Juraj Hledik

Correlated Assets and Contagious Defaults

Paper

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Abstract

We study systemic risk in a network model of the interbank market where the asset returns of the banks in the network are correlated. In this way we can study the interaction of two important channels for systemic risk (correlation of asset returns and contagion due direct financial linkages). We carry out a simulation study that determines the probability of a systemic crisis in the banking network as a function of both the asset correlation, and the connectivity and structure of the financial network. An important observation is the fact that the relation between asset correlation and the probability of a systemic crisis is hump-shaped; in particular, lowering the correlation between the asset returns of different banks does not always imply a lower probability of a systemic crisis.

Keywords: Systemic risk, Contagion, Financial Networks, Asset Correlations

1 Introduction

The availability of modern risk-transfer tools enables banks to diversify away idiosyncratic risk concentrations in their portfolios. However, diversification at the level of individual banks might lead to more similar asset positions across banks and thus to a higher correlation of bank's asset returns. This has sparked a debate on the impact of increasing asset return correlations on financial stability. Prior to the financial crisis risk transfer between banks and diversification at the individual bank level was generally regarded as something positive. This view is for instance embodied in the following quote from a 2002-speech of Alan Greenspan (then chairman of the FED) to the council of foreign relations, see Greenspan (2002).

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> [In the past year] I, particularly, have been focusing on innovations in the management of risk and some of the implications of those innovations for our global economic and financial system. The development of our paradigms for containing risk has emphasized dispersion of risk to those willing, and presumably able, to bear it. If risk is properly dispersed, shocks to the overall economic systems will be better absorbed and less likely to create cascading failures that could threaten financial stability.

Note that Greenspan explicitly entertains the idea that the default of any given financial institution may result in "cascading failures" of other banks via a network of direct credit relationships. Reducing idiosyncratic risk concentrations may thus be beneficial as it reduces the likelihood that individual banks default in the first place.

After the financial crisis, diversification at the level of individual-bank level and the potential increase in the correlation of banks' asset portfolios were seen much more critical. For instance, Wagner (2010) argues that while diversification may indeed reduce the default probability of individual banks, the ensuing rise in asset correlations increases the likelihood of a systemic banking crisis (an event where many banks fail simultaneously). However, in his analysis network effects and direct business links between financial institutions are neglected. Other contributions criticize a high level of correlation between banks' asset portfolios on different grounds. In particular, Acharya and Yorulmazer (2007) argue that banks have an incentive to engage in *herding* to induce possible government bailouts.

Given these different views, in the present paper we study the impact of correlated asset positions on financial stability in a *network model* for financial institutions. The network represents direct business links between banks such as a borrower-lender relationship. This permits us to include two important sources for a systemic banking crisis[1] in a single model: first we consider correlation between the asset positions of different banks (the so-called *correlation channel* for systemic risk); second we consider a contagious spreading of defaults through the financial network (the so-called *contagion channel* for systemic risk). We find that the correlation channel and the contagion channel are tightly connected; in particular,

[1]There are many different definitions of systemic risk. For our purposes, a systemic crisis is considered as an event where a significant proportion of financial institutions in the system default. The risk of such an event happening is then referred to as systemic risk.

the impact of an increase in asset return correlation on financial stability is ambiguous and depends on the structure of the financial network. It turns out that the stability of the system decreases with its density as well as with the asset return correlation only up to a certain threshold. After that, increasing the level of correlation of external assets as well as increasing the borrower-lender network density becomes beneficial in terms of financial stability. This is due to the role that connections play in our model. They represent direct credit transfers, such that if a bank goes bankrupt initially, it can cause further defaults in a contagious cascade. Therefore, as external assets become more correlated in terms of their portfolio holdings, the probability of observing isolated incidents of these initial defaults decreases. In a perfectly correlated system, where everybody holds the same portfolio, we either observe no defaults or a full systemic failure. In such case, the network amplification channel is completely eliminated. This line of reasoning can explain the *contagion puzzle* by a simple interplay of the two channels. Where both correlation and contagion might be insufficient to cause systemic crisis on their own, their joint effect can be potentially devastating.

We use a simulation approach for our analysis. We randomly draw a financial network from a set of networks with given probabilistic characteristics that reflect stylized facts observed in real-world interbank networks. Subsequently we generate a set of asset returns for the banks in the network. We assume that a bank defaults if confronted with a large enough negative asset return. In that case all its creditor banks suffer a loss. If this loss is sufficiently large, some of the creditor banks default as well, which then leads to further losses and possibly to a whole cascade of contagious defaults. The use of randomly generated networks serves to robustify our analysis with respect to the details of the network topology. This is important since the exact structure of real-world financial networks is hard to observe due to a shortage of relevant data on financial linkages.

Despite this inconvenience, we were able to obtain the interbank exposure data for the Austrian market. We show the viability of our framework on this dataset and show that both the qualitative and quantitative aspects of our results stay unchanged.

The present paper contributes also to the growing literature on network models and contagious defaults. The vast majority of papers in this area uses a two-step procedure. In the first step, they arrive at a network either by direct observation or by estimation on the basis of disclosed financial statements. Alternative approaches for network generation rely on micro-founded formation games (see Tardos and Wexler (2007)), asymptotic derivations for large and homogeneous networks (see Battiston et al. (2012)) or on simulation methods (see Hurd et al. (2014) or Hurd and Gleeson (2011)). In the second step, it is assumed that an exogenously chosen set of banks (called initially defaulting banks) fails, and the effect on the system is analyzed. Models of this type are frequently used by regulators. Examples include Elsinger et al. (2006) (Austria), Upper and Worms (2004) (Germany), **?** (UK) Degryse and Nguyen (2007) (Belgium), Blavarg and Nimander (2002) (Sweden), Mistrulli (2007) (Italy) or Lublóy (2004) (Hungary). Our setup differs from these contributions since we generate the set of initial defaulting banks by an economically relevant mechanism and since we study the interaction of the asset correlation channel and of the direct contagion channel.

Influential early papers in the academic literature on contagion and financial networks include Allen and Gale (2000) and Eisenberg and Noe (2001). Moreover, network models are becoming increasingly more popular in other areas of economics; see for example Braumolle et al. (2014) or a paper by Elliott et al. (2014) where the authors model crossholdings via a network model applied to European sovereign debt data.

2 Model and methodology

2.1 An example

Before we unveil the full scope of our model, we present a simple example on four banks to highlight the mechanics. By considering only four banks for this exercise, we are able to explain how correlated shocks and contagion interact together in a simple setting. Assume the following:

- Initially, bank $k \in \{1, 2, 3, 4\}$ receives a return r_k on its investment where r_k is a weighted sum of a market return r_M and a bank-specific idiosyncratic return ϵ_k: $r_k = \rho r_M + (1 - \rho)\epsilon_k$. Assume the same ρ for every bank and that $r_m, \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4$ are uniformly distributed on {-1,0,1}. Therefore, it is equally likely that a given return will be positive, negative, or zero.

- We say that a bank suffers a fundamental default if $r_k < 0$.
- We further assume that banks are connected in a network of their loans. If a bank defaults, it spreads the shock to all its creditors in a contagious fashion. If more than half of bank's debtors default, so does the bank itself. We call this a contagious default.

This simple framework serves as an introduction to the general setup from Section 2.2. There, we consider an arbitrary number of institutions, continuous return distribution and a full balance sheet structure for each bank in the system. Still, this simplified setting already allows us to observe how fundamental defaults give rise to contagious defaults and how this process depends on the structure of the system. For the purpose of this stylized setting, we always assume that network is symmetric, e.g. bank A is a debtor of bank B if and only if bank B is a debtor of bank A[2]. We consider all possible loan networks that can arise on a set of four banks as depicted in Figure 1.

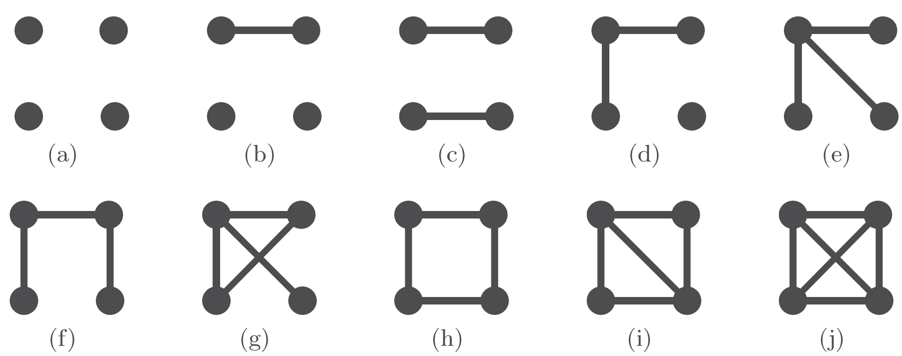


Figure 1: All possible loan network structures on a set of four banks.

We say that we observe a systemic crisis in the system if more than 50% of the banks in the system (which means 3 or 4 banks) are in default (fundamental or due to contagion). Since we know the distribution of market- and idiosyncratic returns, it is possible to quantify the probability of such event. For example, in network (d), probability of crisis in zero asset correlation case ($\rho = 0$) is 0.4074. To obtain this number, one needs to see

[2]This assumption will be relaxed in the general setting where asymmetric loan structures are considered.

that to get more than two defaults (our definition of systemic crisis), there are only two options. Option one: bank in top left corner defaults and contagiously spreads the default to banks in top right and bottom left corners. Option two: banks in top left and bottom right corners default simultaneously, forcing bank in the top left corner to default as well. Probability of the first scenario is 1/3, probability of the second is (1/3)*(1/3)=1/9 (zero correlation means that fundamental defaults are independent) and their joint probability is (1/3)*(1/3)*(1/3)=1/27. Therefore, the probability of systemic crisis is 1/3 + 1/9 - 1/27 $\backsimeq$ 0.4074[3]. In Table 1, probability of systemic crisis is evaluated in the same spirit for all networks (a)-(j) and values of rho equal to 0 (no asset correlation), 0.5 (intermediate level of correlation) and 1 (full correlation). Parameter C in the second column represents the average number of connections of a bank in the given network.

Table 1: Probability of systemic crisis as a function of network topology (see Figure 1) and return correlation.

	C	$\rho = 0$	$\rho = 0.5$	$\rho = 1$
(a)	0	0.1111	0.2346	0.3333
(b)	0.5	0.3086	0.3663	0.3333
(c)	1	0.3086	0.3663	0.3333
(d)	1	0.4074	0.4074	0.3333
(e)	1.5	0.5062	0.4733	0.3333
(f)	1.5	0.2593	0.3333	0.3333
(g)	2	0.3580	0.3992	0.3333
(h)	2	0.2099	0.3004	0.3333
(i)	2.5	0.4074	0.4321	0.3333
(j)	3	0.4074	0.4321	0.3333

We observe that for most network structures, there is a hump-shaped behavior in the correlation dimension. This is not true for an empty network, as we will see also in our general setup. The reason for this observation is the fact that without the network structure as an amplification channel, it is extremely unlikely to observe many fundamental defaults happening at the same time. This is the main point of the paper. By introducing the contagion channel, it is suddenly possible to observe large crises even for much lower level of asset correlation. In other words, intermediate levels of asset correlation are the most

[3]It is obvious that in this simplified setting, initial return is only important to determine whether a bank experiences a fundamental default or not. However, in the general framework, the level of initial return will affect a bank's capital buffer, therefore making it vulnerable to future shocks even in the case it survives initially.

dangerous. The rationale behind this observation is simple: In the case of a perfect asset correlation ($\rho = 1$), the loan network does not play a role and if we observe a systemic crisis, then it automatically means that everybody has suffered a fundamental default. This happens with probability 1/3, when the market return realization is -1. However, in the case of imperfect correlation, it is more likely that at least one of the banks in the system experiences a fundamental default. As a consequence, this bank can spread the shock in a wave of contagious defaults. Apparently, the worst case arises when these two channels interact ($\rho = 0.5$). In this case, banks' returns are somewhat correlated, but not perfectly such that the contagion channel still plays a role. This is also the point of our general setup.

2.2 The Model

The financial network. We will represent the network of interbank exposures by a directed graph $\mathbb{G}$ consisting of N nodes. Each nodes represents an unspecified financial institution, while edges between them represent their (directed) credit exposures. Alas, an edge from bank i to bank j means that bank j has a credit exposure towards bank i. [4]

The graph[5] $\mathbb{G}$ is described by an adjacency matrix $E_{\mathbb{G}}$ with elements e_{ij} satisfying:

$$e_{ij} = \begin{cases} 1 & \text{if } i \text{ is a debtor of } j\,, \\ 0 & \text{if } i \text{ is not a debtor of } j\,. \end{cases} \tag{1}$$

We use the following notation to describe the balance sheet of the banks in the network. The total asset value of bank k by is denoted A_k; the nominal value of the loans made to other banks in the system is denoted by A_k^{IB} (short for interbank); the external assets (e.g. loans to non-banks) are denoted by A_k^{EX}; finally, L_k^{IB} and L_k^{EX} represent the interbank liabilities and the external liabilities (e.g. customer deposits) of bank k, so that total liabilities are equal to $L_k = L_k^{IB} + L_k^{EX}$. The equity of bank k is then given by $E_k = A_k - L_k$ and E_k/A_k is the *capital ratio* of the bank. These quantities are illustrated in Table 2.

[4]There are different possible interpretations for the role of connections in our model, ranging from direct debt to derivative exposures.

[5]Throughout the paper we use the terms *graph* and *network*. When talking about a graph, we are concerned with the structure of financial linkages, whereas when referring to a network, we mean not just connections themselves but also balance sheet quantities of individual banks.

Table 2: Balance sheet of bank k

assets	liabilities
interbank assets A_k^{IB}	interbank liabilities L_k^{IB}
external assets A_k^{EX}	external liabilities L_k^{EX}
	equity E_k
total assets A_k	total liabilities L_k

Following assumptions allow us to create the financial network from a given adjacency matrix $E_{\mathbb{G}}$.

Assumption 1 *All loans in the system are of the same size, normalized to one*[6].

Under Assumption 1 bank k's interbank assets A_k^{IB} are given by the number of its debtors and the interbank liabilities L_k^{IB} are equal to the number of its creditors, that is

$$A_k^{IB} = \sum_{i=1}^{N} e_{ik} \text{ and } L_k^{IB} = \sum_{j=1}^{N} e_{kj}, \tag{2}$$

where e_{ij} are elements of $E_{\mathbb{G}}$ as described in equation (1).

The next assumption can be viewed as a stylized version of the risk capital requirements imposed under the current Basel regulations.

Assumption 2 *The capital ratio E_k/A_k of every bank is equal to an exogenously given constant $\gamma_k < 1$.*

Finally, we make an assumption on A_k^{IB}/A_k, the ratio of bank k's interbank assets and of its total assets; following the language of Elliott et al. (2014) we refer to this ratio as the level of *integration* of bank k into the network. Loosely speaking we assume that the level of integration is equal to an exogenously given constant $\kappa > 0$ or, equivalently, that $A_k = \frac{1}{\kappa} A_k^{IB}$. However, under Assumption 2 this is not always consistent with the requirement that the external liabilities are nonnegative. In fact, the requirement that $L_k^{EX} \geq 0$ gives that

$$\gamma_k A_k = E_k = A_k - L_k^{IB} - L_k^{EX} \leq A_k - L_k^{IB},$$

[6]Equivalently, one could relax the assumption of equal loan sizes, assuming equal balance sheet size instead. These two models are identical from the technical point of view.

and hence the inequality $A_k \geq L_k^{IB}/(1-\gamma_k)$. Motivated by these considerations we make the following assumption on A_k.

Assumption 3 *The total asset value of the banks in the network is given by*[7].

$$A_k = \max\left\{ \frac{1}{\kappa_k} A_k^{IB}, \frac{1}{1-\gamma_k} L_k^{IB} \right\}, \quad k = 1, \dots, N. \tag{3}$$

For typical parameterizations of the model $\frac{1}{\kappa_k}$ is significantly larger than $\frac{1}{1-\gamma_k}$. In that case if $A_k^{IB} \approx L_k^{IB}$ the first term from (3) is binding so that $\kappa_k A_k = A_k^{IB}$. If L_k^{IB} is much larger than A_k^{IB} the second inequality is binding and ensures that the total balance sheet size is not lower than the sum of interbank liabilities and equity

The external assets (liabilities) are finally given by the difference between total assets (liabilities) and interbank assets (liabilities plus capital buffer). This gives

$$A_k^{EX} = A_k - A_k^{IB} \qquad \text{and} \qquad L_k^{EX} = A_k - E_k - L_k^{IB} = (1-\gamma_k)L_k - L_k^{IB}. \tag{4}$$

To summarize, we have created a balance sheet structure from a given adjacency matrix $E_{\mathbb{G}}$ along the following steps:

1. Assign the value of interbank assets A_k^{IB} and liabilities L_k^{IB} of every bank in the network according to equation (2).

2. Determine the asset value A_k, $k = 1, \dots, N$, of the banks according to (3).

3. Define A_k^{EX} and L_k^{EX} according to equation (4).

Initial defaults. We randomly draw the return on bank ks external assets, denoted r_k, from a given distribution. Consequently, a sufficiently negative return can force a bank to default which we refer to as the *initial default*. Specifically, an initial default occurs if the

[7] In a special case where a bank has no connections at all, we assume that it puts a portion κ of its total assets into some riskless investment (such as government bonds) and 1-κ into risky external assets. We also assume that the size of the balance sheet A_k is 1. This is completely harmless since a bank with no connections cannot spread contagion to the rest of the system. This is a purely technical workaround: otherwise, according to the definition, we would see that banks without connections have their balance sheet size equal to zero.

bank k asset value after the return realization is lower than the liabilities of bank k, that is if

$$A_k^{IB} + A_k^{EX}(1 + r_k) < L_k\,.$$

Since $L_k = A_k - E_k = A_k(1 - \gamma_k)$, an initial default thus occurs if $r_k < -\gamma A_k / A_k^{EX}$. For banks with level of integration equal to κ (the typical case) this can be rewritten as $r_k < -\gamma/(1 - \kappa)$.

Correlation of asset returns. We assume that the random variables $r_1, \ldots, r_N$ follow the following simple one-factor model:

$$r_k = \mu + \sqrt{\rho} r^M + \sqrt{(1 - \rho)}\epsilon_k, \quad 1 \leq k \leq N. \tag{5}$$

Here r^M is a *market return* that is common for all banks in the system and ϵ_k is an *idiosyncratic return* that differs across banks. We assume that r^M and $\epsilon_1, \ldots, \epsilon_N$ are independent and normally distributed with mean zero and standard deviation $\sigma = 0.2\sqrt{dt}$ with $dt = 1/252$. The parameter μ in equation (5) is set equal to $\mu = 0.05dt$

Under the factor model (5) the correlation between the asset value change r_k and r_l of two different banks is equal to ρ; in particular, for $\rho = 0$ the sensitivity of any bank to the market return is zero such that its solvency is only driven by its own idiosyncratic return. On the other hand, for $\rho = 1$ there are no individual shocks and every bank faces the same return on its external assets. Note that (5) implies that the marginal distribution of r_k and hence the probability of an initial default is not affected if the correlation parameter ρ is varied. This is in stark contrast to the analysis of Wagner (2010), where a higher level of correlation of different banks is associated with a lower volatility of banks' asset returns and hence with a lower probability of initial defaults. We come back to this issue in Section **??** below.

Table 3 illustrates the impact of ρ on the distribution of initial defaults. We see that lowering the value of ρ has two effects: the probability of observing at least one initial default is increased, while the probability that a large fraction of the banks in the system default decreases. These effects are well-known in the literature on portfolio credit risk

	$\rho = 0$	$\rho = 0.3$	$\rho = 0.5$	$\rho = 0.9$
P(k is in default)	0.024%	0.024%	0.024%	0.024%
P(at least one default)	2.40%	1.97%	1.39%	0.25%
P(at least 20% in default)	$4.9\ 10^{-13}$%	0.0043%	0.0077%	0.0088%

Table 3: Individual default probability of bank k, probability of observing at least one initial default and probability that more than 20% of the banks in the system default initially for varying correlation parameter ρ ($N = 100$ banks, $\gamma = 0.035$, $\kappa = 0.2$).

models; see for instance Frey and McNeil (2003).[8]

2.3 The Contagion Channel

We assume that a defaulted bank is unable to fulfill its obligations, which results in a reduction of the interbank assets of its creditors. If these losses are big enough it may cause some of the creditors to default as well. For simplicity we assume zero recovery on defaulted interbank loans.[9].

For a financial network with given adjacency matrix $E_{\mathbb{G}}$, balance sheet quantities $A_k^{IB}, A_k^{EX}, L_k^{IB}, L_k^{EX}, A_k, \gamma_k$ and given return realization r_k for every bank k, the mechanism that (potentially) generates a default cascade is described as follows:

1. Shock the external assets A_k^{EX} of each bank k by the return realization r_k.
2. If any of the banks defaults, propagate the shock to the asset side of its creditors. The new amount of interbank assets satisfies: $A_k^{IB}(\text{new}) = A_k^{IB}(\text{old}) - \sum_i e_{ik} \mathbf{1}_{\{i \text{ is in default}\}}$
3. If the total value of bank k's assets falls below its liabilities, that is $A_k < L_k$, bank k defaults.
4. Repeat steps 2 and 3 until there are no further defaults.

[8] We have also conducted all of our simulations on a set of multivariate t-distributed returns to account for tail risk. The probability of a systemic crisis has a different magnitude in this case, but the qualitative results stay the same as in the normally distributed returns scenario.

[9] Thanks to this assumption, there is no need for a settlement algorithm in the spirit of Eisenberg and Noe (2001). Assuming non-zero recovery rate on distressed loans would however not change the overall quantitative nature of our results.

2.4 Simulation procedure and network generation

We conduct a two-layer Monte Carlo analysis. In the inner layer, we generate $K = 500$ return realizations that follow the factor model (5). This whole layer is embedded in the outer layer where 1000 random networks are created. For this we use two different probabilistic models, namely a homogeneous Erdos-Renyi random graph and a inhomogeneous model that generates graphs with a core-periphery structure[10].

2.4.1 Homogeneous (Erdos-Renyi) random graphs

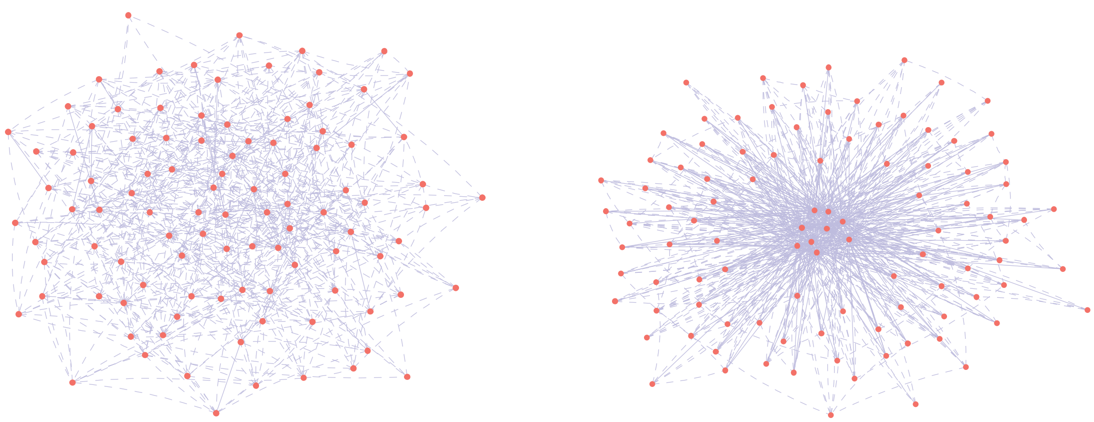
In the Erdos-Renyi model, a random graph is generated such that the probability that there is an edge between any two nodes in the graph is a constant number p_{ER}; or put differently, every Erdos-Renyi random graph is parameterized only by two numbers - the number of nodes in the graph N and the probability p_{ER} that any two of them are connected. As a result, connections are formed independently, such that the elements e_{ij}, $1 \leq i, j \leq N$ of $E_{\mathbb{G}}$ are iid Bernoulli random variables. For $p_{ER} = 1$ we get a complete directed graph in which every bank is connected to every other bank and vice versa, while for $p_{ER} = 0$ there are no links between the banks in the system.

2.4.2 Inhomogeneous (core-periphery) random graphs

According to Soramäki et al. (2006), Bech and Atalay (2010), Iori et al. (2008) and others, a typical financial network exhibits a significant degree of so-called *disassortativity*, that is small banks tend to be connected to large ones and vice versa. An interpretation of this finding is that large banks act as intermediaries for smaller ones. This structure is in contrast to the structure of social networks that tend to be assortative (people with few friends tend to be connected with other people having a small number of friends). To account for the observed disassortativity, we extend the Erdos-Renyi setting by making each bank belong either to a group called *core* with probability p_{core} or to a group called *periphery* with a probability $1 - p_{core}$. The difference between these two groups of institutions lies in

[10]A core-periphery structure is a typical scenario for a banking network structure which includes bigger banks connected heavily amongst themselves and smaller banks connected to bigger banks only. Then the set of tightly connected banks is referred to as the core, while the set of smaller, less connected banks is referred to as periphery.

the probability of forming connections with other banks. A core bank has a large probability of establishing a connection both with other core banks and with other peripherals while a connection between two peripherals is less likely. In this paper we take the probability of a connection between two core banks equal to $p_{CC} = 0.9$; the probability of a connection between two peripheral banks is set to $p_{PP} = 0.005$; the probability of a connection between a core bank and a peripheral in either direction is set to $p_{CP} = p_{PC} = 0.5$. Given the type of the banks in the system, connections are formed independent of each other. In this way we end up with an assortative network that we refer to as a *core-periphery* structure. The resulting network exhibits a star shape with few banks tightly connected in the center and the rest on the periphery. In financial terms core banks can be interpreted as (large) dealer banks that act as an intermediary for the other banks in the network. The difference between an Erdos-Renyi and a core-periphery network is illustrated in Figure 2.



(a) Erdos-Renyi random graph

(b) Core-periphery random graph

Figure 2: One realization of a random graph for $N = 100$ banks; left panel Erdos-Renyi network; right panel core-periphery network.

Note that since a core bank has on average more connections than a peripheral bank a higher value of p_{core} leads to a higher density of the ensuing network. In particular, for $p_{core} = 1$, the whole network is formed by core banks so we actually get a very dense Erdos-Renyi setting (identical to the case where $p_{ER} = 0.9$) whereas for $p_{core} = 0$, every bank is peripheral. Since peripherals are connected with probability $p_{PP} = 0.005$, we get a sparse Erdos-Renyi setting (corresponding to $p_{ER} = 0.005$). Therefore, an intermediate level of p_{core} corresponds to a network which lies between two homogeneous Erdos-Renyi extremes. In the Monte Carlo simulations, the probability p_{core} of belonging to the core is varied between 0 and 20%.

3 Results

We now present the results of a simulation study that illustrates the impact of the asset return correlation and of the density/connectivity of the network on financial stability. We measure the density of a given network by the expected number of counterparties of a randomly chosen bank in the system.[11] From now on, we will call this quantity *connectivity* and denote it by C. In the Erdos-Renyi random graph, connectivity is given by $C = p_{ER}(N-1)$; in the case of a core-periphery network, connectivity is easily seen to be

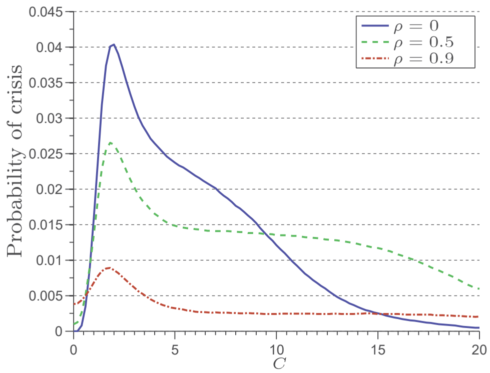
$$C = (N-1)(p_{core}^2 p_{CC} + p_{core}(1-p_{core})(p_{CP} + p_{PC}) + (1-p_{core})^2\, p_{PP}).$$

The output variable in our analysis is the relative frequency of scenarios in the simulation in which a systemic crisis occurred. Here a scenario is viewed as one realization of random network together with one realization of random returns, and a systemic crisis is defined as a scenario where more than 20% of all banks in the network are in default at the end of the default cascade. In the sequel we will call this relative frequency simply the *probability of a systemic crisis.* Note that the exact value of the threshold in the definition of a systemic crisis (20% or different) is irrelevant. In fact, for all but very small values of the connectivity parameter C we observed a *dichotomous* behavior: in a given scenario there are either very few defaults or the network is wiped out (almost) entirely. This behavior was observed for both network types and for all values of ρ.

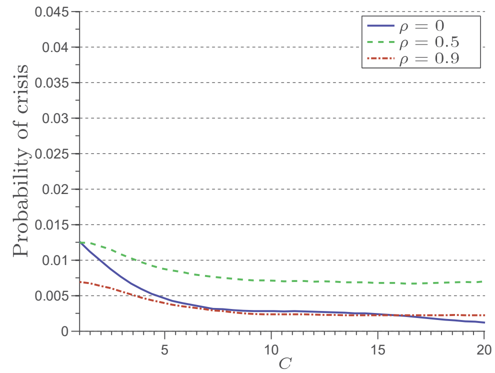
3.1 Erdos-Renyi networks.

The results for Erdos-Renyi random networks are illustrated in Figures 3a and 4a. Figure 3a gives the probability of a systemic crisis for fixed ρ and varying C; Figure 4a depicts sections for fixed C and varying ρ.

[11] In graph theoretic literature, this is known as the average graph degree.

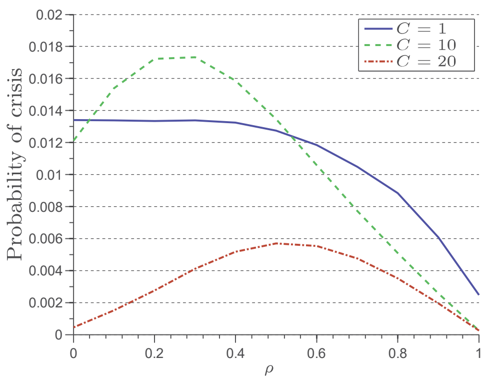


(a) Erdos-Renyi network

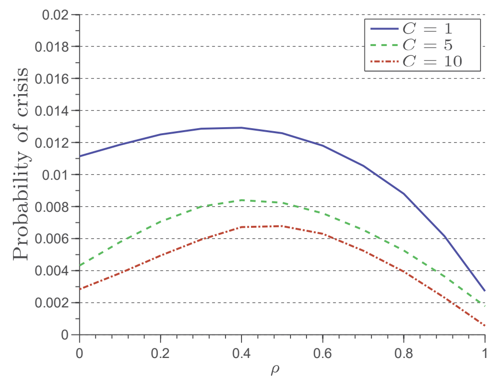


(b) Core-periphery network

Figure 3: Probability of systemic crisis for both network structures on $N = 100$ nodes as a function of network connectivity C for particular levels of correlation ρ. Bank equity ratio $\gamma = 0.035$ and integration $\kappa = 0.2$.



(a) Erdos-Renyi network



(b) Core-periphery network

Figure 4: Share of scenarios with systemic crisis for both network structures on $N = 100$ nodes as a function of asset correlation ρ for particular levels of connectivity C. Bank equity ratio $\gamma = 0.035$ and integration $\kappa = 0.2$.

Connectivity. First we discuss the impact of variations in network connectivity C (see Figure 3a). Here we observe a hump-shaped behavior: for small values of C, the probability of a systemic crisis is small. Intuitively, this is due to the fact that in a very sparse network the contagion channel is inactive since there is almost no opportunity for shock propagation. As C increases the likelihood of a systemic crisis increases up to a maximum at which the system is most vulnerable. Beyond that maximum the probability of a systemic crisis decreases again, and banking networks with a high connectivity appear to be fairly resilient. This resilience is due to enhanced hedging opportunities of the institutions in the system (if a bank has more counterparties, the loss caused by its default is borne by more banks). Finally we see that for ρ close to one the probability of a crisis is relatively insensitive with respect to network connectivity. This is due to the fact that for ρ large the occurrence of a systemic crisis is determined to a large extent by the realization of the common return factor r^M, independent of the structure of the financial network.

Note finally that the hump-shaped form of the relation between C and the probability of a systemic crisis in Erdos-Renyi graphs is in line with findings from other recent papers in the network literature, see for instance Hurd et al. (2014), **?**, or Elliott et al. (2014).

Correlation. Next we consider the impact of varying the asset return correlation ρ (see Figure 4a.) For medium and high values of C we observe a hump-shaped behavior. For ρ close to zero the probability of a systemic crisis is *increasing* in ρ. This is of course due to the fact that by increasing ρ we increase the probability that a large part of the system defaults initially, see Table 3. However, if ρ exceeds a certain threshold $\bar{\rho}$, the probability of a systemic crisis is *decreasing* in ρ. In order to understand this behavior we make recourse to the argument of Greenspan (2002) mentioned in the introduction of the paper: with direct links between banks there can be default cascades during which the initial default of a few financial institutions spreads through a large part of the financial system. Such a cascade is more likely if many of the "initial survivors" are also close to default because they were hit by a negative shock on their asset returns. This is in turn more likely for ρ large since in that case a negative market-return shock substantially weakens all banks in the system. In fact, for ρ sufficiently high a single initial default may be enough to generate a systemic crisis via the contagion channel. Moreover, we know from Table 3 that the probability of observing *at least one* initial default is decreasing in ρ. Taken together, these arguments explain the hump-shaped nature of the relation between ρ and the probability of a systemic crisis. For low values of C the network is very fragile (recall our discussion of Figure 3a), so that the "Greenspan effect" (the fact that a higher ρ may decrease the probability of a systemic crisis) kicks in already for relatively low values of ρ; in the extreme case $C = 1$ the probability of a systemic crisis is even decreasing in ρ for all values of ρ.

3.2 Core-periphery networks.

We repeat the same analysis for core-periphery networks. The results are depicted in Figures 3b and 4b. As in the Erdos-Renyi case the probability of a systemic crisis is a nonlinear function of ρ and C. The relation between ρ and the probability of a systemic crisis is of the same hump-shaped form as in the Erdos-Renyi network, with a similar interpretation (compare Figures 4a and 4b.) In the core periphery networks the probability of a systemic crisis is generally lower than in the Erdos-Renyi case. These findings are in line with the general claim that heterogeneous network structures are relatively resilient, see for instance Gai et al. (2011) and Simon (1962). Moreover, they lend support to regulatory attempts to generate networks with a high degree of connectivity, for instance by limiting the amount of direct lending between any two financial institutions.

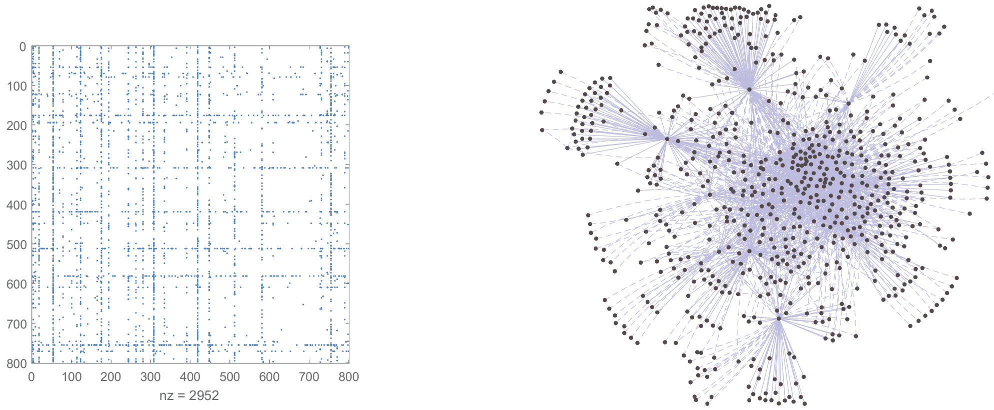
We also consider a variant of the model where the equity capital ratio γ_{core} of core institutions (institutions that have many links and that can therefore be regarded as systemically important) is higher than the equity ratio of peripherals. We found that this modification significantly reduces the probability of a systemic crisis, which obviously supports proposals to regulate systemically important institutions more tightly.

4 Empirical Analysis

In addition to our theoretical findings, we also provide an application of our framework on the real world data. For this purpose, we are using the data generously provided by the Austrian National Bank (OeNB). It contains quarterly observations of interbank exposures across 800 biggest Austrian banks which we aggregate to a single network. For further details about the dataset, please refer to Puhr et al. (2014).

We keep all assumptions in our model unchanged and run our contagion algorithm for this particular network structure. Naturally, since we are dealing with a given network structure, we do not need to resort to any network generating algorithms as we did before. Consequently, we can only observe the effect of correlated assets on the probability of crisis, the network effect is no longer relevant.

Figure 5 shows the structure of the network both as an adjacency matrix as well as



(a) Adjacency matrix

(b) Plot of the network snapshot

Figure 5: Adjacency matrix for the OeNB, consisting of 2952 edges represented as dots (a) and a graphical representation of the network snapshot for the nodes with at least one connection (b).

a graphical representation. As expected, we can observe that it exhibits a core-periphery structure which is typical for financial networks.

With the network structure given, we only run a single layer of the Monte Carlo simulation to generate random returns as explained in Section 2.2 (Eq. 5). Again, we look at the probability of systemic crisis and plot the results in Figure 6. It should not come as a surprise that we again observe the same hump-shaped behavior as in the case of our simulated networks. The order of magnitude of crisis probability stays unchanged which strengthens the robustness of our results. With additional data on bank asset correlation, one could use our framework to estimate this probability with even higher precision.

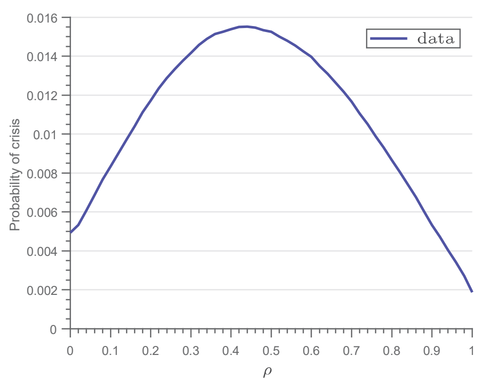


Figure 6: Share of scenarios with systemic crisis for the Austrian interbank market as a function of asset correlation ρ. Bank equity ratio $\gamma = 0.035$ and integration $\kappa = 0.2$.

5 Conclusion

We have presented a simulation study that is concerned with the joint effect of correlated asset positions and of the network structure of a banking system on financial stability. Both a simple case of a homogeneous Erdos-Renyi network and a more realistic scenario of inhomogeneous core-periphery network structure were examined in the process.

We conclude that in order to judge the implications of correlation on the magnitude of systemic risk, one needs to take the underlying network structure into account. Most dangerous are homogeneous networks of intermediate density since they are dense enough to propagate shocks but not dense enough to hedge off potential risk. Moreover, we found that lower values of asset correlation do not always reduce the probability of a systemic crisis Furthermore, we present results for the more realistic case of core-periphery networks. There, the probability of a crisis is generally lower than in the homogeneous Erdos-Renyi network, which indicates high resiliency of such networks. This is further documented on the dataset of actual interbank exposures in Austria where same conclusions can be drawn with respect to the level of correlation among financial institutions.

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