The Structure of the Tax-system and the Estimation of Labor Supply Models

Lennart Flood

Working Paper in Economics No. 1
April 1998

School of Economics and Commercial Law
Göteborg University

Abstract: *Using a repeated cross section for married prime age Swedish males for the years 1984, 1986 and 1988 produce drastically different labor supply elasticities. From 1984 to 1988 the Swedish tax system has reduced both tax levels and degree of progressivity, the numbers of kink-points have dropped from 18 to only 3. The numbers of individuals close to or at a kink point have a large influence on the estimated parameters, more individuals close to a kink imply larger estimated incentive effects. More kinks in the tax system imply a higher probability of finding individuals close to a kink.*

Key words: Labor supply and taxes, piece-wise linear taxes, kink points, differentiable approximation, Jacobean, measurement error, compensated elasticities.

JEL-classification: J22

Göteborg University
Department of Economics
405 30 Gothenburg
Box 640
Sweden

Tel:	+46-31-7731331
Fax:	+46-31-7731326
E-mail:	Lennart.Flood@economics.gu.se

1 Introduction.

During the last two decades there has been intensive research in the area of labor supply and taxes. Sophisticated estimation methods have been developed that use information about the whole budget set of each individual in a sample. Originally these methods were introduced by Burtless & Hausman (1978) and since then a large amount of empirical research has been based on this approach. The Burtless & Hausman approach is characterized by a detailed description of the income tax system. Since most income tax systems consist of piecewise-linear segments this results in piecewise-linear segments in the budget set. Therefore, this approach is sometimes called "piecewise-linear," PWL.

In this paper the question of the relationship between estimated income and wage responses and the structure of the tax system will be analyzed. The specific question in focus is whether a more progressive tax system implies higher estimated labor supply responses? The empirical base for this question is a series of labor supply estimates based on a Swedish data set, HINK, consisting of repeated cross sections from 1984 to 1992. During these years the Swedish tax system has changed dramatically from a system characterized by high marginal tax rates and a high degree of progressivity to a system with considerably lower levels and progressivity.

According to our results for a sample of prime age males, the estimated labor supply parameters change drastically over these years. From high positive compensated elasticities in 1984 to negative values in 1988 and thereafter. The purpose of this paper is to offer an explanation of these findings. The first part of the paper describes the characteristics of the modern approach to labor supply; the emphasis is on the role played by the tax system. Three approaches will be discussed. The first two are based on the differential approach, which approximate the tax system by a differentiable function and the third one is based on the PWL approach. The second part of the paper deals with an empirical illustration.

According to our findings the progressivity of the tax system affects the estimated wage and income parameters. Higher progressivity implies, ceteris paribus, higher estimated incentive effects.

2 Labor supply and taxes

The derivation of the PWL-model is a standard exercise and need not to be repeated here. The classic references are Burtless & Hausman (1978) and Blomquist (1983). An analytically convenient alternative to the PWL approach is based on a differentiable approximation to the tax system. This approach has been introduced by MaCurdy et. al. (1990) and applied on Swedish data in Flood & MaCurdy (1993). As demonstrated in Flood & MaCurdy and in a recent Monte Carlo study by Ericsson & Flood (1995), this approach is similar to the PWL-approach and if the approximation to the tax system is good the results will coincide with the results of the PWL-approach.

As a first step the differential approach without measurement error in hours will be considered. Using this simple framework will clearly demonstrate the role played by the tax structure in the estimation. Then, the assumption of no measurement error is dropped and again the structure of taxes is studied. Finally, the traditional PWL approach is discussed.

2.1 Differential approach, no measurement error.

To be able to exploit the role played by the tax structure, the simplest possible framework will be used. Assume that each individual choose hours of work by maximizing utility subject to a budget constraint

$$C = wh + Y + V - t(I) \qquad (1)$$

where earnings are given by hourly gross wage, w, times yearly hours of work, h. Non labor income consist of a taxable, Y, and a non-taxable, V, component. Finally the function t determines taxes based on taxable income, I = Y + wh -d, where d denotes deductions.

Utility maximization with quasi-concave preferences and a convex budget set determines the functional form of the labor supply function. Hours of work are given by

$$h=g(w',y,\nu) \qquad \nu\sim N(0,\sigma_\nu) \qquad (2)$$

Thus, preferred hours are given as a function of marginal wage rates, w', and virtual incomes, y; ν represents heterogeneity in preferences. This paper is based on the random intercept model, thus ν enters additively and both $(\partial h/\partial\nu)$ and $(\partial\nu/\partial h)$ are positive.

The marginal wage rate is given as $w(1-\tau')$, where τ' is the approximated marginal tax rate. The virtual income, y, is given as $V+Y+\tau'wh-\tau$, where τ is the total tax rate obtained by integration of τ', ($\tau = \int \tau'\, dI$).

Since no measurement errors in hours are assumed, observed hours, h*, are equal to preferred or optimal hours, h.

The likelihood function for h (for one individual) is given as

$$L = J\, f_\nu \qquad (3)$$

where f_ν is the ordinary density

$$f_\nu = \frac{1}{\sigma_\nu} \phi \left(\frac{\nu}{\sigma_\nu} \right) \qquad (4)$$

and J is the Jacobean given as

$$J \equiv \frac{\partial \nu}{\partial h} = 1 + \left(\frac{\partial g}{\partial w'} - \frac{\partial g}{\partial y} h \right) \tau'' w^2 \qquad (5)$$

where τ'' is the second derivative of the tax function τ.

Thus, the likelihood consists of two components. The first one, the Jacobean, corrects for endogeneity and the second one is the ordinary density.

How does the structure of the tax system enter this likelihood? Of course, taxes enter into the variables marginal wages and virtual income, but, more important, also through the variable τ'', which enters the Jacobean. The second derivative of the tax system, τ'', is a measure of the degree of progressiveness on the part of the tax system where the individual is observed.

Given $[(\partial g/\partial w') - (\partial g/\partial y)h] > 0$ (the Slutsky condition) and $\tau''>0$ (convex budget set), it follows that an increase in τ'' implies an increase in the likelihood function.

It is instructive to consider the value of τ'' when the tax system is piecewise linear. For simplicity, consider a tax system with one kink. (Not very unlike the Swedish system after 1990). Using the approach suggested in MaCurdy et. al. we define the following parameters: a lower tax limit, 0.3, and a higher, 0.5. The break point where the tax rate changes is given for incomes equal to 180000 kr (≈ $ 27 000). Finally define a parameter, σ, that indicates how fast the tax rate changes at the break point.

Then the approximated marginal tax rate is

$$\tau'=\{\Phi(I/\sigma)-\Phi([I-180]/\sigma)\}0.3+\Phi([I-180]/\sigma)0.5 \qquad (6)$$

where Φ is the normal distribution function. The functions, τ and τ'', are given by integration and differentiation respectively of τ' with respect to I.

For all individuals on an internal segment τ'' will be zero and J is therefore equal to one. Thus, if no individuals are observed at a kink the model reduces to a specification without heterogeneity and there will be no correction for endogeneity. Of course if all individuals are observed on an internal segment, w' and y are not endogenous and there is no need to include a correction term. On the other hand for individuals at a kink, the value of τ" can be large. The size of τ" is partly decided by the σ-parameter; for σ approaching zero, τ" will approach infinity. Thus, in the limit we have the situation of a kink. Consequently the likelihood function for these individuals, at or very close to a kink, will be much higher than for individuals on an internal segment.

(Figure 1. About here)

Figure 1, shows the approximation around the kink point using two different values of σ. A value like 0.2 has been used in previous work (e.g Flood & MaCurdy) and this results in a very close approximation. As a result τ" is non zero at a very limited range, (179 500 - 180 500 kr). Thus, given a wage of 100 kr/hour the range in hours is just 10 hours/year. Thus, very few individuals could be expected in this small range. But, for these individuals the value of τ" can be substantial and consequently these individuals will have a large contribution to the likelihood function. A larger value of σ results in a much smoother approximation and also in a larger range of non-zero τ". The values of τ" are smaller but more individuals can be expected to have a positive value.

The Jacobean serves as a penalty function giving individuals different weights depending on the distance from a kink. Further, not only do individuals close to a kink obtain a higher likelihood but also, as has been discussed in e.g MaCurdy et. al., the Slutsky constraint should be fulfilled for these individuals in order for the likelihood to be coherent.

It is instructive to consider the likelihood for individuals at a kink and individuals on an internal segment. For individuals at (or close to) a kink the likelihood is given by (3) but for other individuals the likelihood is reduced to the density part (4) only. Thus, in the typical case, the Jacobean will enter the likelihood only for a very few individuals (only very few individuals could be expected to be close to a kink). Furthermore, there will be a tendency for these few individuals to have a large impact on the estimated α and β parameters. The reason for this, as said above, is that they typically will have higher likelihoods (weights) than other individuals. However, it is not only the case that they will have a large impact, but it is also possible to say something about the expected effects on the estimated parameters. The linear model, which has been used in much research, can serve as an illustration. Here the Jacobean is equal to $1+(\alpha-\beta h)\tau''w^2$ and obviously it increases with a positive α and a negative β, (the Jacobean is in fact maximized for $\alpha=\infty$ and $\beta=-\infty$), individuals at a kink will generate higher incentive effects.

The maximum likelihood procedure tries to find those α and β parameters that maximize the likelihood function. In this maximizing process certain individuals will be "more important" than others; these individuals also produce parameters with the "right" signs. The individuals at a kink drive the results. This does not imply that there could be no incentive effects even if individuals close to a kink are not included. However, individuals close to a kink result in higher estimated incentive effects.

To summarize, the crucial characteristic of the tax system is the number of kinks. The more kinks that are present in the tax system, the higher will be the probability of observing individuals at or close to a kink and a higher proportion of individuals at a kink implies larger estimated incentive effects. This result can also be generalized for a smooth and differentiable tax system. More individuals located on a part where the degree of progressivity is higher implies larger estimated incentive effects.

The discussion above implies at least three results with an important empirical consequence. First, since many western countries have experienced tax reforms that have reduced the degree of progressivity, an expected result from using repeated cross sectional data is that the estimated incentive effects become smaller over time. The second result relates to the choice of σ-parameter. The choice of σ is important for the estimation results and the crucial question is how the choice of σ affects the J" term. A larger σ should reduce the number of individuals with $\tau'' =0$ but it will also reduce the size of the largest τ''-values, therefore the outcome is indeterminate. The final result relates to the sensitivity of the estimated incentive effects. Since the incentive results are driven by the number of individuals close to a kink the results can be very sensitive depending on dropping a few individuals at or close to a kink.

The last point is directly related to the assumption of no measurement error in hours. If measurement error is allowed for the result should be less sensitive to deleting a few individuals close to a kink. Even if measurement error is allowed for in most studies; An interesting example is found in a recent study by Brännäs & Karlsson (1996) in which a simple labor supply model is estimated without measurement error and also without the heterogeneity assumption. Note that the structure of the tax system enters because of the heterogeneity assumption. The findings in this paper supports the discussion above. Allowing for heterogeneity but not for measurement error produces positive compensated elasticities. However, when the model is reestimated without the heterogeneity assumption the elasticities become negative. Since, dropping the heterogeneity assumption means that the Jacobean is not included in the likelihood function the tax system does not produce the ”right” sign on elasticities.

Thus, one suggestion for obtaining more "robust" results would be to drop the assumption of heterogeneity. However, this results in misspecified models. Both heterogeneity and measurement error is an integral part of a labor supply model, and according to many empirical results they both have a significant effect.

2.2 Differential approach and measurement error.

Assuming a labor supply model with heterogeneity $\nu \sim N(0,\sigma_\nu^2)$ and additive measurement error, $h^*=h+\varepsilon$ and $\varepsilon \sim N(0,\sigma_\varepsilon^2)$, the likelihood function is obtained by integration over the total range of hours.

$$L = \int_0^{\max(h)} f_\varepsilon \, J \, f_\nu \, dh \tag{7}$$

where max(h) is some upper limit in hours of work and f_ε is the density

$$f_\varepsilon = \frac{1}{\sigma_\varepsilon} \phi \left(\frac{h^* - h}{\sigma_\varepsilon} \right) \tag{8}$$

This likelihood consists of three components, two densities and the Jacobean. Again, the Jacobean plays a strategic role and again the likelihood is completely different depending on the distance from a kink.

In order to illustrate the shape of the function two different examples (individuals) are considered; first an individual observed at an internal segment (far below the kink) and secondly an individual close to the kink.

Assume a linear model and the two-segment tax system introduced above. The following parameters are used; α=0.03, β=-0.02, σ_ν=0.5, σ_ε=0.5, ν=0.2 and ε=0.3. The distance from a kink can be decided by choosing different wage rates. For each hour, h, in the range 0-3000 define:

1. Taxable income as I = wh
2. Approximated tax rates τ(h), τ'(h) and τ''(h)
3. Marginal wage rate, w'(h) and virtual income, y(h)
4. The function $f(h)=f_\varepsilon(h)\ J(h)\ f_\nu(h)$

Figure 2 plots the function f(h) for a gross wage of 60 kr/hour. Since this choice of the gross wage gives optimal hours far below a kink for the range in hours from 0-3000 the second derivative τ''(h) is always zero and therefore the Jacobean plays no role at all. This explains the smooth shape of f(h).

(Figure 2. About here)

In sharp contrast to this smooth shape study figure 3, which shows the situation when optimal hours and observed hours are close to a kink. By selecting w=90, optimal hours are to be found just to the left of the kink.

(Figure 3. About here)

As discussed above, the shape of f(h) is now dominated by the Jacobean. The τ''-term will be positive close to the kink and therefore $1+(\alpha-\beta h)\ \tau''w^2$ will be a large positive number (since w^2 is a large number).

However, the likelihood is the area under the f(h)-function and even if there is a peak the area can be small. This is exactly the reason why measurement error reduce the sensitivity of observations close to a kink. However, typically the likelihood is still larger for individuals at a kink . As an illustration, for the first case the area is 0.439 and for the second 0.667.

Again, the σ-parameter is important and figure 3 is based on σ=1.0. Choosing a smaller σ generates a more concentrated distribution around the kink.[1]

To summarize, measurement error reduces the role played by the Jacobean. However, the influence can still be dramatic and therefore the main result holds. A progressive tax system implies larger estimated incentive effects.

2.3 The PWL approach

The PWL approach can be considered as a special case of the differential approach. The differential approach collapses to the PWL case as σ goes to zero.

The PWL-likelihood for the two segment tax system is given as

$$L = \int_{-\hat{h}_1}^{H-\hat{h}_1} f(h^*-\hat{h}_1,\nu)d\nu \int_{H-\hat{h}_1}^{H-\hat{h}_2} g(\varepsilon,\nu)d\nu \int_{H-\hat{h}_2}^{H_{max}-\hat{h}_2} f(h^*-\hat{h}_2,\nu)d\nu \qquad (9)$$

where the three terms are probabilities for the first segment, kink and second segment respectively.

[1] This also raises a practical problem of the numerical evaluation of the integral. It is difficult to obtain an accurate value of the integral for small σ - values. Two fortan subroutines that seem to work are DCADRE in the NAG-library and DQDAGS in IMSL. However, for small values of σ (like 0.2) these routines often indicate that the precision in the calculated integral is low. This is one argument for using relatively large σ-values, which of course result in a less accurate approximation of the tax system.

H is kink hours, f(.,.) and g(.,.) are bivariate densities and $\hat{h}_i = \alpha w'_i + \beta y_i$, i=1,2.

The interesting term from our perspective is the probability of a kink. Since ε and ν are assumed independent the kink-term can be written

$$p(kink) = \frac{1}{\sigma_\varepsilon}\phi\left(\frac{h^* - H}{\sigma_\varepsilon}\right)\left[\Phi\left(\frac{H - \hat{h}_2}{\sigma_\nu}\right) - \Phi\left(\frac{H - \hat{h}_1}{\sigma_\nu}\right)\right] \qquad (10)$$

For what values of α and β is this probability maximized? That is, then is the difference $(H - \hat{h}_2) - (H - \hat{h}_1)$ maximized?

Substitution and simplification gives

$$\alpha(w'_1 - w'_2) - \beta(y_2 - y_1)$$

using the definition of virtual income gives

$$\alpha(w'_1 - w'_2) - \beta(w'_1 - w'_2)H$$

and finally

$$(w'_1 - w'_2)(\alpha - \beta H) \qquad (11)$$

Given convexity, the first term is positive, and the second term is the Slutsky effect. Thus, maximization of the kink probability is obtained by maximizing the Slutsky effect.

A further result, that is discussed in MaCurdy et. al. is that the kink probability is only defined for a positive Slutsky effect. Thus, statistical testing is difficult in the PWL-framework. The differential approach differs slightly regarding the Slutsky constraint.

$$(\alpha - \beta h) \geq -\left(\frac{\partial \tau'}{\partial I} w^2 \right)^{-1} \qquad (12)$$

Thus, in contrast to the PWL-method, the Slutsky term can be negative, but not too negative. If σ approaches zero the differential approach coincide with the PWL.

3 Estimation of male labor supply

The data comes from the Swedish HINK-survey from Statistics Sweden and here the years 1984, 1986, and 1988 will be used. We concentrate on the subsample of employed prime aged married males. The sample size is relatively large, 1843, 2022 and 2166 respectively for the different years. The tax-system has changed considerably during these years. Since 1984, a year characterized by a high level of taxes and a high degree of progressivity (18 internal kinks), both the level and progressivity have gone down considerably. In 1986 the number of internal kinks were 8 and finally in 1988 only 3. Since we are particularly interested in the number of kinks and their effects on estimated coefficients these changes in the tax structure provide an ideal setting for such a study.

Based on these data the following functional form has been used

$$h^* = \mu + \alpha \log(w') + \beta(y/w') + Z\gamma + \nu \qquad \nu \sim N(0,\sigma_\nu^2) \qquad (13)$$

The Z-vector includes variables such as children, age, region, unemployment and sector. The variable sector refers to whether the individual is employed in the state, municipal or private sector. To obtain a more flexible specification the " parameter is also a function of the three sectors.

The marginal tax rates have been approximated by a differentiable function. For 1984 and 1986 a part of the tax system has been approximated using a third degree polynomial and for 1988 the approximation is obtained without the use of a polynomial. Note, that the approximations used have been obtained by using $\sigma = 0.2$, but in the estimations we also use $\sigma = 1.0$.

The first sets of results are presented in Table 1 and are based on the differential approach without measurement error.

The results in row (1) give the estimated elasticities for the two different σ-values, for the three different years. The most striking result is the large decrease in the estimated compensated elasticities. For σ=0.2 the elasticities drop from 0.078 in 1984 to 0.038 in 1986 and finally to zero in 1988. The result for 1988 has been obtained by imposing the Slutsky restriction. Similar results, but with somewhat larger compensated elasticities, is also found when σ=1.0 is used. Our interpretation of this result is that it reflects directly the number of kinks (the degree of progressivity) in the tax system.

It is instructive to compare the distribution of the τ''-variable over the years. Table 2 shows the frequencies of τ'' for the different years using the two σ-values. For the first two years a minor portion is equal to zero, about 10%, but for the last year most individuals (88%) have a zero value. In 1984 and 1986 most individuals fall on the polynomial part of the tax system; this results in relatively small but positive τ''. If σ=1.0 is used, no individuals have a value of $\tau''>0.2$ and if σ=0.2 is used only very few individuals have values higher than 0.2 (six in 1984 and 1988 and one in 1986).

From Table 1 it follows that using σ=1.0 always results in larger compensated elasticities. An explanation for this result is that the number of individuals with $\tau''=0$ decreases when the larger σ-value is used.

The importance of the tax system is evident from the second row in Table 1. The model has been estimated for the years 1986 and 1988, given the 1984 tax system. This results in much larger compensated elasticities. For 1986 the compensated elasticity increases from 0.038 to 0.087 (for σ=0.2); the corresponding result for 1988 is an increase from zero to 0.031. Thus, when the tax system for 1984 is used the results for both 86 and 88 are more in agreement with the 1984 results.

Finally, in row three, the impact of the individuals close to a kink is shown. All individuals with a value of $\tau''>0.1$ have been deleted. The results indicate an extreme sensitivity. By deleting nine individuals out of 1843 in 1984, the results change from a (significant) compensated elasticity of 0.078 to a (insignificant) value of 0.003. Thus by deleting less than a half percent of the sample the results change almost 100%. Similar results also apply for the other years.

The next set of results refers to the model with measurement error and these results are displayed in Table 3. The results follow a similar pattern as the results reported in Table 1. The incentive effects drop for later years and using 1984 taxes results in higher incentive effects. It is interesting that the results in Table 3 are similar to the results in Table 1; this is particularly the case when σ=1.0 is used. According to our estimation results the measurement component is smaller than the heterogeneity component. This is of course a result that is specific for the data set used in this study. However, if a linear model is used instead, measurement error turns out to be quite important. But the linear model with measurement error also produce higher incentive effects compared to the model without measurement error.

Finally consider the question of individuals close to a kink. It is not as straightforward to investigate the impact of "outliers" for the model with measurement error. In general it should be expected that the influence of individuals close to a kink should be smaller compared to the model without measurement errors.

The final set of results refers to the PWL-approach and are presented in Table 4. As expected the results are quite similar to the differentiable approach with σ=0.2.

4 Summary and conclusion

The results in this study suggest that the crucial characteristic of the tax system is the number of kinks. The more kinks that are present in the tax system, the higher will be the probability of observing individuals at or close to a kink and a higher portion of individuals at a kink implies larger incentive effects.

It is possible to expand a bit on the importance of the functional form and the influences of the kinks. An expected result would be that the linear model is more "robust" with respect to individuals at the kinks. The reason would be that the Jacobean does not play the same dominating role in the likelihood function because the density component is larger (the fit is worse for the linear model). The important matter is the size of the Jacobean relative to the density term and since a less flexible form should increase the density term of the likelihood, the influence of the Jacobean should be less for the linear model. This result is also supported by our data. However, it does not imply that the linear model is a better choice since this model is very restrictive. For instance the linear model excludes backward bending labor supply curves.

Finally, it is possible to give an interpretation for earlier unpublished results found on our data. The compensated elasticities for females have been found to be negative for different subgroups and years. Thus, even for a year like 1984 with a high degree of progressivity, our results indicate negative compensated elasticities for females and for the years 1986 and 1988 these elasticities become more negative. This contradicts one of the main findings in the literature. However the tax system in 1984 was characterized by a relatively long initial segment. Females to a much larger degree than males fall on this segment because they have lower earnings. Thus, as expected, a large portion of females has a value of τ '' equal to zero. In 1984 and given σ=0.2, 13.2% of the males had a τ ''-value equal to zero; the corresponding results for females are almost 48%.

This leads to a concluding comment that not only does the number of kinks matters but also their location in the tax system. A long initial segment followed by a large degree of progressivity (large number of kinks) implies very different estimated supply elasticities compared to a system that is more progressive for smaller incomes.

References

Blomquist N,S., 1983, The Effect of Income on Male Labor Supply in Sweden. Journal of Public Economics, 22.

Brännäs, K & Karlsson., 1996, Estimating the Perceived Tax Scale within a Labour Supply Model, Economics Letters 52.

Burtless, G & Hausman, J., 1978, The Effect of Taxes on Labour Supply, Journal of Political Economy, 86

Ericson, P., & Flood L.R., 1995, A Monte Carlo Evaluation of Labor Supply Models, Working paper, Department of Economics University of Gothenburg.

Flood L.R. & MaCurdy, T., 1993, Work Disincentive Effects of Taxes: An Empirical Analysis of Swedish Men. Empirical Economics, Volume 22, issue 3, pages 431-460.

MaCurdy T & Green D & Paarsch H.,1990, Assessing Empirical Approaches for Analyzing Taxes and Labor Supply. Journal of Human Resources 25.

Table 1. Mean values of male labor supply elasticities 1984-88
Differential approach no measurement error

Standard errors in parentheses

(1) Elasticities

(2) Elasticities using 1984-year taxes

(3) Elasticities deleting individuals which has τ'' > 0.1

	1984			1986			1988		
	e_w (1)	e_y (2)	e_c (3)	e_w (4)	e_y (5)	e_c (6)	e_w (7)	e_y (8)	e_c (9)
(1) Elasticities σ = 0.2	0.056 (0.015)	-0.041 (0.012)	0.078 (0.017)	0.018 (0.019)	-0.036 (0.013)	0.038 (0.022)	-0.002 (0.002)	-0.003 (0.004)	(*) 0.000 (0.001)
σ = 1.0	0.076 (0.019)	-0.046 (0.012)	0.101 (0.022)	0.053 (0.021)	-0.048 (0.014)	0.080 (0.025)	0.000 (0.003)	-0.007 (0.006)	(*) 0.004 (0.004)
(2) 84 year taxes σ = 0.2	---	---	---	0.063 (0.018)	-0.043 (0.012)	0.087 (0.021)	0.017 (0.013)	-0.025 (0.008)	0.031 (0.015)
σ = 1.0	---	---	---	0.104 (0.021)	-0.055 (0.013)	0.135 (0.025)	0.034 (0.017)	-0.029 (0.008)	0.051 (0.018)
(3) J'' > 0.1 deleted σ = 0.2	-0.010 (0.019)	-0.024 (0.011)	0.003 (0.022)	-0.009 (0.020)	-0.028 (0.013)	0.006 (0.024)	-0.002 (0.002)	-0.003 (0.003)	(*) 0.000 (0.001)
σ = 1.0	0.058 (0.019)	-0.042 (0.012)	0.081 (0.022)	0.039 (0.021)	-0.044 (0.013)	0.064 (0.025)	0.000 (0.003)	-0.006 (0.005)	(*) 0.004 (0.003)

Note: All elasticities have been evaluated at h=2.0, w'=25 and y=90

e_w is the uncompensated wage elasticity
e_y is the income elasticity
e_c is the compensated elasticity

(*) The Slutsky constraint, α-β(h_i - y_i/w'_i) ≥ 0, has been imposed.

Table 2. Frequency of τ''-values

	1984		1986		1988	
	σ=0.2	**σ=1.0**	**σ=0.2**	**σ=1.0**	**σ=0.2**	**σ=1.0**
$\tau'' = 0$	244	207	196	160	2123	1913
$0 < \tau'' < 0.1$	1590	1635	1822	1861	37	253
$0.1 < \tau'' < 0.2$	3	1	3	1	0	0
$0.2 < \tau'' < 0.3$	5	0	0	0	6	0
$0.3 < \tau'' < 0.4$	0	0	0	0	0	0
$0.4 < \tau'' < 0.5$	0	0	1	0	0	0
$0.5 < \tau'' < 0.6$	1	0	0	0	0	0

Table 3. Mean values of male labor supply elasticities 1984-88
Differential approach measurement error
Standard errors in parentheses
(1) Elasticities
(2) Elasticities using 1984-year taxes
(3) Elasticities deleting individuals which has $\tau'' > 0.1$

	1984			1986			1988		
	e_w (1)	e_y (2)	e_c (3)	e_w (4)	e_y (5)	e_c (6)	e_w (7)	e_y (8)	e_c (9)
(1) Elasticities $\sigma = 0.2$	0.060 (0.017)	-0.040 (0.011)	0.083 (0.021)	0.023 (0.208)	-0.042 (0.013)	0.046 (0.024)	-0.002 (0.002	-0.001 (0.001	(*) -0.002 (0.002
$\sigma = 1.0$	0.078 (0.028)	-0.042 (0.012)	0.102 (0.030)	0.056 (0.022)	-0.051 (0.015)	0.084 (0.027)	-0.003 (0.003)	-0.001 (0.001)	(*) -0.002 (0.002)
(2) 84 year taxes $\sigma = 0.2$	______	______	______	0.088 (0.018)	-0.052 (0.012)	0.117 (0.021)	0.043 (0.025)	-0.037 (0.015)	0.063 (0.022)
$\sigma = 1.0$	______	______	______	0.104 (0.021)	-0.055 (0.013)	0.135 (0.025)	0.080 (0.015)	-0.044 (0.011)	0.104 (0.020)

Note: All elasticities have been evaluated at h=2.0, w'=25 and y=90

e_w is the uncompensated wage elasticity
e_y is the income elasticity
e_c is the compensated elasticity

(*) The Slutsky constraint, α-$\beta(h_i - y_i/w'_i) \geq 0$, has been imposed.

Table 4. Mean values of male labor supply elasticities 1984-88 PWL model with measurement error

	1984			1986			1988		
	e_w (1)	e_y (2)	e_c (3)	e_w (4)	e_y (5)	e_c (6)	e_w (7)	e_y (8)	e_c (9)
(1) Elasticities	0.089 (0.023)	-0.006 (0.018)	0.090 (0.032)	0.061 (0.009)	-0.006 (0.009)	0.064 (0.014)	0.011 (0.033)	-0.016 (0.035)	(*) 0.019 (0.052)

Note: All elasticities have been evaluated at h=2.0, w'=25 and y=90

e_w is the uncompensated wage elasticity
e_y is the income elasticity
e_c is the compensated elasticity

(*) The Slutsky constraint, $\alpha-\beta(h_i - y_i/w'_i) \geq 0$, has been imposed.

Table 5. Mean values elasticities generated data 1984-88
Differential approach no measurement error

Standard errors in parentheses

(2) Elasticities sample size 1000
(3) Elasticities sample size 2000
(4) Elasticities sample size 5000

	1984			1986			1988		
	e_w (1)	e_y (2)	e_c (3)	e_w (4)	e_y (5)	e_c (6)	e_w (7)	e_y (8)	e_c (9)
(2) 1000 observations σ = 0.2	-0.015 (0.021)	-0.011 (0.013)	0.002 (0.001)	-0.058 (0.015)	-0.020 (0.012)	-0.087 (0.014)	-0.012 (0.015)	-0.011 (0.011)	0.004 (0.005)
σ = 1.0	-0.015 (0.025)	-0.011 (0.014)	0.002 (0.023)	-0.056 (0.021)	0.009 (0.011)	-0.069 (0.018)	-0.007 (0.015)	-0.004 (0.012)	-0.001 (0.009)
(3) 2000 observations σ = 0.2	0.001 (0.017)	-0.010 (0.010)	0.016 (0.013)	-0.010 (0.013)	-0.002 (0.007)	-0.007 (0.007)	-0.004 (0.010)	-0.005 (0.008)	0.003 (0.004)
σ = 1.0	0.016 (0.020)	-0.016 (0.011)	0.040 (0.022)	-0.031 (0.014)	-0.007 (0.008)	-0.042 (0.015)	0.001 (0.011)	0.001 (0.009)	-0.001 (0.007)
(4) 5000 observations σ = 0.2	0.015 (0.010)	0.003 (0.006)	0.010 (0.007)	0.003 (0.007)	0.007 (0.003)	-0.008 (0.004)	0.009 (0.006)	0.004 (0.005)	0.003 (0.002)
σ = 1.0	0.030 (0.012)	-0.004 (0.007)	0.035 (0.014)	-0.006 (0.010)	0.010 (0.005)	-0.022 (0.011)	0.011 (0.007)	0.007 (0.005)	0.007 (0.004)

Note: All elasticities have been evaluated at h=2.0, w'=25 and y=90

e_w is the uncompensated wage elasticity
e_y is the income elasticity
e_c is the compensated elasticity

(*) The Slutsky constraint, α-β(h_i - y_i/w'_i) ≥ 0, has been imposed.

Table 6. Mean values elasticities generated data 1984-88
Differential approach measurement error

Standard errors in parentheses

(5) Elasticities sample size 1000

(6) Elasticities sample size 2000

(7) Elasticities sample size 5000

	1984			1986		
	e_w (1)	e_y (2)	e_c (3)	e_w (4)	e_y (5)	e_c (6)
(3) 1000 observations						
σ = 0.2	0.040 (0.040)	-0.025 (0.020)	0.077 (0.054)	-0.0 (0.015)	-0.0 (0.012)	-0.0 (0.014)
σ = 1.0	0.242 (0.071)	-0.104 (0.032)	0.398 (0.108)	-0.003 (0.073)	-0.004 (0.024)	-0.004 (0.103)
(4) 2000 observations						
σ = 0.2	0.072 (0.033)	-0.035 (0.016)	0.124 (0.047)	-0.019 (0.018)	0.004 (0.009)	-0.025 (0.021)
σ = 1.0	0.283 (0.052)	-0.118 (0.024)	0.460 (0.080)	-0.051 (0.028)	0.013 (0.011)	-0.070 (0.038)
(5) 5000 observations						
σ = 0.2	0.059 (0.017)	-0.014 (0.008)	0.081 (0.022)	0.002 (0.014)	0.008 (0.006)	-0.010 (0.017)
σ = 1.0	0.229 (0.028)	-0.085 (0.013)	0.356 (0.043)	-0.033 (0.026)	0.019 (0.009)	-0.062 (0.036)

Note: All elasticities have been evaluated at h=2.0, w'=25 and y=90

e_w is the uncompensated wage elasticity
e_y is the income elasticity
e_c is the compensated elasticity

(*) The Slutsky constraint, $\alpha-\beta(h_i - y_i/w'_i) \geq 0$, has been imposed.

Figure 1. Differential approximation to a piecewise linear tax system

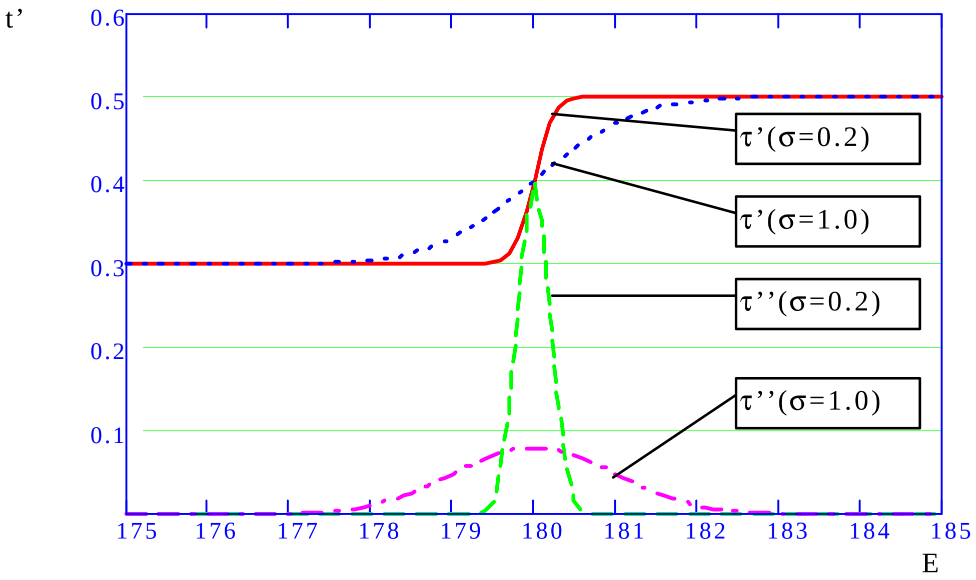

Figure 2. Optimal and observed hours far from a kink

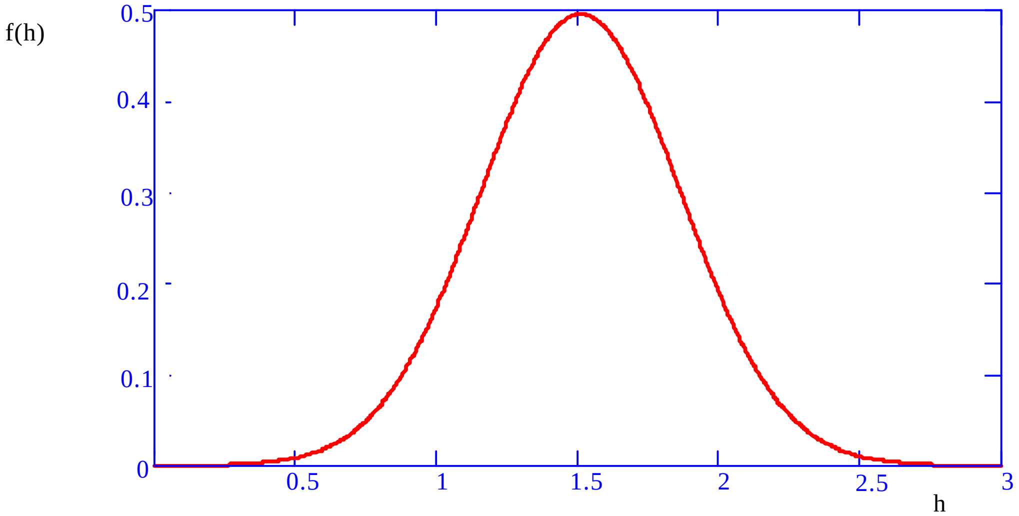

Figure 3. Optimal and observed hours close to a kink