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# Drucker-Prager yield criterion application to study the behavior of CFRP confined concrete under compression

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**Abstract**
This paper presents a theoretical study of the behavior of ∅150x300 mm concrete specimens confined under centered compression. A new graphical and didactical work presents the cone that represents the triaxial Drucker-Prager yielding criterion under different confinement conditions. A sensitivity analysis is performed to evaluate the influence of the internal friction angle and the Poisson's ratio of the concrete at the ultimate strength level for confined concrete. Similarly a sensitivity analysis is performed to evaluate the influence of the level of confinement on the strength capacity of the specimens. Finally a comparison of these analytical results with experimental results from confined concrete cylinders with CFRP is presented, where the ultimate strength and ductility level reached with this strengthening is evaluated.

## 1 Introduction

The strength capacity improvement of the confined concrete is well-known. First results were presented by Richard et al. [1] in 1928 and the Mander curves [2] for confined concrete are the first results implemented in standards. The Drucker-Prager yield criterion [3] has been used by several authors to simulate the triaxial behavior of concrete, both in confined concrete with steel tubes [4,5], and confined concrete by means of FRP [6-10]. This work presents a graphical study of this confinement effect for specimens under uniaxial compression. Finally it is experimentally analyzed the effect of this strengthening with CF under constant loads.

The Drucker–Prager yield criterion (DP) is a pressure-dependent model for determining whether a material has failed or undergone plastic yielding. The yielding surface of the DP criterion may be considered depending on the internal friction angle of the material and its cohesion. In the space defined by the principal stresses, it is expressed as:

$$-\frac{2\cdot sen\phi}{(3-sen\phi)}\cdot I_1+\sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right]}-\frac{6c\cdot\cos\phi}{(3-sen\phi)}=0 \qquad (1)$$

where, $I_1 = \sigma_1 + \sigma_2 + \sigma_3$, $c$= cohesion, $\phi$= internal friction angle.

With the purpose of obtaining the classical representation, this yielding surface can be expressed in the following way:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 - A^2 (I_1 + B)^2 = 0 \qquad (2)$$

Where: $A = \dfrac{2 \cdot \sqrt{2} \cdot sen\phi}{3 - sen\phi}$, $B = \dfrac{3 c \cdot \cos\phi}{sen\phi}$ and compression stresses are considered positives in the main stresses space.

Figure 1 represents the classical quadric surface for a conventional concrete ($f_c$=25 MPa) with $c$ = *4.47 MPa*, $\phi$ = *30º*. The axis of the cone takes the direction of the vector (1,1,1).

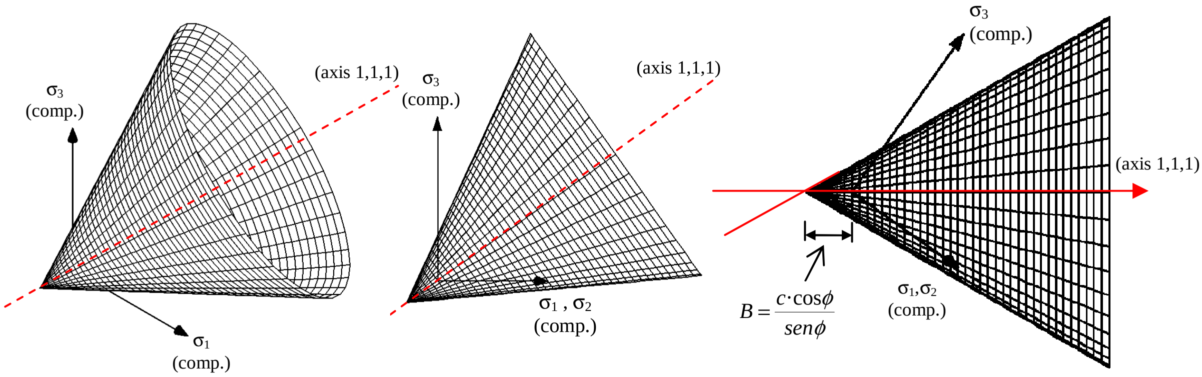
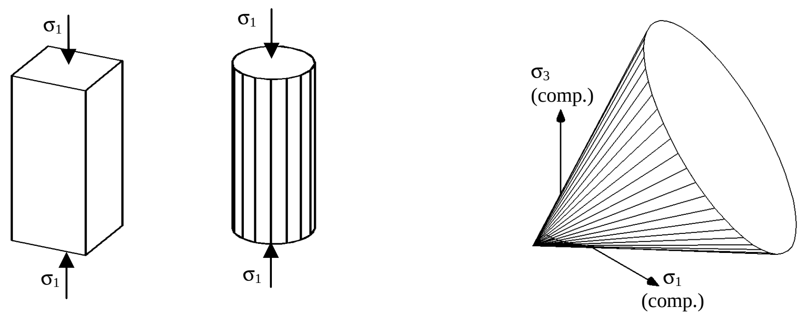



Figure 1: Graphic representation of the DP criterion.

## 2 Analytical study

### 2.1 Unconfined specimen. Monotonic compression

This is a situation in which the plane of the plane stress state ($\sigma_3$=0) intersects the yielding surface and also $\sigma_2$=0: the intersection with the axis $\sigma_1$. A free lateral expansion exists.



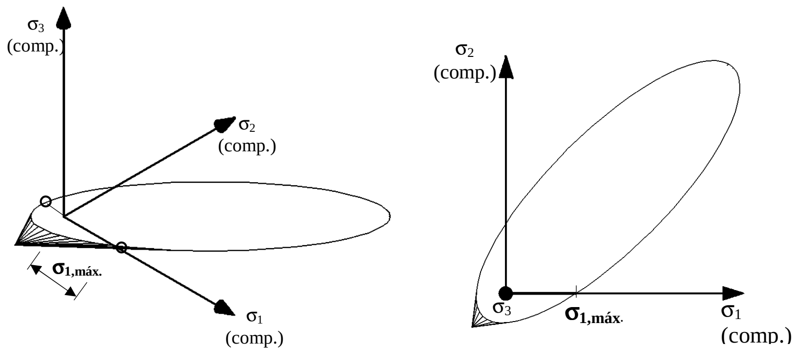



Figure 2: Graphic representation of the DP criterion for monotonic stresses.

The maximum admissible compression for this situation is:

$$\sigma_1 = \frac{6 \cdot c \cdot \cos\phi}{(1 - \frac{2 \cdot sen\phi}{(3 - sen\phi)})(3 - sen\phi)} = \frac{2c \cdot \cos\phi}{(1 - sen\phi)} \qquad (3)$$

## 2.2 Specimen with a perfect theoretical confinement under uniaxial compression

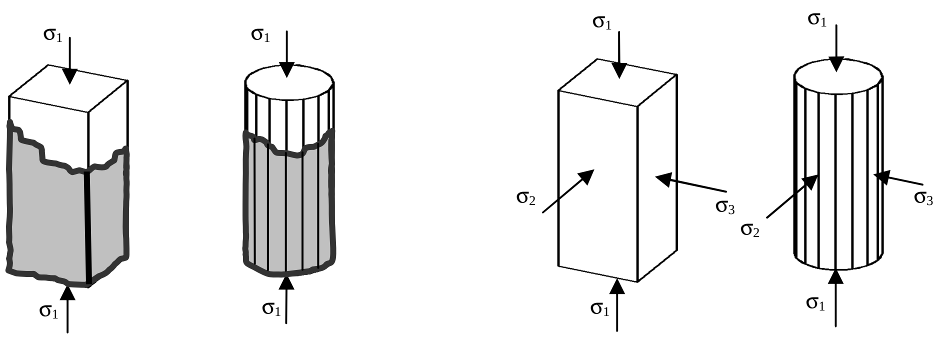



Figure 3: Confined specimen: Triaxial stresses

Admitting that the exterior jacketing does not allow any type of lateral expansion and considering that the yielding point is not reached, the application of the Hooke's law allows concluding that the relationship among the normal stresses for each section of the column are:

$$\left.\begin{aligned} \sigma_1 &= \frac{1-\nu}{2 \cdot \nu} \cdot (\sigma_2 + \sigma_3) \\ \sigma_2 &= \sigma_3 \end{aligned}\right\} \quad \sigma_1 = \frac{1-\nu}{\nu} \cdot \sigma_2 \qquad (4)$$

Figure 4 presents the intersection between the plane presented in equation (4) and the DP cone.

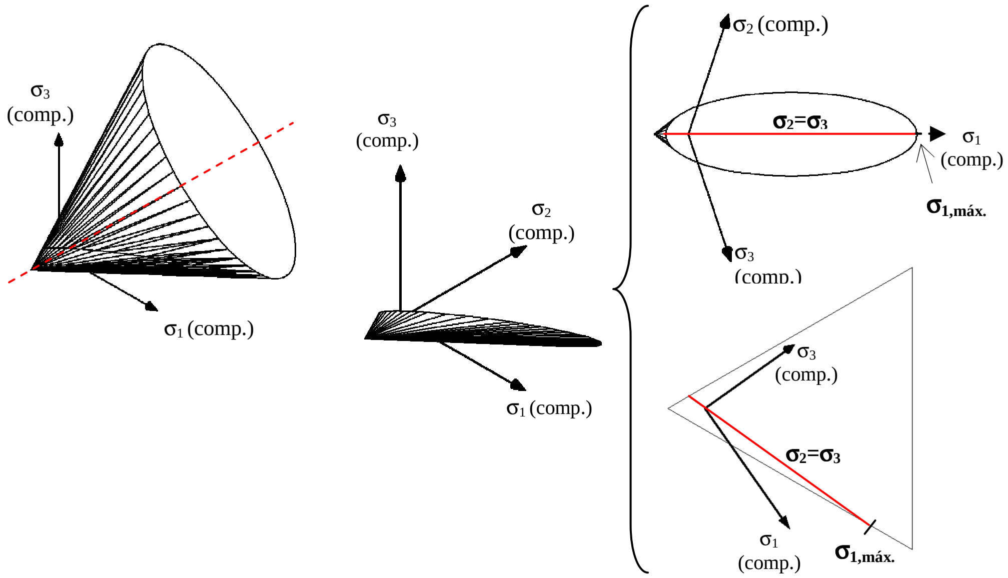



Figure 4: Graphic representation of the DP criterion for a confined specimen.

The maximum admissible compression can be obtained from equation (5):

$$\left(2\cdot\left(\frac{1-2\nu}{1-\nu}\right)-A^2\left(\frac{1+\nu}{1-\nu}\right)\right)\sigma_1^2-2\cdot A^2\cdot B\cdot\left(\frac{1+\nu}{1-\nu}\right)\sigma_1-A^2\,B^2= \tag{5}$$

The solution of this second grade equation will only have as a valid solution the positive solution, because the negative one would correspond to the theoretical tension limit.

The comparison between the maximum theoretical compression strength of a confined pillar with and without confinement, by means of the application of DP for a conventional concrete with an internal friction angle of 30º, a Poisson ratio of 0.2 and a cohesion, is given by the expression $c = f_{co}^{\prime}\cdot\dfrac{(1-\sin\phi)}{2\cdot\cos\phi}$ [12], which is **four** times the limit load por a non confined column. This situation is presented in figure 5.

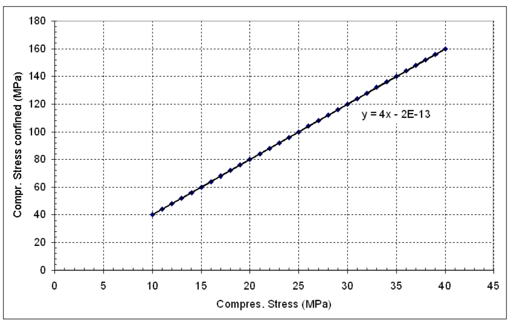



Figure 5: Maximum theoretical compression strength of a perfectly confined column.

# 3 Sensitivity study of Poisson's ratio

This analysis is carried out maintaining constant the cohesion and the internal friction angle. Figure 6 shows the significant influence of the value of the Poisson ratio in the maximum admissible theoretical compression for a column with ideal jacketing. For a value of 0.25 the capacity of the column could be infinite.

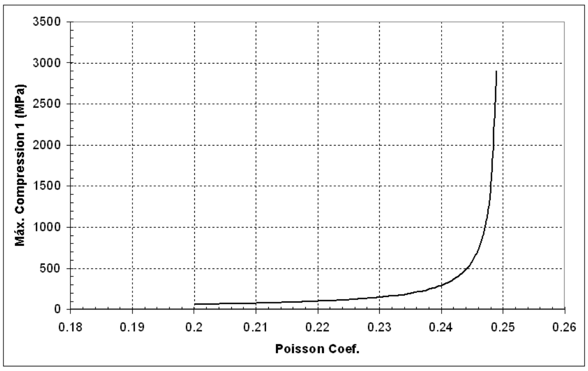



Figure 6: Poisson ratio influence for the maximum theoretical resistance of a confined column under monotonic forces.

Figure 7 graphically analyzes this limit of 0.25. For a material with a Poisson ratio of 0.25, it is observed that the intersection of the DP surface with the plane originated by eq. 4 generates a parabola -plane parallel to a generatrix of the cone-. From a theoretical point of view the material would never reach the yielding point.

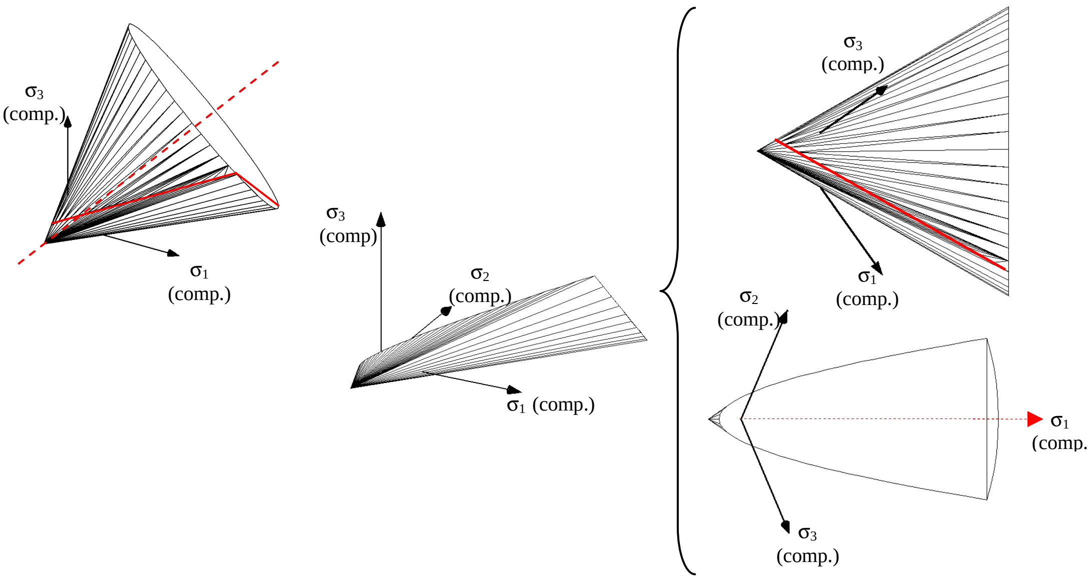



Figure 7: Graphic representation of the DP criterion for a confined specimen with a Poisson ratio of 0.25

# 4 Sensitivity study for the internal friction angle.

This study is carried out with the purpose of evaluating the influence of the internal friction angle in the increase of the strength capacity of the specimen perfectly confined. The figure 8 shows the important influence of this parameter, since for a Poisson's ratio equal to 0.25 it is observed that it can increase up to 5 times the strength capacity of the concrete specimen, diminishing this increment as the internal friction angle decreases.

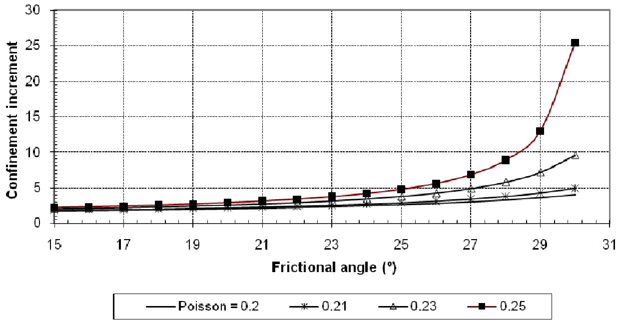



Figure 8: Frictional angle influence for the maximum theoretical resistance of a confined column under monotonic forces.

# 5 Specimen under uniaxial stress state with non-ideal confinement.

Assuming membrane behaviour for the confinement, and elasticity for both materials (concrete and confinement), we have (fig. 9):

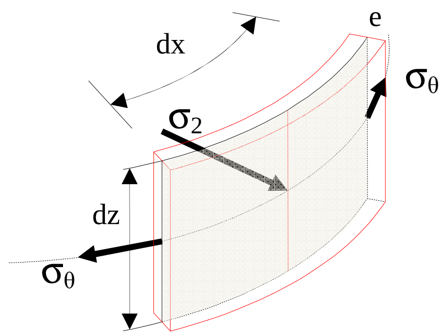



Figure 9: Theoretical scheme to evaluate confinement strength

Equilibrium: $N_\theta = -R\sigma_2$ (6)

Compatibility: $\varepsilon_\theta = \varepsilon_2 = \varepsilon_3$ (7)

Elastic behaviour: $\varepsilon_\theta = \dfrac{N_\theta}{tE_R}$ (8)

$$\varepsilon_2 = \varepsilon_3 = [\sigma_2 - \nu_C(\sigma_3 + \sigma_1)] / E_C = [(1-\nu_C)\sigma_2 - \nu_C\sigma_1] / E_C \quad (9)$$

where: $t$ is the thickness of the strengthening, $E_R$ is the elastic modulus of the strengthening, $E_C$ is the elastic modulus of the concrete and $\nu_C$ is the Poisson ratio of the concrete.
Replacing (6) in (8), (8) and (9) in (6):

$$-\frac{R\sigma_2}{tE_R} = [(1-\nu_C)\sigma_2 - \nu_C\sigma_1]/E_C$$

and radial stress $\sigma_2$ can be explicited as:

$$\sigma_2 = \frac{\nu_C\sigma_1}{1-\nu_C+\frac{R}{t}\frac{E_C}{E_R}} = \alpha\sigma_1 \qquad (10)$$

that returns to (4) when $E_R/E_C \to \infty$

Replacing (10) in (2) we have for yield point:

$$2\sigma_1^2(1-\alpha)^2 - A^2[\sigma_1(1+2\alpha)+B]^2 = 0$$
$$\sqrt{2}\sigma_1(1-\alpha) = \pm A[\sigma_1(1+2\alpha)+B]$$

Negative value lies in the lower generatrix ($\sigma_1$ in tension) of the DP cone; then, for compression, at the yielding point for the concrete:

$$\sigma_{1y} = \frac{AB}{\sqrt{2}-A-\alpha(\sqrt{2}+2A)} \qquad (11)$$

Recalling (10) for $\alpha$ and substituting A and B for the tested concrete, we can graph (fig 10)

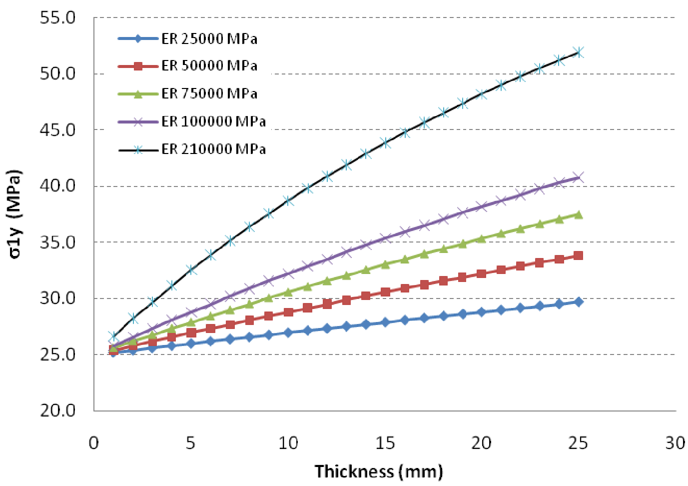



Figure 10 $\sigma_{1y}$ yielding values versus thickness $t$ (mm) and elastic modulus $E_R$ (N/mm$^2$) for the strengthening (R=75 mm, HA-25, $\phi$=30º, $\nu_c$=0.2, c=7.22 MPa)

In the frequent case of strengthening using FRP with of $E_C \approx E_R$ *and* $R >> t$, (10) simplifies to an approximate value:

$$\sigma_2 = \sigma_1 \nu_C \frac{t}{R}\frac{E_R}{E_C} = \alpha' \sigma_1$$

This approximation gives (eq. 11) $\sigma_{1y} = 25.20\ MPa$ and (eq. 10): $\sigma_2 = 0.0027 \cdot \sigma_{1y}$

The value for $\sigma_{1y}$ is somewhat higher than the one corresponding to the specimen without strengthening, but lower than the test results for the strengthened one. The value for $\sigma_2$ is insignificant. This result leads to think that concrete is plastically loaded significantly beyond the yielding point, until the strengthening fails. Theoretical evaluation of the corresponding $\sigma_1$ value requires a more complicated fully plastic analysis that will not be included here.

# 6 Experimental results and discussion

With the purpose of carrying out an experimental validation of theoretical studies presented for confined concrete with CFRP, cylindrical test specimens of ∅150 mm and 300 mm high have been made. The compression strength at 28 days has been evaluated, obtaining a value of 40 MPa. The same concrete test specimens have been subjected to a preload of 65% and 85% of the maximum compression load, maintaining constant this load during 24 h. After this preload a CFRP jacketing strengthening was introduced and finally each specimen was broken.

CFRP (fig 11a) is compound of uniaxial carbon fiber and an epoxy resin like matrix. The CF has an elastic modulus of 234 GPa and a thickness of 0.131 mm. The resulting 1 mm thick CFRP has an elastic modulus of 25 GPa, that verifies the mixtures rule.


(a) (b)

Figure 11: (a) CFRP confinement preparation. (b) Specimen prepared to be preloaded at 85% of maximum load capacity. (c) Specimen broken with a preload of 65% of the maximum load capacity.

First test results are shown in figure 12. All analyzed specimens present values of ultimate compression capacity higher than the values for the unstrengthened specimen. When the specimen has an initial preload, concrete creep is observed and all preloaded specimens have a ultimate compression capacity higher than the corresponding to non preloaded specimens.

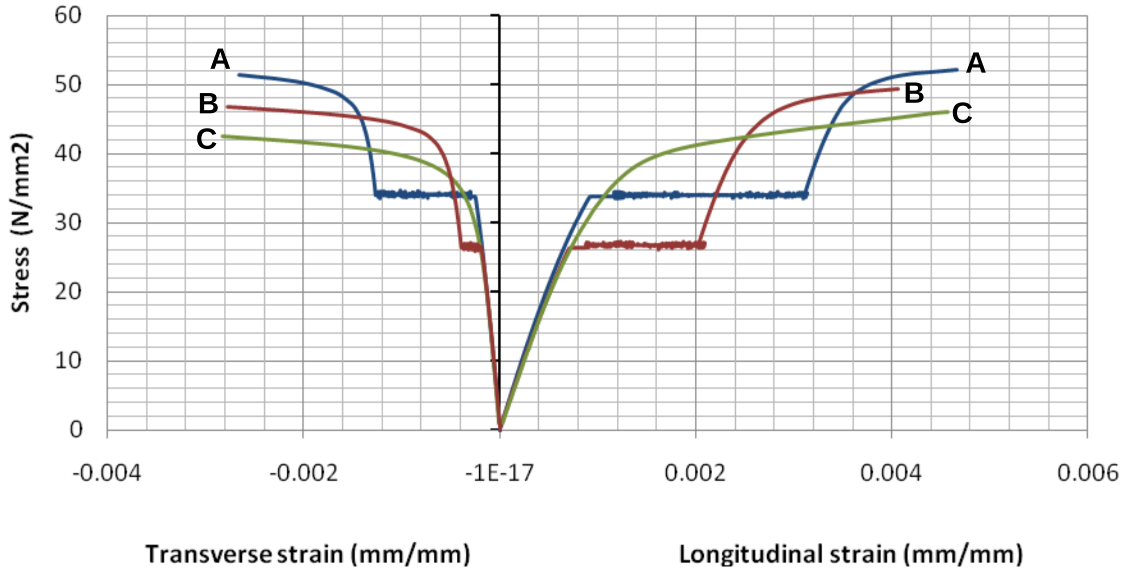



Figure 12: Stress-strain diagram, specimen reinforced with one layer of FRP. (A) No preload. (B) Preloaded at 65% of maximum load (26 MPa). (C) Preloaded at 65% of maximum load (34 MPa)

The observed increment in all the specimens of the CFRP strengthening 1 mm thick –fig. 12- is higher than 10% of the strength capacity of the initial concrete specimen. This increment disagree with the elastic analyses outlined in previous sections where only an increment around 1% can be justified.

The theoretical evaluation of this 10% increment requires a more complicated fully plastic analysis that is not included in this paper.

It can be concluded that to get this increment in the strength capacity of the concrete, it should be completely yielded, justifying the final strength of the specimen by the final strength capacity of CFRP, since the specimen presents an explosive failure.

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