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A note on the boundedness in a chemotaxis-growth system with nonlinear sensitivity

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Abstract. This paper deals with a parabolic-elliptic chemotaxis-growth system with nonlinear sensitivity

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (\psi(u) \nabla v) + f(u), & (x,t) \in \Omega \times (0,\infty), \\ 0 = \Delta v - v + g(u), & (x,t) \in \Omega \times (0,\infty), \end{cases}$$

under homogeneous Neumann boundary conditions in a smooth bounded domain $\Omega \subset \mathbb{R}^n$ $(n \geq 1)$, where $\chi > 0$, the chemotactic sensitivity $\psi(u) \leq (u+1)^q$ with $q > 0$, $g(u) \leq (u+1)^l$ with $l \in \mathbb{R}$ and $f(u)$ is a logistic source. The main goal of this paper is to extend a previous result on global boundedness by Zheng et al. [*J. Math. Anal. Appl.* **424**(2015), 509–522] under the condition that $1 \leq q + l < \frac{2}{n} + 1$ to the case $q + l < 1$.

Keywords: boundedness, chemotaxis-growth system, chemotactic sensitivity.

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1 Introduction

In this paper, we study the following Keller–Segel chemotaxis-growth system with nonlinear sensitivity under homogeneous Neumann boundary conditions

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (\psi(u) \nabla v) + f(u), & (x,t) \in \Omega \times (0,\infty), \\ 0 = \Delta v - v + g(u), & (x,t) \in \Omega \times (0,\infty), \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0, & (x,t) \in \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^n$ $(n \geq 1)$ is a smooth bounded domain, $\frac{\partial}{\partial \nu}$ denotes the differentiation with respect to the outward normal derivative on $\partial\Omega$, $\chi > 0$ is a parameter referred to as chemosensitivity, and $u(x,t)$, $v(x,t)$ denote the density of the cells population and the concentration

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of the chemoattractant, respectively. $\psi(u)$ and $g(u)$ describe the chemotactic sensitivity of cell population and production rate of the chemoattractant, respectively. Throughout this paper, we assume that the nonnegative functions $\psi \in C^2([0,\infty))$ and $g \in C^2([0,\infty))$ satisfy the conditions that there exist some constants $q > 0$ and $l \in \mathbb{R}$ such that

$$\psi(s) \leq (s+1)^q \quad \text{and} \quad g(s) \leq (s+1)^l \quad \text{for all } s \geq 0. \tag{1.2}$$

Moreover, the logistic source $f \in C^0([0,\infty)) \cap C^1((0,\infty))$ fulfills

$$f(s) \leq a - bs^k \quad \text{with } a \geq 0,\ b > 0,\ k > 1 \text{ and } f(0) \geq 0. \tag{1.3}$$

Chemotaxis is the oriented movement of biological cells or organisms in response to gradients of the concentration of chemical signal substance in their environment, where the chemical signal substance may be produced or consumed by the cells themselves. The most interesting situation related to self-organization phenomenon takes place when cells detect and response to a chemical which is secreted by themselves. The pioneering works of chemotaxis model were introduced by Patlak [13] in 1953 and Keller and Segel [9] in 1970, and we refer the reader to the surveys [5–7] where a comprehensive information of further examples illustrating the outstanding biological relevance of chemotaxis can be found.

In order to understand (1.1), let us mention some previous contributions in this direction. In recent years, the following initial boundary value problems have been studied by many authors

$$\begin{cases} u_t = \nabla\cdot(\varphi(u)\nabla u) - \chi\nabla\cdot(\psi(u)\nabla v) + f(u), & (x,t) \in \Omega\times(0,\infty), \\ 0 = \Delta v - v + g(u), & (x,t) \in \Omega\times(0,\infty), \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0, & (x,t) \in \partial\Omega\times(0,\infty), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.4}$$

where $f(u) \leq a - bu^k$ with $a \geq 0$, $b > 0$ and $k > 1$, $\chi > 0$, $\Omega \subset \mathbb{R}^n$ $(n \geq 1)$ is a bounded domain with smooth boundary $\partial\Omega$. For the special case $\varphi(u) = 1$, $\psi(u) = g(u) = u$ and $k = 2$ in (1.4), Tello and Winkler [15] proved that the solutions of (1.4) are global and bounded provided that either $n \leq 2$, or $n \geq 3$ and $b > \frac{(n-2)\chi}{n}$ with $\chi > 0$. Moreover, for any $n \geq 1$ and $b > 0$, the existence of global weak solution was shown under some additional conditions. Furthermore, if $k > 2 - \frac{1}{n}$, some global very weak solutions of semilinear parabolic-elliptic model (1.4) were constructed by Winkler [19]. When $\psi(u) = g(u) = u$, $\varphi(u) \geq c(u+1)^p$ with $c > 0$, $p \in \mathbb{R}$, $k = 2$ and $b > \left(1 - \frac{2}{n(1-p)_+}\right)\chi$ with $\chi > 0$, Cao and Zheng [2] proved that model (1.4) has a unique global classic solution, which is uniformly bounded. Wang et al. in [18] investigated the boundedness and asymptotic behavior for model (1.4) with the special case $\psi(u) = g(u) = u$ and $\varphi(u) \geq C_\varphi u^{m-1}$ $(m \geq 1)$ under other additional technique conditions. Recently, Zheng [24] and Xie–Xiang [23] improved the results of [18] by using different methods, respectively. In the recent paper [21], for the case of $f(u) = ru - \mu u^2$ with $r \geq 0$ and $\mu > 0$, in one-dimensional case, Winkler proved that going beyond carrying capacities actually is a genuinely dynamical feature of the simplified parabolic-elliptic system provided that $\mu < 1$ and diffusion is sufficiently weak, moreover, he investigated global boundedness and finite-time blow-up for a corresponding hyperbolic-elliptic limit problem. Furthermore, Lankeit [10] extended the results of Winkler [21] to the higher dimensional radially symmetric case. Moreover, Viglialoro and Woolley [16] derived the eventual smoothness and asymptotic behavior of solutions for the corresponding fully parabolic (1.4) with $\varphi(u) = 1$, $\psi = u$ and $g(u) = u$ in three dimensional case. For the case $f(u) = \kappa u - \mu u^2$, Lankeit [11] showed that in the three-dimensional setting, after some time, these solutions become classical solutions, provided that κ is not too large. In this case, he also considered their large-time behaviour and proved decay if $\kappa \leq 0$ and the existence of an absorbing set if $\kappa > 0$ is sufficiently small. When $f(u) = 0$, Egger et al. [4] investigated the identification of these nonlinear parameter functions for problem (1.1). Furthermore, this is underlined in [12] by a recent result on global existence and boundedness in a fully parabolic counterpart of (1.4) involving general signal production under the assumption that $f(u) = 0$ and $g(u) \leq Ku^{\kappa}$ for all $u \geq 1$ with some $\kappa < \frac{2}{n}$. To the best of our knowledge, when $\varphi(u) = 1$, $\psi(u) = u$ in (1.4), where the second equation in (1.4) is replaced by $0 = \Delta v - m(t) + u$ with $m(t) = \frac{1}{|\Omega|}\int_\Omega u(x,t)dx$, the only result obtained by Winkler in [20] for (1.4) with logistic source $f(u)$ is about finite-time blow-up in the higher-dimensional case under some additional conditions. Furthermore, for the general cases φ and ψ in (1.4), Zheng et al. [27] studied the global boundedness and finite-time blow-up for the solution under different conditions about the parameter functions. When $\varphi(u) = 1, g(u) \leq u^l$ in (1.4), Zheng et al. [28] proved that model (1.1) possesses a unique nonnegative global bounded classical solution (u, v), provided that one of the cases holds: (i) $1 \leq q + l < \frac{2}{n} + 1$ and $k > 1$; (ii) $q + l \geq \frac{2}{n} + 1$, $b > \frac{(q+l-1)n-2}{(q+l-1)n}\chi$, $q \geq 1$ and $k \geq q + l$. Moreover, other variants of the corresponding parabolic-parabolic types have been studied by some authors [1, 14, 17, 25, 26, 29].

In the present paper, motivated by the ideas in [24], our main purpose is to extend a previous result on global boundedness by Zheng et al. [28] under the condition that $1 \leq q + l < \frac{2}{n} + 1$ to the case $q + l < 1$. Our main result in this paper is stated as follows.

Theorem 1.1. *Let $\Omega \subset \mathbb{R}^n$, $n \geq 1$ be a bounded domain with smooth boundary. Assume that $\psi(u)$ and $g(u)$ satisfy (1.2) with $q + l < 1$, and $f(u)$ fulfills (1.3). Then for any nonnegative initial data $u_0 \in C^1(\overline{\Omega})$, model (1.1) possesses a nonnegative global classical solution (u, v) which is uniformly bounded in time in the sense that there exists $C > 0$ such that*

$$\|u(\cdot, t)\|_{L^\infty(\Omega)} \leq C \quad \text{for all } t > 0.$$

This paper is organized as follows. In the next section, we prove our main result by means of the iteration technique.

2 Proof of Theorem 1.1

In this section, we shall prove our boundedness result by an iteration procedure used in [24]. Firstly, we state one result concerning local existence of a classical solution to (1.1).

Lemma 2.1. *Let $\Omega \subset \mathbb{R}^n$ $(n \geq 1)$ be a bounded domain with smooth boundary. Assume that the functions ψ and g belong to $C^2([0,\infty))$ and satisfy $\psi \geq 0$ and $g \geq 0$ in $[0,\infty)$, and $f \in C^0([0,\infty)) \cap C^1((0,\infty))$ fulfills $f(0) \geq 0$. Then for any nonnegative initial data $u_0 \in C^1(\overline{\Omega})$, there exists a maximal existence time $T_{\max} \in (0,\infty]$ and a pair of nonnegative functions $(u,v) \in \left(C^0\left(\overline{\Omega} \times [0, T_{\max})\right) \cap C^{2,1}(\overline{\Omega} \times (0, T_{\max}))\right)^2$ such that (u, v) is a classical solution of (1.1) in $\Omega \times (0, T_{\max})$. Moreover, if $T_{\max} < +\infty$, then*

$$\lim_{t \nearrow T_{\max}} \|u(\cdot, t)\|_{L^\infty(\Omega)} = \infty. \tag{2.1}$$

Proof. The local-in-time existence of classical solution to (1.1) is well-established by a fixed point theorem in the context of Keller–Segel-type chemotaxis systems. The proof is quite standard, for the details, we refer the readers to [3, 8, 15, 18, 22, 24]. □

Now let us pick any $s \in (0, T_{\max})$ and $s \leq 1$, then by the regularity property asserted in Lemma 2.1, we derive $(u(\cdot,s), v(\cdot,s)) \in C^2(\overline{\Omega})$ with $\frac{\partial u(\cdot,s)}{\partial \nu} = \frac{\partial v(\cdot,s)}{\partial \nu} = 0$ on $\partial\Omega$, so that in particular we can take $M > 0$ such that

$$\sup_{0\leq\tau\leq s} \|u(\cdot,\tau)\|_{L^\infty(\Omega)} + \sup_{0\leq\tau\leq s} \|v(\cdot,\tau)\|_{L^\infty(\Omega)} \leq M. \tag{2.2}$$

Lemma 2.2. *Let (u, v) be a solution to (1.1) on $(0, T_{\max})$. Assume that $\psi(u)$ and $g(u)$ satisfy (1.2) with $q + l < 1$, and $f(u)$ fulfills (1.3). Then there exist positive constants K_0 and K, depending only on a, b, q, l, k, M and Ω, such that*

$$\int_\Omega u^{\mu_i}(x,t)dx \leq K_0 K^{\mu_i}(T+1) \quad \text{for all } t \in (s, T), \tag{2.3}$$

where

$$\mu_i = 2^i + 1 - q - l \quad \text{and} \quad i \geq 1. \tag{2.4}$$

Proof. Multiplying the first equation in (1.1) by $(1+u)^{\mu_i - 1}$ and integrating by parts, we have

$$\begin{aligned}\frac{1}{\mu_i}\frac{d}{dt}\int_\Omega (1+u)^{\mu_i}dx =& -(\mu_i - 1)\int_\Omega (1+u)^{\mu_i-2}|\nabla u|^2 dx + \chi(\mu_i - 1)\\ &\times \int_\Omega (1+u)^{\mu_i-2}\psi(u)\nabla u\cdot\nabla v dx + \int_\Omega (1+u)^{\mu_i-1}f(u)dx\\ =:& I + II + III.\end{aligned} \tag{2.5}$$

Let

$$\Psi(u) = \int_0^u (1+\sigma)^{\mu_i-2}\psi(\sigma)d\sigma, \tag{2.6}$$

then

$$\Psi(u) \leq \int_0^u (1+\sigma)^{\mu_i+q-2}d\sigma \leq \frac{1}{\mu_i+q-1}(1+u)^{\mu_i+q-1} \tag{2.7}$$

due to the condition (1.2).

By the second equation in (1.1) and (2.7), we derive from $q > 0$ that

$$\begin{aligned} II &= \chi(\mu_i-1)\int_\Omega (1+u)^{\mu_i-2}\psi(u)\nabla u\cdot\nabla v dx\\ &= -\chi(\mu_i-1)\int_\Omega \Psi(u)\Delta v dx\\ &= \chi(\mu_i-1)\int_\Omega \Psi(u)(g(u)-v)dx\\ &\leq \chi(\mu_i-1)\int_\Omega \Psi(u)g(u)dx\\ &\leq \frac{\chi(\mu_i-1)}{\mu_i+q-1}\int_\Omega (1+u)^{\mu_i+q+l-1}dx\\ &\leq \chi\int_\Omega (1+u)^{\mu_i+q+l-1}dx.\end{aligned} \tag{2.8}$$

By using Young's inequality, we infer from $k > 1$ that

$$\begin{aligned} III &= \int_\Omega (1+u)^{\mu_i-1}f(u)dx\\ &\leq \int_\Omega (1+u)^{\mu_i-1}(a-bu^k)dx\\ &\leq \int_\Omega (1+u)^{\mu_i-1}(a-b+kb-kbu)dx\\ &= (a-b+2kb)\int_\Omega (1+u)^{\mu_i-1}dx - kb\int_\Omega (1+u)^{\mu_i}dx.\end{aligned} \tag{2.9}$$

Hence, it follows from (2.5), (2.8) and (2.9) that

$$\begin{aligned}\frac{1}{\mu_i}\frac{d}{dt}\int_\Omega(1+u)^{\mu_i}dx \le & -(\mu_i-1)\int_\Omega(1+u)^{\mu_i-2}|\nabla u|^2dx+\chi\int_\Omega(1+u)^{\mu_i+q+l-1}dx\\ &+(a-b+2kb)\int_\Omega(1+u)^{\mu_i-1}dx-kb\int_\Omega(1+u)^{\mu_i}dx.\end{aligned}\tag{2.10}$$

By $q+l<1$ and Young's inequality twice again, we see

$$\chi\int_\Omega(1+u)^{\mu_i+q+l-1}dx\le\frac{kb}{4}\int_\Omega(1+u)^{\mu_i}dx+C_1\tag{2.11}$$

and

$$(a-b+2kb)\int_\Omega(1+u)^{\mu_i-1}dx\le\frac{kb}{4}\int_\Omega(1+u)^{\mu_i}dx+C_2,\tag{2.12}$$

where

$$\begin{aligned}C_1&=\frac{1-q-l}{\mu_i}\left(\frac{kb}{4}\cdot\frac{\mu_i}{\mu_i+q+l-1}\right)^{-\frac{\mu_i+q+l-1}{1-q-l}}\chi^{\frac{\mu_i}{1-q-l}}|\Omega|\\ &=\frac{1-q-l}{\mu_i}\frac{kb}{4}\left(1+\frac{1-q-l}{\mu_i+q+l-1}\right)^{-\frac{\mu_i+q+l-1}{1-q-l}}\left[\left(\frac{4\chi}{kb}\right)^{\frac{1}{1-q-l}}\right]^{\mu_i}|\Omega|\\ &\le\frac{1-q-l}{\mu_i}\frac{kb|\Omega|}{4}\left[\left(\frac{4\chi}{kb}\right)^{\frac{1}{1-q-l}}\right]^{\mu_i}\end{aligned}\tag{2.13}$$

and

$$\begin{aligned}C_2&=\frac{1}{\mu_i}(a-b+kb)^{\mu_i}\left(\frac{kb}{4}\cdot\frac{\mu_i}{\mu_i-1}\right)^{-(\mu_i-1)}|\Omega|\\ &=\frac{1}{\mu_i}\frac{kb|\Omega|}{4}\left(1+\frac{1}{\mu_i-1}\right)^{-(\mu_i-1)}\left(\frac{4(a-b+kb)}{kb}\right)^{\mu_i}\\ &\le\frac{1}{\mu_i}\frac{kb|\Omega|}{4}\left(\frac{4(a-b+kb)}{kb}\right)^{\mu_i}.\end{aligned}\tag{2.14}$$

Now, taking

$$K_1=\frac{kb|\Omega|}{4}\max\{1-q-l,1\}$$

and

$$K_2=\max\left\{1+\left(\frac{4\chi}{kb}\right)^{\frac{1}{1-q-l}},\ 1+\frac{4(a-b+kb)}{kb}\right\},$$

it follows from (2.10)–(2.14), we derive

$$\frac{1}{\mu_i}\frac{d}{dt}\int_\Omega(1+u)^{\mu_i}dx+\frac{kb}{2}\int_\Omega(1+u)^{\mu_i}dx\le\frac{2K_1K_2^{\mu_i}}{\mu_i}.\tag{2.15}$$

Integrating (2.15) over (s,t) for all $t<T$, we have

$$\int_\Omega(1+u(x,t))^{\mu_i}dx\le\int_\Omega(1+u(x,s))^{\mu_i}dx+2K_1K_2^{\mu_i}T.\tag{2.16}$$

According to (2.2), we derive

$$\begin{aligned}\int_\Omega (1+u(x,t))^{\mu_i}dx &\le (1+M)^{\mu_i}|\Omega| + 2K_1K_2^{\mu_i}T \\ &\le K_0K^{\mu_i}(T+1)\end{aligned} \tag{2.17}$$

where $K_0 = 2K_1 + |\Omega|$ and $K = K_2 + M + 1$. The proof of Lemma 2.2 is complete. □

Now, we establish an iteration procedure to derive L^∞-estimate of $u(\cdot,t)$ for all $t \in (0,T)$, where $T \in (0, T_{\max})$.

Lemma 2.3. *Let (u,v) be a solution to (1.1) on $(0,T_{\max})$. Assume that $\psi(u)$ and $g(u)$ satisfy (1.2) with $q+l<1$, and $f(u)$ fulfills (1.3). Then there exists a positive constant $C>0$ such that*

$$\|u(\cdot,t)\|_{L^\infty(\Omega)} \le C \quad \textit{for all } t \in (0,T),$$

where $T \in (0,T_{\max})$.

Proof. Let

$$\mu_i = 2^i + 1 - q - l \quad \text{and} \quad i \ge 1, \tag{2.18}$$

it follows from Lemma 2.2 that

$$\int_\Omega u^{\mu_i}(x,t)dx \le K_0K^{\mu_i}(T+1) \quad \text{for all } t \in (s,T), \tag{2.19}$$

which implies

$$\|u(\cdot,t)\|_{L^{\mu_i}(\Omega)} \le K_0^{\frac{1}{\mu_i}} K(T+1)^{\frac{1}{\mu_i}} \quad \text{for all } t \in (s,T) \text{ and } i \ge 1, \tag{2.20}$$

where s, K_0 and K are given by (2.2) and Lemma 2.2, respectively.

Due to $q+l<1$, it follows that $\mu_i \to \infty$ as $i \to \infty$. Hence, letting $i \to \infty$ on both sides of (2.20), we have

$$\|u(\cdot,t)\|_{L^\infty(\Omega)} \le K \quad \text{for all } t \in (s,T). \tag{2.21}$$

On the other hand, we derive from Lemma 2.1 that

$$\|u(\cdot,t)\|_{L^\infty(\Omega)} \le M \quad \text{for all } t \in (0,s]. \tag{2.22}$$

Now, selecting $C := \max\{K, M\}$, it is easy to see that Lemma 2.3 holds. □

Now we begin with the proof of Theorem 1.1.

Proof of Theorem 1.1. With the aid of the blow up criterion (2.1) and Lemma 2.3, it follows that $T_{\max} = \infty$. Therefore, according to Lemma 2.1 and Lemma 2.3, we obtain the desired result. The proof of Theorem 1.1 is complete. □

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