An Algebraic Control Approach Based on the Estimation of an Ultra-Local Broïda Model

Hajer Thabet[1], Mounir Ayadi[1] and Frédéric Rotella[2]

***Abstract*— This paper deals with a new parameter estimation method for an ultra-local model of Broïda. The proposed approach is based on the algebraic derivation techniques and the linear system resolution method, in order to estimate the ultra-local model parameters, such that the variable time-delay. The closed-loop control is achieved via an adaptive PI controller to reject the influence of noises and disturbances. A simulation results of a thermal process application are given to validate the effectiveness of the proposed strategy. A performance comparison with a classical PID controller is achieved.**

***Index Terms*— Ultra-local model control, Adaptive PI controller, Algebraic derivation, Time-delay, Linear system resolution method, Parameter estimation, Robustness, Trajectory tracking.**

I. INTRODUCTION

Writing down simple and reliable differential equations for describing a concrete plant is almost always a difficult task. For this reason, the industrial world is not willing to employ most techniques based on a precise mathematical modeling in spite of considerable advances in the last years. This unfortunate situation is overcome thanks to the ultra-local model control which is recently introduced by M. Fliess and C. Join [6], [7], [8], [28], [29]. The concept of this approach is based on an elementary continuously updated local modeling via the unique knowledge of the input-output behavior. This recent control strategy has already applied in several applications [8], [10], [15], [16]. For the sake of simplicity presentation, we assume that the system is single-input and single-output. The input-output behavior is assumed to be well approximated within its operating range by an ordinary differential equation $E\left(y, \dot{y}, \ldots, y^{(a)}, u, \dot{u}, \ldots, u^{(b)}\right) = 0$. Replace this unknown, or at least poorly known, system equation by the ultra-local model that can be written as:

$$y^{(\nu)}(t) = F(t) + \alpha(t)\, u(t) \tag{1}$$

which is continuously updated, where:

- The time-varying functions $F(t)$ and $\alpha(t)$ which are estimated via the input and the output measurement, subsume the structural properties.
- The order $\nu \in \mathbb{N}$ is necessarily a design parameter that can be arbitrarily chosen. This order of derivation is always chosen to be equal to 1 or 2.

Assume that $\nu = 1$, the desired behavior is achieved via an adaptive Proportional-Integral controller, or *a*-PI. Based on the knowledge of the ultra-local model parameters (2), the control input is defined as follows:

$$u(t) = \frac{-\hat{F}(t) + \dot{y}^d(t) + K_P e(t) + K_I \int e(t)}{\hat{\alpha}(t)} \tag{2}$$

where:

- $y^d(t)$ is the output reference trajectory, which is obtained according to the rules of flatness-based trajectory planning [11], [26].
- $e(t) = y^d(t) - y(t)$ is the tracking error.
- K_P and K_I are the usual PI gains.

The ultra-local model is updated via the online algebraic and non-asymptotic techniques [9], [12], [13], [14], [27] which offer a quite simple and considering robust estimates. These recent techniques have attracted a lot of attention in several concrete applications [2], [22].

The occurrence of delays is manifold in engineering systems such as transport, communication, biomedicine, signal processing. The real time delay identification is one of the most crucial open problems in the field of delay systems (see, *e.g.*, [25]), and several on line estimation methods have been suggested in the literature for the identification of delay. Recent developments in [3], [4] have considered the on line identification of delay systems with an algebraic approach.

The delays have been taken into account by the model-free control strategy [6], [7], [8]. However, this control approach has shown that the delayed terms are found in the unknown variables of the ultra-local model, which implies that the identification of delays is not necessary.

In this work, the identification of the delay can be possible by using the algebraic derivation techniques. For this, a temporally variable delay, $T_R(t)$, is introduced in the control input of the ultra-local model (1). The expression of the ultra-local model with delay becomes:

$$y^{(\nu)}(t) = F(t) + \alpha(t)\, u(t - T_R(t)) \tag{3}$$

The online estimation of delay and the others parameters of ultra-local model, presents the main contribution of this paper. The proposed control strategy and their corresponding adaptive PID controller allow to estimate the *ultra-local model of Broïda* using the algebraic derivation techniques with the linear system resolution method.

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This paper is organized as follows. Section II presents the algebraic derivation of noisy signals. The proposed parameter identification approach to estimate the ultra-local model of Broïda is developed in Section III. Section IV describes some numerical simulations of a thermal process with the proposed algebraic approach, and several simulation results are provided. Some concluding remarks may be found in section V.

II. ALGEBRAIC DERIVATION OF NOISY SIGNALS

The algebraic derivation techniques for nonlinear estimation and identification boils down to the obtention of fast and robust derivative estimations of noisy time signals. These techniques of estimation consist in the design of FIR filters by resolving a classical polynomial approximation of signals. From the algebraic manipulation of signals in the operational domain, the polynomial approximation is obtained. The objective is to estimate the derivative of time signal $s(t)$ which is defined by the power series:

$$s(t) = \sum_{n=0}^{\infty} s^{(n)}(0) \frac{t^n}{n!} \tag{4}$$

This analytic time function (4) is assumed to be convergent around 0. Approximate $s(t)$ by the truncated Taylor expansion, of degree N, as follows:

$$s_N(t) = \sum_{n=0}^{N} s^{(n)}(0) \frac{t^n}{n!} \tag{5}$$

The usual notations of operational calculus [19], [30] yield:

$$S_N(p) = \sum_{n=0}^{N} \frac{s^{(n)}(0)}{s^{n+1}} \tag{6}$$

Multiply both sides by positive powers of $\frac{d}{dp}$. The quantities $s^{(n)}(0)$, $n = 0, 1, \ldots, N$, which are linearly identifiable [12], [13], satisfy the following triangular system of linear equations:

$$\frac{d^\kappa p^{N+1} S_N}{dp^\kappa} = \frac{d^\kappa}{dp^\kappa} \left(\sum_{n=0}^{N} s^{(n)}(0) p^{N-n} \right) \tag{7}$$

where $0 \leqslant \kappa \leqslant N-1$. Noting that it is possible to isolate each coefficient $s^{(n)}(0)$ appearing in the previous expression (7) by applying a convenient operator to $S_N(p)$ [17], [18]. Multiplying both sides of (7) by p^{-M}, $M > N$ is sufficiently large, permit to get rid of time derivatives, *i.e.*, of $p^\sigma \frac{d^\iota S_N}{dp^\iota}$, $\sigma = 1, \ldots, N$, $0 \leqslant \iota \leqslant N$.
The conversion to the temporal domain is based on the two following formulas of operational calculus [19], [30]:

- The operator $\frac{1}{p^k}$, $k \geqslant 1$, corresponds to the function $t \to \frac{t^{k-1}}{(k-1)!}$.
- The operator $\frac{d}{dp}$ corresponds to the multiplication by $-t$.

Moreover, the Cauchy's integral formula to transform a multiple integral to simple integral is defined by:

$$\begin{aligned} &\int_0^T \int_0^{\tau_{k-1}} \ldots \int_0^{\tau_1} (-1)^n \mu^n s(\mu)\, d\mu d\tau_1 \ldots d\tau_{k-1} = \\ &\qquad \frac{(-1)^n}{(k-1)!} \int_0^T (T-\mu)^{k-1} \mu^n s(\mu)\, d\mu \end{aligned} \tag{8}$$

Consequently, the calculation of $s^{(i)}(0)$ is obtained, from an integral on the time interval $[0, T]$ for a given small $T > 0$, as follows:

$$s^{(i)}(0) = \int_0^T P(\mu, T)\, s_N(\mu)\, d\mu \tag{9}$$

where $P(\mu, T)$ is polynomial in μ and T [23], [24]. As $\left.\frac{d^i s(t-\mu)}{d\mu^i}\right|_{\mu=0} = (-1)^i s^{(i)}(t)$, the derivative i of the signal $s(t)$ can be expressed as an integral which implies the values of s_N on the time interval $[t-T, t]$:

$$s^{(i)}(t) = (-1)^i \int_0^T P(\mu, T)\, s_N(t-\mu)\, d\mu \tag{10}$$

Noting that the integral operation plays the role of low-pass filter and reduces the noise that corrupts the signal $s(t)$. The choice of T and N results such that: the larger is T, the smaller is the effect of the noise (the better integrals low pass filtering) and the larger is the error due to truncation. The larger is N, the smaller is the error due to truncation and the larger is the error due to noise.

III. PROPOSED ALGEBRAIC APPROACH

The proposed control approach, also called *generalized Broïda*, allows to estimate the ultra-local model of Broïda. Indeed, the linear model of Broïda is defined by the following operator transfer [1], [21]:

$$F(p) = \frac{K}{(1+\tau p)^\nu} e^{-T_R p} \tag{11}$$

where K is the model gain, τ the model time constant and T_R the model time-delay. The expression of ultra-local model with time-delay (3) can be generalized from the ultra-local model of Broïda which is defined by the differential equation (12). This equation is deduced from (11), in the temporal domain, as follows:

$$y^{(\nu)}(t) = -\frac{1}{\tau(t)} y(t) + \frac{K(t)}{\tau(t)} u(t - T_R(t)) \tag{12}$$

The correspondence between the two equations (3) and (12) aids to identify the parameters of the ultra-local model of Broïda from those of the ultra-local model with time-delay, such that:

$$K(t) = -\frac{\alpha(t)}{F(t)} y(t), \quad \tau(t) = -\frac{y(t)}{F(t)} \tag{13}$$

In order to estimate the parameters of the ultra-local model with time-delay (3), assuming that $F(t)$, $\alpha(t)$ and $T_R(t)$ are constant. Then, we obtain the following relation for $\nu = 1$:

$$\dot{y}(t) = F + \alpha u(t - T_R) \tag{14}$$

The delay can be expressed in the operational domain by the simplified Pade approximation such as $e^{-T_R p} \simeq \dfrac{1 - T_R p/2}{1 + T_R p/2}$, hence, the operational form of the equation (14) is written as follows:

$$\begin{aligned} py - y(0) + \frac{T_R}{2}\left(p^2 y - py(0) - \dot{y}(0)\right) \\ = \frac{F}{p} + \alpha u - \alpha \frac{T_R}{2}(pu - u(0)) \end{aligned} \tag{15}$$

Take three derivatives of (15) with respect to the complex variable p to get rid of the initial conditions $y(0)$, $\dot{y}(0)$ and $u(0)$:

$$\begin{aligned} p\frac{d^2 y}{dp^2} + 2\frac{dy}{dp} = -\frac{T_R}{2}\left(2y + 4p\frac{dy}{dp} + p^2\frac{d^2 y}{dp^2}\right) \\ +2\frac{F}{p^3} + \alpha\frac{d^2 u}{dp^2} - \alpha\frac{T_R}{2}\left(p\frac{d^2 u}{dp^2} + 2\frac{du}{dp}\right) \end{aligned} \tag{16}$$

Multiplying the equation (16) by p^3 gives:

$$\begin{aligned} p^4\frac{d^2 y}{dp^2} + 2p^3\frac{dy}{dp} = -\frac{T_R}{2}\left(2p^3 y + 4p^4\frac{dy}{dp} + p^5\frac{d^2 y}{dp^2}\right) \\ +2F + \alpha p^3\frac{d^2 u}{dp^2} - \alpha T_R\left(p^4\frac{d^2 u}{dp^2} + 2p^3\frac{du}{dp}\right) \end{aligned} \tag{17}$$

Applying the operator $\dfrac{d}{dp}$ with the aim of eliminating the parameter F from the equation (17):

$$\begin{aligned} p^4\frac{d^3 y}{dp^3} + 6p^3\frac{d^2 y}{dp^2} + 6p^2\frac{dy}{dp} \\ = -\frac{T_R}{2}\left(p^5\frac{d^3 y}{dp^3} + 9p^4\frac{d^2 y}{dp^2} + 18p^3\frac{dy}{dp} + 6p^2 y\right) \\ +\alpha\left(p^3\frac{d^3 u}{dp^3} + 3p^2\frac{d^2 u}{dp^2}\right) \\ -\alpha\frac{T_R}{2}\left(p^4\frac{d^3 u}{dp^3} + 6p^3\frac{d^2 u}{dp^2} + 6p^2\frac{du}{dp}\right) \end{aligned} \tag{18}$$

Finally, Multiplying both sides by p^{-6} in order to eliminate any non causal term and to avoid differentiations with respect to time:

$$\begin{aligned} \frac{1}{p^2}\frac{d^3 y}{dp^3} + 6\frac{1}{p^3}\frac{d^2 y}{dp^2} + 6\frac{1}{p^4}\frac{dy}{dp} \\ = -\frac{T_R}{2}\left(\frac{1}{p}\frac{d^3 y}{dp^3} + 9\frac{1}{p^2}\frac{d^2 y}{dp^2} + 18\frac{1}{p^3}\frac{dy}{dp} + 6\frac{1}{p^4}y\right) \\ +\alpha\left(\frac{1}{p^3}\frac{d^3 u}{dp^3} + 3\frac{1}{p^4}\frac{d^2 u}{dp^2}\right) \\ -\alpha\frac{T_R}{2}\left(\frac{1}{p^2}\frac{d^3 u}{dp^3} + 6\frac{1}{p^3}\frac{d^2 u}{dp^2} + 6\frac{1}{p^2}\frac{du}{dp}\right) \end{aligned} \tag{19}$$

Returning to the time domain, the previous relation (19) can be represented as follows:

$$Y(t) = -\frac{T_R}{2}h_1(t) + \alpha h_2(t) - \alpha\frac{T_R}{2}h_3(t) \tag{20}$$

where:

$$\begin{aligned} Y(t) &= \int_0^T \left[5\mu^4 - 10T\mu^3 + 6T^2\mu^2 - T^3\mu\right] y(\mu)\,d\mu \\ h_1(t) &= \int_0^T \left[-20\mu^3 + T^3 - 12T^2\mu + T\left(12\mu^2 + 18\mu^3\right)\right] \\ &\quad y(\mu)\,d\mu \\ h_2(t) &= \int_0^T \left[\mu^5 + \frac{5}{2}T\mu^4 - 2T^2\mu^3 + \frac{1}{2}T^3\mu^2\right] u(\mu)\,d\mu \\ h_3(t) &= \int_0^T \left[5\mu^4 - 4T\mu^3 - T^3\mu + 6T^2\mu^2\right] u(\mu)\,d\mu \end{aligned}$$

Noting that $[0, T]$, for $T > 0$, represents the time interval of integration. We can generate a linear system from the equations of (20), that is:

$$Y = H\theta \tag{21}$$

where $\theta^T = \begin{bmatrix} -\dfrac{T_R}{2} & \alpha & -\alpha\dfrac{T_R}{2} \end{bmatrix}$, and the vector $H = \begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix}$.

In the present approach, the parameters θ in (21) are updated at each sampling time T_e from the measurements of the output and the knowledge of the input. At sampling time k (*i.e.* $t = kT_e$), the unknown parameters are estimated by solving the linear system $Y_k = H_k\theta_k$ such that the general expression of estimation is defined as:

$$\theta_k = H_k^{\{1\}}Y_k + \left(I_m - H_k^{\{1\}}H_k\right)\lambda_k \tag{22}$$

where:

- H_k is a vector of size $(1 \times m)$;
- $H_k^{\{1\}}$, which verifies $H_k = H_k X H_k$, denotes any generalized inverse of H_k;
- λ_k is an arbitrary column vector of size $(m \times 1)$. The coefficients of the vector λ_k can be used to satisfy other constraints of the system control.

After the estimation of the two parameters $\hat{\alpha}$ and $\hat{T}_R$ of the ultra-local model using the algebraic derivation techniques, it remains to estimate the third parameter $\hat{F}$ which is given by:

$$\hat{F} = \widehat{\dot{y}} - \hat{\alpha}u\left(t - \hat{T}_R\right) \tag{23}$$

where $\widehat{\dot{y}}$ presents the estimate of the first derivative of the output which is obtained, by applying the algebraic derivations, as follows:

$$\widehat{\dot{y}} = -\frac{3!}{T^3}\int_0^T (T - 2\mu)\,y(\mu)\,d\mu \tag{24}$$

where the identification window T is chosen such that $T \geqslant 2T_e$, which is very short in order to get an estimate at each instant.

The principle of the proposed control approach is presented in the figure 1. The ultra-local model parameters in (3) (F, α, T_R) are estimated, from the measurements of the output and the input, based on the algebraic derivation techniques and the linear system resolution method. Thanks to these estimated parameters, we identify the parameters of the ultra-local model of Broïda in (12) (K, τ, T_R) which allow the setting of the PID controller gains (K_P, K_I, K_D) in an adaptive way in order to obtain the feedback control.

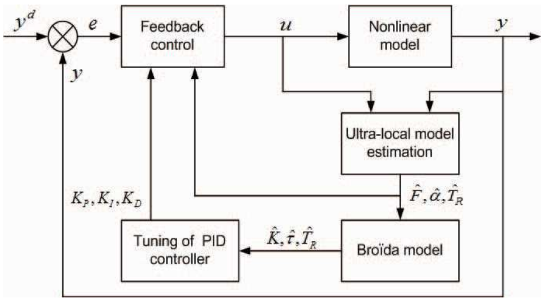

Fig. 1. Principle of proposed control approach.

IV. APPLICATION: THERMAL PROCESS

Figure 2 shows the considered thermal process which is constituted by a tube of constant volume $V\left[m^3\right]$ and a heating resistor $R_c\,[Ohm]$ connected to a DC power supply $u\,(t)$. The parameter $C\left[J.m^{-3}.^{\circ}K^{-1}\right]$ is the specific heat constant of air. The voltage $u\,(t)$, applied to the resistance, allows to heat the air entering at the tube by Joule effect [20]. Indeed, $T_E\,[^{\circ}K]$ is the ambient temperature, and $f_j\left[m^3.s^{-1}\right]$ is the air rate flow entering according to the valve opening angle j. The purpose of the control system is to regulate the temperature $T_S\,[^{\circ}K]$ of the outgoing air at the constant temperature, given that the air flows into the tube with an initial temperature $T_E\,[^{\circ}K]$ and at the flow rate $f_j\left[m^3.s^{-1}\right]$.

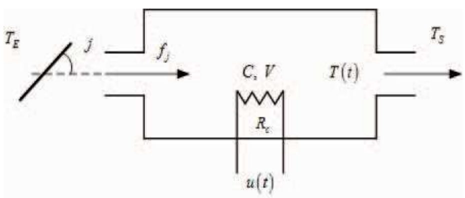

Fig. 2. Simplified schema of thermal process [5].

The flow rate signal is assumed to piecewise constant and can be vary by changing the throttle position j. By applying a variation to the amplifier input, two phenomena are noted:

- The heat capacity of the resistor which is an abrupt voltage change translates into a slower evolution of the resistor temperature. This phenomenon is defined by a transfer function of first order characterized by the time constant τ.
- The delay of the temperature measurement due to the distance between the resistor and the thermistor measurement. This phenomenon is reflected by a time delay T_R in the transfer function.

This leads to a first approximation whose the theoretical transfer function of the model is given by [5]:

$$H\,(p) = K\frac{e^{-T_R p}}{1+\tau p} \tag{25}$$

where K is the overall static gain, τ is the time constant and T_R is the time-delay. The numerical parameters values of the considered thermal process, for an ambient temperature equal to $20^{\circ}C$, are given in the Table I.

TABLE I
PARAMETER VALUES OF THE CONSIDERED SYSTEM [5].

Parameter	**V**alue
K	0.86
τ	0.49 s
T_R	0.27 s

The most well-known tuning method of the PID controller parameters is that of Ziegler-Nichols (Z-N) which is suggested in [21]. The tuning parameters of adaptive PI controller, using Z-N method, is given by the following expressions:

$$K_P = \frac{0.9\tau}{KT_R}, \quad K_I = \frac{K_P}{3.33T_R} \tag{26}$$

In order to show the robustness of the proposed algebraic approach, a performance comparison with a classical PID controller is implemented. The PID controller parameters, $K_P = 1.83$, $K_I = 3.72$ and $K_D = 0.2$, are settled by applying the Z-N method.

In the simulations, a desired trajectory $y^d\,(t)$ is generated to satisfy the system constraints. The integrals used in the parameter estimation, are numerically approximated by the trapezoidal rule. For this, we choose a sample time $T_e = 0.1$ s and a sliding identification window $T = 5T_e$. A centered white noise with variance of 0.001 is added to the system output in order to test the robustness of proposed controller. At $t = 125$ s, a disturbance given by the sensor of 0.5 V is applied to the output temperature measurement.

The simulation results given in the figures 3, 4, 5 and 6 underline the importance of the proposed adaptive PI controller in terms of robustness and trajectory tracking. In fact, we notice that the tracking error (see figure 4) in the case of classical PID control is significant compared with that obtained by ultra-local model control. Thus, the on line estimation of time-delay and the others parameters is very interesting for the development of a closed-loop system control with good performances. Figure 3 shows that the thermal perturbation is rejected faster by the a-PI controller than the PID one. Moreover, the control inputs depicted in the figure 5 show that the ultra-local model control input is smoother. In spite of the system output is noisy and disturbed, the proposed algebraic approach allows to estimate the variable time-delay (see figure 6) thanks to the algebraic derivation techniques and the adaptive controller.

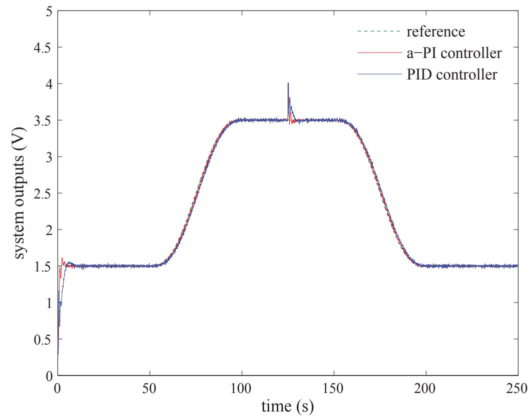

Fig. 3. Reference trajectory and noisy system outputs.

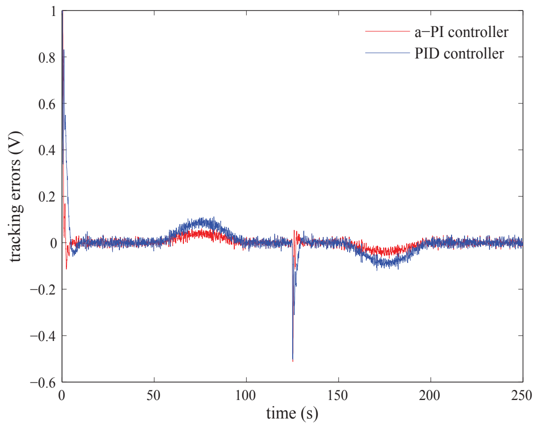

Fig. 4. Tracking errors.

V. CONCLUSIONS

The contribution of this paper is the resolution of the online time-delay estimation problem for the ultra-local model control, using the algebraic derivation techniques. The proposed algebraic approach leads to a closed-loop behavior which is robust with respect to noises, disturbances and unmodeled dynamics. The comparison with the PID controller shows that the designed controller is better in terms of robustness and trajectory tracking performances. Also, the adaptive character of our proposed control method provides best results with respect to the classical PID control.

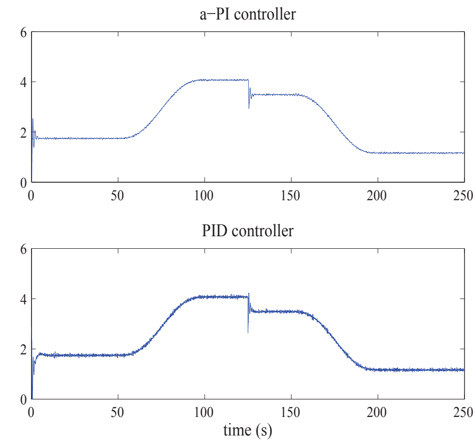

Fig. 5. Control inputs.

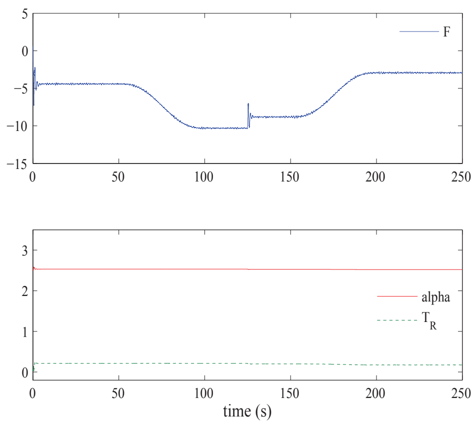

Fig. 6. Parameter estimation in the case of algebraic derivation method.

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