Efficient CAD Tool of Complex Passive Devices composed of Arbitrarily Shaped Waveguides using Nyström and BI-RME Methods

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***Abstract*— This paper presents a novel CAD tool for the complete characterization of complex passive microwave devices in waveguide technology. The analysis is based on an Integral Equation (IE) technique that provides a very efficient characterization of discontinuities between waveguides with an arbitrary cross-section composed of linear, circular and elliptical arcs. The modal analysis of such waveguides is based on a modified version of the well-known Boundary Integral - Resonant Mode Expansion (BI-RME) method using the Nyström approach, instead of using the traditional Galerkin version of the Method of Moments (MoM), thus providing significant savings on computational costs. Previous results concerning this Nyström BI-RME approach only compared the loss of accuracy in computing the modal spectrum of arbitrary waveguides. In this work, new comparative benchmarks between the results provided by the new technique and the original BI-RME method are successfully presented for real complex passive waveguide devices.**

I. INTRODUCTION

Arbitrary waveguides that are composed of linear, circular and elliptical arcs are widely used in many microwave devices. For instance, ridged rectangular [1] as well as cross-shaped irises [2] are frequently used in dual-mode empty or dielectric loaded resonator filters. Recently, due to the mechanization effects of most common manufacturing techniques of waveguide components, the presence of rounded corners in rectangular waveguides has been under investigation in guided applications [3]. Another example of great practical interest is the elliptical waveguide, which has found increasing application in many passive microwave components, such as dual-mode and triple-mode filters and circular waveguide polarizers [4].

Many passive devices used in real applications can be considered as the cascade of step discontinuities between two such waveguides with arbitrary cross-sections [5]. The electrical behavior of waveguide discontinuities may be obtained through different multimodal methods, such as those based on the generalized scattering, impedance or admittance matrices. In any case, such methods always need the complete modal characterization of each arbitrary waveguide and the coupling integrals between such waveguides [5].

The BI-RME method is a good choice for the calculation of the modal spectrum of an arbitrary cross-section waveguide because rapidly convergent expressions for the Green functions may be employed [6]. Furthermore, the BI-RME method permits to calculate the coupling coefficients between such arbitrary waveguide and a standard rectangular one very easily [7]. However, the problem could be even more efficiently solved following the Nyström method instead of the traditional Galerkin approach. This alternative method would be of great practical usage in space mapping design strategies [8], where a fast and not very accurate analysis tool is always required. The authors firstly proposed the Nyström method for such purposes in [9], where it revealed to be a very efficient method for the characterization of arbitrary waveguides. However, the impact of the accumulated error due to a less accurate discretization technique should be assessed through the analysis of complete microwave devices. These structures need not only the modal chart of an arbitrary waveguide, but also the coupling integrals between two of such perturbed waveguides.

Then, in this paper, the fast and simple implementation of the BI-RME method based on the Nyström technique is applied for the first time to the full wave characterization of complex microwave devices involving several cascaded discontinuities between arbitrary waveguides (several examples of such devices are shown in [10]). The arbitrary cross-section of these waveguides can be composed of linear, circular and for elliptical arcs. Comparisons between the numerical results obtained by the new efficient technique and the results obtained by the classical BI-RME approach in [10] fully validate the novel Nyström implementation. The gain in efficiency and the accuracy loss have been measured and presented for microwave devices composed of complex junctions connected in cascade.

II. BASIC THEORY

The BI-RME method computes the cut-off frequencies and the field patterns of arbitrarily shaped waveguides. In this work, the Nyström method is used to solve the IEs that arise in the BI-RME method for the TE and TM modes [5]-[6]. If the Nyström method is followed to solve such problems, two linear matrix eigenvalue problems arise. The key step in this procedure is directly related to the rigorous treatment of the singular behavior of the integral equations kernels, which must be carefully considered for both kinds of modes.

The elements of the $\mathbf{L}'$ matrix (see [5]-[6]) corresponding to the TM case must be computed as follows:

$$L'_{ij} = \omega'_{ij}\, g(\vec{s_i}, \vec{s_j}) \qquad (1)$$

where ω'_{ij} are the Nyström quadrature weights and g is the scalar Green function [5]-[6], which is singular when the discrete observation and source points coincide ($\vec{s_i} = \vec{s_j}$). In such case, the singular contribution of the kernel must be approximated by its Taylor expansion, which gives place to regular g_r and singular g_s terms (see [10] for more details). In that case, the elements are solved as:

$$L'_{ii} = \omega'_{ii}\, g_r(\overrightarrow{s_i}, \overrightarrow{s_i}) + \int_{\sigma_i} g_s(\overrightarrow{s}, \overrightarrow{s}')dl' \qquad (2)$$

where the regular term is directly computed, and the singular integral is analytically solved. It must be observed that the integral solution depends on the geometry under consideration σ_i (linear, circular or elliptical arcs). Further details about such solution will be presented during the talk.

For the TE case, the elements of the **L** and **C** matrices (see [6]) are computed in the following way:

$$L_{ij} = \omega_{ij}\, \widehat{\mathbf{t}}(\overrightarrow{s_i}) \cdot \overline{G}_{st}(\overrightarrow{s_i}, \overrightarrow{s_j}) \cdot \widehat{\mathbf{t}}(\overrightarrow{s_j}) \qquad (3)$$

$$C_{ij} = \omega_{ij} \frac{\partial^2 g(\overrightarrow{s_i}, \overrightarrow{s_j})}{\partial l \partial l'} \qquad (4)$$

where $\overline{G}_{st}$ is the solenoidal dyadic Green function [6], which is again singular when $\overrightarrow{s_i}$ is equal to $\overrightarrow{s_j}$. When computing the L_{ii} elements according to (3), the singular contribution of the function $\overline{G}_{st}$ can be avoided following the same technique proposed before for the TM case, but considering the unitary tangent vector $\widehat{\mathbf{t}}$ in (3).

On the other hand, the evaluation of the **C** matrix elements using (4) requires the numerical computation of a double partial derivative of g with respect to the observation and source contour parameters l and l'. Such double derivative presents a hypersingularity, which has been solved via the traditional method of adding and subtracting an asymptotic term. Proceeding in this way, the diagonal terms of the **C** matrix are finally obtained as:

$$C_{ii} = -\sum_{j \neq i} \omega_{ij} \frac{\partial^2 g(\overrightarrow{s_i}, \overrightarrow{s_j})}{\partial l \partial l'} \qquad (5)$$

where this expression is only valid in two scenarios. The first one is the consideration of closed contours inside the standard waveguide. The second one applies when the arbitrary insertions have their edges on the boundary of the standard waveguide.

Once the cut-off frequencies of the modal solutions of both arbitrary waveguides (AWs) are obtained, the coupling integral coefficients between such two sets of modes must be efficiently obtained. If the same auxiliary rectangular waveguide is used to solve the two AWs involved in the discontinuity, the required modal coupling coefficients are easily computed through the expression:

$$\langle \mathbf{e}_i^{(AW_1)}, \mathbf{e}_j^{(AW_2)} \rangle = \sum_{n=1}^{N_{(RW)}} \langle \mathbf{e}_i^{(AW_1)}, \mathbf{e}_n^{(RW)} \rangle \langle \mathbf{e}_n^{(RW)}, \mathbf{e}_j^{(AW_2)} \rangle \qquad (6)$$

where $\mathbf{e}_i^{(AW)}$ and $\mathbf{e}_n^{(RW)}$ are the i-th and n-th normalized electric modal vectors of the arbitrarily and the rectangular waveguide, respectively. The evaluation of the $\langle \mathbf{e}_i^{(AW)}, \mathbf{e}_n^{(RW)} \rangle$ terms is easily performed by post-processing some of the matrices already used in the mode determination procedure (see [7]). Note that the choice of the same auxiliary contour for solving both modal problems allows to compute (6) by only one summation, thus substantially improving the computational efficiency of this technique.

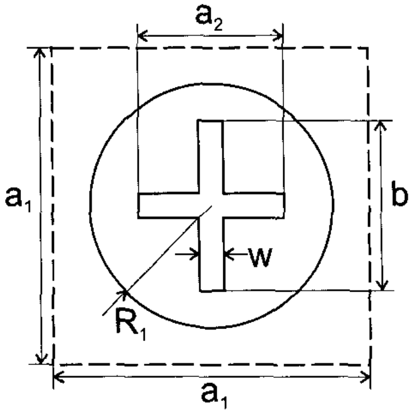

Fig. 1. Cross-shaped iris (a_1 = 25 mm, a_2 = 15.3 mm, b = 17.3 mm, w = 2 mm and R_1 = 12.0 mm).

Once the modal spectra are obtained, the integral equation technique proposed in [11] is used to solve every cascaded discontinuity. Then, the new Nyström method can be directly evaluated for the full-wave analysis of modern passive waveguide devices.

III. RESULTS

In order to show the gain in efficiency of the novel CAD tool proposed in this paper, CPU computation times for three practical examples using Galerkin and Nyström approaches are compared. The first one is the analysis of a junction between a cross-shaped iris and a circular waveguide. The second and third results correspond to the S parameters of two complex devices involving discontinuities between circular and elliptical waveguides. All CPU costs offered in this section have been obtained with a Pentium IV platform at 2.4 GHz with 1 GB DDRAM.

A. *Cross-shaped Iris*

The first example is a complex discontinuity between a cross-shaped iris and a circular waveguide shown in Fig. 1. This iris is commonly used in circular waveguide dual-mode filters, which are widely used for space applications.

Comparisons between Nyström and Galerkin solutions for the cut-off frequencies of the cross-shaped iris and the coupling integrals of the considered junction are shown in Tables I and II. In both tables, the Galerkin results used as a reference to compute the errors are taken from [10].

In Table I, results for the first 50 modes (35 TE and 15 TM) of the cross-shaped iris have been computed using 2000 modes of the auxiliary square box. The relative error has been kept below 1.2%, meanwhile the average error is preserved below 0.5%. The total CPU effort required is of 250 sec., which is rather well compared with the 410 sec. related to the Galerkin technique reported in [10], thus providing a substantial time reduction of 38%.

Table II shows the coupling integral coefficients between the first three modes of the cross-shaped waveguide and several modes of the circular waveguide computed with the proposed Nyström method. The associate absolute error when compared to the coefficients provided by [10] is also included in italic letters.

TABLE I

CUT-OFF FREQUENCIES

Order	Mode Type (TE/TM)	Nyström (GHz)	Relative Error (%)
1	TE	8.9519	1.07
5	TE	26.5069	1.12
9	TE	44.4153	0.19
13	TM	61.1482	0.33
20	TE	75.9002	0.01
25	TM	78.7285	0.24
30	TM	82.4485	0.14
35	TE	86.9649	0.17
38	TM	90.7506	0.09
48	TE	99.9278	0.37

TABLE II

COUPLING COEFFICIENTS

Cross-shaped waveguide modes		
TE_1	TE_2	TE_3
0.32405 (TE_{11c}) $4.2 \cdot 10^{-3}$	-0.206476 (TE_{21s}) $1 \cdot 10^{-2}$	0.31346 (TE_{11s}) $5.5 \cdot 10^{-3}$
-0.34725 (TM_{11s}) $2.5 \cdot 10^{-2}$	-0.04357 (TE_{01}) $2 \cdot 10^{-3}$	0.33972 (TM_{11c}) $2.4 \cdot 10^{-2}$
-0.08361 (TE_{31c}) $1.3 \cdot 10^{-3}$	-0.36261 (TM_{21c}) $7.5 \cdot 10^{-3}$	0.06662 (TE_{31s}) $1.7 \cdot 10^{-3}$
0.16011 (TE_{21c}) $1 \cdot 10^{-4}$	0.01705 (TE_{41s}) $2 \cdot 10^{-4}$	0.20261 (TE_{12s}) $5 \cdot 10^{-4}$
0.16958 (TM_{31s}) $1.2 \cdot 10^{-3}$	-0.20477 (TE_{22s}) $2.3 \cdot 10^{-3}$	0.14096 (TM_{31c}) $1 \cdot 10^{-4}$

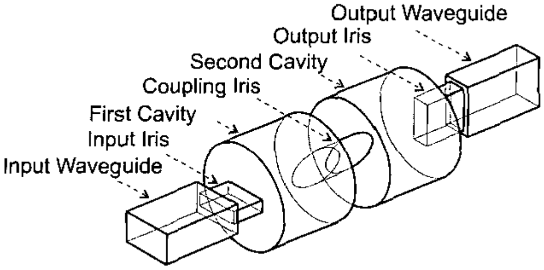

Fig. 2. Prototype with two circular waveguide cavities (radius 11.70 mm, length 100.0 mm) coupled through an elliptical iris (semi-axis of 9.0 and 7.0 mm, rotation angle of 45°, length 0.95 mm) and connected to the input and output WR-75 waveguides by two rectangular irises (input iris: 18.0 × 1.10 mm, length 0.50 mm; output iris: 1.1 × 14.10 mm, length 0.50 mm).

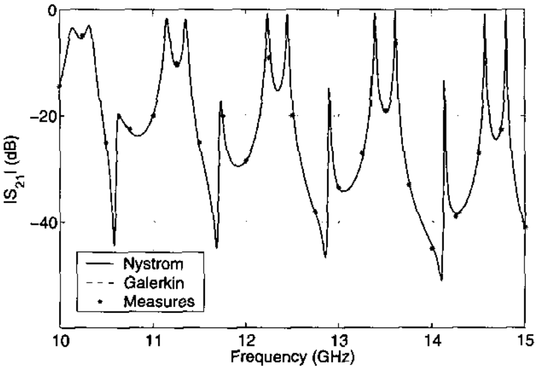

Fig. 3. Magnitude of the transmission S_{21} coefficient of the circular waveguide prototype shown in Fig. 2.

B. Circular waveguide prototype

The first validation example considered has been a verification prototype described in [12], which essentially consists of two circular waveguide cavities coupled through a rotated elliptical iris and fed to the input/output standard WR-75 waveguides by two rectangular irises (see Fig. 2). The simulated transmission coefficient of this device is successfully compared in Fig. 3 with the experimental results given in [12].

In order to obtain these very accurate results, the integral equation technique implemented has required a total number of 10 accessible modes, 40 basis functions and 400 terms in the kernels for solving each discontinuity. The complete full-wave analysis of this example has required 382.4 sec. for the Galerkin approach and 277.5 sec. for the Nyström method, which involves a time reduction equal to 27.4%.

C. Twister with Circular and Elliptical waveguides

Finally, a much more complex structure, solved in [10] using the Galerkin technique, has been simulated following the new Nyström BI-RME approach. This device is a 2-pole 90° twist component for K-band applications, which is intended to operate at 26.3 GHz with a wide bandwidth of around 2 GHz.

Photographs of this prototype, as well as of their integrating pieces, are displayed in Figs. 4- 6, where the geometric dimensions are given. The simulated scattering parameters of this complex two-port device are shown in Fig. 4, where they are successfully compared with the Galerkin approach and the measurements of a manufactured prototype. Such results were obtained using 20 accessible modes, 50 basis functions and 400 kernel terms in the integral equations related to each discontinuity. These simulating parameters involved a total CPU effort of 660.8 sec. for Galerkin method and 495.3 sec. for Nyström approach. In this example, the gain in computational cost is about 25%, which clearly validates the Nyström method as a good alternative to the Galerkin approach in order to provide rather accurate results in lower CPU times.

IV. CONCLUSION

A fast and rather accurate Nyström BI-RME algorithm has been applied to the complete modal characterization of complex microwave devices composed of cascaded arbitrary waveguides. This novel method offers some advantages compared to the standard Galerkin BI-RME method: the first one is the simplicity of the implementation and the second improvement is the reduction of the computational time. Furthermore, the arbitrary cross-section of the analyzed waveguides may be composed of linear, circular and elliptical arcs. The new simulation tool has been thoroughly verified through several application examples of great practical interest. The simulation tool has been employed to obtain the electrical response of two complex waveguide devices involving junctions between circular and elliptical waveguides. CPU times have been included to validate the efficiency improvement of the new CAD tool, which can be used in modern design procedures based on the well-known Aggressive Space Mapping technique.

Fig. 4. Photograph of a 90° twist component prototype manufactured in standard WR-34 waveguide. The internal pieces are: two square waveguides (size 8.636 mm and length 1.8 mm), a central piece with two circular waveguides (radius 6.1 mm and length 1.6 mm) and an inner elliptical iris (semi-axis of 6.0 mm and 3.9 mm, rotation angle of 45.0° and length 0.3 mm).

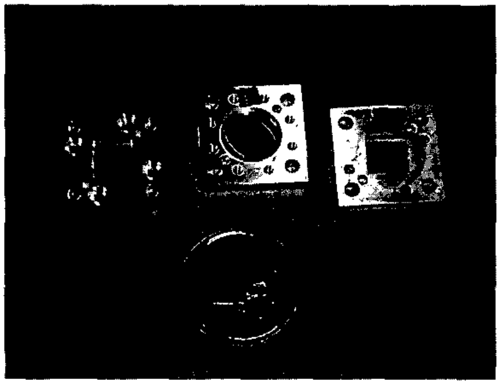

Fig. 5. Photograph of the internal pieces of the 90° twist prototype shown in Fig. 6.

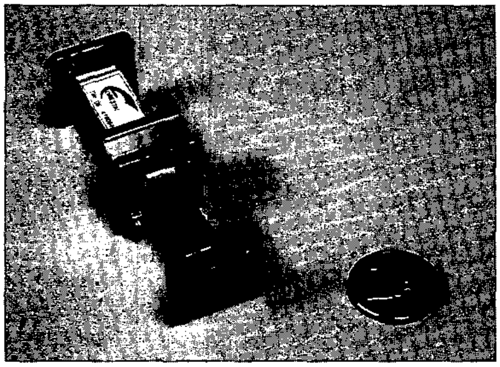

Fig. 6. Photograph of the complete and mounted structure for the 90° twist prototype.

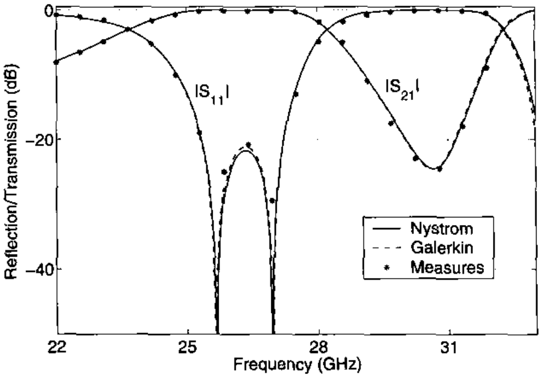

Fig. 7. Magnitude of the reflection S_{11} and transmission S_{21} coefficients of the 90° twist component shown in Fig. 4.

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