Ultra-Local Model Control Based on an Adaptive Observer

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***Abstract*— In this paper, a new ultra-local model control approach is proposed. The concept is based on the linear adaptive observer to estimate the ultra-local model parameters instead of algebraic derivation technique. The importance of adaptive observer is deduced in the join estimation of state and unknown parameters of parametric systems. The closed-loop control is implemented via an adaptive PID controller to reject disturbances due to exogenous parameter uncertainties. In this paper, a performance comparison between the adaptive observer based method and the algebraic derivation technique is developed to show the efficiency of the proposed control strategy. The approaches are applied to a two-tank system for the water level control. Several successful simulation results are shown to demonstrate the effectiveness of the proposed controller.**

***Index Terms*— Ultra-local model control, Adaptive PID controller, Linear adaptive observer, Numerical derivation, Parameter estimation, Robustness analysis, Two-tank-system.**

I. INTRODUCTION

It is well known that the overwhelming majority of the industrial control applications is based on PID (Proportional-Integral-Derivative) controllers (see, e.g., [1], [19], and the references therein). The PID control is often applied in the industry even if PID controllers could render poor results when a process has a large operating domain. In this case, instead of relying on a more accurate knowledge structure of the controlled system model, the ultra-local model control has been recently introduced with the model-free control notion [3], [4], [5], [8]. This approach does not necessitate any mathematical modeling. The unknown dynamics is approximated on a very small time interval by a very simple model which is continuously updated using the online estimation techniques ([6], [9]). The loop is closed thanks to an adaptive PID, which provides the feedforward compensation and is easily tuned. The ultra-local model control has already led to a number of exciting applications and several works have been made [3], [7], [15], [16].

Talking about the state and parameter estimation, the Luenberger observer, Kalman filter and asymptotic observer (see [2]) are well known solutions for state estimation in linear dynamic systems. For joint estimation of state and unknown parameters, some results are also known under the name of *adaptive observer*, see, *e.g.*, [13], [18], [23]). For single-input-single-output (SISO) time invariant system, some results can be found in [14], [17]. Recently, adaptive observers for multi-input-multi-output (MIMO) linear time varying (LTV) systems have been developed in [22], [23], [24]. Some results on truly nonlinear systems have also been reported ([25]). The adaptive observer for MIMO LTV systems is conceptually simple and computationally efficient.

The ultra-local model control consists in trying to estimate via the input and the output measurements what can be compensated by control in order to achieve a good output trajectory tracking. In the works [3], [4], the estimation of a single parameter by the algebraic derivation technique is insufficient to obtain the desired performance when the estimation of the second parameter is required. For this reason, a new ultra-local model control approach is proposed in this work to improve these performances. The main contribution of this paper is to design a new adaptive PID controller based on an adaptive observer to estimate the both ultra-local model variables. A comparison between the algebraic derivation technique and the adaptive observer based method is given to estimate the ultra-local model parameters. The aim is to clarify the performance improvement and effectiveness of the proposed controller design. In this paper, the ultra-local model control is applied to a two-tank water system which is considered as a nonlinear system of first-order.

The paper is organized as follows. The concepts of the ultra-local model control and of the corresponding adaptive PIDs are presented in Section 2. Section 3 develops two different methods of ultra-local model parameter identification: algebraic derivation method and adaptive observer based method. Section 4 deals with the ultra-local model control of two-tank-system and gives simulation results. Section 5 presents some concluding remarks.

II. ULTRA-LOCAL MODEL CONTROL

A. Basic Idea

For simplicity's sake, we are restricting ourselves to single-input single-output systems. The control input is denoted by u and the output is denoted by y. The input-output behavior of the plant is assumed to be well approximated within its operating range by an ordinary differential equation:

$$E\left(y(t), \dot{y}(t), \ldots, y^{(a)}(t), u(t), \dot{u}(t), \ldots, u^{(b)}(t)\right) = 0 \tag{1}$$

which is nonlinear in general and unknown or at least poorly known. Replace it by the ultra-local model which is continuously updated and given by:

$$y^{(\nu)}(t) = F(t) + \alpha(t)\, u(t) \tag{2}$$

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The order of derivation ν is in practice 1 or 2. The two quantities $F(t)$ and $\alpha(t)$ representing the unknown parameters of the ultra-local model, contain all structural information including disturbances and their derivatives.

B. Adaptive Controllers

Consider again the ultra-local model (2). The desired behavior for a derivation order $\nu = 1$ in the equation (2) is obtained thanks to an adaptive Proportional Integral controller, or *a*-PI controller, as follows:

$$u(t) = \frac{-\hat{F}(t) + \dot{y}^d(t) + K_P e(t) + K_I \int e(t)}{\hat{\alpha}(t)} \tag{3}$$

where:

- $y^d(t)$ is the output reference trajectory, obtained according to the precepts of the flatness-based control [11], [20];
- $e(t) = y^d(t) - y(t)$ is the tracking error;
- K_P and K_I are the usual tuning gains [1], [19].

Combining the equations (2) and (3), we obtain the following equation:

$$\ddot{e}(t) + K_P \dot{e}(t) + K_I e(t) = 0 \tag{4}$$

Noting that the parameters $F(t)$ and $\alpha(t)$ don't appear anymore in the equation (4). We are therefore left with a linear differential equation with constant coefficients of order 2. The tracking condition is therefore easily fulfilled by an appropriate tuning of K_P and K_I.

III. PARAMETER IDENTIFICATION METHODS OF ULTRA-LOCAL MODEL

A. Algebraic Derivation Method

In the previous works [3], [4], [8], if $\nu = 1$, the numerical value of $F(t)$, which contains the whole structural information, is determined thanks to the knowledge of $u(t)$, α and of the estimate of the derivative $\dot{y}(t)$. Based on the algebraic parameters identification developed in [9], the estimation of the noisy signal $\dot{y}$ can be written in the following integral form:

$$\widehat{\dot{y}} = -\frac{3!}{T^3} \int_0^T (T - 2t)\, y(t)\, dt \tag{5}$$

where the integration window $[-T, 0]$ is, in practice, very short. This window is sliding to obtain the estimated at each instant.
The setting parameters of the filter (the size of the window and the sampling time T_e) are directly related to its cutoff frequency and therefore can be readily adapted to the process signal dynamics.
Before applying the control input, it is necessary to propose an estimation of the quantity $F(t)$ in real-time. At the sampling time kT_e (noted k), the estimation of F is written as follows:

$$\hat{F}_k = \widehat{\dot{y}}_k - \alpha u_{k-1} \tag{6}$$

where $\widehat{\dot{y}}_k$ is the estimate of the derivative of the system output that can be provided at the instant k, α is a constant design parameter, and u_{k-1} is the control input that has been applied to the system during the previous sampling time. The estimate of F leads to the following ultra-local model control principle:

$$u(t) = \frac{-\hat{F}(t) + \dot{y}^d(t) + K_P e(t) + K_I \int e(t)}{\alpha} \tag{7}$$

where α is a non-physical parameter which must be chosen such that $F(t)$ and $\alpha u(t)$ have the same order of magnitude.

The identification of both parameters F and α is the most important task of this work, in particular, if the estimate of the second parameter α is necessary. The algebraic derivation is no longer used in the model-free approach, it has been replaced by an easier identification procedure. In this case, a new technique of ultra-local model parameters estimation using an adaptive observer is proposed in the following.

B. Adaptive Observer Based Method

1) Problem Formulation: In the case where $\nu = 1$, the ultra-local model (2) is written as follows:

$$\begin{aligned} \dot{y}(t) &= F + \alpha u(t) \\ &= F + (\alpha - 1)\, u(t) + u(t) \\ &= u(t) + \begin{bmatrix} 1 & u(t) \end{bmatrix} \begin{bmatrix} F \\ \alpha - 1 \end{bmatrix} \end{aligned} \tag{8}$$

From the equation (8), the ultra-local model (2) can be represented in the form of a linear time-invariant SISO state-space system as follows (see [23] for more details about these systems):

$$\begin{aligned} \dot{x}(t) &= Bu(t) + \Psi(t)\,\theta \\ y(t) &= Cx(t) \end{aligned} \tag{9}$$

where:

- $x(t) \in \mathbb{R}$, $u(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are respectively the state, input and output of the system,
- $A = 0$ and $B = C = 1$. In this case, the output $y(t)$ is the state of the system $x(t)$,
- $\theta = \begin{bmatrix} F \\ \alpha - 1 \end{bmatrix} \in \mathbb{R}^p$ is a column vector of parameters assumed unknown,
- $\Psi(t) = \begin{bmatrix} 1 & u(t) \end{bmatrix} \in \mathbb{R}^{1\times p}$ is a vector of measured signals.

The problem considered in this note is the joint estimation of $x(t)$ and θ from measured $u(t)$, $y(t)$ and $\Psi(t)$.
Now, consider the case where $\nu = 2$, and assuming the following state vector $x(t) \in \mathbb{R}^2$:

$$x(t) = \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix}$$

The ultra-local model (2) is transformed in the following matrix form:

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ F \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} u(t) \\ &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ &\quad + \begin{bmatrix} 0 & 0 \\ 1 & u(t) \end{bmatrix} \begin{bmatrix} F \\ \alpha - 1 \end{bmatrix} \end{aligned} \tag{10}$$

With the previous relation (10), the following linear time-invariant SISO state-space system is obtained. This formalism allows us to apply the adaptive observer developed in [22]:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) + \Psi(t)\theta \\ y(t) &= Cx(t)\end{aligned} \tag{11}$$

where:

- the matrix A and the vectors B and C are defined by:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

- the matrix of measured signals $\Psi(t)$ and the vector of parameters are given as follows:

$$\Psi(t) = \begin{bmatrix} 0 & 0 \\ 1 & u(t) \end{bmatrix}, \quad \theta = \begin{bmatrix} F \\ \alpha - 1 \end{bmatrix}$$

The design of an adaptive observer is studied in the following in order to estimate the state $x(t)$ and the parameters θ from the measured signals $u(t)$, $y(t)$, $\Psi(t)$, and the matrices A, B, C.

2) Linear Adaptive Observer Design: Consider the SISO linear time-invariant state-space system given in Equation (11), where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}$, $y(t) \in \mathbb{R}$, $\theta \in \mathbb{R}^p$ and $\Psi(t) \in \mathbb{R}^{n \times p}$. Given the following assumptions:

Assumption 1: Assume that the matrix pair (A, C) in system (11) is such that there exists a vector of constant gain $K \in \mathbb{R}^n$ so that the system

$$\dot{\eta}(t) = [A - KC]\,\eta(t) \tag{12}$$

is globally exponentially stable.

Assumption 2: Let $\Upsilon(t) \in \mathbb{R}^n \times \mathbb{R}^p$ be a matrix of signals generated by a stable filter such as:

$$\dot{\Upsilon}(t) = [A - KC]\,\Upsilon(t) + \Psi(t) \tag{13}$$

Assume that $\Psi(t)$ is persistently exciting so that there exist two positive constants δ, L and a positive gain Σ such that, for all t, the following inequality is satisfied:

$$\int_t^{t+L} \Upsilon^T(\tau)\, C^T \Sigma C \Upsilon(\tau)\, d\tau \geqslant \delta I \tag{14}$$

with $I \in \mathbb{R}^p \times \mathbb{R}^p$ the identity matrix.

Assumption 1 states that for any given parameter θ, a state observer with exponential convergence can be designed for system (11). The gain K sets the estimator dynamics. Assumption 2 is a persistent excitation condition, typically required for system identification.

Let $\Gamma \in \mathbb{R}^p \times \mathbb{R}^p$ be any symmetric positive definite matrix. Therefore, under Assumptions 1 and 2, the following system of ordinary differential equations:

$$\dot{\Upsilon}(t) = [A - KC]\,\Upsilon(t) + \Psi(t) \tag{15}$$

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + \Psi(t)\hat{\theta}(t) \\ &+ \left[K + \Upsilon(t)\Gamma\Upsilon^T(t)\,C^T\Sigma\right][y(t) - C\hat{x}(t)]\end{aligned} \tag{16}$$

$$\dot{\hat{\theta}}(t) = \Gamma\Upsilon^T(t)\,C^T\Sigma\,[y(t) - C\hat{x}(t)] \tag{17}$$

is a global exponential adaptive observer for the system (11). Remark that the matrix $\Upsilon(t)$ is generated by a stable linear filtering of $\Psi(t)$ (for more details, see [23], [24]). Typically, the gain vector K is chosen only to ensure the stability of $A - KC$, the total gain for the the state estimation being $K + \Upsilon(t)\Upsilon^T(t)C^T$. Γ allows to set the rate of convergence between the state and the parameters.

Remark 1: No matter the initial conditions $x(t_0)$, $\hat{x}(t_0)$ and $\hat{\theta}(t_0)$, the convergence of the product $y(t) - C\hat{x}(t)$ to 0 remains always valid when $t \to \infty$.

In the following, a two-tank system is studied in the case of two different ultra-local model control approaches. In the both control techniques, the loop is closed by an adaptive PI controller (*i.e.*, the design parameter $\nu = 1$).

IV. A TWO-TANK SYSTEM STUDY

A. Model Description

Consider the two-tank system described in the Figure 1 which is constituted by two identical water tanks that have the same section S. Denote by $h_1(t)$ the water level in the upper tank, which also represents the system output, $h_2(t)$ the water level in the lower tank, $q_1(t)$ the input flow of the upper tank, $q_2(t)$ the output flow of the upper tank and $q_3(t)$ the output flow of the lower tank. In the steady state, the conservation of the total volume of water leads to $q_1(t) = q_3(t)$. The nonlinear model of the considered system is as follows:

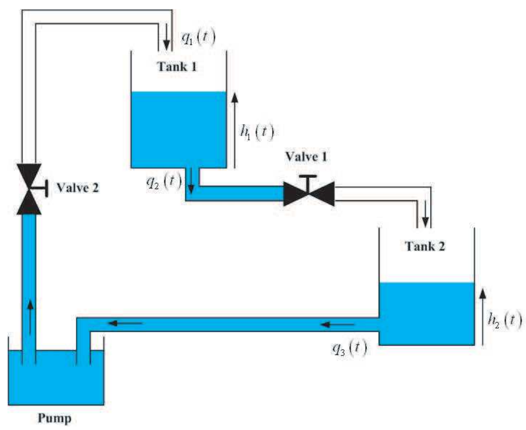

Fig. 1. Two-tank system.

$$\begin{aligned}S\dot{h}_1(t) &= q_1(t) - q_2(t) \\ S\dot{h}_2(t) &= q_2(t) - q_3(t)\end{aligned} \tag{18}$$

with $q_2(t) = k_1\sqrt{h_1(t)}$ and $q_3(t) = k_2\sqrt{h_2(t)}$.

The term $k_i\sqrt{h_i(t)}$, $i = 1, 2$, comes from the turbulent regime of the water discharge by the valves. The two parameters k_1 and k_2 represent the coefficients of the canalization restriction.

We obtain then the following model:

$$\begin{aligned}\dot{h}_1(t) &= -\frac{k_1}{S}\sqrt{h_1(t)} + \frac{1}{S}q_1(t) \\ \dot{h}_2(t) &= \frac{k_1}{S}\sqrt{h_1(t)} - \frac{k_2}{S}\sqrt{h_2(t)}\end{aligned} \tag{19}$$

These two equations are nonlinear due to the presence of the term $\sqrt{h(t)}$, hence the most difficult task in the control of this considered system will be the control of the water level $h_1(t)$ in different operating conditions.

B. Control Design

In the simulations, we choose to generate a desired trajectory $h_1^d(t)$ satisfying the system constraints based on the flatness concept [11], [20]. Our reference trajectory ensures a transition from $h_1^d(t_0) = 2$ cm to $h_1^d(t_f) = 7$ cm. The two transition instants are chosen $t_0 = 50$ s and $t_f = 150$ s, and the reference trajectory is generated by a polynomial of order 5.
The principle of ultra-local model control proposed in this work, is illustrated in Figure 2. This technique is based on a linear adaptive observer for joint estimation of the ultra-local model parameters and the system output which also represents the system state.

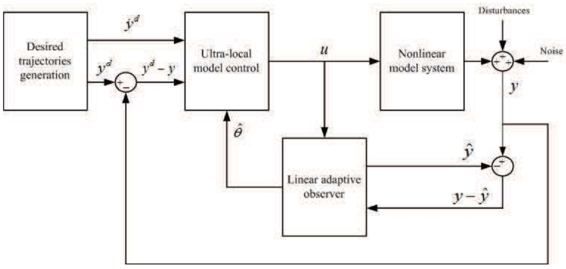

Fig. 2. Functional diagram of the global simulation control with an adaptive observer.

In the numerical simulations, the new control approach is applied to a two-tank system whose the parameter values are given in the table I.

TABLE I
PARAMETER VALUES OF THE CONSIDERED SYSTEM.

Parameter	Value
S	332.5 cm^2
k_1	42.1 $cm^{5/2}/s$
k_2	42.1 $cm^{5/2}/s$

A performance comparison is carried out in this work between the two techniques of the ultra-local model control. The first technique is based on the algebraic derivation (AD) method, developed in [6], [9], and the second is based on the adaptive observer (AO) method. The state-space system of ultra-local model, given in equation (9), is considered in the control approach based on an adaptive observer.
The parameters of the adaptive observer are chosen $\Sigma = 1$, $\Gamma = \text{diag}([6, 0.1])$ and $K = 13$ is computed as the Kalman gain. For the algebraic derivation method, we chose $\alpha = 1$, the sample time $T_e = 0.01$ s. In order to attenuate the influence of the quick fluctuations of $q_1(t)$, a low pass filter is added whose the time constant T should not be too large. We set $T = 40T_e$.

The *a*-PI controller (3) is selected in such a way that the polynomial $p^2 + K_P p + K_I$ has the negative roots $(-0.528, -9.472)$, which corresponds to a response time of 9 s, in the case of adaptive observer method. Moreover, we obtain the negative roots $(-1.774, -0.225)$, which the response time is 10 s, in the case of algebraic derivation method. The gain values are given in Table II.

TABLE II
ADAPTIVE CONTROLLER PARAMETERS.

Gain	Adaptive PI (AD method)	Adaptive PI (AO method)
K_P	2	10
K_I	0.4	5

C. Simulation Results

In the simulations, a centred white noise (normal law N(0,0.001)) is added to the system output in order to test the robustness of numerical simulations of this work. At $t = 190$ s, a level water disturbance of 0.7 cm, which simulates a problem in the sensor, is applied to the system.
The simulation results are given in the following figures (3, 4, 5 and 6) which show the best performance obtained by the proposed approach in terms of reference trajectory tracking and robustness with respect to external disturbances and noises. It is clear that the method of parameter estimation by an adaptive observer is more effective than that by algebraic derivation. With an adaptive observer, a tracking error close to zero is obtained in transient state (see Figure 4). It is clear that, in Figure 5, the addition of a filter with a time constant T has an effect on the control theory. The filter effect on the noise control is clear in the case of the control with algebraic derivation method.
To properly compare the performance of these two control techniques, the system dynamics is tested in the case of parameter uncertainties. In this case, the parameters S and k_1 are increased by 50% when the time $t > 100$ s. The figures 7, 8, 9 and 10 clearly show the robustness of the ultra-local model control technique based on adaptive observer with respect to the system parameters uncertainties.

V. CONCLUSIONS

The above numerical simulation results show that the linear adaptive observer method yields better performances than the algebraic derivation method. The proposed approach has allowed the design of a new water level controller, which is able to ensure good trajectory tracking even in various operating conditions.
The proposed ultra-local model controller is more robust with respect to corrupting noises, external disturbances and parameter uncertainties. A performance improvement in terms of robustness and trajectory tracking is obtained thanks to the both parameter estimation which represents the most important benefit of the proposed adaptive controller.

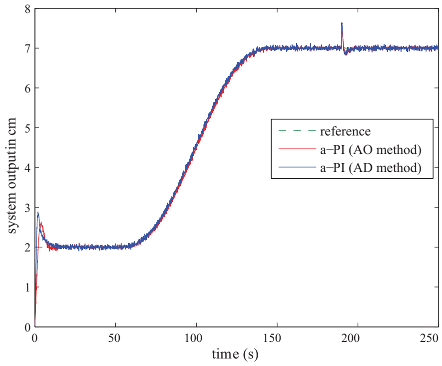

Fig. 3. Noisy system outputs in the case of adaptive observer (AO) method and algebraic derivation (AD) method.

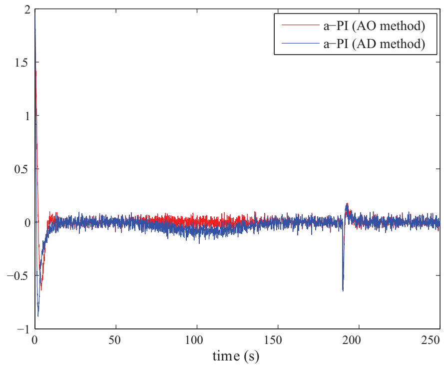

Fig. 4. Tracking errors in the case of the two different methods.

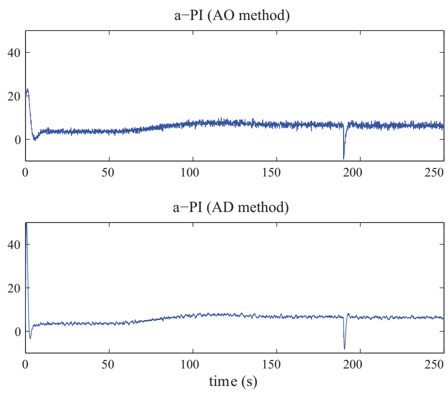

Fig. 5. Control inputs in the case of the two different methods.

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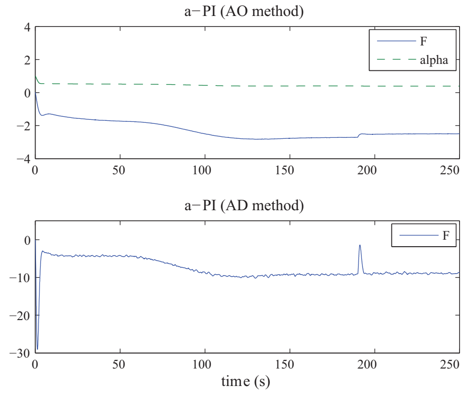

Fig. 6. Parameter estimation in the case of adaptive observer method.

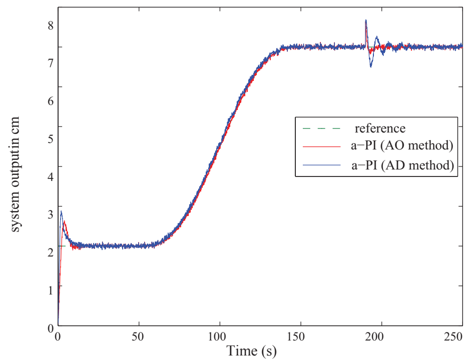

Fig. 7. Noisy system outputs in the case of adaptive observer (AO) method and algebraic derivation (AD) method - parameter uncertainties (50% of S and k_1).

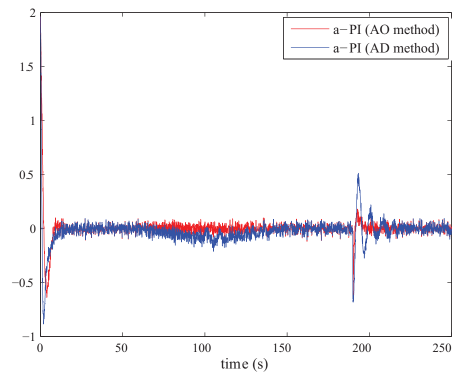

Fig. 8. Tracking errors in the case of the two different methods - parameter uncertainties (50% of S and k_1).

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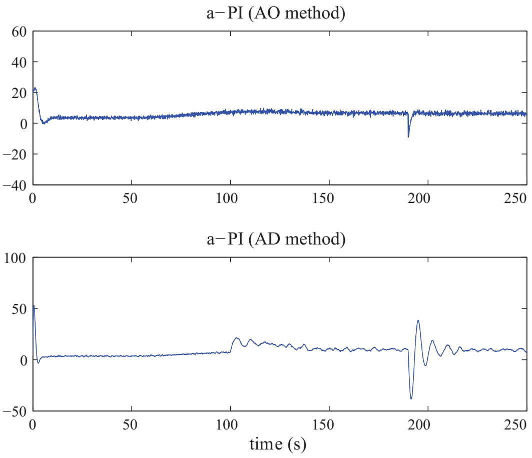

Fig. 9. Control inputs in the case of the two different methods - parameter uncertainties (50% of S and k_1).

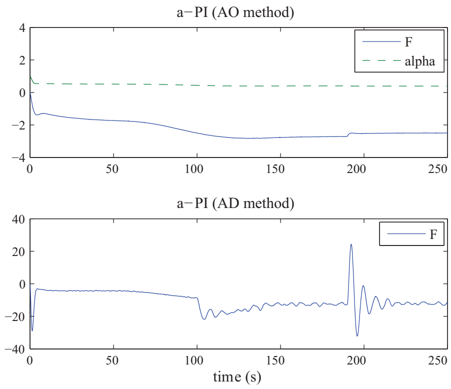

Fig. 10. Parameter estimation in the case of adaptive observer method - parameter uncertainties (50% of S and k_1).

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