Miskolc Mathematical Notes
Vol. 16 (2015), No 1, pp. 361-368
HU e-ISSN 1787-2413
DOI: 10.18514/MMN.2015.1260

Approximately algebraic tensor products

Ismail Nikoufar and Themistocles M. Rassias

Miskolc Mathematical Notes
Vol. 16 (2015), No. 1, pp. 361–368

HU e-ISSN 1787-2413

APPROXIMATELY ALGEBRAIC TENSOR PRODUCTS

ISMAIL NIKOUFAR AND THEMISTOCLES M. RASSIAS

Received 23 May, 2014

Abstract. Let X and Y be normed spaces over a complete field $\mathbb{F}$ with dual spaces X' and Y' respectively. Under certain hypotheses, for given $x \in X$, $y \in Y$ and a mapping u from $X' \times Y'$ to $\mathbb{F}$, we apply Hyers–Ulam approach to find a unique bounded bilinear mapping v near to u such that $||v|| = ||x \otimes y||$.

2010 *Mathematics Subject Classification:* 46B10; 47L50; 39B72

Keywords: algebraic tensor product, dual space, stability

1. Introduction

Let X, Y, and Z be normed linear spaces over the same field $\mathbb{F}$. A mapping $\phi : X \times Y \longrightarrow Z$ is said to be bilinear if the mappings $x \longmapsto \phi(x,y)$ and $y \longmapsto \phi(x,y)$ are linear. A bilinear mapping $\phi : X \times Y \longrightarrow Z$ is said to be bounded if there exists $M > 0$ such that $||\phi(x,y)|| \leq M||x||||y||$ for all $x \in X$ and $y \in Y$. The norm of ϕ is then defined by

$$||\phi|| := \sup\{||\phi(x,y)|| : (x,y) \in \mathcal{B}_X \times \mathcal{B}_Y\},$$

where $\mathcal{B}_X := \{x \in X : ||x|| \leq 1\}$. The set of all bounded bilinear mappings from $X \times Y$ to Z is denoted by $\mathcal{BL}(X \times Y, Z)$. Let X' and Y' be dual spaces of X and Y respectively. For given $x \in X$ and $y \in Y$, $x \otimes y$ is an element of $\mathcal{BL}(X' \times Y', \mathbb{F})$ defined by $x \otimes y(f,g) := f(x)g(y)$ for all $f \in X'$ and $g \in Y'$. The algebraic tensor product of X and Y, $X \otimes Y$, is defined to be the linear span of $\{x \otimes y : x \in X, y \in Y\}$ in $\mathcal{BL}(X' \times Y', \mathbb{F})$ (see [3]).

A classical question in the theory of functional equations is the following (see [4], [6], [7], [9], [10], [8], [12], [14], [15], [20], [19], [17], [18], [21], [13], [22]): *When is it true that a function which approximately satisfies a functional equation ζ must be close to an exact solution of ζ?*

If the problem accepts a solution, we say that the equation ζ is stable. There are cases in which each approximate solution is actually a true solution. In such cases, we call the equation ζ superstable.

The first stability problem concerning group homomorphisms was raised by Ulam [22] during his talk before a Mathematical Colloquium at the University of Wisconsin in 1940. Ulam's problem was partially solved by Hyers [7] for mappings between Banach spaces. Hyers' Theorem was generalized by Aoki [1] for additive mappings and by Th. M. Rassias [16] for linear mappings by considering an unbounded Cauchy difference. The paper of Th. M. Rassias [16] has provided a lot of influence in the development of what is called the generalized Hyers-Ulam stability or the Hyers-Ulam-Rassias stability of functional equations. A generalization of the Th. M. Rassias theorem was obtained by Gavruta [5] in 1994 by replacing the unbounded Cauchy difference by a general control function in the spirit of Th. M. Rassias' approach. Badora [2] proved the generalized Hyers-Ulam stability of ring homomorphisms, which generalizes the result of D. G. Bourgin. Miura [11] proved the generalized Hyers-Ulam stability of Jordan homomorphisms.

In this paper, under certain hypotheses and using Hyers–Ulam approach, we find a unique bounded bilinear mapping v near to a given mapping $u : X' \times Y' \longrightarrow \mathbb{F}$ such that $||v|| = ||x \otimes y||$ for $x \in X$, $y \in Y$. Throughout this paper, it is assumed that X and Y are normed spaces over a complete field $\mathbb{F}$ with dual spaces X' and Y' respectively.

2. Results

Theorem 1. *Let $u : X' \times Y' \to \mathbb{F}$ be a mapping for which there exist positive real valued functions φ_1, φ_2, and φ on $X' \times X' \times Y'$, $X' \times Y' \times Y'$, and $X' \times Y'$, respectively such that*

$$\tilde{\varphi}(f,g) := \sum_{i=0}^{\infty} \frac{1}{2^{i+1}} \varphi_1(2^i f, 2^i f, g) < \infty, \tag{2.1}$$

$$\lim_{n\to\infty} \frac{1}{2^n} \varphi_1(2^n f_1, 2^n f_2, g) = \lim_{n\to\infty} \frac{1}{2^n} \varphi_2(2^n f, g_1, g_2) = \lim_{n\to\infty} \frac{1}{2^n} \varphi(2^n f, g) = 0, \tag{2.2}$$

$$|u(cf_1 + f_2, g) - cu(f_1, g) - u(f_2, g)| \le \varphi_1(f_1, f_2, g), \tag{2.3}$$

$$|u(f, cg_1 + g_2) - cu(f, g_1) - u(f, g_2)| \le \varphi_2(f, g_1, g_2) \tag{2.4}$$

for all $f, f_1, f_2 \in X'$, $g, g_1, g_2 \in Y'$, and $c \in \mathbb{F}$. Then, there exists a unique bilinear mapping v from $X' \times Y'$ to $\mathbb{F}$ such that

$$|u(f,g) - v(f,g)| \le \tilde{\varphi}(f,g) \quad (f \in X', g \in Y'). \tag{2.5}$$

Moreover, if the mapping u satisfies

$$||u(f,g)| - |f(x)g(y)|| \le \varphi(f,g) \tag{2.6}$$

for some fixed $x \in X$ and $y \in Y$, then $||v|| = ||x \otimes y||$ and so in particular v is bounded.

Proof. Putting $c = 1$ and replacing f_1 and f_2 in (2.3) by f and dividing both sides by 2, we get

$$|\frac{1}{2}u(2f, g) - u(f, g)| \leq \frac{1}{2}\varphi_1(f, f, g) \tag{2.7}$$

for all $f \in X'$ and $g \in Y'$. Replacing f by $2f$ in (2.7) and dividing both sides by 2, we find that

$$|\frac{1}{2^2}u(2^2 f, g) - \frac{1}{2}u(2f, g)| \leq \frac{1}{2^2}\varphi_1(2f, 2f, g) \tag{2.8}$$

for all $f \in X'$ and $g \in Y'$. Combining (2.7) with (2.8), we obtain

$$|\frac{1}{2^2}u(2^2 f, g) - u(f, g)| \leq \frac{1}{2}\varphi_1(f, f, g) + \frac{1}{2^2}\varphi_1(2f, 2f, g)$$

for all $f \in X'$ and $g \in Y'$. By induction on n, we conclude that

$$|\frac{1}{2^n}u(2^n f, g) - u(f, g)| \leq \sum_{i=0}^{n-1} \frac{1}{2^{i+1}}\varphi_1(2^i f, 2^i f, g) \tag{2.9}$$

for all $f \in X'$ and $g \in Y'$. We now turn to use the Cauchy convergence criterion. Replace f by $2^k f$ in (2.9) and divide both sides by 2^k, where k is an arbitrary positive integer, to get

$$|\frac{1}{2^{n+k}}u(2^{n+k} f, g) - \frac{1}{2^k}u(2^k f, g)| \leq \sum_{i=k}^{n+k-1} \frac{1}{2^{i+1}}\varphi_1(2^i f, 2^i f, g)$$

for all $f \in X'$, $g \in Y'$, and all positive integers $n \geq k$. It follows from the last inequality and (2.1) that the sequence $\{\frac{1}{2^n}u(2^n f, g)\}$ is a Cauchy sequence for all $f \in X'$ and $g \in Y'$. Since $\mathbb{F}$ is a complete field, this sequence converges. Define $v(f, g) := \lim_{n\to\infty} \frac{1}{2^n}u(2^n f, g)$. Taking the limit as $n \to \infty$ in (2.9), we find that the inequality (2.5) holds for all $f \in X'$ and $g \in Y'$. Replace f_1 and f_2 in (2.3) by $2^n f_1$ and $2^n f_2$ respectively and divide both sides by 2^n and take the limit as $n \to \infty$ and apply then (2.2) to get the mapping $f \longmapsto v(f, g)$ is linear. By a similar way one can replace f in (2.4) by $2^n f$ and divide both sides by 2^n to deduce that the mapping $g \longmapsto v(f, g)$ is linear. Consequently, the mapping v is bilinear. Our next claim is to prove that v is unique. Let v' be another mapping satisfying (2.5). Hence,

$$\begin{aligned}
|v(f, g) - v'(f, g)| &= \frac{1}{2^k}|v(2^k f, g) - v'(2^k f, g)| \\
&\leq \frac{2}{2^k}\tilde{\varphi}(2^k f, g) \\
&= 2\sum_{i=k}^{\infty} \frac{1}{2^{i+1}}\varphi_1(2^i f, 2^i f, g)
\end{aligned}$$

for all $f \in X'$ and $g \in Y'$. Passing to the limit as $k \to \infty$, we conclude that v is unique. Replace f by $2^n f$ in (2.6) and divide both sides by 2^n, to arrive at

$$\left| \frac{1}{2^n} |u(2^n f, g)| - |f(x)g(y)| \right| \le \frac{1}{2^n} \varphi(2^n f, g) \tag{2.10}$$

for all $f \in X'$ and $g \in Y'$. Taking the limit as $n \to \infty$ in (2.10) and applying the definition of the norm, we conclude that $||v|| = ||x \otimes y||$ and so v is bounded. □

Remark 1. Under the same hypotheses of Theorem 1, with (2.1) and (2.2) replaced by

$$\tilde{\varphi}(f, g) := \sum_{i=0}^{\infty} \frac{1}{2^{i+1}} \varphi_2(f, 2^i g, 2^i g) < \infty, \tag{2.11}$$

$$\lim_{n\to\infty} \frac{1}{2^n} \varphi_1(f_1, f_2, 2^n g) = \lim_{n\to\infty} \frac{1}{2^n} \varphi_2(f, 2^n g_1, 2^n g_2) = \lim_{n\to\infty} \frac{1}{2^n} \varphi(f, 2^n g) = 0, \tag{2.12}$$

there exists a unique mapping $v \in \mathcal{BL}(X' \times Y', \mathbb{F})$ satisfying (2.5). Note that by using (2.4) and the same method as in the proof of Theorem 1, we can define $v(f, g) := \lim_{n\to\infty} \frac{1}{2^n} u(f, 2^n g)$.

In the following corollaries, as a consequence of Theorem 1, we show the Rassias stability of algebraic tensor products.

Corollary 1. *Let $x \in X$, $y \in Y$, and $u : X' \times Y' \to \mathbb{F}$ be a mapping such that*

$$||u(f, g)| - |f(x)g(y)|| \le \alpha + \beta(||f||^p + ||g||^p) + \gamma ||f||^p ||g||^p, \tag{2.13}$$

$$|u(cf_1 + f_2, g) - cu(f_1, g) - u(f_2, g)| \le \alpha + \beta(||f_1||^q + ||f_2||^q + ||g||^q) + \gamma ||f_1||^{\frac{q}{2}} ||f_2||^{\frac{q}{2}} ||g||^q,$$

$$|u(f, cg_1 + g_2) - cu(f, g_1) - u(f, g_2)| \le \alpha + \beta(||f||^r + ||g_1||^r + ||g_2||^r) + \gamma ||f||^r ||g_1||^{\frac{r}{2}} ||g_2||^{\frac{r}{2}}$$

for all $f, f_1, f_2 \in X'$, $g, g_1, g_2 \in Y'$, and $c \in \mathbb{F}$, where p, q, r, α, β, and γ are constants with $0 \le p, q, r < 1$, $\alpha > 0$, and $\beta, \gamma \ge 0$. Then, there exists a unique mapping $v \in \mathcal{BL}(X' \times Y', \mathbb{F})$ such that $||v|| = ||x \otimes y||$ and

$$|u(f, g) - v(f, g)| \le \alpha + \beta(2k||f||^q + ||g||^q) + \gamma k |f||^q ||g||^q \tag{2.14}$$

for all $f \in X'$ and $g \in Y'$, where $k = \frac{1}{2-2^q}$.

Remark 2. Under the hypotheses of Corollary 1 and using Remark 1, there exists a unique mapping $v \in \mathcal{BL}(X' \times Y', \mathbb{F})$ such that $||v|| = ||x \otimes y||$ and

$$|u(f, g) - v(f, g)| \le \alpha + \beta(||f||^r + 2k||g||^r) + \gamma k |f||^r ||g||^r$$

for all $f \in X'$ and $g \in Y'$, where $k = \frac{1}{2-2^r}$.

Theorem 2. *Let* $\{x_i\}_{i=1}^m$ *and* $\{y_i\}_{i=1}^m$ *be linearly independent sets in* X *and* Y *respectively and* u *be a mapping from* $X'\times Y'$ *to* $\mathbb{F}$ *for which there exist mappings* $\varphi_1 : X'\times X'\times Y' \longrightarrow \mathbb{R}^+$, $\varphi_2 : X'\times Y'\times Y' \longrightarrow \mathbb{R}^+$, *and* $\varphi : X'\times Y' \longrightarrow \mathbb{R}^+$ *satisfying* (2.1), (2.2), (2.3), (2.4) *and*

$$\left| |u(f,g)| - \sum_{i=1}^{m} |f(x_i)g(y_i)| \right| \le \varphi(f,g) \tag{2.15}$$

for all $f \in X'$, $g \in Y'$. *Then, there exists a unique mapping* $v \in \mathcal{BL}(X'\times Y',\mathbb{F})$ *such that*

$$|u(f,g)-v(f,g)| \le \tilde{\varphi}(f,g) \ \ (f \in X', g \in Y'), \ \ ||v|| \le \sum_{i=1}^{m} ||x_i \otimes y_i||. \tag{2.16}$$

In the following our interest is to provide a dual for Theorem 1.

Theorem 3. *Let* $x \in X$, $y \in Y$, *and let* $u : X'\times Y' \to \mathbb{F}$ *be a mapping for which there exist mappings* $\varphi_1 : X'\times X'\times Y' \longrightarrow \mathbb{R}^+$, $\varphi_2 : X'\times Y'\times Y' \longrightarrow \mathbb{R}^+$, *and* $\varphi : X'\times Y' \longrightarrow \mathbb{R}^+$ *satisfying* (2.3), (2.4), (2.6), *and*

$$\tilde{\varphi}(f,g) := \sum_{i=0}^{\infty} 2^i \varphi_1(\frac{f}{2^{i+1}}, \frac{f}{2^{i+1}}, g) < \infty, \tag{2.17}$$

$$\lim_{n\to\infty} 2^n \varphi_1(\frac{f_1}{2^n}, \frac{f_2}{2^n}, g) = \lim_{n\to\infty} 2^n \varphi_2(\frac{f}{2^n}, g_1, g_2) = \lim_{n\to\infty} 2^n \varphi(\frac{f}{2^n}, g) = 0 \tag{2.18}$$

for all $f, f_1, f_2 \in X'$, $g, g_1, g_2 \in Y'$. *Then, there exists a unique mapping* $v \in \mathcal{BL}(X'\times Y',\mathbb{F})$ *satisfying* (2.5).

Proof. By induction on n, we conclude that

$$|u(f,g) - 2^n u(\frac{f}{2^n}, g)| \le \sum_{i=0}^{n-1} 2^i \varphi_1(\frac{f}{2^{i+1}}, \frac{f}{2^{i+1}}, g) \tag{2.19}$$

for all $f \in X'$ and $g \in Y'$. Replace f by $\frac{f}{2^k}$ in (2.19) and multiply both sides by 2^k, where k is an arbitrary positive integer, to get

$$|2^k u(\frac{f}{2^k}, g) - 2^{n+k} u(\frac{f}{2^{n+k}}, g)| \le \sum_{i=k}^{n+k-1} 2^i \varphi_1(\frac{f}{2^{i+1}}, \frac{f}{2^{i+1}}, g)$$

for all $f \in X'$, $g \in Y'$, and all positive integers $n \ge k$. In order to use the Cauchy convergence criterion, the last inequality and (2.17) imply the sequence $\{2^n u(\frac{f}{2^n}, g)\}$ is a Cauchy sequence for all $f \in X'$ and $g \in Y'$. Due to completeness of $\mathbb{F}$, this sequence converges. Define $v(f,g) := \lim_{n\to\infty} 2^n u(\frac{f}{2^n}, g)$. Taking the limit as $n \to \infty$ in (2.19), we deduce that the inequality (2.5) holds for all $f \in X'$ and $g \in Y'$. The rest of the proof is similar to that of Theorem 1. □

Remark 3. Under the same hypotheses of Theorem 3, with (2.17) and (2.18) replaced by

$$\tilde{\varphi}(f,g) := \sum_{i=0}^{\infty} 2^i \varphi_2(f, \frac{g}{2^{i+1}}, \frac{g}{2^{i+1}}) < \infty, \tag{2.20}$$

$$\lim_{n\to\infty} 2^n \varphi_1(f_1, f_2, \frac{g}{2^n}) = \lim_{n\to\infty} 2^n \varphi_2(f, \frac{g_1}{2^n}, \frac{g_2}{2^n}) = \lim_{n\to\infty} 2^n \varphi(f, \frac{g}{2^n}) = 0, \tag{2.21}$$

there exists a unique mapping $v \in \mathcal{BL}(X' \times Y', \mathbb{F})$ satisfying (2.5). We remark that by using (2.4) and the same method as in the proof of Theorem 3, one can define $v(f,g) := \lim_{n\to\infty} 2^n u(f, \frac{g}{2^n})$.

Corollary 2. *Let $x \in X$, $y \in Y$, and $u : X' \times Y' \to \mathbb{F}$ be a mapping such that*

$$||u(f,g)| - |f(x)g(y)|| \le \alpha ||f||^p ||g||^p, \tag{2.22}$$

$$|u(cf_1 + f_2, g) - cu(f_1, g) - u(f_2, g)| \le \beta ||f_1||^{\frac{q}{2}} ||f_2||^{\frac{q}{2}} ||g||^q,$$

$$|u(f, cg_1 + g_2) - cu(f, g_1) - u(f, g_2)| \le \gamma ||f||^r ||g_1||^{\frac{r}{2}} ||g_2||^{\frac{r}{2}}$$

for all $f, f_1, f_2 \in X'$, $g, g_1, g_2 \in Y'$, and $c \in \mathbb{F}$, where $p,q,r > 1$, and $\alpha, \beta, \gamma > 0$. Then, there exists a unique mapping $v \in \mathcal{BL}(X' \times Y', \mathbb{F})$ such that $||v|| = ||x \otimes y||$ and

$$|u(f,g) - v(f,g)| \le \frac{\beta}{2^q - 2} ||f||^q ||g||^q \quad (f \in X', g \in Y').$$

Proof. It is enough to define $\varphi(f,g) := \alpha ||f||^p ||g||^p$, $\varphi_1(f_1, f_2, g) := \beta ||f_1||^{\frac{q}{2}} ||f_2||^{\frac{q}{2}} ||g||^q$, and $\varphi_2(f, g_1, g_2) := \gamma ||f||^r ||g_1||^{\frac{r}{2}} ||g_2||^{\frac{r}{2}}$ for all $f, f_1, f_2 \in X'$ and $g, g_1, g_2 \in Y'$ and then apply Theorem 3. □

Remark 4. Under the hypotheses of Corollary 2 and using Remark 3, there exists a unique mapping $v \in \mathcal{BL}(X' \times Y', \mathbb{F})$ such that $||v|| = ||x \otimes y||$ and

$$|u(f,g) - v(f,g)| \le \frac{\gamma}{2^r - 2} ||f||^r ||g||^r \quad (f \in X', g \in Y').$$

Theorem 4. *Let $\{x_i\}_{i=1}^m$ and $\{y_i\}_{i=1}^m$ be linearly independent sets in X and Y respectively and u be a mapping from $X' \times Y'$ to $\mathbb{F}$ for which there exist mappings $\varphi_1 : X' \times X' \times Y' \longrightarrow \mathbb{R}^+$, $\varphi_2 : X' \times Y' \times Y' \longrightarrow \mathbb{R}^+$, and $\varphi : X' \times Y' \longrightarrow \mathbb{R}^+$ satisfying* (2.17), (2.18), (2.15), (2.3), (2.4). *Then, there exists a unique mapping $v \in \mathcal{BL}(X' \times Y', \mathbb{F})$ satisfying* (2.16).

ACKNOWLEDGEMENT

The authors would like to thank the anonymous referee for his/her valuable comments concerning the statement of Theorem 1.

REFERENCES

[1] T. Aoki, "On the stability of the linear transformation in Banach spaces," *J. Math. Soc. Japan.*, vol. 2, pp. 64–66, 1950.

[2] R. Badora, "On approximate derivations," *Math. Inequal. Appl.*, vol. 9, pp. 167–173, 2006.

[3] F. F. Bonsal and J. Duncan, *Complete normed algebra*. New York, Heidelberg, Berlin: Springer-Verlag, 1973.

[4] V. A. Faizev, T. M. Rassias, and P. K. Sahoo, "The space of (ψ,γ)-additive mappings on semigroups," *Trans. Amer. Math. Soc.*, vol. 354, no. 11, pp. 4455–4472, 2002.

[5] P. Gavruta, "Generalization of the Hyers–Ulam–Rassias stability of approximately additive mappings," *J. Math. Anal. Appl.*, vol. 184, pp. 431–436, 1994.

[6] P. M. Gruber, "Stability of isometries," *Trans. Amer. Math. Soc.*, vol. 245, pp. 263–277, 1978.

[7] D. H. Hyers, "On the stability of the linear functional equation," *Proc. Nat. Acad. Sci. U.S.A.*, vol. 27, pp. 222–224, 1941.

[8] D. H. Hyers, G. Isac, and T. M. Rassias, "On the asymptoticity aspect of Hyers–Ulam stability of mappings," *Proc. Amer. Math. Soc.*, vol. 162, no. 2, pp. 425–430, 1998.

[9] D. H. Hyers, G. Isac, and T. M. Rassias, *Stability of functional equations in several variables*. Boston: Birkhauser, 1998.

[10] D. H. Hyers and T. M. Rassias, "Approximate homomorphisms," *Aequations Mathematicae*, vol. 44, pp. 125–153, 1992.

[11] T. Miura, S. E. Takahashi, and G. Hirasawa, "Hyers–Ulam–Rassias stability of Jordan homomorphisms on Banach algebras," *J. Inequal. Appl.*, vol. 1, pp. 435–441, 2004.

[12] M. S. Moslehian and T. M. Rassias, "Stability of functional equations in non-Archimedean spaces," *Applicable Analysis and Discrete Mathematics*, vol. 1, no. 2, pp. 325–334, 2007.

[13] A. Najati and T. M. Rassias, "Stability of a mixed functional equation in several variables on Banach modules," *Nonlinear Analysis*, vol. 72, no. 3–4, pp. 1755–1767, 2010.

[14] A. Prastaro and T. M. Rassias, "Ulam stability in geometry of PDE's," *Nonlinear Func. Anal. Appl.*, vol. 8, no. 2, pp. 259–278, 2003.

[15] T. M. Rassias, "On a modified Hyers-Ulam sequence," *J. Math. Anal. Appl.*, vol. 158, pp. 106–113, 1991.

[16] T. M. Rassias, "On the stability of the linear mapping in Banach spaces," *Proc. Amer. Math. Soc.*, vol. 72, pp. 297–300, 1998.

[17] T. M. Rassias, "On the stability of functional equations and a problem of Ulam," *Acta. Appl. Math.*, vol. 62, no. 1, pp. 23–130, 2000.

[18] T. M. Rassias, "On the stability of functional equations in Banach spaces," *J. Math. Anal. Appl.*, vol. 251, no. 1, pp. 264–284, 2000.

[19] T. M. Rassias, "The problem of S. M. Ulam for approximately multiplicative mappings," *J. Math. Anal. Appl.*, vol. 246, no. 2, pp. 352–378, 2000.

[20] T. M. Rassias and P. Semrl, "On the Hyers–Ulam stability of linear mappings," *J. Math. Anal. Appl.*, vol. 173, pp. 325–338, 1993.

[21] T. M. Rassias, "On the stability of functional equations originated by a problem of Ulam," *Mathematica*, vol. 44, no. 1, pp. 39–75, 2002.

[22] S. M. Ulam, *A collection of mathematical problems*. New York: Interscience Publisher, 1960.

Authors' addresses

Ismail Nikoufar
Department of Mathematics, Payame Noor University, P.O. BOX 19395-3697 Tehran, Iran
E-mail address: nikoufar@pnu.ac.ir

Themistocles M. Rassias
Department of Mathematics, National Technical University of Athens, Zografou, Campus 15780 Athens, Greece
E-mail address: trassias@math.ntua.gr