Short communication

Fresnel diffraction in a theoretical eye: a fractional Fourier transform approach

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Abstract. In this work, we have applied the fractional Fourier transform to obtain the Fresnel diffraction patterns in a theoretical eye. The FRT approach to Fresnel diffraction is easily implemented in a Gullstrand–Emsley theoretical eye, and it allows us to obtain the retinal image and then to derive the modulation transfer function of the eye, which can be used in the determination of optical performance of the eye.

1. Introduction

The use of theoretical eye models is widely introduced in ophthalmic and optometric work in order to predict the retinal image quality or the effect of changes of ocular parameters. In the literature, the most commonly used models assumed the paraxial approach and apply simplifications of the actual ocular parameters to represent the cornea and lens by a pair of spherical surfaces. In this way, the earliest models were introduced by Gullstrand and von Helmholtz [1, 2]. These models are a series of spherical surfaces that predict the properties of eye. Gullstrand's eye model [1] was designed to be anatomically accurate: the cornea consisted of two surfaces whose spherical curvatures were obtained by studying a variety of biological eyes, the crystalline lens is modelled as a lower index shell with a high index core to approximate the gradient index structure of actual lenses. Von Helmholtz's model [2] is simpler and assumes that the cornea is a single surface and the lens has a uniform effective index. Le Grand and El Hage [3] similarly simplified Gulltrand's schematic eye by making the lens of uniform index, but maintaining the two surfaces of the cornea. These models are still relatively popular today for studying some of the properties of the eye, but all of them fail to predict the aberration content on biological eyes, i.e. they cannot predict the measured values of eye aberrations.

Usually, optical performance of these theoretical eyes is tested by ray-tracing methods, determining the main aberrations through a modulation transfer function study [4–8].

More sophisticated models used the total wavefront aberration of the eye derived from experimental measurements to evaluate the imaging capabilities of the eye [9, 10].

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The use of Fresnel diffraction theory to characterize the optical behaviour of a theoretical eye, although it must be the most powerful method, has several methodological problems when it is applied to an arbitrary short axial distance. These problems appear if we calculate the Fresnel pattern by the classical fast Fourier transform (FFT) algorithms [11].

The aim of this work is to study the application of the fractional Fourier transform (FRT) to obtain the Fresnel diffraction patterns in a theoretical eye in order to derive the optical performance of this eye by the calculation of the modulation transfer function.

2. Methods

The FRT was recently introduced by Mendlovic and Ozaktas [12, 13]. They stated an operational definition of the FRT by the use of propagation in a gradient-index (GRIN) medium that, combining self-focusing and propagation, provides the Fourier transform of an input plane at a given distance. Lohmann [14] gave a different definition of the FRT based on the Wigner distribution. He proposed two optical set-ups for FRT implementation. Both definitions were proven to be identical, as shown in Mendlovic *et al.* [15]. The FRT can be easily related to the optical wave propagation in a free medium and the light distributions at intermediary planes can be observed as a FRT of different orders [16–16].

The integral definition for the Fourier transform of fractional p is as follows [14],

$$u_p(x) = C_1 \int_{-\infty}^{+\infty} u(x_0) \exp\left[\frac{i\pi}{f_1 \lambda \tan \phi}(x_0^2 + x^2)\right] \exp\left(\frac{-2i\pi}{f_1 \lambda \sin \phi} x_0 x\right) dx_0, \qquad (1)$$

where $\phi = p(\pi/2)$; p is the fractional order, λ is the wavelength, f_1 is a scaling factor of the transformed function and C_1 is a constant that equals

$$C_1 = \exp\left\{-i\left[\frac{\pi \operatorname{sgn}(\sin \phi)}{4} - \frac{\phi}{2}\right]\right\} |\sin \phi|^{-1/2}. \qquad (2)$$

In general, the calculation of this integral is complicated and several algorithms have been reported. In this paper we will use the fast algorithm recently developed by García *et al.* [11]. This algorithm is based on the fast Fourier transform and can be easily implemented in our case. The process involves two FFTs in cascade and is valid for fractional orders varying from -1 to 1. In Mas *et al.* [20] it is shown that the Fresnel integral and the FRT are closely related. In fact it is shown there that the FRT can be interpreted as a geometrical image of a Fresnel pattern, plus an additional quadratic phase factor. Thus, the algorithms for FRT calculation can be easily modified to calculate diffraction patterns.

For the simulation of Fresnel diffraction in a theoretical eye, we can depart from the Gullstrand–Emsley model [1]. This model represents the cornea by a single surface of curvature radius r_c, and the lens by two spherical surfaces of curvature r_{L1} and r_{L2} (see figure 1).

The object for our simulation was a beam of parallel light incident on the cornea with intensity I_0. Moreover, it acts as the entrance pupil. If we assume that d_c is the thickness of the corneal lens and $z(x, y)$ describes the shape of the cornea,

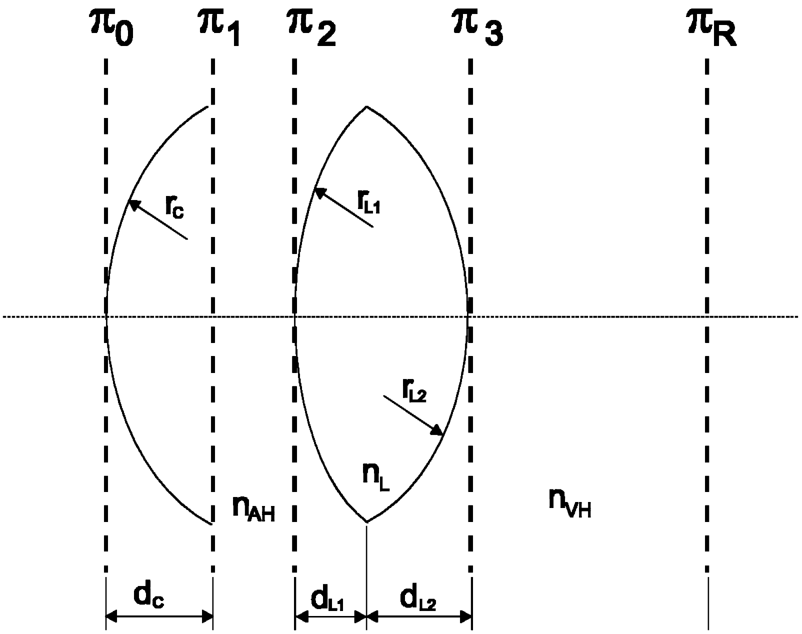

Figure 1. Schematic diagram of the Gullstrand–Emsley theoretical eye.

the action of the cornea can be characterized by means of a transmittance function $t_c(x, y)$ defined as follows:

$$t_c(x_0, y_0) = \exp\{i2\pi[d_c + (n_c - 1)z(x_0, y_0)]\}. \tag{3}$$

So, the amplitude distribution in the immediate plane after the lens is

$$A_1(x_1, y_1) = I_0^{1/2} t_c(x_0, y_0). \tag{4}$$

The propagation of this field distribution from plane π_0 to plane π_1, can be evaluated through the FRT of order p_1, where p_1 is dependent on the axial distance z_{HA}. Then, after applying the Fresnel-through-FRT algorithm [20], the distribution in the desired plane (π_2) is

$$A_2(x_2, y_2) = \text{Fresnel}[A_1(x_1, y_1)], \tag{5}$$

where 'Fresnel' means the calculation of a diffraction pattern.

The lens can be characterized by two spherical surfaces of radii r_{L1} and r_{L2}, which introduces a transmittance function t_L of the form:

$$\begin{aligned} t_L(x_2, y_2) = \exp\{&i2\pi[n_{AH}d_{L1} + n_{VH}d_{L2} + (n_L - n_{AH})z_{L1}(x_2, y_2) \\ &+ (n_L + n_{VH})z_{L2}(x_2, y_2)]\}, \end{aligned} \tag{6}$$

where n_{AH}, n_{VH} and n_L are the refractive index of the aqueous humour, vitreous humour and lens respectively; d_{L1} and d_{L2} are the thicknesses of the first lens surface and the second lens surface respectively and z_{L1} and z_{L2} are the functions which describe both lens surfaces.

At plane π_3, the field distribution must take into account the pupil function of the iris, $P_i(x_i, y_i)$ and the transmittance function of the lens as t_L, so the field distribution in the immediate plane after the crystalline lens, π_3, can be written as

$$A_3(x_3, y_3) = A_2(x_2, y_2)\, P_i(x_i, y_i)\, t_L(x_2, y_2). \quad (7)$$

The propagation of this field from plane π_3 to retina can be evaluated through a FRT of order p_2, p_2 being dependent on the distance between the lens and the retina. The amplitude at the retinal plane will be

$$A_R(x_R, y_R) = \mathrm{Fresnel}[A_3(x_3, y_3)] \quad (8)$$

and the retinal image can be calculated by obtaining the intensity distribution at the retinal plane:

$$I_R(x_R, y_R) = |A_R(x_R, y_R)|^2. \quad (9)$$

3. Results

Figure 2 shows different images obtained with the proposed model at four different intermediate planes. The figure represents how the beam is focused on the retina, and from the last image of this figure we can calculate the MTF (Modulation Transfer Function) assuming that the retinal image is the PSF (Point Spread Function) of the optics system of the eye (in fact, this figure represents the image of a point object placed at infinity). Figure 3 shows the MTF obtained from

Figure 2. Images at different planes during the propagation through the eye. The last image represents the focused beam, i.e. the retinal image.

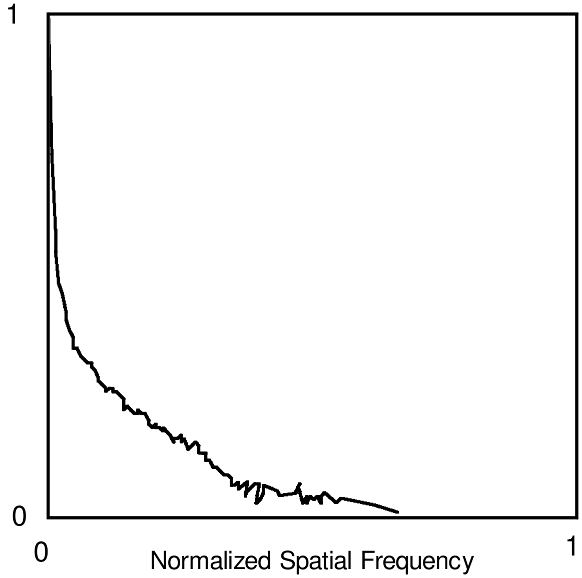

Figure 3. Modulation transfer function of the retinal image for an emmetropic observer.

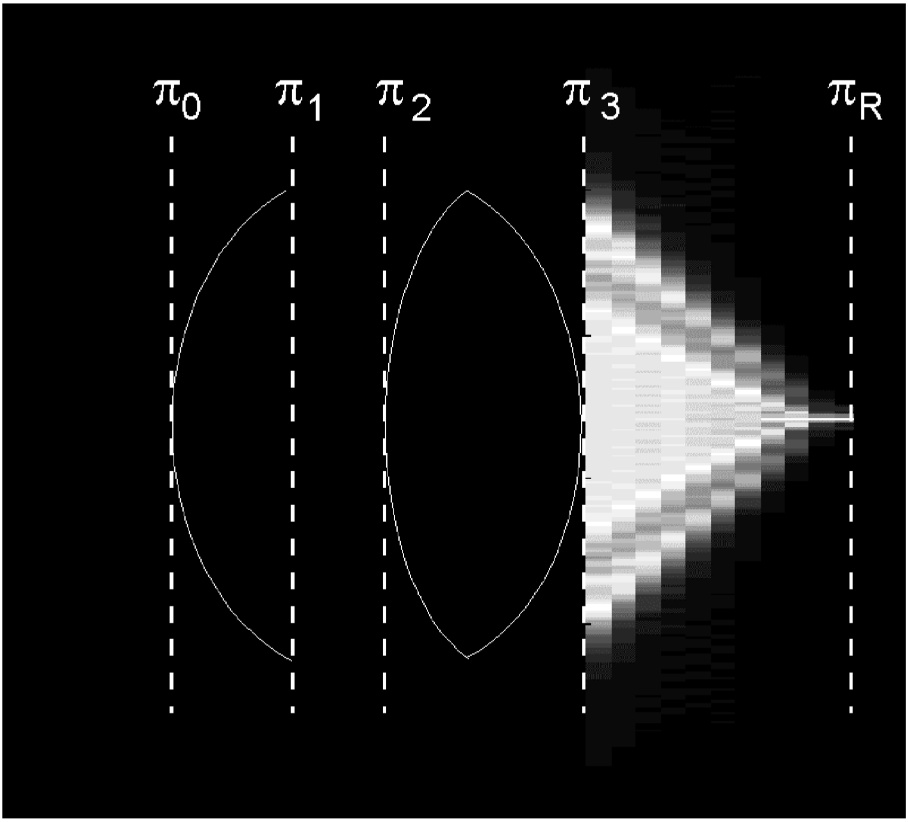

Figure 4. Wavefront propagation in an emmetropic eye.

this image. This result is very similar to that obtained empirically *in vivo* by double pass methods [21].

One of the main advantages of the Fresnel-through-FRT algorithm is the possibility of visualizing the beam propagation in an optical system. Figure 4 shows the evolution of the wave front through the ocular media in an emmetropic observer from a plane just after the crystalline lens to the retinal plane. Similar results can be obtained for a hyperopic eye and in a myopic eye (figures 5 (*a*) and (*b*)). We can observe how the beam is focused before the retinal plane in the myopic eye and how in the hyperopic eye the focus is after the retinal plane. This result can be easily applied to the study of light propagation in ocular media when the crystalline lens has been replaced by an intraocular lens.

4. Conclusions

The fractional Fourier transform represents a powerful tool for Fresnel diffraction pattern analysis and can be easily implemented in a theoretical eye for the evaluation of the optical performance of an eye. From this model it is

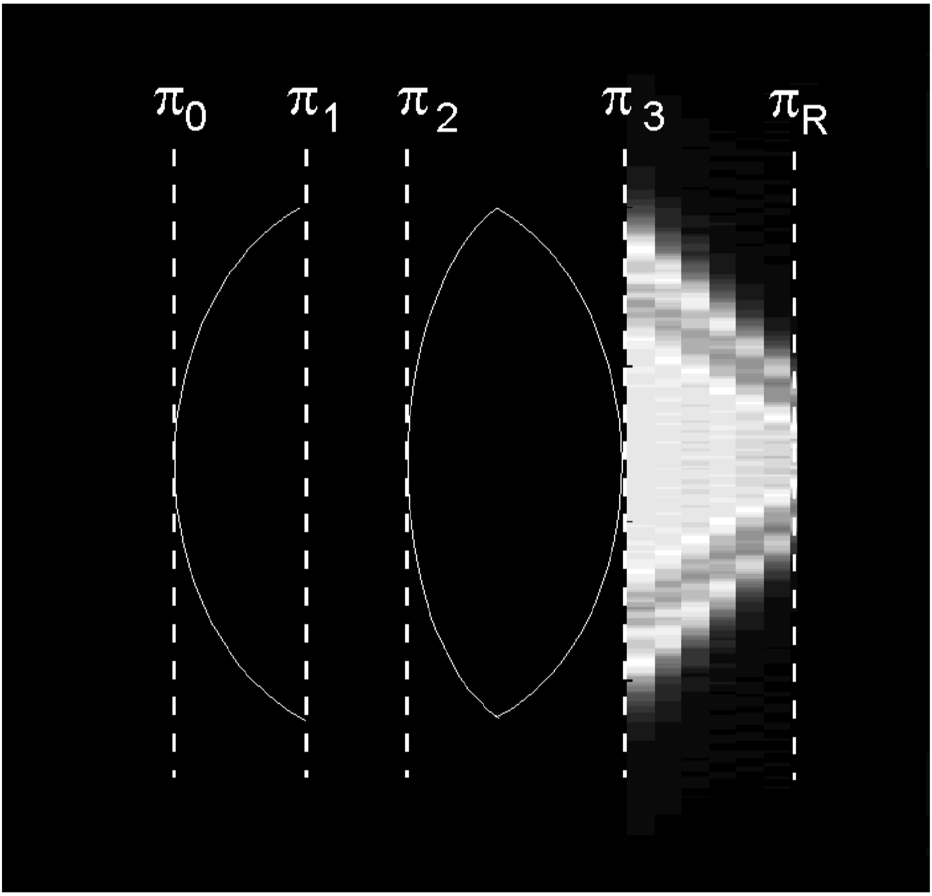

Figure 5. Evolution of the wave front (*a*) in a hyperopic eye.

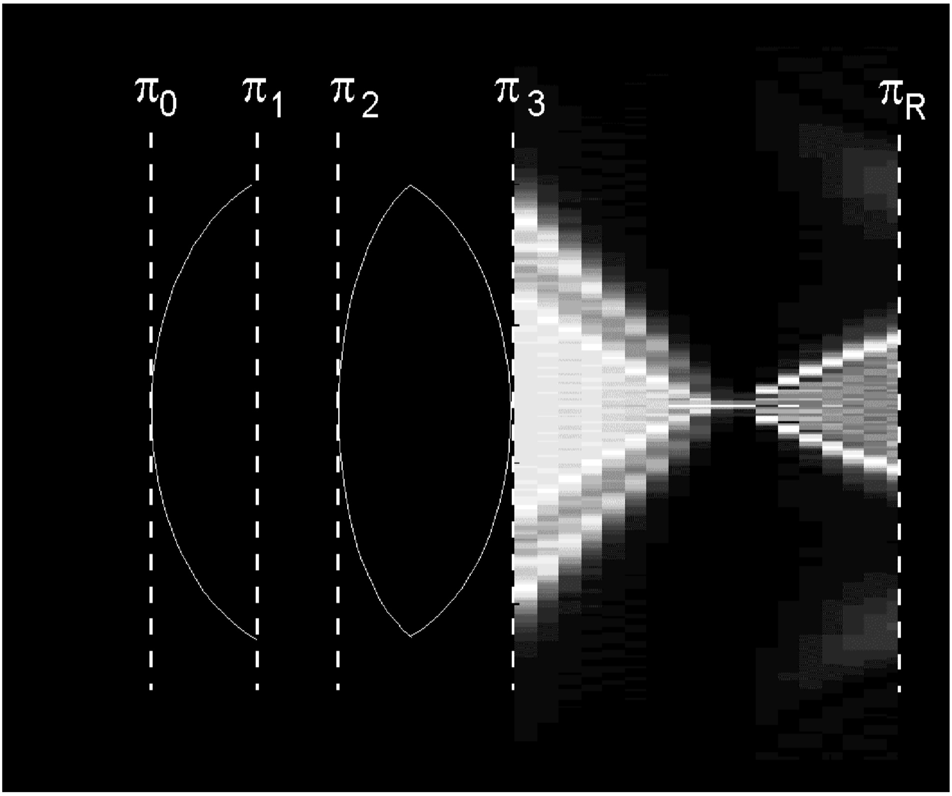

Figure 5(*b*). In a myopic eye.

possible to obtain theoretically the Fresnel diffraction in real eyes if we know the geometrical morphology of the eye. This methodology can be extended to the study of aberration properties of the human eye, using aspheric surfaces. To obtain more information about the optics of the eye, the shell structure (gradient of the refractive index) of the crystalline lens had to be incorporated [22, 23] The method introduced here can also be applied to the design and evaluation of ophthalmic compensations.

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