Reliability-Based Optimum Design of a Square Box Column Constructed from Cellular Plates

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Abstract. Cellular plates can be calculated as isotropic ones, bending moments and deflections being determined by using the classic results for various loads and support types. A cantilever stub column of a square box section composed of welded cellular plates is optimized. The column is subject to compression and bending and is constructed from four equal cellular side plates. The constraints on overall buckling are formulated according to the Det Norske Veritas design rules. The horizontal displacement of the column top is limited. The cost function to be minimized includes the costs of the materials, assembly, welding and painting.

Randomness is considered both in loading and material properties. A level II reliability method (FORM) is employed.

Keywords: cellular plates, reliability, optimization, box column.

1 Introduction

Box beams and columns of large load-carrying capacity are widely applied in bridges, buildings, highway piers and pylons. Since the thickness required for an unstiffened box column can be too large, stiffened plate elements or cellular plates should be used. The strength is considerably larger than that of a plate stiffened on one side by open section ribs because of the larger torsional stiffness of the cellular plate (Farkas and Jármai 2007). The stiffening presented here consists of rectangular hollow sections (RHS) applied as an orthogonal grid.

In this work a maximum probability of failure is stipulated for design and the reliability is evaluated by using a level II procedure FOSM (first order second order reliability method) (Hasofer and Lind 1974), the sensitivity information being obtained analytically. The overall probability of failure which account for the interaction by correlating the modes of failure is considered.

A branch and bound strategy coupled with a entropy-based algorithm is used to solve the reliability-based optimization. The entropy-based procedure is employed to find optimum continuous design variables giving lower bounds on the decision tree and the discrete solutions are found by implicit enumeration.

2 Characteristics of Cellular Plates with RHS Stiffeners

2.1 Design Variables

The optimal sizes and number of RHS stiffeners in both directions as well as the deck plate thicknesses and the width of the box column sections are sought The unknowns were the dimensions of the column, width, thicknesses, number of stiffeners: b_0 column width, t_h plate thickness, b_x,b_y are stiffener heights in x and y directions, c_x, c_y stiffener widths in x and y directions t_b stiffener thickness, n_x and n_y number of stiffeners in x and y direction. It is stipulated symmetry on RHS, $t_{bx}=t_{by}$, $c_x=c_y$ and $b_x=b_y$. The number of independent design variables is 7. For the sake of practical design all the variables are discrete. It is necessary to stipulate lower limits for c_x, c_y, t_{bx},t_{by} and t_h

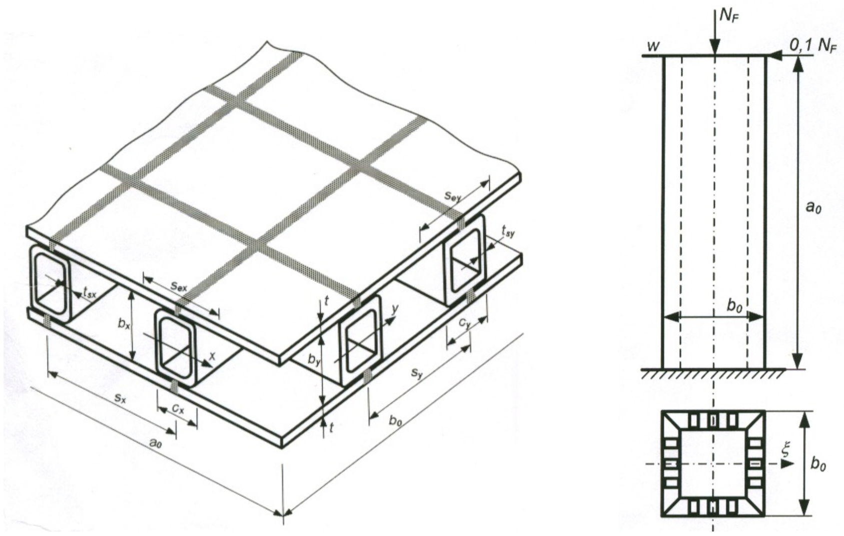

Fig. 1. Cellular plate with RHS stiffeners and cantilever column of square box cross section. The walls are constructed from cellular plates with RHS stiffeners.

2.2 Geometric Characteristics, Bending Moments and Deflections

The bending and torsional stiffness used in the Huber equation are:

$$B_x = \frac{E_1 I_y}{s_y} \quad ; \quad B_y = \frac{E_1 I_x}{s_x} \quad ; \quad E_1 = \frac{E}{1-\upsilon^2} \quad ; \quad H = \frac{E_1}{2}\left(\frac{I_y}{s_y} + \frac{I_x}{s_x}\right) \tag{1}$$

Effective plate width,

$$s_{ey} = C_y s_y \quad , \quad s_{ex} = C_x s_x \tag{2}$$

where,

$$s_y = b_0/n_y \quad , \quad s_x = a_0 / n_x \tag{3}$$

Cross sectional area of a stiffener with upper and bottom base plate parts,

$$C_y = 1 \quad if \quad \lambda_y = \frac{s_y}{56.84t\varepsilon} < 0.673 \;;\; C_y = \frac{\lambda_y - 0.22}{\lambda_y^2} \quad if \quad \lambda_y \geq 0.673 \tag{4}$$

Effective cross sectional areas,

$$A_{ey} = A_{RHSy} + 2s_{ey}t \quad , \quad A_{ex} = A_{RHSx} + 2s_{ex}t \tag{5}$$

Moment of inertia,

$$I_y = I_{RHSy} + 2s_{ey}t\left(\frac{b_y + t}{2}\right)^2 \quad , \quad I_x = I_{RHSx} + 2s_{ex}t\left(\frac{b_x + t}{2}\right)^2 \tag{6}$$

2.3 Constraints

The buckling constraints are formulated according to the Det Norske Veritas (1995):

$$\sigma = \frac{N_F}{4A_{ey}(n_y - 1)} + \frac{0.1N_F a_0}{W_\xi} \leq \sigma_{cr} = \frac{f_{y1}}{\sqrt{1+\lambda^4}} \tag{7}$$

$$\sigma_E = \frac{N_E s_y}{A_{ey}}; \quad \lambda = \sqrt{\frac{f_{y1}}{\sigma_E}} \quad ; W_\xi = \frac{2I_\xi}{b_0} \tag{8}$$

$$I_\xi = 2I_y\left(n_y - 1\right) + 2\left(n_y - 1\right)A_{ey}\left(\frac{b_0}{2}\right)^2 + 2I_{\xi S} \tag{9}$$

where the moment of inertia of RHS stiffeners is given by,

$$I_{\xi S} = I_{RHSz}\left(n_y - 1\right) + 2A_{ey}s_y^2 n_y \frac{n_y^2 - 1}{24} \tag{10}$$

Deflection constraint

$$w_{\max} = H_F \frac{L^3}{3EI_\xi} \leq w_{allow} = \frac{L}{\phi} \quad , \quad \phi = 500 - 1000 \tag{11}$$

Limitation of the distance between stiffener flanges must be imposed to allow the welding of the stiffener web to the upper base plate.

2.4 Cost Function

Here the fabrication consists of two phases:

(1) fabrication of four cellular plates: (a) welding of the grid of RHS stiffeners (b) welding of the deck plate elements to the grid (c) welding of the base plate elements to the grid, except the two outermost plate strips to make it possible to weld the transverse stiffeners to the corner diagonal plates.

(2) fabrication of the whole square box column from four cellular plates: (a) welding of the deck plates and the transverse stiffeners to the four corner diagonal plates (b) welding of the outermost base plate strips to the corner plates.

The cost functions are formulated according to these fabrication phases. For each phase the number of assembled elements, the volume of the assembled structure, the characteristics of used welds (size, type, welding methods and weld length) should be determined as shown in (12)

$$K_w = k_w \left[C_1 \Theta \sqrt{\kappa \rho V} + 1.3 \sum_i C_{wi} a w i^n C_p L_{wi} \right] \tag{12}$$

where k_w is the welding cost factor C_1 is the factor for the assembly usually taken as C_1=1min/kg$^{0.5}$, Θ is the factor expressing the complexity of the assembly the first member calculates the time of assembly, κ is the number of structural parts to be assembled ρV is the mass of the assembled structure, the second member estimates the time of welding Cw and n are the constants given for the specific welding technology and weld type. C_{pi} is the factor for welding position (down 1, vertical 2, overhead 3) L_w is the weld length, the multiplier 1.3 takes into account the additional welding times (deslagging, chipping, changing the electrode). More detailed description can be found in (Jármai and Farkas 2010).

3 Reliability-Based Optimization

The following assumptions are considered: (1) the general configuration including the length of all members is specified in a fixed (deterministic) manner; (2) the failure modes are overall buckling, local buckling and maximum deflections; (3) the magnitudes of the static loads that form the load vector are random, but their locations deterministic; (4) the allowable stresses and displacements are random, but their position is deterministic.

The reliability index has the geometrical interpretation as the smallest distance from the line (or the hyperplane) forming the boundary between the safe domain and the failure domain. The evaluation of the probability of failure reduces to simple evaluations in terms of mean values and standard deviations of the basic random variables. Ditlevsen´s method (Ditlevsen 1979) which incorporates the effects of statistical dependence between any two failure modes, narrows considerably the bounds on the system failure probability and is used here.

4 Optimization Strategy

4.1 Branch and Bound

The problem is non-linear and the design variables are discrete. Given the small number of discrete design variables an implicit branch and bound strategy was adopted to find the least cost solution. The two main ingredients are a combinatorial tree with appropriately defined nodes and some upper and lower bounds to the optimum solution associated the nodes of the tree. It is then possible to eliminate a large number of potential solutions without evaluating them. Any leaf of the tree whose bound is strictly less than the incumbent is active. Otherwise it is designated as terminated and need not to be considered further. The B&B tree is developed until every leaf is terminated. The branching strategy adopted was breadth first, consisting of choosing the node with the lower bound (Simões 1987).

4.2 Optimum Design with Continuous Design Variables

For solving each relaxed problem with continuous design variables the simultaneous minimization of the cost and constraints is sought. All these goals are cast in a normalized form. If a reference cost is specified, this goal should be improved in the following iteration unless other criteria become dominant.

Another goal arise from the reliability constraint on overall buckling for the square box column,

The third goal deals with the reliability constraint arising from the horizontal displacement at the top column to be exceeded,

The remaining goals deal with the local buckling in the square box column which can be solved by deterministic means and the probability that the shear stress at the corners exceeds the allowable values.

The objective of this Pareto optimization is to obtain an unbiased improvement of the current design, which can be found by the unconstrained minimization of the convex scalar function (Simões and Templeman 1989).

$$F(t,h) = \frac{1}{\rho}.1n\left[\sum_{j=l}^{3} \exp \rho\big(g(t,h)\big)\right] \tag{13}$$

This form leads to a convex conservative approximation of the objective and constraint boundaries. Accuracy increases with ρ.

5 Numerical Results and Discussion

a_0=15000mm, density $\rho=7.85\text{x}10^{-6}$kg/mm^3, Poisson ratio $\nu=0.3$. Yield stress of steel represented by a Gaussian distribution with mean stress 440 MPa the coefficient of variation being 0.10. Gaussian distribution was also adopted for the design axial load of N_x=5.607 10^7 N and a coefficient of variation of 0.15. The specified minimum probability of failure is 10^{-5}. The randomness of the Young modulus, Poisson ratio and a_0 were not considered for the sake of simplicity. The number of constraints is 11. They include upper and lower size limits of the unknowns such as minimum and maximum thickness. The design variables in the algorithm were considered discrete except b_0 which. n_y determines a maximum c_x,c_y to avoid overlapping. The total costs and design variables are summarized in Table 1.

The general conclusion is that the solution is almost independent on c_x; an increase in t_{bx} reduces the stress, but increases cost; b_x increasing leads to higher critical stress allowing for more feasible solutions. n_y increasing lead to a smaller stress and larger critical stress. Some of the variables are at their lower limits.

Table 1. Optimum solutions

ϕ	t_h	n_y	n_x	b_x	c_x	t_{bx}	b_o	Cost
300	8	13	2	140	10	2	3274	56307
1000	6	16	2	145	10	2	4660	61813

6 Conclusions

Design fabrication and economy are the three components of an optimum design. If we consider the analytical aspects of the design, the effect of different welding and other technologies on the cost of the structure than one can reach a minimum cost solution using efficient optimization techniques. A stiffened column with cellular structure is shown. When stiffened cellular columns with flat stiffeners or half I-beam (Simões et al 2009) are compared with the hollow type stiffeners, the best construction is the later.

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