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OPTIMUM DESIGN OF A TRANSMISSION LINE TOWER
Welded tubular truss structure

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INTRODUCTION

The trusses of transmission line towers are usually constructed from rods of angle profile with bolted connections, as it is used by Rao [1] and Silva et al [2]. These rods have a poor overall buckling strength. The aim of this study is to show the advantages of trusses constructed from circular hollow section (CHS) rods with welded nodes. Taniwaki and Ohkubo [3] have used CHS rods, they have considered special Japanese problems of seismic-design and cost of land as well as special mathematical methods.
Another aim is to solve the following optimization problem: determine the slope angle (sprawling) of the four main rods of the truss tower and the cross-sectional areas of rods, which minimize the structural volume or cost and fulfil the design and fabrication constraints.
Design constraints relate to the tensile stress and overall buckling strength of rods. Fabrication constraints prescribe the minimum angle between CHS rods to ease the welding of nodes.
For the numerical optimization process a tower of 45 m height is selected and the loads are determined according to the rules of the Hungarian Standard for transmission lines MSZ 151 [4,5].
The cost function contents the cost of material, cutting and grinding of the ends of CHS rods, assembly, welding and painting.
More groups of rods having the same cross-sectional area are selected. Approximate formulae are used instead of overall buckling formulae of Eurocode 3 (EC3) [6], which allow for expressing the cross-sectional area of compressed rods explicitly.
To obtain comparable optima the required cross-sectional areas are not rounded to available profiles.
Formulae are developed for the minimization of structural volume considering the displacement constraint. The rod forces are calculated using the finite element method.

1 LOADS

The tower has two main parts. The upper part solves for the fixing of conductors. The whole height of the tower is 45 m, the height of the upper part is 21 m. The present study treats the optimum design of the lower part with the height of 24 m. The loads acting from the upper part are calculated according to the MSZ 151 [4.5]. The governing load combination is as follows: in the one side of the tower the whole tension and on the other side the half of the tension of conductors and rime without wind load. The distance of towers: 400 m.

Weight of two lightning conductors: 2x712x0.4x9.81 = 5587 N. Weight of 12 electric conductors: 12x1935x0.4x9.81 = 91115 N. Weight of the upper part of the tower: approximately 40 kN.
Additional load according to the Hungarian standard MSZ 151-1:2000 [4] the weight of rime $z = 3.25 + 0.25d$, where d is the wire-diameter.
For the lightning conductors with d = 16 mm z = 7.25 N/m, for electric conductors with d = 31.05 z = 11.025 N/m. For 400 m distance it is 2900 N and 4405 N, respectively.
Vertical load from the upper part of the tower is multiplied by a safety factor of 1.1:
$$V = 1.1(91.115 + 5.5587 + 40) + 12x4.405 + 2x2.9 = 209.03kN$$
The allowable tensile stress of a 95/55 steel lightning conductor is 140 N/mm^2 and that of a 500/66 aluminium electric conductor is 85 N/mm^2. The tensile force of a lightning conductor is

$(96.5+56.3)140 = 21392N$
and that of an electric conductor
$(504.7+65.4)85 = 48458N$.
The governing load combination is the half of the tensile force of conductors. The horizontal force acting on the top of the lower part of the tower:
$H_0 = (2x21.392+12x48.4585)0.5 = 312.143$ kN
and the bending moment from the tensile forces
$M = 21.392x21.8+2x48.4585(16.4+8.2) = 2850.5$ kNm.
It is supposed that the tower is square symmetric in plane.

Vertical loads acting on the half lower part of the tower (width of the tower a_1 = 3.7 m) (*Fig. 1*):

$$F_{10} = \frac{209.03}{4} + \frac{2850.5}{2x3.7} = 437.46 \text{ kN}; \qquad F_{20} = \frac{2850.5}{2x3.7} - \frac{209.03}{4} = 332.94 \text{ kN}; \tag{1}$$

Loads acting on the inclined tower plane (*Fig. 2*)

$$F_1 = 437.46\Theta/\Omega, F_2 = 332.94\Theta/\Omega; \qquad H = \frac{H_0}{2} - \frac{437.46+332.94}{\Omega} = 156.07 - \frac{770.4}{\Omega}; \tag{2}$$

2 GEOMETRIC DATA (*Figs 1, 2*)

Factors for the transformation of loads from vertical to inclined plane:

$$\Omega = \sqrt{\theta^2 - 1}, \Theta = \sqrt{\theta^2+1}; \text{ where } \theta = \tan\beta; \qquad c = L\frac{\Theta}{\Omega}, d = \frac{L}{\Omega}, f = \theta d; \tag{3}$$

$$h_1 = 6000f/L, h_2 = 7000f/L, h_3 = 4500f/L, h_4 = 4500f/L, h_5 = 2000f/L; \tag{4}$$

$$d_i = h_i/\theta, \; i=1,\ldots,5; \; \tan\alpha_1 = \frac{h_1}{d+a_1/2}, \tan\alpha_2 = \frac{h_2}{-d_2+a_3/2}; \tag{5}$$

$$a_2 = a_1 + 2d, a_3 = a_2 - 2d_1, a_4 = a_3 - 2d_2, a_5 = a_4 - 2d_3, a_6 = a_5 - 2d_4; \tag{6}$$

$$\tan\alpha_3 = 2h_3/a_4, \tan\alpha_4 = \frac{h_4}{-d_4+a_5/2}; \qquad \tan\alpha_5 = 2h_5/a_6; \tag{7}$$

$$\gamma_1 = \beta - \alpha_1, \gamma_2 = 180 - \beta - \alpha_2; \qquad \gamma_3 = \beta - \alpha_3, \gamma_4 = 180 - \beta - \alpha_4; \qquad \gamma_5 = \beta - \alpha_5 \tag{8}$$

Rod lengths: $L_i = h_i c/f, i = 1...5$; $\qquad L_6 = L_1, L_7 = L_2, L_8 = L_3, L_9 = L_4, L_{10} = L_5$; (9)

$$L_{11} = L_{19} = \sqrt{h_1^2 + \frac{a_2^2}{4}}, L_{12} = L_{20} = \frac{a_3}{2}, L_{13} = L_{21} = \sqrt{h_2^2 + \frac{a_4^2}{4}} \tag{10}$$

$$L_{14} = L_{22} = \sqrt{h_3^2 + \frac{a_4^2}{4}}, L_{15} = L_{23} = \frac{a_5}{2}, L_{16} = L_{24} = \sqrt{h_4^2 + \frac{a_6^2}{4}} \tag{11}$$

$$L_{17} = L_{25} = \sqrt{h_5^2 + \frac{a_6^2}{4}}, L_{18} = L_{26} = \frac{a_6}{2} \tag{12}$$

3 ROD FORCES FROM A HORIZONTAL FORCE *F* = 1

$$S_{17} = F\frac{L_{17}}{a_6}, S_{25} = -S_{17}; \qquad S_4 = \frac{F(h_4+h_5)}{a_5 \sin\beta}; \tag{13}$$

$$S_{16} = \frac{S_{17}\cos\alpha_5 - S_4\cos\beta}{\cos\alpha_4}, S_{24} = -S_{16}; \qquad S_9 = -S_4; \tag{14}$$

$$S_{14} = S_{16}\frac{a_6 L_{14}}{L_{16} a_4}, S_{22} = -S_{14}; \qquad S_{13} = -S_{14}\frac{\theta\cos\alpha_3 - \sin\alpha_3}{\theta\cos\alpha_2 + \sin\alpha_2}, S_{21} = -S_{13}; \tag{15}$$

$$S_{11} = S_{13}\frac{a_4 L_{11}}{L_{13} a_2}, S_{19} = -S_{11}; \qquad S_2 = \frac{F(h_2 + h_3 + h_4 + h_5)}{a_3 \sin\beta}; \tag{16}$$

$$\sin\beta = \frac{\theta}{\sqrt{\theta^2 + 1}}, \; S_7 = -S_2; \tag{17}$$

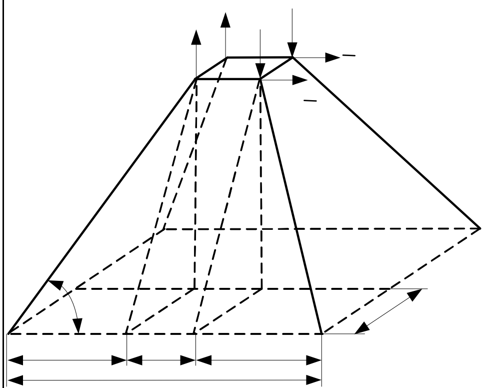

Fig. 1. Bottom part of the tower with loads from the upper part

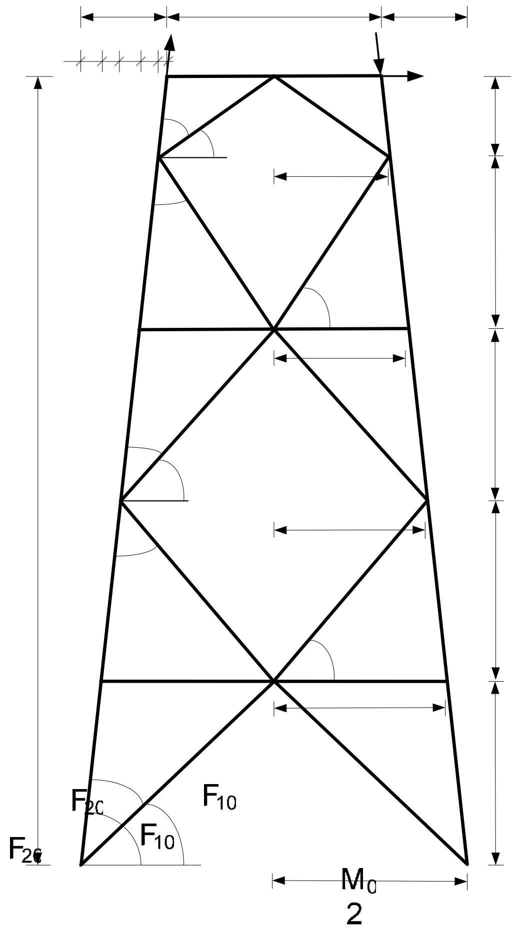

Fig. 2. The trussed inclined plan of the bottom part

4 ROD FORCES FROM H, F_1 AND F_2

$$H = 156070 - \frac{770400}{\Omega}, \; F_1 = 437460\frac{\Theta}{\Omega}; \qquad F_2 = 332940\frac{\Theta}{\Omega}; \tag{18}$$

$N_2 = HS_2 + F_2$, $N_4 = HS_4 + F_2$; $\quad N_7 = -HS_2 - F_1$, $N_9 = -HS_4 - F_1$ (19)

$N_{11} = HS_{11}$, $N_{13} = HS_{13}$; $\quad N_{14} = HS_{14}$, $N_{16} = -HS_{16}$; (20)

$N_{17} = HS_{17}$, $N_{19} = -N_{11}$; $\quad N_{21} = -N_{13}$, $N_{22} = -N_{14}$; (21)

$N_{24} = -N_{16}$, $N_{25} = -N_{17}$; $\quad N_{18} = 332940/\Omega, N_{26} = 156070 - 437460/\Omega$ (22)

$N_{10} = -F_1$, $N_5 = F_2$ (23)

5 OPTIMIZATION PROCESS

(1) Selection of a preliminary slope angle: $\beta_{opt} = 80^0$. (2) Determination of rod forces for 80^0.

(3) Determination of rod groups having the same cross-sectional area on the basis of rod forces. The selected rod groups are as follows:

(a) lower chords 1-2-6-7, governing rod: 7,
(b) upper chords: 3-4-5-8-9-10, governing rod: 9,
(c) braces 11-13-14-16-19-21-22-24-18-26 governing rod: 11,
(d) upper braces 17-25 governing rod: 17.

6 DESIGN OF CROSS-SECTIONAL AREAS OF GOVERNING RODS

For check of overall buckling the approximate formulae of the Japan Road Association (JRA) [7] can be used instead of Eurocode 3 [6] curve (b). In this case closed formulae can be given for cross-sectional sizes.

$$N / A \le \chi f_y \tag{24}$$

$$\chi = 1 \qquad \text{for } 0 \le \bar{\lambda} \le 0.2; \tag{25}$$

$$\chi = 1.109 - 0.545\,\bar{\lambda} \qquad \text{for } 0.2 \le \bar{\lambda} \le 1; \tag{26}$$

$$\chi = \frac{1}{0.773 + \bar{\lambda}^2} \qquad \text{for } \bar{\lambda} \ge 1. \tag{27}$$

Introducing the symbol

$$\vartheta = 100 D / L; \tag{28}$$

and using $\bar{\lambda} = c / \vartheta$ the closed formulae are as follows.
For $0.2\vartheta \le c \le \vartheta$

$$\vartheta = 0.24572\, c \left[1 + \sqrt{1 + \frac{14.93475 \nu}{c^2}} \right]; \tag{29}$$

and for $\vartheta \le c$

$$\vartheta = \left\{ 0.3865 \nu \left[1 + \sqrt{1 + \frac{6.69424 c^2}{\nu}} \right] \right\}^{1/2}; \tag{30}$$

for circular hollow sections (CHS)

$$c = \frac{100 K \sqrt{8}}{\lambda_E}, \nu = \frac{10^4 S}{L^2} \cdot \frac{\delta}{\pi f_y}; \tag{31}$$

where the variable value of $\delta = D/t = 15\text{--}40$ is used, taking into account that for transmission towers the minimum section thickness is 4 mm.

$$D = \frac{\vartheta L}{100}, A = \frac{\pi D^2}{\delta}. \tag{32}$$

In the case of very long struts with small compressive force, the limitation of the strut slenderness can be governing. From the limitation of λ the required radius of gyration is

$$\lambda = KL / r \le \lambda_{\max}; \; r \ge KL / \lambda_{\max} \tag{33}$$

According to BS 5400 (1982)[8] $\lambda_{\max} = 180$.
K is the strut end restraint factor, for chords $K = 0.9$, for verticals and diagonals $K = 0.75$ [9].

7 FORMULAE FOR VOLUME *V* AND COST *K* OF THE TRUSS IN THE FUNCTION OF *B*

$$V = 2A_7\left(L_6 + L_7\right) + 2A_9\left(2L_9 + L_5\right) + V_1 \quad V_1 = 2A_{11}\left(L_{11} + L_{21} + L_{14} + L_{16} + L_{18}\right) + 2A_{17}L_{17}. \tag{34}$$

The cost function contents the cost of material, cutting and grinding of CHS strut ends, assembly, welding and painting.
The cost of material is given by

$$K_M = k_M \rho V; \tag{35}$$

where an average specific cost of k_M = 1.0 \$/kg is considered, ρ = 7.85x10^{-6} kg/mm^3 for steel.
The cost of cutting and grinding of CHS strut ends is calculated with a formula proposed by Glijnis [6]

$$K_{CG}(\$) = k_F \Theta_{CG} \frac{2.5\pi D}{(350-2t)0.3\sin\alpha}; \tag{36}$$

where k_F = 1.0 \$/min is the specific fabrication cost, $\Theta_{CG} = 3$ is a factor for work complexity, 350 mm/min is the cutting speed, 0.3 is the efficiency factor, diameter D and thickness t are in mm, α is the inclination angle of diagonal braces.
In our case

$$K_{CG} = \frac{5k_F\Theta_{CG}\pi}{0.3}(G_1+G_2+G_3); \qquad G_1 = \frac{D_{11}}{350-t_{11}}\left[\sum_{i=1}^{4}\left(\frac{1}{\sin\alpha_i}+\frac{1}{\sin\gamma_i}\right)\right]; \tag{37}$$

$$G_2 = \frac{D_{11}}{350-t_{11}}\frac{1}{\sin\beta}; \qquad G_3 = \frac{D_{17}}{350-t_{17}}\left(\frac{1}{\sin\alpha_5}+\frac{1}{\sin\gamma_5}\right); \tag{38}$$

The general formula for the welding cost is as follows [10,11,12]

$$K_w = k_F\left(C_1\Theta\sqrt{\kappa\rho V} + 1.3\sum_i C_{wi}a_{wi}^n C_{pi}L_{wi}\right). \tag{39}$$

where C_1 is the factor for the assembly usually taken as C_1 = 1 min/kg$^{0.5}$, Θ is the factor expressing the complexity of assembly, the first member calculates the time of the assembly, κ is the number of structural parts to be assembled, ρV is the mass of the assembled structure, the second member estimates the time of welding, C_w and n are the constants given for the specified welding technology and weld type.
Furthermore C_{pi} is the factor for the welding position (download 1, vertical 2, overhead 3), L_w is the weld length, the multiplier 1.3 takes into account the additional welding times (deslagging, chipping, changing the electrode).
In our case k_F = 1.0 \$/min, Θ = 3,

$$K_W = k_F\left[\Theta\sqrt{\kappa\rho V} + 1.3x0.7889x10^{-3}(T_1+T_2+T_3)\right]; \tag{40}$$

$$T_1 = 2D_{11}t_{11}^2\left[\sum_{i=1}^{4}\left(\frac{1}{\sin\alpha_i}+\frac{1}{\sin\gamma_i}\right)\right]; \quad T_2 = \frac{2D_{11}t_{11}^2}{\sin\beta}; \tag{41}$$

$$T_3 = 2D_{17}t_{17}^2\left(\frac{1}{\sin\alpha_5}+\frac{1}{\sin\gamma_5}\right); \quad \kappa = 15. \tag{42}$$

The cost of painting is calculated as

$$K_P = k_P S_P, k_P = 28.8x10^{-6} \text{ \$/mm}^2. \tag{43}$$

The superficies to be painted is

$$S_p = S_{p1} + S_{p2} + S_{p3}; \tag{44}$$

$$S_{p1} = 2\pi D_7(L_1+L_2) + 2\pi D_9(L_3+L_4+L_5); \tag{45}$$

$$S_{p2} = 2\pi D_{11}(L_{11}+L_{13}+L_{14}+L_{16}+L_{18}) \tag{46}$$

$$S_{p3} = 2\pi D_{17}L_{17} \tag{47}$$

8 SEARCH FOR B_{OPT} FOR V_{MIN} AND K_{MIN}

The search is performed by using a MathCAD algorithm. The results are given is Table 1.
It can be seen that the constraint for the angles between the rods is active for angle γ_3, thus the optimum truss angle is β = 80^0. Disregarding the angle constraint the optimum for minimum volume would be 82^0 and for minimum cost 84^0.

Table 1. Optimum truss angle for minimum volume and cost

β^0	$\gamma_3^{\,0}$	$10^{-8}V$ [mm^3]	K [\$]
79	30.3	1.902	3952
80	**29.9**	**1.874**	**3903**
81	29.4	1.854	3855
82	28.9	*1.844*	3820
83	28.4	1.845	3800
84	*27.6*	*1.857*	*3796*
85	26.9	1.883	3812

9 SELECTION OF AVAILABLE PROFILES AND A MASS COMPARISON

The available profiles: rod 7: 177.8/6, rod 9: 168.8/4.5, rod 11: 114.3/5, rod 17: 39.7/4. The optimum mass of the whole tower is 9351 kg. Rao [1] has optimized a 400 kV tower of high 44.3 m with lightning conductors of diameter 11 mm and electric conductors of diameter 31.77 mm, the ground clearance a_2 = 8.84 m, rods of L-shaped angles with a bolted type construction. The total mass was 11400 kg. Since the tower of Rao is very similar to the present tower, the comparison is realistic. It can be concluded that using CHS profiles instead of angles and optimizing the clearance a saving in mass of (11400-9351)/11400x100 = 18% can be achieved.

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