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A 2D Finite Element Formulation for the Study of the High Frequency Behaviour of Wound Components

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Abstract—When the frequency of power converters increases, the wound components like inductors or transformers can no longer be supposed ideal. In this paper a formulation using 2D finite elements is presented for studying the high frequency coupled magnetic and electric field problem (eddy currents, proximity effects and stray capacitance). This formulation is based on a particular division of the 3D spatial domain which enables a 2D calculation.

I. INTRODUCTION

As power converters can work at the megahertz frequency level, the parasitic elements have to be taken into account in wound components like inductors and transformers. This paper is concerned with the formulation of the magnetic and electric field coupled phenomena (eddy currents, copper losses, stray capacitance). These phenomena are the cause of a large part of the high frequency parasitic behaviour of wound components.

A large number of papers about wound components for high frequency applications, have been published in the last years. There are different methods to study the magnetic and the electric phenomenon: the analytical approach which requires a simple geometry of the component or heavy geometrical approximation [1], the experimental approach which is based on measurements and requires prototypes [1], the field calculation approach which is based on the electric or the magnetic field calculation by a finite element method [2].

In most papers the electric and magnetic phenomena are studied separately. And yet these two phenomena are strongly coupled in magnetic components of high frequency converters. The purpose of this paper is to present a 2D finite element formulation which allows to study these coupled phenomena by solving the electric and magnetic field equations simultaneously. The electromagnetic structures considered here possess a translational symmetry. This method can be applied to those possessing an axial symmetry.

II. MAXWELL EQUATIONS

All the electromagnetic phenomena can be modelled by Maxwell equations:

$$\vec{\nabla} \times \vec{H} - \vec{J} - \frac{\partial \vec{D}}{\partial t} = \vec{0} \quad (1) \quad \vec{\nabla} \cdot \vec{B} = 0 \quad (3)$$

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = \vec{0} \quad (2) \quad \vec{\nabla} \cdot \vec{D} = \rho \quad (4)$$

Equations (1) and (3) allow to define fields vectors in terms of the well known magnetic vector potential \vec{A} and the electric scalar potential Φ .

The relations describing materials must be added to these relations:

$$\vec{D} = \epsilon \vec{E} \quad (5) \quad \vec{J} = \sigma \vec{E} \quad (6)$$

$$\vec{B} = \mu \vec{H} \quad (7)$$

where \vec{E} is the electric field, \vec{D} is the electric displacement, \vec{H} is the magnetic field, \vec{B} is the magnetic flux density, \vec{J} is the current density, σ is the electric conductivity, ϵ is the electric permittivity and μ is the magnetic permeability.

III. LOW FREQUENCY EQUATION

In general at low frequency the displacement current term can be neglected and following potentials equations are obtained:

$$\vec{\nabla} \times (\nu \vec{\nabla} \times \vec{A}) + \sigma \left(\frac{\partial}{\partial t} \vec{A} + \vec{\nabla} \Phi \right) = \vec{0} \quad (8)$$

ν is the inverse of permeability μ .

When the studied electromagnetic device is fed by a voltage supply, the equation of the electric circuits must be added. Consider for instance a device fed by a simple electric circuit with a supply voltage V_{ext} current in serie with an external R_{ext} and an inductance L_{ext} . The external circuit equation is:

$$V_{ext} = R_{ext} I + L_{ext} \frac{\partial I}{\partial t} + V_{wind} \quad (9)$$

where V_{wind} is the actual voltage applied on the winding of the device.

If the geometry of the device can be considered as translationally symmetric along the axis Oz a 2D formulation can be employed. The axis Oz is also the axis along which the current flows and only the component of the current density on this axis is not equal to zero. In this case the vector potential is expected to have the same direction as the current density and has only a unique non-zero component. The equation which governs the vector potential then is:

$$-\vec{\nabla}_{xy} \cdot (\nu \vec{\nabla}_{xy} A) = -\sigma \left(\frac{\partial A}{\partial t} + \frac{\partial}{\partial z} \Phi \right) = J \quad (10)$$

where A and J are the components of the vector potential and the current density along the z axis.

Now let consider the winding shown on figure 1. This winding is formed of 6 conductors and each conductor is numbered. Because all the conductors are connected in series, the total current in each conductor is equal to the current I supplying the winding. Two relations can be written to relate the local magnetic equation and the circuit equation:

$$I = \iint_{Sc(ic)} J dx dy \quad (11)$$

$$\frac{\partial \Phi}{\partial z} = -\frac{U(ic)}{L} \quad (12)$$

$U(ic)$ is the voltage between the two ends of the conductor (ic) of length L , $Sc(ic)$ is the cross-sectionnal area occupied by conductor (ic) [3]. From equations (10), (11) and (12) we can write:

$$I = \frac{U(ic)}{R(ic)} - \iint_{Sc(ic)} \sigma \frac{\partial A}{\partial t} dx dy \quad (13)$$

where $R(ic)$ is the dc-resistance of conductor (ic). And equation (10) can be written:

$$-\vec{\nabla}_{xy} \cdot (\nu \vec{\nabla}_{xy} A) + \sigma \left(\frac{\partial A}{\partial t} - \frac{U(ic)}{L} \right) = 0 \quad (14)$$

And finally the sum of the voltage drop on each conductor is equal to the voltage drop on the winding:

$$V_{wind} = \sum_{ic=1}^{nco} U(ic) \quad (15)$$

where nco is the number of conductors.

When a 2D finite element method is used to discretise the space, we obtain the global system of differential equations:

$$\begin{cases} [M_{mag}] \{A\} + [N_{mag}] \frac{\partial}{\partial t} \{A\} + [P_{mag}] \{U\} = 0 \\ [IR_{dc}] \{U\} - [Q_{mag}] \frac{\partial}{\partial t} \{A\} - I = 0 \\ [D] \{U\} + R_{ext} I + L_{ext} \frac{\partial}{\partial t} I = V_{ext} \end{cases} \quad (16)$$

Matrices $[M_{mag}]$, $[N_{mag}]$ are well-known matrices. They are derived from equation (14) by the standard Galerkin's method [3]. The contribution of each element and each conductor to the coupling matrix $[P_{mag}]$ is:

$$P_{mag}(in, ic)_e = \iint_{D^e} \sigma \frac{N_{in}}{L} dx dy \quad (17)$$

where (e) is the number of the element containing node (in) , (ic) is the conductor containing element (e) , N_{in} is the shape function associated to node (in) and element (e) , D^e is the area of element (e) . $[IR_{dc}]$ is the inverse of the diagonal matrix $[R_{dc}]$ containing the dc resistances of conductors. The elementary contribution to the coupling matrix $[Q_{mag}]$ is:

$$Q_{mag}(ic, in)_e = \iint_{D^e} \sigma N_{in} dx dy \quad (18)$$

Matrix $[D]$ is equal to the one-line matrix of nco coefficients equal to 1, $\{A\}$ is the vector of the nodal values of the z component of the vector potential A , and $\{U\}$ is the vector of the voltage drop on each conductor.

With this model we have simulated the behavior of the winding shown on figure 1, which contains 6 conductors. Figure 2 shows the variation of the ac resistance of the winding in function of the square of the excitation frequency. Figure 3 shows the voltage drop on each conductor at a frequency equal to 20 kHz.

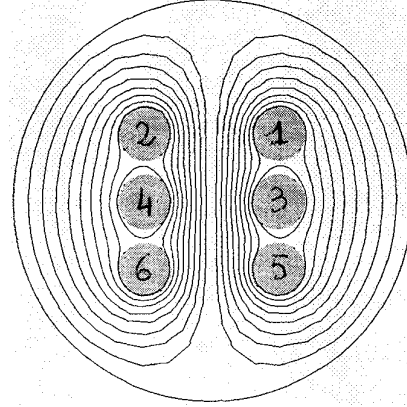


Fig. 1. The studied winding with 6 conductors in series and the magnetic field calculated at low frequency.

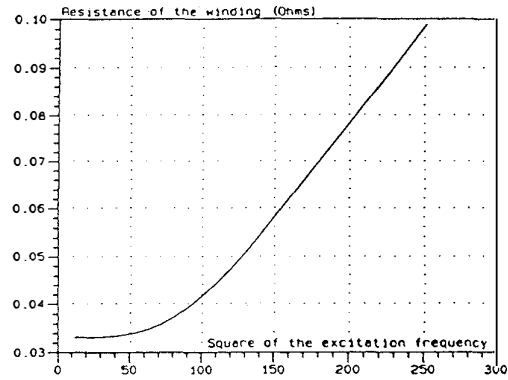


Fig. 2. The ac resistance of the studied winding in function of the square of the excitation frequency.

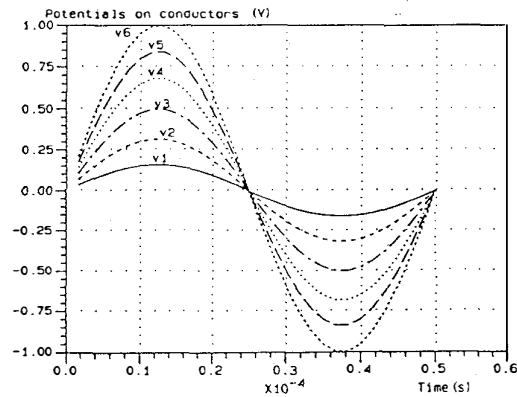


Fig. 3. The potential on each conductor of the studied winding calculated at a frequency of 20 kHz and for an applied voltage $V_{wind}=1$ V, $R_{ext}=0$, $L_{ext}=0$.

Qualitatively the response frequency calculated seems to be good and reproduce the characteristics of skin and proximity effects observed in transformers and inductances. Now consider the results on figure 3. It can be seen that the voltages on each conductor are not equal and the voltages are not distributed linearly along the winding. The differences between the voltages drop on conductors increase with the frequency.

If the conductors are not at the same voltage it means that between them an electric field is created (Fig. 3). The media between conductors having a permittivity ϵ , to this electric field corresponds an electric vector displacement \vec{D} . Since the voltage on conductors vary with time, a current displacement is created. This current is responsible of the capacitive effects between conductors. So in order to calculate these capacitive effects we must take into account this term which is often neglected in most of applications.

IV. MODELLING THE CAPACITIVE EFFECT

To model capacitive phenomena in the windings of electrical devices we suppose that the vector potential is parallel to the z axis. This hypothesis is correct as long as the main current flows along the z axis.

The electric field can be decomposed in two orthogonal vectors:

$$\vec{E} = \vec{E}_z + \vec{E}_{xy} \quad (19)$$

E_z is the component along the translation axis and E_{xy} is the component in the x,y plane perpendicular to Oz. For the E_z component we have:

$$E_z = -\frac{\partial}{\partial t} A - \frac{\partial \Phi}{\partial z} \quad (20)$$

And since the components A_x and A_y of the potential vector are neglected, we have:

$$\vec{E}_{xy} = -(\vec{\nabla} \Phi)_{x,y} \quad (21)$$

And if we rewrite equation (4) in the x,y plane we obtain:

$$\vec{\nabla} \cdot (\vec{\nabla} \Phi)_{x,y} = \rho \quad (22)$$

Outside conductors the charge density ρ is equal to zero. In conductors, where the coefficient $\tau = \epsilon/\sigma$, called electric relaxation time, is very small, we can consider that the charge density ρ is equal to zero too. Then each conductor is an equipotential domain in the x,y plane.

The potential of a conductor is not constant along the z axis. To take this fact into account, the z axis must be subdivided in sections. Each section corresponds to an x,y plane perpendicular to Oz.

Two electric vectors displacements must be considered: the z component $J_d z$ due to E_z and the x,y component $J_d xy$ due to E_{xy} . The component $J_d z$ flows along the translation axis z and the vector potential created by it remains in this axis. The component $J_d xy$ component creates a magnetic field in the z direction. Since this component of the induction is parallel to the axis of the conductor it does not induce any flux in the conductor nor voltage drop due to the variation of flux. We can then neglect the influence of this component on the global behavior of the device and not take

it into account. In this case the magnetic field has only two components and the vector potential only one along the z axis.

We can summarize all the hypothesis done:

- the vector potential A has a unique component along the axis of translation z,
- the charge density is null;
- the vector potential A and the scalar potential Φ are constant along z in a section.

V. HIGH FREQUENCY EQUATIONS

If we take into account these hypothesis we obtain the following local equations on each section:

$$\begin{cases} -\vec{\nabla} \cdot (\nu \vec{\nabla} A) = \sigma \left(\frac{\Delta U}{\Delta z} - \frac{\partial}{\partial t} A \right) + \epsilon \left(\frac{\partial}{\partial t} \frac{\Delta U}{\Delta z} - \frac{\partial^2}{\partial t^2} A \right) \\ \vec{\nabla} \cdot (\epsilon \vec{\nabla}_{x,y} \Phi) = 0 \end{cases} \quad (23)$$

where Δz is the length of one section, ΔU is the voltage drop between the ends of each conductor in each section.

After discretization the application of the Gallerkin's method to the first local equation gives the following system of differential equations:

$$[M_{\text{mag}}] \{A\} + [N_{\text{mag}}] \frac{\partial}{\partial t} \{A\} + [P_{\text{mag}}] \{\Delta U\} + [N_{\text{elec}}] \frac{\partial^2}{\partial t^2} \{A\} + [P_{\text{elec}}] \frac{\partial}{\partial t} \{\Delta U\} = 0 \quad (24)$$

which extends the similar equations (16) obtained at low frequency to higher frequency. $[M_{\text{mag}}]$, $[N_{\text{mag}}]$ and $[P_{\text{mag}}]$ have exactly the same expression as their homonymes in equation (16) except that here they are calculated only on one section. $[N_{\text{elec}}]$ and $[P_{\text{elec}}]$ are very similar to respectively matrices $[N_{\text{mag}}]$ and $[P_{\text{mag}}]$ only coefficients σ or μ are replaced by coefficient ϵ .

Consider now the winding on figure 1. Let's suppose that the z axis is subdivided in sections. The voltage drop $\delta U(ic)$ on each conductor (ic) is equal to:

$$\delta U(ic) = \sum_{it=1}^{nst} \Delta U(it, ic) \quad (25)$$

where $\Delta U(it, ic)$ is the voltage drop along the conductor (ic) in the (it) section and nst is the number of sections along the z axis. The potential in the middle of the section (it) of the conductor (ic) is equal to:

$$U(ic, it) = \sum_{i=1}^{ic-1} \delta U(i) + \sum_{kt=1}^{it-1} \Delta U(ic, kt) + \frac{1}{2} \Delta U(ic, it) + U_0 \quad (26)$$

U_0 is the reference potential of the external power supply. Since in each section a conductor is an equipotential domain, the second equation of (23) gives after a finite element discretization:

$$[M_{\text{elec}}] \{\Phi\} + [C_{\Delta}] \{\Delta U\} + [C_{\bullet}] U_0 = 0 \quad (27)$$

Matrix $[M_{\text{elec}}]$ is obtain from the second equation of (23) by the standard Gallerkin's method and is very similar to $[M_{\text{mag}}]$. $[C_{\Delta}]$ and $[C_0]$ are obtained from equations (23) and

(26), knowing that $U(ic, it)$ are the values of the potentials on sections of conductors, $\{\Phi\}$ is the vector of the nodal values of the electric potential scalar, $\{\Delta U\}$ is the vector of the voltage drop between the two ends of each section of conductor.

The current $Im(ic, it)$ flowing through each cross-sectional area of conductor is equal to the total current along the axis of the conductor which is parallel to z:

$$Im(ic, it) = \iint_{Sc(ic, it)} (J_z + \frac{\partial}{\partial t} D_z) dx dy \quad (28)$$

$Sc(ic, it)$ is the cross-sectional area of section (it) of conductor (ic). We obtain:

$$Im(ic, it) = \iint_{Sc(ic, it)} \sigma \left(\frac{\Delta U(ic, it)}{\Delta z} - \frac{\partial}{\partial t} A \right) + \varepsilon \left(\frac{\partial}{\partial t} \frac{\Delta U}{\Delta z} - \frac{\partial^2}{\partial t^2} A \right) dx dy \quad (29)$$

After discretization we obtain the following equation:

$$\{Im\} = [IR_{dc}]\{\Delta U\} + [C_{cc}] \frac{\partial}{\partial t} \{\Delta U\} - [Q_{mag}] \frac{\partial}{\partial t} \{A\} - [Q_{elec}] \frac{\partial^2}{\partial t^2} \{A\} \quad (30)$$

where $[IR_{dc}]$ is the inverse of the diagonal matrix of dc resistances of sections of conductors. $[C_{cc}]$ is the diagonal matrix of the capacitances between the two ends of sections of conductors. $[Q_{mag}]$ and $[Q_{elec}]$ are the coupling matrices whose generic component is similar to equation (18) giving the generic component of $[Q_{mag}]$ in the case of low frequency. $\{Im\}$ is the vector of the currents flowing through the cross-sectional area of sections of conductors.

Between two sections of conductors there is a loss of current flowing outside the conductor by capacitive effect. This loss of current can be evaluated by the relation:

$$Im(ic, it+1) - Im(ic, it) = - \iint_{\Sigma(ic, it)} \frac{\partial}{\partial t} \vec{D} \cdot \vec{n} d\Sigma \quad (31)$$

$\Sigma(ic, it)$ is the cylindrical external surface of section (it) of conductor (ic). Since the vector displacement is constant along z in a section of conductor:

$$Im(ic, it+1) - Im(ic, it) = - \Delta z \oint_{\Gamma(ic, it)} \varepsilon \frac{\partial}{\partial t} \vec{E}_{xy} \cdot \vec{n} dl \quad (32)$$

$\Gamma(ic, it)$ is the trace of $\Sigma(ic, it)$ in the plane x,y and Δz is the length of the section (it). After discretization the following equation is obtained:

$$[PP]\{Im\} + [FF] \frac{\partial}{\partial t} \{\Phi\} = 0 \quad (33)$$

The matrix $[PP]$ can be deduced easily from equation (32). The second term of the left-side of equation (33) is equal to the derivative of charge accumulated in the sections of conductors.

To all these equations we must add the circuit equation (9). The voltage drop between the two ends of the windings is equal to the sum of all the voltages applied on sections of conductors $\Delta U(ic, it)$:

$$V_{wind} = \sum_{ic=1}^{nc} \sum_{it=1}^{nst} \Delta U(ic, it) \quad (34)$$

The matrix equation obtained is similar to the last matrix equation of global system (16) obtained at low frequency.

The final global system of differential equations (35) is an extension of the system of equations (16). I is the total current in the winding. All of the equations of system (35) are formally the extension of system (16) except the fourth one. In fact this equation is the extension of relations between the currents on conductors. In low frequency they are all equal to the current I , at higher frequency they are different because of the capacitive effect.

$$\left\{ \begin{array}{l} [M_{mag}]\{A\} + [N_{mag}] \frac{\partial}{\partial t} \{A\} + [P_{mag}]\{\Delta U\} + \\ [N_{elec}] \frac{\partial^2}{\partial t^2} \{A\} + [P_{elec}]\{\Delta U\} = 0 \\ [M_{elec}]\{\Phi\} + [C_{\Delta}]\{\Delta U\} + [C_0]U_0 = 0 \\ [IR_{dc}]\{\Delta U\} - [Q_{mag}] \frac{\partial}{\partial t} \{A\} - \\ [Q_{elec}] \frac{\partial^2}{\partial t^2} \{A\} + [C_{cc}] \frac{\partial}{\partial t} \{\Delta U\} - \{I_m\} = 0 \\ [PP]\{I_m\} + [FF] \frac{\partial}{\partial t} \{\Phi\} = 0 \\ [D]\{\Delta U\} + R_{ext}I + L_{ext} \frac{\partial}{\partial t} I = V_{ext} \end{array} \right. \quad (35)$$

VI. CONCLUSION

For studying the high frequency parasitic behavior of wound components, we have developed a 2D finite element formulation, based on a particular division of the spatial study domain and three basic hypothesis. This modelisation allows to take into account the magnetic and the electric field coupled phenomena that appears in windings of electrical devices when they are supplied by high frequency voltages. This method must enable the calculation of the electric scalar potential, the magnetic vector potential and the global quantities like currents and voltages along the windings.

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