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Importance of direction of vibration on the onset of Soret-driven convection under gravity or weightlessness

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Abstract. This paper considers the influence of the direction of vibration on the stability threshold of two-dimensional Soret-driven convection. The configuration is an infinite layer filled with a binary mixture, which can be heated from below or from above. The limiting case of high-frequency and small-amplitude vibration is considered for which the time-averaged formulation has been adopted. The linear stability analysis of the quasi-mechanical equilibrium shows that the problem depends on five non-dimensional parameters. These include the thermal Rayleigh number (Ra_{T}), the vibrational parameter (R), the Prandtl number (Pr), the Lewis number (Le), the separation ratio (S) and the orientation of vibration with respect to the horizontal heated plate (α). For different sets of parameters, the bifurcation diagrams are plotted $Ra_{\mathrm{c}} = f(S)$ and $k_{\mathrm{c}} = g(S)$, which are the critical thermal Rayleigh and the critical wave numbers, respectively. Our results indicate that, relative to the classical case of static gravity, vibration may affect all regions in Ra_{c}-S stability diagram. In the case of mono-cellular convection, by using a regular perturbation method, a closed-form relation for the critical Rayleigh number is found. Several physical situations in the presence or in the absence of gravity (micro-gravity) are discussed.

1 Introduction

One of the most recent subjects in space science is the research on the behaviour of materials in the micro-gravity environment. Zero-gravity hydrodynamics has emerged as a new field of fundamental studies and has continued to develop intensively. The production of new materials forms the basis of most technological improvements. Consequently, possibilities of experiments in weightlessness may help to improve the production of traditional materials and may create opportunities for production of new materials.

It was thought that thermosolutal convection, which can produce undesirable effects in solidification of binary alloys and solutions, would be totally eliminated in a space laboratory. However, early studies under micro-gravity conditions cast doubt on this idea [1].

It has been shown that a spacecraft in orbit is subject to many disturbing influences of human as well as equipment origin. These influences result in the appearance of residual accelerations, which are commonly called "g-jitter". As a first step, g-jitter may be modelled as mono-frequency oscillations [2].

In broad terms, the subject of thermo-vibration convection concerns the appearance of a mean flow in a confined cavity filled with a fluid having temperature non-homogeneities [3]. This kind of convection induced by time-dependent body forces is distinguished from the gravity-induced convection in that it may exist under micro-gravity conditions.

The influence of time-dependent gravitational acceleration on the onset of Rayleigh-Bénard convection has been the subject of numerous theoretical, numerical and experimental studies. For a review of Russian research on this domain see, for example, the comprehensive book of Gershuni and Lyubimov [4] and for some early development of this subject the reviews of Davis [5], Ostrach [6] and Nelson [7] are highly recommended. The study of convection in a binary mixture with Soret effect provides us with more interesting instability mechanisms and pattern formation phenomena, which are absent in a single-component fluid. Under the Soret effect a concentration gradient is established as a result of a temperature gradient [8].

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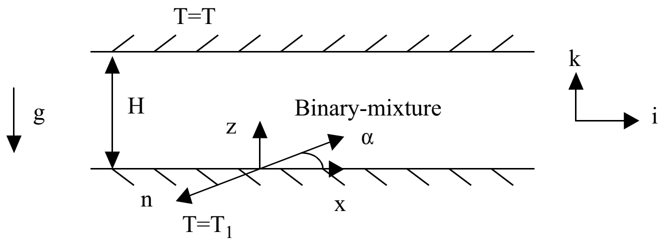

Fig. 1. Problem geometry.

Terrones and Chen [9] consider the two-component Rayleigh-Bénard problem under the effect of vertical vibration. They consider different boundary conditions (stress-free and rigid), and the cross-diffusion effects are included in their studies. Gershuni *et al.* [10,11] consider the same problem under the effect of high frequency and small amplitude of vibration. They find that, for a small wave number, the stability is stationary and they obtain an analytical relation for the stability threshold. Myznikova and Smorodin [12] and Smorodin *et al.* [13] investigate the same problem under finite frequency.

It should be indicated that Saunders *et al.* [14] studied the same problem without Soret effect; they further considered the stress-free case. A list of useful papers on the applied aspect of solidifications under the effect of vibration may be found in [14]. Also, Jue and Ramaswarmy [15] study numerically the thermo-solutal convection in a confined cavity in the Rayleigh-Bénard geometry.

It has been suggested elsewhere [7] that not only the amplitude of g-jitter but also its orientation contributes significantly to the acceleration environment in a space laboratory. In the present paper we study the influence of directions of vibration on the stability threshold of a horizontal binary mixture layer with the Soret effect. Different situations based on vibrational and gravitational mechanisms are considered. The results are presented in stability diagrams for stationary or oscillatory bifurcations.

2 Governing equations

The geometry of the problem consists of an infinite horizontal layer containing a binary mixture, Figure 1. The horizontal boundaries are assumed rigid, impermeable and are kept at different constant temperatures. The Soret effect, which describes the coupling between a temperature gradient and a resulting mass flux in a multi-component system, is taken into account. The layer is subjected to a harmonic mono-frequency vibration, which is characterized by amplitude b, frequency ω, and its direction $\mathbf{n}$. The governing equations are written in the coordinate system linked to the oscillating layer which allows us to replace the gravitational acceleration $\mathbf{g}$ by $\mathbf{g} + b\omega^2 \sin\omega t\, \mathbf{n}$. In the case of high-frequency and small-amplitude vibration, the asymptotic behaviour of the system may be approximated with the superposition of fields having slow evolution with time and fields having rapid evolution with respect to time [16]. It is important to note that the average of fields with rapid evolution with respect to time over the vibration period $(2\pi/\omega)$ is zero. The fields with rapid time evolution (v', T', c') may be expressed as a function of slow fields by using some hypotheses [4].

The time-averaged system of equations in dimensionless form in the framework of the Boussinesq approximation can be written as

$$\begin{aligned}
&\boldsymbol{\nabla}\cdot\mathbf{V}^* = 0\,,\\
&\frac{\partial \mathbf{V}^*}{\partial t^*} + (\mathbf{V}^*\cdot\boldsymbol{\nabla})\mathbf{V}^* = -\boldsymbol{\nabla}P^*\\
&\quad + Pr\,\boldsymbol{\nabla}^2\mathbf{V}^* + Ra\,Pr(T^*+C^*)\mathbf{k}\\
&\quad + Ra_{\mathrm{v}}\,Pr(\mathbf{W}\cdot\boldsymbol{\nabla})\left[(T^*+C^*)\mathbf{n}-\mathbf{W}\right]\,,\\
&\frac{\partial T^*}{\partial t^*} + \mathbf{V}^*\cdot\boldsymbol{\nabla}T^* = \boldsymbol{\nabla}^2 T^*\,,\\
&\frac{\partial C^*}{\partial t^*} + \mathbf{V}^*\cdot\boldsymbol{\nabla}C^* = \frac{1}{Le}(\boldsymbol{\nabla}^2C^* - S\boldsymbol{\nabla}^2T^*)\,,\\
&\boldsymbol{\nabla}\cdot\mathbf{W}^* = 0\,,\\
&\boldsymbol{\nabla}\times\mathbf{W}^* = \boldsymbol{\nabla}(T^*+C^*)\times\mathbf{n}\,,
\end{aligned} \tag{1}$$

where $\mathbf{V}^*$, T^* and C^* are velocity, temperature and mass fraction fields and $\mathbf{W}^*$ represents the solenoidal vector.

The corresponding boundary conditions are

$$\begin{aligned}
&z^*=0,\ 1: V^*=0, \qquad W_z^*=0,\\
&\frac{\partial C^*}{\partial z^*} - S\frac{\partial T^*}{\partial z^*} = 0\,,\\
&T^*(x, z^*=0)=1, \qquad T^*(x^*, z^*=1)=0.
\end{aligned} \tag{2}$$

The non-dimensional parameters are defined as

$$Ra_{\mathrm{T}} = \frac{g\beta_{\mathrm{T}}\Delta T H^3}{\nu a}, \quad Ra_{\mathrm{v}} = \frac{(b\omega\beta_{\mathrm{T}}\Delta T H)^2}{2\nu a} = Ra_{\mathrm{T}}^2\, R^2\,,$$

$$S = -\frac{D_{\mathrm{T}}\beta_{\mathrm{c}}}{\beta_{\mathrm{T}}}, \quad Pr = \frac{\nu}{a}, \quad Le = \frac{a}{D}\,.$$

Ra_{T} is the thermal Rayleigh number, Ra_{v} is the vibrational Rayleigh number, S the separation factor, Pr the Prandtl number and Le is the Lewis number. In these non-dimensional parameters, β_{T}, ν, a, β_{c}, D_{T} and D represent the coefficient of thermal expansion, kinematic viscosity, the thermal diffusivity, the composition expansion coefficient, the thermo-diffusion coefficient and finally the mass diffusivity, respectively.

3 Linear stability analysis

The orientation of vibration relative to the temperature gradient plays an important role in the existence of mechanical equilibrium. If the axis of vibration is not parallel to the temperature gradient, a mechanical quasi-equilibrium solution exists for an infinite horizontal layer. This solution is characterized by

$$\begin{aligned}
T_0 &= 1 - z^*,\\
C_0 &= -S\,z^* + \mathrm{const}\,,\\
W_{0x^*} &= -\,(1+S)\ (z^*-1/2)\ \cos\alpha\,,\\
W_{0z^*} &= 0\,,\\
V_0 &= 0\,.
\end{aligned} \tag{3}$$

For the linear stability analysis, the temperature, concentration and solenoidal fields are perturbed around this quasi-equilibrium state. Two-dimensional disturbances are considered which are developed in normal modes:

$$(\psi, T', C', F) = (\phi(z)\,, \qquad \theta(z)\,, \qquad \xi(z)\,, \\ f(z)) \exp(-\lambda t + ikx).$$

In the above equation, ϕ, θ, ξ and f represent the amplitude of the velocity stream function, the temperature, the concentration and the solenoidal stream function disturbances, respectively;

$$\begin{aligned} -\lambda D\phi &= Pr\ D^2\phi - ikRa\,Pr(\theta+\xi) \\ &\quad -ikRa_{\mathrm{v}}\ Pr(1{+}S)\left[(\theta{+}\xi)\cos\alpha - \frac{\mathrm{d}f}{\mathrm{d}z}\right]\cos\alpha \\ &\quad +k^2 Ra_{\mathrm{v}}\ Pr(1+S)f\sin\alpha\ , \\ &\quad -\lambda\theta + ik\phi = D\theta\ , \\ -\lambda\xi + ikS\phi &= \frac{1}{Le}D(\xi - S\theta)\ , \\ Df &= \left(\frac{\mathrm{d}\theta}{\mathrm{d}z} + \frac{\mathrm{d}\xi}{\mathrm{d}z}\right)\cos\alpha - ik(\theta+\xi)\sin\alpha\ . \end{aligned} \tag{4}$$

The corresponding boundary conditions are

$$z = 0, \quad 1: \phi = \phi' = 0, \quad \theta = 0, \quad \frac{\mathrm{d}\xi}{\mathrm{d}z} - S\frac{\mathrm{d}\theta}{\mathrm{d}z} = 0\ . \tag{5}$$

A Galerkin method is used to solve (4-5). The trial functions are chosen as follows:

$$\begin{aligned} \phi &= \sum_{i=1}^{N} \phi_i (z - z^2)^2 z^{i-1}, \\ \theta &= \sum_{i=1}^{N} \theta_i (z - z^2) z^{i-1}, \\ \xi &= \xi_0 + \sum_{i=1}^{N} \xi_i z^i \left(\frac{z}{i+1} - \frac{z^2}{i+2}\right), \\ q &= \sum_{i=1}^{N} q_i z^{i-1}(z - z^2)\ . \end{aligned} \tag{6}$$

It should be noted that in order to facilitate the numerical study, the transformation $q = \xi - S\theta$ is made. The solution of system (4) with boundary conditions (5) leads to a spectral problem in which λ is related to the important parameters of the problem:

$$\lambda = \lambda(Ra_{\mathrm{T}}, Ra_{\mathrm{v}}, Pr, Le, S, k, \alpha)\ .$$

Generally, λ is a complex number ($\lambda = \lambda_r + i\lambda_i$). For a stationary bifurcation, the stability domain is determined by setting $\lambda = 0$. In the case of an oscillatory bifurcation, the stability domain is determined by considering $\lambda_r = 0$ (λ_i represents the frequency of oscillatory bifurcation).

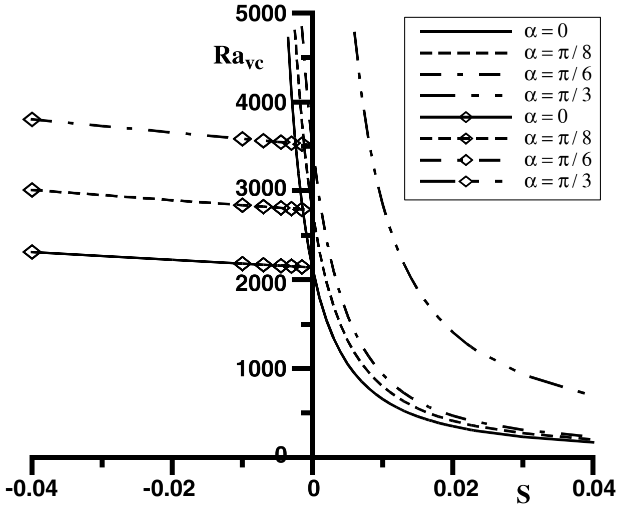

Fig. 2. Stability diagram for the onset of thermo-solutal convection under micro-gravity conditions for different directions of vibration for $Le = 100$, $Pr = 6.7$.

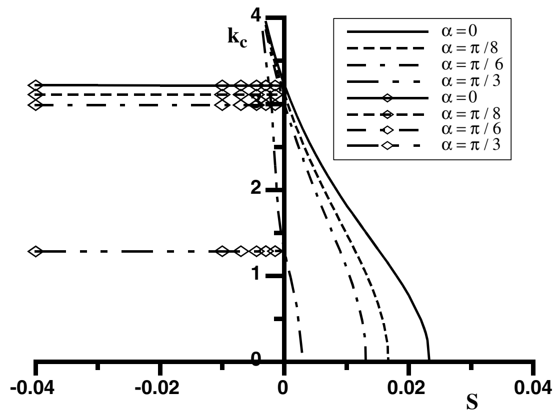

Fig. 3. Effect of the direction of vibration on the critical wave number in micro-gravity, $Le = 100$, $Pr = 6.7$.

4 Thermo-solutal convection under micro-gravity conditions ($\mathrm{Ra_T} = 0$)

In this case the convective-induced mechanism is due to mechanical vibration. The results of stability analysis under high-frequency and small-amplitude vibration are presented in Figure 2 (the truncation order for calculation is four and the validation is made with the result of Gershuni [4] for the onset of convection under microgravity, for example for $\alpha = 0$ the precision is $2.56\ 10^{-5}$). It may be concluded from this figure that a positive separation factor ($S > 0$) has a destabilizing effect on the quasi-equilibrium solution. Further, it is obvious that the direction of vibration has an important effect on the stability of the problem. Increasing the direction of vibration (α) has a stabilizing effect on the onset of convection under micro-gravity conditions. Figure 3 shows that if S is increased, the critical wave number decreases. For this region, increasing the direction of vibration reduces the wave number. It can be concluded from the results of stability analysis that the

Table 1. Influence of the direction of vibration on the critical parameters of oscillatory bifurcation for $S = -0.001$, $Le = 100$ and $Pr = 6.7$.

α	Ra_{Tco}	k_{co}	ω_0
0	2142.94	3.21	0.59
$\pi/8$	2788.08	3.11	0.57
$\pi/6$	3521.53	2.98	0.56
$\pi/4$	8284.81	2.33	0.49

long-wave mode is the preferred convective pattern. For this reason, a regular perturbation analysis with the wave number as a small parameter is performed (the details are given in App. A). The result of this analysis can be presented in the following form:

$$Ra_{\mathrm{vc}} = \frac{720}{LeS\,(1+S)\;\cos^2\alpha} \quad (k \to 0)\,. \tag{7}$$

For the direction of vibration perpendicular to the temperature gradient, we obtain the result given by Gershuni [10].

In order to know from which value of S the long-wave mode ($k \to 0$) may be achieved, we followed the procedure proposed by Nield [17] and Knobloch [18], which leads us to the following relation:

$$S_{k\to 0} = \frac{1}{Le\left(\dfrac{831}{1900} + \dfrac{189}{190}\tan^2\alpha\right) - 1}\,. \tag{8}$$

The theoretical results are in good agreement with the numerical results given in Figure 3. For negative separation ratios, the onset of convection can be in the form of stationary or oscillatory bifurcation. Figure 2 shows that the oscillatory bifurcation takes form before the stationary bifurcation. In the interval of $S \in (0, -0.04)$, which is chosen for the reason of a realizable experiment, the Ra_{Tco} increases slightly with increasing absolute value of S. From a pattern formation point of view, it is evident that in the interval studied the critical wave number is almost insensitive to the separation factor, cf. Figure 3. It should be added that increasing the direction of vibration has a stabilizing effect. At the same time the oscillatory frequency decreases with increasing the direction of vibration. Some of the results are summarized in Table 1.

5 Thermo-solutal convection under the simultaneous action of vibration and gravitation

When the direction of vibration is not vertical ($\alpha \neq \pi/2$), convective motion in all regions of the stability diagram is possible. Two different cases are considered, namely the stationary bifurcation and the oscillatory bifurcation.

5.1 Stationary bifurcation

The results of linear stability analysis for a layer heated from below are presented in the stability diagram Ra_{Tc}-S,

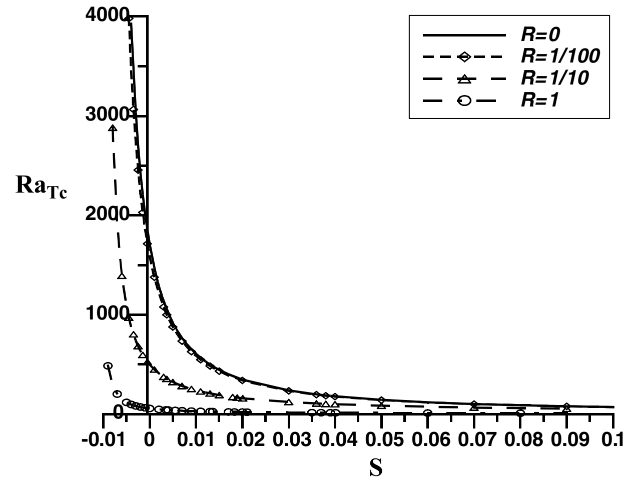

Fig. 4. Stability diagram for the onset of thermo-solutal convection for different vibrational parameters for $Le = 100$ and $\alpha = \pi/6$.

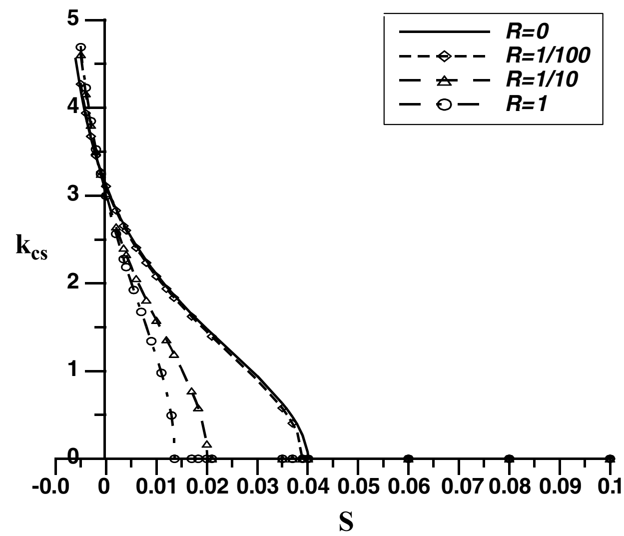

Fig. 5. Influence of vibrational parameter on the critical wave number for $Le = 100$ and $\alpha = \pi/6$.

see Figure 4. The influence of the vibration parameter R on the threshold of stability is shown for $Le = 100$ and $\alpha = \pi/6$. It is clear that increasing R has a destabilizing effect on the quasi-equilibrium solution. For the same range of parameters, Figure 5 illustrates the influence of R on the critical wave number in the k_{c}-S diagram. One can conclude that increasing the vibrational parameter reduces the critical wave number. As the long-wave mode is the dominant convection mode in this situation, by following the method described in Section 4, we obtain the following relation for the onset of convection for $k \to 0$ (for details refer to App. A):

$$Ra_{\mathrm{Tc}} + (1+S)Ra_{\mathrm{v}}\cos^2\alpha = \frac{720}{SLe}, \qquad (Ra_{\mathrm{v}} = R^2 Ra_{\mathrm{T}}^2). \tag{9}$$

The influence of the angle of vibration with heated boundary for a fixed value of the vibration parameter R is shown

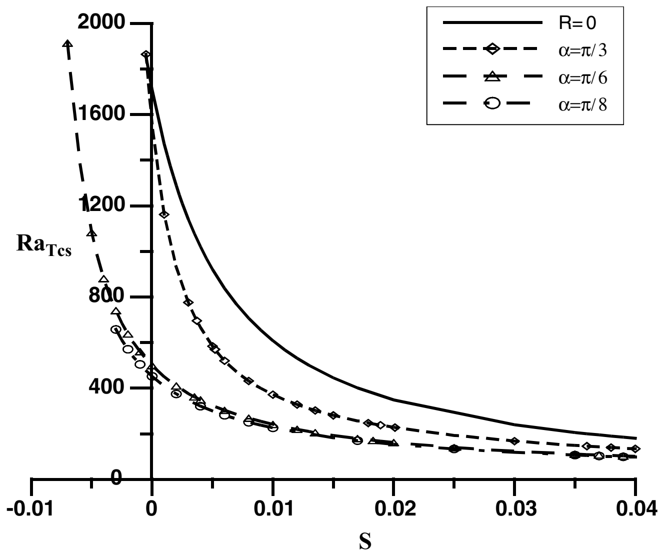

Fig. 6. Influence of the directions of vibration on the quasi-equilibrium solution in the presence of the gravity field for the layer heated from below for $Le = 100$, $Pr = 6.7$ and $R = 1$.

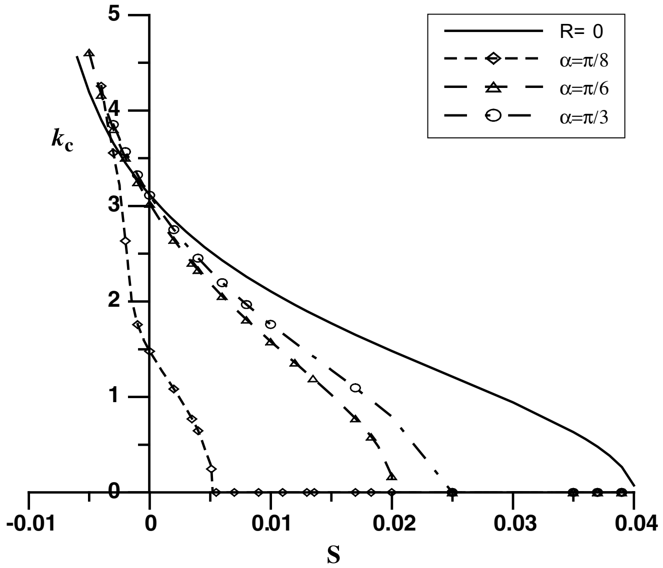

Fig. 7. Influence of the directions of vibration in the presence of the gravity field on the critical wave number for the layer heated from below for $Le = 100$, $Pr = 6.7$ and $R = 1$.

in Figure 6. The results indicate that increasing the direction of vibration can stabilize the quasi-equilibrium solution. The formation of the long-wave mode may be delayed by increasing the direction of vibration, see Figure 7.

5.2 The oscillatory bifurcation

The results of the linear stability analysis are presented in Table 2, which shows that for negative values of S, there exists an oscillatory bifurcation. It may be observed that the critical values of the Rayleigh number for the oscillatory bifurcation (Ra_{Tco}) are less than the critical

Table 2. Influence of S on the onset of stationary and oscillatory bifurcations for $\alpha = \pi/6$, $Le = 100$, $Pr = 6.7$ and $R = 1/100$.

S	k_{cst}	Ra_{Tst}	k_{co}	Ra_{Tco}	ω_0
−0.0005	3.1845	1714.6193	3.100	1600.6593	0.3750
−0.001	3.2687	1860.14065	3.1014	1603.0991	0.5569
−0.0015	3.3589	2028.4102	3.1024	1604.6990	0.6973
−0.002	3.4558	2224.7157	3.103	1605.9551	0.8155
−0.0025	3.5607	2456.0176	3.1035	1607.0415	0.9195
−0.003	3.6749	2731.6365	3.1038	1608.0329	1.0133

Table 3. Influence of S on the onset of stationary and oscillatory bifurcations for $\alpha = \pi/8$, $Le = 100$, $Pr = 6.7$ and $R = 1/100$.

S	k_{cst}	Ra_{Tst}	k_{co}	Ra_{Tco}	ω_0
−0.0005	3.194	1681.9239	3.1095	1571.9292	0.3763
−0.001	3.2793	1822.0779	3.1109	1577.2811	0.5587
−0.0015	3.3701	1983.7151	3.1119	1575.8300	0.6995
−0.002	3.4678	2171.7224	3.1126	1577.0504	0.8180
−0.0025	3.5736	2392.5063	3.1130	1578.1088	0.9222
−0.003	3.688	2654.5781	3.1134	1579.0765	1.0162

values of Rayleigh number for the stationary bifurcations (Ra_{Tcst}). The results are calculated for $\alpha = \pi/6$, $Le = 100$, $Pr = 6.7$ and $R = 1/100$. Reducing the direction of vibration destabilizes the quasi-equilibrium solution and slightly increases the oscillatory frequency, cf. Table 3.

6 Conclusions

In this paper, the influence of the direction of vibration on the onset of thermo-solutal convection is considered. The geometry of the problem is an infinite layer with rigid and impermeable boundaries; the layer can be heated from below or above. The Soret effect is included in governing equations. Linear stability analysis is performed which shows that the thermal stability of the problem depends on the following parameters: Ra_{T}, Ra_{v}, Le, Pr, α and S. In the case of micro-gravity conditions for different sets of parameters, the bifurcation diagrams are plotted as $Ra_{\mathrm{vc}} = F(S)$ and $k_{\mathrm{c}} = G(S)$, which are the critical vibrational Rayleigh and the critical wave numbers respectively. It is proved analytically that in the region $S \in [-1, 0]$, there is no possibility of long-wave mode. It is shown that in this region, there is a possibility of multi-cellular oscillatory convection, which is formed before the stationary convection. The negative Soret coefficient has a stabilizing effect. The effect of the direction of vibration in this region is studied and its effect on the frequency of oscillatory bifurcation is discussed. For the region with $S > 0$, the onset of convection is in the form of stationary multi-cellular or mono-cellular convection. Increasing the direction of vibration has a stabilizing effect on the quasi-equilibrium solution. In the case of vibrational effect in the presence of gravity field, our results show that for the direction of vibration, which is not, parallel to the temperature gradient, increasing the vibrational parameter R has a destabilizing effect on the quasi-equilibrium solution. In addition, it is shown that the long-wave mode is the dominant convection pattern. We find an analytical relation, which is in good agreement with the numerical results. As the vibration is of high frequency, the oscillatory frequency may be easily distinguished. We have equally investigated the effect of the direction of vibration on the oscillatory onset of convection. Our results show that increasing the direction of vibration has a stabilizing effect on the quasi-equilibrium solution and the oscillatory frequency is diminished.

Appendix A. Onset of convection at vanishing value of the wave number

In this section, the stability analysis of the problem in the case of long-wave mode is analyzed. As was explained earlier, the Soret effect influences drastically the critical value of the thermal Rayleigh number. Indeed, it is possible to be much more precise and to find analytically the critical Rayleigh number as a function of (Le, Ra_{v}, S, α). In order to obtain the criterion for the onset of convection in long-wave mode, the regular perturbation method is used. The eigenvalue equation (4) is expanded in powers of k. For this reason, the variables are developed as follows:

$$\phi(z) = \sum_{n=0}^{\infty} k^n \phi_n(z)\,, \qquad \theta(z) = \sum_{n=0}^{\infty} k^n \theta_n(z),$$
$$\xi(z) = \sum_{n=0}^{\infty} k^n \xi_n\,,$$
$$f(z) = \sum_{n=0}^{\infty} k^n f_n(z)\,, \qquad \lambda(z) = \sum_{n=0}^{\infty} k^n \lambda_n(z)\,.$$

For (k^0), we obtain the following system:

$$(k^0)\ :\ \begin{cases} -\lambda_0 \dfrac{\mathrm{d}^2\phi_0}{\mathrm{d}z^2} = Pr\dfrac{\mathrm{d}^4\phi_0}{\mathrm{d}z^4}\,, \\ -\lambda_0\theta_0 = \dfrac{\mathrm{d}^2\theta_0}{\mathrm{d}z^2}\,, \\ -\lambda_0\xi_0 = \dfrac{1}{Le}\left(\dfrac{\mathrm{d}^2\xi_0}{\mathrm{d}z^2} - S\dfrac{\mathrm{d}^2\theta_0}{\mathrm{d}z^2}\right), \\ \dfrac{\mathrm{d}^2 f_0}{\mathrm{d}z^2} = \left(\dfrac{\mathrm{d}\theta_0}{\mathrm{d}z} + \dfrac{\mathrm{d}\xi_0}{\mathrm{d}z}\right)\cos\alpha\,, \end{cases}$$

$z = 0,\ 1 : \phi_0 = \frac{\mathrm{d}\phi_0}{\mathrm{d}z} = \theta_0 = f_0,\quad \frac{\mathrm{d}\xi_0}{\mathrm{d}z} - S\frac{\mathrm{d}\theta_0}{\mathrm{d}z} = 0$, which results in $\lambda_0 = 0$, $\phi_0 = 0$, $\theta_0 = 0$, $f_0 = 0$ and $\xi_0 = \text{const}$.

For (k^1), the system reads

$$(k^1)\ :\ \begin{cases} \dfrac{\mathrm{d}^4\phi_1}{\mathrm{d}z^4} = i\xi_0\left[Ra_{\mathrm{T}} + Ra_{\mathrm{v}}(1+S)\cos^2\alpha\right]\,, \\ \dfrac{\mathrm{d}^2\theta_1}{\mathrm{d}z^2} = 0\,, \\ \dfrac{1}{Le}\left(\dfrac{\mathrm{d}^2\xi_1}{\mathrm{d}z^2} - S\dfrac{\mathrm{d}^2\theta_1}{\mathrm{d}z^2}\right) = -\lambda_1\xi_0\,, \\ \dfrac{\mathrm{d}^2 f_1}{\mathrm{d}z^2} = \left(\dfrac{\mathrm{d}\theta_1}{\mathrm{d}z} + \dfrac{\mathrm{d}\xi_1}{\mathrm{d}z}\right)\cos\alpha - i\xi_0\sin\alpha\,, \end{cases}$$

for which we may find

$$\lambda_1 = 0, \quad \theta_1 = 0, \quad \xi_1 = \text{const}, \quad f_1 = i\xi_0 z(1-z)\sin\alpha/2$$

and $\phi_1 = i\xi_0 z^2(z-1)^2[Ra + (1+\varepsilon)Ra_{\mathrm{v}}\cos^2\alpha]/24$.

For (k^2), the following system of equation can be obtained:

$$(k^2)\ :\ \begin{cases} \dfrac{\mathrm{d}^4\phi_2}{\mathrm{d}z^4} = i\xi_1\left[Ra_{\mathrm{T}} + Ra_{\mathrm{v}}(1+S)\cos^2\alpha\right]\,, \\ \dfrac{\mathrm{d}^2\theta_2}{\mathrm{d}z^2} = i\phi_1\,, \\ \dfrac{1}{Le}\left(\dfrac{\mathrm{d}^2\xi_2}{\mathrm{d}z^2} - S\dfrac{\mathrm{d}^2\theta_2}{\mathrm{d}z^2}\right) = \left(\dfrac{1}{Le} - \lambda_2\right)\xi_0 + iS\phi_1\,, \\ \dfrac{\mathrm{d}^2 f_2}{\mathrm{d}z^2} = \left(\dfrac{\mathrm{d}\theta_2}{\mathrm{d}z} + \dfrac{\mathrm{d}\xi_2}{\mathrm{d}z}\right)\cos\alpha - i\xi_1\sin\alpha\,. \end{cases}$$

By invoking the solvability condition $(\frac{1}{Le} - \lambda_2)\xi_0 + i\,S\int_0^1 \phi_1 \mathrm{d}z = 0$ and using the ϕ_1 expression, we find

$$\lambda_2 = \frac{1}{Le} - \frac{S}{720}\left[Ra_{\mathrm{T}} + (1+S)Ra_{\mathrm{v}}\cos^2\alpha\right]\,.$$

We note that $\lambda_2 \in \Re$, which means that the instability is of stationary type; for marginal stability λ_2 is set to zero and we obtain

$$Ra_{\mathrm{Tc}} + (1+S)Ra_{\mathrm{v}}\cos^2\alpha = \frac{720}{SLe}.$$

For the particular case of the mono-cellular convection in microgravity ($Ra_{\mathrm{T}} = 0$) we obtain

$$Ra_{\mathrm{vc}} = \frac{720}{LeS\,(1+S)\,\cos^2\alpha}\,.$$

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