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**Stability analysis in milling by taking into account the influence of forced vibrations on the actual tool-workpiece engagement conditions**Giovanni Totis<sup>a,\*</sup>, Tamas Insperger<sup>b</sup>, Gabor Stepan<sup>b</sup>, Marco Sortino<sup>a</sup><sup>a</sup>*Polytechnic Department of Engineering and Architecture, University of Udine, Via delle Scienze 206, 33100 Udine, Italy*<sup>b</sup>*Department of Applied Mechanics, Budapest University of Technology and Economics, Budapest, Hungary*\* Corresponding author. Tel.: +39 0432 558258; fax: +39 0432 558251. E-mail address: [giovanni.totis@uniud.it](mailto:giovanni.totis@uniud.it)**Abstract**

In order to increase the material removal rate in milling, advanced cutting tools with complex geometry are typically applied under extreme cutting conditions which may trigger undesired chatter vibrations of the machining system. Recently some dynamic milling models were proposed in the literature which take into account the higher geometrical complexity of these tools. In these works, the tool-workpiece engagement conditions are computed from a purely geometric-kinematic analysis of the milling operation. Moreover, they are kept constant throughout the stability analysis, independently from any possible increase of the axial depth of cut. In many cases the experimental validation of the proposed models is incomplete. In this work a novel methodology for assessing milling stability is presented, which is based on the correct linearization of the regenerative perturbations around the actual steady state forced vibrations. When the axial depth of cut is progressively increased, the resulting forced vibrations may cause a variation of each tooth-workpiece contact conditions, thus influencing the process dynamic behavior. This effect is more dominant when the degree of symmetry is poor as in the case of variable pitch cutters, when there is significant teeth runout, and when the average chip thickness is concurrently very small as in peripheral milling. The proposed approach for chatter prediction consists of an incremental linear stability analysis which does progressively adapt to the gradually increasing depth of cut up to the stability border. The concept was successfully verified with experimental cutting tests.

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**Keywords:** Milling, Tool, Dynamics, Chatter, Prediction, Forced Vibrations, Regenerative Vibrations**1. Introduction and aim**

Recently, some advanced dynamic milling models and stability analysis methodologies were developed for understanding and predicting the dynamic behavior of imperfect and/or asymmetrical cutter geometries which are frequently applied in practice. Specifically, they focused on the effect of tool runout, variable pitch cutters, helicoid teeth with variable helix and serrated cutting edges on the effective cutting conditions and on milling dynamics. They showed that entrance and exit angles of each cutting tooth as well as its nominal chip thickness may be strongly influenced by teeth runout and by teeth angular distribution [1] [2] [3]. Besides, multiple delays are generated by complex cutter geometries, which are eventually responsible for T-periodic steady state forced vibrations, where T is the main spindle revolution period [1]. Eventually, some unexpected bifurcation types may arise in these conditions [4].

Nevertheless, in all these works the tool-workpiece engagement conditions were computed in static conditions. In other words, entrance and exit angles of each cutting tooth were determined from a purely geometric-kinematic analysis of the cut-

ting process by excluding the influence of forced vibrations. Tool-workpiece engagement conditions were assumed to be unaffected by the increase of depth of cut, which is responsible for an increase of forced vibrations. Moreover, in most cases experimental validation was missing or poor.

This work is based on the key idea that forced vibrations may considerably influence the actual tool-workpiece engagement conditions, especially in the presence of teeth runout and/or with uneven angular pitch cutters. Accordingly, the actual chip thickness, cutting forces and machining system vibrations may exhibit strongly non-linear evolutions when the axial depth of cut is increased, due to the strong dynamic coupling between forced vibrations and tool-workpiece engagement conditions. In the following a novel stability prediction approach will be sketched and tested by some preliminary cutting tests.

**2. New approach for stability analysis**

Let us consider a generic milling operation performed by a cylindrical end mill or by a slender face shoulder cutter. Let us assume that milling dynamics are expressed in the state space

form

$$\begin{cases} \dot{q}(t) = A_{dyn}q(t) + B_{dyn}F(t) \\ u(t) = C_{dyn}q(t) \end{cases} \quad (1)$$

where  $F$  is the  $2 \times 1$  resultant cutting force vector in the  $XY$  plane orthogonal to the main spindle axis;  $u$  is the  $2 \times 1$  tool tip barycentre displacement vector;  $q$  is the  $n \times 1$  state space vector while  $A_{dyn}$ ,  $B_{dyn}$  and  $C_{dyn}$  are the time invariant matrices representing cutter dynamics in the  $XY$  plane, which are derived from modal analysis performed on the tooling system.

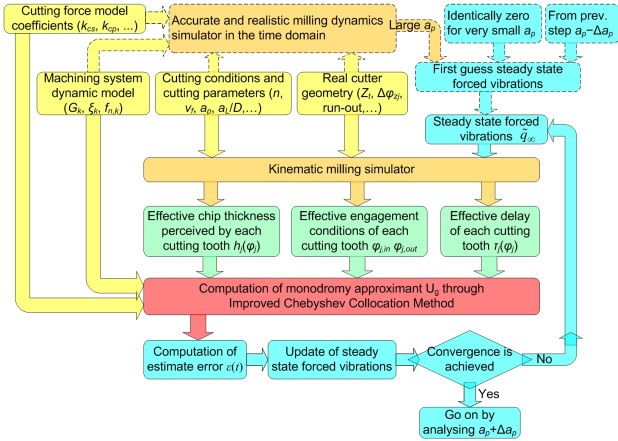


Fig. 1. Concept of the new incremental algorithm for milling stability analysis.

Let  $T$  be the main spindle revolution period. Let  $\tau \cong T/z_t$  be the fundamental delay between subsequent teeth, where  $z_t$  is teeth number. Assume we know the  $T$ -periodic steady state (stable) forced vibrations  $q_{\infty,0}(t)$  for a given depth of cut  $a_p$ , corresponding to the effective  $T$ -periodic cutting forces  $F_0(t)$ . Let us now slightly increase the axial depth of cut to  $a_p + \Delta a_p$ . Total vibrations can be decomposed into the sum

$$q(t) = q_{\infty}(t) + \delta q(t) \quad (2)$$

where the first term are the new  $T$ -periodic steady state vibrations arising at  $a_p + \Delta a_p$  while the second term is any possible regenerative perturbation with respect to such  $T$ -periodic behavior. Steady state vibrations can be further decomposed into

$$q_{\infty}(t) = \tilde{q}_{\infty}(t) + \varepsilon(t) \quad (3)$$

where the first term is a steady state estimate while the second is the estimate error with respect to the true steady state. At first guess the steady state estimate should be

$$\tilde{q}_{\infty}(t) \approx \left(1 + \frac{\Delta a_p}{a_p}\right) q_{\infty,0}(t) \quad (4)$$

i.e. approximately proportional to the steady state forced vibrations found at the previous step (when depth of cut was  $a_p$ ), by exploiting the classical linearity between forced vibrations and depth of cut.

In short, the total vibrations can be exactly represented by the sum

$$q(t) = \tilde{q}_{\infty}(t) + \underbrace{[\varepsilon(t) + \delta q(t)]}_{\zeta(t)} \quad (5)$$

The term  $\zeta(t)$  is supposed to be sufficiently small to allow a linearization of the system (1) around the tool-workpiece contact conditions and chip thickness trends corresponding to  $\tilde{q}_{\infty}$ .

After some algebraic manipulations, one may finally arrive at the following system of Delay Differential Equations

$$\dot{\zeta}(t) = A(t)\zeta(t) + \sum_{j=1}^{z_t} B_j(t)\zeta(t - \tau_j(t)) + B_0(\tilde{q}_{\infty}, t) \quad (6)$$

If we are observing the behavior of a dissipative stable system far from the stability boundaries, the regenerative perturbation should decay to zero after a sufficiently high number of spindle revolutions. Therefore  $\zeta(t) \rightarrow \varepsilon(t)$  as  $t \rightarrow \infty$ , i.e. the estimate error  $\varepsilon(t)$  is the steady state solution of (6) and therefore it can be determined numerically.

This term can be now used to improve the steady state estimate, by updating it with the sum  $\tilde{q}_{\infty}(t) + \varepsilon(t)$ .

The new estimate can be used to recompute the effective tool-workpiece engagement conditions. In this work this task was accomplished by using an accurate geometric-kinematic numerical simulator. The linearization can be performed again around the new cutting conditions and a new estimate error will be found. If we are able to progressively decrease such estimate error to zero, then the steady state estimate will converge to the true steady state estimate  $\tilde{q}_{\infty}(t) \rightarrow q_{\infty}(t)$ .

When a sufficiently good estimate of  $q_{\infty}(t)$  will be available, it will be possible to correctly linearize milling dynamics around the actual tool-workpiece engagement conditions. The spectral radius can be now computed in order to correctly assess system stability for  $a_p + \Delta a_p$ . In case the spectral radius is still below unity, we can continue increasing the depth of cut until the critical stability boundary will be finally encountered.

In the present work the discretization of system (6) was performed by an Improved Chebyshev Collocation Method, whose details will be deferred to other publications.

The incremental innovative algorithm for stability analysis is briefly represented in Figure 1.

In order to show the effectiveness of the proposed methodology for explaining and predicting milling dynamics, a large set of experimental tests was carried out. Here we will illustrate only a part of such measurements, which will be adequately described in other research publications.

### 3. Experimental observations and model validation

All the experimental tests were carried out at the Laboratory for Advanced Mechatronics - LAMA FVG - located in Udine, Italy, by using a 3 axes CNC milling machine Haas VF2-TR. Several sensors were installed into the machine tool for modal analysis, cutting force measurement and chatter detection. All sensor signals were sampled at 30 kHz by a National Instruments Data Acquisition device (cDAQ-9178 with NI9215 modules) and stored on a PC for further analysis, which was carried out in the MathWorks MATLAB environment. In all cases the workpiece material was Ck45 carbon steel, with about 198HB, which was machined in dry cutting conditions.

First of all, cutting force model identification was carried out by using a modular Sandik Coromant tooling system composed of a spindle adapter (C5-390B.140-40 040), an intermediate adapter (C5-391.02-32 060A) and a face shoulder milling cutter (R390-025C3-11M050) with external diameter  $D = 25$  mm, see Figure 2 (a). The cutter had three equally spaced teeth ( $z_t = 3$ ) consisting of face milling inserts (R390-11 T3 04E-PL 1030) with  $r_{\varepsilon} = 0.4$ , cutting edge angle  $\chi = 90^\circ$  and axial rake

angle  $\gamma_a \leq 15^\circ$ . Cutting tests were performed according to a full factorial Design of Experiments by varying feed per tooth on three levels ( $f_z = 0.06, 0.08, 0.10$  mm) and depth of cut on four levels ( $a_p = 0.4, 0.8, 1.2, 1.6$  mm). Down milling with 6% of radial tool-workpiece immersion was adopted. Cutting speed  $v_c$  was kept constant and equal to 225 m/min. Cutting forces were measured by applying a special plate dynamometer [5]. By using this device, instantaneous and average cutting forces were measured and then analyzed in order to identify the following cutting force model coefficients: main shearing  $k_{cs} = 2497$  N/mm<sup>2</sup>, normal shearing  $k_{ns} = 1492$  N/mm<sup>2</sup>, main ploughing  $k_{cp} = 29.3$  N/mm, normal ploughing  $k_{np} = 33.8$  N/mm. According to the linear regression routine the 95% confidence interval of each coefficient was within  $\pm 4\%$  the estimated value.

Afterwards, a slender tooling system with a variable pitch cutter was assembled and tested, see Figure 2 (b) and (c). Specifically, the same spindle adapter and intermediate element used for cutting force model identification were applied, together with additional tool extender modules (C3-391.01-32 080A and C3-391.01-32 060A) and a face shoulder cutter (R390-032C3-11M050) with external diameter  $D = 32$  mm. In this configuration the total tooling system overhang - from cutter tip to spindle nose - was about 290 mm. The cutter had three teeth of the same type specified above. However, in this case the angular pitch was uneven:  $\Delta\varphi_{1,2} = 120^\circ$ ,  $\Delta\varphi_{2,3} = 126.67^\circ$ ,  $\Delta\varphi_{3,1} = 113.33^\circ$ . Tool radial runout was also checked periodically during the chatter tests. On average the second tooth was slightly more protruding than tooth 1 ( $R_2 \approx R_1 + 5\mu\text{m}$ ) while tooth 3 was slightly less protruding than tooth 1 ( $R_3 \approx R_1 - 5\mu\text{m}$ ).

Modal analysis was carried out on the slender tooling system by means of pulse testing technique. An eddy current sensor (Micro-Epsilon type ES1 with sensitivity  $\approx 10$  mV/ $\mu\text{m}$ ) together with a triaxial piezoelectric accelerometer Kistler 8763B100AB (sensitivity  $\approx 50$  mV/g) were applied for measuring cutter vibrations along the transversal directions  $X$  and  $Y$ . Impulsive forces were applied by means of an instrumented impact hammer type Dytran 5800B4 (sensitivity 2.41 mV/N).

There was only one dominant resonance peak for each direction. Therefore, we were tempted to consider only a single harmonic oscillator for each direction. Nevertheless, the maximum nominal chip thickness  $h_{j,max}$  was about 0.04 mm during chatter tests, while the amplitude of the steady state forced vibrations did easily exceed 0.1 mm in such conditions. Therefore, a very accurate model of machining system dynamics was required in order to obtain a satisfactory estimate of chip thickness, which is the result of the effective forced vibrations overlapped to the trochoidal cutter movement.

Therefore, it was necessary to include all the most significant vibration modes in the final dynamic model. Specifically, the vibration modes listed in Table 1 were identified, where  $G_k$  is the static compliance,  $f_k$  is the natural frequency and  $\xi_k$  is the damping coefficient of the generic mode  $k$ .

Chatter tests were eventually carried out by performing down milling passes with 2% of radial immersion. Feed per tooth was kept constant during the tests ( $f_z = 0.08$  mm). Several spindle speeds were tested in the range [1500, 2851] rpm. For each spindle speed, depth of cut was increased from 0.5 mm up to 4 mm. The test was stopped when severe chatter

Table 1. Estimated modal parameters of the tooling system under examination.

Param.	1	2	3	4	5	6
$G_{x,k}$ [ $\mu\text{m}/\text{N}$ ]	0.038	0.287	0.235	0.045	0.043	0.016
$f_{x,k}$ [Hz]	155	311	324	429	670	1222
$\xi_{x,k}$ [ ]	0.046	0.025	0.021	0.039	0.534	0.101
$G_{y,k}$ [ $\mu\text{m}/\text{N}$ ]	0.086	0.551	0.062	0.018	–	–
$f_{y,k}$ [Hz]	265	304	364	943	–	–
$\xi_{y,k}$ [ ]	0.566	0.031	0.068	0.033	–	–

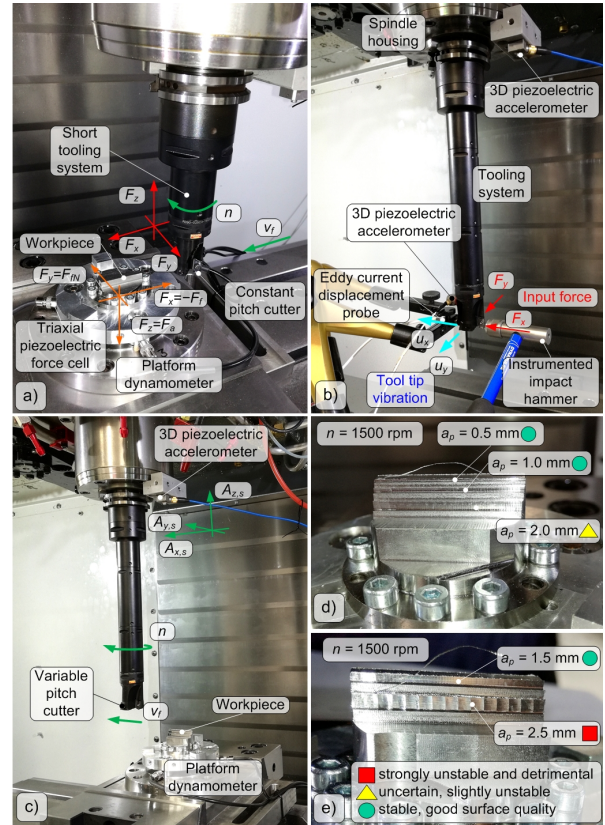


Fig. 2. a) Cutting force model identification by using a constant pitch cutter; b) modal analysis on the slender tooling system; c) chatter tests by using a variable pitch cutter; d) and e) surface quality and cutting conditions observed at 1500 rpm in 2% down milling.

occurred. The stability of the system was assessed both from visual inspection of the machined surface and by calculating quantitative chatter indicators [6] obtained from the triaxial accelerometer (Kistler 8764B50 with sensitivity  $\approx 100$  mV/g) installed on spindle housing, as illustrated in Figure 2 (c),(d) and (e).

For the sake of brevity, here only the case with  $n = 1500$  is reported. In Figure 3 (a), the chip thickness behavior simulated by the classic approach is compared to that obtained through the new approach. Already at  $a_p = 0.5$  a considerable difference between the two simulations is visible. The classic approach does not take into account the influence of forced vibrations on the nominal chip thickness and on the effective tool-workpiece engagement conditions, which are on the contrary strongly affected by forced vibrations. Similarly, the effective time delay  $\tau_j$  perceived by tooth  $j$  is radically dependent on forced vibrations, as can be noticed in Figure 3 (b). Moreover, the cutting conditions may evolve in an unexpected and non-linear

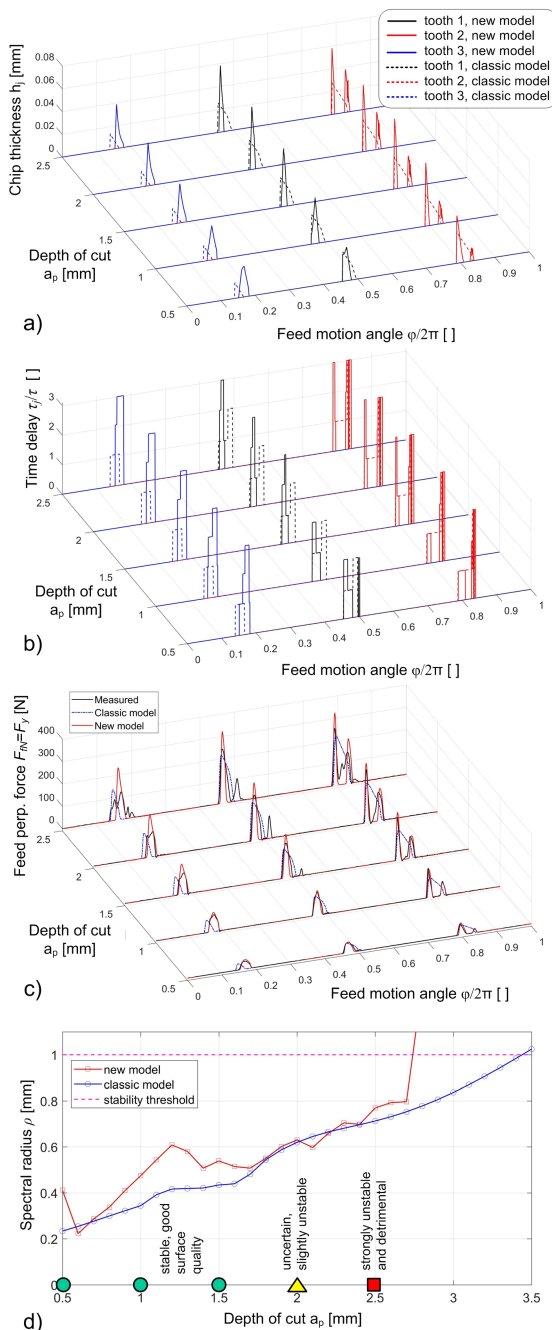


Fig. 3. Comparison of classic and new approach: a) simulated chip thickness; b) simulated time delay; c) feed perpendicular force (simulated vs measured); d) spectral radius (predicted vs experimental points).

way when depth of cut is increased.

The accuracy and effectiveness of the proposed methodology for modeling milling dynamics is confirmed in Figure 3 (c), where the feed perpendicular force simulated by the new method is well correlated to the measured cutting force trend, which reveals radically different tool-workpiece engagement conditions with respect to those derived from the classic approach.

Eventually, the spectral radius obtained from milling dynamics linearization around the (correct) tool-workpiece engagement conditions is on average in better agreement with the experimental points obtained both at 1500 rpm (see Figure 3 (d)) and at other spindle speed levels, thus further confirming

the potential usefulness and progress provided by the proposed methodology.

## 4. Conclusions

The proposed incremental method for linear stability analysis around the actual tool-workpiece engagement conditions has demonstrated that the effect of forced vibrations should not be neglected when their amplitude is comparable to the average chip thickness. Forced vibrations and tool-workpiece engagement conditions are dynamically coupled and they influence each other giving rise to a non-linear evolution of chip thickness, tool-workpiece engagement conditions, cutting forces and machining system vibrations. This fact is particularly important when considering peripheral milling with variable pitch cutters and/or tool runout, which are responsible for an initial asymmetry of the dynamic system. This can be greatly amplified when depth of cut is increased, because the interconnection between cutting conditions and forced vibrations becomes stronger. Some preliminary experimental tests confirmed these novel theoretical discoveries.

It is worth noting that the proposed methodology is not just equivalent to a time domain simulation. The geometric-kinematic numerical simulation is necessary but it is dramatically shorter than a classical time domain simulation since only a few rounds need to be computed. Afterwards, when the convergence to the final steady state solution is fast enough, the method can be much faster globally than a typical time domain simulation. Moreover, no direct estimate of the spectral radius can be derived from a pure time domain simulation unless some discretization as well as some linear stability analysis are also performed around the obtained steady state solution.

The proposed algorithm incorporates the best advantages of both the pseudospectral linear methods and the time domain simulations.

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