Miskolc Mathematical Notes
Vol. 8 (2007), No 2, pp. 109-115

HU e-ISSN 1787-2413
DOI: 10.18514/MMN.2007.110

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HU e-ISSN 1787-2413

SOME COMMON FIXED POINT THEOREMS FOR A CLASS OF FUZZY CONTRACTIVE MAPPINGS

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Received 11 November, 2004

Abstract. The purpose of this paper is to state and prove a new lemma generalizing Lemma 3.1 of Arora and Sharma [1] and Proposition 3.2 of Lee and Cho [10]. Some common fixed point theorems for a type of fuzzy contractive mappings are also established. These theorems extend and generalize several previous results [3, 14, 21, 22].

1991 *Mathematics Subject Classification:* 47H10, 54H25

Keywords: fuzzy sets, fuzzy map, fuzzy contractive mappings, common fixed points.

1. INTRODUCTION

Common fixed point theorems have been applied to diverse problems during the last few decades. These theorems provide techniques for solving a variety of applied problems in mathematical science and in dynamic programming (see, e. g., [4, 15, 16]). Extensions of the Banach contraction principle to multivalued mappings were initiated independently by Markin [11] and Nadler [13]. Therefore, results on fixed points of contractive type multivalued mappings have been carried out by many authors (see, for example, [2, 17, 21]).

The theory of fuzzy sets was investigated by Zadeh [24] in 1965. Some applications on results in this theory are discussed (see [9, 23]). In 1981, Heilpern [7] first introduced the concept of fuzzy contractive mappings and proved a fixed point theorem for these mappings in metric linear spaces. His result is a generalization of the fixed point theorem for point-to-set maps of Nadler [13]. Later, several fixed point theorems for types of fuzzy contractive mappings appeared (see, for instance, [1, 18–20]).

In this paper, we state and prove a new lemma generalizing Lemma 3.1 of Arora and Sharma [1] and Proposition 3.2 of Lee and Cho [10]. Two common fixed point theorems of a type of fuzzy contractive mappings are established. These theorems generalize and extend results in [3, 14, 21, 22]. Finally, we state a conclusion containing a brief of our results and future research.

2. Basic preliminaries

The definitions and terminologies for further discussions are taken from Heilpern [7]. Let (X,d) be a metric linear space. A *fuzzy set* in X is a function with domain X and values in $[0,1]$. If A is a fuzzy set and $x \in X$, then the function-value $A(x)$ is called the *grade of membership* of x in A. The collection of all fuzzy sets in X is denoted by $\mathfrak{F}(X)$.

Let $A \in \mathfrak{F}(X)$ and $\alpha \in [0,1]$. The *α-level set* of A, denoted by A_α, is defined by the formula

$$A_\alpha = \begin{cases} \{x : A(x) \geq \alpha\} & \text{if } \alpha \in (0,1], \\ \overline{\{x : A(x) > 0\}} & \text{if } \alpha = 0. \end{cases} \tag{2.1}$$

where $\overline{B}$ is the closure of a (nonfuzzy) set B.

Definition 1. A fuzzy set A in X is an *approximate quantity* if and only if its α-level set is a nonempty compact convex (nonfuzzy) subset of X for each $\alpha \in [0,1]$ and $\sup_{x \in X} A(x) = 1$.

The set of all approximate quantities, denoted $W(X)$, is a subcollection of $\mathfrak{F}(X)$.

Definition 2. Let $A, B \in W(X)$, $\alpha \in [0,1]$ and $CP(X)$ be the set of all nonempty compact subsets of X. Then one puts $p_\alpha(A,B) = \inf_{x \in A_\alpha, y \in B_\alpha} d(x,y)$, $\delta_\alpha(A,B) = \sup_{x \in A_\alpha, y \in B_\alpha} d(x,y)$, and $D_\alpha(A,B) = H(A_\alpha, B_\alpha)$, where H is the *Hausdorff metric* between two sets in the collection $CP(X)$.

We define the functions $p(A,B) = \sup_\alpha p_\alpha(A,B)$, $\delta(A,B) = \sup_\alpha \delta_\alpha(A,B)$, and $D(A,B) = \sup_\alpha D_\alpha(A,B)$.

Note that p_α is nondecreasing function of α.

Definition 3. Let $A, B \in W(X)$. Then A is said to be *more accurate* than B (or B includes A), denoted by $A \subset B$, if and only if $A(x) \leq B(x)$ for each $x \in X$.

The relation $\subset$ induces a partial ordering on $W(X)$.

Definition 4. Let X be an arbitrary set and Y be a metric linear space. F is said to be a *fuzzy mapping* if and only if F is a mapping from the set X into $W(Y)$, i. e., $F(x) \in W(Y)$ for each $x \in X$.

The following lemma and proposition are used in the sequel.

Lemma 1 ([12]). *Suppose that $\gamma : [0,\infty) \to [0,\infty)$ is a right continuous function such that $\gamma(t) < t$ for all $t > 0$. Then for every $t > 0$, $\lim_{n\to\infty} \gamma^n(t) = 0$, where γ^n is the nth iterate of γ, $n \in \mathbb{N} \cup \{0\}$.**

Proposition 1 ([13]). *If $A, B \in CP(X)$ and $a \in A$, then there exists $b \in B$ such that $d(a,b) \leq H(A,B)$.*

*$\mathbb{N}$ is the set of all positive integers

We consider the set Φ of all functions $\phi : [0,\infty)^5 \to [0,\infty)$ with the following properties:

(i) ϕ is nondecreasing with respect to each variable,
(ii) ϕ is right continuous with respect to each variable,
(iii) for each $t > 0$, $\Psi(t) = \max\{\phi(t,t,t,t,t), \phi(t,t,t,2t,0), \phi(t,t,t,0,2t)\} < t$.

3. MAIN RESULTS

Throughout this paper, let (X,d) be a metric space. We consider a subcollection of $\mathfrak{F}(X)$ denoted by $W^*(X)$. Each fuzzy set $A \in W^*(x)$, its α-level set is a nonempty compact (nonfuzzy) subset of X for each $\alpha \in [0,1]$. It is obvious that each element $A \in W(X)$ leads one to $A \in W^*(X)$ but the converse is not true. Now, we introduce the improvements of the lemmas in Heilpern [7] as follows.

Lemma 2. *If $\{x_0\} \subset A$ for each $A \in W^*(X)$ and $x_0 \in X$, then $p_\alpha(x_0, B) \leq D_\alpha(A,B)$ for each $B \in W^*(X)$.*

Lemma 3. *$p_\alpha(x,A) \leq d(x,y) + p_\alpha(y,A)$ for all $x, y \in X$ and $A \in W^*(X)$.*

Lemma 4. *Let $x \in X, A \in W^*(X)$ and $\{x\}$ be a fuzzy set with membership function equal to a characteristic function of the set $\{x\}$. Then $\{x\} \subset A$ if and only if $p_\alpha(x,A) = 0$ for each $\alpha \in [0,1]$.*

Proof. If $\{x\} \subset A$, then $x \in A_\alpha$ for each $\alpha \in [0,1]$. This implies that $p_\alpha(x,A) = \inf_{y \in A_\alpha} d(x,y) = 0$ for any $\alpha \in [0,1]$. Conversely, if $p_\alpha(x,A) = 0$, then we have $\inf_{y \in A_\alpha} d(x,y) = 0$. It follows that $x \in \overline{A_\alpha} = A_\alpha$ for an arbitrary $\alpha \in [0,1]$. Then $\{x\} \subset A$. □

Also, we state and prove a new lemma in the following way.

Lemma 5. *Let (X,d) be a complete metric space, $F : X \to W^*(X)$ be a fuzzy map and $x_0 \in X$. Then there exists $x_1 \in X$ such that $\{x_1\} \subset F(x_0)$.*

Proof. For $n \in \mathbb{N}$, $((F(x_0))_{n/(n+1)})$ is a decreasing sequence of nonempty compact subsets of X. Thus we have from Proposition 11.4 and Remark 11.5 of [25, pp. 495–496] that $\bigcap_{n=1}^{\infty}(F(x_0))_{n/(n+1)}$ is nonempty and compact.

Let $x_1 \in \bigcap_{n=1}^{\infty}(F(x_0))_{n/(n+1)}$. Then $\frac{n}{n+1} \leq (F(x_0))(x_1) \leq 1$. As $n \to \infty$, we get that $(F(x_0))(x_1) = 1$. This implies that $\{x_1\} \subset F(x_0)$. □

Remark 1. It is clear that Lemma 5 is a generalization of Lemma 3.1 of Arora and Sharma [1] and Proposition 3.2 of Lee and Cho [10].

Now, we are ready to prove our main theorems.

Theorem 1. *Let (X,d) be a complete metric space and F_1, F_2 be fuzzy mappings from X into $W^*(X)$. If there is a $\phi \in \Phi$ such that for all $x, y \in X$,*

$$D(F_1(x), F_2(y)) \leq \phi(d(x,y), p(x, F_1(x)), p(y, F_2(y)), p(x, F_2(y)), p(y, F_1(x))), \quad (3.1)$$

then there exists $z \in X$ such that $\{z\} \subset F_1(z)$ and $\{z\} \subset F_2(z)$.

Proof. Let $x_0 \in X$. Then by Lemma 5, there exists $x_1 \in X$ such that $\{x_1\} \subset F_1(x_0)$. For $x_1 \in X$, the set $(F_2(x_1))_1$ is nonempty compact subset of X. Since $(F_1(x_0))_1$ and $(F_2(x_1))_1$ belong to $CP(X)$ and $x_1 \in (F_1(x_0))_1$, Proposition 1 asserts that there exists $x_2 \in (F_2(x_1))_1$ such that $d(x_1, x_2) \leq D_1(F_1(x_0), F_2(x_1))$. So, we have from Lemma 4 and the property (i) of ϕ that

$$\begin{aligned} d(x_1,x_2) &\leq D_1(F_1(x_0), F_2(x_1)) \leq D(F_1(x_0), F_2(x_1)) \\ &\leq \phi(d(x_0,x_1), \\ &\qquad p(x_0, F_1(x_0)), p(x_1, F_2(x_1)), p(x_0, F_2(x_1)), p(x_1, F_1(x_0))) \\ &\leq \phi(d(x_0,x_1), d(x_0,x_1), d(x_1,x_2), d(x_0,x_1) + d(x_1,x_2), 0). \end{aligned}$$

If $d(x_1,x_2) > d(x_0,x_1)$, then

$$d(x_1,x_2) \leq \phi(d(x_1,x_2), d(x_1,x_2), d(x_1,x_2), 2d(x_1,x_2), 0) < d(x_1,x_2).$$

This contradiction demands that

$$d(x_1,x_2) \leq \phi(d(x_0,x_1), d(x_0,x_1), d(x_0,x_1), 2d(x_0,x_1), 0).$$

Similarly, one can deduce that

$$d(x_2,x_3) \leq \phi(d(x_1,x_2), d(x_1,x_2), d(x_1,x_2), 0, 2d(x_1,x_2)).$$

By induction, we have a sequence (x_n) of points in X such that, for all $n \in \mathbb{N} \cup \{0\}$,

$$\{x_{2n+1}\} \subset F_1(x_{2n}), \quad \{x_{2n+2}\} \subset F_2(x_{2n+1}).$$

It follows by induction that $d(x_n, x_{n+1}) \leq \Psi^n(d(x_0,x_1))$, where Ψ is defined in the property (iii) of ϕ. Then, Lemma 1 gives that $\lim_{n\to\infty} d(x_n, x_{n+1}) = 0$. Since

$$d(x_n,x_m) \leq d(x_n,x_{n+1}) + d(x_{n+1},x_{n+2}) + ... + d(x_{m-1},x_m),$$

then $\lim_{n,m\to\infty} d(x_n,x_m) = 0$. Therefore, (x_n) is a Cauchy sequence. Since X is a complete metric space, then there exists $z \in X$ such that $\lim_{n\to\infty} x_n = z$. Next, we show that $\{z\} \subset F_i(z), i = 1, 2$. Now, we get from Lemma 2 and Lemma 3 that

$$\begin{aligned} p_\alpha(z, F_2(z)) &\leq d(z, x_{2n+1}) + p_\alpha(x_{2n+1}, F_2(z)) \\ &\leq d(z, x_{2n+1}) + D_\alpha(F_1(x_{2n}), F_2(z)), \end{aligned}$$

for each $\alpha \in [0,1]$. Taking the supremum on α in the last inequality, we obtain from the property (i) of ϕ that

$$\begin{aligned} p(z, F_2(z)) &\leq d(z, x_{2n+1}) + D(F_1(x_{2n}), F_2(z)) \\ &\leq d(z, x_{2n+1}) + \phi(d(x_{2n}, z), p(x_{2n}, F_1(x_{2n})), p(z, F_2(z)), \\ &\qquad p(x_{2n}, F_2(z)), p(z, F_1(x_{2n}))) \\ &\leq d(z, x_{2n+1}) + \phi(d(x_{2n}, z), d(x_{2n}, x_{2n+1}), p(z, F_2(z)), \\ &\qquad p(x_{2n}, F_2(z)), d(z, x_{2n+1})). \end{aligned}$$

As $n \to \infty$, we have from the properties (i), (ii) and (iii) of ϕ with $p(z, F_2(z)) \neq 0$ that

$$\begin{aligned} p(z, F_2(z)) &\leq \phi(0, 0, p(z, F_2(z)), p(z, F_2(z)), 0) \\ &\leq \phi(p(z, F_2(z)), p(z, F_2(z)), p(z, F_2(z)), p(z, F_2(z)), p(z, F_2(z))) \\ &< p(z, F_2(z)). \end{aligned}$$

This contradiction yields $p(z, F_2(z)) = 0$. We then get from Lemma 4 that $\{z\} \subset F_2(z)$. Similarly, one can show that $\{z\} \subset F_1(z)$. □

Example 1. Let $X = [0,1]$ endowed with the metric d defined by $d(x,y) = |x-y|$. It is clear that (X,d) is a complete metric space. Assume that $\phi(t_1,t_2,t_3,t_4,t_5) = \frac{3}{4}t_1$ for arbitrary $t_i \in [0,\infty)$, $i = \overline{1,5}$. It is obvious that $\Psi(t) < t$ for all $t > 0$. Let $F_1 = F_2 = F$. Define a fuzzy mapping F on X such that for all $x \in X$, $F(x)$ is the characteristic function for $\{\frac{3}{4}x\}$. For each $x, y \in X$,

$$\begin{aligned} D(F(x), F(y)) &= \frac{3}{4} d(x,y) \\ &= \phi(d(x,y), p(x, F(x)), p(y, F(y)), p(x, F(y)), p(y, F(x))). \end{aligned} \tag{3.2}$$

The characteristic function for $\{0\}$ is the fixed point of F.

As corollaries of Theorem 1, we get the following statements.

Corollary 1. *Let (X,d) be a complete metric space and F_1, F_2 be fuzzy mappings from X into $W^*(X)$ satisfying the following conditions: for any x, y in X,*

$$\begin{aligned} D(F_1(x), F_2(y)) \leq a_1 p(x, F_1(x)) + a_2 p(y, F_2(y)) + a_3 p(y, F_1(x)) \\ + a_4 p(y, F_1(x)) + a_5 d(x,y), \end{aligned} \tag{3.3}$$

where a_1, a_2, a_3, a_4, and a_5 are non-negative real numbers, $\sum_{j=1}^{5} a_i < 1$ and $a_1 = a_2$ or $a_3 = a_5$. Then there exists $z \in X$ such that $\{z\} \subset F_1(z)$ and $\{z\} \subset F_2(z)$.

Proof. We consider the function $\phi : [0,\infty)^5 \to [0,\infty)$ defined by the formula

$$\phi(x_1, x_2, x_3, x_4, x_5) = a_1 x_2 + a_2 x_3 + a_3 x_5 + a_4 x_4 + a_5 x_1, \tag{3.4}$$

where $\sum_{i=1}^{i=5} a_i < 1$ such that $a_1 = a_2$ or $a_3 = a_4$. Since $\phi \in \Phi$, we have from Theorem 1 that there exists $z \in X$ such that $\{z\} \subset F_1(z)$ and $\{z\} \subset F_2(z)$. □

The following corollary is a fuzzy version of the fixed point theorem of Singh and Whitfield [21] for multivalued mappings.

Corollary 2. *Let (X,d) be a complete metric space and F_1, F_2 be fuzzy mappings from X into $W^*(X)$. If there is a constant α, $0 \le \alpha < 1$, such that, for each $x, y \in X$,*

$$D(F_1(x), F_2(y)) \le \alpha \max\left\{d(x,y), \frac{1}{2}[p(x,F_1(x)) + p(y,F_2(y))], \right.$$
$$\left. \frac{1}{2}[p(x,F_2(y)) + p(y,F_1(x))]\right\}, \quad (3.5)$$

then there exists $z \in X$ such that $\{z\} \subset F_1(z)$ and $\{z\} \subset F_2(z)$.

Proof. We consider the function $\phi : [0,\infty)^5 \to [0,\infty)$ defined by

$$\phi(x_1,x_2,x_3,x_4,x_5) = \alpha \max\left\{x_1, \frac{1}{2}[x_2+x_3], \frac{1}{2}[x_4+x_5]\right\}. \quad (3.6)$$

Since $\phi \in \Phi$, we get from Theorem 1 that there exists $z \in X$ such that $\{z\} \subset F_1(z)$ and $\{z\} \subset F_2(z)$. □

Remark 2. (1) If there is a $\phi \in \Phi$ such that, for each $x, y \in X$,

$$\delta(F_1(x), F_2(y)) \le \phi(d(x,y), p(x,F_1(x)), p(y,F_2(y)),$$
$$p(x,F_2(y)), p(y,F_1(x))), \quad (3.7)$$

then the conclusion of Theorem 1 remains valid. This result is considered as a special case of Theorem 1 because $D(F_1(x), F_2(y)) \le \delta(F_1(x), F_2(y))$ [8, p. 414]. Moreover, this result generalizes Theorem 3.3 of Park and Jeong [14].

(2) Corollary 1 is [22, Theorem 3.1] without condition (a), where condition (a) reads as follows: "for each $x \in X$, there exists $\alpha(x) \in (0,1]$ such that $(F_1(x))_{\alpha(x)}$ and $(F_2(x))_{\alpha(x)}$ are nonempty closed bounded subsets of $\mathfrak{F}(X)$." Also, Corollary 1 generalizes [3, Theorem 3.1].

(3) Theorems 3.1 and 3.4 of Park and Jeong [14] are special cases of Theorem 1.

The following theorem generalizes Theorem 1 to a sequence of fuzzy contractive mappings.

Theorem 2. *Let $(F_n : n \in \mathbb{N} \cup \{0\})$ be a sequence of fuzzy mappings from a complete metric space (X,d) into $W^*(X)$. If there is a $\phi \in \Phi$ such that, for all $x, y \in X$,*

$$D(F_0(x), F_n(y)) \le \phi(d(x,y), p(x,F_0(x)), p(y,F_n(y)),$$
$$p(x,F_n(y)), p(y,F_0(x))) \qquad \forall (n \in \mathbb{N}), \quad (3.8)$$

then there exists a common fixed point of the family $(F_n : n \in \mathbb{N} \cup \{0\})$.

Proof. Putting $F_1 = F_0$ and $F_2 = F_n$ for all $n \in \mathbb{N}$ in Theorem 1. Then there exists a common fixed point of the family $(F_n : n \in \mathbb{N} \cup \{0\})$. □

Remark 3. If there is a $\phi \in \Phi$ such that, for all $x, y \in X$,

$$\delta(F_0(x), F_n(y)) \le \phi(d(x,y), p(x, F_0(x)), p(y, F_n(y)), p(x, F_n(y)), p(y, F_0(x))) \qquad (\forall n \in \mathbb{N}), \quad (3.9)$$

then the conclusion of Theorem 2 remains valid. This result is considered as a special case of Theorem 2 for the same reason as in Remark 2 (1).

4. Conclusion

This paper presents an improvement of some results in [1, 7, 10]. Also, it presents two common fixed point theorems for a type of fuzzy contractive mappings. These theorems generalize and extend results in [3, 14, 22] and [21], respectively. A fixed point theorem for fuzzy contractive mappings is stated generalizing [1, Theorem 3.5]. Many applications of our main theorems are possible, e. g., for differential and integral equations. In view of the references [5, 6], some future research can be done, for example:

(1) I believe that our results can be hold for $FC(X)$, where $FC(X) = \{A \in \mathfrak{F}(X) : A_\alpha$ is a nonempty closed (nonfuzzy) subset of X for each $\alpha \in [0,1]\}$,

(2) it is also possible to generalize our results to quasi-metric spaces.

Acknowledgement

I wish to thank Prof. B. E. Rhoades from the Indiana University (USA) and Dr. F. M. Zeyada for their critical reading of the manuscript and valuable comments.

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