Anamorphic and spatial frequency dependent phase modulation on liquid crystal displays. Optimization of the modulation diffraction efficiency

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Abstract: In this work we present experimental evidence of an anamorphic and spatial frequency dependent phase modulation in commercially available twisted nematic liquid crystal spatial light modulators. We have found that the phase modulation depth depends on the magnitude of the local spatial frequency component along the horizontal direction. Along the vertical direction the phase modulation depth does not depend on the spatial frequency. This phenomenon is related with the electronics driving the device and in no way related to liquid crystal physics. It causes a reduction of the optical efficiency of a diffractive optical element displayed onto this type of modulator. We present an algorithm to correct this effect and more efficiently display a diffractive optical element. We apply it to the particular case of a Fresnel lens. Experimental results that confirm the improvements in the efficiency of the displayed diffractive lens are presented.

OCIS codes: (50.1970) Diffractive optics; (230.3720) Liquid-crystal devices; (230.6120) Spatial Light Modulators; (220.3620) Lens design; (70.4560) Optical data processing.

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1. Introduction

Spatial light modulators (SLM) are optical devices useful for the application in optical image processing, programmable diffractive optics and adaptive optics [1-3]. Among the different technologies, liquid crystal displays (LCD) have become the most available and employed SLM for these applications [4]. For instance, twisted nematic LCDs can be extracted from projection devices, and they can be used to act as a phase-mostly SLM when properly selecting input and output polarization configurations [5]. These phase-mostly SLMs are very interesting devices for diffractive applications, where a designed phase-only mask is displayed onto the liquid crystal display. One case of particular interest is Fresnel lenses. They have long been proposed as replacements or complement to conventional refractive lenses [6], and they actually play an important role in modern optical technology. Real-time diffractive Fresnel lenses have been produced using LCDs in refs. [7,8].

One key parameter for the realization of such diffractive optical elements is its diffraction efficiency [9]. The pixelated structure of the display produces a characteristic 2D diffraction grid pattern, which strongly reduces the efficiency of the displayed element (its reconstruction is replicated on every point of the grid pattern). In spite of this effect, the use of pixelated SLMs is still very interesting for diffractive applications. The low spatial resolution of electronically addressed SLMs permits to restrict the efficiency analysis of the displayed diffractive elements to the scalar theory [10]. This limited spatial resolution also reduces, in general, the diffraction efficiency of the displayed lenses and induces a quantization of the phase levels at the lens edges [11]. If the focal length is too short, a lens array effect is produced, which degrades the efficiency of the designed lens [12-14]. Here we restrict our analysis to lenses with long enough focal length to avoid this effect. In this situation, the diffraction efficiency of the displayed phase-only diffractive element is reduced when the modulation produced by the display is not a perfect linear phase-only modulation with 2π phase depth [15]. We have recently defined a modulation diffraction efficiency parameter and

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developed a model to numerically evaluate it as a function of the complex modulation provided by the LCD [16].

In recent experiments we have observed that the LCD phase modulation strongly depends on the orientation and spatial frequency components of the displayed image. We have found that this phenomenon is related with the electrical signal addressing scheme for large content information displays, i.e. it is in no way related to liquid crystal physics. Basically, as the LCD is fed a video signal, there is a low frequency in the vertical direction in the video signal and a high frequency in the horizontal direction, as is also usual with television circuits. When the frequency is very high along the horizontal direction the signal can be affected by the bandwidth of the electronic circuitry. This electrical analysis will be discussed with greater detail in a future paper. Now in this paper we present measurements of the phase modulation for our LCD working in a phase-mostly configuration that evidences an anamorphic behavior. The phase modulation remains invariant for the whole range of vertical frequencies (rows per period), and for low horizontal frequencies (columns per period). However, the phase modulation depth for high horizontal frequencies is seriously reduced. In general, a limited phase modulation depth seriously reduces the modulation diffraction efficiency. Therefore this anamorphic effect degrades the efficiency of any diffractive phase-only element displayed onto the LCD. Another consequence is that the shape of the diffractive element may get distorted, e.g. losing the rotational symmetry in the case of a Fresnel lens.

We demonstrated in Ref. [16] that the application of a projection of the complex values based on the minimum Euclidean distance principle [17,18] leads to important improvement of the efficiency of a diffractive phase element, even if the phase modulation depth provided by the display is seriously reduced. The technique is valid for phase-only functions with values distributed over a 2π range with a constant probability density function. This is the case of a Fresnel lens. Here we present an algorithm for the design of the displayed phase-only diffractive element that compensates the anamorphic and spatial frequency dependent behavior of the phase modulator. The algorithm is based on a different Euclidean projection for spatial frequencies with different orientations. We apply this technique for the optimization of the efficiency of displayed Fresnel lenses and experimentally demonstrate an improvement that is in agreement with the predictions of the theory.

The paper is organized as follows. In Section 2 we present the measurement of the phase modulation obtained from gratings displayed onto the LCD with different orientations and frequencies; they demonstrate the anamorphic behavior mentioned above. In Section 3 the concept of modulation efficiency is reviewed and the algorithm to correct the anamorphic phase modulation is presented. In Section 4 we present experimental results of the efficiency of the displayed lenses that show the improvement when the anamorphic phase modulation is corrected. Finally, in Section 5 the conclusions of the work are presented.

2. Anamorphic and spatial frequency dependent phase modulation depth

Liquid crystal displays modulate an input light beam through the change in the orientation of the birefringent liquid crystal molecules when applying a voltage. When inserted between two polarizers they generally produce a complex amplitude modulation [19]. We have recently demonstrated that phase-mostly modulation can be obtained with this kind of displays when a proper elliptically polarized light configuration is employed [5,20]. For this purpose we use a combination of polarizers and wave-plates in front and behind the LCD, in order to optimize the response of the display to achieve a phase-mostly modulation.

In the experiments we use a twisted nematic LCD from Sony, Model LCX012BL, with a VGA resolution (640 columns $\times$ 480 rows), extracted from a video projector Sony VPL-V500. We use a short wavelength λ=458 nm, in order to achieve a large value of the phase modulation depth. Figure 1 shows the intensity of the transmitted light as a function of the addressed grey level in the phase-mostly modulation configuration. This polarization configuration has been obtained following the procedure exposed in Ref. [20]. These data are obtained by addressing a uniform image to the display and measuring the transmission with

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respect to the addressed gray level, which ranges from 0 to 255. The intensity transmission in Fig. 1 has been normalized with respect to the maximum value. A quite flat curve is obtained.

In this polarization configuration we also measure the phase modulation produced by the display. We use a method proposed in Ref. [21], where a binary grating is addressed to the display and the ratio between the first and zero diffraction orders is measured. This ratio depends on the phase difference between the two levels of the grating. Since the intensity transmission remains almost flat on the entire range (Fig. 1), the LCD approximately produces a binary phase grating. We have observed that the LCD response strongly depends on the orientation and spatial frequency components of the displayed image. We have displayed gratings in the vertical and horizontal directions, with different periods.

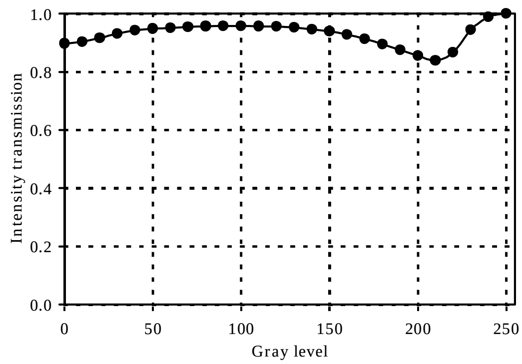

Fig. 1. Normalized intensity transmission for the phase-only modulation configuration of the twisted nematic LCD.

Figure 2 shows the intensity of the zero and first diffraction orders for different binary gratings displayed on the LCD. The gratings are generated with a fixed gray level g_1=0 and a second gray level g_2 variable in the range from 0 to 255. In Figs. 2(a) and 2(b) the grating is displayed along the vertical direction, and the period is of 64 and 2 rows/period respectively. As the gray level g_2 increases, the intensity of zero order (DC) is reduced while the first order increases. When g_2 reaches a value around 180 the intensity of the first order shows a maximum and the zero order is strongly reduced, indicating a π phase difference between both gray levels. For higher values the DC term increases again while the first diffraction order is reduced, showing that the phase modulation approaches 2π for g_2=255. Both gratings, with 64 and 2 rows/period show very similar behavior, proving that the phase modulation is not being affected by the spatial frequency in this direction.

This is not the situation when the grating is displayed along the horizontal direction. In Fig. 2(c) and 2(d) the binary grating has spatial frequency of 64 and 2 columns/period respectively. In Fig. 2(c) the results are equivalent to those of the previous gratings. However, when the period of the grating is of only 2 columns the evolution of the diffracted orders is very different. In Fig. 2(d) it is observed a maximum phase depth of only around π radians for the maximum gray level (note that the first order intensity is equal to zero for this gray level).

We tested this asymmetric behavior of the phase modulation depth for gratings with other periods. Figure 3 shows the phase modulation as a function of the addressed gray level. Figure 3(a) corresponds to gratings with vertical frequencies of 64, 32, 16, 8, 4 and 2 rows/period. The phase modulation remains invariant for the whole range of frequencies, reaching a maximum phase depth very close to 2π. Thus it represents an ideal phase-mostly modulation device for diffractive optics. Figure 3(b) shows the phase modulation now measured using gratings with horizontal frequencies of 64, 32, 16, 8, 4 and 2 columns/period. These results show that for low frequencies the phase modulation is equivalent to that produced for vertical frequencies. However, as the spatial frequency increases, the maximum phase modulation depth is being reduced, reaching only π radians when the period is of only 2 columns.

Therefore, it is concluded that the device is not so efficient to display diffractive optical elements with high horizontal frequencies as it is with low horizontal frequencies or vertical frequencies. If a regular spherical Fresnel lens is displayed onto this LCD, the modulation diffraction efficiency for vertical direction is higher than for the horizontal direction.

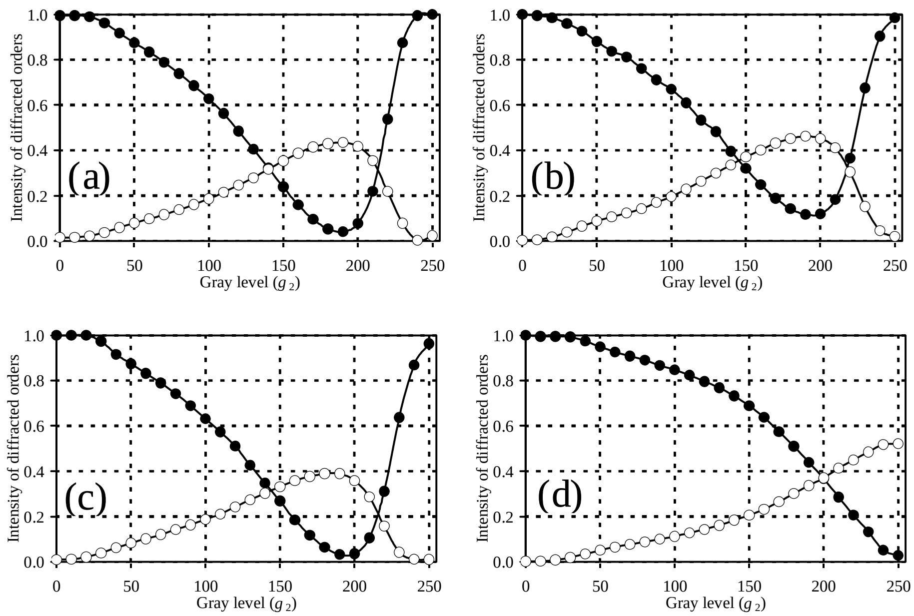

Fig. 2. Intensity of zero (●) and first (○) diffracted orders generated by a binary grating displayed on the LCD with gray levels 0 and g_2, as a function of g_2. (a) 64 rows/period (vertical frequency). (b) 2 rows/period (vertical frequency). (c) 64 columns/period (horizontal frequency). (d) 2 columns/period (horizontal frequency).

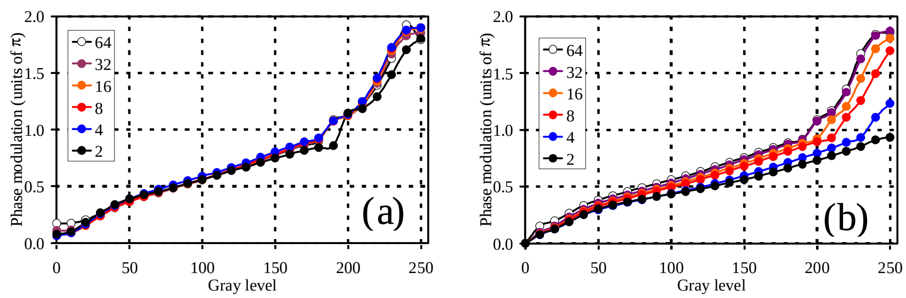

Fig. 3. Phase modulation as a function of the addressed gray level and for binary gratings with period of 64, 32, 16, 8, 4 and 2 pixels. (a) Vertical frequency (rows/period). (b) Horizontal frequency (columns/period).

3. Encoding algorithm for phase-only diffractive elements with high modulation efficiency

In this section we present a method to encode a diffractive optical element in this anamorphic phase modulator with optimal modulation diffraction efficiency. Firstly we briefly review the method based on the minimum Euclidean distance to encode phase-only diffractive elements onto a phase-only modulator with limited phase modulation depth. Then we extend the method to incorporate the anamorphic phase modulation described above.

3.1. Minimum Euclidean projection

Let us consider a designed spatial phase-only distribution φ(*x*,*y*) to be displayed onto the LCD. The display produces in general a complex modulation *m*(φ)=*a*(φ)exp(*ip*(φ)) which is a function of the addressed phase φ. The minimum Euclidean projection states that the most efficient way of projecting a complex diffractive mask onto a restricted modulation domain is to assign the available complex value closest in the complex plane [17,18]. In Refs. [15,16] we applied this technique to the realization of a phase-only diffractive optical element with a pure phase-only modulator with a maximum phase depth smaller than 2π. The maximum phase depth is ε=2π(1-*c*), where the mismatch parameter *c* is in the range [0,1]. In this case, the optimal realization given by the minimum Euclidean projection involves the following encoding of the phases *p* as a function of the addressed phase φ [16]:

$$p(\varphi)=\begin{cases}\varphi & \text{if } \varphi<\varepsilon\\ \varepsilon & \text{if } \varepsilon<\varphi<\frac{\varepsilon}{2}+\pi\\ 0 & \text{if } \varphi>\frac{\varepsilon}{2}+\pi\end{cases}, \tag{1}$$

where we omitted the dependence on the spatial coordinates (*x*,*y*) for simplicity. If the phase-only function φ(*x*,*y*) has values distributed over a 2π range with a constant probability density function, the modulation diffraction efficiency (η_m) can be defined as the coefficient of the first term in the Fourier expansion of the function m(φ) [16]. Next we evaluate the modulation diffraction efficiency (η_m) for the encoding in Eq. (1), and the resulting expression is [16]:

$$\eta_m=\left\{\frac{\varepsilon}{2\pi}+\frac{1}{\pi}\sin\left(\frac{\varepsilon}{2}\right)\right\}^2=\left\{1-c+\frac{1}{\pi}\sin\left(\pi(1-c)\right)\right\}^2. \tag{2}$$

A Fresnel lens is a particular case of this situation, where the spatial dependence φ(*x*,*y*) of the designed phase-only function is given by

$$\varphi(x,y)=-\frac{\pi r^2}{\lambda f}. \tag{3}$$

being $r=\sqrt{x^2+y^2}$ the radial coordinate and *f* the focal length of the lens. In Ref. [16] we demonstrated that a great improvement in the modulation diffraction efficiency of the displayed lens is obtained when the minimum Euclidean projection is applied.

3.2. Algorithm for the correction of the phase modulation

As we previously described, the anamorphic behavior of the LCD reduces the efficiency of the displayed diffractive elements. To overcome this drawback we propose an algorithm for the correction of the phase modulation. In principle, the algorithm is valid for diffractive optical elements (DOE) exhibiting a phase variation slow enough so that it makes sense to define a local spatial frequency at each point of the DOE [22]. In Section 2 it is shown that the LCD response is almost uniform for vertical spatial frequencies but it is strongly dependent on the horizontal frequency values. Therefore, a correction must be introduced to compensate this effect.

The first step is to analyze, for each pixel of the image, what is the horizontal frequency value in its neighborhood, i.e. we calculate the local frequency component ν_x along the horizontal direction. This local frequency is given by [22],

$$\nu_x=\frac{1}{2\pi}\frac{\partial\varphi(x,y)}{\partial x} \tag{4}$$

In principle, the concept of local spatial frequency makes sense if the variation of the phase function $\varphi(x,y)$ is slow enough [22]. The result in the case of Fresnel lenses is given by,

$$\nu_x = \frac{-x}{\lambda f} \tag{5}$$

where we obtain the known result that the local spatial frequency for a quadratic phase function has a linear dependence with the coordinate. Once this value is established we have to know the maximum phase modulation (ε) achieved for that frequency. In Fig. 4 it is shown the maximum phase modulation depth as a function of the horizontal period. As the next step, if the pixel phase value (φ) is larger than ε, according to Eq. (1) we have to assign the phase ε if $\varepsilon < \varphi < \varepsilon/2 + \pi$ or the phase 0 if $\varphi > \varepsilon/2 + \pi$. If $\varphi < \varepsilon$ we have to apply a look up table that connects the phase value φ with a gray value. Figure 5 shows these curves for the different spatial frequencies for which the phase modulation has been measured. Actually these curves are the inverse of those plotted in Fig. 3(b). For frequency values different from the measured ones a linear interpolation between the nearest frequencies is applied. Figure 5 sketches an example of this situation. A phase φ is desired for a spatial period of 6 pixels in the horizontal direction. Since this frequency has not been calibrated, it is assigned a gray level *g* obtained from the interpolation between curves for periods of 4 and 8 pixels.

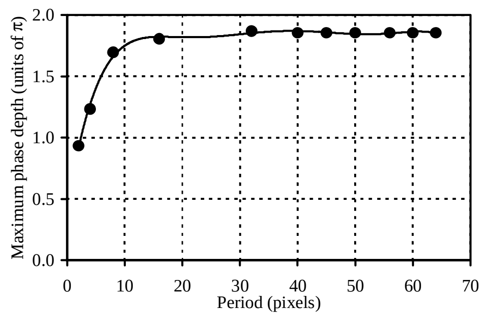

Fig. 4. Maximum phase modulation as a function of the vertical period, measured in pixels.

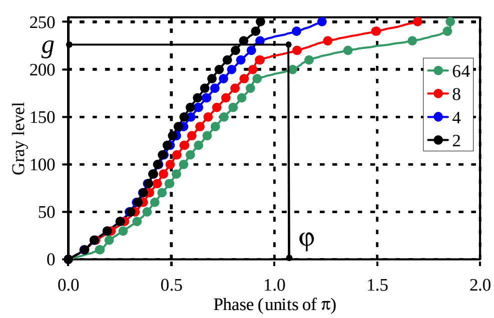

Fig. 5. Look up tables that assign a gray value to a phase value for different horizontal frequencies.

4. Experimental results

To prove the enhancement produced by the proposed technique, we generate diffractive lenses to be displayed onto the LCD. The efficiency of the lens is directly given by the modulation diffraction efficiency η_m in Eq. (2). We first compare this theoretical modulation diffraction efficiency with the experimental results of cylindrical lenses, and we demonstrate the differences for horizontal and vertical directions. Although our LCD reaches almost 2π phase modulation for vertical frequencies, we generate lenses on the computer assuming different

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values of ε, in order to verify Eq. (2). In this way it is possible to experimentally reproduce the situation for modulators that have a smaller phase modulation depth.

To study this dependence we generate a series of different lenses as follows: first the ideal lens phase-only function modulo 2π is calculated. From this function we generate a series of copies. A different maximum phase value $\varepsilon=2\pi(1-c)$ is assigned to each copy, and each pixel phase value is encoded following the minimum Euclidean projection given by Eq. (1). Then, these numerical functions have to be displayed onto the LCD. For cylindrical lenses with axis along the horizontal direction, the phase modulation depth is not dependent on the spatial frequency, and the phase-only functions are directly addressed to the display. For cylindrical lenses with axis along the vertical direction we compare two procedures. In the first one the horizontal frequency (rows/period) dependence of the modulation is not taken into account and we consider that the phase modulation curve is the one corresponding to the vertical or the low horizontal frequencies. In the second procedure, the frequency dependence is taken into account and the algorithm described in Section 3.2 is applied before we can send the phase-only function to the LCD.

Figure 6 shows the modulation efficiency both for vertical and horizontal cylindrical lenses. The continuous line shows the theoretical evolution of the modulation diffraction efficiency as a function of the mismatch parameter c given by Eq. (2). When $c=0$ the maximum phase modulation depth is $\varepsilon=2\pi$ and therefore the modulation efficiency is perfect. As the value of c increases the modulation efficiency reduces. However the use of the minimum Euclidean projection makes this reduction slow, and for instance η_m is still around 0.7 for a $c=0.5$ (maximum phase $\varepsilon=\pi$).

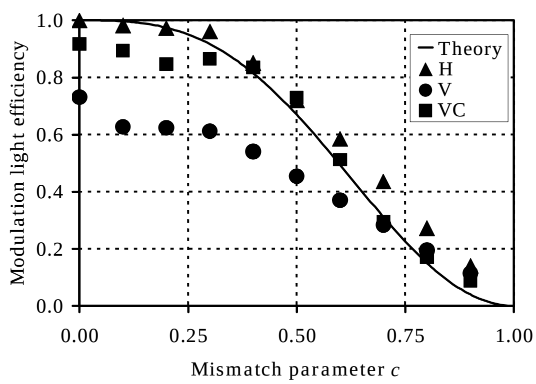

Fig. 6. Modulation diffraction efficiency for cylindrical lenses as a function of the mismatch parameter c of a modulator with limited phase depth. The line shows the theoretical efficiency (Eq. (2)). H: horizontal lenses. V: vertical lenses. VC: corrected vertical lenses.

The experimental modulation efficiency is calculated by measuring the intensity of the focalized light. The experimental data are normalized to the value obtained for the ideal lens, i.e., the one reaching 2π maximum phase modulation. This is obtained for the horizontal cylindrical lens when the maximum phase range is available ($c=0$). It is noticeable the good agreement among the theoretical curve given by Eq. (2) and the experimental data corresponding to horizontal lenses with different phase depths (see Fig. 6, triangles). However, when the vertical lenses are displayed, the efficiency is much less (Fig. 6, circles), due to the phase modulation changes as a function of the frequency. The modulation efficiency of the vertical lenses is around a 75% of the equivalent horizontal lens. However, when these vertical lenses are corrected according to the procedure described in the previous section, the efficiency of the lenses is noticeably enhanced (Fig. 6, squares), approaching that of the horizontal lenses. This proves the enhancement that can be obtained with the proposed correction method.

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Then we generate spherical diffractive lenses and we repeat the procedure. Figure 7 shows the equivalent results. Again the lenses are generated with a limited maximum phase, and then they are encoded following the encoding given by Eq. (1). Here we compare two situations: first we consider lenses with no correction of the anamorphic phase modulation (white circles), and secondly we apply the algorithm described the Section 3.2 with the correction of the anamorphic phase modulation (black circles). Again the correction results in an improvement in the efficiency around a factor 25% for values of *c* up to 0.5.

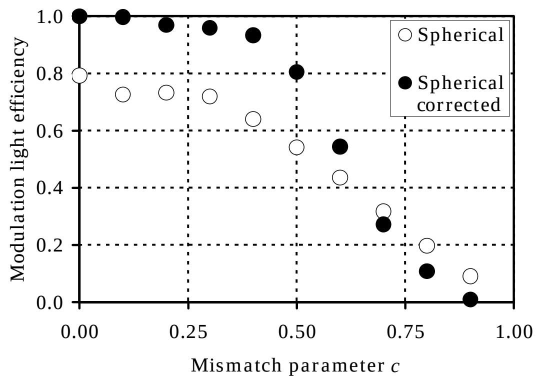

Fig. 7. Modulation diffraction efficiency of spherical lenses as a function of the mismatch parameter *c*. White dots correspond to lenses without correction and black dots to equivalent lenses with correction of the anamorphic phase modulation.

5. Conclusions

In summary, we have analyzed the modulation diffraction efficiency of liquid crystal SLM which present an anamorphic and spatial frequency dependent behavior of the phase modulation depth. We have shown experimental evidence that a LCD may produce this kind of phase modulation and we have studied how this situation affects the efficiency of a displayed phase-only diffractive optical element. We have used a twisted nematic LCD operating in a phase-mostly modulation regime. We have measured the phase modulation as a function of the spatial frequency, and we have shown that it strongly depends on its value for horizontal frequencies (rows/period). We have developed an encoding algorithm based on the minimum Euclidean projection to display the diffractive element with an optimal modulation diffraction efficiency. In principle, the algorithm is valid for DOEs exhibiting a phase variation slow enough so that it makes sense to define a local spatial frequency at each point of the DOE. We have applied the algorithm to the case of Fresnel lenses. Experimental results have been included to verify the developed theory. We have displayed cylindrical lenses and different diffraction efficiency is produced depending on their orientation. This asymmetry in the phase modulation with the frequency orientation produces a lack of rotational symmetry in spherical lenses displayed onto the LCD. We have demonstrated that the application of the proposed correction algorithm results in an important enhancement in the efficiency of the lens.

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