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Saeed Shabani and Abdolrahman Razani

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GLOBAL OPTIMAL SOLUTIONS FOR NONCYCLIC MAPPINGS IN G-METRIC SPACES

SAEED SHABANI AND ABDOLRAHMAN RAZANI

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Abstract. In this paper, the existence of solutions of some minimization problems for noncyclic mappings in $G-$metric spaces is studied. Our results can be considered as an extension of Abkar and Gabeleh's result [*Global Optimal Solutions of Noncyclic Mappings in Metric Spaces,* J. Optim. Theory. Appl. **153** (2011), 298–305] to the case of $G-$metric spaces.

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1. INTRODUCTION

In 2011, Abkar et al. [2] studied the existence of solutions of some specific minimization problems for noncyclic mappings in metric spaces. In 2006, Mustafa et al. [11] introduced the $G-$metric spaces as a generalization of the notion of metric spaces. Fixed point results and other results in $G-$metric spaces have been proved by a number of authors, see, e.g., [1,3–5,12,14,15]. In this paper we investigate some minimization problems for noncyclic mappings in $G-$metric spaces. This work extends results of Abkar et al. [2] to the case of $G-$metric spaces.

2. PRELIMINARIES

Throughout this paper, N is the set of all natural numbers and R is the set of all real numbers. Generalizations of the notion of a metric space have been proposed by Gabler [8,9] and by Dhage [6,7]. Mustafa et al. [11] introduced a more appropriate notion of a generalized metric space as following.

Definition 1. Let X be a nonempty set, and $G : X \times X \times X \to R^+$ be a function satisfying the following conditions:

(1) $G(x,y,z) = 0$ if $x = y = z$,
(2) $0 < G(x,x,y)$ for all $x, y \in X$ with $x \neq y$,

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(3) $G(x,x,y) \leq G(x,y,z)$ for all $x,y,z \in X$ with $y \neq z$,
(4) $G(x,y,z) = G(x,z,y) = G(y,z,x) = \cdots$,
(5) $G(x,y,z) \leq G(x,w,w) + G(w,y,z)$ for all $x,y,z,w \in X$,

The function G is called a generalized metric, or, a $G-$metric on X, and the pair (X,G) is called a $G-$metric space.

Example 1. ([11, Example 6.3]) Let (X,d) be a metric space and define the functions G_s and G_m with

$$G_s(x,y,z) = d(x,y) + d(y,z) + d(x,z), \quad \forall x,y,z \in X$$

$$G_m(x,y,z) = \max\{d(x,y), d(y,z), d(x,z)\}, \quad \forall x,y,z \in X$$

Then (X,G_s) and (X,G_m) are $G-$metric space.

Now, we recall some of the basic concepts for $G-$metric spaces from ([11]).

Definition 2. Let (X,G) be a $G-$metric space, and $\{x_n\}$ be a sequence of points of X, we say that $\{x_n\}$ is $G-$convergent to x and write $x_n \xrightarrow{G} x$ if $\lim_{n,m\to\infty} G(x, x_n, x_m) = 0$, that is, for any $\epsilon > 0$, there exists $n_0 \in N$ such that $G(x,x_n,x_m) < \epsilon$, for all $n,m \geq n_0$.

Proposition 1. *Let (X,G) be a $G-$metric space, then the following are equivalent.*

(1) *$\{x_n\}$ is $G-$convergent to x.*
(2) $\lim_{n\to\infty} G(x,x_n,x_n) = 0$.
(3) $\lim_{n\to\infty} G(x,x,x_n) = 0$.

Definition 3. Let (X,G) be a $G-$metric space, a $\{x_n\}$ is called $G-$Cauchy for any $\epsilon > 0$, there exists $n_0 \in N$ such that $G(x_n,x_m,x_l) < \epsilon$, for all $n,m,l \geq n_0$ that is $\lim_{n,m,l\to\infty} G(x_n,x_m,x_l) = 0$

Proposition 2. *Let (X,G) be a $G-$metric space, then the following are equivalent.*

(1) *$\{x_n\}$ is $G-$Cauchy.*
(2) *For any $\epsilon > 0$, there exists $n_0 \in N$ such that $G(x_n,x_m,x_m) < \epsilon$, for all $n,m \geq n_0$*

Definition 4. Let (X_1,G_1) and (X_2,G_2) be $G-$metric spaces. A function $f : (X_1,G_1) \to (X_2,G_2)$ is $G-$continuous at a point $a \in X$ if for any $\epsilon > 0$, there exists $\delta > 0$ such that $x,y \in X_1$, $G_1(a,x,y) < \delta$ implies $G_2(f(a), f(x), f(y)) < \epsilon$. A function f is $G-$continuous on X if and only if it is $G-$continuous at all $a \in X$.

Proposition 3. *Let (X_1,G_1) and (X_2,G_2) be $G-$metric spaces. A function $f : (X_1,G_1) \to (X_2,G_2)$ is $G-$continuous at a point $x \in X$ if and only if whenever $\{x_n\}$ is $G-$convergent to x, $\{f(x_n)\}$ is $G-$convergent to $f(x)$.*

Definition 5. A $G-$metric space (X,G) is said to be $G-$complete if every G-Cauchy sequence in (X,G) is $G-$convergent in (X,G).

Definition 6. Let (X,G) be a $G-$metric space. A $G-$Ball with center x_0 and radius r is

$$B_G(x_0,r)=\{x\in X:G(x_0,y,y)<r\}.$$

Definition 7. Let (X,G) be a $G-$metric space and $\epsilon>0$ be given, then a set $A\subset X$ is called $\epsilon-$net of (X,G) if given any x there is at last one point $a\in A$ such that $x\in B_G(a,\epsilon)$. If the A is finite then A is called a finite $\epsilon-$net of (X,G). Note that if A is an $\epsilon-$net then $X=\bigcup_{a\in A}B_G(a,\epsilon)$.

Definition 8. A $G-$metric space (X,G) is called $G-$totally bounded if for every $\epsilon>0$ there exists a finite $\epsilon-$net.

Definition 9. A $G-$metric space (X,G) is called $G-$compact space if it is $G-$complete and $G-$totally bounded.

Proposition 4. *Let (X,G) be a $G-$metric space, then the following are equivalent.*

(1) *(X,G) is a $G-$compact space.*
(2) *(X,G) is $G-$sequentially compact, that is, if the sequence $\{x_n\}\subset X$ is such that* $\sup\{G(x_n,x_m,x_l):n,m,l\in N\}<\infty$, *then $\{x_n\}$ has a $G-$convergent subsequence.*

Theorem 1 ([12], Theorem 2.1)**.** *Let (X,G) be a $G-$metric space and $T:X\to X$ be a mapping which satisfies the following condition, for all $x,y,z\in X$,*

$$\begin{aligned}G(T(x),T(y),T(z))\le\ & k\max\{G(x,y,z),G(x,T(x),T(x)),G(y,T(y),T(y)),\\ & G(z,T(z),T(z)),G(x,T(y),T(y)),\\ & G(y,T(z),T(z)),G(z,T(x),T(x))\},\end{aligned}\tag{2.1}$$

where $k\in[0,1/2)$. *Then T has a unique fixed point (say u) and T is G-continuous at u.*

Definition 10. Let A,B,C be subsets of a $G-$metric space (X,G). A mapping $T:A\cup B\cup C\to A\cup B\cup C$ is called relatively $G-$nonexpansive if

$$G(T(x),T(y),T(z))\le G(x,y,z),\quad \forall(x,y,z)\in A\times B\times C.$$

Definition 11. Let (X,G) be a $G-$metric space and $A,B,C\subset X$, then

$$dist(A,B,C)=\inf\{G(a,b,c):a\in A,\ b\in B,\ c\in C\}.$$

Example 2. Let R be equipped with the usual metric, and $A=[-1,0]$ and $B=N_o$ and $C=N_e$ where N_o and N_e are the set of odd natural numbers and even natural numbers, respectively. Let $G_m(x,y,z)=\max\{|x-y|,|x-z|,|y-z|\}$, then $dist(A,B,C)=2$.

Definition 12. Let (X,G) be a $G-$metric space and $A,B,C\subset X$, $T:A\cup B\cup C\to A\cup B\cup C$ is said noncyclic mapping, if

$$T(A)\subset A,\quad T(B)\subset B,\quad T(C)\subset C.$$

We consider the following minimization problem: Find

$$\min_{a\in A}\{G(a,T(a),T(a))\},\qquad \min_{b\in B}\{G(b,T(b),T(b))\},$$

$$\min_{b\in B}\{G(c,T(c),T(c))\},\qquad \min_{(a,b,c)\in A\times B\times C}\{G(a,b,c)\} \tag{2.2}$$

We say that $(x^\star,y^\star,z^\star)\in A\times B\times C$ is a solution of above problem, if

$$Tx^\star=x^\star,\quad Ty^\star=y^\star,\quad Tz^\star=z^\star,$$

and

$$G(x^\star,y^\star,z^\star)=dist(A,B,C).$$

Definition 13. Let (X,G) be a $G-$metric space and $A,B,C\subset X$, we set

$$A_0=\{a\in A:G(a,b,c)=dist(A,B,C),\ \ for\ some\ \ b\in B,c\in C\}$$

$$B_0=\{b\in B:G(a,b,c)=dist(A,B,C),\ \ for\ some\ \ a\in A,c\in C\}$$

$$C_0=\{c\in C:G(a,b,c)=dist(A,B,C),\ \ for\ some\ \ a\in A,b\in B\}$$

Definition 14. Let (X,G) be a $G-$metric space and A,B,C be nonempty subsets of X, with $A_0\neq\varnothing$. We say that A,B,C have $P-$property iff

$$\begin{cases} G(x_1,y_1,z_1)=dist(A,B,C)\\ G(x_2,y_2,z_2)=dist(A,B,C)\\ G(x_3,y_3,z_3)=dist(A,B,C)\end{cases}$$

then

$$G(x_1,x_2,x_3)=G(y_1,y_2,y_3)=G(z_1,z_2,z_3),$$

where $x_1,x_2,x_3\in A_0$ and $y_1,y_2,y_3\in B_0$ and $z_1,z_2,z_3\in C_0$.

The above definition were found in the case of metric space in ([13]).

Example 3. Let A,B,C be nonempty subsets of a $G-$metric space (X,G) such that $A_0\neq\varnothing$ and $dist(A,B,C)=0$, then A,B,C have $P-$property.

Definition 15. Let (X,G) be a $G-$metric space and $T:X\to X$ be a mapping. T is called expansive if for all $x,y,z\in X$,

$$G(T(x),T(y),T(z))\geq G(x,y,z).$$

Definition 16. Let (X,G) be a $G-$metric space and $T:X\to X$ be a mapping. T is said to be asymptotically regular iff $\lim_{n\to\infty}G(T^nx,T^{n+1}x,T^{n+1}x)=0$, for all $x\in X$.

3. MAIN RESULTS

We start this section with the following theorem.

Theorem 2. *Let A,B,C be nonempty and closed subsets of a $G-$complete space (X,G) such that $A_0 \neq \varnothing$ and A,B,C satisfies the $P-$property. Let $T : A\cup B\cup C \to A\cup B\cup C$ be a noncyclic mapping. Suppose that*

(1) *$T|_A$ be a mapping which satisfies in (2.1).*
(2) *T is relatively $G-$nonexpansive.*

Then the minimization problem (2.2) has a solution.

Proof. If $x \in A_0$, then there exist $y \in B$ and $z \in C$ such that $G(x,y,z)$ $= dist(A,B,C)$. Since T is relatively $G-$nonexpansive then

$$G(T(x),T(y),T(z)) \leq G(x,y,z) = dist(A,B,C)$$

Hence $Tx \in A_0$.

Let $x_0 \in A_0$ by Theorem 1 if $x_n = T^n(x_0)$ then $x_n \xrightarrow{G} x^\star$ where $x^\star$ is unique fixed point of T in A. Since $x_0 \in A_0$ there exist $y_0 \in B$ and $z_0 \in C$ such that $G(x_0,y_0,z_0) = dist(A,B,C)$. Since $x_1 = Tx_0 \in A_0$, there exist $y_1 \in B$ and $z_1 \in C$ such that $G(x_1,y_1,z_1) = dist(A,B,C)$. Using this process, we have a sequence $\{y_n\}$ in B and $\{z_n\}$ in C such that

$$G(x_n,y_n,z_n) = dist(A,B,C) \quad \forall n \in N\cup\{0\}.$$

Since A,B,C have the $P-$property, we have for all $m,n,l \in N\cup\{0\}$

$$G(x_n,x_m,x_l) = G(y_n,y_m,y_l) = G(z_n,z_m,z_l).$$

This implies that $\{y_n\}$ and $\{z_n\}$ are $G-$Cauchy sequences, and there exist $y^\star \in B$ and $z^\star \in C$ such that $y_n \xrightarrow{G} y^\star$ and $z_n \xrightarrow{G} z^\star$ Thus

$$G(x^\star,y^\star,z^\star) = \lim_{n\to\infty} G(x_n,y_n,z_n) = dist(A,B,C)$$

Since

$$G(T(x^\star),T(y^\star),T(z^\star)) \leq G(x^\star,y^\star,z^\star) = dist(A,B,C)$$

Therefore by the $P-$property, we have

$$G(x^\star,T(x^\star),T(x^\star)) = G(y^\star,T(y^\star),T(y^\star)) = G(z^\star,T(z^\star),T(z^\star))$$

Thus $(x^\star,y^\star,z^\star) \in A\cup B\cup C$ is a solution of the minimization problem (2.2). □

Example 4. Let R be equipped with the usual metric, and $G_m(x,y,z) = \max\{|x-y|,|x-z|,|y-z|\}$. Let $A = [-2,0]$ and $B = \{1\}$ and $C = [2,3]$. It is obvious that

$A_0 = \{0\}, B_0 = \{1\}, C_0 = \{2\}$. Define $T : A\cup B\cup C \to A\cup B\cup C$ with

$$T(x) = \begin{cases} \frac{x}{4} & x \in A \\ 1 & x \in B \\ \frac{x+2}{2} & x \in C \end{cases}$$

It is easy to check that all the conditions of Theorem 2 hold. Therefore, the minimization problem (2.2) has a solution $(x^\star, y^\star, z^\star) = (0, 1, 2)$.

Theorem 3. *Let A, B, C be nonempty subsets of a $G-$complete space (X, G) such that A is G-compact and B and C are $G-$closed. Let $A_0 \neq \emptyset$ and A, B, C satisfy the $P-$property. Let $T : A\cup B\cup C \to A\cup B\cup C$ be a noncyclic mapping. Then the minimization problem (2.2) has a solution provided that the following conditions are satisfied:*

1. *T is relatively $G-$nonexpansive.*
2. *$T|_A$ is a $G-$expansive.*
3. *$T|_B$ and $T|_C$ be mappings which satisfy in (2.1).*

Proof. If $x \in A_0$, and $x_{n+1} = Tx_n, (n \in N\cup\{0\})$. By argument similar in the proof of Theorem 2 we obtain that $T(A_0) \subset A_0$ and there exist y_n in B and z_n in C such that

$$G(x_n, y_n, z_n) = dist(A, B, C) \quad \forall n \in N\cup\{0\}.$$

Since A is $G-$compact, by Proposition 4 there exist a subsequence $\{x_{n_k}\}$ of the $\{x_n\}$ such that $x_{n_k} \xrightarrow{G} x^\star \in A$. Since A, B, C satisfy the $P-$property,

$$G(x_{n_k}, x_{n_s}, x_{n_l}) = G(y_{n_k}, y_{n_s}, y_{n_l}) = G(z_{n_k}, z_{n_s}, z_{n_l}), \quad (k, s, l \in N).$$

This implies that $\{y_n\}$ and $\{z_n\}$ are $G-$Cauchy sequences and there exist $y^\star \in B$ and $z^\star \in C$ such that $y_{n_k} \xrightarrow{G} y^\star$ and $z_{n_k} \xrightarrow{G} z^\star$. Thus

$$G(x^\star, y^\star, z^\star) = \lim_{n\to\infty} G(x_{n_k}, y_{n_k}, z_{n_k}) = dist(A, B, C)$$

Now we prove that $x^\star, y^\star, z^\star \in F(T)$. Since T is relatively $G-$nonexpansive,

$$G(T^2(x^\star), T^2(y^\star), T^2(z^\star)) = G(T(x^\star), T(y^\star), T(z^\star)) = dist(A, B, C).$$

Since A, B, C satisfy the $P-$property, we have

$$G(x^\star, T(x^\star), T(x^\star)) = G(y^\star, T(y^\star), T(y^\star)) = G(z^\star, T(z^\star), T(z^\star)),$$

and

$$\begin{aligned} G(T(x^\star), T^2(x^\star), T^2(x^\star)) &= G(T(y^\star), T^2(y^\star), T^2(y^\star)) \\ &= G(T(z^\star), T^2(z^\star), T^2(z^\star)). \end{aligned}$$

Now let $Ty^\star \neq T^2y^\star$, since $T|_B$ satisfies in (2.1),

$$G(T(y^\star), T(T(y^\star)), T(T(y^\star))) \leq kG(y^\star, T(y^\star), T(y^\star))$$

Thus since $T|_A$ is a $G-$expansive, we have

$$\begin{aligned} G(T(y^\star),T^2(y^\star),T^2(y^\star)) = &\quad G(T(y^\star),T(T(y^\star)),T(T(y^\star))) \\ \leq &\quad kG(y^\star,T(y^\star),T(y^\star)) \\ = &\quad kG(x^\star,T(x^\star),T(x^\star)) \\ \leq &\quad kG(T(x^\star),T^2(x^\star),T^2(x^\star)) \\ = &\quad kG(T(y^\star),T^2(y^\star),T^2(y^\star)), \end{aligned}$$

which is a contraction. Therefore $Ty^\star = T^2y^\star$. A similar argument implies that $Tz^\star = T^2z^\star$. Thus $x^\star = T(x^\star)$ and $y^\star = T(y^\star)$ and $z^\star = T(z^\star)$. □

Example 5. Let $X = R^3$ and

$$\begin{aligned} G((x_1,y_1,z_1),(x_2,y_2,z_2),(x_3,y_3,z_3)) = &\quad \max\{G_m(x_1,x_2,x_3), \\ &\quad G_m(y_1,y_2,y_3),G_m(z_1,z_2,z_3)\}, \end{aligned}$$

where $G_m(x,y,z) = \max\{|x-y|,|x-z|,|y-z|\}$. Let $A = \{(x,0,0) : -1 \leq x \leq 0\}$ and $B = \{(0,y,0) : 0 \leq x \leq 1\}$ and $C = \{(0,0,z) : -1 \leq z \leq 1\}$. It is obvious that $A_0 = B_0 = C_0 = \{(0,0,0)\}$ and $dist(A,B,C) = 0$, therefore A,B,C have the $P-$property. Define $T : A\cup B\cup C \to A\cup B\cup C$ with

$$T(x,0,0) = (-x,0,0),\ \ T(0,y,0) = (0,\frac{y}{4},0)\ \ and\ \ T(0,0,z) = (0,0,\frac{z}{4}).$$

It is easy to check that all the conditions of Theorm 3 hold. Therefore the minimization problem (2.2) has a solution $x^\star = y^\star = z^\star = (0,0,0)$.

Theorem 4. *Let A,B,C be nonempty subsets of a $G-$complete space (X,G) such that A is $G-compact$ and B and C are $G-$closed.Let $A_0 \neq \varnothing$ and A,B,C satisfy the $P-$property. Let $T : A\cup B\cup C \to A\cup B\cup C$ be a noncyclic mapping. Then the minimization problem (2.2) has a solution provided that the following conditions are satisfied:*

(1) *T is relatively $G-$nonexpansive.*
(2) *$T|_A$ is $G-$continuous and asymptotically regular.*

Proof. Let $\{x_n\}$, $\{y_n\}$, $\{z_n\}$, $\{x_{n_k}\}$, $\{y_{n_k}\}$, $\{z_{n_k}\}$, $x^\star$, $y^\star$ and $z^\star$ be as in Theorem 3. We have $x_{n_k}\xrightarrow{G}x^\star \in A$, $y_{n_k}\xrightarrow{G}y^\star \in B$, $z_{n_k}\xrightarrow{G}z^\star \in C$ and $G(x^\star,y^\star,z^\star) = dist(A,B,C)$. From Proposition 3, since $T|_A$ is $G-$continuous, we have

$$x_{n_k+1} = T(x_{n_k})\xrightarrow{G}T(x^\star).$$

Also by the asymptotic regularly of $T|_A$, we obtain

$$\begin{aligned} G(x^\star,T(x^\star),T(x^\star)) = &\quad \lim_{k\to\infty} G(x_{n_k},T(x_{n_k}),T(x_{n_k})) \\ = &\quad \lim_{k\to\infty} G(T^{n_k}(x_0),T^{n_k+1}(x_0),T^{n_k+1}(x_0)) \\ = &\quad 0. \end{aligned}$$

This implies that $T(x^\star) = x^\star$. Since T is relatively $G-$nonexpansive, we have

$$G(T(x^\star), T(y^\star), T(z^\star)) \leq G(x^\star, y^\star, z^\star) = dist(A, B, C)$$

Therefore by the $P-$property, we have

$$G(x^\star, T(x^\star), T(x^\star)) = G(y^\star, T(y^\star), T(y^\star)) = G(z^\star, T(z^\star), T(z^\star))$$

Hence $T(y^\star) = y^\star$ and $T(z^\star) = z^\star$. □

QUESTION: In 2011, Karapinar [10] obtain some common fixed point results in partial metric spaces. Can one study the minimization problem (2.2) for two mappings in partial metric spaces?

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Authors' addresses

Saeed Shabani
Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.
E-mail address: s.shabani@srbiau.ac.ir, shabani60@gmail.com

Abdolrahman Razani
Department of Mathematics, Faculty of Science, Imam Khomeini International University, P.O.Box 34149-16818, Qazvin, Iran.
E-mail address: razani@ikiu.ac.ir