OPTIMAL DESIGN OF SHELL-AND-TUBE HEAT EXCHANGERS

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1. INTRODUCTION

The heat exchanger is a heat transfer device that exchanges heat between two or more process fluids. These devices used in the chemical and energy industry and also in the households. Heat exchangers have lots of types, such as double pipe, shell-and-tube or plate heat exchangers, air coolers, graphite block heat exchanger for example. We can group them according to the structural material: steel, stainless steel, copper, aluminium, graphite, rarer titanium, zirconium or nickel alloys. According to the flow arrangement we class parallel flow, counter-flow or cross-flow. As an engineer, our objective is choosing the construction that able to transfer the necessary heat and has the lowest cost all of.

2. DRIVING FORCES

2.1. FUNDAMENTAL EQUATION OF SURFACE HEAT EXCHANGERS

The heat transfer performance of a surface heat exchanger depends on three factors: the mean temperature difference, the heat transfer area and the heat transfer coefficient.

$$Q = k \cdot A \cdot \Delta T_{LOG} \tag{1}$$

where:

- Q: necessary heat [W],
- k: heat transfer coefficient [$W/m^2 \cdot °C$],
- A: heat transfer area [m^2],
- ΔT_{LOG}: mean temperature difference [°C].

2.2. MEAN TEMPERATURE DIFFERENCE

The process fluids in shell-and-tube heat exchanger are entering in the ends of the device. The driving force of heat transfer depends on the inlet and outlet temperatures. In case of counter-flow, the mean difference is higher than parallel flow. The calculation:

$$\Delta T_{LOG} = \frac{\Delta N - \Delta K}{ln\frac{\Delta N}{\Delta K}} \tag{2}$$

where:

- ΔN: the bigger temperature difference,
- ΔK: the less temperature difference.

2.3. HEAT TRANSFER AREA

The heat transfer between the process fluids realize across the tubes. The surface depends on the medium diameter, length and number of the tubes. Bigger the surface, bigger the performance, but mean bigger material cost too, what is not acceptable. We can use ribbed or finned tubes that also mean a higher cost.

$$A = L \cdot z \cdot \pi \cdot d_{med} \tag{3}$$

where:

- L: length of the tubes [m],
- z: number of the tubes [-],
- d_{med}: medium diameter of the tubes (arithmetic mean of the internal and external diameter) [m].

2.4. HEAT TRANSFER COEFFICIENT

The heat transfer coefficient is the third factor, and this calculation is the hardest. The coefficient depends on the internal and external convection heat transfer coefficient and the conductivity in the wall of the tube.

$$k = \frac{1}{\frac{1}{\alpha_i} + \frac{s_{wall}}{\lambda_{wall}} + \frac{1}{\alpha_e}} = U = \frac{1}{\frac{1}{h_i} + \frac{dx_{wall}}{k_{wall}} + \frac{1}{h_e}} \tag{4}$$

where:

- α_i (h_i): individual convection heat transfer coefficients [W/m^2·K],
- s_{wall} (dx_{wall}): thickness of the wall [m],
- λ_{wall} (k_{wall}): heat conductivity of the wall [W/m·K],
- note: the sign in the brackets is the English notation.

The conductivity depends on the material of the tube. The copper and graphite have the highest conductivities (~400W/m·K), and the stainless steel has the lowest (~15W/m·K). The heat transfer coefficient is less, than the least value of these three items. In the engineering practice, one of the convection heat transfer coefficients will be the least, so we would like to improve these factors.
Typical values of convection heat transfer coefficients:

Conditions of heat transfer	Value of coefficients [W/m^2·K]
gases in free convection	5-37
water in free convection	100-1200
water flowing in the tubes	1000-4000
water boiling	4000-8000
Condensation of water vapor	5000-12000

1. Table: Typical values of convection

3. INDIVIDUAL CONVECTION HEAT TRANSFER COEFFICIENTS

We must know the type of the flow to calculate the convection heat transfer coefficient. The connection depends on the phase change (yes or no), type of the flow (laminar, tubular or transient flow), inside or outside the tube. We use experimental and constraint equations.

3.1. CONDENSATION

In case of condensation, we could calculate the convection heat transfer coefficient directly. We use Nusselt's equation:

$$\alpha = 0{,}943 \cdot \sqrt[4]{\frac{\lambda^3 \cdot \rho^2 \cdot r \cdot g}{\eta \cdot \Delta t_{cond} \cdot H}}, \qquad (5)$$

where:

- ρ: density of process material [kg/m^3],
- λ: heat conductivity of process material [W/m·K],
- r: latent heat of process material [J/kg],
- g: acceleration of gravity [9,81 m/s^2],
- η: dynamic viscosity of process material [Pa·s],
- Δt_{cond}: difference between the wall and the condensation temperature [°C],
- H: the specific geometry. In case of vertical wall or tube H is the height of the wall. In case of horizontal pipelines:

$$H = Z^{2/3} \cdot d_{med}, \qquad (6)$$

- where Z means the number of tubes under each other.

3.2. BOILING

If we boil a mixture, we can calculate also directly the convection heat transfer coefficient. The empirical formula by György Fábry is:

$$\alpha = 88 \cdot \Delta t_{boil}{}^2 \cdot p^{0,6} \cdot C_f, \qquad (7)$$

where:
Δt_{boil}: difference between the wall and the boiling point [°C],
p: pressure [bar],
C_f: correction factor in case of substances other than water [-].

$$C_f = \frac{\rho}{\rho_w} \cdot \left(\frac{c \cdot \lambda \cdot r_w \cdot \sigma_w}{c_w \cdot \lambda_w \cdot r \cdot \sigma}\right)^{1/2} \cdot \left(\frac{\rho" \cdot \eta}{\rho"_w \cdot \eta_w}\right)^{-1/4} \qquad (8)$$

where:

- c: specific heat of process material[J/kg·K],
- σ: surface tension of process material [N/m],
- ρ”: vapor density of process material [kg/m^3],
- *note*$_1$: the index w concern to the water, measures without index concern to the boiling substance,
- *note*$_2$: material properties replaced in the formula at the boiling point,
- *note*$_3$: if the process substance is a mixture, the material properties shall be weighted with the mole fractions.

3.3. WITHOUT PHASE CHANGE

Without phase change we use constraints and empirical formulas. The type of the flow must be investigated; the formulas depend on the flow type. If the flow is laminar, the Nusselt number depends on the Pèclet number, in case of transient or tubular flow this depends on the Prandtl and Reynolds numbers. The material properties must be calculated on the average temperature. If we have calculated the Nusselt number, then we calculate the individual heat transfer coefficient.
In case of tubular flow, the value of the convection heat transfer coefficient is much larger, than the value in laminar flow, so we have to create tubular flow. The numbers of tubes, the form of the tubes, values of cooling substance have modified.
The formula in the case of tubular flow is as follows:

$$Nu = 0{,}023 \cdot Re^{0{,}8} \cdot Pr^{1/3} \tag{9}$$

where:

- Nu: Nusselt number [-],
- Re: Reynolds number [-],
- Pr: Prandtl number [-].

If the value of Nusselt number is known, the convectional heat transfer coefficient can be calculated:

$$Nu = \frac{\alpha \cdot L}{\lambda} \tag{10}$$

where:

- L: specific geometric [m].

4. OPTIMUM DESIGN

During optimization, we search the construction, that able to transfer the necessary heat and it has the lowest cost. We can consider material, production, maintenance and operational cost.

- Material costs:
  - tubes

- shell
- tube sheet
- horizontal and vertical baffles
- rear and front ends

- Production costs:
  - cutting of tubes
  - welding and welding preparation
  - sheet rolling
  - tube sheet drilling
- Maintenance costs:
  - periodical maintenance
  - mounting
  - cleaning
- Operational costs:
  - cost of electricity

First of all we have calculated the optimum for the minimum weight.

4.1. CONSTRAINTS

When we have specified the conditions, we have taken care on fluid mechanics, producing and practice viewpoints.

4.1.1. Length of the tube

In the trade turnover, we can purchase 6m long tubes. If we do not want to get a lot of waste, we must design 6, 3, 2, 1.5, 1.2, 1 or 0.5 m long heat exchanger.

4.1.2. Tubes

Just like the previous section, we could not purchase tubes with whatever size. We should create a table with the available sizes. Furthermore, the number of the tubes must be an integer.

4.1.3. Fluid mechanics

First of all, we must maximize the value of the fluid speed. In case of liquid flow, this value is 1.5-2 m/s (in gas of gas, this is about 8-10 m/s). This is necessary, because if the speed is too high, the friction and erosion could make leaks in the wall of the tubes. Secondly, we must create a condition about the turbulence too. In case of tubular flow, the convection is higher than other flow types and the formula changes too. So, the Reynolds number must be more than 10000. (In the future, we would like to investigate tube with special geometry, how changes the tubular condition.)

4.1.4. Thermal conditions

Least but not last, the heat flow must be constant between the internal and external heat convection and the conductivity.

$$q = \alpha_b \cdot \left(t_{i,med} - t_{i,wall}\right) = \frac{\lambda_{wall}}{s_{wall}} \cdot \left(t_{i,wall} - t_{e,wall}\right) = \alpha_e \cdot \left(t_{e,wall} - t_{e,med}\right) \quad (11)$$

4.2. OBJECTIVE FUNCTION

In case of optimal design, we look for the minimum of the next function next to the conditions in the previous paragraph:

$$V = \left[\left(\frac{d_e^2 \cdot \pi}{4} - \frac{(d_e - 2 \cdot s_t)^2}{4}\right) \cdot N \cdot L\right] + \left[\left(\frac{D_{shell}^2 \cdot \pi}{4} - \frac{(D_{shell} - 0{,}01)^2}{4}\right) \cdot N \cdot L\right] \quad (12)$$

5. SOLUTION

If we would like to optimize a heat exchanger, we must put it in a technology. In my exercise, the specifications are:

- technology fluid:
    - material: water,
    - mass flow: 10 kg/s,
    - inlet temperature: 60°C,
    - outlet temperature: 30°C.
- cooling fluid:
    - material: water,
    - inlet temperature: 10°C.

The optimization is calculated with the help of the Excel Solver. A spreadsheet has been created, then the setup of the constraints, unknowns and the objective function.

5.1. ONE-PASS HEAT EXCHANGER

In the first step, we have calculated with the easiest construction. The objective was the minimum weight, the changing parameters was the number, the length and the external diameter of the tubes, outlet temperature of the cooling fluid and the internal temperature of the wall.

Heat exchanger	(without phase change)						**Geometry**						
Process substa	víz		Coolong subs	water			Internal diam	16	mm	0,016	m	Acsb	
mass flow	10	kg/s	mass flow	20,22391	kg/s		wall thicknes	2	mm	0,002	m	0,000201	m2
Inlet temp.	60	°C	Inlet temp.	10	°C		External diam	20	mm	0,02	m	Acsk	
Outlet temp.	30	°C	Outlet temp.	24,82387	°C		numbers of tu	32	lambdatube	50	W/mK	0,000314	m2
tköz	45	°C	tköz	17,41194	°C		z	3		de	0,019877		
deltat	30	°C	deltat	14,82387	°C		**n**	5					
c(t)	4180,149402	J/kgK	c(t)	4182,984	J/kgK		Dköpeny	0,17	m			Adshell	
Q1	1254044,821	J/s	Q2	1254045	J/s							0,022698	m2
deltatlog	26,87774856	°C			tcsk	45	°C		q1	55116,48	j/s		
					tcsf	37,32142	°C		q2	55116,48	j/s		
vtube	1,568899774	m/s	OK		tkf	35,11676	°C		diff	8,86E-07			
Retube	41881,06182	-	OK		tkk	17,41194	°C						
Nutube	180,4512001	-											
alfatube	7177,954402	W/m2K											
					k	1997,838	W/m2K		V1	0,046708	m3	tube	
vshell	1,6	m/s	OK		A	23,35393	m2		V2	0,000506	m3	tube sheet	
Reshell	29970,70296	-	OK		L	12,90588	m2		V3	0,072981		shell	
Nushell	104,1428752	-											
alfashell	3113,076664	W/m2K							Vtotal	0,120195	m3		
									mtotal	943,5281	kg		
								result:	more pass need				

Figure 1: The spreadsheet of the optimization of one-pass heat exchanger

Result: 32 pieces, 20x2 mm, 12.9 m long tubes, 170 mm diameter shell and 943.5 kg minimum weight. The length of the tubes is too long (Figure 1).

5.2. MORE-PASS HEAT EXCHANGER

In this calculation we have used the previous table, but we modified a little bit. If we use a two-pass construction, the flow section will be the half of the original section. (In case of four-pass, the section will be the quarter of the original.) I do not manipulate the original conditions.
Result: 114 pieces, 21x2.1 mm, 3.49 m long tubes, 303.5 mm diameter shell diameter and 670 kg minimum weight (Figure 2). All of the conditions are satisfied, and the needed cooling water is increased from 20.22 kg/s to 52.7 kg/s (operational cost decreasing).

Heat exchanger	(without phase change)						**Geometry**						
Process substance	water		Coolong subs	water			Internal diameter	16,8	mm	0,0168	m	Acsin	
mass flow	10	kg/s	mass flow	52,5992	kg/s		wall thickness	2,1	mm	0,0021	m	0,000221671	m2
Inlet temp.	60	°C	Inlet temp.	10	°C		External diameter	21	mm	0,021	m	Acsex	
Outlet temp.	30	°C	Outlet temp.	15,6967	°C		numbers of tubes	114	lambdatube	50	W/mK	0,000346361	m2
tmed	45	°C	tmed	12,8483	°C		z	6		de	0,01551		
deltat	30	°C	deltat	5,69669	°C		n	11					
c(t)	4180,149402	J/kgK	c(t)	4185,16	J/kgK		Dshell	0,3035	m			Adshell	
Q1	1254044,821	J/s	Q2	1254045	J/s							0,072344792	m2
							Numbers of pass	4	1,2,4 lehet				
vtube	1,597797439	m/s	OK										
Retube	44785,09535	-	OK		tcsk	45	°C		q1	55816,2	j/s		
Nutube	190,393637	-			tcsf	37,2615	°C		q2	55816,2	j/s		
alfatube	7212,802682	W/m2K			tkf	34,9172	°C		diff	7,6E-08			
					tkk	12,8483	°C						
vshell	1,6	m/s	OK										
Reshell	28007,93182	-	OK										
Nushell	89,95497505	-			k	1736,03	W/m2K		V1	0,04964	m3	tube	
alfashell	2529,177585	W/m2K			A	23,6394	m2		V2	0,00131	m3	tube sheet	
					L*	3,49237	m		V3	0,0344	m3	shell	
									Vtotal	0,08535	m3		
									mtotal	670,022	kg		

Figure 2: The spreadsheet of the optimization of more-pass heat exchanger

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