoula 2002, 2003, 2005a,b; O'Neilss & Dubinski 2003; Debattista et al. 2004; Martinez-Valpuesta, Shlosman & Heller 2004; Debattista et al. 2006; Martinez-Valpuesta, Shlosman & Heller 2006), where the orbital families of bars are changed in such a manner that leads to a vertical thickening of the bar, the so called boxy/peanut structure. In recent studies (Martinez-Valpuesta, Shlosman & Heller 2006; Athanassoula 2006) multiple buckling effects have also been discussed. Buckling is expected to be a natural part of the evolution of bars, being particularly important in strong bars. The presence of boxy/peanut bulges for edge-on galaxies has previously been shown both kinematically (Kuijken & Merrifield 1995; Chung & Bureau 2004) and using surface photometry (Lütticke, Dettmar & Pohlen 2000; Bureau et al. 2006). Lütticke, Dettmar & Pohlen found that 54% of the bulges of all morphological types among edge-on galaxies have boxy/peanut bulges, which implies that they should be common also among the less inclined galaxies.

When a bar forms in numerical simulations it is thin, but soon develops a vertically thick inner part and a vertically thick more extended middle component of the bar (Athanassoula 2003, 2005). In the surface brightness profiles both vertically thick components can be interpreted as pseudobulges. In the simulation models the surface brightness profile of the innermost component of the bar is nearly exponential, whereas for the middle component it depends on the viewing angle: when viewed edge-on the profile takes a Freeman type II profile shape. Also the length of the peanut structure depends on the viewing angle so that it is longest in the edge-on galaxies (Athanassoula & Beaton 2006). The strength of the peanut structure depends on bar strength so that it is strongest in strong bars (Athanassoula & Misioritis 2002; Bureau & Athanassoula 2005). Strong bars in simulation models are also found to have frequently ansae-type morphologies (Athanassoula & Misioritis 2002; Athanassoula, Lambert & Dehnen 2005; Athanassoula & Beaton 2006; Martinez-Valpuesta, Shlosman & Heller 2006).

However, a comparison of the observations with the simulation models is difficult because different parts of the bars are seen in face-on and in edge-on views. This has been demonstrated for example by Athanassoula & Beaton (2006), who showed that although the boxy/peanut structure is visible at an inclination of 77^0, the length of this structure is shorter at this inclination than in the edge-on view. Also, in face-on view the boxy/peanut structure is barely visible. Based on these difficulties we cannot argue that the bulges we see in the early-type galaxies are boxy/peanut structures produced by buckling effects. However, we have shown strong evidence that bars in the early-type galaxies are evolved systems, which is also the case with the boxy/peanut shaped bar/bulges in the simulation models. Namely, we find that 90% of early-type galaxies have either flat or intermediate-type Fourier amplitude profiles, and 40% have ansae bar morphologies. In comparison, the amplitude profiles in spiral galaxies are mostly exponential and only 12% show ansae-type morphologies (14 with ansae among 115 OSUBGS galaxies). [3], Also, by comparing the Fourier amplitude profiles and the multicomponent decompositions for the same galaxies we identify inner structures that cannot be explained by the classical bulges or ovals. Buta et al. (2006) showed that the observed flat-top amplitude profiles are at least consistent with models with large dark matter halos, but other factors as centrally concentrated halo mass profiles or enough mass in the resonances might also lead to similar evolved amplitude profiles.

Therefore, both the morphological analysis of bars and bar strengths in S0-S0/a galaxies hint to evolved bars. However, what still needs to be explained in this picture is why bars in the S0-S0/a galaxies are repeatedly found to be fast? And also, why bars, particularly in the early-type S0s, are less prominent than bars in the later type S0s or in S0/a galaxies? None of this is expected in models where bars evolve due to a significant transfer of the angular momentum from the bar to the halo. It has been shown by Martinez-Valpuesta, Shlosman & Heller (2006) that the connection between the different properties of bars might actually be a complicated process, so that during the buckling process Ω_p sharply increases, followed by a sudden decrease in bar length. However, even in this model the bar should both grow in length and slow down in the long run, so that after the second buckling the bar should be more prominent and more slowly rotating than after the first buckling.

There are other models, however, like those by Athanassoula, Lambert and Dehnen (2005), that can account better for the observed properties of bars in the S0 galaxies. They showed that if central mass concentrations like realistic secondary bars or nuclear disks are used in the models, bars become shorter, less massive and faster than in models without any centrally concentrated mass components. Nuclear bars and inner disks appear in more than 50% of the studied S0-S0/a galaxies in our sample, implying that the models by Athanassoula, Lambert & Dehnen are a promis-

[3] a similar result for the early-type galaxies has been obtained also by Martinez-Valpuesta & Knapen 2007, private communication

ing explanation for the type of bars we find in the very early-type galaxies. In the context of evolution it would be natural to expect that the early-type S0 galaxies are older than the late-type S0s or S0/a galaxies, which implies that their bars have had more time to evolve: in this case to become less prominent. Ultimately, the efficiency of nuclear bars and disks in redistributing matter in the disk is related to the problem of cuspy halos: if the halos are cuspy as assumed in present cosmological models, the central parts of the galaxies are halo dominated, thus reducing the effects discussed by Athanassoula, Lambert & Dehnen (2005). But if the halos have constant density cores then their models might efficiently produce the type of bars found in S0s.

Although it seems that bars in early-type galaxies have many characteristics of evolved systems, the details of the mechanism of the angular momentum transfer between the bar and the dark matter halo remains unsolved. Even if bars evolve due to the angular momentum transfer between the bar and the halo the time scale of the evolution can be either rapid or more slow, which is expected to have imcplications to the ages of bars and the masses of dark matter haloes. The masses of the halos in the early-type galaxies are poorly known, but the rotation curve observations by Mathieu, Merrifield & Kuijken (2002, see also Romanowsky 2006) seem to indicate that the halo masses in S0 galaxies might actually be quite small. Bulges are expected to affect the evolution of bars in a similar manner as the dark matter halos (Athanassoula 2003), but this has not yet been tested using realistic B/T-ratios for galaxies in the different Hubble type bins.

6 CONCLUSIONS

Properties of bars and bulges in the Hubble sequence are discussed, based on the analysis of 216 disk galaxies, selected from the OSUBGS and NIRS0S samples. Our main emphasis has been to combine the various properties of bars and bulges, presented for the individual galaxies in a series of papers by us, and to discuss the implications of these measurements in the Hubble sequence. The properties of bars were derived mainly by Fourier techniques and the properties of bulges by applying a multicomponent decomposition code. The analysis results were compared with various dynamical models in the literature. We find strong photometric and kinematic evidence of pseudobulges in the S0-S0/a galaxies. However, most probably pseudobulges in the early and late-type galaxies have a different origin, which would also make more understandable why pseudobulges are frequently found in such gas poor galaxies as in S0s.

Our main results are the following:

(1) We show both photometric and kinematic evidence that the bulges in many S0-S0/a galaxies have characteristics of pseudobulges: they have on average small shape parameters and 56% of them have inner components as nuclear bars or nuclear rings, confirming the earlier results by Laurikainen et al. (2006). Kinematic data collected from the literature is in many cases consistant with rotationally supported bulges.

(2) We find evidence showing that pseudobulges in the late-type galaxies were formed by star formation in the disk, while in the early-type galaxies (S0-S0/a) they are mainly bar-related structures. This interpretation is based on the decompositions, which show that these components have nearly exponential surface brightness profiles, and on Fourier analysis showing flat-top/double-peaked amplitude profiles, where the higher Fourier modes are also important.

(3) We find that bars with flat-top/double peaked amplitude profiles are stronger that the more simple bars, using all bar strength indicators: bar torque (Q_g), the ellipticity of a bar (f_{bar}), the relative mass a bar (A_2) and the length of a bar. Galaxies with flat-top/double-peaked amplitude profiles have also slightly more exponential bulges and lower B/T flux ratios than the more simple bars.

(4) Strong bars are found to have the following characteristics: 1) flat-top/double peaked amplitude profiles 2) sharp outer cut-offs, 3) a presence of higher Fourier modes, and 4) in many cases ansae-type bar morphologies. We found that 40% of S0-S0/a galaxies have ansae, in contrast to 13% in spiral galaxies. Flat-top/double or intermediate-type amplitude profiles are found in 92% of S0-S0/a galaxies, whereas amplitude profiles of bars in spiral galaxies are largely exponential.

(5) The distributions of bar strength in different Hubble type bins are shown using four different bar strength indicators, Q_g, A_2, f_{bar} and bar length, which is expected to give important clues for understanding the mechanism of how bars lose their angular momentum and thus evolve over time. For example, it needs to be understood why bars in the early-type S0 galaxies are less prominent than bars in the later type S0s or in S0/a galaxies.

(7) Even 70 % of S0-S0/a galaxies are found to have ovals or lenses, confirming the earlier result by Laurikainen et al. (2006). According to dynamical models by Athanassoula (2003) weak ovals may form in hot halos and more extended ovals in hot disks. We find both weak and expended ovals and thin classical bars in the same galaxies, which needs to be explained by the theoretical models.

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Table 1. Stellar kinematical properties of bulges.

Gal.	T	INC/PA [degrees]	V_{max} $[km/sec]$	σ $[km/sec]$	ϵ	r [arcsec]	ref.
NGC 210	3	49.2/162.7	83.3	120	0.1	10"	Pizzella et al. 2004
NGC 936	-1	42.4/123	80.2	175	0.15	5"	Kormendy (1983)
NGC 1400	-3	23.8/37	90.0	266	0.1	5"	Bertin et al. 1994
NGC 1553	-2	41.5/150	99.4	162	0.4	10"	Kormendy 1984
NGC 1574	-3	16/31	39.9	180	0.1	6"	Jarvis et al. (1988)
NGC 2681	0	24.2/102(2)	20.1	111	0.2	5"	McElroy 2004; Moiseev et al.2004
NGC 2775	2	36.8/166.5	125.2	175	0.11	<20"	Corsini et al. 1999
			100.0	140	0.11	<20'	Shapiro et al. 2003
NGC 2855	0	33.1/107	120.0	190	0.17	10"	Corsini et al. 2002
NGC 2983	-1	54.8/91	54.7	160	0.2	2"	Bettoni et al.1988
NGC 3169	1	39.1/58.0	112.0	171	0.36	10"	Heraudeau & Simien 1998
NGC 3626	-1	48.0/159	150.0	142	0.4	5"	Haynes et al.2000
NGC 3706	-4	50.65/76	139.9	281	0.38	10"	Carollo et al.1994; 1993
NGC 3810	5	47.2/23.1	50.0	73	0.27	15"	Heraudeau et al. 1999
NGC 3941	-2	49.4/7	60.0	131/150	0.2	5"	Fisher 1997;Denicolo2005
NGC 4138	1	53.3/148.2	100.0	161	0.21	5"	Jore et al. 1996
NGC 4340	-1	56.2/99	48.2	115	0.07	5"	Simien,Prugniel,1997
NGC 4450	2	43.9/2.8	45.0	126	0.25	10"	Fillmore et al. 1986
NGC 4579	2	38.5/95.3	55.0	174	0.22	5-10"	Palacios et al. 1997, Heraudeau & Simien 1998
NGC 4596	0	44.3/116	45.0	149	0.15	5"	Bettoni & Gallaher, 1997
NGC 4643	0	33.3/50	79.6	167	0.1	5"	Magrelli et al.1992
NGC 5005	3	63.6/67.9	110.0	213	0.25	5"	Batcheldor et al. 2005
NGC 7727	1	26.9/159.8	35.0	181	0.27	8"	Simien & Prugniel 1997
ESO 208-G21	-4	43.9/109	110.0	150	0.52	5"	Carollo et al. (1993)

Table 2. The ellipticies of the barred NIRS0s galaxies, using the parameter f_{bar} as defined by Whyte et al. (2002).

NGC	f_{bar}
718	0.124
936	0.273
1022	0.281
1079	0.262
1317	0.107
1326	0.220
1350	0.234
1387	0.117
1415	0.239
1440	0.173
1452	0.297
1512	0.306
1533	0.167
1574	0.120
2273	0.249
2681	0.098
2781	0.070
2859	0.235
2983	0.262
3081	0.256
3358	0.132
3626	0.142
3941	0.077
4245	0.164
4340	0.283
4596	0.241
4608	0.243
4643	0.255

Table 3. Comparison of galaxies having single and double gaussian bars. The errors are mean errors.

	SG	DG
$< M_K >$	23.4 ± 0.2	23.4 ± 0.3
$< Q_g >$	0.09 ± 0.01	0.23 ± 0.03
$< f_{bar} >$	0.17 ± 0.09	0.22 ± 0.09
$< A_2 >$	0.39 ± 0.03	0.61 ± 0.05
$< barlen/h_R >$	1.14 ± 0.11	1.56 ± 0.12
$< B/T >$	0.27 ± 0.04	0.17 ± 0.02
$< n >$	1.9 ± 0.2	1.4 ± 0.2

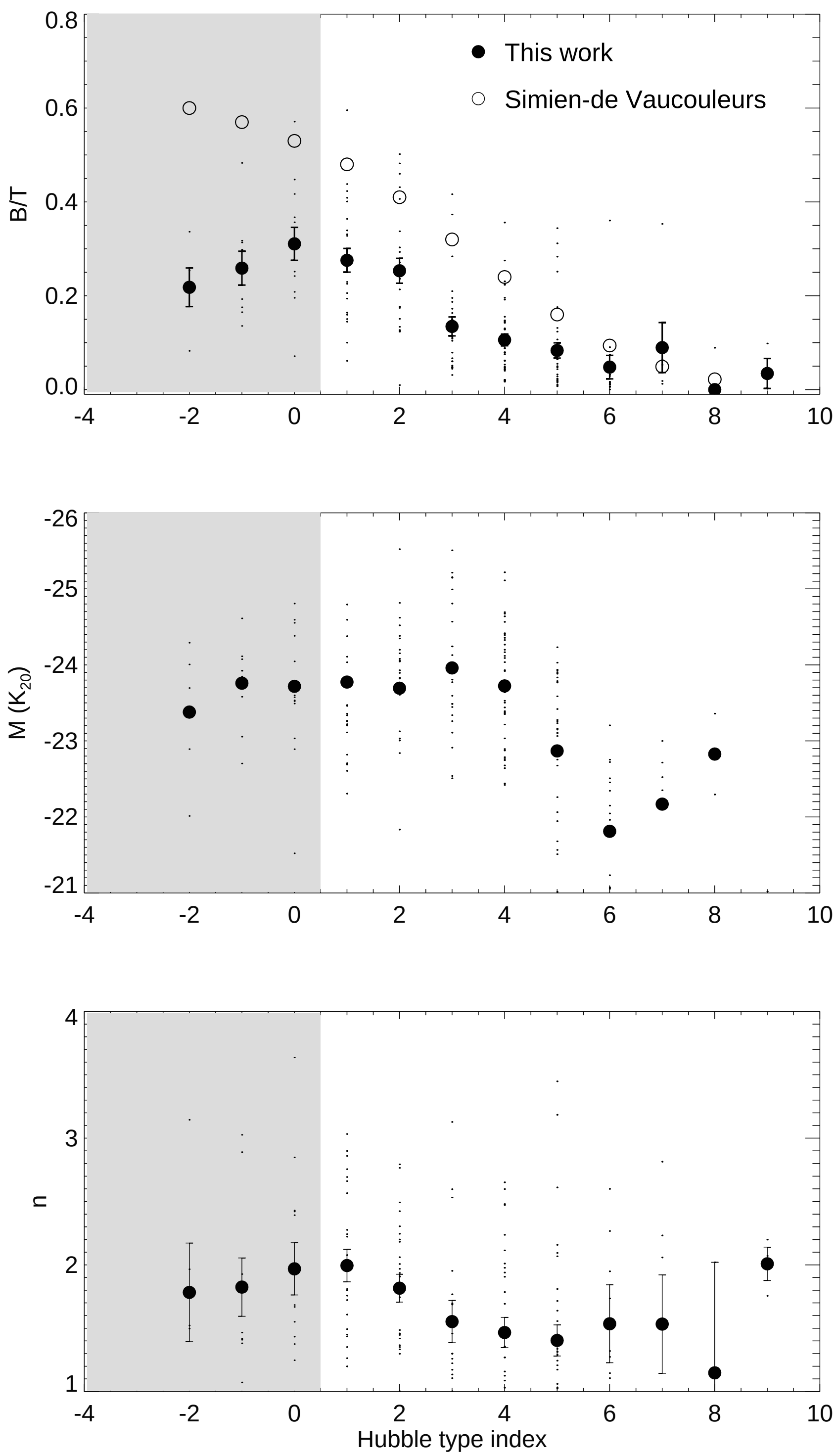


Figure 1. In this figure are shown the bulge-to-total flux ratio, B/T (upper panel), the absolute K-band galaxy luminosity using the magnitudes to the surface brightness of 20 $mag/arcsec^2$ taken from the NED and corrected for Galactic extinction (middle panel), and the shape parameter of the bulge, n corresponding to Sersic's function (lower panel), as a function of the Hubble type index. These parameters are calculated for our sample of 216 galaxies. The large symbols indicate mean values and the error bars are standard deviations of the mean. For comparison, in the upper panel the measurements by Simien & de Vaucouleurs, obtained in the B-band (1986) are also shown. Our parameters of the bulges are derived from K_s and H-band images.

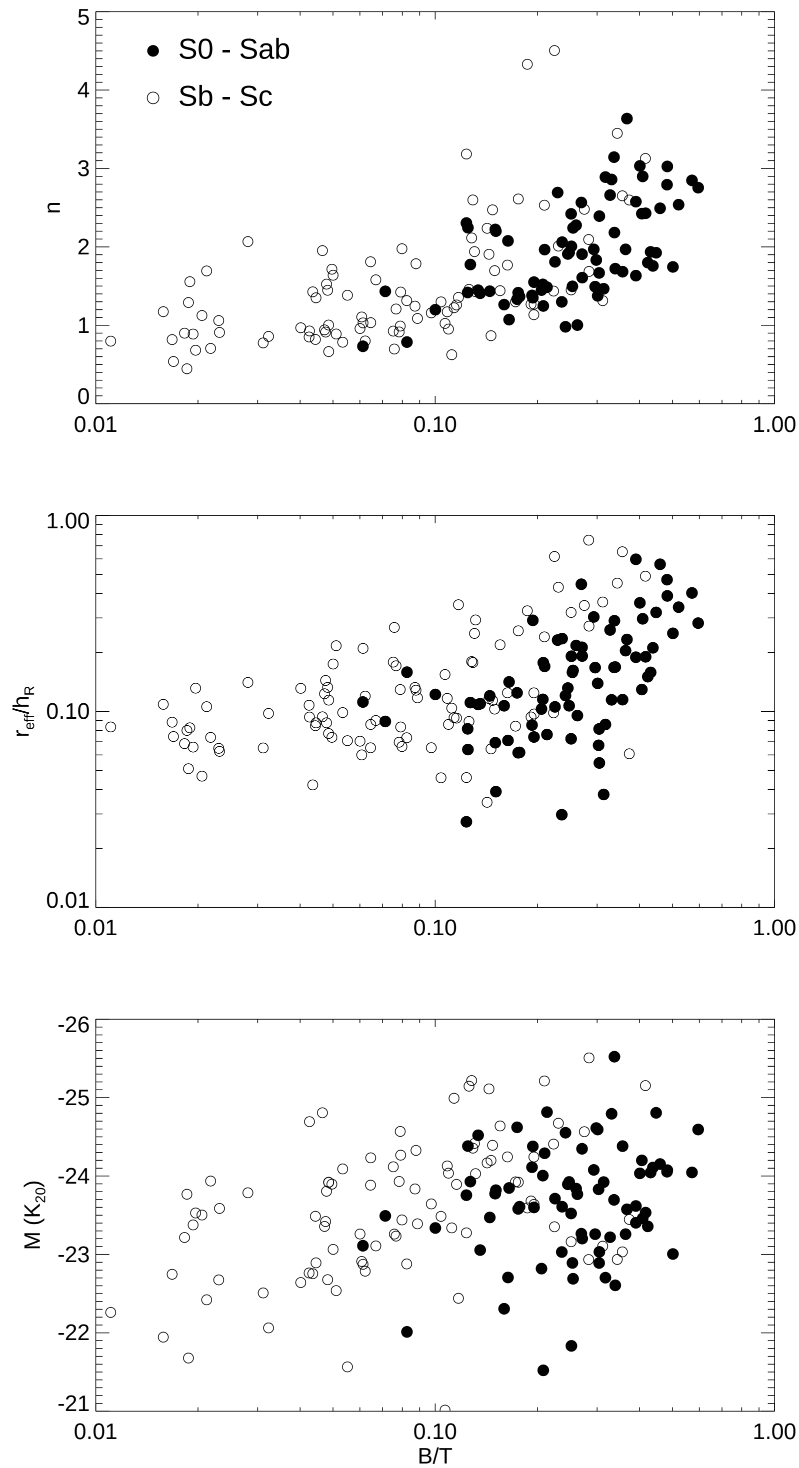


Figure 2. For the same sample are shown the shape parameter of the bulge (upper panel), the effective radius of the bulge, scaled to the scale length of the disk (middle panel), and the absolute galaxy luminosity (lower panel), as a function of B/T. The scale lengths are measured from the near-IR images by applying a multicomponent decompositions. The parameters are shown separately for the S0-Sab galaxies, and for the Sb-Sc type spirals.

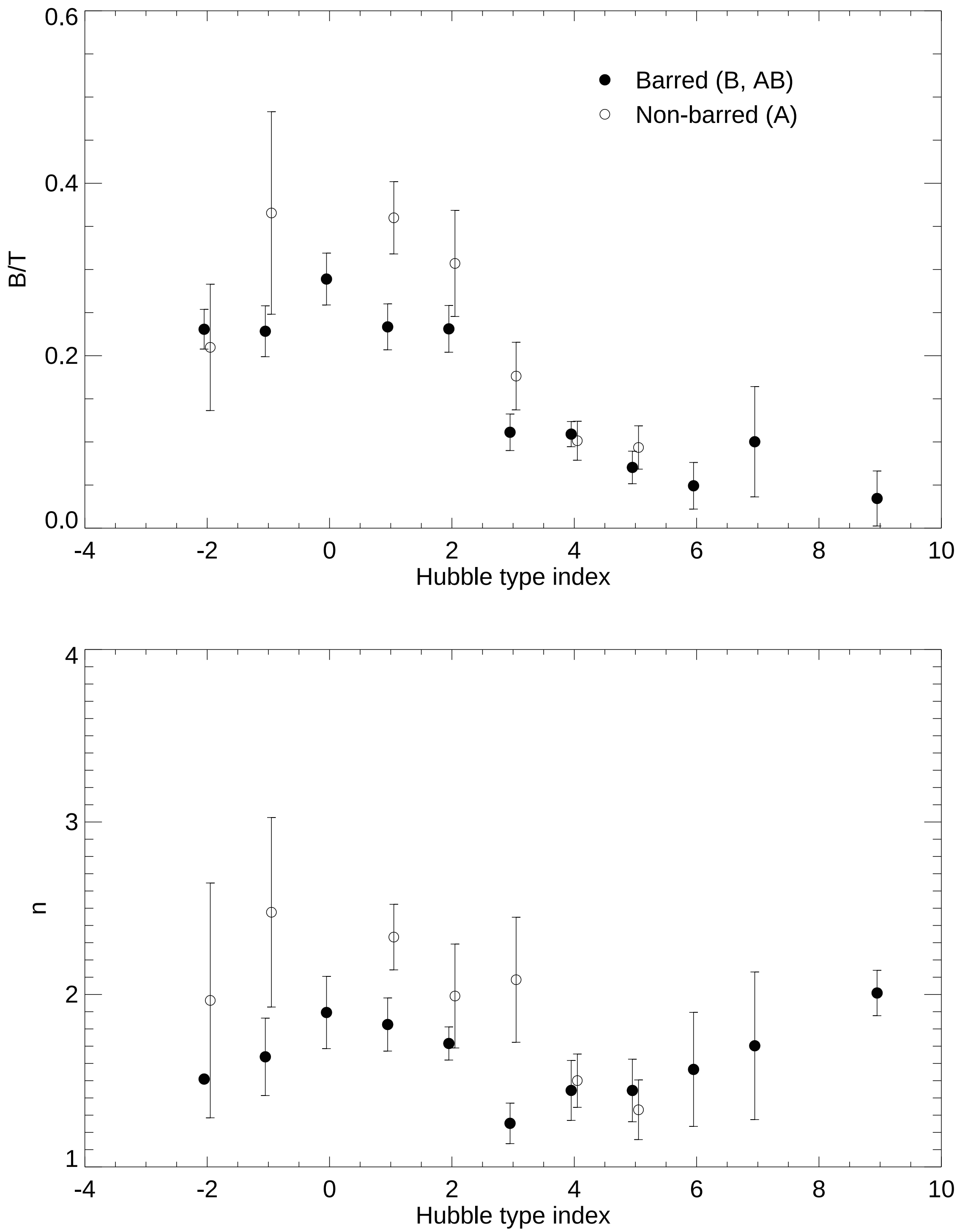


Figure 3. The parameters of the bulge, B/T and n, are shown separately for barred and non-barred galaxies as a function of the Hubble type index. The classification of barred/non-barred is taken from "The de Vaucouleurs Atlas of Galaxies" by Buta et al. (2007). Again, the symbols are mean values and the error bars indicate the standard deviations of the mean values.

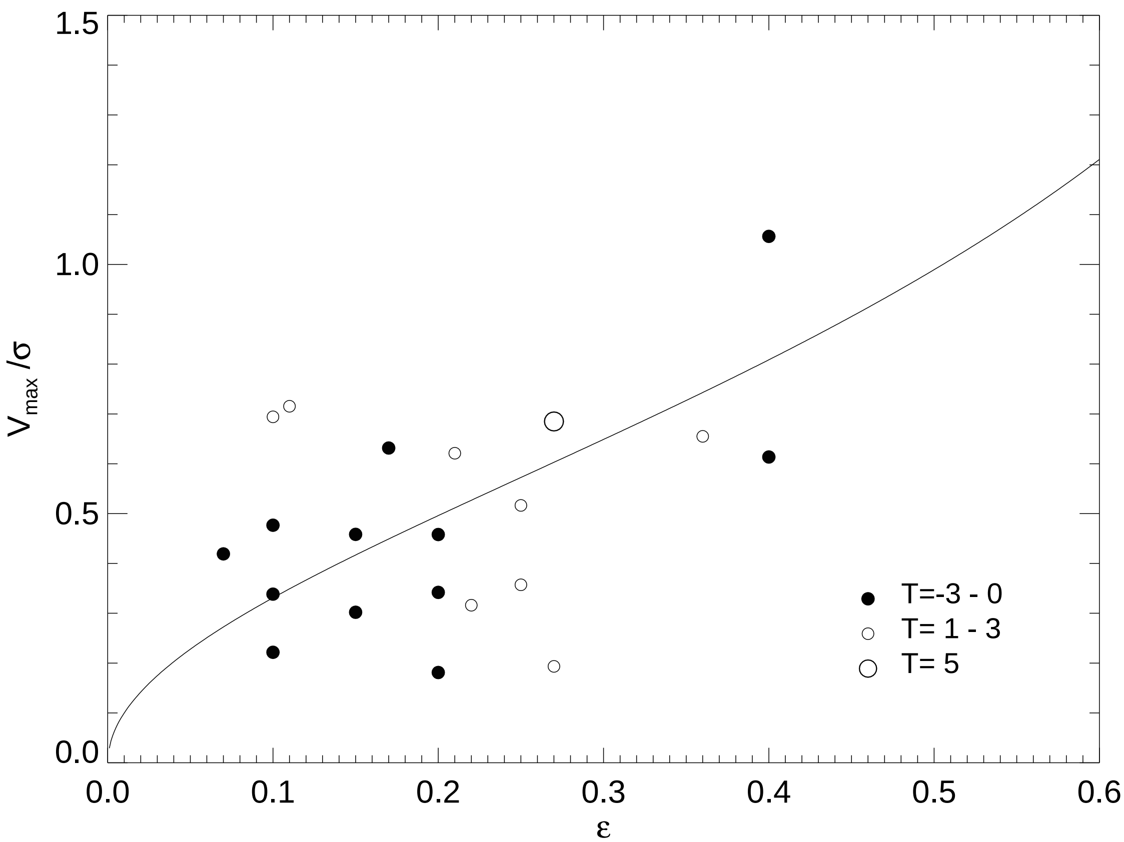
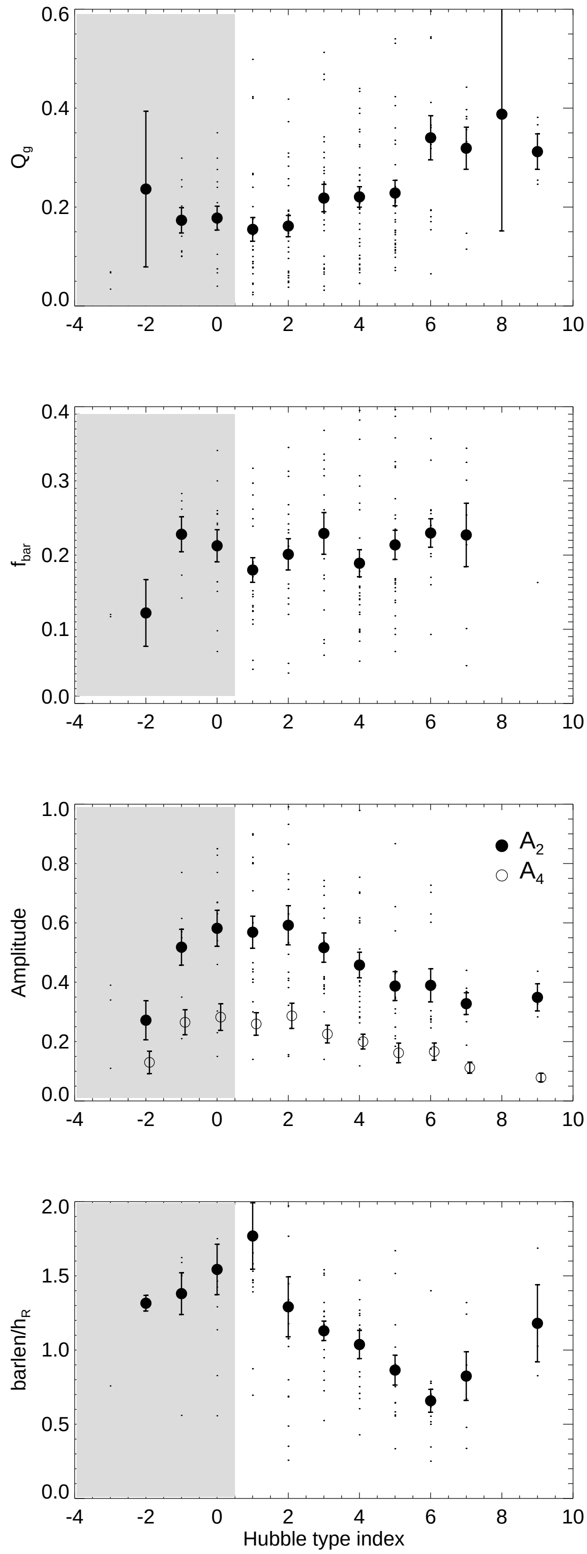


Figure 4. The kinematical properties of the bulges for a subsample of galaxies. V_{max} is the maximum line of sight rotation velocity of the bulge measured from the absorption lines, σ is the line of sight stellar velocity dispersion of the bulge just outside the nucleus, and ϵ is the characteristic ellipticity in the region interior to the radius of V_{max}. The values are taken from our Table 1, and the different symbols represent different Hubble type indexes. Inclination of the disk would shift the data points almost along the oblate rotator line, as predicted by the models by Binney (2005).



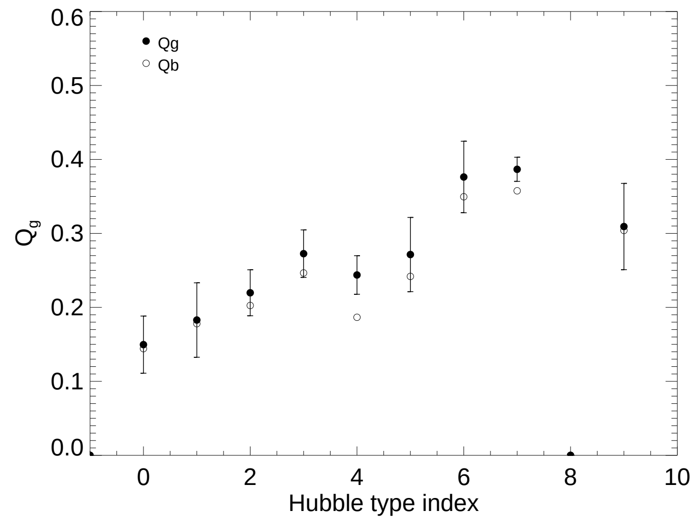


Figure 6. Comparison of the mean bar torques with (Q_b, from Buta, Laurikainen & Salo 2004) and without (Q_g, from Buta et al. 2005) the correction of the spiral arms in the OSUBGS sample. Both values are calculated adding the Fourier modes up to m=10. Only those galaxies are included for which it was possible to apply the bar/spiral separation approach.

Figure 5. Four different estimates of bar strength are calculated for our sample of 216 galaxies and shown as a function of the Hubble type index. Q_g is a bar torque indicator, which is the bar induced maximum tangential force, divided by the azimuthally averaged radial axisymmetric force field. The parameter f_{bar} is a measure of the ellipticity of the bar, as defined by Whyte et al. (2002), and explained in more detail in the text. The A_2 and A_4 are amplitudes of density for the m=2 and m=4 Fourier modes. Bar length has been estimated from the phases of the A_2 Fourier

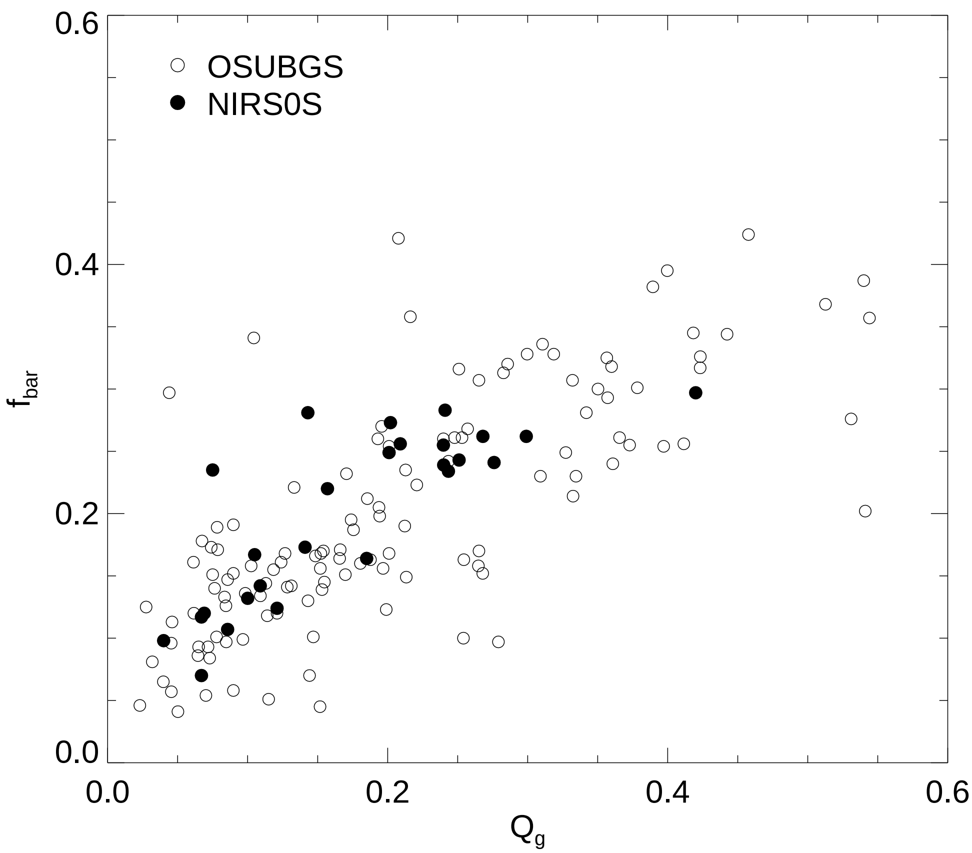


Figure 7. A correlation between the ellipticity of the bar, f_{bar} and the bar torque, Q_g. The open circles show the galaxies in the OSUBGS sample, whereas the filled circles show the galaxies in the NIRS0S sample.

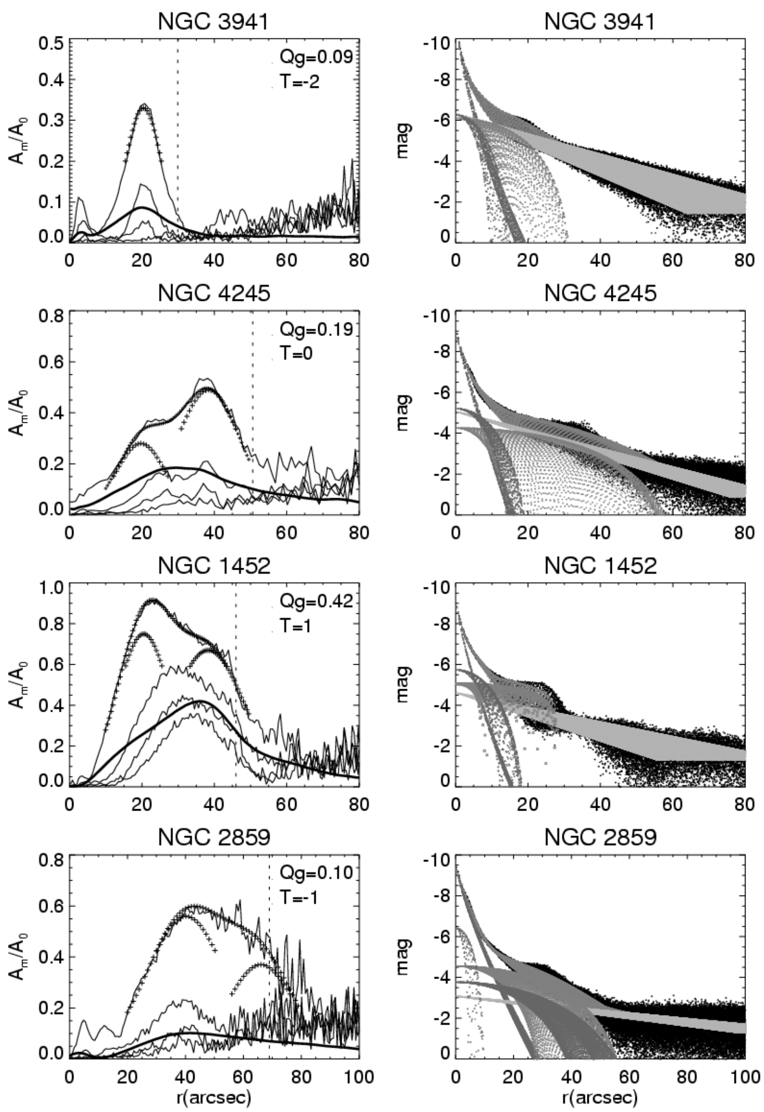


Figure 8. Examples of early-type disk galaxies in our sample. In the left row are shown the Fourier amplitude profiles (thin lines) and the Q_T profiles (thick lines) for these galaxies. The crosses show simple double Gaussian fits to the A_2 profiles, for which fits the parameters were taken from the Table XX in Buta et al. (2006): shown separately are the two peaks and the overall fit to the observed profile. The vertical dashed line shows the length of the bar, estimated from the phases of the A_2 Fourier amplitudes. In the upper right corners of these figures are indicated the values of the bar torques and the Hubble type indexes. In the y-axis the densities A_m of each mode are divided by the axisymmetric, azimuthally averaged densities A_0. In the right row of the figure are shown the results of the multicomponent decompositions for the surface brightness profiles for the same galaxies. The disk was fitted by an exponential function, the bulge by a Sersic's function, and the bars typically by a Ferrers function and the ovals and inner components of bars by a Sersic's function. A more detailed description of these decompositions can be found in Laurikainen et al. (2006).

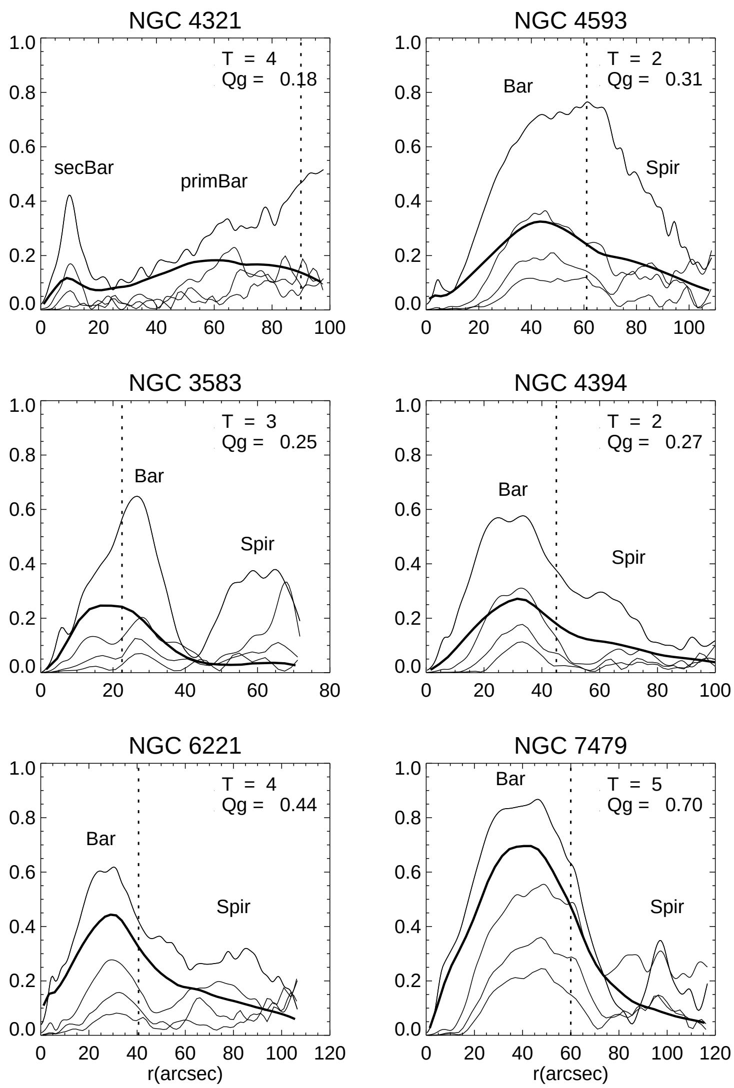


Figure 9. Characteristic examples of spiral galaxies in our sample. These figures are similar to those shown in the left row in Figure 7. The symbols are also the same as in Figure 7.