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ORIGINAL STUDY

The 2D time-dependent similarity transformation model as a tool for deformation monitoring

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Abstract Besides the methodology of triangulation and geodetic networks nowadays, the permanent stations of satellite receivers exist, giving extra asset to geodetic daily practice. Permanent stations perform observations incessantly for the visible satellites. However, the coordinates of these stations are often changing over time due to geophysical and tectonic processes. Consequently, these changes are perceived to modern observations. So, along with the coordinates of geodetic points in a given epoch, their changes over time (e.g. the velocities of their movements) are also considered. Furthermore, any change to the reference system definition or/and to the network's geometry can significantly impact the estimated coordinates and velocities. This paper investigates the reference datum definition problem (or datum problem, or zero order design problem) in a such network over time, which is later generalized for the study of the deformation control-networks. Emphasis is given to techniques of time-dependent 2D transformation models, with numerical tests on a simulation network.

Keywords Velocities · Deformations · Monitoring · Control networks · Time-depended transformation models · Simulations

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1 Introduction

The 2D similarity transformation has a wide range of applications to classical and engineering geodesy. Its main scope is the connection between two different reference systems in terms of translations, scale and orientation. An advantage of the 2D transformation is the absence of the height information and thus there is no assumption for the associated height system (orthometric or/and geometric one) required. This can be useful e.g. when we combine classical with GNSS networks, respectively. In advance, the GNSS-derived heights are less accurate compared to the horizontal coordinates, respectively. On the other hand, the 2D similarity transformation cannot be applied rigorously for relatively large areas (e.g. more than few kilometers). While the spatial (coordinate) transformation is a straightforward procedure, the associated model for 2D velocity transformation is usually omitted. However, in various geodetic monitoring applications the 2D velocity transformation can be needed (e.g. Doukas et al. 2004).

The main issue is to connect in an optimal way the coordinates and the estimated velocities through a unified algorithm. For example, this may be required in cases that:

- we have two or more different network realizations at the same area, such an old geodetic monitoring network (realized from classical observations) and a GNSS network which share only a number of common points. Especially for the GNSS network, one should pay great attention, since the observations have completely different nature from the classical one.
- some of the control benchmarks of a network are missing and there is the need of handling and restoring the monitoring information.
- we want to connect two different realizations of the same network: For example, if we aim to quantify the spatial and dynamic inconsistencies from different constraints handling (e.g. Rossikopoulos 1986; Dermanis 1987; Rossikopoulos 1999).
- we test if one or both of the networks carries some systematic errors or blunders.

Thus, we should build a methodology on the optimal combination of the coordinates and velocities of different network realizations. The present study is dedicated to obtain a new methodology dealing with the simultaneous estimation of the spatial and dynamic parameters. All the necessary formulas are given and in addition a numerical example is performed.

We must note that till now, there is no published mathematical model for the transformation of the 2D (horizontal) velocities. In the majority of the cases, the dynamic part of the network (the velocities) is ignored, and only the classical spatial 2D similarity transformation is applied. This fact can lead to misinterpretations, since the estimated parameters absorb not only the spatial change but in addition, the deformation part which is related to the velocity estimation. The present paper aims to treat this problem, by providing all the mathematical tools and pointing out some crucial theoretical aspects.

The novelty of the proposed approach/strategy lies in the fact that it allows the simultaneous 2D similarity transformation both for coordinates and velocities. Till now, the similarity transformation refers only to the full 3D case (the so-called time-dependent 3D transformation, see e.g. Altamimi et al. 2002). In addition, the described approach gives for the first time the necessary mathematical formulas for the 2D similarity transformation of the (plane) velocities. Finally, we should mention that the use of the 3D time-dependent transformation can be problematic in relative small areas, due to the high correlations among the translations, rotations and scale (and their rates). On the other hand, the 2D similarity transformation fits better to limited areas. Thus, the 2D time-dependent transformation can stand as an optimal choice for deformation networks in relative small areas.

2 Methodology

The classical 2D similarity transformation has the well known form (e.g. Torge 2001):

$$
\begin{aligned}
X_i &= \mu \cos \theta x_i + \mu \sin \theta y_i + t_x \\
Y_i &= -\mu \sin \theta x_i + \mu \cos \theta y_i + t_y \\
&\vdots \\
X_n &= \mu \cos \theta x_n + \mu \sin \theta y_n + t_x \\
Y_n &= -\mu \sin \theta x_n + \mu \cos \theta y_n + t_y
\end{aligned}
\tag{1}
$$

where $(x_i, y_i), (X_i, Y_i)$ are the planar coordinates in the initial and the final reference system, respectively at a point i, t_x, t_y are the translations of the initial reference system with respect to the final one, μ and θ are the scale and the rotation parameters, respectively. For simplicity reasons, we can combine the scale and rotation terms as follows (Dermanis and Fotiou 1992):

$$
c = \mu \cos \theta \tag{2a}
$$

$$
d = \mu \sin \theta \tag{2b}
$$

However, the coordinates of both reference systems are inevitably affected from errors. Thus, the observed planar coordinates are expressed through the following relations (pointwise):

$$
X_i^b = X_i + e_{X_i} \tag{3a}
$$

$$
Y_i^b = Y_i + e_{Y_i} \tag{3b}
$$

$$
x_i^b = x_i + e_{x_i} \tag{3c}
$$

$$
y_i^b = y_i + e_{y_i} \tag{3d}
$$

where the term e stands for the error of each coordinates component and the superscript b denotes the observed quantities, respectively. Combining Eqs. (2) and (3), Eq. (1) now yields:

$$
\begin{aligned}
X_i^b - e_{X_i} &= c\left(x_i^b - e_{x_i}\right) + d\left(y_i^b - e_{y_i}\right) + t_x \\
Y_i^b - e_{Y_i} &= -d\left(x_i^b - e_{x_i}\right) + c\left(y_i^b - e_{y_i}\right) + t_y \\
&\vdots \\
X_n^b - e_{X_n} &= c\left(x_n^b - e_{x_n}\right) + d\left(y_n^b - e_{y_n}\right) + t_x \\
Y_n^b - e_{Y_n} &= -d\left(x_n^b - e_{x_n}\right) + c\left(y_n^b - e_{y_n}\right) + t_y
\end{aligned}
\tag{4}
$$

The velocities are the derivatives of the coordinates with respect to the time (pointwise):

$$
\dot{x}_i = \frac{dx_i}{dt} \tag{5a}
$$

$$
\dot{y}_i = \frac{dy_i}{dt} \tag{5b}
$$

$$\dot{X}_i = \frac{dX_i}{dt} \tag{5c}$$

$$\dot{Y}_i = \frac{dY_i}{dt} \tag{5d}$$

According to the same conceptual manner for the effect of the errors to the coordinates (Eq. 3), we can write the same relations for the velocities:

$$\dot{X}_i^b = \dot{X}_i + e_{\dot{X}_i} \tag{6a}$$

$$\dot{Y}_i^b = \dot{Y}_i + e_{\dot{Y}_i} \tag{6b}$$

$$\dot{x}_i^b = \dot{x}_i + e_{\dot{x}_i} \tag{6c}$$

$$\dot{y}_i^b = \dot{y}_i + e_{\dot{y}_i} \tag{6d}$$

Following the classical differentiating rules for the Eq. 4 and taking into account the Eq. (6), we can express the velocity transformation as follows:

$$
\begin{aligned}
\dot{X}_i^b - e_{\dot{X}_i} &= \dot{c}\left(x_i^b - e_{x_i}\right) + \dot{d}\left(y_b^i - e_{y_i}\right) + c\left(\dot{x}_i^b - e_{\dot{x}_i}\right) + d\left(\dot{y}_i^b - e_{\dot{y}_i}\right) + \dot{t}_x \\
\dot{Y}_i^b - e_{\dot{Y}_i} &= -\dot{d}\left(x_i^b - e_{x_i}\right) + \dot{c}\left(y_b^i - e_{y_i}\right) - d\left(\dot{x}_i^b - e_{\dot{x}_i}\right) + c\left(\dot{y}_i^b - e_{\dot{y}_i}\right) + \dot{t}_y \\
&\vdots \\
\dot{X}_n^b - e_{\dot{X}_n} &= \dot{c}\left(x_n^b - e_{x_n}\right) + \dot{d}\left(y_n^b - e_{y_n}\right) + c\left(\dot{x}_n^b - e_{\dot{x}_n}\right) + d\left(\dot{y}_n^b - e_{\dot{y}_n}\right) + \dot{t}_x \\
\dot{Y}_n^b - e_{\dot{Y}_n} &= -\dot{d}\left(x_n^b - e_{x_n}\right) + \dot{c}\left(y_n^b - e_{y_n}\right) - d\left(\dot{x}_n^b - e_{\dot{x}_n}\right) + c\left(\dot{y}_n^b - e_{\dot{y}_n}\right) + \dot{t}_y
\end{aligned}
\tag{7}
$$

The dot signs denote the dynamic parameters (four extra parameters). In order to proceed with the least squares adjustment, it is needed to choose which of the existing methodologies (observation equations, condition equations or mixed model) will apply. A fruitful description of the different adjustment methodologies is given by Dermanis (1976). In our case, since the observations and the parameters are correlated, we should employ the mixed model, which is generally described as follows:

$$\mathbf{l}(\psi, \mathbf{u}) = \mathbf{0} \tag{8}$$

where $\mathbf{l}$ refers to a function of the observation vector $\psi = \begin{bmatrix} \mathbf{X}^T & \dot{\mathbf{X}}^T & \mathbf{x}^T & \dot{\mathbf{x}}^T \end{bmatrix}^T$ and the vector of the unknowns $\mathbf{u} = \begin{bmatrix} c & d & t_x & t_y & \dot{c} & \dot{d} & \dot{t}_x & \dot{t}_y \end{bmatrix}^T$. In addition, we

have: $\mathbf{X} = \begin{bmatrix} X_i^b \\ Y_i^b \\ \vdots \\ \vdots \\ X_n^b \\ Y_n^b \end{bmatrix}, \dot{\mathbf{X}} = \begin{bmatrix} \dot{X}_i^b \\ \dot{Y}_i^b \\ \vdots \\ \vdots \\ \dot{X}_n^b \\ \dot{Y}_n^b \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_i^b \\ y_i^b \\ \vdots \\ \vdots \\ x_n^b \\ y_n^b \end{bmatrix}, \dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_i^b \\ \dot{y}_i^b \\ \vdots \\ \vdots \\ \dot{x}_n^b \\ \dot{y}_n^b \end{bmatrix}$ The final linearized combined model

(for the coordinates and the velocities) has the following form:

$$\mathbf{w} = \mathbf{K}\delta\mathbf{u} + \mathbf{B}\mathbf{e} \tag{9}$$

where $\mathbf{w} = \mathbf{l}(\psi, \mathbf{u}^o)$ is the loop closure vector (the superscript o stands for the approximate values of the unknowns) $\delta\mathbf{u}$ the corrections term, $\mathbf{B}$ the Jacobian matrix $\frac{\partial \mathbf{l}}{\partial \psi}$, $\mathbf{e} =$

$\begin{bmatrix} \mathbf{e}_{\mathbf{x}}^T & \mathbf{e}_{\dot{\mathbf{x}}}^T & \mathbf{e}_{\mathbf{x}}^T & \mathbf{e}_{\dot{\mathbf{x}}}^T \end{bmatrix}^T$ the vector of the observations errors, $\mathbf{K} = \frac{\partial \mathbf{l}}{\partial \mathbf{u}} = \begin{bmatrix} \tilde{\mathbf{E}}^T & \mathbf{G}^T \end{bmatrix}^T$, and

$$\tilde{\mathbf{E}} = \begin{bmatrix} \mathbf{E}^T & \mathbf{Z}^T \end{bmatrix}^T, \ \mathbf{Z} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \ \text{In addition, } \mathbf{E} = \begin{bmatrix} x_i^b & y_i^b & 1 & 0 \\ -x_i^b & y_i^b & 0 & -1 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ x_n^b & y_n^b & 1 & 0 \\ -x_n^b & y_n^b & 0 & -1 \end{bmatrix} \text{ and}$$

$$\mathbf{G} = \begin{bmatrix} \dot{x}_i^b & \dot{y}_i^b & 0 & 0 & x_i^b & y_i^b & 1 & 0 \\ -\dot{x}_i^b & \dot{y}_i^b & 0 & 0 & -x_i^b & y_i^b & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \dot{x}_n^b & \dot{y}_n^b & 0 & 0 & x_n^b & y_n^b & 1 & 0 \\ -\dot{x}_n^b & \dot{y}_n^b & 0 & 0 & -x_n^b & y_n^b & 0 & 1 \end{bmatrix}. \ \text{In our case, for each point } i \text{ we have}$$

the following four equations:

$$l_{1_i} : X_i^b - c^o x_i^b - d^o y_i^b - t_x^o = w_{1_i} \tag{10a}$$

$$l_{2_i} : Y_i^b + d^o x_i^b - c^o y_i^b - t_y^o = w_{2_i} \tag{10b}$$

$$l_{3_i} : \dot{X}_i^b - c^o \dot{x}_i^b - \dot{c}^o x_i^b - d^o \dot{y}_i^b - \dot{d}^o y_i^b - \dot{t}_x^o = w_{3_i} \tag{10c}$$

$$l_{4_i} : \dot{Y}_i^b + d^o \dot{x}_i^b + \dot{d}^o x_i^b - c^o \dot{y}_i^b - \dot{c}^o y_i^b - \dot{t}_y^o = w_{4_i} \tag{10d}$$

Our final model comprises eight transformation parameters: The classical spatially related parameters of the similarity transformation, in addition to their associated transformation rates. Note, the velocities are mutually dependent to the eight transformation parameters, though the coordinates are blind to the rate parameters. For the approximate values of the parameters one can initially set all of them as zero, except for the c, which can be one. Of course, after a number of iterations the optimal parameters are estimated (setting each time the previous estimated values as the approximate one).

2.1 The mathematical model for the least squares adjustment

Applying the least squares principle (e.g. Koch 1987), the estimated unknown parameters, adjusted observations and adjusted residuals (four for the coordinates and four for the velocities) are derived as follows (e.g. Dermanis and Fotiou 1992 for the mixed model):

$$\delta\hat{\mathbf{u}} = -\left(\mathbf{K}^T \mathbf{M}^{-1} \mathbf{K}\right)^{-1} \mathbf{K}^T \mathbf{M}^{-1} \mathbf{w} \tag{11}$$

the associated covariance matrix is:

$$\mathbf{C}_{\hat{\mathbf{u}}} = \hat{\sigma}^2 \left(\mathbf{K}^T \mathbf{M}^{-1} \mathbf{K}\right)^{-1} \tag{12}$$

The estimated errors,

$$\hat{\mathbf{e}} = \mathbf{C}_\mathbf{e} \mathbf{B}^T \mathbf{M}^{-1} (\mathbf{w} + \mathbf{K}\delta\hat{\mathbf{u}}) \tag{13}$$

and $\hat{\sigma}^2 = \frac{\hat{e}^T C_e^{-1} \hat{e}}{4n-8}$ the a posteriori variance (n the number of the points). Finally, $\mathbf{M} =$

$\mathbf{BC_e^{-1}B}^T$ with $\mathbf{C_e} = \begin{bmatrix} \mathbf{C_{ex}} & & & \\ & \mathbf{C_{e\dot{x}}} & & \\ & & \mathbf{C_{e_x}} & \\ & & & \mathbf{C_{e_{\dot{x}}}} \end{bmatrix}$ the $4n \times 4n$ covariance matrix of the

observation errors (coordinates and velocities). We assume implicitly, that the coordinates and velocities are not correlated. This is a realistic assumption, since the velocities are estimated mainly, by individual time series of the coordinates.

If we want to express the covariance error matrix to a more correct form, we have:

$$\mathbf{C_e} = \begin{bmatrix} \sigma_1^2 \mathbf{Q_{e_x}} & & & \\ & \sigma_2^2 \mathbf{Q_{e_{\dot{x}}}} & & \\ & & \sigma_3^2 \mathbf{Q_{e_x}} & \\ & & & \sigma_4^2 \mathbf{Q_{e_{\dot{x}}}} \end{bmatrix} \tag{14}$$

where $\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2$ are the (unknown) variances of each group of the observations. They can be estimated using the so-called Variance Component Analysis (VCA- see e.g. Bähr et al. 2007).

2.2 The case of different realization epochs

It is possible that the two reference systems are not realized in the same epoch. For example, this can occur when the initial network was measured many years before the final one. In this case one should take into account the coordinate change throughout the time. In this case, all the coordinate related quantities, presented in the matrices $\mathbf{E}$ and $\mathbf{G}$ (Eq. 9), respectively should be changed, according to the following expressions:

$$x_i = x_i^{t_0} + \dot{x}_i(T_o - t_0) \tag{15a}$$

$$y_i = y_i^{t_0} + \dot{y}_i(T_o - t_0) \tag{15b}$$

where $x_i^{t_0}, y_i^{t_0}$ the 2D coordinates of the initial frame at the epoch t_o of its realization and T_o the reference epoch of the final reference frame. By this manipulation, we express the coordinates of the initial system to the realization epoch of the final one. If we ignore this critical modification, the estimated parameters would absorb a kind of pseudo information, which in fact is a bias due to the different realization epochs. This bias can be significant, if the time difference between them is relatively large (e.g. decades) or/and the velocities are also large.

2.3 Some remarks regarding the practical application of the 2D time dependent similarity transformation

(1)   The above equations are used for the simultaneous estimation of the parameters connecting the coordinates and the velocities of two different reference systems. We underline again that the 2D similarity transformation is well defined only for limited areas. If one wants to apply the 3D similarity transformations both for coordinates and velocities, Altamimi et al. (2002) give all the necessary mathematical formulas for it. The present study should be considered as the special case of the 3D time-dependent transformation given by Altamimi et al. (2002).

(2)   The same concept can be easily implemented to the vertical networks. In this case, we have only two estimated parameters: the offset and its rate.

(3)   The time dependent 2D similarity transformation could be expanded for the affine or polynomial transformations, respectively. However, the model will include relative many unknown parameters.

(4)   We should be aware of the realization epochs. As we discussed previously, this can distort our results. If the reference epoch is not explicitly defined, we should re-compute the associated networks per epoch. Then, it is needed to define a reference epoch (e.g. in the mean epoch of the observation). Finally, the velocity estimation using e.g. time series analysis should be estimated.

3 Numerical example

Our new approach for the optimal estimation of the spatial and dynamic parameters of the 2D similarity transformation is tested through a simulation paradigm.

1.   Initially, we create a random network of 9 points (Fig. 1). We set an initial network of well distributed stations, covering an area of approximately 1 km^2. From this network, we extract 40 horizontal distances and 26 angles as they derived using the Pythagorean theorem and the the differences of the computed azimuths, respectively. In the case of the horizontal distances, we add zero mean noise with 1.0 mm/yr standard deviation, while for the angles we assume 0.2 mgons standard deviation (again zero mean). For both cases, we used the Matlab random noise generator. The noise level reflects adequately the observational uncertainty of the modern and precise terrestrial measurements used for the deformation monitoring.

2.   For these 9 points, we also consider 2D velocities. We artificially generate 18 velocities (x and y components of the 9 points) with the following statistical characteristics: mean average 10 mm/yr and standard deviation 3.0 mm/yr.
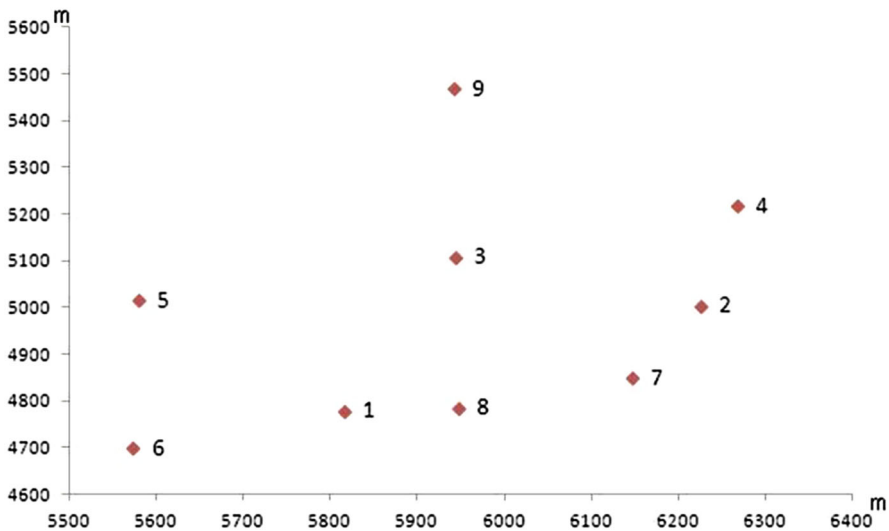

Fig. 1 The simulated network

3. The simulated network (initial network) is solved imposing partial inner constraints to the points 1, 3 and 9. The mean standard deviations (mean average of the squared diagonal elements of the covariance matrix of the estimated unknowns, Eq. 12, ibid.) are 0.96 mm and 0.13 mm/yr for the coordinates and velocities, respectively.

4. In order to compute the final network, we apply the eight similarity transformation parameters (spatial and dynamic). These parameters are designed in a way that realize a reference system which is relatively "close" to the initial one, in terms of translations, scale and orientation (and for their associated rates). Then, we add random noise to the coordinates and velocities for the final reference system. The statistical characteristics of the added noise are zero mean (for both coordinates and velocities) and standard deviations of 1.5 mm and 1.0 mm/yr, respectively. Furthermore, for 2 x-components and 2 y-components of the initial network, we add a bias of 5 mm to their noised values. This bias is induced in order to take account of some systematic effects. We assumed both noise and bias for the final network, in order to have a realistic simulation scenario. Tables 1 and 2 refer to the coordinates and velocities of the initial and final reference system, respectively. Figure 2 depicts the velocities with respect to the two systems.

We also apply statistical tests to our results (Koch 1987). We implement the two-sides F-test (testing the variances) and the Student test (t test) for outlier detection for the initial and the final network, respectively.

Table 1 The coordinates and the velocities of the initial reference system

Point	X (m)	Y (m)	V_x (m/y)	V_y (m/y)
1	5815.301	4782.201	0.0086	−0.0102
2	6224.399	5006.500	0.0105	−0.0103
3	5942.299	5110.501	0.0092	−0.0104
4	6266.600	5220.100	0.0097	−0.0100
5	5579.299	5017.201	0.0110	−0.0090
6	5573.299	4701.701	0.0089	−0.0082
7	6146.501	4851.500	0.0113	−0.0094
8	5946.800	4787.699	0.0107	−0.0103
9	5941.200	5470.099	0.0099	−0.0093

The coordinates are in meters and the velocities are in meters/yr

Table 2 The coordinates and the velocities of the final reference system

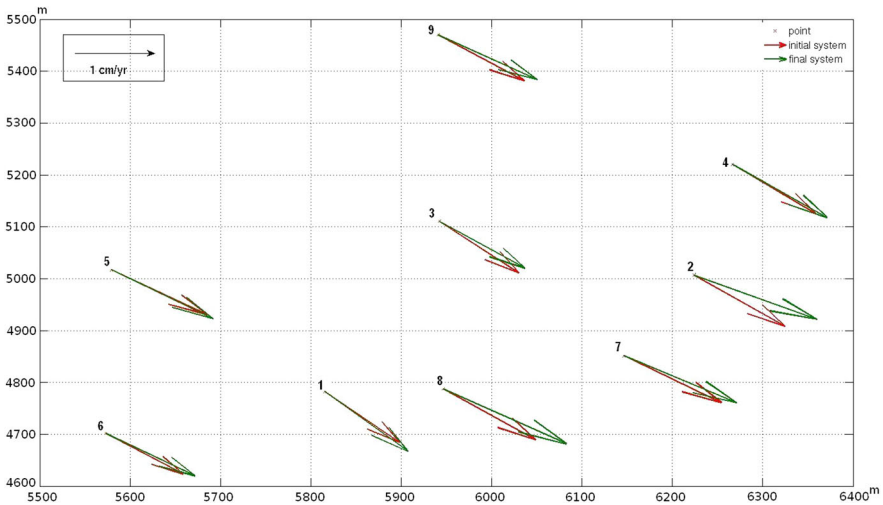| Point	X (m)	Y (m)	V_X (m/y)	V_Y (m/y)
1	5815.303	4782.205	0.0082	−0.0102
2	6224.402	5006.505	0.0120	−0.0085
3	5942.302	5110.502	0.0081	−0.0080
4	6266.607	5220.101	0.0092	−0.0092
5	5579.303	5017.205	0.0099	−0.0083
6	5573.307	4701.706	0.0087	−0.0072
7	6146.503	4851.502	0.0110	−0.0080
8	5946.806	4787.704	0.0120	−0.0094
9	5941.203	5470.103	0.0097	−0.0076

Fig. 2 The simulated velocities of the two systems

Table 3 The estimated parameters and their associated formal errors

Parameter	Values
c	$0.99999720486 \pm 2\mathrm{e}{-}07$
d	$-1.7277\mathrm{e}{-}07 \pm 5\mathrm{e}{-}07$
t_x	0.0219 ± 0.015 m
t_y	0.0164 ± 0.012 m
$\dot{c}$	$2.2681\,\mathrm{e}{-}06 \pm 3.2\mathrm{e}{-}07$
$\dot{d}$	$-3.419\mathrm{e}{-}08 \pm 1.2\mathrm{e}{-}08$
$\dot{t}_x$	0.0067 ± 0.0028 m/yr
$\dot{t}_y$	-0.0103 ± 0.009 m/yr

Regarding the covariance matrix of the observation errors, we use the covariance matrix of estimated coordinates, and a unity matrix for the velocities. For the velocities, the unity matrix was multiplied by a factor of 1×10^{-6}, assuming implicitly that the velocities were estimated with a formal error of 1 mm/yr. We should note here that since the network is solved using inner constraints, the associated covariance matrix of the coordinates is singular (e.g. Koch 1987). We overcame this problem, by slightly modifying the covariance matrix by adding a diagonal matrix, as follows (Bjerhammar 1973; Sillard and Boucher 2001; Rossikopoulos 2001):

$$\mathbf{C}_{\hat{\mathbf{x}}} = \mathbf{C}_{\hat{\mathbf{x}}} + \delta \cdot \mathbf{I} \tag{16}$$

where $\mathbf{C}_{\hat{\mathbf{x}}}$ is the covariance matrix of the estimated coordinates and $0 < \delta \ll 1$. In our case, we have $\delta = 0.00001$.

The results of the implementation of the combined 2D similarity transformation are presented in the following tables. Table 3 shows the estimated parameters and their associated formal errors, while Table 4 refers to the residual statistics for the coordinates and velocities.

Table 4 The residuals of the coordinates and the velocities, after the application of the 2D time dependent similarity transformation

Quantity	Coordinates	Velocities
Min	−0.0027	−0.0021
Max	0.0027	0.0027
Std	0.0017	0.0010
Mean	0.0001	0.0000

The coordinates are in meters and the velocities are in meters/yr

The standard deviation of the residuals (0.0017 m and 0.0010 m/yr, respectively) is at the level of our initial formal error assumptions (the assumptions regarding the added random noise to the observations). Furthermore, the mean residual average is practical zero.

Finally, Fig. 3 shows the correlations between the estimated parameters. Correlations are estimated directly from the covariance matrix of the estimated unknown parameters.

Observing Fig. 3, we imply that there are strong correlations (>0.8) between some parameters. The largest correlations are found between:

- the parameter c and the translations t_x, t_y
- the $\dot{c}$ and the dynamic translations $\dot{t}_x, \dot{t}_y$
- the $\dot{d}$ and the dynamic translation $\dot{t}_y$

These correlations might be caused due to the relatively small network area and in top of that, the origin of the reference system is almost 6 km away from the network. It is worth to mention that there is no significant cross-correlation between the spatial (referring to the coordinates) and the dynamical (referring to the velocities) parameters. We should always take into consideration the correlation between the estimated parameters. Large correlation could reveal some problems which probably distort the transformation.
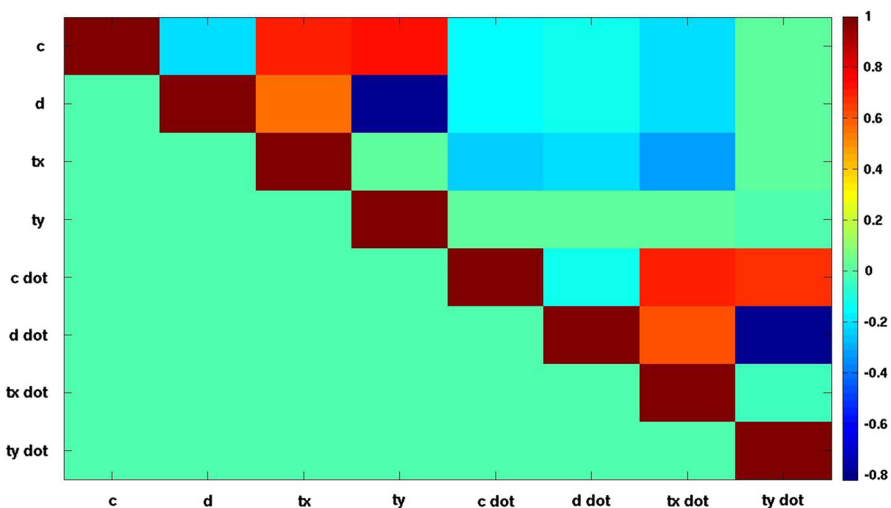

Fig. 3 The correlations between the estimated parameters

However, we must keep in mind that the 2D similarity transformation is applied to limited areas, a fact that increases the correlations.

4 Conclusions and further investigations

At the present study, we discussed a new approach on the optimal 2D similarity transformation, using coordinates and velocities. We provide all the necessary formulas and we test them through a simulation network. The new methodology could be useful in various geodetic and surveying applications, like:

1. Connecting two different dynamic networks established for geodetic monitoring (e.g. fault or dam monitoring or even for an industrial application). This could be useful, because for the aforementioned cases, the velocities play a crucial role for the safety of the people and the constructions.
2. Assimilating the information from a tectonic plate model, with results derived from geodetic measurements. This could lead to the improvement of local deformation models through the use of global solutions. In addition, it can be considered as a tool on the assessment of the tectonic plate models by the use of local precise geodetic networks.
3. In cases where the height information is rather problematic and it is possible to distort the results. Our methodology does not imply any height handling and thus it can be applied independently. Furthermore, the precise height estimation needs for the most of the cases geometric levelling measurements which increase the time and the cost of our work.
4. For the comparison of the results between the 3D and 2D time dependent transformation models regarding the horizontal velocities. E.g. how the height related information affects on the horizontal plane deformation estimation. This can be done by applying independently the 3D and 2D time-dependent transformation, respectively. Then one can compare the horizontal coordinate residuals after the implementation of the two different transformations mentioned before.
5. In cases where it is necessary to investigate if there is consistency between different types of observations from which the horizontal deformations are determined. E.g. if GNSS scale (and its rate) is consistent to the total station derived one, respectively. This can be important, since the older classical networks define their scale using terrestrial distance measurements.

The new methodology was tested through a simulation network. The simulated network was designed as a precise network for deformation monitoring. We followed rigorous statistical tests in order to reject possible blunders. Applying the 2D similarity time-dependent transformation, we conclude that it provides the necessary information for the reliable spatial and dynamic connection between two reference systems. The standard deviation of the coordinate residuals and velocities lay at 1.7 mm and 1.0 mm/yr, respectively.

Of course, the new methodology should be implemented with real observations and reference systems in order to clarify and further investigate the results, especially the correlations of the estimated parameters. We should again note that the 2D similarity transformation offers reliable results only in limited areas. The results of the proposed

approach should be definitely compared with those of the other strategies that are already applied for the deformation studies.

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