

# Numerical Determination of Distillation Boundaries for Multicomponent Homogeneous and Heterogeneous Azeotropic Systems

**Juan A. Reyes-Labarta\***, J.A. Caballero, A. Marcilla  
([ja.reyes@ua.es](mailto:ja.reyes@ua.es))

Assistant Professor  
Chemical Engineering Department  
Institute of Chemical Process Engineering  
University of Alicante. Spain  
(<http://iq.ua.es/gcef.htm>)



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## outlines

- Introduction

- ✓ Relevance of the topological analysis:

- ✓ Topology of the Gibbs Energy Function (LLE, LLSE, LLS<sub>h</sub>E)
- ✓ Topology of azeotropic LV Equilibrium Surfaces

- Distillation Boundary Condition

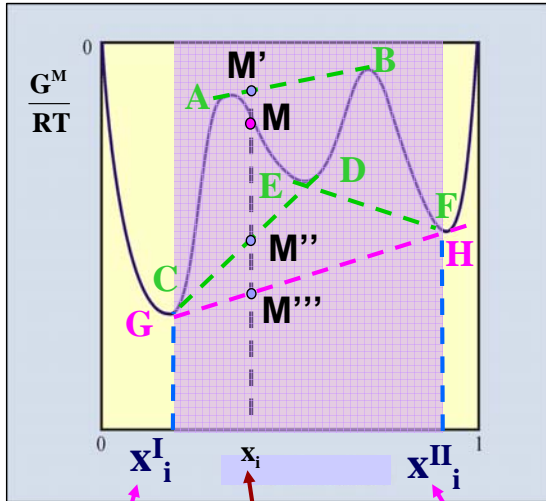
- Mathematical algorithm

- Numerical examples

- Conclusions



1) Possibility of different false solutions

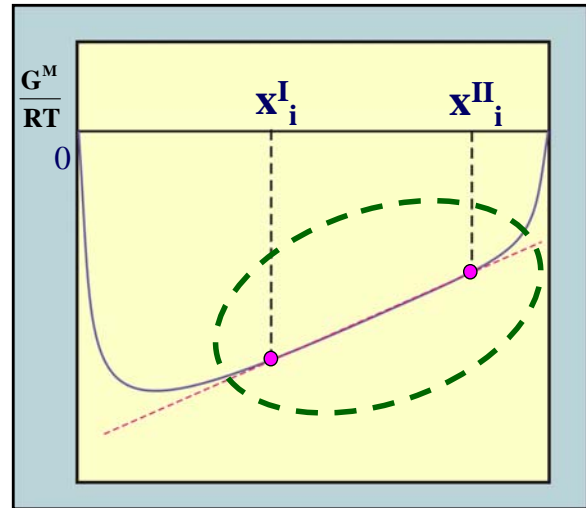


Liquid phases in equilibrium

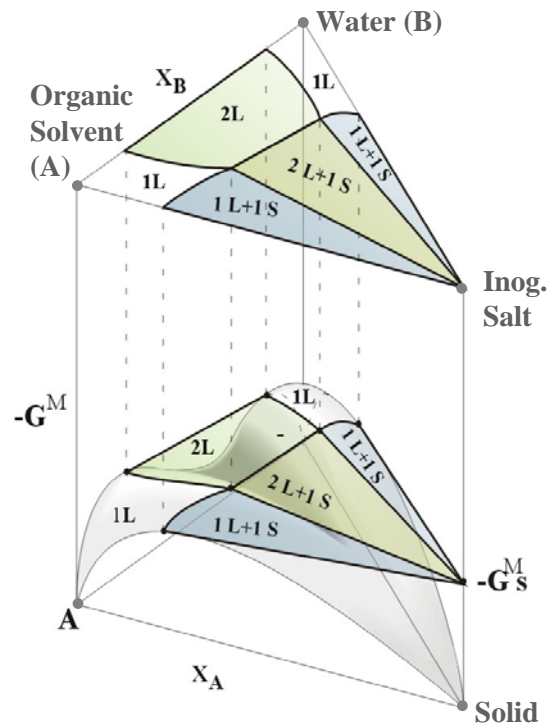
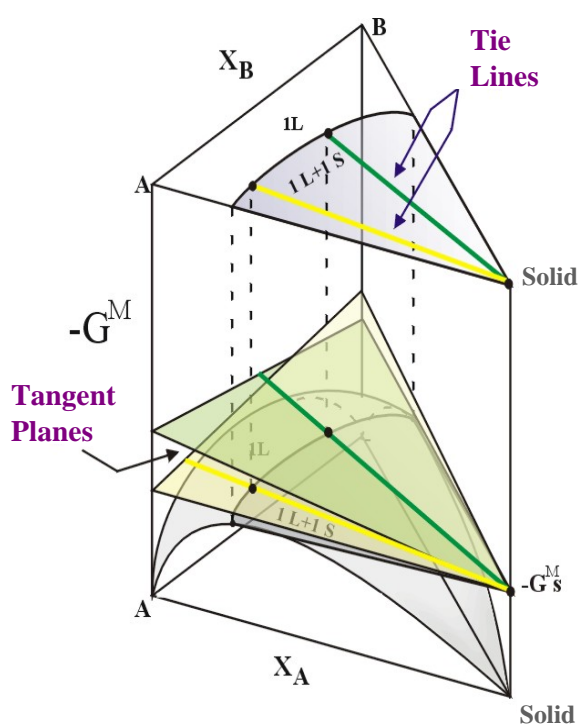
Global Mixture

TYPICAL PROBLEMS!!

2) Uncertainty in the final solution

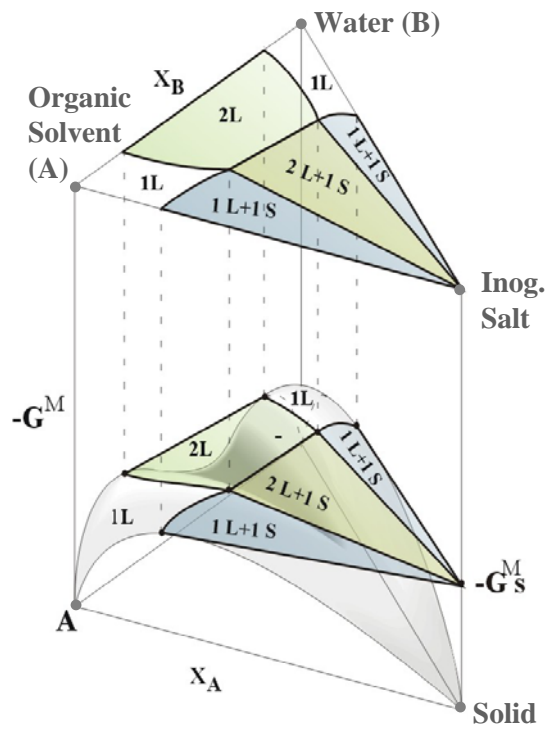
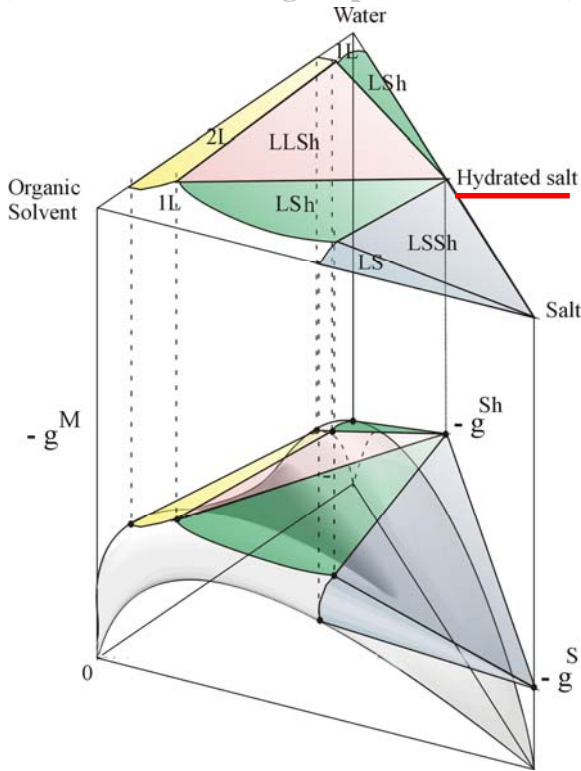


(minor common tangent plane criterion)

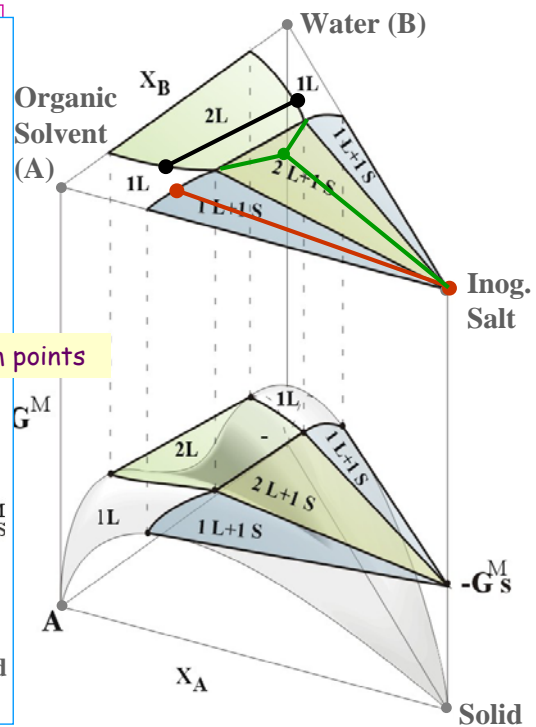
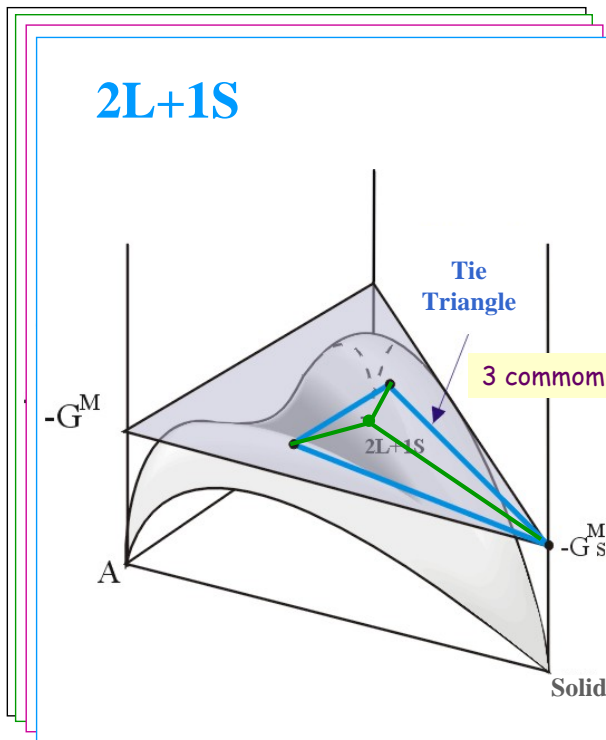




(minor common tangent plane criterion)



(minor common tangent plane criterion)





NEW STRATEGIES

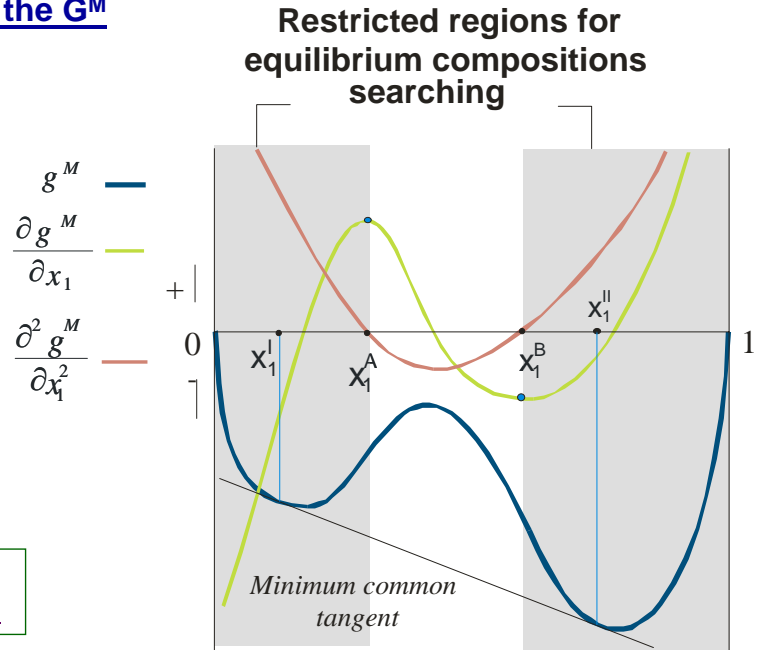
◆ Limit the equilibrium composition space for the LLE root determination

a) Using the second derivative of the  $G^M$

Advantages:

- ✓ Less time consuming
- ✓ Trivial solution is avoided

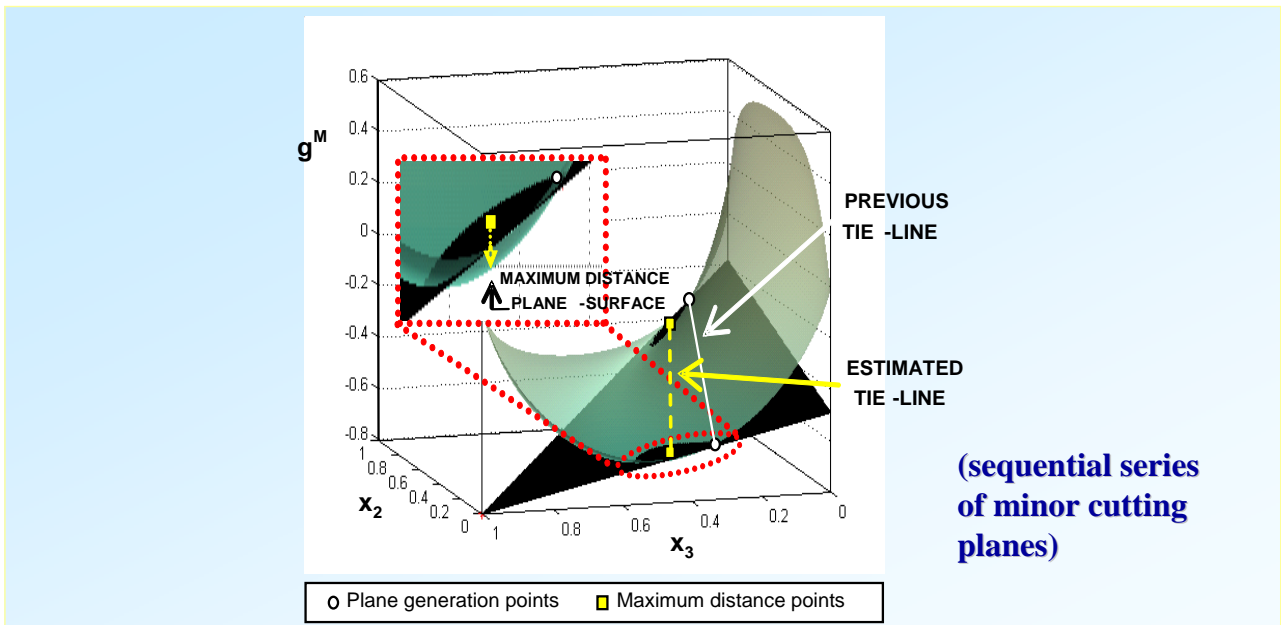
Marcilla et al. Fluid Phase Equilibria (2010)  
<http://dx.doi.org/10.1016/j.fluid.2009.12.026>



NEW STRATEGIES

◆ Limit the equilibrium composition space for the LLE root determination

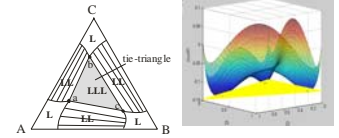
b) Using a geometrical approach → very good approximation to the ELL solution



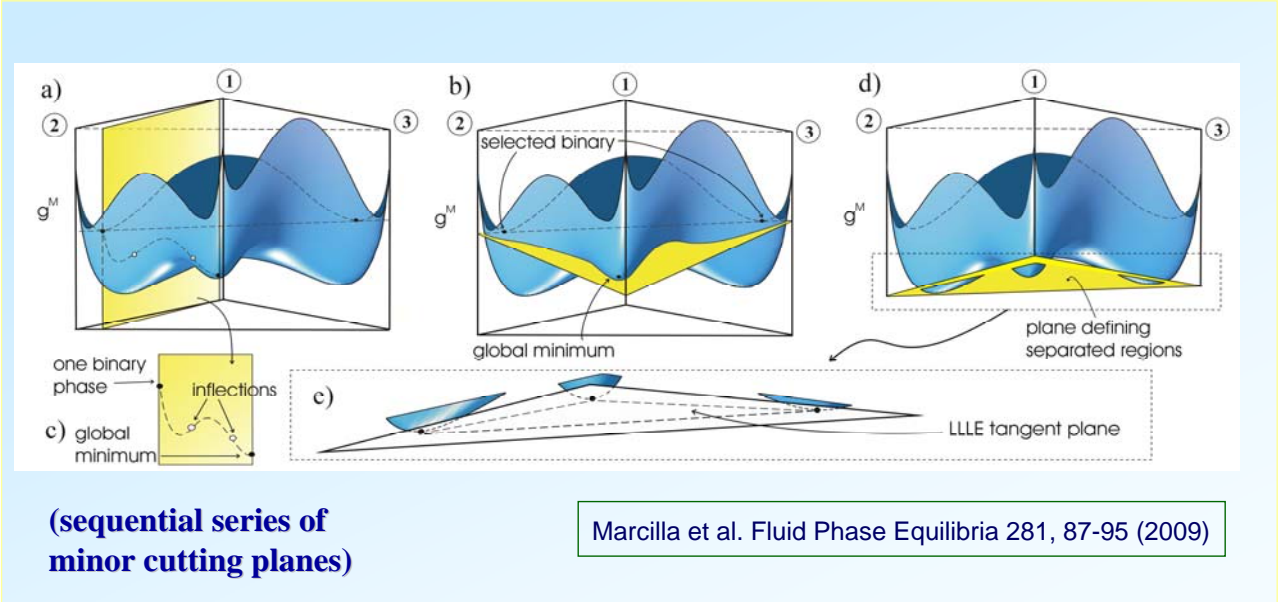


NEW STRATEGIES

◆ Limited composition space for the LLE root determination



b) Using a geometrical approach → very good approximation to the ELLL solution



NEW STRATEGIES

◆ Mathematical constraints for binary parameters

Border line between L and LL regions for the NRTL model

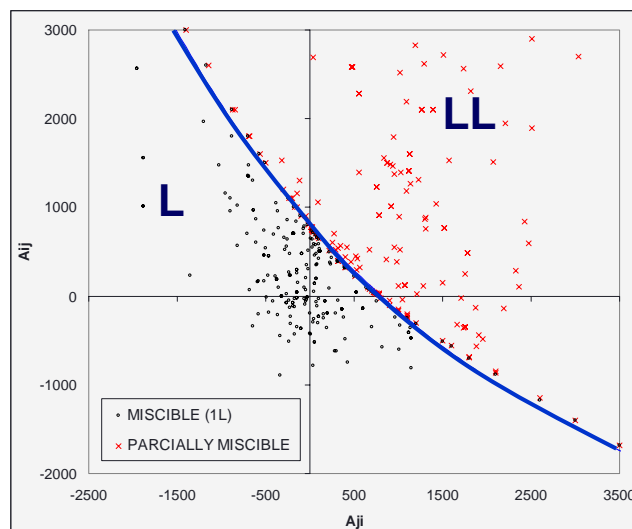
$$A_{ij} = f(A_{ji}) = -4.46564 \cdot 10^{-8} A_{ji}^3 + 2.95745 \cdot 10^{-4} A_{ji}^2 + 1.20662 A_{ji} + 766.908$$

Homogeneous (L)  $A_{ij} < f(A_{ji})$

Heterogeneous (LL)  $A_{ij} > f(A_{ji})$

Type island ternary systems:

- A12+ A21 < 0 (dissimilar binary pair)
- A13+ A31 > 0
- A23+ A32 > 0





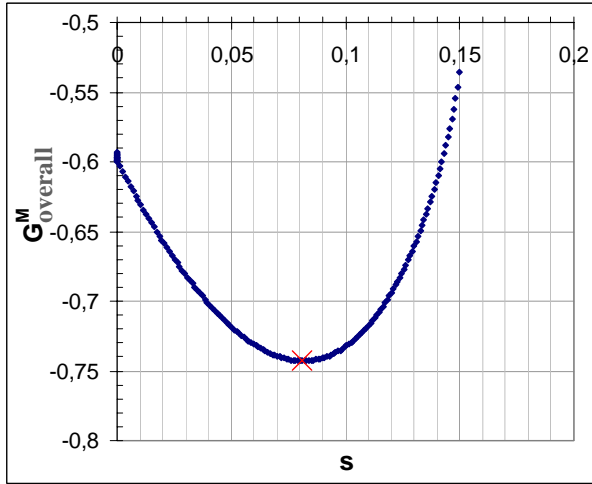
NEW STRATEGIES

◆ Different objective functions

a) Minimum of the overall Gibbs Energy of mixing

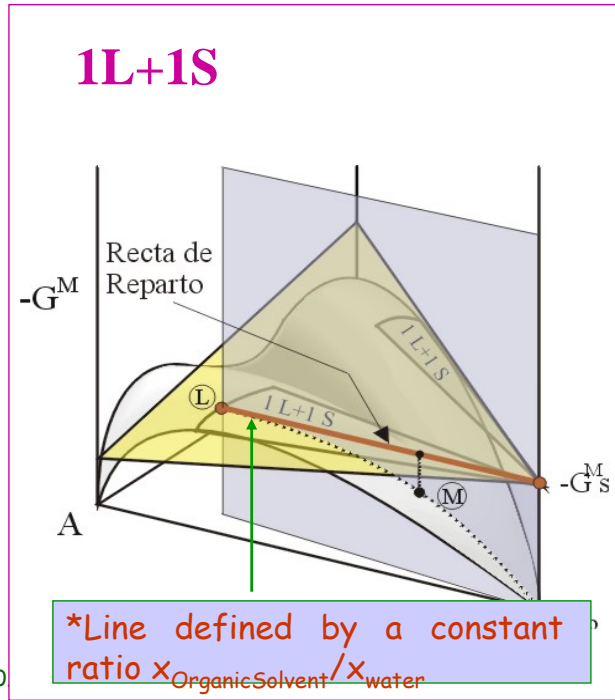
$$\frac{G_{\text{overall}}^M}{RT} = \frac{G^S}{RT} + \frac{G_{\text{liquid}}^M}{RT} = s \cdot \frac{\Delta\mu^S}{RT} + (1-s) \cdot \sum_{i=1}^c x_i^L \cdot \frac{\Delta\mu_i^L}{RT}$$

Calculation of the Minimum of the overall Gibbs Energy of mixing, along a concrete line\* for each experimental point.



Reyes et al. IEC&R 40,902-907 (2001)

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NEW STRATEGIES

◆ Different objective functions

a) Minimum of the overall Gibbs Energy of mixing

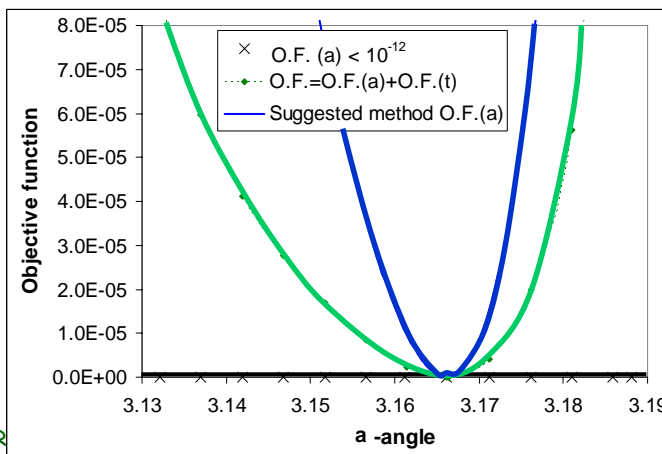
$$\frac{G_{\text{overall}}^M}{RT} = \frac{G^S}{RT} + \frac{G_{\text{liquid}}^M}{RT} = s \cdot \frac{\Delta\mu^S}{RT} + (1-s) \cdot \sum_{i=1}^c x_i^L \cdot \frac{\Delta\mu_i^L}{RT}$$

b) Isoactivity criterium  $O.F.(a) = \sum_{i=1}^3 (a_i^I - a_i^{II})^2 = 0$

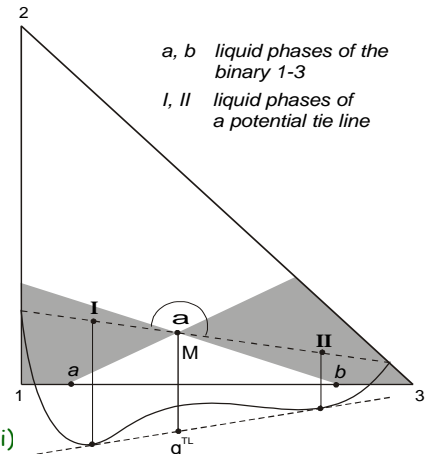
c) Isoactivity + Minor common tangent condition (Iglesias Silva et al., 2003)

d) A modification of the initial vector method of Eubank et al., 1992 (Olaya et al., 2007)

Olaya et al. Chem.Eng.Education 41(3), 218-224 (2007)



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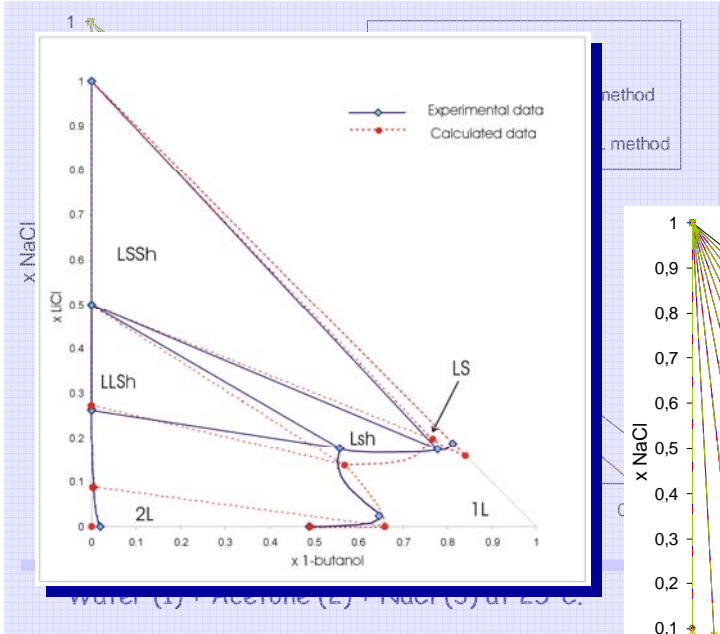


12



## NEW STRATEGIE RESULTS:

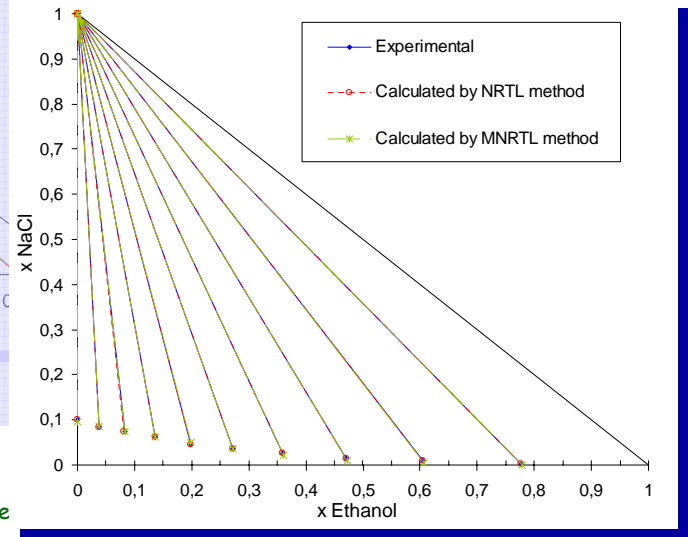
- **SIMULTANEOUS CORRELATION (including all equilibrium regions) OF LLS SYSTEMS (NRTL and E-NRTL)**



Olaya et al. IEC&R 46, 7030-7037 (2007)

Marcilla et al. IEC&R 47, 2100-2108 (2008)

- **Water(1) + Ethanol (2) + NaCl (3) at 25°C.**



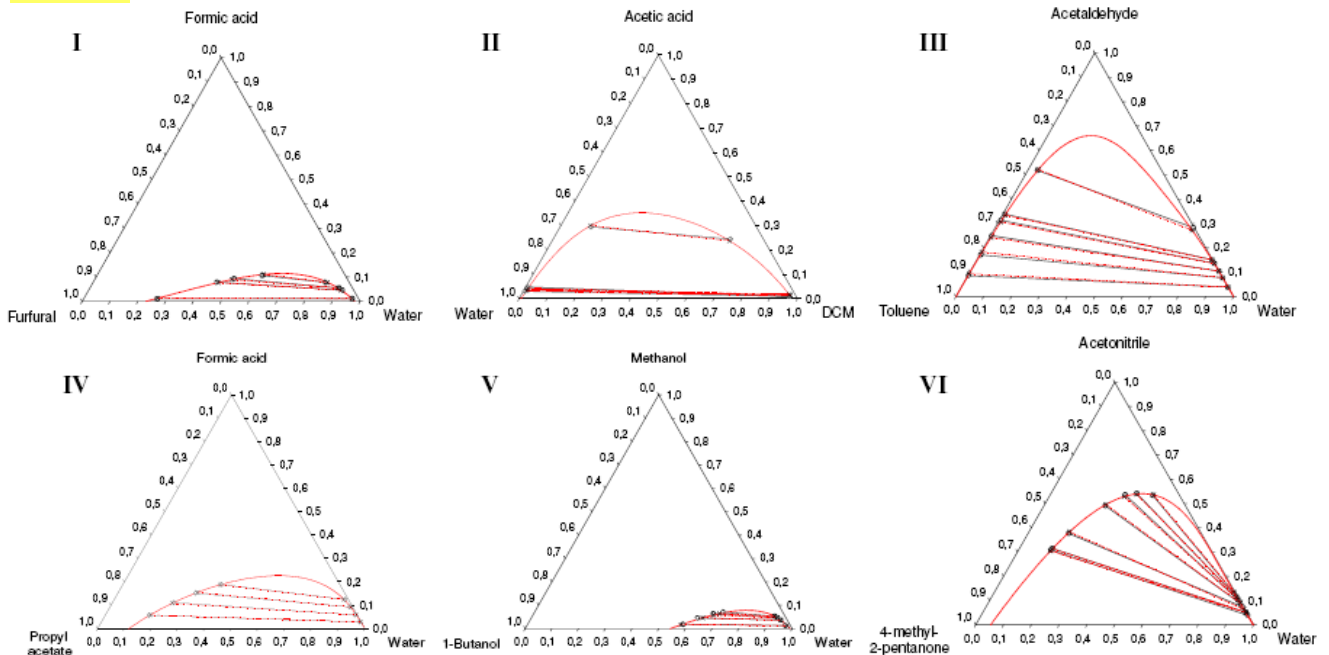
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## NEW STRATEGIE RESULTS:

- **CORRELATION OF (uncorrelated) COMPLEX LL SYSTEMS (NRTL)**

### TYPE I



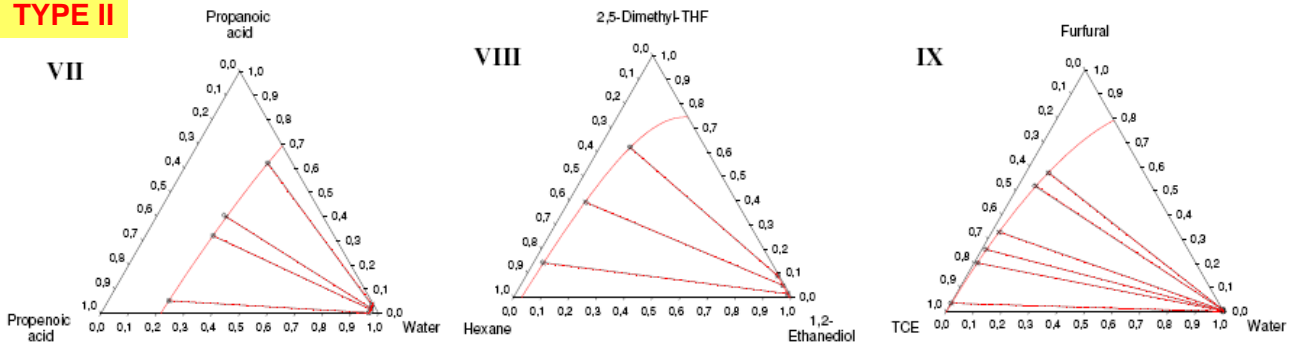
Reyes-Labarta et al. Fluid Phase Equilibria 278, 9-14 (2009)



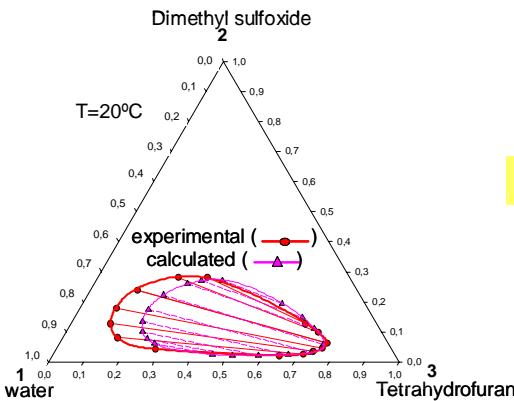
## NEW STRATEGIE RESULTS:

### •CORRELATION OF(uncorrelated) COMPLEX LL SYSTEMS (NRTL)

#### TYPE II

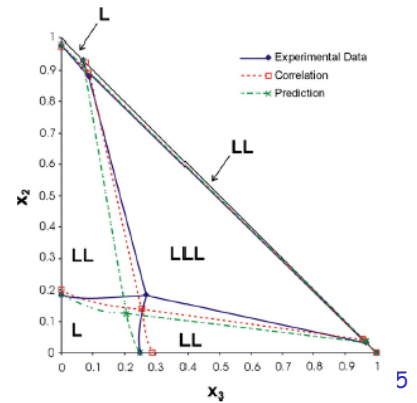


#### TYPE 0



Olaya et al.  
Fluid Phase  
Equilibria  
265, 184-191  
(2008)

#### TYPE III

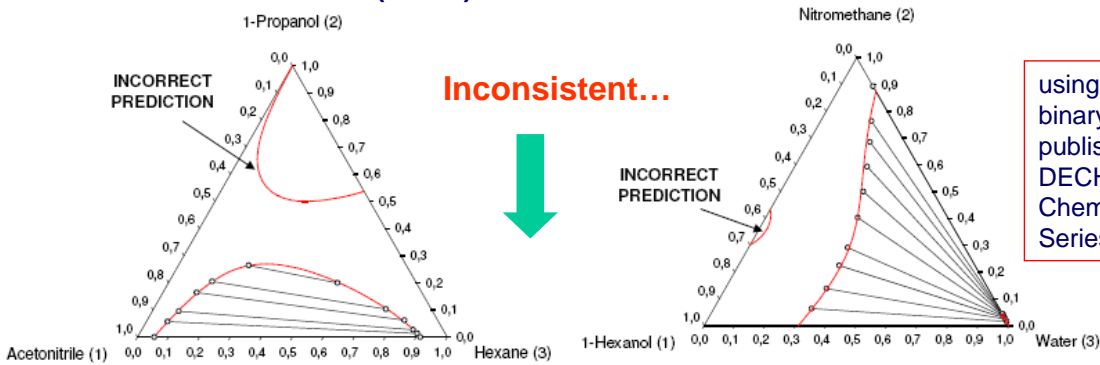


Marcilla et al.  
Fluid Phase  
Equilibria  
281, 87-95  
(2009)



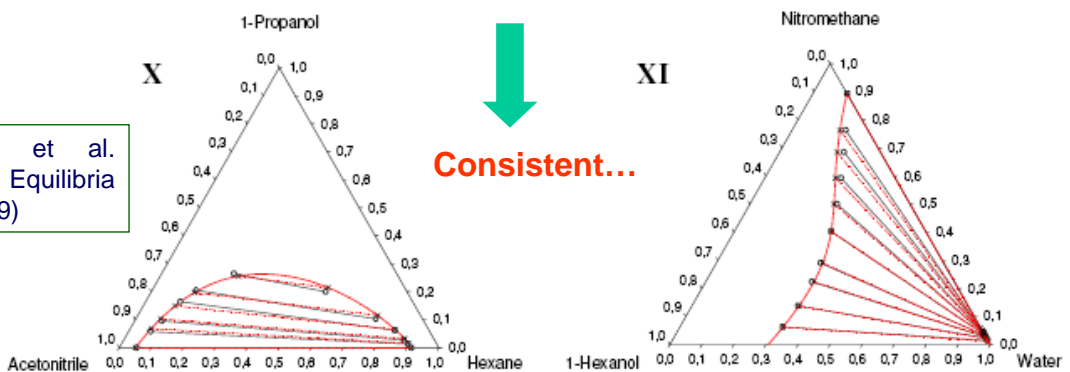
## NEW STRATEGIE RESULTS:

### • CORRECTION OF SOME (NRTL) INCONSISTENT IN LL (type I-II) SYSTEMS



using the NRTL  
binary parameters  
published in the  
DECHEMA  
Chemistry Data  
Series

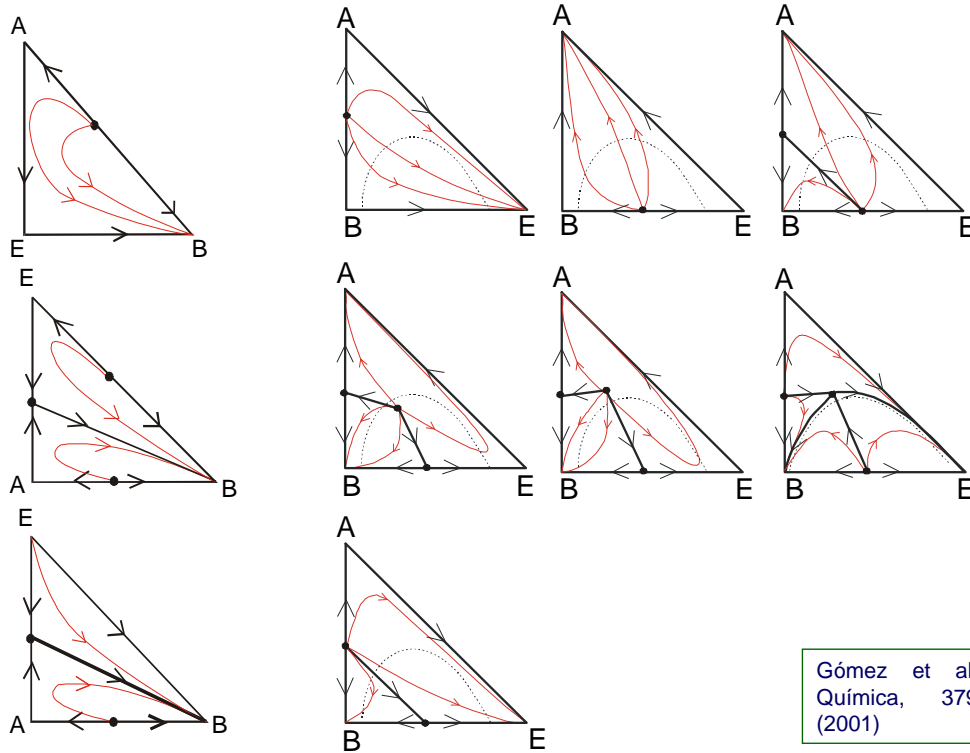
Reyes-Labarta et al.  
Fluid Phase Equilibria  
278, 9-14 (2009)







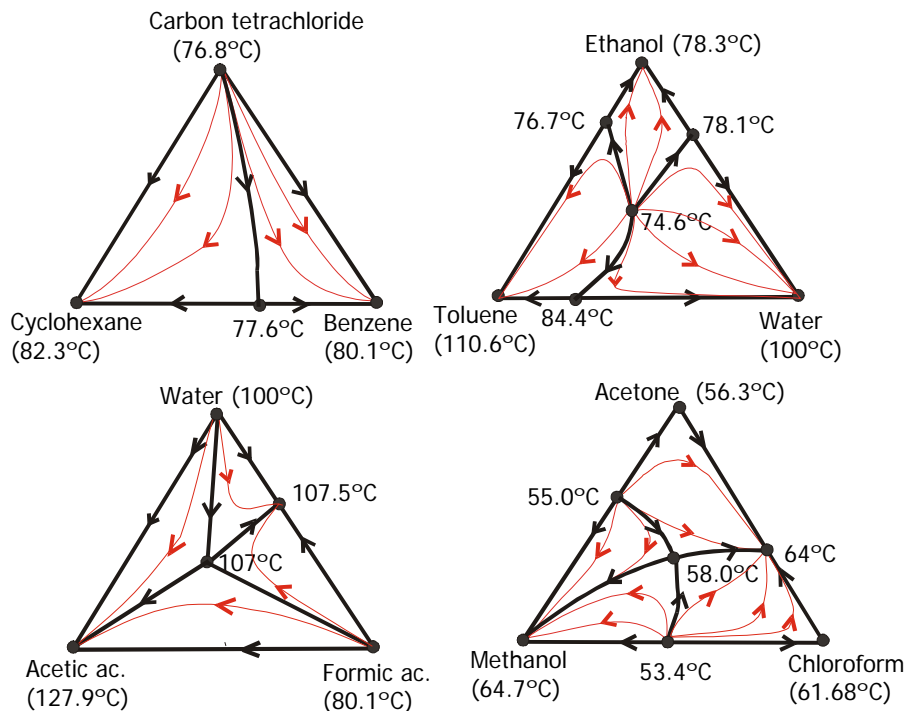
## • LV Equilibria (P = cte). Homogeneous Ternary Azeotropic Systems



Gómez et al. Ingeniería Química, 379, 253-262 (2001)



## • LV Equilibria (P = cte). Homogeneous Ternary Azeotropic Systems

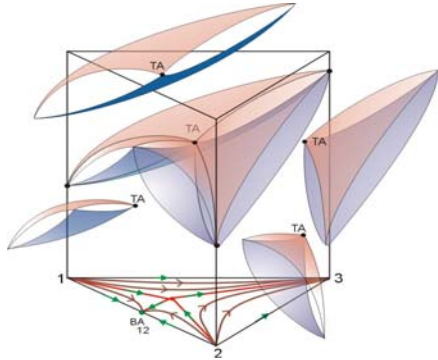


Gómez et al. Ingeniería Química, 379, 253-262 (2001)

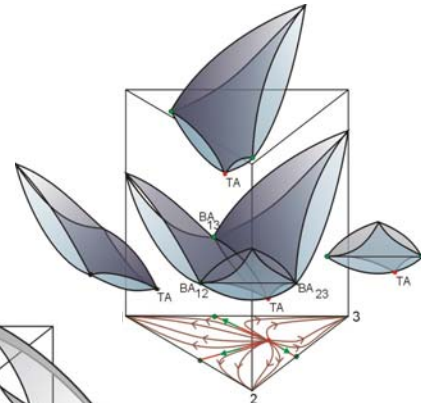


• LV Equilibria (T vs. x,y at P = cte). Homogeneous Ternary Azeotropic Systems

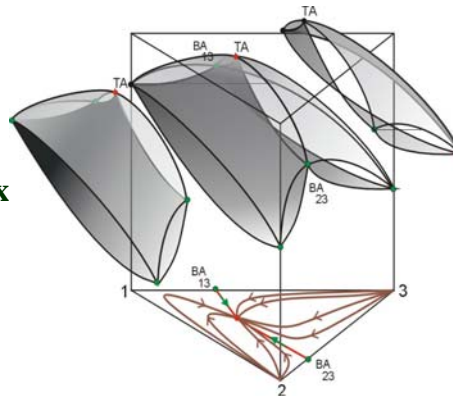
1 BAMax + 1 TAsaddle



3 BAmin + 1 TAmin



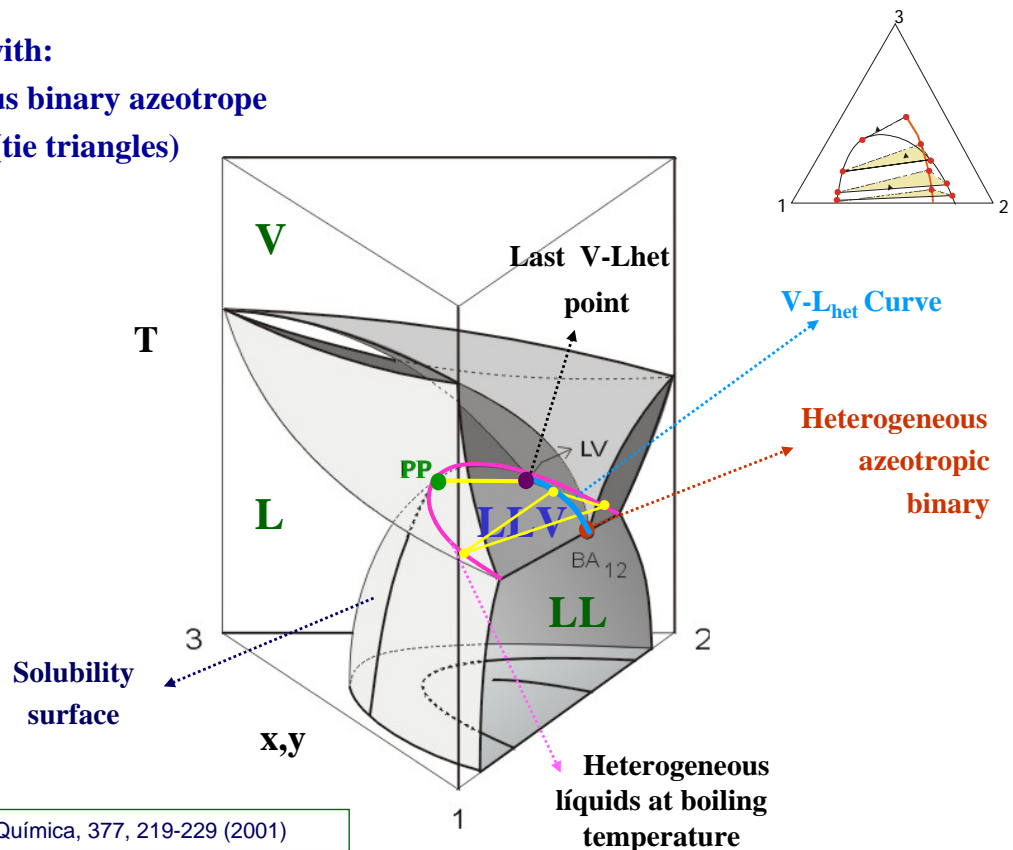
2 BMax + 1 TAMax



Gómez et al. Ingeniería Química, 377, 219-229 (2001)



Ternary system with:  
1 heterogeneous binary azeotrope  
1 LLV region (tie triangles)

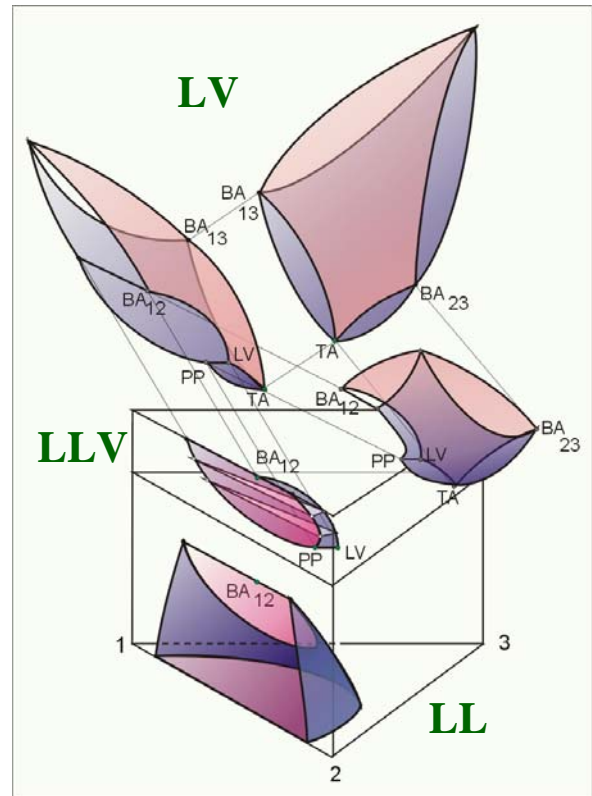
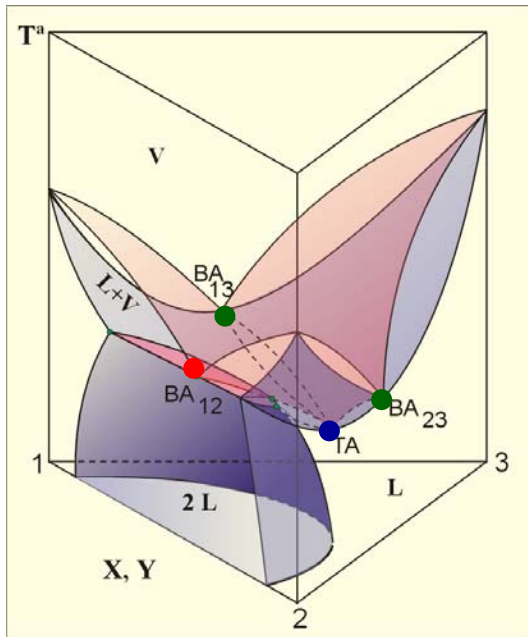


Gómez et al. Ingeniería Química, 377, 219-229 (2001)



Ternary system with:

- 1 heterogeneous binary azeotrope
- 2 homogeneous binary azeotrope
- 1 homogeneous ternary azeotrope

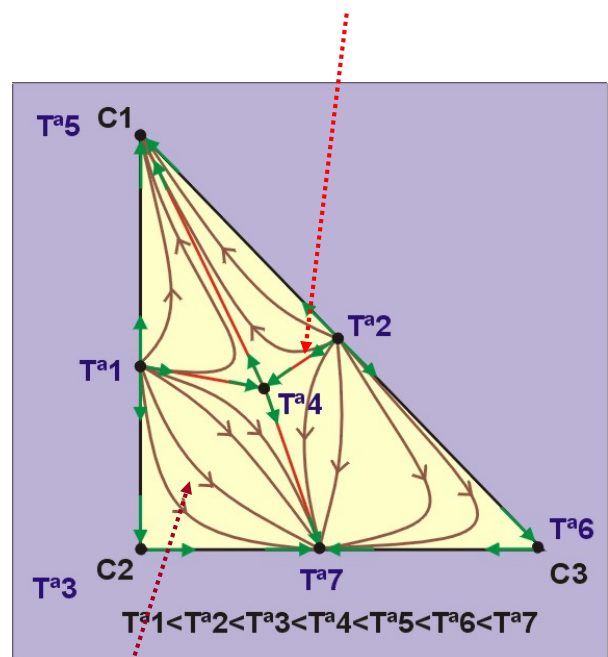
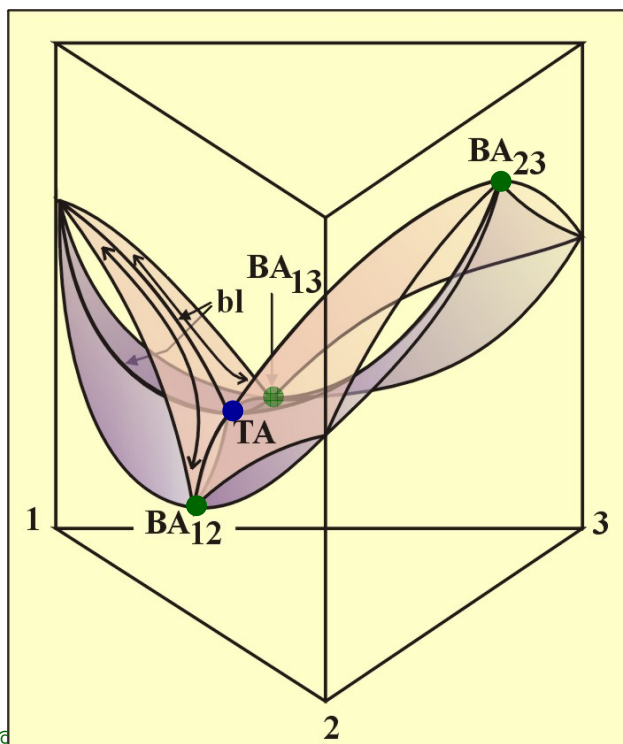


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Ternary system with:

- 3 azeotropic binary compositions (2 min, 1 max) **Distillation Boundary**
- 1 azeotropic ternary composition

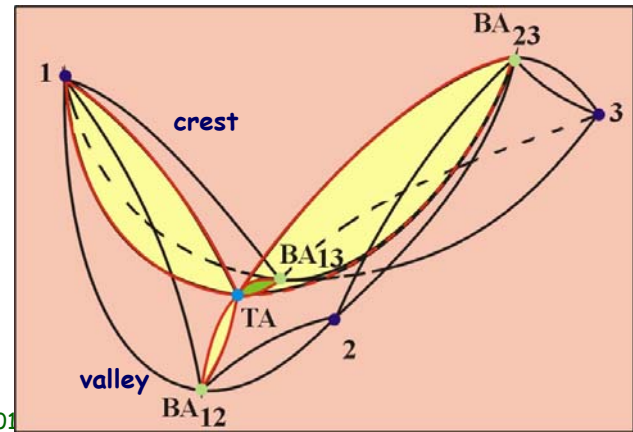
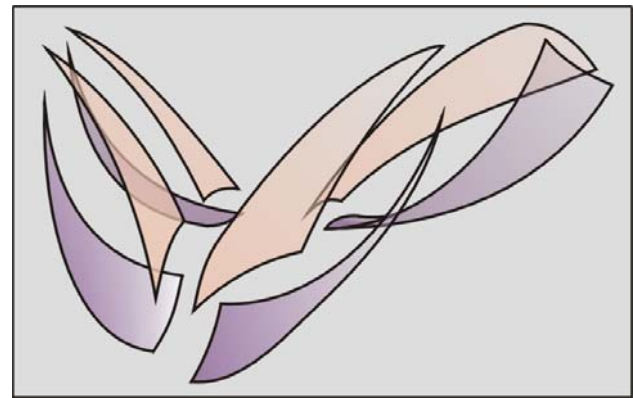
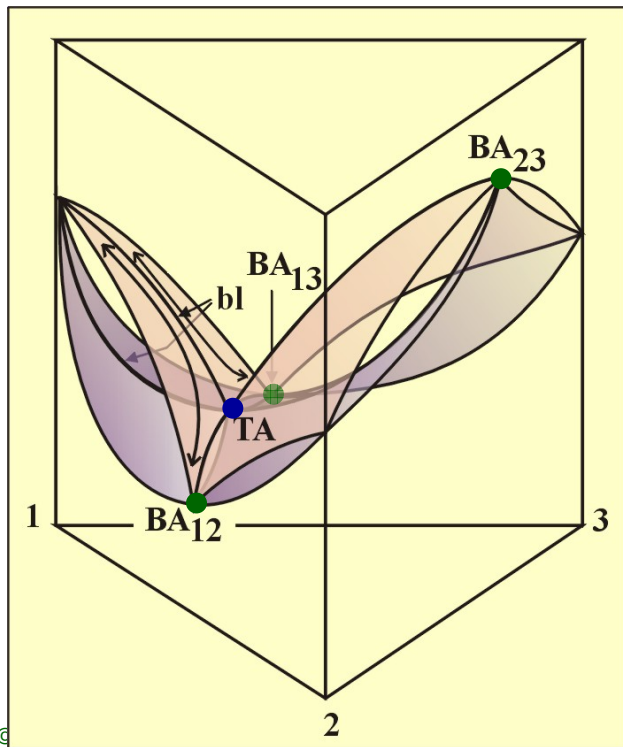


Trajectory distillation

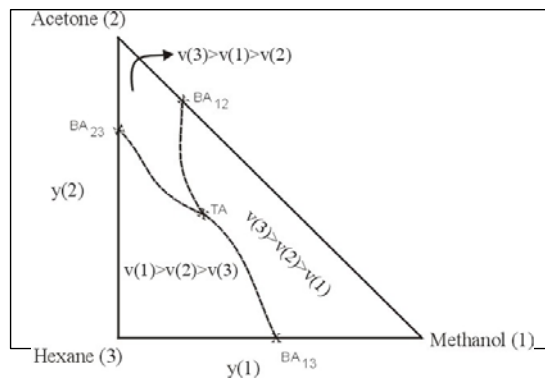
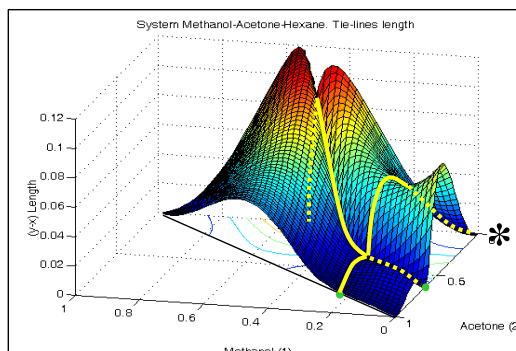
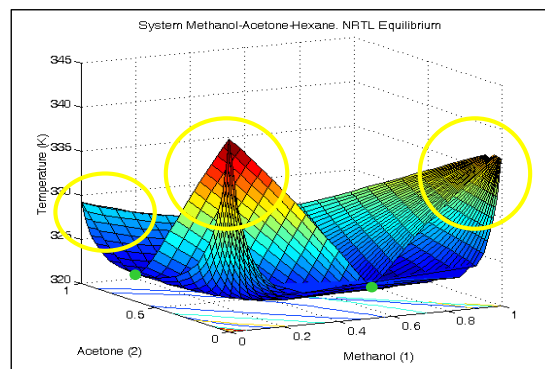
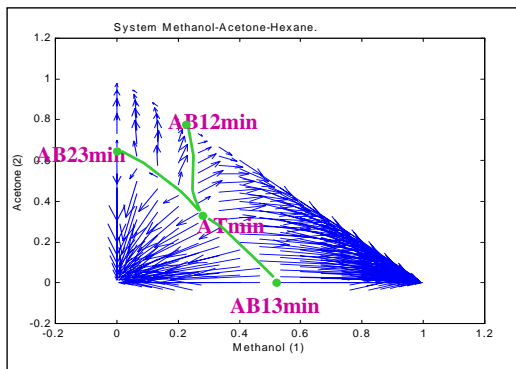


Ternary system with:

- 3 azeotropic binary compositions
- 1 azeotropic ternary composition



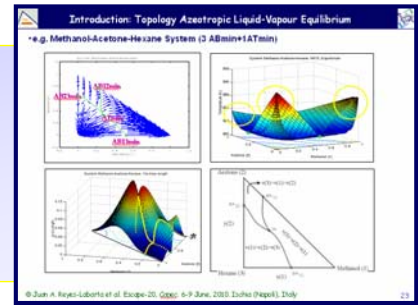
•e.g. Methanol-Acetone-Hexane System (3 ABmin+1ATmin)





## • A Distillation boundary needs:

- A valley or crest in the T vs x,y diagram
- A minimum in the LV tie line size
- Inversion at least of one relative volatility



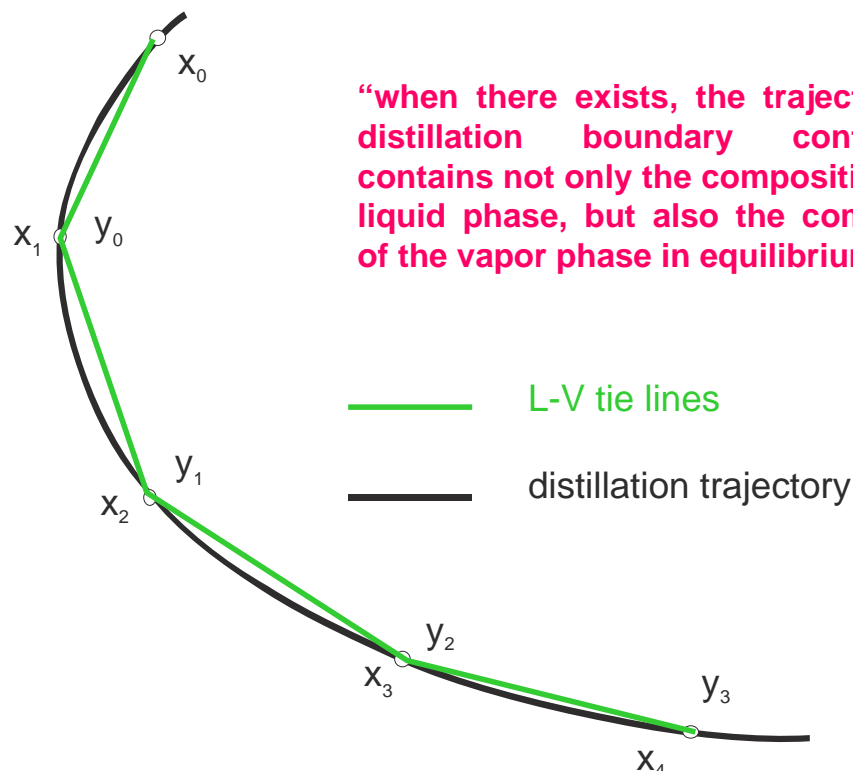
- Any of these conditions is sufficient by itself.
- The simultaneous inversion of two relative volatilities seems to guarantee the existence of a distillation boundary.



## DISTILLATION BOUNDARIES CALCULATION



### • Topological concept used:

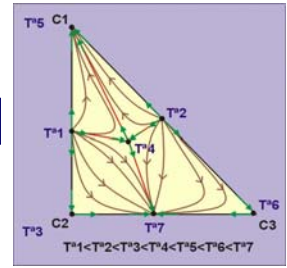
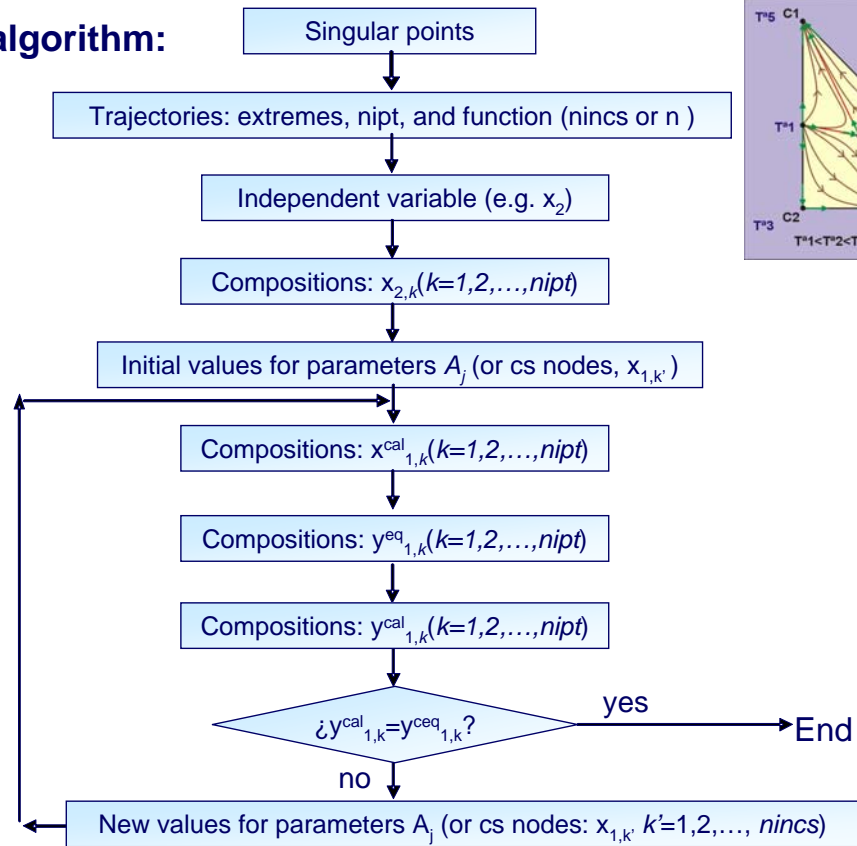




# DISTILLATION BOUNDARIES CALCULATION



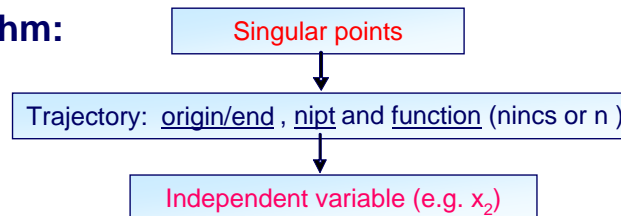
## •Mathematical algorithm:



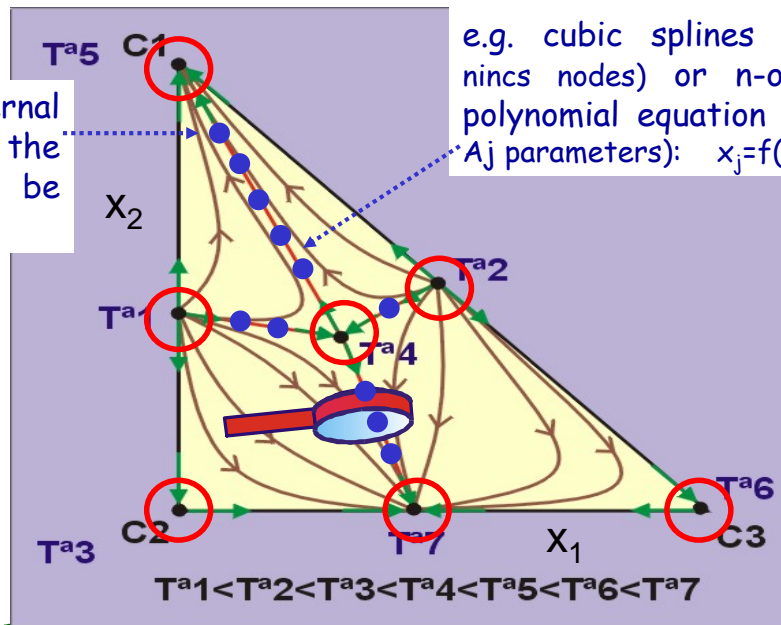
# DISTILLATION BOUNDARIES CALCULATION



## •Mathematical algorithm:



nipt = number of internal points along the trajectory k to be calculated



e.g. cubic splines (with nincs nodes) or n-order polynomial equation (with Aj parameters):  $x_j = f(x_i)$

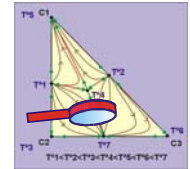
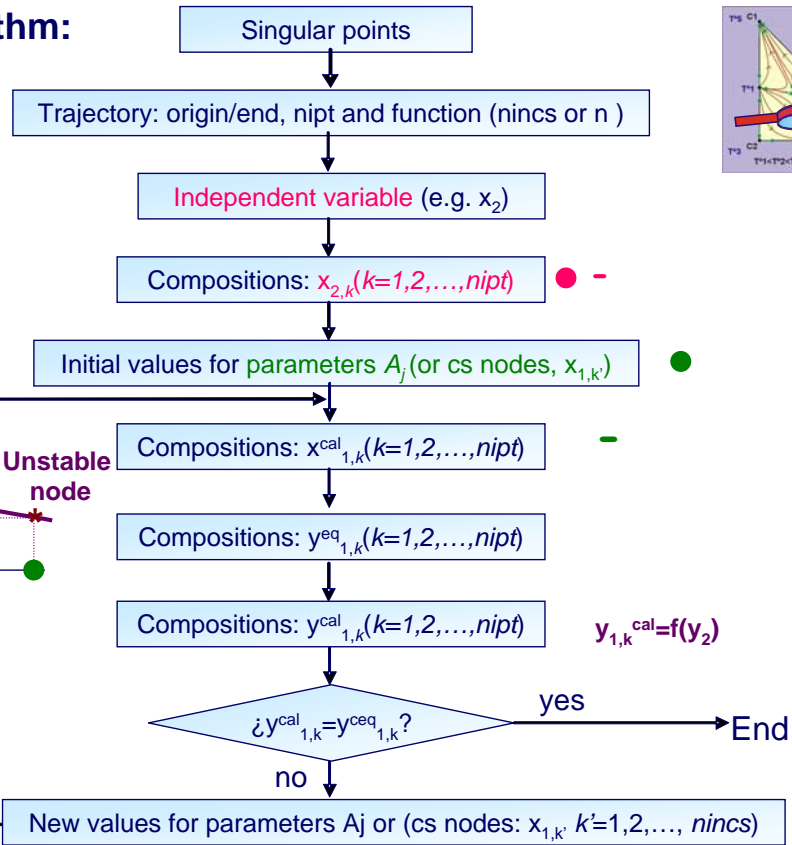
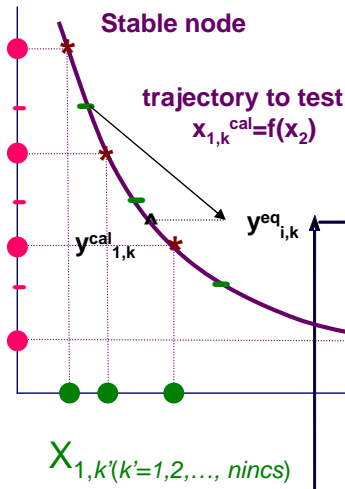


# DISTILLATION BOUNDARIES CALCULATION



## Mathematical algorithm:

$x_{2,k} (k=1,2,\dots,n_{ipt})$

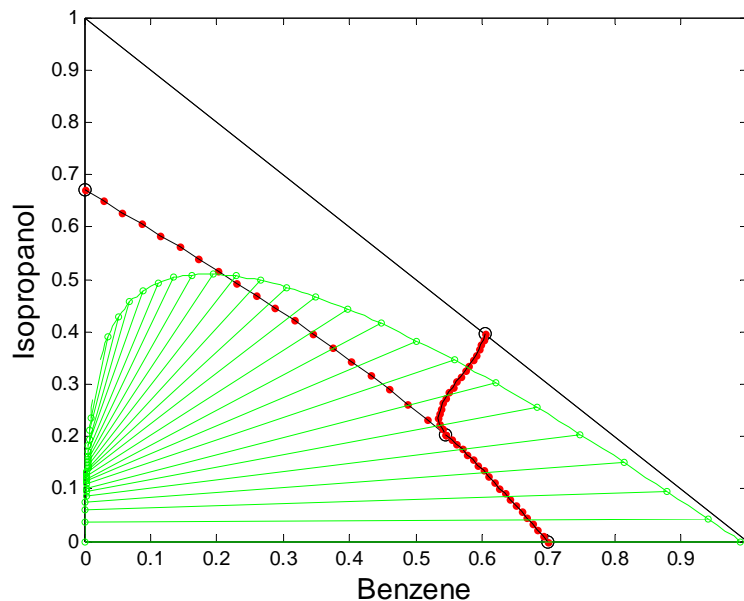


# NUMERICAL EXAMPLES: Ternary Distillation Boundaries (LLV)



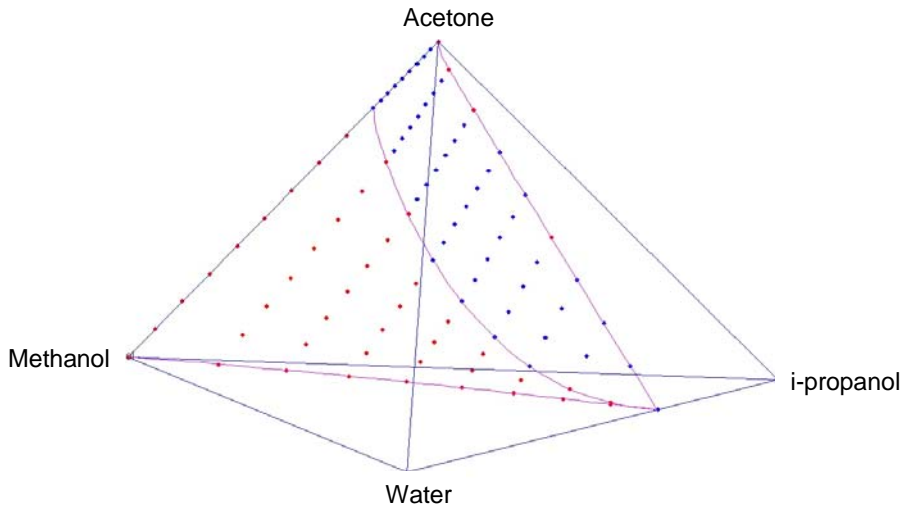
Heterogeneous ternary system with:

- 1 heterogeneous azeotropic binary composition
- 2 homogeneous azeotropic binary compositions
- 1 homogeneous azeotropic ternary composition





Homogeneous quaternary system with:  
2 homogeneous azeotropic binary compositions



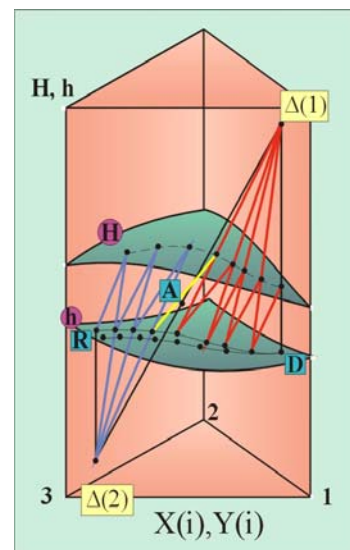
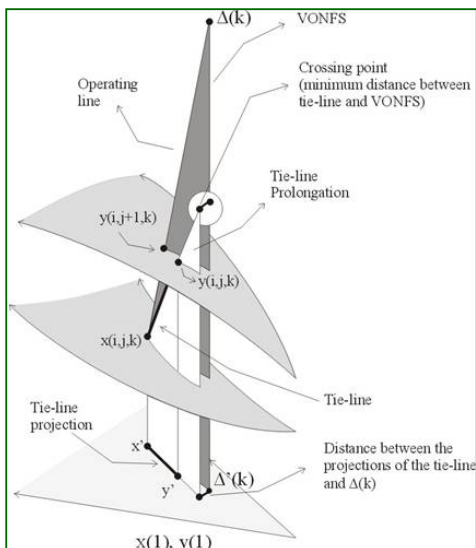
➤ In this case, the distillation boundary is formed by the two different surfaces, that intersect in one curve.

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•Conclusions

✓ Topological analysis can really help to equilibrium calculations in complex systems, and therefore in the calculation of unit operation such as multicomponent distillation columns (including minimum reflux calculation)



Reyes-Labarta et al. IEC&R 39(10),3912-3919 (2000)

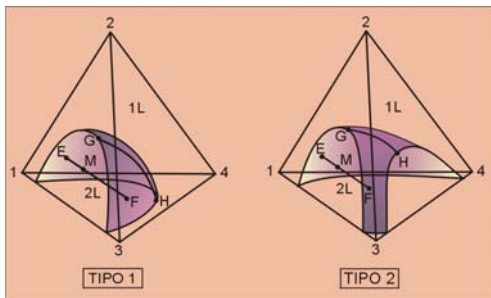
Marcilla et al. Latin American Applied Research and International Journal of Chemical Engineering, 27, 51-60 (1997)



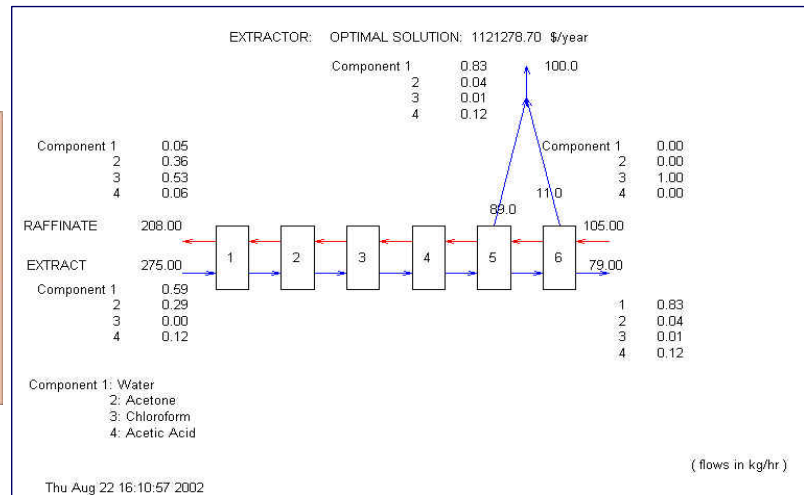


•Conclusions

✓ Topological analysis can really help to equilibrium calculations in complex systems, and therefore in the calculation of unit operation such as multicomponent distillation columns (including minimum reflux calculation) or extraction columns...



Marcilla et al. IEC&R, 38, 3083-3095 (1999)



Reyes-Labarta & Grossmann, AIChE 47(10), 2243-2252 (2001)

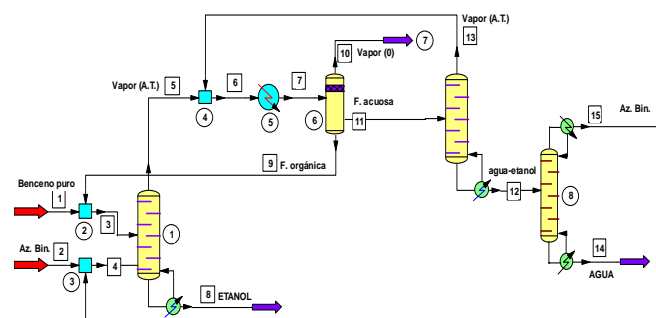


•Conclusions

✓ Topological analysis can really help to equilibrium calculations in complex systems, and therefore in the calculation of unit operation such as multicomponent distillation or extraction columns...

✓ The proposed new method is very easy to model, gives very good results and has also the possibility of using different equilibrium equations (thermodynamical or empirical)

✓ The prediction of distillation boundaries is essential in the design and simulation of complex chemical processes, including distillation column sequences, etc.





-Reyes-Labarta, J.A. "Diseño de Columnas de Rectificación Y Extracción Multicomponentes". Biblioteca Virtual Miguel de Cervantes (Universidad de Alicante), 1998.

<http://www.cervantesvirtual.com/FichaObra.html?Ref=4845&ext=pdf>

-A. Marcilla, A. Gómez, J.A. Reyes, M.M. Olaya.; New Method for Quaternary Systems Liquid-liquid Extraction Tray to Tray Design. *Industrial & Engineering Chemistry Research*, 38, 3083-3095 (1999).

-A. Marcilla, A. Gómez, J.A. Reyes; New Methods for Designing Distillation Columns of Multicomponent Mixtures. *Latin American Applied Research and International Journal of Chemical Engineering*, 27, 51-60 (1997).

-Reyes, J.A.; Gomez, A.; Marcilla, A. Graphical concepts to orient the minimum reflux ratio calculation on ternary mixtures distillation. *Industrial & Engineering Chemistry Research* 39(10),3912-3919 (2000).

-J.A. Reyes-Labarta, I.E. Grossmann; Disjunctive Programming Models for the Optimal Design Of Liquid-liquid Multistage Extractors And Separation Sequences. *AIChE Journal*. 2001, 47 (10), 2243-2252.

-J.A. Reyes-Labarta y I.E. Grossmann. Optimal Synthesis of Liquid-liquid Multistage Extractors. *Escape-11 (European Symposium of Computer Aided Process Engineering)*, Capec (Computer Aided Process Engineering Center). ISBN: 0-444-50709-4 (Dinamarca, 2001).



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**Juan A. Reyes-Labarta**

e-mail: [ja.reyes@ua.es](mailto:ja.reyes@ua.es)

web: <http://iq.ua.es/gcef.htm>

web: <http://iq.ua.es/~jareyes/>

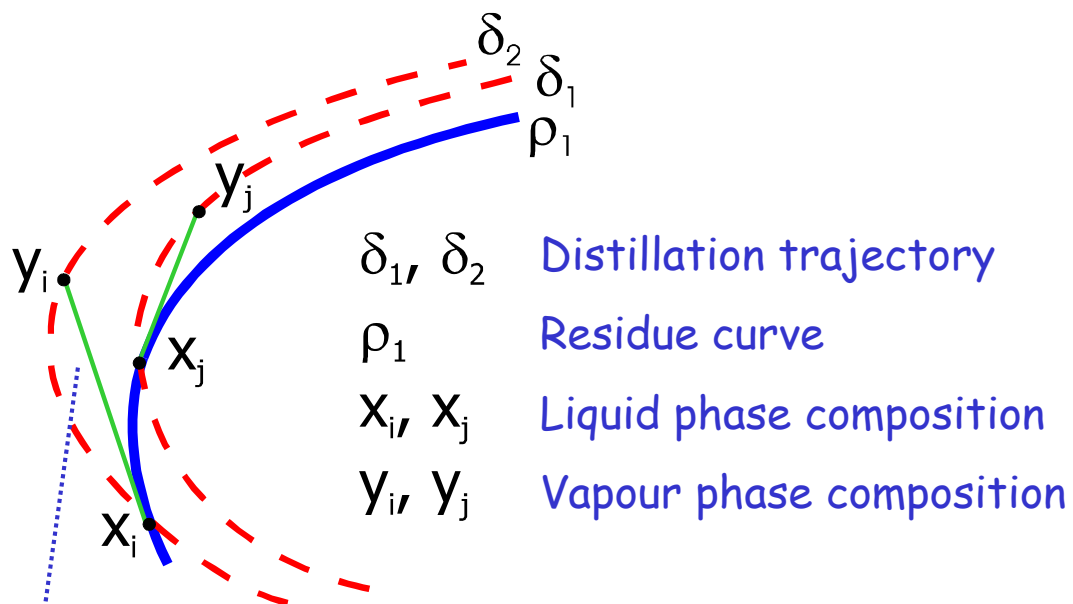


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## Introduction: Topology Azeotropic Liquid-Vapour Equilibrium



LV tie line (tangent to the residue curve)

Gómez et al. Ingeniería Química, 377, 219-229 (2001)

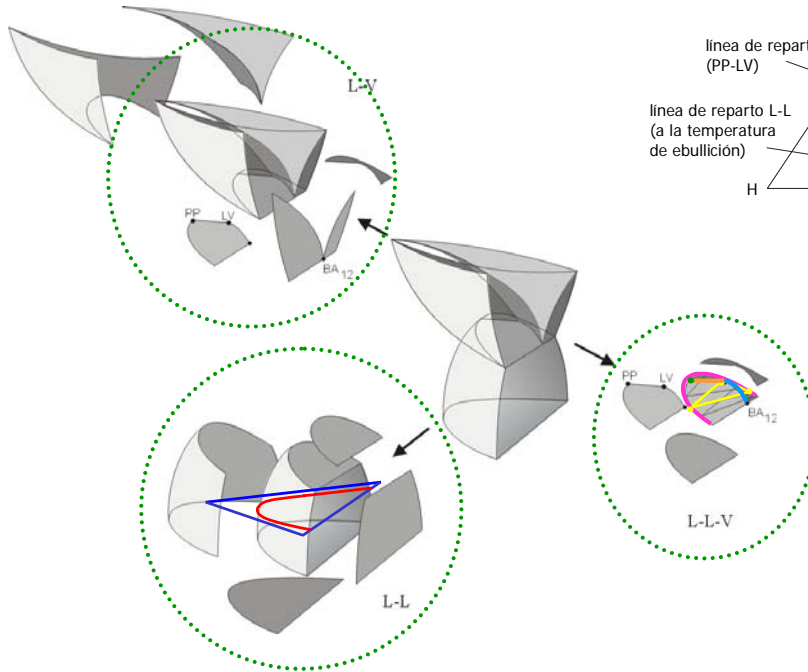




Ternary system with:

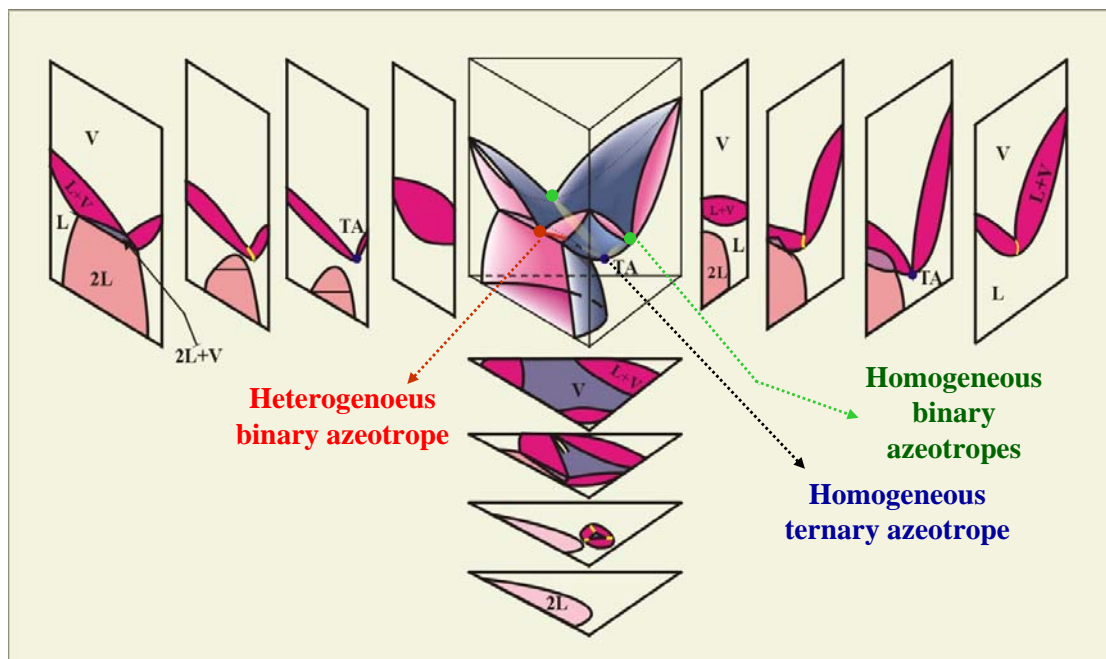
1 heterogeneous azeotropic binary composition

- I Componente de volatilidad intermedia
- H Componente menos volátil
- L Componente más volátil



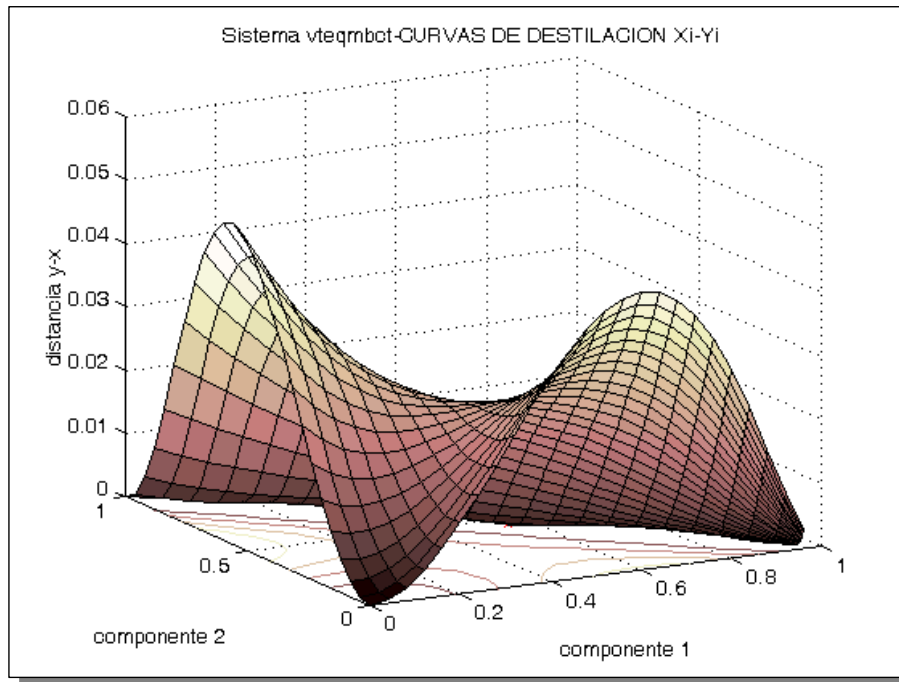
Ternary system with:

- 1 heterogeneous azeotropic binary composition
- 2 homogeneous azeotropic binary compositions
- 1 homogeneous azeotropic ternary composition





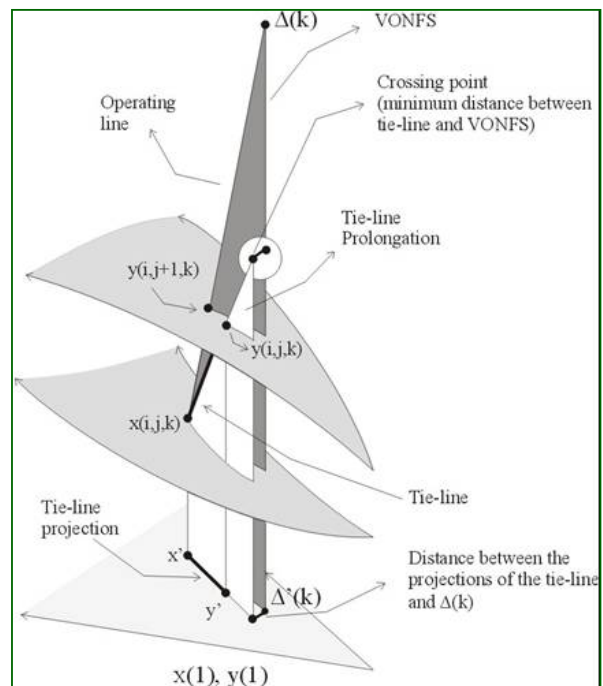
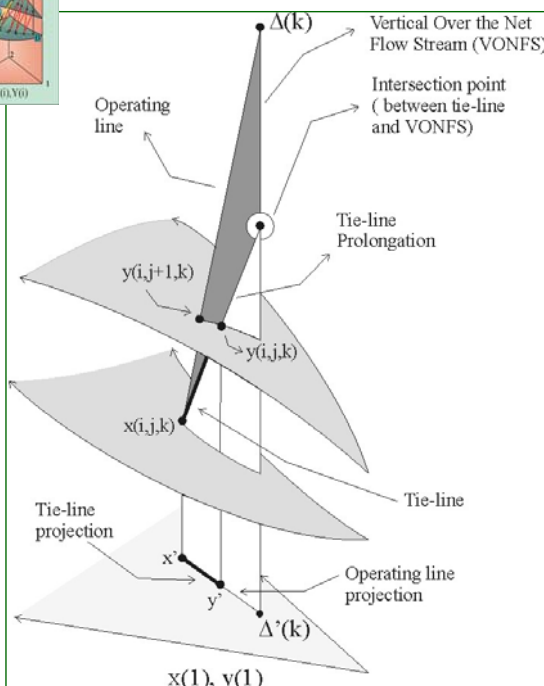
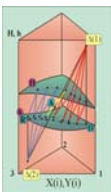
•e.g. Benzene-Ciclohexane-Toluene System (1 ABmin)



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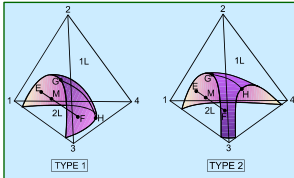


• GRAPHICAL CONCEPTS TO ORIENT THE MINIMUM REFLUX RATIO CALCULATION ON TERNARY MIXTURES DISTILLATION



Reyes-Labarta et al. IEC&R 39(10),3912-3919 (2000)

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• EMPIRICAL CORRELATIONS

Marcilla et al. IEC&R 38(8), 3083-3095 (1999)

$$\log\left(\frac{y'_k}{y'_p}\right) = \left\{ \left[ a_{k,p} + b_{k,p} \cdot \left(\frac{x'_4}{x'_2}\right) + c_{k,p} \cdot \left(\frac{x'_4}{x'_2}\right)^2 \right] + \left[ d_{k,p} + e_{k,p} \cdot \left(\frac{x'_4}{x'_2}\right) + f_{k,p} \cdot \left(\frac{x'_4}{x'_2}\right)^2 \right] \cdot \log\left(\frac{x'_2}{x'_1}\right) + \left[ g_{k,p} + h_{k,p} \cdot \left(\frac{x'_4}{x'_2}\right) + i_{k,p} \cdot \left(\frac{x'_4}{x'_2}\right)^2 \right] \cdot \left[ \log\left(\frac{x'_2}{x'_1}\right) \right]^2 \right\}$$

$$\log\left(\frac{y'(3)}{y'(2)}\right) = k_1 \qquad \log\left(\frac{y'(2)}{y'(1)}\right) = k_2 \qquad \log\left(\frac{y'(4)}{y'(3)}\right) = k_3$$

$$y'(1) + y'(2) + y'(3) + y'(4) = 1 + 4 \cdot C$$



$$y'(i) = y(i) + C$$

Four equations with four variables !!



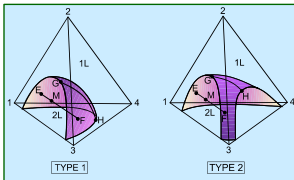
If you have:

- a concrete multicomponent azeotropic system,
- a specified separation and/or recovery of solutes in the product streams (for the distillation process).

The design problem needs the determination of:

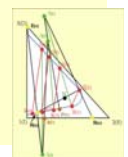
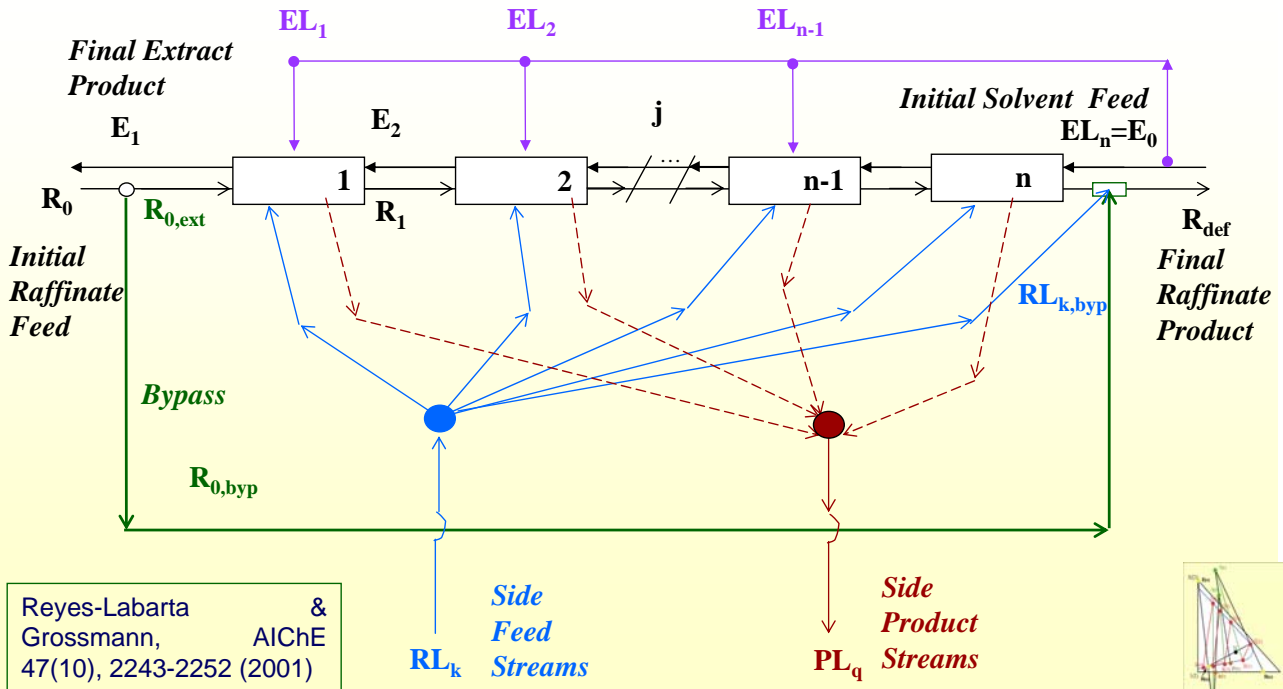
- possible distillation boundaries and distillation regions present in the system,

The objective is to make the correct design of the distillation column (that can not work in different distillation regions)



• **COMPLEX EXTRACTOR DESIGN**  
General Disjunctive Programming

Side Solvent Feeds



• **COMPLEX EXTRACTOR DESIGN (GDP)**

<http://newton.cheme.cmu.edu/interfaces/extractor/main.html>

