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A New Adaptive Controller for a Two-Tank-System Based on Algebraic Derivation Techniques

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***Abstract*—In this paper, a new adaptive controller is proposed for ultra-local model based control. The concept is based on the algebraic derivation techniques to estimate the two ultra-local model parameters. A comparison study with the control method of Fliess-Join which is based on some elementary differential algebra is developed. The robustness of the ultra-local model controllers with respect to noises, external disturbances and parameter uncertainties is highlighted. The control strategies are successfully tested via the control of a two-tank nonlinear system. Different numerical simulations show the effectiveness of the proposed controller.**

***Index Terms*—Ultra-local model control, Adaptive PI controller, Parameter estimation, Algebraic derivation, Trajectory tracking, Robustness, Two-tank-system.**

I. Introduction

Most of the existing control approaches, like adaptive control, predictive control, nonlinear control, PIDs [2], [22], have been employed and tested in several areas [18]. Let us emphasize that writing down a good model, *i.e.*, a model combining simplicity and exactness, is a most difficult task which has perhaps not yet been and might never be achieved in a satisfactory way. Therefore, instead of relying on a more accurate knowledge structure of the controlled system model, the ultra-local model control has been recently introduced with the model-free control notion [4], [5], [6], [9], [27], [28]. This paper develops a new ultra-local model setting and the corresponding adaptive PIDs which have already been successfully applied in several case-studies in different fields [1], [4], [8], [14], [16], [17].

The numerical differentiation, *i.e.*, the derivatives estimation of noisy time signals, is an important but difficult ill-posed theoretical problem [19], [20]. It has attracted a lot of attention in many fields of engineering and applied mathematics. In spite of recent significant advances on the numerical differentiation of noisy signals, this task remains quite complex and sometimes difficult to implement. This technique is replaced by recent, but quite simple, algebraic and non-asymptotic techniques for online parameter identification [26], [7], [11], [12], [13] which have been applied in several concrete case-studies [1], [3].

The ultra-local model based control consists in trying to estimate via the input-output behavior what can be compensated by control in order to achieve a good trajectory tracking performance. The main contribution of this work is to design a new adaptive PID controller able to estimate the two ultra-local model parameters. A large literature exists on the algebraic estimation of derivatives for noisy signals. However, a new approach based on the algebraic derivation has been implemented in this work in order to increase the control performances. A comparison between the algebraic derivation method of Fliess-Join and that proposed in this paper for parameter estimation is developed. The new control approach is applied to a two-tank water system which is considered as nonlinear system.

This paper is organized as follows. A short review of ultra-local model control is presented in Section 2. The general principal and the adaptive PI controller are given in this section. Section 3 defines, via online algebraic estimation technique, a new adaptive controller which takes into account the estimation of both ultra-local model parameters. A two-tank system case-study is considered in Section 4, where numerical computer simulations are displayed. Some concluding remarks are provided in Section 5.

II. ULTRA-LOCAL MODEL CONTROL

A. Basic Idea

The ultra-local model control is based on a local modeling, constantly updated, from the only knowledge of the input-output behavior. With the following unknown differential equation:

$$E\left(y\left(t\right), \dot{y}\left(t\right), \ldots, y^{(a)}\left(t\right), u\left(t\right), \dot{u}\left(t\right), \ldots, u^{(b)}\left(t\right)\right) = 0 \tag{1}$$

which is linear or nonlinear and where E is a sufficiently smooth function of its arguments. We assume that for an integer, $0 < \nu < a$, $\frac{\partial E}{\partial y^{(\nu)}} \neq 0$, the implicit function theorem allows to write, at least locally, the following equation:

$$y^{(\nu)} = \varphi\left(t, y, \dot{y}, \ldots, y^{(\nu-1)}, y^{(\nu+1)}, \ldots, y^{(a)}, u, \dot{u}, \ldots, u^{(b)}\right) \tag{2}$$

which approximately describes the input-output behavior. The ultra-local model, valid in a very short part of time, is defined by the following differential equation:

$$y^{(\nu)}\left(t\right) = F\left(t\right) + \alpha\left(t\right) u\left(t\right) \tag{3}$$

The order of derivation ν is generally equal to 1 or 2. $F(t)$ and $\alpha(t)$ represent the ultra-local model parameters to identify in real time. These two parameters contain all the structural information, including disturbances and their derivatives.

B. Adaptive PI Controller

The desired behavior is obtained in the closed loop, if, *e.g.*, $\nu = 1$ in the equation (1), thanks to the adaptive PI controller or, abbreviated as *a*-PI:

$$u(t) = \frac{-\hat{F}(t) + \dot{y}^d(t) + K_p e(t) + K_I \int e(t)}{\hat{\alpha}(t)} \tag{4}$$

where:

- $y^d(t)$ is the output reference trajectory, obtained according to the precepts of the flatness-based control which is well suited to solve the trajectory planning problems [10], [15], [24], [25].
- $e(t) = y^d(t) - y(t)$ is the tracking error.
- K_P and K_I are the two tuned gains.

Combining the two equations (3) and (4), for $\nu = 1$, gives:

$$\ddot{e}(t) + K_P \dot{e}(t) + K_I e(t) = 0 \tag{5}$$

where $F(t)$ and $\alpha(t)$ don't appear in (5), as well as the parts and unknown system disturbances are eliminated by compensation. Therefore, the tracking condition becomes easily achieved by a linear differential equation of order 2 with constant coefficients K_P and K_I.

III. PARAMETER IDENTIFICATION METHODS

A. Algebraic Derivations

Remember the estimation basics of derivatives (for more details, see [20]). In order to apply the technique of Fliess-Join, considering the signal $s(t)$ approached around 0 by the truncated Taylor development:

$$s(t) = \sum_{n=0}^{\infty} \frac{s^{(n)}(0)}{n!} t^n \tag{6}$$

Approaching $s(t)$ in the interval $[0, T]$, $T > 0$, by the polynomial $s_N(t) = \sum_{n=0}^{N} \frac{s^{(n)}(0)}{n!} t^n$ of degree N, the operational form $S_N(p)$ of $s_N(t)$ is written as follows [21]:

$$S_N(p) = \frac{s(0)}{p} + \frac{\dot{s}(0)}{p^2} + \cdots + \frac{s^{(N)}(0)}{p^{N+1}} \tag{7}$$

where $\frac{1}{p}$ represents the integration operator:

$$\frac{1}{p} s(t) = \int_0^T s(\tau) d\tau \tag{8}$$

It is possible to isolate each coefficient $s^{(i)}(0)$ appearing in the above expression by applying a practice operator to $S_N(p)$. The two following formulas of operational calculus are very useful [21]:

- The operator $\frac{1}{p^k}$ corresponds to the function $t \rightarrow \frac{t^{k-1}}{(k-1)!}$.
- The operator $\frac{d}{dp}$ corresponds to the multiplication in the temporal domain by $-t$.

In addition, Cauchy's integral formula for converting a multiple integral to simple integral, is given by:

$$\int_0^T \int_0^{t_{k-1}} \cdots \int_0^{t_1} f(\tau) d\tau dt_1 \ldots dt_{k-1} = \int_0^T \frac{(T-\tau)^{k-1}}{(k-1)!} f(\tau) d\tau \tag{9}$$

Therefore, the expression $s^{(i)}(0)$ in the temporal domain is obtained as follows:

$$s^{(i)}(0) = \int_0^T P(\tau, T) s_N(\tau) d\tau \tag{10}$$

where $P(\tau, T)$ is a polynomial in terms of τ and T [23]. Noting that (10) gives the calculation of $s^{(i)}(0)$ from an integral over the time interval $[0, T]$ for a given small $T > 0$. The results obtained above thus give an estimate $s^{(i)}(t)$ at time $t = 0$ from the polynomial signal $s(t)$, see (6), taken on the interval $[0, T]$.

B. Algebraic Derivation Method of Fliess-Join

The method of Fliess-Join [4], [6], [8], consists in imposing the parameter α (such as it exists an infinity solutions can be always do it). Therefore, the estimation of the parameter $F(t)$ is obtained thanks to the knowledge of the control input $u(t)$, the imposed parameter α and the estimate of the derivative $\dot{y}(t)$.

The estimate of the first order derivative of a noisy signal y is written as follows [13]:

$$\widehat{\dot{y}} = -\frac{3!}{T^3} \int_0^T (T - 2t) y(t) dt \tag{11}$$

where $[0, T]$ is quite short time window. This window is sliding in order to get this estimate at each time instant.

The robustness with respect to noises is ensured by the integral which is the simplest low-pass filter. The data filtering is achieved in two ways. On the one hand by the temporal integrations and we can add, on the other hand by varying on the length T of the sliding integration window.

At the sampling time kT_e (noted k for simplicity), the estimation of F is written as follows:

$$\hat{F}_k = \widehat{\dot{y}}_k - \alpha u_{k-1} \tag{12}$$

where $\widehat{\dot{y}}_k$ is the estimate of the derivative of the system output that can be provided at the instant k, α is a constant design parameter, and u_{k-1} is the control input that has been applied to the system during the previous sampling time.

With strong constraint on the parameter α, the set of solutions is transformed in a single solution. The identification of both parameters $\hat{F}$ and $\hat{\alpha}$ is the most important contribution of this paper. Therefore, a new algebraic derivation method to estimate the ultra-local model parameters is proposed in the following.

C. Proposed Algebraic Derivation Method

Considering the ultra-local model (3), by assuming that the two parameters F and α are constant. For $\nu = 1$, the following relation is obtained:

$$\dot{y}(t) = F + \alpha u(t) \tag{13}$$

Writing the operational form of the relation (13), gives:

$$py = y(0) + \frac{F}{p} + \alpha u \tag{14}$$

Drifting with respect to p in order to eliminate the initial condition, we obtain:

$$p\frac{dy}{dp} + y = -\frac{F}{p^2} + \alpha\frac{du}{dp} \tag{15}$$

Multiplied by p^2 gives:

$$p^3\frac{dy}{dp} + p^2 y = -F + \alpha p^2\frac{du}{dp} \tag{16}$$

Drifting with respect to p:

$$p^3\frac{d^2y}{dp^2} + 4p^2\frac{dy}{dp} + 2py = \alpha\left(p^2\frac{d^2u}{dp^2} + 2p\frac{du}{dp}\right) \tag{17}$$

Divided by p^4 to have only the integrals, gives:

$$\frac{1}{p}\frac{d^2y}{dp^2} + 4\frac{1}{p^2}\frac{dy}{dp} + 2\frac{1}{p^3}y = \alpha\left(\frac{1}{p^2}\frac{d^2u}{dp^2} + 2\frac{1}{p^3}\frac{du}{dp}\right) \tag{18}$$

The temporal interpretation of operations leads to the algebraic identification of ultra-local model. Indeed, the temporal interpretations of operations involved in the expression (18), for an integration window $[0, T]$, T is small, give the estimate of $\hat{\alpha}$ as follows:

$$\hat{\alpha} = \frac{\displaystyle\int_0^T \left[6\tau^2 - 6\tau T + T^2\right] y(\tau)\, d\tau}{\displaystyle\int_0^T \left[-2\tau^3 + 3T\tau^2 - \tau T^2\right] u(\tau)\, d\tau} \tag{19}$$

The equation (11) allows to estimate the first order derivative of a noisy signal y over a short time window $[0, T]$, and from the relation (19), the estimate of $\hat{F}$ is obtained as follows:

$$\hat{F} = \hat{\dot{y}}(t) - \hat{\alpha} u(t) \tag{20}$$

In the following, both ultra-local model control approaches are applied to a two-tank system in order to test the effectiveness of our proposed technique.

IV. Case Study: Two-Tank-System

A. Model description

The system considered in this work is the two-tank-system described in the Figure 1 which is constituted by two identical water tanks that have the same section S. Denote by $h_1(t)$ the water level in the upper tank, which also represents the system output, $h_2(t)$ the water level in the lower tank, $q_1(t)$ the input flow of the upper tank, $q_2(t)$ the output flow of the upper tank and $q_3(t)$ the output flow of the lower tank. In the steady state, the conservation of the total volume of water leads to $q_1(t) = q_3(t)$.

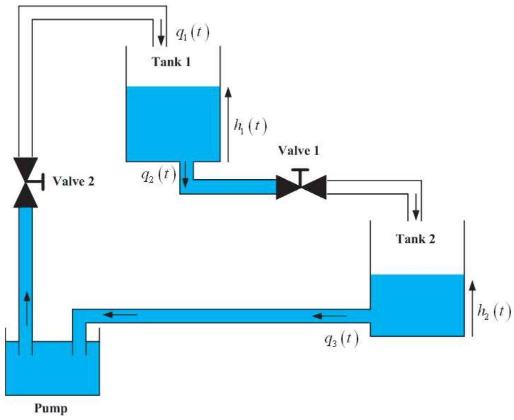

Fig. 1. Two-tank system.

The nonlinear model of the considered system is as follows:

$$\begin{aligned} S\dot{h}_1(t) &= q_1(t) - q_2(t) \\ S\dot{h}_2(t) &= q_2(t) - q_3(t) \end{aligned} \tag{21}$$

with $q_2(t) = k_1\sqrt{h_1(t)}$ and $q_3(t) = k_2\sqrt{h_2(t)}$.
The term $k_i\sqrt{h_i(t)}$, $i = 1, 2$, comes from the gravity flow. Both parameters k_1 and k_2 represent the coefficients of the canalization restriction.
We obtain then the following model:

$$\begin{aligned} \dot{h}_1(t) &= -\frac{k_1}{S}\sqrt{h_1(t)} + \frac{1}{S}q_1(t) \\ \dot{h}_2(t) &= \frac{k_1}{S}\sqrt{h_1(t)} - \frac{k_2}{S}\sqrt{h_2(t)} \end{aligned} \tag{22}$$

These two equations are nonlinear due to the presence of the term $\sqrt{h(t)}$, hence the most difficult task in the control of this considered system will be the control of the water level $h_1(t)$ in the upper tank.

B. Control Design

We choose to generate a desired trajectory $h_1^d(t)$ satisfying the system constraints, ensuring a transition from $h_1^d(t_0) = 2$ V to $h_1^d(t_f) = 5$ V. The two transition instants are chosen $t_0 = 50$ s and $t_f = 150$ s, and the reference trajectory is generated by a polynomial of order 5. The new control approach is applied to a two-tank system whose parameter values are given in the table I.

TABLE I
PARAMETER VALUES OF THE CONSIDERED SYSTEM.

Parameter	Value
S	332.5 cm^2
k_1	42.1 $cm^{5/2}/s$
k_2	42.1 $cm^{5/2}/s$

A performance comparison is carried out in this work between two different ultra-local model control strategies. The first one is the method of Fliess-Join (FJ) developed in [7], [12]. Whereas, the second is our proposed algebraic derivation (AD) method. For the two control techniques, the adaptive controllers have the same parameters where $K_P = 10$ and $K_I = 5$. These gains are determined by a placement of two poles in the functional equation (5) in order to stabilize the tracking error with good dynamics.
In the case of ultra-local model control based on the method of Fliess-Join, we have fixed the parameter $\alpha = 1$. In addition, we have chosen the sampling time $T_e = 0.1$ s and the sliding window $T = 5T_e$ in the two studied control cases.

C. Simulation Results

A centered white noise, with standard deviation equal to 0.001, is added to the system output. At $t = 200$ s, a level water perturbation of 0.4 V, which simulates a default sensor, is applied to the system in order to test the robustness of our proposed approach. The block diagram presented in the figure 2, summarizes the general principle of ultra-local model based control. Indeed, the online parameter estimation is shown in this figure.

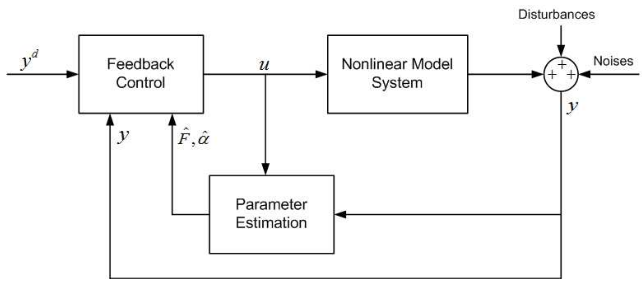

Fig. 2. General structure of ultra-local model control.

The simulation results are given in the following figures 3, 4, 5, 6 and 7. The best performances obtained thanks to the our proposed approach are shown in the two figures 3 and 4. An improvement in terms of trajectory tracking and robustness with respect to external disturbances, noises and parameter uncertainties is given by the proposed algebraic derivation method. It is clear that, with the proposed adaptive PI controller, the consequence of the level water perturbation is smaller and rejected faster than that developed by Fliess-Join. We can observe, in the figure 4, that the tracking error for a-PI control converges to zero despite the severe operating conditions. The figures 6 and 7 present the parameter estimation of ultra-local model. We can see that the aim of this paper is not the system parameter identification but to obtain in each instant parameters which satisfy the ultra-local model.

To properly compare the two different approaches, the system dynamics is tested in the case of parameter uncertainties. For this, the system parameters S and k_1 are increased by 50% when the time $t > 150$ s. The figures 8, 9, 10, 11 and 12 clearly show the robustness of the ultra-local model control technique based on adaptive observer with respect to the system parameter uncertainties. In the figures 8 and 9, we can see that the effect of parameter uncertainties is more significant in the case of a-PI control based on the method of Fliess-Join.
Consequently, the simulation results show the superiority of our proposed control approach thanks to the online estimation of the two ultra-local model parameters.

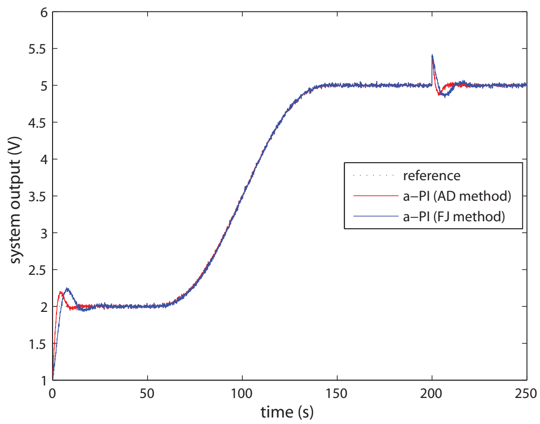

Fig. 3. Reference trajectory and noisy system outputs.

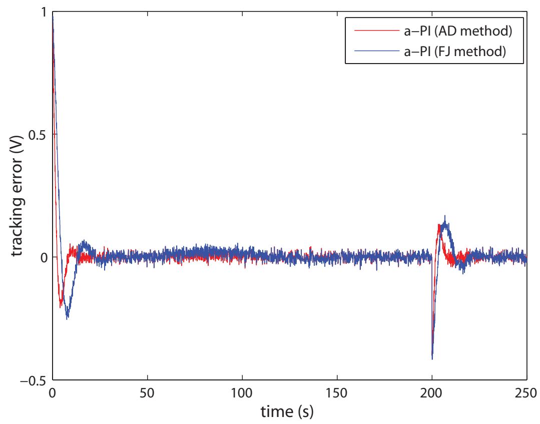

Fig. 4. Tracking errors.

V. CONCLUSION

In this paper, an adaptive controllers are developed based on the ultra-local model concept which can be easily applied for a

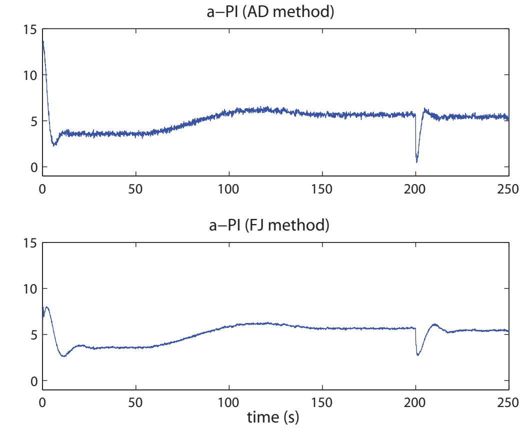

Fig. 5. Control inputs.

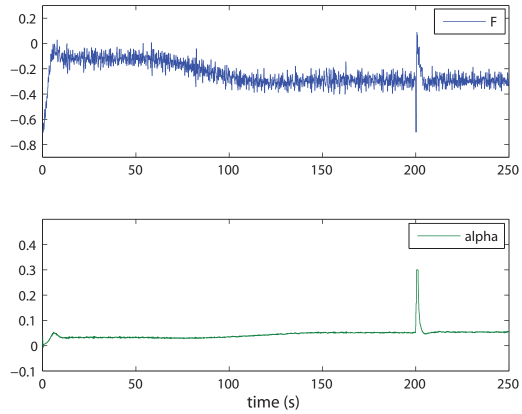

Fig. 6. Parameter estimation in the case of algebraic derivation (AD) method.

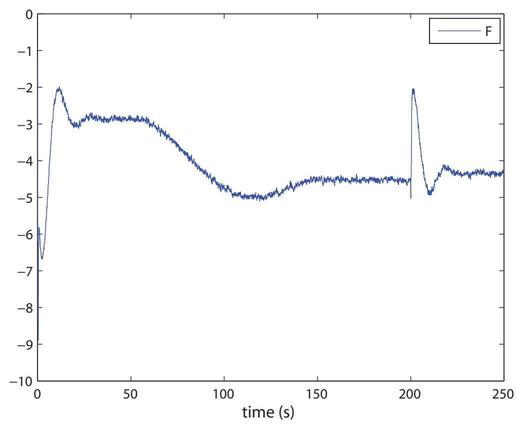

Fig. 7. Parameter estimation in the case of Fliess-Join (FJ) method.

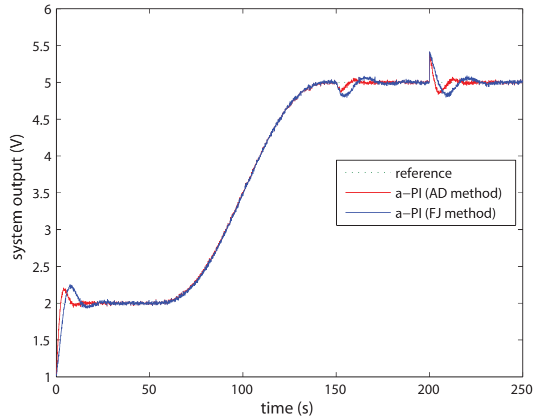

Fig. 8. Reference trajectory and noisy system outputs - parameter uncertainties (50% of S and k_1).

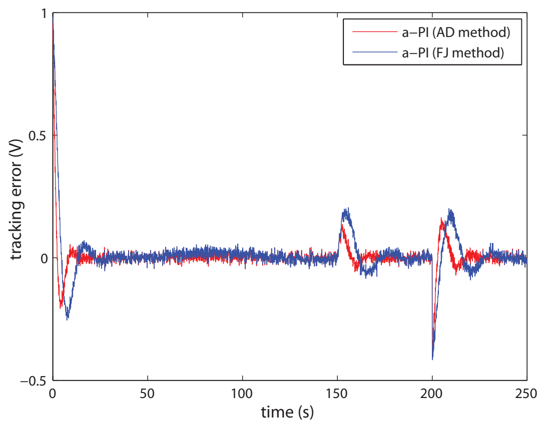

Fig. 9. Tracking errors - parameter uncertainties (50% of S and k_1).

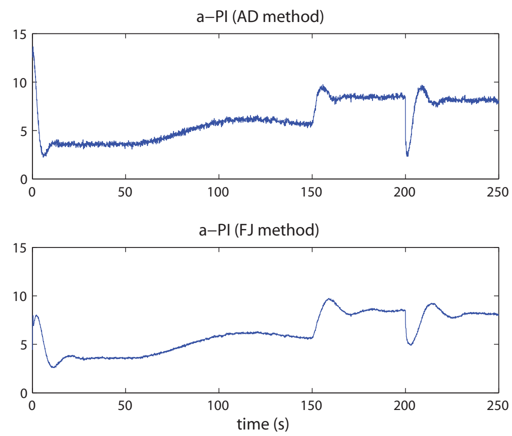

Fig. 10. Control inputs - parameter uncertainties (50% of S and k_1).

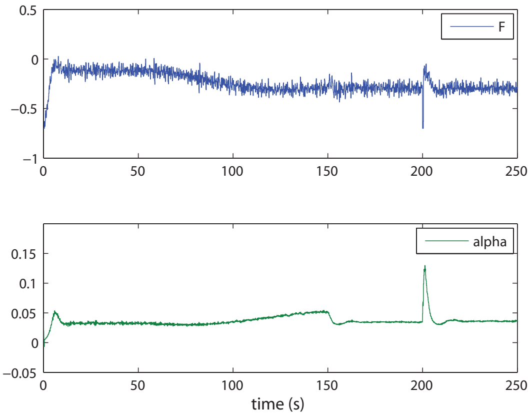

Fig. 11. Parameter estimation in the case of algebraic derivation (AD) method - parameter uncertainties (50% of S and k_1).

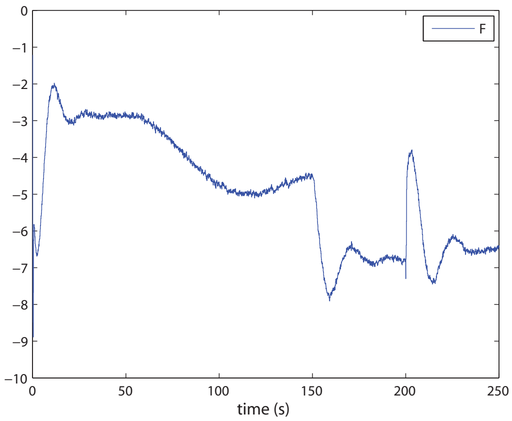

Fig. 12. Parameter estimation in the case of Fliess-Join (FJ) method - parameter uncertainties (50% of S and k_1).

large class of nonlinear systems. The proposed control strategy, extended from the algebraic derivation techniques, provides an improvement in terms of robustness and trajectory tracking performances. The proposed algebraic derivation method presents the contribution of this paper. The online estimation of the two ultra-local model parameters allows to obtain a control approach more robust and effective. Therefore, this new control technique helps to find the desired behavior with better robustness performances.

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