# Volume and cost minimization
# of a displacement-constrained tubular truss



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*ABSTRACT*: A simply supported planar truss with N-type bracing is optimized for minimum volume and cost. The lower chord of the truss is horizontal, but the symmetric upper chord parts are non-parallel and their inclination angle as well as the cross-sectional area of CHS (circular hollow section) rods are optimized. For the calculation of required cross-sectional area of compression struts closed formulae are used as a good approximation of Eurocode 3 buckling curve. A special method is developed for the minimum volume design considering the deflection constraint. In the case of a strong displacement constraint the cross-sectional areas required for the allowed deflection are larger than those required for stress and buckling constraints. The cost function includes the cost of material, cutting and grinding of CHS strut ends, assembly, welding and painting. Special mathematical methods are used to find the optima in the case of a numerical problem.

*Keywords:* tubular truss, structural optimization, overall buckling, displacement constraint, minimum cost design

## 1. INTRODUCTION

The aim of the present study is to solve the optimum design problem of a truss subject to a strong displacement constraint. In the case of stress constraints the tension rods are designed for yield stress by using a safety factor for loading and the compression rods are designed for overall buckling. In the case of a strong displacement constraint the required cross-sectional areas are larger than those required for stress constraints.

In the optimum design process of a truss the optimal value of the cross-sectional areas of struts and the geometric characteristics of the truss are sought which minimize an objective function and fulfil the design and fabrication constraints. The objective function can be the volume (weight) or cost, the design constraints are the limitation of stress and displacement, the fabrication constraints ease the manufacture (welding) process.

In the case of an active displacement constraint a special method is developed to calculate the required cross-sectional areas and the truss geometry.

It is shown that the non-parallel chords are more economic than the beam with parallel chords. Thus, in our case the angle of the upper chord (unknowns $h_9$ and $h_{13}$ in Figure 1) is optimized.

Another problem is the grouping of rods having the same cross-sectional area. The design of all the rods having different cross-sectional areas can cause difficulties in fabrication, but the design of all the

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rods with the same cross-sectional area would be uneconomic. Thus, the economy depends on grouping of rods. In our case four groups are used.

For the minimization of structural volume or cost the cross-sectional areas of rods is needed. The cross-sectional area of compression rods cannot be calculated from the Eurocode 3 buckling formulae. Therefore approximate formulae of Japan Road Association are used. Stress and buckling constraints are calculated using factored forces, whilst the deflection is calculated with forces without a safety factor.

To obtain comparable optima the required cross-sectional areas are not rounded to available profiles and the most economic $\delta = D/t = 50$ slenderness of CHS is used.

The limitation of the angle between CHS struts (minimum 30°) is taken into account as a fabrication constraint. Another fabrication constraint is that the diameters of the chords should be larger than those of verticals and diagonals of the bracing.

The effect of self mass in this comparative study is neglected.

## 2. SURVEY OF SELECTED LITERATURE

In order to illustrate the literature of the optimum design of trusses, the characteristics of some articles are summarized in Table 1.

Table 1. Literature survey of selected journal articles about the optimization of trusses.

Abbreviations: AISC American Institute of Steel Construction, CHS Circular Hollow Section, AASHTO American Assoc. of State and Highway Transportation Officials, EC3 Eurocode 3 [11], W – American wide flange beam, PSO particle swarm optimizer, ACO ant colony strategy, HS harmony search, MINLP mixed-integer nonlinear programming, alum – aluminium.

| Author(s) | Examples | Math. Method | Mate-rial | Buckling calculation | Cross-section | Constraint |
|---|---|---|---|---|---|---|
| Gil 2001 [1] | non-parallel chords | conjugates gradient | Steel | EC3 | | stress and geometrical |
| Tong 2001 [2] | 10-,25-bar | combina-torial | alum. | | | stress and fundamental frequency |
| Makris 2002 [3] | 3-,10-,25-,60-and 132-bar | strain-energy-density | alum. | no buckling | | Displacement |
| Hasancebi 2002 [4] | 224-bar 3D pyramid, simply supported | simulated annealing | steel | AISC | CHS, W-section | layout optimization |
| Kripakaran 2007 [5] | 10-,18-,21-bar | new algorithm | steel, alum. | AASHTO, Euler | CHS | minimum cost |
| Lamberti 2008 [6] | 18-bar cantilever, 25-bar 3D, 45-,72- and 200-bar | simulated annealing | steel, alum. | Euler | | stress, nodal displacement |
| Silih 2008 [7] | non-parallel chords | MINLP | steel | EC3 | CHS | minimum mass or cost |
| Kaveh 2009 [8] | 10-,25-,120-,200-, and 244-bar trans-mission tower | PSO, ACO,HS | steel, alum. | AISC | | stress, nodal displacements |
| Jármai 2004 [9] | Simply supported, parallel chords, 5, 8 spacing | Leap-frog, dynamic-Q | steel | EC3 | CHS | optimum height, effect of loads, min. volume |

Remarks. (1) In trusses the compression members should be designed against overall buckling. The use of Euler-formula gives unsafe design, since it does not take into account the effect of initial imperfections and residual stresses. Therefore buckling formulae of Eurocode 3 or another up-to-date improved buckling formulae should be used.

(2) The type of the investigated cross section should be given, since it has been shown [10] that the cross-sectional form affects the optima significantly.

## 3. MINIMUM VOLUME DESIGN OF THE TUBULAR TRUSS WITH NON-PARALLEL CHORDS

Relatively simple formulae can be derived for trusses to minimize the structural volume and fulfil a displacement constraint.

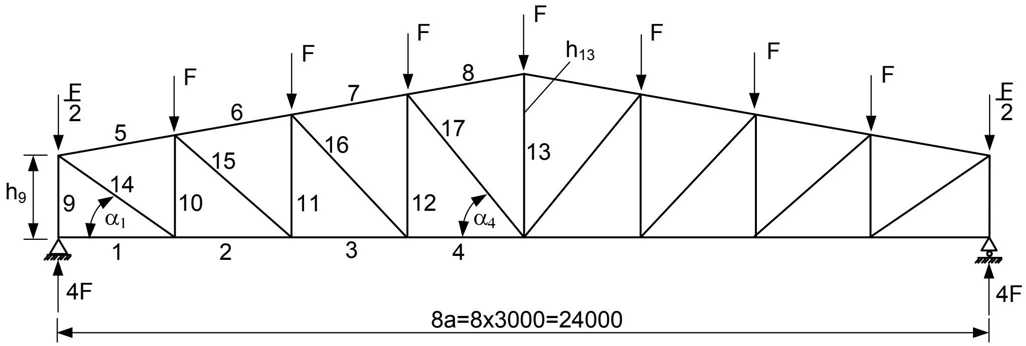


Figure 1. The simply supported truss with non-parallel chords.

The truss rods are divided into $n$-groups having the same cross-sectional areas, so

$$A_i = \mu_i A \qquad i = 1 \ldots n \tag{1}$$

and the displacement constraint is given by

$$w = \frac{1}{EA}\sum_i \frac{S_i s_i L_i}{\mu_i} \le w_0 \tag{2}$$

where $E$ is the elastic modulus, $S_i$ is the rod force, $s_i$ is the rod force from the unit force acting at the midspan, $L_i$ is the rod length, $w_o$ is the admissible deflection.

From equation (2) one obtains

$$A \ge \frac{1}{Ew_0}\sum_i \frac{S_i s_i L_i}{\mu_i} \tag{3}$$

The structural volume is calculated as

$$V = \sum_i A_i L_i = \frac{1}{Ew_0}\sum_i \mu_i L_i \sum_i \frac{S_i s_i L_i}{\mu_i} = \frac{v_1 v_2}{Ew_0} \tag{4}$$

where

$$v_1 = \sum_i \mu_i L_i, v_2 = \sum_i \frac{S_i s_i L_i}{\mu_i} \tag{5}$$

In the minimum volume design the truss geometry is sought, which minimizes

$$V_1 = v_1 v_2 \tag{6}$$

In the case of the simply supported truss shown in Figure 1 the spacing is constant, the non-parallel upper chord is determined by variable heights $h_9$ and $h_{13}$. The truss is subject to a set of vertical static forces $F$ acting on the upper nodes. The displacement of the central lower node is prescribed. It is supposed that all the truss nodes are restrained against transverse deformation.

The variables to be optimized are the heights $h_9$ and $h_{13}$ as well as the cross sectional areas of rods ($A$ and $\mu_i$).

The calculations show that, in the case of a strong displacement constraint the necessary rod cross-sectional areas are so large that the stress constraints on tension and overall buckling are fulfilled. In spite of this fact these constraints should be checked.

To facilitate the welding of nodes for tubular trusses a geometric fabrication constraint should be considered that the minimal angle between rods should be $30^0$, in our case (Figure 1)

$$\tan\alpha_1 = \frac{h_9}{a} \geq \tan 30° \tag{7}$$

from which

$$h_9 \geq a \tan 30° = 1732 \text{ mm} \tag{8}$$

and

$$\tan\alpha_4 \leq 60° = \frac{\pi}{3} \tag{9}$$

These constraints are in our case always active.

The rod forces and lengths ($S_i$, $s_i$, $L_i$) are expressed in function of $h_9$ and the inclination angle of the upper chords α.

$$\tan\alpha = \frac{h_{13} - h_9}{4a}, \cos\alpha = \frac{1}{\sqrt{(\tan\alpha)^2 + 1}}, \sin\alpha = \sqrt{1 - (\cos\alpha)^2} \tag{10}$$

The formulae for $S_i$, $s_i$ and $L_i$ are given in Tables 2, 3, 4 and 5.

Table 2. Characteristics of rods in the lower chord.

| $i$ | $S_i$ | $s_i$ | $L_i$ |
|---|---|---|---|
| 1 | 0 | 0 | $a$ |
| 2 | $3.5Fa/h_{10}$ | $0.5a/h_{10}$ | $a$ |
| 3 | $6Fa/h_{11}$ | $a/h_{11}$ | $a$ |
| 4 | $7.5Fa/h_{12}$ | $1.5a/h_{12}$ | $a$ |

Table 3. Characteristics of rods in the upper chord.

| $i$ | $S_i$ | $s_i$ | $L_i$ |
|---|---|---|---|
| 5 | $\frac{3.5Fa}{h_{10}\cos\alpha}$ | $\frac{0.5a}{h_{10}\cos\alpha}$ | $\frac{a}{\cos\alpha}$ |
| 6 | $\frac{6Fa}{h_{11}\cos\alpha}$ | $\frac{a}{h_{11}\cos\alpha}$ | $\frac{a}{\cos\alpha}$ |
| 7 | $\frac{7.5Fa}{h_{12}\cos\alpha}$ | $\frac{1.5a}{h_{12}\cos\alpha}$ | $\frac{a}{\cos\alpha}$ |
| 8 | $\frac{8Fa}{h_{13}\cos\alpha}$ | $\frac{2a}{h_{13}\cos\alpha}$ | $\frac{a}{\cos\alpha}$ |

Table 4. Characteristics of verticals.

| $i$ | $S_i$ | $s_i$ | $L_i$ |
|---|---|---|---|
| 9 | $4F$ | 0.5 | $h_9$ |
| 10 | $-3.5F+S_5\sin\alpha$ | $-0.5+s_5\sin\alpha$ | $h_{10}=h_9+a\tan\alpha$ |
| 11 | $-2.5F+S_6\sin\alpha$ | $-0.5+s_6\sin\alpha$ | $h_{11}=h_9+2a\tan\alpha$ |
| 12 | $-1.5F+S_7\sin\alpha$ | $-0.5+s_7\sin\alpha$ | $h_{12}=h_9+3a\tan\alpha$ |
| 13 | $-F+2S_8\sin\alpha$ | $2s_8\sin\alpha$ | $h_{13}$ |

Table 5. Characteristics of diagonals.

| $i$ | $S_i$ | $s_i$ | $L_i$ |
|---|---|---|---|
| 14 | $S_5L_{14}\cos\alpha/a$ | $s_5L_{14}\cos\alpha/a$ | $\sqrt{h_9^2+a^2}$ |
| 15 | $(2.5F - S_6\sin\alpha)L_{15}/h_{10}$ | $(0.5 - s_6\sin\alpha)L_{15}/h_{10}$ | $\sqrt{h_{10}^2+a^2}$ |
| 16 | $(1.5F - S_7\sin\alpha)L_{16}/h_{11}$ | $(0.5 - s_7\sin\alpha)L_{16}/h_{11}$ | $\sqrt{h_{11}^2+a^2}$ |
| 17 | $(0.5F - S_8\sin\alpha)L_{17}/h_{12}$ | $(0.5 - s_8\sin\alpha)L_{17}/h_{12}$ | $\sqrt{h_{12}^2+a^2}$ |

The rods are divided to four groups having the same cross-section): lower chord (1, 2, 3, 4), upper chord (5, 6, 7, 8), verticals (9, 10, 11, 12, 13) and diagonals (14, 15, 16, 17).

In order to facilitate the fabrication, the lower and upper chords have the same cross-section ($\mu_1 = \mu_2 = 1$) and the optimal values of $\mu_3$ (multiplier for verticals) and $\mu_4$ (multiplier for diagonals) are sought, which should be smaller than $\mu_1$.

The components of $V_1 = v_1 v_2$ to be minimized are as follows.

$$v_1 = 8(a + L_7) + 2\mu_3\sum_{i=9}^{12} h_i + \mu_3 h_{13} + 2\mu_4\sum_{i=14}^{17} L_i \tag{11}$$

$$v_2 = 2a\sum_{i=2}^{4} S_i s_i + 2L_7\sum_{i=5}^{8} S_i s_i + \frac{2}{\mu_3}\sum_{i=9}^{12} S_i s_i h_i + \frac{S_{13}s_{13}h_{13}}{\mu_3} + \frac{2}{\mu_4}\sum_{i=14}^{17} S_i s_i L_i \tag{12}$$

With the optimum values of $h_9$, $h_{13}$, $\mu_3$ and $\mu_4$

$$A_1 = A_2 = \frac{v_{2opt}}{Ew_{adm}}, A_3 = \mu_{3opt}A_1, A_4 = \mu_{4opt}A_1 \tag{13}$$

The minimum structural volume is

$$V_{min} = v_1 A_1 \tag{14}$$

For a circular hollow section (CHS)

$$A = \pi D t = \pi D^2/\delta, \delta = D/t \tag{15}$$

from which

$$D = \sqrt{\frac{A\delta}{\pi}}, t = \frac{D}{\delta} \tag{16}$$

In the design we should use the maximum value of $\delta$, but it is limited to 50 [12]. In the case of available CHS profiles according to [13] $\delta$ is varied between 10-50. In order to obtain realistic optima in all cases the optimum $\delta = 50$ is used.

## 4. CHECK OF THE COMPRESSION RODS FOR OVERALL BUCKLING

For check of overall buckling the approximate formulae of the Japan Road Association (JRA) [14] can be used instead of EC3 curve (b). In this case closed formulae can be given for cross-sectional sizes.

$$N/A \le \chi f_y \tag{17}$$

$$\chi = 1 \quad \text{for} \quad 0 \le \bar{\lambda} \le 0.2 \tag{18a}$$

$$\chi = 1.109 - 0.545\bar{\lambda} \quad \text{for} \quad 0.2 \le \bar{\lambda} \le 1 \tag{18b}$$

$$\chi = \frac{1}{0.773 + \bar{\lambda}^2} \quad \text{for} \quad \bar{\lambda} \geq 1 \tag{18c}$$

Introducing the symbol

$$\vartheta = 100D/L \tag{19}$$

and using $\bar{\lambda} = c/\vartheta$ the closed formulae are as follows.

For $0.2\vartheta \leq c \leq \vartheta$

$$\vartheta = 0.24572c\left[1 + \sqrt{1 + \frac{14.93475\nu}{c^2}}\right] \tag{20a}$$

and for $\vartheta \leq c$

$$\vartheta = \left\{0.3865\nu\left[1 + \sqrt{1 + \frac{6.69424c^2}{\nu}}\right]\right\}^{1/2} \tag{20b}$$

for CHS

$$c = \frac{100K\sqrt{8}}{\lambda_E}, \nu = \frac{10^4 S}{L^2} \cdot \frac{\delta}{\pi f_y} \tag{21}$$

where the limiting value of $\delta = D/t = 50$ is used.

$$D = \frac{\vartheta L}{100} \tag{22}$$

In the case of very long struts with small compressive force, the limitation of the strut slenderness can be governing. From the limitation of

$$\lambda = K_R L/r \leq \lambda_{\max} \tag{23}$$

the required radius of gyration is

$$r \geq K_R L/\lambda_{\max} . \tag{24}$$

According to [15] $\lambda_{\max} = 180$.

$K_R$ is the strut end restraint factor, for chords $K_R = 0.9$, for verticals and diagonals $K_R = 0.75$ [16].

For the check of overall buckling the following constraint should be fulfilled for all compression rods

$$A_i \geq \frac{\pi D_i}{\delta} \tag{25}$$

where $A_i$ is the optimum cross-sectional area for displacement constraint and $D_i$ is the required diameter from overall buckling calculation.

## 5. THE COST FUNCTION

The cost function contents the cost of material, cutting and grinding of CHS strut ends, assembly, welding and painting.

The cost of material is given by

$$K_M = k_M \rho V_2 \tag{26}$$

where an average specific cost of $k_M = 1.0$ \$/kg is considered, $\rho = 7.85\times10^{-6}$ kg/mm$^3$ for steel. $V_2$ is the actual structural volume (see equation (35)).

The cost of cutting and grinding of CHS strut ends is calculated with a formula proposed by Glijnis [17]

$$K_{CG}(\$) = k_F \Theta_{CG} \frac{2.5\pi D}{(350-2t)0.3\sin\alpha} \tag{27}$$

where $k_F$ = 1.0 \$/min is the specific fabrication cost, $\Theta_{CG} = 3$ is a factor for work complexity, 350mm/min is the cutting speed, 0.3 is the efficiency factor, diameter $D$ and thickness $t$ are in mm, $\alpha$ is the inclination angle of diagonal braces.

In our case for verticals

$$K_{CG} = \Theta_{CG} 2.5\pi 9 D_3 \frac{1+\frac{1}{\cos\alpha}}{(350-2t_3)0.3} \tag{28}$$

For diagonals at the lower strut ends

$$K_{CG1} = \Theta_{CG} 2.5\pi 2 D_4 \frac{\sum_{i=1}^{4} \frac{1}{\cos\alpha_i}}{(350-2t_4)0.3} \tag{29}$$

where

$$\tan\alpha_1 = h_9/a, \tan\alpha_2 = h_{10}/a, \tan\alpha_3 = h_{11}/a, \tan\alpha_{.4} = h_{12}/a \tag{30}$$

For diagonals at the upper strut ends

$$K_{CG2} = \Theta_{CG} 2.5\pi 2 D_4 \frac{\sum_{i=1}^{4} \frac{1}{\cos\beta_i}}{(350-2t_4)0.3} \tag{31}$$

where

$$\beta_i = 90° - \alpha - \alpha_i \text{, } i = 1, 2, 3, 4 \tag{32}$$

The general formula for the welding cost is as follows [10,17,18]]

$$K_w = k_w \left( C_1 \Theta \sqrt{\kappa \rho V} + 1.3 \sum_i C_{wi} a_{wi}^n C_{pi} L_{wi} \right) \tag{33}$$

where $k_w$ [\$/min] is the welding cost factor, $C_1$ is the factor for the assembly usually taken as $C_1$ = 1 min/kg$^{0.5}$, $\Theta$ is the factor expressing the complexity of assembly, the first member calculates the time of the assembly, $\kappa$ is the number of structural parts to be assembled, $\rho V$ is the mass of the assembled structure, the second member estimates the time of welding, $C_w$ and n are the constants given for the specified welding technology and weld type.

Furthermore $C_{pi}$ is the factor for the welding position (download 1, vertical 2, overhead 3), $L_w$ is the weld length, the multiplier 1.3 takes into account the additional welding times (deslagging, chipping, changing the electrode).

In our case $k_w$ = 1.0 \$/min, $\Theta$ = 3, the cost of assembly and welding using SMAW (shielded metal arc welding) fillet welds is given by for verticals

$$K_W = k_W \left[ \Theta \sqrt{21 \rho V_2} + 1.3x0.7889x10^{-3} x 9\pi D_3 \left(1+\frac{1}{\cos\alpha}\right) t_3^2 \right] \tag{34}$$

$$V_2 = 8aA_1 + 8L_7 A_2 + 2A_3 \sum_{i=9}^{12} h_i + A_3 h_{13} + 2A_4 \sum_{i=14}^{17} L_i \tag{35}$$

For diagonals at the lower strut ends

$$K_{W1} = 1.3x0.7889x10^{-3}x2\pi D_4 t_4^2 \sum_{i=1}^{4} \frac{1}{\cos \alpha_i} \tag{36}$$

For diagonals at the upper strut ends

$$K_{W1} = 1.3x0.7889x10^{-3}x2\pi D_4 t_4^2 \sum_{i=1}^{4} \frac{1}{\cos \beta_i} \tag{37}$$

The cost of painting is calculated as

$$K_P = k_P S_P, k_P = 28.8x10^{-6} \text{ \$/mm}^2 \tag{38}$$

The superficies to be painted is

$$S_P = 8a\pi D_1 + 8L_7\pi D_2 + 2\pi D_3 \sum_{i=9}^{12} h_i + \pi D_3 h_{13} + 2\pi D_4 \sum_{i=14}^{17} L_i \tag{39}$$

The total cost is given by

$$K = K_M + K_{CG} + K_{CG1} + K_{CG2} + K_W + K_{w1} + K_{w2} + K_P \tag{40}$$

6. NUMERICAL DATA

Loads for displacement calculation (without safety factor) $F$ = 120,000 N, for stress and buckling constraints $F_0$ = 1.5, $F$ = 180,000 N (safety factor of 1.5). Yield stress of steel $f_y$ = 355 MPa, elastic modulus $E = 2.1\times10^5$ MPa, span length $L$ = 24 m, allowable displacement at the middle of the span $w_0$ = 32 mm = $L$/750.

7. THE OPTIMIZATION PROCESS

Calculate the optimum values of $h_9$, $h_{13}$, $\mu_3$ and $\mu_4$ to obtain $V_{min}$ or $K_{min}$ and fulfil the constraints on displacement, on minimum angle $\alpha_1$ [equation (8)], maximum angle $\alpha_4$ [equation (9)] as well as on stress and overall buckling.

The ranges of unknowns are as follows: $1732 < h_9 < 5000$, $4000 < h_{13} < 8000$ and $h_9 < h_{13}$, $0.5 < \mu_3 < 1$, $0.5 < \mu_4 < 1$.

In the case of minimum volume design equations (13) and (14) give the results and equations(25) should be fulfilled. In the case of minimum cost equation (40) should be minimized, for which equations (11), (12), (13), (16) and (35) should be used.

8. RESULTS OF THE OPTIMIZATION

The fabrication constraints [equation (7) and (8)] determine the optimal pair of unknowns $h_9$ and $h_{13}$ as follows: for a given $h_9$ a value of $h_{13}$ smaller than $h_{13opt}$ gives larger $v_1v_2$, larger does not fulfil the fabrication constraint equation (8). Table 6 shows the max $h_{13}$ in function of $h_9$.

Table 6. Maximum $h_{13}$ values in function of $h_9$. Values in mm.

| $h_9$ | 1750 | 1850 | 1950 | 2000 | 2100 | 2200 | 2300 |
|---|---|---|---|---|---|---|---|
| $h_{13OPT}$ | 6340 | 6310 | 6280 | 6260 | 6220 | 6190 | 6160 |

Using a MathCAD algorithm the following optima are determined: in the case of $\mu_3 = \mu_4 = 0.6$, $h_{9opt}$ = 1950, $h_{13opt}$ = 6280, $v_1v_{2min} = 2.321\times10^{15}$, $V_{min} = 3.454\times10^8$ mm$^3$, $K_{min}$ = 7825 \$, $A_1 = A_2$ = 3708, $A_3 = A_4$ = 2225 mm$^2$.

The cross-sectional areas required for stress and buckling constraints are as follows: $A_1 = A_2 = 2195$, $A_3 = 2084$, $A_4 = 2094$ mm$^2$. It can be seen that the cross-sectional areas determined for a strong displacement constraint are larger than those required for stress or buckling constraints.

For comparison the optimum data for the truss of parallel chords: $h_{9opt} = h_{13opt} = 5000$ mm, $V_{min} = 5.852\times10^8$ mm$^3$. $K_{min} = 11350$ \$. It can be seen that the truss of non-parallel chords is much more economic than the truss of parallel chords.

## 9. CONCLUSIONS

The optimization problem to be solved is the following: found the optimal geometry and cross-sectional areas of rods which minimize the structural volume or cost for a simply supported tubular truss with non-parallel chords for a strong displacement constraint.

For the solution of this problem a developed calculation method is used. Besides the displacement constraint the rods are checked for tension stress and overall buckling. It is shown that, in the case of a strong displacement constraint the cross-sectional areas are larger than those required for constraints on stress and buckling.

The fabrication (welding) constraints on minimal angle between tubular rods (30°) have been also active. In the calculation of overall buckling the Eurocode 3 formulae are approximated by formulae of Japan Road Association enabling the explicit expression of the necessary cross-sectional area.

Special formulae are used for the cost calculation. The cost function expresses the cost of material, cutting and grinding of the tubular (CHS) rod ends, assembly, welding and painting. It is shown that, in this case, the structural optima for minimum volume and minimum cost are the same.

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