



Open Archive Toulouse Archive Ouverte

OATAO is an open access repository that collects the work of Toulouse researchers and makes it freely available over the web where possible

This is an author's version published in: <http://oatao.univ-toulouse.fr/20082>

To cite this version:

Tahiri, Ayoub and Ladevèze, David and Chiron, Pascale[✉] and Archimède, Bernard[✉] and Lhuissier, Ludovic *Reservoir management using a network flow optimization model considering quadratic convex cost functions on arcs.* (2018) *Water Resources Management*, 32 (10). 3505-3518. ISSN 0920-4741

Any correspondence concerning this service should be sent to the repository administrator: tech-oatao@listes-diff.inp-toulouse.fr

Reservoir Management Using a Network Flow Optimization Model Considering Quadratic Convex Cost Functions on Arcs

Ayoub Tahiri¹  · David Ladeveze² · Pascale Chiron¹ · Bernard Archimede¹ · Ludovic Lhuissier²

Abstract The allocation of water resources between different users is a hard task for water managers because they must deal with conflicting objectives. The main objective is to obtain the most accurate distribution of the resource and the associated circulating flows through the system. This induces the need for a river basin optimization model that provides optimized results. This article presents a network flow optimization model to solve the water allocation problem in water resource systems. Managing a water system consists in providing water in the right proportion, at the right place and at the right time. Time expanded network allows to take into consideration the temporal dimension in the decision making. Since linear cost functions on arcs present many limitations and are not realistic, quadratic convex cost functions on arcs are considered here. The optimization algorithm developed herein extend the cycle canceling algorithm developed for linear cost functions. The methodology is applied to manage the three reservoirs of *La Haute-Vilaine's* watershed located in the north west of France to protect a three vulnerable areas from flooding. The results obtained with the algorithm are compared to a reference scenario which consists in considering reservoirs transparent. The results show that the algorithm succeeds in managing the reservoir releases efficiently and keeps the flow rates below the vigilance flow in the vulnerable areas.

Keywords Optimization models · Quadratic convex minimum cost network flow problem · Reservoir operations · Water resources planning and management

✉ Ayoub Tahiri
ayoub.tahiri@enit.fr

¹ Ecole nationale d'ingénieurs de Tarbes, Tarbes, France

² Compagnie d'Aménagement des Coteaux de Gascogne, Tarbes, France

1 Introduction

Nowadays, more consideration is given to save water and adopt better water management policies instead of developing new sources of water. This trend is actually due to the expensive cost of water infrastructures and their negative impact on the environment (Labadie John 2004), and the fact that many storage projects worldwide are not economically beneficial because of their mismanagement (World Commission on Dams 2000). Thus, water management is a pertinent strategy for meeting growing water demands, improving performance and efficiency of water systems, and preserving the environment (Higgins John and Gary 1999).

The allocation of water resources between different users is a hard task for water managers because they must satisfy both water demands for human activities and environmental goals. Managing a complex water resource system often involves conflicting objectives, which implies the use of integrated operational strategies. The main objective is to obtain the most accurate distribution of the resource and the associated circulating flows through the system. The optimal management and operations of a water resources system consists in maximizing benefits, minimizing costs, satisfying the required flows in the river and storing water in reservoirs, responding to water demands, avoiding floods, and preserving the quality of water. These management requirements cause a need for river basin optimization model that provides prescriptive results as well as allows the system managers to easily modify operating policy.

Intensive research on the use of optimization models to reservoir systems has been done. Labadie John (2004) assessed the state of the art in optimization of reservoir system management and operations with a strong emphasis on optimization of multireservoir systems. Wurbs Ralph (1993) reviewed reservoir system simulation and optimization models and contributed to a better understanding of modeling tools which could help the practitioner in choosing the appropriate model. Deepti and Madalena (2010) discussed simulation, optimization and combined simulation–optimization modeling approaches and provide an overview of their applications reported in the literature. Simonovic Slobodan (1992) discussed in details reservoir system methods and explained the reasons for the gap between theory and practice, and suggested some solutions to overcome them.

In the literature, different optimization techniques were used for the optimization of complex water resource systems. Linear programming (LP) are widely used for optimization of complex water resource systems in both management cases: drought (Tu et al. 2003) and flood (Needham et al. 2000). Dynamic programming (DP) is also widespread used in complex water system (Kumar Nagesh et al. 2010). Its formulation is based on the Bellman principle of optimality (Bellman 1956). The inconvenient of DP is the combinatorial explosion, also called the “dimensionality curse” occurring for large systems (Labadie John 2004). Other optimization techniques were used for complex water resource systems, such as stochastic optimization (Uditha and Ricardo 2007) or genetic algorithm (Daniel and Mays Larry 2015).

Optimal coordination of multiple objectives for river basin systems requires the assistance of computer modeling tools to make rational management decisions. Various softwares exist today to solve the problems associated with water management such as AQUATOOL (Andreu et al. 1996) and MODSIM (Labadie John et al. 2000).

Network optimization algorithms (Evans 1992; Ahuja Ravindra et al. 1988; Du and Pardalos Panos 1993) are often used for a large number of real-world applications, for example: communications, computer science, transportation, construction projects, and supply chain management. Network models are also used for water management because they are

intuitive, can be solved very quickly and suitable for solving large multi-reservoir multi-period allocation problems (Kuczera 1993). The water balance optimization problem can be represented as a directed graph on which graph theory algorithms may be applied. Nodes stand for convergence points, diversion points, demand locations and water sources, and arcs represent reservoir releases, channel flows, carryover storage and withdrawals. Moreover, flow values on arcs may change over time, and are referred as “dynamic flows” in the literature (Fulkerson Delbert 1966), and additionally, flows do not travel instantaneously through a network but require a certain amount of time to travel each arc. Managing a water system consists in providing water in the right proportion, at the right place and at the right time. Time expanded network (Fulkerson Delbert 1966) takes into consideration the temporal dimension in the decision making. Solving the dynamic flow problem may be reduced to solving a static flow problem in an extended graph (Skutella 2009): nodes are duplicated at each time step and the arcs link the nodes according to the transit time. Water management objectives are modeled with cost functions over arcs, and the optimal management corresponds to the distribution of minimum-cost.

The minimum-cost flow problem corresponds to finding a flow of minimum cost in a network whose arcs have flow capacities and costs per unit of flow. Different types of unit cost can be considered, such as: constant costs, linear costs and quadratic costs. It's necessary to distinguish between the unit cost and the real cost of a flow over an arc. If the unit cost is a function of the flow (linear function for example), the real cost is obtained by integrating the unit cost function between zero and the value of the flow on the arc. For example, constant unit costs induce linear costs over arcs.

Different modeling of the hydraulic system are possible, depending on the constraints to be considered such as taking into account: the presence of hydraulic structures, evaporation, infiltration. Haro et al. (2012) presented a generalized model with three different network flow algorithms (Out-of-Kilter, RELAX-IV and NETFLO) for solving the optimal allocation of water resources taking into account two non-network constraints: evaporation from reservoirs and returns from demands. Schardong and Simonovic (2015) presented a coupled self-adaptive multiobjective differential evolution and network flow algorithm for the optimal operation of complex multipurpose reservoir systems. An Out-of-Kilter method for minimal-cost flow problems was used to optimize the water resource system. Houda et al. (2013) used expanded network flow with fixed time delay to model three flood diversion areas on a river, and the Min-Cost-Max-Flow optimization algorithm with a linear programming formulation to achieve flood attenuation. The optimization is computed over a time horizon. The use of such a model requires that transfer times are considered static. Houda et al. (2016) also suggested a reduced graph coupled with a time delay matrix so that the time delay has no longer to be static. The optimization is computed over multiple time period. Links between static networks are represented through a matrix and thus the network flow communicates with this matrix where the values of delayed flows are stored.

Most of the work developed on network flow optimization for water management consider constant unit cost functions on arcs in order to use linear programming algorithms. However, considering weight factors to represent allocation priorities presents limitations: (1) if some objectives have the same weight factors, the resource can not be distributed equally over them; (2) the representation of the relation flow/cost in reservoirs is too simple to be realistic (for example the unit cost for a cubic meter of water in an almost empty reservoir is not the same as the one for a reservoir in the process of overflowing); (3) the weight factors must be updated according to the management type (for example: drought or flood management). In order to better model the economic evaluation of the resource, a cost function that evolves in terms of flow is required. In this paper, a reservoir optimization

algorithm considering piecewise increasing linear unit cost functions is presented. These type of unit cost functions allow to model profits and losses corresponding to the presence of a resource in a location and can balance the available resource on the set of objectives present in the network. The integration of unit cost functions induce quadratic convex cost functions on arcs. The optimal management corresponds to the distribution of minimum-cost. This problem is called the minimum quadratic convex-cost network flow problem (Minoux 1984).

2 Network Flow Model

2.1 Framework

The hydraulic system is modeled with a directed single source network $G = (V, E)$, possibly containing parallel arcs (directed multigraph), with node set V and arc set E . Let S be the source node and U the sink node of the network. In order to take into account the transfer time, a time-expanded network is considered: the nodes are duplicated at each time step over the duration of the simulation, and the transit times are implicit in arcs linking those copies. For an arc e_{ij} , i is the origin node, and j is the target node. Let $\gamma(n)$ and $\gamma^{-1}(n)$ respectively denote the sets of the outgoing and incoming arcs of a node n . Each arc e is associated with a positive flow $\phi_e \in [0; u_e]$, where u_e is the maximum capacity of the arc. For each node n of V , except for S and U , the conservation flow law is verified:

$$\forall n \in V \setminus \{S, U\} \quad \sum_{e \in \gamma(n)} \phi_e = \sum_{e \in \gamma^{-1}(n)} \phi_e \quad \text{and} \quad \sum_{e \in \gamma(S)} \phi_e = \sum_{e \in \gamma^{-1}(U)} \phi_e \quad (1)$$

Each arc e is associated with a unit cost function UC_e which represents the unit cost of transport. Unit cost functions, defined on arcs, are increasing piecewise linear functions:

$$UC_e(\phi) = a_e \phi + b_e \quad ; a_e \geq 0 \quad (2)$$

An increasing linear unit cost function that changes sign from negative to positive represents the economic value of a unit of water. The latter, models that a unit of water can generate profit (negative cost) as well as generate damage (positive cost) depending on the position relative to the objective flow.

The cost of a flow ϕ_e on an arc e is expressed as:

$$\int_0^{\phi_e} UC_e(\phi) d\phi \quad (3)$$

Geometrically, the cost of a flow ϕ_e on an arc e corresponds to the blue surface in Fig. 1 . The cost is minimal when ϕ_e is at the intersection of the unit cost function with the abscissa axis.

The cost of the graph corresponds to the sum of the costs on the set E :

$$C_{graph} = \sum_{e \in E} \int_0^{\phi_e} UC_e(\phi) d\phi \quad (4)$$

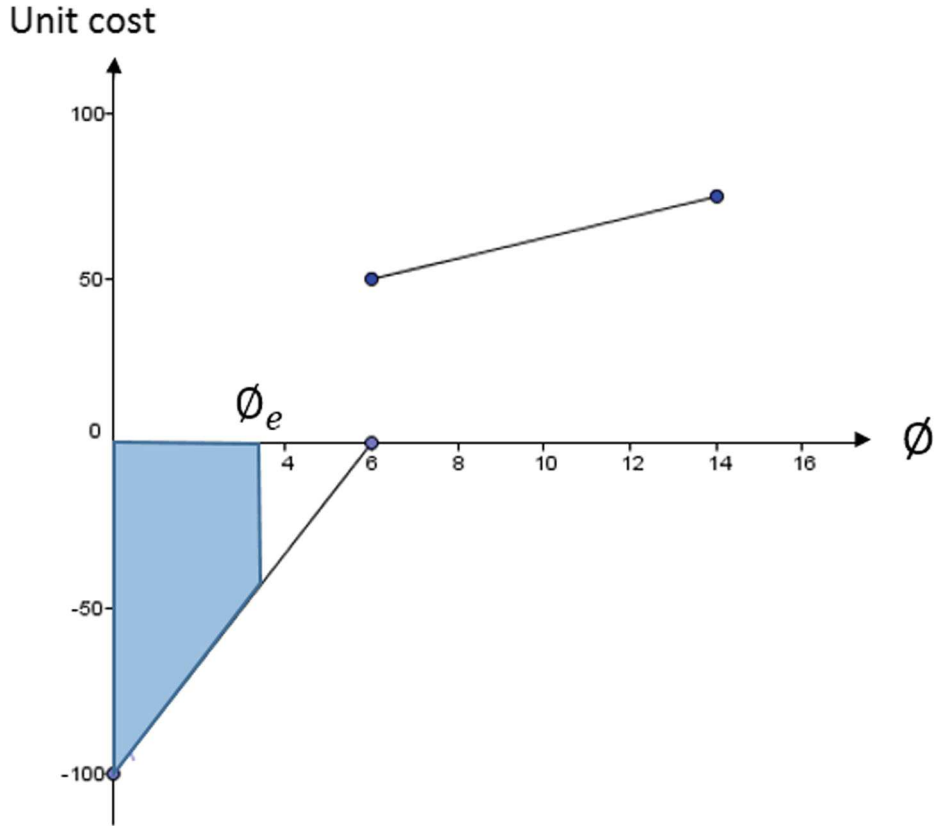


Fig. 1 Unit cost

2.2 Objective Function

The minimum-cost flow algorithm search to distribute the resource from the source node S to the sink node U so that the sum of the costs on arcs be minimal.

The minimum-cost network flow problem can be stated formally as follows:

$$\min \sum_{e \in E} \int_0^{\phi_e} UC_e(\phi) d\phi \quad (5)$$

$$\sum_{e \in \gamma(S)} \phi_e = \text{available resource}$$

$$\forall e \in E \quad 0 \leq \phi_e \leq u_e$$

$$\forall n \in V \setminus \{S, U\} \quad \sum_{e \in \gamma(n)} \phi_e = \sum_{e \in \gamma^{-1}(n)} \phi_e$$

$$\sum_{e \in \gamma(S)} \phi_e = \sum_{e \in \gamma^{-1}(U)} \phi_e$$

The integral $\int_0^{\phi_e} UC_e(\phi) d\phi$ is quadratic and convex because the slope of the unit cost function is positive. Therefore the objective function is quadratic and convex.

3 Optimization Algorithm

3.1 Residual Network

In order to take into account the particularities of the cost functions considered in this work, the classical definition of a residual network (Evans 1992) is extended. For each arc $e \in E$, let \bar{e} denotes its reverse arc. Let $\bar{E} = \{\bar{e} | e \in E\}$ denotes the set of the reverse arcs. The arcs of E are called forward arcs and those of \bar{E} backward arcs. The arc \bar{e} represents the ability of pushing flow back on the arc e .

For a flow ϕ_e on arc e , the residual capacity of arcs e and \bar{e} are $ur_e = \min(u_e - \phi_e, w)$, with w the value of the first discontinuity encountered on the unit cost function over the interval $[\phi_e; u_e]$ in the increasing direction, and $ur_{\bar{e}} = \min(\phi_e, v)$, with v the value of the first discontinuity encountered on the unit cost function over the interval $[0; \phi_e]$ in the decreasing direction (see Fig. 2).

The residual capacity of e represents the amount of flow that can be added to e until the saturation of the arc or crossing a discontinuity and that of \bar{e} stands for the flow that can be removed without crossing a discontinuity.

The unit cost on e and \bar{e} are expressed as follows: $C_e = \lim_{\substack{\phi \rightarrow \phi_e \\ \phi > \phi_e}} UC_e(\phi)$ and $C_{\bar{e}} = - \lim_{\substack{\phi \rightarrow \phi_e \\ \phi < \phi_e}} UC_e(\phi)$, respectively. If UC_e is a constant integer, we meet the classical definition of a residual graph. The graph G is associated to the residual graph G' composed of forward arcs and backward arcs.

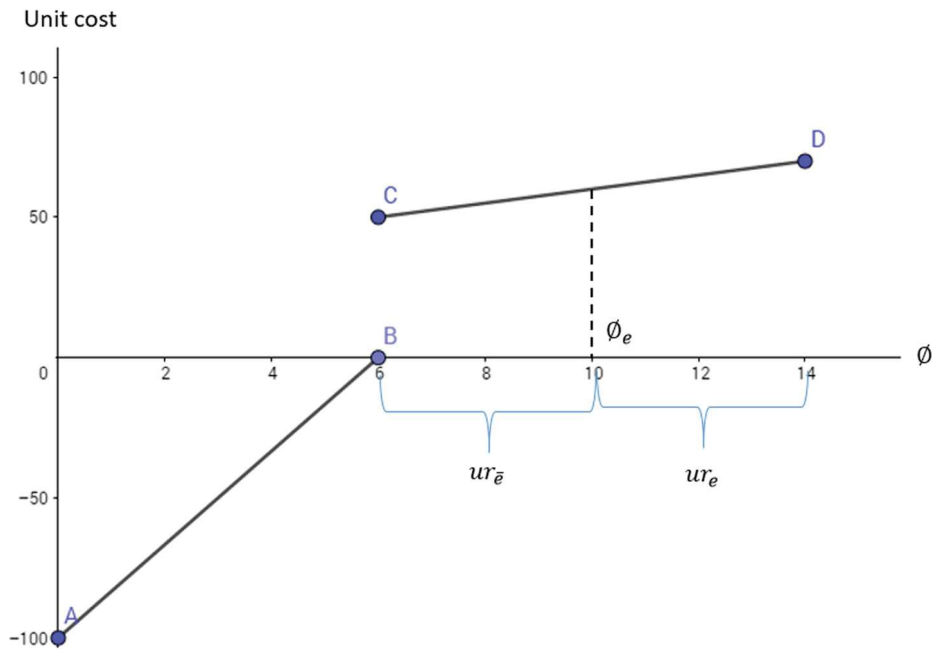


Fig. 2 Capacities on arcs

3.2 Optimality Conditions

The cost of a feasible flow f is optimal if and only if the residual network contains no negative cost cycles (Busacker and Saaty 1966). A cycle is a path that contains at least two arcs, such that all arcs of the sequence are different and whose ends coincide. The cost of a cycle is the sum of the unit costs of its arcs. A residual cycle is negative if its cost is negative. The existence of a negative cost residual cycle implies that the current flow can be improved by displacing a portion of the flow through the cycle. Canceling a cycle with an important negative cost and with a large residual capacity would significantly improve the objective function. The concept of the method developed herein for looking for optimality extends the work developed by Klein (1967) for constant positive unit costs over arcs. The basic scheme is described by the algorithm 1.

Algorithm 1 Search for minimum cost distribution

```

while Residual graph contains negative cost cycles do

    Detect a negative cost cycle

    Cancel the detected negative cost cycle

end while

```

Many algorithms exist to find negative cost cycles in a directed graph, such as : Bellman–Ford algorithm (Bellman 1958), Floyd-Warshall algorithm (Floyd Robert 1962), A* algorithm (Hart Peter et al. 1968). Herein, the Bellman-Ford algorithm is implemented since it was observed that it's the most efficient for our situation.

3.3 Canceling a Negative Cost Cycle

To cancel a negative cost cycle, a computed amount of the flow is displaced around the cycle to obtain a zero cost cycle, or to saturate an arc of the cycle.

Let $\xi = (V' : E')$ be a negative directed cycle in the residual network composed of forward and backward arcs. V' and E' are sub-groups of V and $E \cup \bar{E}$, respectively. In order to distinguish forward arcs and backward arcs, let us consider E_1 and \bar{E}_2 as groups of forward arcs and backward arcs, respectively, with $E_1 \cup \bar{E}_2 = E'$.

The set $E_1 \cup E_2$ is an independent subgraph of G . In other words, any exchange of flow between E_1 and E_2 does not impact the flows of the arcs belonging to: $G \setminus (E_1 \cup E_2)$.

To cancel a negative cost cycle, the flow in the direction of the cycle is increased until the cost of the cycle becomes zero or until the saturation of one of the arcs of the cycle (the unit cost depends on the flow). This is equivalent to moving a flow $\delta\phi \geq 0$ from E_2 to E_1 .

An example of a cycle is illustrated in Fig. 3: $V' = \{A, B, C, D, E\}$, $E' = \{e_1, e_2, \bar{e}_3, \bar{e}_4, \bar{e}_5\}$, $E_1 = \{e_1, e_2\}$, $E_2 = \{e_3, e_4, e_5\}$ and $\bar{E}_2 = \{\bar{e}_3, \bar{e}_4, \bar{e}_5\}$.

The cost of the subgraph $E_1 \cup E_2$ is:

$$C = \sum_{e \in E_1} \left[\int_0^{\phi_e} UC_e(\phi) d\phi \right] + \sum_{e \in E_2} \left[\int_0^{\phi_e} UC_e(\phi) d\phi \right] \quad (6)$$

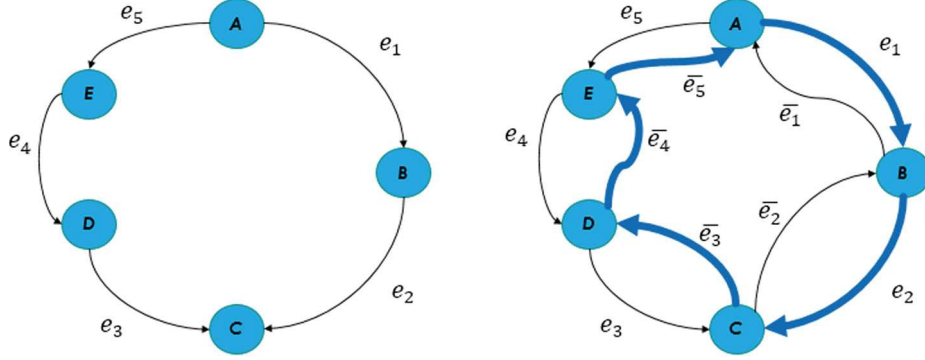


Fig. 3 An example of a network and its residual network

Let y denotes the flow exchanged between E_1 and E_2 , and $C(y)$ the variation of the cost of the subgraph corresponding to this exchange. $C(y)$ is given by the formula:

$$C(y) = \sum_{e \in E_1} \left[\int_{\phi_e}^{\phi_e + y} UC_e(\phi) d\phi \right] + \sum_{e \in E_2} \left[\int_{\phi_e}^{\phi_e - y} UC_e(\phi) d\phi \right] \quad (7)$$

$$= \sum_{e \in E_1 \cup E_2} \left[\int_{\phi_e}^{\phi_e + \epsilon_e y} UC_e(\phi) d\phi \right] \quad (8)$$

with $\epsilon_e = 1$ if $e \in E_1$, and $\epsilon_e = -1$ if $e \in E_2$

$$C(y) = \sum_{e \in E_1 \cup E_2} \left[\int_{\phi_e}^{\phi_e + \epsilon_e y} (a_e \phi + b_e) d\phi \right] \quad (9)$$

$$= \sum_{e \in E_1 \cup E_2} \left[\frac{a_e}{2} y^2 + \epsilon_e y (a_e \phi_e + b_e) \right] \quad (10)$$

$$= \sum_{e \in E_1 \cup E_2} \left[\frac{a_e}{2} y^2 + \epsilon_e y UC_e(\phi_e) \right] \quad (11)$$

$$= \frac{1}{2} \left[\sum_{e \in E_1 \cup E_2} a_e \right] y^2 + \left[\sum_{e \in E_1 \cup E_2} \epsilon_e UC_e(\phi_e) \right] y \quad (12)$$

$$= \frac{1}{2} \theta y^2 + \zeta y \quad (13)$$

with $\theta = \sum_{e \in E_1 \cup E_2} a_e$, the sum of the slopes of the unit cost functions of the arcs in $E_1 \cup E_2$, and $\zeta = \sum_{e \in E_1 \cup E_2} \epsilon_e UC_e(\phi_e)$ the cost of the cycle ξ .

Figure 4 shows the evolution of the cost of the subgraph. $C(y)$ is a polynomial of second degree and is convex because θ is positive. Its minimum is reached for $y = -\frac{\zeta}{\theta}$. The optimum flow $\delta\phi$ to displace on the cycle ξ in order to optimize the cost of the corresponding subgraph with respect to the capacities of the arcs is given by:

$$\delta\phi = \min \left(-\frac{\zeta}{\theta}, \min(ur_e | e \in E') \right) \quad (14)$$

At each elimination of a cycle, the cost of the graph decreases of $\delta C \in [-\frac{\zeta^2}{\theta}; 0[$. The optimum is reached when no more negative cycle exists.

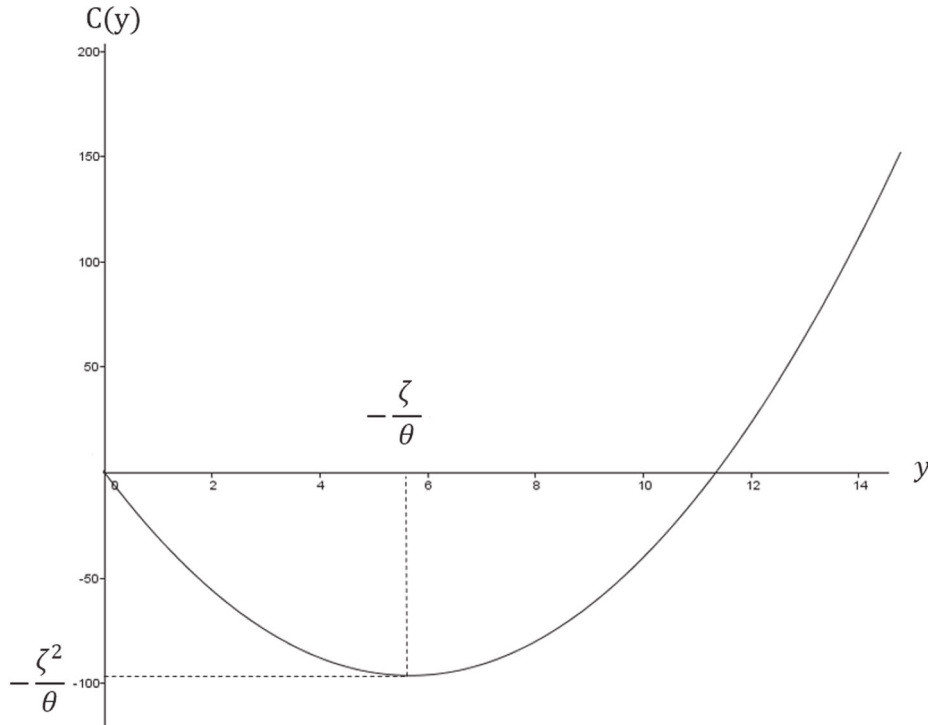


Fig. 4 Evolution of the cost of a subgraph corresponding to a cycle

4 Case Study

4.1 The Studied Hydraulic System

The herein described model and the corresponding proposed algorithm have been applied to the water system of the *Haute-Vilaine's* watershed situated in the department of *Ille et vilaine*, which is located in the region of Brittany in the northwest of France.

Figure 5 presents a synoptic of the *Haute-Vilaine* watershed, and the 3 reservoirs to be managed: *Haute-Vilaine*, *Cantache* and *Valière*. Vulnerable areas to flooding in the *Vilaine* valley are: *Vitrè*, *Châteaubourg* and *Cesson-Sévigné*.

A situation of vigilance occurs when the flow reaches : $7m^3/s$ at *Vitrè*, $36m^3/s$ at *Châteaubourg* and $50m^3/s$ at *Cesson-Sévigné*. Maintaining the flows below this thresholds induce strong constraints to respect the flows released downstream of the three reservoirs during floods. Moreover, the intermediates flows not controlled by the dams bring a significant flood flow to the sensitive areas studied. Because of the location of vulnerable areas regarding to the reservoirs, the proportions of the catchment area uncontrolled by the reservoirs are: 17% of the *Haute-Vilaine's* watershed for *Vitrè*, 41% of the *Haute-Vilaine*, *Valière* and *Cantache* watersheds for *Châteaubourg* and 60% of the *Haute-Vilaine*, *Valière* and *Cantache* watersheds for *Cesson-Sévigné*.

Physical characteristics of the 3 reservoirs are listed in Fig. 6:

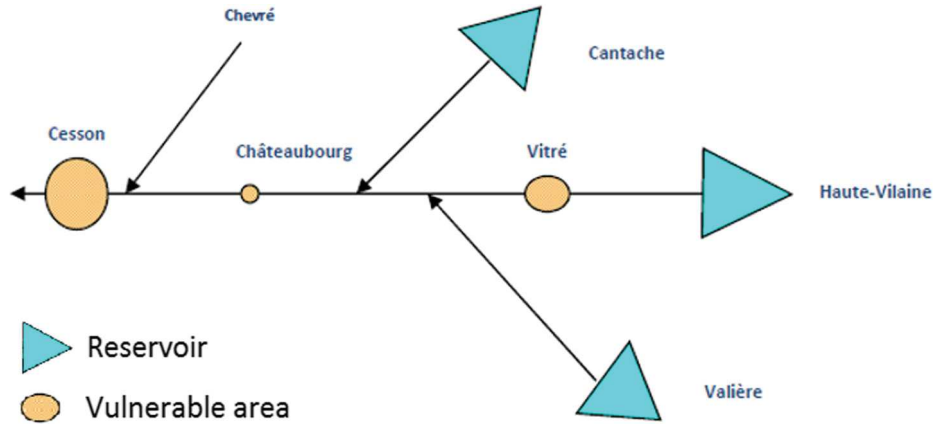


Fig. 5 Hydraulic system

4.2 Network Model

The network model corresponding to the studied hydraulic system is provided in Fig. 7. Node S corresponds to the source node. It's a fictive node that supplies the 3 reservoirs with water. The nodes A, B, C correspond to the 3 reservoirs: *Cantache*, *Haute-Vilaine* and *Valière* respectively. The nodes D, E, F correspond to the 3 vulnerable areas *Vitré*, *Châteaubourg* and *Cesson-Sévigé* respectively. The nodes I, G, H corresponds to the intersections with *Vilaine's* river. The node U is a fictive node to respect the conservation flow law. The time transfer delays (unit hours) of the arcs are denoted in red in Fig. 7.

For the simulation, the flood hydrograph measured at *Cantach* (see Fig. 8) is considered. The incoming flows at *Haute-Vilaine*, *Valière* and at the three vulnerable areas are estimated from this hydrograph according to their size of catchment. Hence, the flood hydrographs at *Haute-Vilaine*, *Valière*, *vitrè*, *Châteaubourg* and *Chevré* represent respectively: 88.5%, 47.8%, 17%, 97% and 141.8% of the flood hydrograph at *Cantach*. We consider that the 3 reservoirs are initially 80% full.

The purpose of the simulation is to apply the herein *proposed management strategy* for reservoir releases in order to respect the maximum flow thresholds targeted at the vulnerable areas. The results of this strategy are compared with a *reference strategy*, in which the reservoirs release the exact amount of water they receive (transparent management). In order

Reservoir's name	Capacity (Mm^3)	Watershed Surface (Km^2)	capacity of the drain valve (m^3/s)
Valiere	5.1	67	8
Haute Vilaine	6.5	124	10
Cantache	6.3	140	37

Fig. 6 Physical characteristics

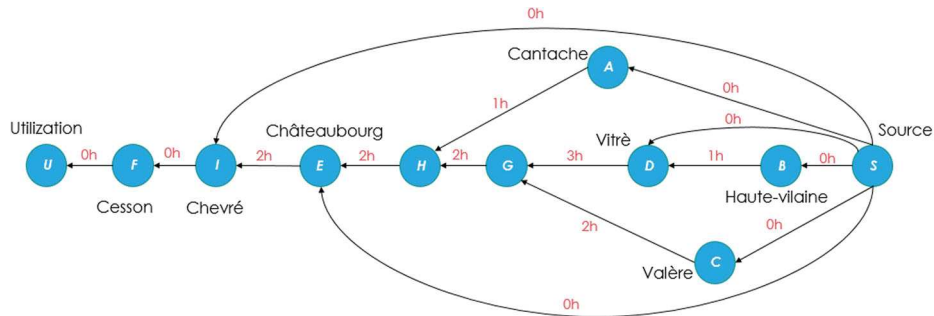


Fig. 7 Network model

to compare the two management strategies, the storage available volume should be the same. Hence, a new constraint is considered: the final reservoir storage level should be 80%. The results of the simulation are shown in Fig. 9.

The proposed algorithm realize a management over the whole horizon. In order to protect from the 2nd peak of the flood, this algorithm firstly proceed to a release from the reservoirs so that the available storage volume is increased (from the first hour to the tenth hour, see Fig. 9a). The 3 reservoirs work in group to protect the 3 vulnerable areas, and the final filling rate corresponds to the setted objective (i.e. 80%). Flow rates at the 3 vulnerable areas doesn't exceed the objective threshold flow objective except for *Cesson-Séviigné* where the flow reaches $53m^3/s$ (see Fig. 9b, c and d). This is due to the fact that the catchment area uncontrolled by the reservoirs is large in *Cesson-Séviigné*.

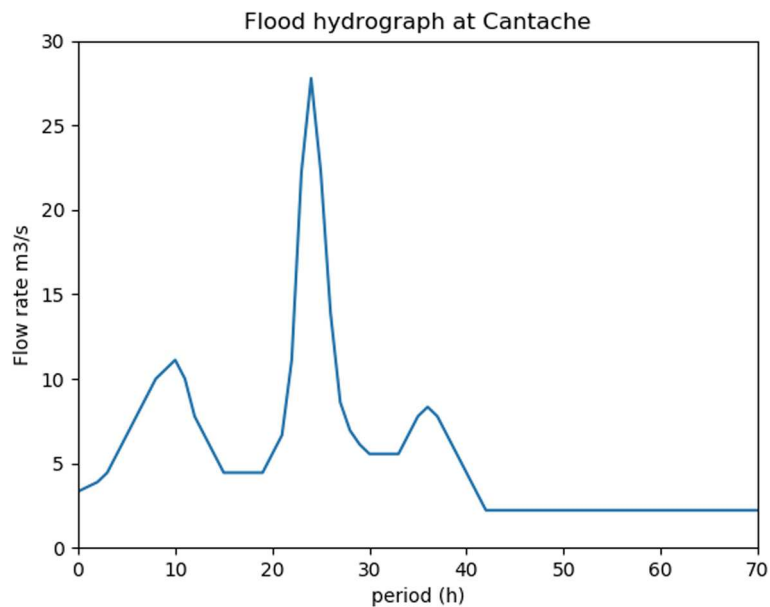


Fig. 8 Flood hydrograph at Cantache

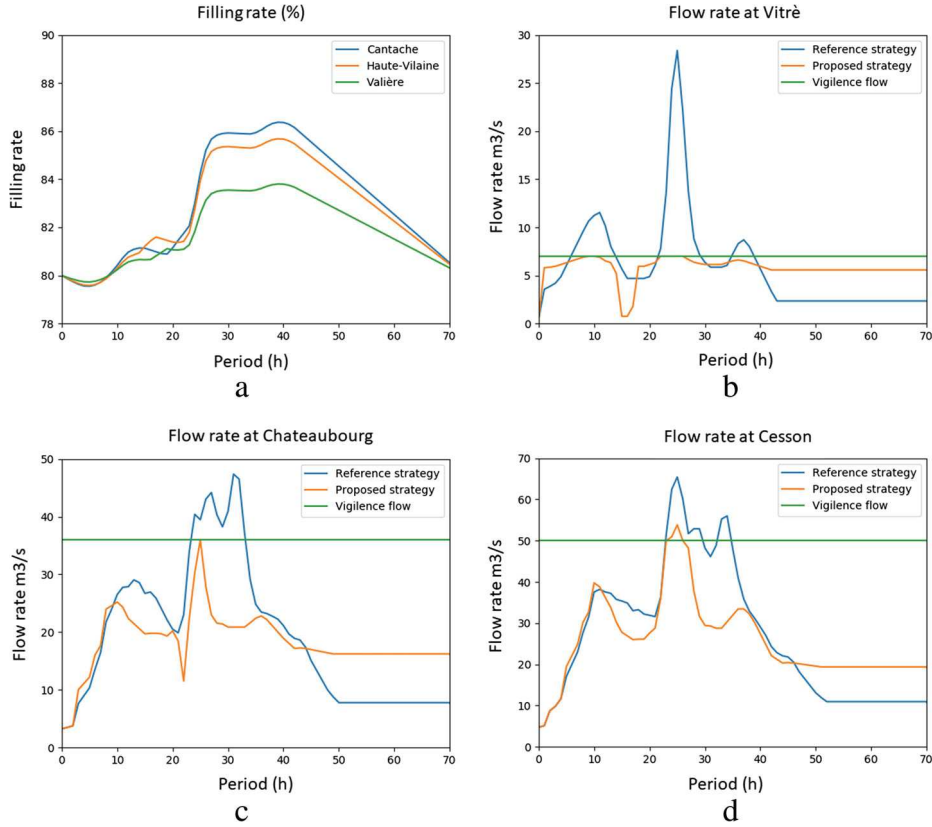


Fig. 9 Results of the simulation

The network considered consists of 650 nodes, 1080 arcs, 219 cost functions and 657 discontinuities. The computation time is less than 30 seconds on a computer with the following characteristics: 2.5 GHz, Core(TM) i7, 16 Go RAM.

The simulation results analysis shows that the proposed reservoirs management strategy allows to achieve the flow objectives, except for the most downstream point in Cesson-Sévigné, because the intakes at Chevré can not be controlled by the reservoirs. Besides, the reference scenario leads to flows much higher than the thresholds.

5 Concluding Remarks

Herein a heuristic algorithm to solve quadratic convex minimum cost network flow problem is developed and applied to solve the water management problem. The latter is formulated as a control problem, in which reservoir releases represent the decision variables and the optimal solution corresponds to the most accurate distribution. The methodology was tested and provided an excellent results on the *La Haute-Vilaine's* hydraulic system to protect 3 vulnerable areas from flood.

In this work, volumes were considered to be carried as solids, which means that their deformation weren't considered. Future work will focus on the integration of the hydraulic constraints in the model and particularly the water transfer deformation.

Compliance with Ethical Standards

Conflict of interests The authors declare that they have no conflict of interest

References

- Ahuja Ravindra K, Magnanti Thomas L, Orlin James B (1988) Network flows. Technical report, DTIC Document
- Andreu J, Capilla J, Sanchis E (1996) Aquatool, a generalized decision-support system for water-resources planning and operational management. *J Hydrol* 177(3):269–291
- Bellman R (1956) Dynamic programming and lagrange multipliers. *Proc Natl Acad Sci* 42(10):767–769
- Bellman R (1958) On a routing problem. *Q Appl Math* 16(1):87–90
- Busacker RG, Saaty TL (1966) Finite graphs and networks: An introduction with applications
- Daniel C, Mays Larry W (2015) Development of an optimization/simulation model for real-time flood-control operation of river-reservoirs systems. *Water Resour Manag* 29(11):3987–4005
- Du D-Z, Pardalos Panos M (1993) Network optimization problems: algorithms, applications and complexity, vol 2. World Scientific, Singapore
- Evans J (1992) Optimization algorithms for networks and graphs. CRC Press, Boca Raton
- Floyd Robert W (1962) Algorithm 97: shortest path. *Commun ACM* 5(6):345
- Fulkerson Delbert R (1966) Flow networks and combinatorial operations research. *Amer Math Mon* 73(2):115–138
- Haro D, Paredes J, Solera A, Andreu J. (2012) A model for solving the optimal water allocation problem in river basins with network flow programming when introducing non-linearities. *Water Resour Manag* 26(14):4059–4071
- Hart Peter E, Nilsson NJ, Bertram R (1968) A formal basis for the heuristic determination of minimum cost paths. *IEEE Trans Syst Sci Cybern* 4(2):100–107
- Higgins John M, Gary BW (1999) Overview of reservoir release improvements at 20 tva dams. *J Energy Eng* 125(1):1–17
- Klein M (1967) A primal method for minimal cost flows with applications to the assignment and transportation problems. *Manag Sci* 14(3):205–220
- Kuczera G (1993) Network linear programming codes for water-supply headworks modeling. *J Water Resour Plan Manag* 119(3):412–417
- Kumar Nagesh D, Falguni B, Srinivasa RK (2010) Optimal reservoir operation for flood control using folded dynamic programming. *Water Resour Manag* 24(6):1045–1064
- Labadie John W, Baldo Marc L, Modsim LR (2000) Decision support system for river basin management. Available at: modsim. engr. colostate.edu
- Labadie John W (2004) Optimal operation of multireservoir systems: state-of-the-art review. *J Water Resour Plann Manag* 130(2):93–111
- Minoux M (1984) A polynomial algorithm for minimum quadratic cost flow problems. *Eur J Oper Res* 18(3):377–387
- Needham JT, Watkins Jr DW, Lund JR, Nanda SK (2000) Linear programming for flood control in the iowa and des moines rivers. *J Water Resour Plan Manag* 126(3):118–127
- Houda N, Pascale C, Bernard A (2013) A flood lamination strategy based on transportation network with time delay. *Water Sci Technol* 68(8):1688–1696
- Houda N, Pascale C, Bernard A (2016) Contribution to a flood situation management: a supervisory control scheme to reduce disaster impact. *Water Sci Technol Water Supply* 16(3):587–598
- Deepti R, Madalena MM (2010) Simulation–optimization modeling: a survey and potential application in reservoir systems operation. *Water Resour Manag* 24(6):1107–1138
- Uditha R, Ricardo H (2007) Deterministic and stochastic optimization of a reservoir system. *Water Int* 32(1):155–162
- Schardong A, Simonovic SP (2015) Coupled self-adaptive multiobjective differential evolution and network flow algorithm approach for optimal reservoir operation. *J Water Resour Plann Manag* 141(10):04015015

- Skutella M (2009) An introduction to network flows over time. In: *Research Trends in Combinatorial Optimization*. Springer, pp 451–482
- Simonovic Slobodan P (1992) Reservoir systems analysis: closing gap between theory and practice. *J Water Resour Plan Manag* 118(3):262–280
- Tu M-Y, Nien-Sheng H, Yeh William W-G (2003) Optimization of reservoir management and operation with hedging rules. *J Water Resour Plan Manag* 129(2):86–97
- World Commission on Dams (2000) *Dams and Development: A New Framework for Decision-making: the Report of the World Commission on Dams*. Earthscan
- Wurbs Ralph A (1993) Reservoir-system simulation and optimization models. *J Water Resour Plann Manag* 119(4):455–472