Three essays on horizontal product differentiation and price dispersion

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1 Introduction

Simple textbook models of perfect competition and monopoly markets, and their corresponding solutions of equilibrium prices and quantities provide only crude descriptions for the forces at hand that drive the microeconomic behavior of agents in many markets. Consequently, alternative and more detailed approaches need to be developed. This thesis is rooted within this field of research and studies two phenomena that are not satisfactorily explained by classical textbook models: firstly, market entry and competition for a duopoly considering the underlying geometrical and spatial market structure, and secondly, the existence and determinants of price dispersion using empirical data from the Austrian retail gasoline market.

What are the limitations of the textbook models and why are phenomena such as spatial competition and price dispersion not accounted for? The common model of perfect market competition explains observed market prices and produced quantities by the concept of an equilibrium state where supply matches demand based on consumers' preferences and firms' production technology. In this model it is well understood that supply and demand rest upon an aggregation of individual preferences and budget endowments as well as individual production and cost curves. Since the number of firms and consumers is assumed to be sufficiently high, for each individual firm and consumer it is impossible to have an impact on the equilibrium outcome. Put differently, the achieved price and quantity are given for each agent and strategic interaction between any of the many agents can be considered negligible for the equilibrium state (and would therefore not occur). A further implication is that the traded product in a perfectly competitive market would have to be perfectly homogeneous with respect to its physical characteristics and utility for consumption. That is, each individual product needs to be a perfect substitute, otherwise a firm would have arbitrage opportunities and the competitive forces exemplified by the equilibrium price would not be effective. Moreover, the model requires free entry and exit for firms into the perfectly competitive market. The intuition is again to avoid arbitrage, and that for the competitive market forces to work swiftly consumers should be able to switch between different suppliers without any cost.

Now, consider the first of the research interests of this thesis: to model the economic behavior of two firms located at different retail outlets and competing for consumers distributed over a geographical area consisting of a road network the basic conjectures

of perfect market competition have to be adapted. Firstly, the concept of an equilibrium as a result of an aggregation of individual economic parameters (and interests) can not be sustained in a duopoly. Clearly, the strategic interaction between firms represents a determinant for the market outcome. The profit-maximizing price decisions of each agent have to consider the optimal decision of its rival. Consequently, the derivation of an equilibrium outcome requires a detailed examination of possible strategies a firm could devise accounting for mutual dependencies, that is for the rival's response and consecutive actions to be taken. Secondly, since sellers are located at different places their products can not be considered as homogeneous anymore. A reasonable assumption in a spatial market setting is that consumers located closer to a particular retail outlet enjoy an advantage in consumption compared to remotely located consumers. Thus, the geographical proximity to sellers' premises serves as a characteristic that differentiates their products. In this case products are considered to be horizontally differentiated since the valuation of characteristics depends on the particular consumer.[1] The instance of horizontal differentiation is linked to the third assumption since the notion of product heterogeneity is linked to transportation costs incurred by the consumers which are proportional to the distance traveled. Moreover, location and relocation costs for firms are likely to be prohibitively high on certain markets such that free entry and exit as well as free relocation choices may not be generally justified.

Turning to the second research interest of this thesis, the existence of price dispersion breaks with the assumption that in a perfectly competitive market equilibrium one distinct price occurs. Rather, empirical evidence suggests that price dispersion of homogeneous goods is widespread and significant. One important explanation for price dispersion are information asymmetries suggesting that market demand does not consist of perfectly homogeneous consumers. Rather, the asymmetric dissemination of information and the existence of different consumer groups may help to explain the phenomenon of price dispersion. This motivates our work for an investigation of price dispersion with data for the retail gasoline market in Austria.

Besides the common source of the idealized textbook model of perfect market competition, how are the two different research interests of this thesis connected? In a seminal paper in the discipline of industrial economics Salop (1979) showed that a number of different sellers who are located in a symmetrical pattern around a circle all charge the same profit-maximizing price (above marginal cost). In his model sellers are located at different locations but due to the symmetry a unique price level in equilibrium obtains. Furthermore, Economides (1993) demonstrates that for a number of sellers on a bounded line (in a variant of the original model of Hotelling (1929)) a symmetrical location pattern yields a convex, symmetric, U-shaped equilibrium price structure. His explanation rests upon the fact that as a result of the

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[1]For further explanations on the definition of a product space see Tirole (2003), p. 96ff.

geometrical environment sellers have a different degree of market power. In particular, the closer a firm is located to the edge of the market the more it is able to exploit its monopoly power and therefore charges a higher price. In general, it can be concluded from these studies, firstly, that differences in the spatial distribution of sellers critically impact their optimal price decision and thus the overall price distribution in the market, and secondly, that the geometrical set-up is also a determinant for the spatial competition and firms' price decision. This, in a nutshell, defines the 'game plan' of this thesis. In the first part the research interest focuses on the explanation of firms' location decision in the geometrical setting of intersecting roads. As it can be expected that the price distribution shows special characteristics in such a market environment, the second part studies the empirical price distribution on the gasoline market which serves as a representative example for the sort of spatial competition investigated in the theoretical part.

A formal requirement for this thesis is that the research output shall be presented at relevant scientific conferences. This requirement has been achieved. The developed sequential two-stage price-location model has been presented at the 8th International Research Meeting in Business and Management (IRMBAM) on 5th of July 2017 held at the IPAG Business School in Nice, and the XXXII Jornadas de Economia Industrial on 7th of September 2017 held at the University of Navarra in Pamplona. The empirical study on the gasoline price distribution for Austria has been presented at the XXVI Jornadas de Economia Industrial on 16th of September 2011 held at the University of Valencia in Valencia, and at research seminars at the School of Geographical Sciences of the Arizona State University (ASU) on 14th of April 2012 and at the Rijksuniversiteit Groningen in September 2011 in the cause of scholar visits supported and organized by the Network for European and United States Urban and Regional Studies (NEURUS).

To sum up, this thesis examines the effects of an extension of the assumptions of the classical model of perfect market competition. In particular, a spatial competition model for a duopoly with two firms competing under horizontal product differentiation is explored, and an empirical investigation of price dispersion for a market with horizontal product differentiation is conducted. Generally, the outcomes stress that a closer look at the supply side in terms of the strategic interaction between firms, and a closer look at the demand side in terms of heterogeneous consumer groups proves to be valuable to gain insights on determinants of market equilibrium states. More detailed findings are as follows.

The *first* essay provides an introduction into the literature of spatial competition models and studies their predictions on the degree of horizontal product differentiation. For this purpose a selection of articles, mainly from the game theoretical strand of the literature, is re-examined in which each model extends and modifies basic parameters of the original model of Hotelling (1929). The selection is based on

a representation of characteristic determinants to explain spatial product differentiation.

The main determinants are considered to be: consumers' reservation price, transportation costs, the number of firms, the timing and further specifics of the price-location game, characteristics of the consumer distribution, and the market geometry. Furthermore, the literature survey emphasizes that markets consisting of intersecting roads represent a particular fruitful subject of future research. The nature of competition in this market setting is different compared to the linear city exemplified by the importance of asymmetrical location patterns. Consequently, the strategic interaction, firms' profit-maximizing behavior and potential equilibrium outcomes under sequential entry in a market with intersecting roads remain to be an interesting field to study.

The *second* essay addresses this research gap and based on the work of Anderson (1987) studies a two-stage market entry game in a spatially extended Hotelling's duopoly. Particularly, the effect of a demand dependent centrality bonus Z distributed in the middle of the linear city is examined on the reaction functions of an incumbent firm and the strategic entry decision of an entrant firm. A solution is provided for an entry accommodating scenario where both players optimize profits over their strategic variables and the center Z is taken by the incumbent firm. The results further suggest that the entrant is not capable of capturing Z. In addition, the model implies a lower degree of product differentiation as Z increases.

A comparison with the literature shows that these results are well in line with Anderson's model for $Z = 0$. In a business strategy view the outcome supports the thesis of Gelman & Salop (1983), coined by the term 'judo economics', since the entrant earns highest profits by committing himself to a distant location and charging a comparatively lower price than the incumbent.

The *third* essay analyzes the price distribution of diesel in the Austrian retail gasoline market and tests predictions of the impact of the fraction of informed and uninformed consumers on the mean price and price variance. Further, introducing two measures of spatial competition, the relation of local competition between stations and the mean and variance are examined.

In a pooled cross-section analysis a two step approach is followed. Initially, price levels are estimated with respect to the influence of competition, search costs, stations' location and further station-specific characteristics. Controlling for these observable price characteristics, the residuals are used in the second step to investigate the behavior of the price variance. In addition to OLS, to account for spatial spillover effects a Spatial Error Model (SEM) is applied to estimate the price function. Additionally, tests on model specification and robustness checks using different weighting matrices, search cost proxies and dispersion measures are carried out.

The results reveal a negative (positive) correlation between the fraction of informed

(uninformed) consumers and the mean price. Further, price variance shows an inverse U-shape with the fraction of informed consumers. Thus, the variance initially increases as the proportion of informed consumers increases and starts to decline after the share of informed exceeds a threshold of roughly 43%. These findings are in line with predictions of classical search models, most notably Stahl (1989), and empirically support the meaning of consumer search in the context of oligopolistic pricing. Further, the mean price decreases as competition intensifies whereas the price variance increases under increased entry competition (Janssen & Moraga-Gonzalez (2004), Carlson & McAfee (1983)). This suggests stations' tendency to focus more strongly on the lower price segment as competition increases.

2 Centrality and Spatial Differentiation – A Literature Survey

2.1 Introduction

The Hotelling model is an established tool to analyze spatial competition in various market settings. It is appealing because of its analytical tractability and the intuitive predictions that can be deducted. Therefore, the literature that flows from the Hotelling model is vast.

Previous surveys aim at providing a comprehensive overview of the diverse strands of the literature that is linked to spatial competition modeling. For instance, Biscaia & Mota (2013) classify respective articles into groups differentiating the type of competition (Bertrand competition vs. Cournot competition), the shape of the market (circular vs. linear markets), and the existence of incomplete information. Another survey that accounts for a wide range of determinants for product differentiation is provided by Brenner (2001). The categories he defines to distinguish the literature cover the topics of price competition, spatial price discrimination, demand characteristics (price elasticity, customer distribution), the number of firms, collusion, as well as multiple product dimensions.

Comparable to the study of Graitson (1982) and in contrast to the more recent contributions, the present survey has the goal to define a narrow sphere and focus on the basic parameters of the Hotelling model according to its original formulation. The narrow scope definition is justified firstly, by the goal to keep the review tractable and to avoid officious complexity, and secondly, by the ambition to take a deep dive into the mechanical details of the models in order to disclose their economic arguments and gain insights into their predictions on spatial product differentiation.

The selection of articles is admittedly a subjective issue. Inspired by the study of numerous contributions in the field, the choice of the research papers for this survey is motivated by picking out representative articles (mainly with a game theoretical focus) that highlight characteristic determinants of spatial product differentiation which will be summarized in the concluding section. According to this outline, particular streams of the literature that would be interesting to study in detail, however, are not part of the scope. Specifically, in this survey only models with deterministic

and one-dimensional characteristics are considered. Thus, multidimensional product spaces, models where the strategic variables price and location follow probabilistic measures as well as two-dimensional markets with an areal problem set are not explicitly examined.

Essentially, the unique contribution of this survey is two-fold. Firstly, the impact of market geometry and of the distribution of consumers on market equilibrium outcomes shall be highlighted. In particular, markets consisting of intersecting roads deserve our attention since they represent an important link between theoretical models and their predictions on the one side and empirical analyses and suitable econometric methods on the other side. Consequently, in addition to the theoretical model survey a section is included that presents evidence from empirical studies to reveal the consistency between these two fields of research. Secondly, intuition for the importance of the strategic interaction of players for the determination of market equilibria shall be developed. Applying considerations on the strategic interaction to the market setting of intersecting roads leads us to identify an interesting research gap in the present state of the field.

In the examination each research paper is analyzed regarding (i) its basic research question, (ii) its methodology and mathematical arguments, and (iii) its predictions on spatial product differentiation and firms' optimal location decision. In subsection 2.2 articles on the Hotelling model and its extensions with respect to the basic model assumptions are presented. Subsection 2.3 deals with articles concerning different market shapes. To start the examination and provide a concise overview in comparison with linear markets the case of circular markets is presented in subsection 2.3.1. In subsection 2.3.2 models emphasizing the position of a market center either by introducing variations in the consumer distribution or by imposing an alternative market geometry are considered. Subsection 2.4 presents examples from the empirical literature, and in subsection 2.5 a brief overview on models of economic agglomeration is given. Eventually, Subsection 2.6 closes with concluding remarks.

2.2 Spatial differentiation in the Hotelling model

The purpose of this section is to give a thorough overview on the Hotelling model and provide intuition for its possible ramifications. Starting from the original version of Hotelling in 1929 a couple of papers from the classical stream of the literature on the Hotelling model will be re-examined with respect to similarities and differences to the original model structure. Special attention is given to the results of these papers concerning market equilibrium states for sellers' prices and locations and the determinants of respective market equilibria. Thus, the model predictions on sellers' tendency to locate in a cluster or disperse in the market as well as explanations

on the challenges of the existence for equilibrium configurations and the different strategies to solve these issues are presented.

The seminal paper of Hotelling (1929) introduces the notion of oligopolistic market competition in space, even though space is only one of various product characteristics to determine the mechanics of economic decisions. Hotelling motivates his work with the question on the possibility and the conditions of a stable market equilibrium in terms of produced quantities and prices for a few number of sellers that serve a comparatively large consumer base. The point of departure for his model is Cournot's duopoly with quantities as sellers' strategic variables leading to a state of equilibrium given by price levels above marginal cost and a state of mutual readjustment in production if one seller changed his decision for the amount produced. The common critique of the Cournot equilibrium focuses on prices and not quantities as strategic variables. In particular, a price undercutting strategy of one seller proves to be profitable leading to a downward pricing spiral floored with sellers' marginal costs which represent the level of equilibrium prices describing the outcome of a Bertrand price competition game.

Hotelling's basic idea is to set up a model which avoids discontinuities, i.e. a situation where all buyers in the market switch to the seller with the lowest price, but rather provide an explanation of the continuous variation in prices and quantities and the corresponding variation in consumer demand. This leads to the prototype of a monopolistic competition model. A seller does not gain or lose the whole market with a strategic decision on price or quantity, despite of more or less profound price differences sellers are able to exercise a certain degree of monopoly power determined by the unique features of their product. Every competitor serves a defined consumer base and market competition is described by a continuous transition of demand which Hotelling interprets as a degree of stability. This 'stability in competition' is attributable to frictions in the market caused by the unwillingness of certain consumers to switch to the cheapest seller who assess certain product characteristics as particularly valueable, most illustratively the geographical proximity to the seller's store.

Methodologically, equilibrium prices, quantities and profits are determined „when the quantity sold by each is considered as a continuous function of the difference in price." (Hotelling (1929), p. 44). Input parameters for the optimization are the total size of the market (number of consumers), switching costs (i.e. transportation costs per unit distance), marginal cost of production (normalized to zero), sellers market position, and perfect price inelasticity. Local monopoly regions are divided by the indifference condition comparing product prices and incurred transportation costs. Equilibrium conditions are determined by sellers' profit maximization. The case is exemplified for a duopoly in the well-known setting of a linear city of length l and

sellers' location given by the distances of a and b from respective ends of the line. Equilibrium profits reduce to $\Pi_1 = \frac{c}{2}(l + \frac{a-b}{3})^2$ and $\Pi_2 = \frac{c}{2}(l - \frac{a-b}{3})^2$ and are derived under ceteris paribus conditions, thus mutually assuming arbitrary but fixed values for the price of the respective competitor. Subsequently, Π_1 and Π_2 imply that each seller is inclined to gravitate towards his rival which is referred to as the principle of minimum differentiation (PMD). Thus, the Hotelling model suggests that sellers agglomerate in a central position of the market and predicts „*the tendency to make only slight deviations in order to have for the new commodity as many buyers of the old as possible, to get, so to speak, between one's competitors and a mass of consumers.*" (Hotelling (1929), p. 54)

Naturally, a leap forward from the Hotelling model is achieved by relaxing the underlying critical assumptions. The condition of perfectly inelastic demand at every point of the market leads to the PMD and gives away one of the major conclusions as regards sellers' location patters. This finding is challenged by Lerner & Singer (1937) who scrutinize the case of inelastic demand over a price range extending from zero to an upper bound. Smithies (1941) extends the previous approaches assuming a linear elastic demand function over the whole market area and three different conjectural hypotheses on the reaction of the competitors to each other's optimal price and location decision. This enables him to investigate the impact of a variation in the transportation cost per unit of distance (i.e. freight rates) on the market equilibrium and derive critical values for a transportation cost parameter[1] that determines sellers' tendency to locate towards the center. The critical assumption says that „*at every point of the market there can be only one price, and there are identical linear demand functions relating price to quantity sold per unit of time at that point. Thus, the total amount sold at any point is supplied by the competitor charging the lower delivered price at that point.*" (Smithies (1941), p. 425) Clearly, this implies that the demand in competitors' hinterlands reacts to changes in location due to a corresponding variation in freight rates which in turn impacts profits. In a nutshell, this marks the main difference to the Hotelling model where each seller enacts 'absolute' monopoly power over his hinterland passing on the entire freight rates without affecting profits.

In particular, Smithies proposes three states of competition, firstly, sellers choosing the exact same price and location, secondly, sellers choosing the exact same price but compete in locations, and thirdly, competition occurring in price and location excluding the case of market deterrence strategies. For each of these, equilibrium prices, profits and locations are derived. In the first case sellers choose the quartiles of the city so as to maximize their profit. Locating closer to the center does not yield higher profits since then sellers' hinterlands can not be exploited optimally which

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[1] The parameter s is defined as the ratio of the unit transportation cost parameter to the price intercept of the linear demand curve. (cf. Smithies (1941), p. 432)

is attributable to comparatively higher freight rates. By contrast, when introducing competition in locations in the second case the dominant strategy is to locate close to the center. Given equally charged prices, for each seller gains in his hinterland are lower when remaining at the quartile position compared to an increase in demand due to the expansion of his territory.[2] Thus, the profit maximizing equilibrium results in equal seller locations closer to the center than the quartile positions and profit maximizing prices below the equilibrium prices of the quartile solution due to higher average freight charges. Finally, in the third case the notion of price competition intensifies sellers' tendency to locate towards the center. As prices decrease the advantage of an advance in each sellers' territory increases, moreover, each competitor has to face the price decision of his rival. Ceteris paribus, i.e. under the same level of freight rates, an equilibrium results with equal prices lying below and equal locations closer to the center than respective values in the second case of sole competition in locations. (cf. Smithies (1941), table 1, p. 435) Put differently, the threshold for sellers tendency to agglomerate at the center in terms of the freight rate is higher under full competition.[3]

In sum, Smithies major finding concerning the location patterns in spatial markets is that in a setting of elastic demand the level of transportation costs is crucial for determining sellers' tendency to agglomerate at the center. The reason is that sellers are not free to pass on transportation costs without any cost but rather balance the chance of achieving territorial gains with the necessity to impose higher freight charges on their hinterlands. Higher transportation costs imply that sellers are inclined to behave more like local monopolists or as Smithies puts it *„imagining the extreme case of an insuperable wall erected at the center of the market."* (Smithies (1941), p. 434f) By contrast low transportation costs support competitive behavior and emphasize the tendency to locate at the center. Further by distinguishing three conjectural hypotheses, Smithies illustrates that sellers' strategic decision is dependent on the type of market competition.

The paper of Eaton & Lipsey (1975) represents an important building block in the literature on spatial competition models. Except for the structure of the market demand[4] the authors challenge the critical assumptions underlying the Hotelling model, derive conclusions on the range of application of the PMD, and provide further principles on the issue of equilibrium location settings in oligopolistic markets. Particularly, they investigate cases with more than two sellers, consider bounded and unbounded market areas, scrutinize the effect of different consumer density functions on the lo-

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[2]Consider also that a retreat for one competitor from the equilibrium position towards the edge of the city is not profitable since the same reaction can not be expected by his rival.

[3]Setting sellers' relative distance to zero Smithies achieves critical values of $s = \frac{4}{7}$ and $s = \frac{8}{11}$ for location competition and full competition respectively.

[4]This is dealt with, for instance, in Lerner & Singer (1937) and Smithies (1941).

cation equilibrium, and on top, extend the analyses to two-dimensional spaces. This makes it necessary to apply simplifying assumptions on other aspects of the model. These are, firstly, that all firms set the same mill price, and secondly, two types of strategic reaction functions (conjectural variation) are assumed: (i) that firms do not react to the location decision of a competitor and retain their own location (zero conjectural variation, ZCV), and (ii) that a change in location of one firm causes a maximum possible loss to one competitor. Thus, in their pure location model the impact of competitors' strategic price setting on the location patterns is not of interest.

The following summarizes their treatment of four models in one-dimensional markets:

- Model 1 represents the setting of the linear city with ZCV and evenly distributed consumers. The argument is made for two conditions defining location equilibria, namely, „(1.i) no firm's whole market is smaller than any other firm's half market" and „(1.ii) the two peripheral firms are paired" (Eaton & Lipsey (1975), p. 29). Intuition suggests that concerning condition (1.i) there is always the fallback-option for one firm of locating infinitesimally close to the nearest neighbor. As for condition (1.ii) a peripheral firm whose market boundary on one side borders to the edge of the city has the dominant strategy of increasing its market area by shifting its location infinitesimally close to its nearest neighbor and forming a pair.

  As a consequence, for two firms the PMD is fulfilled with both locating at the center. By contrast, for three firms no equilibrium is achieved since both peripheral firms want to form a pair with the interior firm, and clearly, no player accepts to take the interior position.[5] Comparable to the duopoly, four firms form two pairs where in equilibrium each player's market is maximized equally dividing up the market with pairs at the quartiles. For five firms two pairs remain, in the equilibrium configuration the stand-alone player locates in the center and the pairs at $\frac{1}{6}$ and $\frac{5}{6}$ of the market respectively. In this case the interior firm benefits from the fact that pairs are formed due to the inward move of each peripheral firm. A further inward move of the pairs towards the center leads to an increase in the hinterlands of the peripheral firms at the expense of the market area of the centrally-faced, paired firms and thus does not represent an equilibrium state. Finally, for six players no unique equilibrium but a continuous range of equilibrium states exists. The dynamics in this configuration are attributable to the two interior firms. In the extreme cases, these minimize their distance and form a third (interior) pair resulting in equally distributed market shares or maximize their distance taking the double market size of the remaining four competitors (two pairs each comprising one peripheral firm).

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[5] Formally, condition (1.i) is violated.

In sum, this leads to the general conclusion that „*(1) no firm can have a market more than twice as large as any other firm's market; and (2) no firm can have a market smaller than the market length of the firms in the peripheral pairs*" (Eaton & Lipsey (1975), p. 31). The respective bounds for equilibrium market sizes for interior and peripheral firms are a decreasing function in the number of firms. Interestingly, the socially optimal equilibrium configuration corresponds to the state where the difference or the inequality of the distribution in the market sizes between the interior firms and the firms of the outer pairs is greatest, i.e. a state with the interior firms spread evenly. Moreover, it is concluded that the location equilibria critically depend on the spatial characteristics of the market which is illustrated by applying the calculus from the linear city to a circle.[6]

- Model 2 investigates the setting of the linear city under the competitive scenario that the location choice of one firm is made under the conjecture of incurring maximum losses, i.e. that a competitor will form a pair on the long side of its market. As a result, the dominant location strategy of each player is to maximize the short side which is established by choosing the middle position in one's market area. For the two peripheral firms this implies that they locate at a distance of one-third of the distance from the city edge to their rival (with a total market area of two-third of this distance), the interior firms choose the midpoint of the distance between their competitors. In the unique equilibrium with n players an equidistant location configuration with market areas of $\frac{1}{n}$ obtains which minimizes the overall transportation costs and thus corresponds to the socially-optimal state. Moreover, the authors mention only one exception to this result highlighting that a distinction between new market entry and moves of existing firms only matters for the case of a duopoly. (cf. Eaton & Lipsey (1975), p. 33) If two players anticipate that no entry occurs they locate at the center and the PMD applies, thus in this particular case no difference in the results between the case of ZCV and maximum losses obtains. By contrast, if the duopolists know of a third firm to enter the market their dominant loss-minimizing strategy is to take the quartile positions.

- Model 3 uses ZCV but deviates from the assumption of evenly distributed consumers. Consumer density is modeled with the consumer density function $c(X)$ with X denoting the distance from the geometrical market origin, e.g. one of the market's edges. In comparison to model 1 the obvious consequence of introducing $c(X)$ is that the equilibrium conditions apply with respect to the consumer distribution and not with respect to the market geometry. Thus, the critical measure of gaining half of the market and subsequently condition

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[6]This case is more extensively treated in chapter 3.1 of this survey.

(1.i) has to be generalized to the principle (3.i) that *„no firm's whole market is less than another's long-side market"* (Eaton & Lipsey (1975), p. 33).[7] The principle that peripheral firms form a pair remains.[8] Furthermore, two additional principles to define the equilibrium state are considered. The first is that for interior firms the value of the consumer density function at the left- and right-handed market boundaries has to be equal (cp. Eaton & Lipsey (1975), condition 3.iii, p. 34), and the second is that for paired firms the value of the consumer density function at the short-side of the market has to be greater or at least equal to the respective value at the long-side (cp. Eaton & Lipsey (1975), condition 3.iv, p. 34). Intuitively, in equilibrium incentives to move must not prevail, i.e. each interior firm locates at the density-related midpoint of its respective market, and paired firms do not move away from their peripheral neighbor towards regions of higher density. The mathematical explanation is rooted in a comparison of cumulative probabilities if the location is varied by a unit distance.[9]

In sum, the four principles lead to the following conclusions for equilibrium configurations. Firstly, the PMD persists in a duopoly with the two firms locating at the median of the distribution. Secondly, there exists no equilibrium for three firms since peripherals are inclined to form a pair which is at odds with condition (3.i). Thirdly, the general principle applies that in equilibrium the number of firms is restricted by the structure of the distribution, particularly, the number of modes must match (or be greater) than half of the number of firms. This is a direct consequence of (3.iii) and (3.iv) which imply that in the market area of every interior firm a mode has to be located, and that for one firm of any pair the market area must also include a mode. If the number of firms equals twice the number of modes all firms are paired (otherwise no equilibrium is achieved). For a smaller number of firms the equilibrium state depends on the characteristics of the distribution. Moreover, the principle makes clear that a strictly monotonic unimodal density function never provides an equilibrium for a market with more than two firms which is attributable to (3.iv).[10]

- Model 4 represents a combination of model 2 and model 3 incorporating a

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[7] The long side consistently being defined as the market side with the higher number of consumers.

[8] The dominant strategy for a peripheral firm to extend its market area by forming a pair is independent of the consumer distribution since no territory is lost by moving inwards.

[9] The market area for an interior firm i at position X_i is expressed as the difference of the cumulative distribution at the right- and left-handed boundaries: $M_i = \int_{B_L}^{X_i} c(X)dx + \int_{X_i}^{B_R} c(X)dx = C(B_R) - C(B_L)$. Optimizing M_i w.r.t. X_i yields the equivalence relation: $\frac{\partial M_i}{\partial X_i} = 0 \Rightarrow c(B_R) = c(B_L)$. The analogous argument applies for paired firms with short- and long-sided boundaries leading to $\frac{\partial M_i}{\partial X_i} > 0$ if $c(B_{short}) < c(B_{long})$.

[10] The authors give an example for this case in figure 4 (cp. Eaton & Lipsey (1975), p. 34).

variable consumer density function $c(X)$ under the conjecture of players' loss-maximizing strategic behavior. Recall that the dominant strategy of each player then is to maximize the short-side of its market. An equilibrium state is defined as regards moves of a firm with respect to its nearest neighborhood (local equilibrium) and with respect to the whole market area, i.e. firms consider the whole city to be a feasible location area (global equilibrium). For states of local equilibrium two types of equilibrium conditions are defined. The Type I conditions demand firstly that the cumulative consumer density in a firm's market is divided by its location X_i into two equal parts,[11] and secondly that the value of consumer density at X_i exceeds (or matches) half of the value at the respective boundary of the short-side of its market[12]. Under a violation of each of these conditions a firm increased the short side and thus gained a higher number of consumers, if it changed its location. As a consequence, the Type I equilibrium reveals the same result as under an even distribution function (model 2), namely that each firm locates at the middle of its market, i.e. the distribution median. Now, states occur where firms are centrally located but $2c(X_i) < c(B_{short})$. This gives rise to Type II equilibrium conditions which fix $c(X_i)$ to the respective value at the short-sided boundary. (cf. Eaton & Lipsey (1975), conditions 4.iii a-b and 4.iv a-b, p. 37) Concerning the existence of global equilibria no particular conditions are derived, respective configurations depend on the structure of $c(X)$.

The significance of the paper by Eaton & Lipsey (1975) is attributable to the generalization of the framework in spatial competition modeling techniques. As regards projections on the PMD in one-dimensional markets, they confirm that in a duopoly firms locate at the, consistently defined, center of the city unless the two firms anticipate the entry of a third rival under the assumption of a loss maximizing behavior. The general implication of their model is that firms are inclined to form pairs in a wide range of equilibrium and disequilibrium states. In particular, they illustrate that an increase in the number of firms contradicts with the tendency to locate at the center with the interesting result of a continuum of equilibria for six firms and more under ZCV and a rectangular distribution. Also, the authors show that the assumption of a loss minimizing strategy is at odds with sellers' preference to locate at the center. Finally, it is made clear that the characteristics of the consumer distribution function are critical for determining the equilibrium in locations.

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[11] $\int_{B_{left}}^{X_i} c(X)dx = \int_{X_i}^{B_{right}} c(X)dx$ (cf. Eaton & Lipsey (1975), condition (4.i), p. 36)

[12] This pins down to the condition $2c(X_i) \geq c(B_{short})$. For any firm i the market boundaries are given by $B_L = \frac{X_{i-1}+X_i}{2}$ and $B_R = \frac{X_i+X_{i+1}}{2}$ with the market value $M_i = \int_{B_L}^{X_i} c(X)dx + \int_{X_i}^{B_R} c(X)dx = C(B_R) - C(B_L)$. Thus, considering the chain rule a move towards the right-handed boundary changes the market by a rate of $\frac{1}{2}c(B_R) - c(X_i)$ (for the right-handed market), and $c(X_i) - \frac{1}{2}c(B_L)$ (for the left-handed market). The argument applies to the two peripheral and the interior firms (cf. Eaton & Lipsey (1975), conditions (4.ii. a-c), p. 36f).

The paper of Hay (1976) stresses a feature in spatial competition modeling that has not received attention in the previous studies. In particular, he drops the assumption that the relocation for firms is costless and investigates the existence of market equilibria when firms choose their most profitable location in a sequential order.[13] Comparable to Smithies (1941), his model rests on Hotelling's linear city under price elastic and identical linear demand curves. The linear city is not explicitly restricted in size and corresponds to an infinte line. His major assumption, however, is the irrevocability of the location choice, i.e. „once the firm has located itself, its capital equipment is immobile. [...] A consequence of this is that the plant must have a long planning horizon in taking location decisions. [...] So in locating his plant the entrepreneur will seek to secure a sufficient market to give him an adequate return at least over this period." (Hay (1976), p. 243) The starting point of the analysis is to set up the demand and profit functions. Specifically, $q_x = y(a - b(P + x))$ refers to the positive linear demand function for an arbitrary firm at a distance x with P as the price at the point of sale (mill price), and y as the population density (number of consumers) at any point in the market. Subsequently, for a market boundary of z to each of the two nearest neighbors the total demand function is defined as $q = 2 \int_0^z q_x dx = 2yz(a - bP - \frac{b}{2}z)$. The cost function is postulated as $C(q) = kq + X$, and the profit function follows with $\Pi = Pq - kq - X$. A firm chooses between the two strategic variables of setting a profit-maximizing price P and occupying a territory of z to one neighbor, according to the first order condition these are negatively proportional.[14] Further, due to the linear demand function the maximum market area z for any P requires $z = \frac{a}{b} - P$ (i.e. the highest possible price is capped at $\frac{a}{b}$). This implies that for the profit-maximizing price P^* the boundary $z^* = \frac{2}{3}(\frac{a}{b} - k)$ obtains and that a firm always wants to expand its market boundary to z^* since $\frac{\partial \Pi(P^*)}{\partial z} > 0$ requires $z < z^*$.[15]

The subsequent analysis of sequential entry distinguishes between the two scenarios of locating in the neighborhood of two competitors and settling down in a vacant space of the market where no other rivals have located yet.

In the first case the maximum critical distance between two firms is evaluated such that an entrant firm may still locate between the two incumbents. (cf. Hay (1976), appendix 2, p. 255f) Recall that firms' locations are assumed to be immobile; additionally, firms react to entry by setting a post-entry mill price denoted as P_c. The

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[13] His model corresponds to model 2 in Eaton & Lipsey (1975), however, they lay out the assumption that there are no costs of relocation. (cf. p. 28). Furthermore, Eaton & Lipsey (1975) do not provide a treatment of a sequential entry game. In Hay (1976) the number of firms is determined by the corollary of monopolistic market competition that entry occurs until excess profits vanish and the market demand curve of each firm is tangential to the cost curve.

[14] Setting $\frac{\partial \Pi}{\partial P} = 0$ yields $P^* = \frac{a}{2b} + \frac{k}{2} - \frac{z}{4}$.

[15] The minimum for z is determined by the condition $\Pi(P^*) > 0$ such that the arbitrary parameters a, b, k and X at least allow for a tangential relationship between the demand curve and the cost curve. (cp. Hay (1976), p. 254f)

initial market boundary between the incumbents is given at a distance Z_c, whereas the total (minimum) market covered by the entrant (within the area of $2Z_c$) adds up to $2Z_m$. Consequently, incumbents' post-entry profits comprise of a piece attributable to the market towards the entrant's location and one piece contingent upon the market on the far side: $\Pi = (P_c - k)(Z_c - Z_m)(a - bP_c - \frac{b}{2}(Z_c - Z_m)) + (P_c - k)Z_c(a - bP_c - \frac{b}{2}Z_c) - X$.[16] Applying the first order condition $\frac{\partial \Pi}{\partial P_c} = 0$ yields the optimal incumbents' post-entry price P_c^* as a function of Z_c and Z_m, the entrant's profit-maximizing price is given by $P_m^* = \frac{a}{2b} + \frac{k}{2} - \frac{Z_m}{4}$ (cf. footnote 14 above). An expression for the critical spacing is obtained by leveling the entrant's and incumbent's price at the market boundary: $P_c^* + Z_c - Z_m = P_m^* + Z_m$, which reduces to $\frac{Z_c}{Z_m} = 2.215$. Thus, for any Z_c below $2.215Z_m$ it is not profitable for the entrant to locate between the incumbent firms.[17]

The second case emphasizes that the optimal market size for a firm choosing a remote place is in fact given by twice the size of its minimum market area which in turn leads to an orderly spacing of firms. As Hay (1976) argues qualitatively, this outcome critically depends on the firms' interaction since subsequent entrants are required to locate at the minimum distance to the initial firm (and act preemptively to entry). Accordingly, irregularities from a general equidistant equilibrium pattern for the whole market are expected if the location patterns of two distant neighborhoods from different edges of the city collide which generally can not be outruled. (cf. Hay (1976), p. 247)

Eventually, Hay (1976) analyzes two further aspects of market dynamics: an increase in the total market demand by a constant growth rate and a variation in consumer density. Clearly, as the total market size increases the entrant is confronted with the intertemporal trade off that locating closer to a competitor reduces profits in the short run but increases profits in the long run as entry is prevented for a comparatively longer time period. (cf. Hay (1976), p. 248) Then obviously, the location decision results from the maximization of the present value of future pay-offs. More precisely, under ceteris paribus conditions, e.g. identical discount rates, the equidistant equilibrium spacing from the static analysis is confirmed. The effect of a change in consumer density is shown leveling the general profit function at the profit-maximizing price $\Pi(P^*) = 0$ and evaluating the partial derivative of y against the minimum market for entry z_m. (cf. Hay (1976), p. 251) From the bijective relation between y and z_m over the relevant range it is concluded that an increase in consumer density causes the size of the minimum market to decrease, or $\frac{\partial z_m}{\partial y} < 0$. Additionally, it is demonstrated that a rise in y leads to an increase profits. This meets the intuition that *„those segments of the market with a higher y offer larger profits even though the*

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[16]Consumer density is normalized to y=1.

[17]The critical distance is greater than $2Z_m$ (twice the minimum market size) since the incumbents also serve the opposite side of their market (up to the distance of Z_c) and thus charge a lower mill price than the entrant.

centres will be closer together" (Hay (1976), p. 251), and thus, represents a model that suggests spatial clusters contingent upon the consumer density distribution in conjunction with firms' locating according to entry deterring strategies.

In sum, the work of Hay (1976) predicts that under a constant number of consumers and both in a static and dynamic market scenario the assumption of immobile locations leads to regular location patterns where firms tend to be spread out evenly over the whole market area by a measure of the minimum distance z_m. Consequently, as a result of sellers' commitment to a final location decision the PMD is discarded. The variation in consumer density, however, allows for the emergence of spatial clusters.

Comparable to the work of Hay (1976), Prescott & Visscher (1977) provide an analysis of a strictly sequential location process. The model structure is that only one firm enters at a time, where each player is confronted with prohibitively high relocation costs. For the optimal location decision a firm accounts for the profit-maximizing decisions of all competitors who have already taken a position, and importantly, also those firms who will enter later in the sequence: *„Each firm is assumed to choose the profit maximizing market position based on the observed choices of firms already located and the location rules that subsequent, equally rational entrants and potential entrants will use. Thus, each firm takes into consideration the effect of its location decision upon the ultimate configuration of the industry."* (Prescott & Visscher (1977), p. 379) Now, this method is applied to different settings.

Firstly, the case of the Hotelling city is investigated. The assumptions comprise a linear market with unit boundaries [0; 1], a rectangular consumer density distribution (the number of consumers is N), exogenously given and equal prices for every seller, and a fixed number of n firms to enter the market. Firms' optimal locations are determined by an algorithm based on the principle of backward induction. To illustrate the argument, the last firm n as a profit-maximizing agent devises a decision rule to choose the most profitable location based on the present market configuration. In turn, firm $n-1$ accounts for firm n's decision rule as well as for the given market structure of the remaining $n-2$ firms when setting up his own location decision rule. This process is followed up to the first entrant firm 1 who considers all profit-maximizing decision rules of his competitors when taking his optimal position. Clearly, for two players the equilibrium with both locating back-to-back at the center obtains since firm 1 knows that the best reaction is to maximize his short side as firm 2's dominant strategy is to locate as closely as possible on firm 1's long side.[18] Moreover, consider the case of three firms. As was argued by Eaton & Lipsey (1975) in a simultaneous game no equilibrium exists. By constrast, Prescott & Visscher (1977) provide an equilibrium solution for the sequential case. (cf. Prescott & Visscher (1977), p. 382) In a nutshell, firms' decision rules suggests to locate in the

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[18]This supplements the predictions of Eaton & Lipsey (1975) in their model 1 and model 2 for a duopoly.

most vacant spaces accounting for different cases that consider all potential locations of their rivals.[19] The equilibrium yields firm 1 and 2 to be located at the quartiles and firm 3 at $\frac{1}{2}$.

In the second setting, the assumption of a fixed number of firms for the linear city is dropped, that is, market entry is endogenized. The underlying calculus rests upon a function w to quantify the market share for an arbitrary profit-maximizing firm in an equilibrium industry structure. The market share is, evidently, dependent on the distance z to the nearest neighbors.[20] (cf. Prescott & Visscher (1977), equations (1)-(3), p. 383f) The purpose of w is to determine the expected value for entry, thus, to set the condition for profit maximization. Clearly, a higher distance implies higher market shares and consequently higher profits, however, the extension of the market area is limited by upcoming entrants and, as the analysis in figure 1 (Prescott & Visscher (1977), p. 384) illustrates, is capped by the amount of the fixed costs α. As a result, the optimal location x suggests a firm to locate from a distance of α from the respective ends of the city and to choose vacant middle markets and locate at a distance of 2α from the nearest neighbor. (cf. Prescott & Visscher (1977), equation 4, p. 384) The number of firms is restricted by $\frac{1}{\alpha}$, in case that $\frac{1}{\alpha} \geq 4$ is not an integer an additional firm enters and locates at a remaining interval exceeding the value of 2α. In sum, the location algorithm leads to an uniform market structure with firms spaced out evenly by a distance of 2α subject to $\frac{1}{\alpha}$ being an even integer, otherwise one irregularity in the location pattern emerges.

In conclusion, Prescott & Visscher (1977) provide a comprehensive treatment of a sequential location model that, in extension to previous works (e.g. Hay (1976)), explicitly accounts for future expectations on the location choice of subsequent firms to enter the market. Furthermore, they derive a solution for the equilibrium state when entry is endogenized. As regards the PMD the results of Prescott & Visscher (1977) are in line with Hay (1976), firms' dominant location strategy under costly sequential entry leads to a location distribution with firms spacing out which is at odds with a general tendency to agglomerate at the market center. In addition, Prescott & Visscher (1977) show that the location choice is also dependent on the level of entry costs. Particularly, in the case where the number of firms is restricted to $n \leq 3$ a lower level of fixed costs leads the first two entrants to locate closer even when the entry barrier still prohibits the third firm to enter. This illustrates that location

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[19]W.l.o.g. firm 1's location is restricted to one half of the city, i.e. $x_1 < \frac{1}{2}$ (left half), then the decision rules for firm 3 are: locate to the right of firm 2 if it is also settled in the left half (case (i)), locate to the left of firm 1 if it is located far right in his half and firm 2 is in the right half (case (ii)), locate to the right of firm 2 if it is located far left in the right half (case (iii)), and locate between firm 1 and 2 if the middle market is large (case (iv)). Firm 2's decision rules require to locate closer to firm 1 if it is close to the edge and to choose a remote location if it is positioned more centrally.

[20]Note that a distinction between locations at the edge of the city with only one nearest neighbor and between the case of two nearest neighbors has to be made.

acts as a strategic instrument to forestall further entry expressed by the counterintuitive result of higher profits and prices under comparatively higher fixed costs. (cf. Prescott & Visscher (1977), table 2, p. 389) Finally, it has to be noted that Prescott & Visscher (1977) emphasize that the Hotelling model does not provide a complete set of noncooperative Nash equilibrium prices: „*The difficulty with this solution concept [...] is that when locations in Nash are sufficiently close, Nash equilibrium prices will not exist. The nonexistence of equilibrium is a problem that frequently arises when reaction functions are [...] discontinuous. The source of the discontinuity in the price reaction function here is that a lower price by one of the firms does not always gain the firm market share in a smooth continuous fashion. A price sufficiently low can capture the entire market, whereas a price slightly higher loses the rival firm's entire hinterland.*" (Prescott & Visscher (1977), p. 386) However, it was not until the influential note of d'Aspremont et al. (1979) to mathematically prove the nonexistence of Nash price equilibria at every point in the market.

Fifty years after the publication of Hotelling's famous paper d'Aspremont et al. (1979) challenge the general prediction of the PMD and prove that in the simultaneous price competition a Nash equilibrium does not exist over the whole range of the linear city. Specifically, they show that for close locations no equilibria obtain. In their proof the usual assumptions apply, distances a and b are defined from the city edges (line of length l), and firms A and B charge p_1 and p_2, unit transportation costs are denoted with c. The demand functions q_1, q_2 are derived from the position of the indifferent consumer and profits π_1, π_2 obtain as a function of price differences, thus, π_1, π_2 reveal two discontinuities when sellers' prices are undercut.[21] (cf. d'Aspremont et al. (1979), p. 1145f) Now, firstly, it is justified that a price equilibrium can only occur in the competitive region of the profit function, i.e., the price difference is restricted to $|p_1^* - p_2^*| < c(l-a-b)$. Intuitively, outside the range any seller could in any case increase his profits by adapting his price.[22] It follows that equilibrium prices are derived from the parabolic parts of the profit function applying the first order condition which yields $p_1^* = c(l + \frac{1}{3}(a-b))$ and $p_2^* = c(l + \frac{1}{3}(b-a))$. Furthermore, applying the definition of the Nash equilibrium[23] it follows that the equilibrium is fulfilled only for restricted intervals of locations a and b. Intuitively, profits gained under p_1^* must exceed profits under an undercutting strategy. For symmetric locations ($a = b$)

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[21] The seller who undercuts his rival gets the whole market l, the undercut firm earns zero demand and profits. Since linear transportation costs are assumed (following Hotelling (1929)) a seller captures the whole market if the price cut shifts the market boundary to the rival's mill, thus the conditions for the discontinuities are derived setting $q_1 = 1 - b$ and $q_2 = a$ respectively.

[22] The undercut firm would decrease his price, moreover condition $|p_1^* - p_2^*| = c(l-a-b)$ implies that a seller who captures a fraction of the market is inclined to drop his price to the undercutting level.

[23] p_1^* maximizes $\pi_1(p_1, p_2^*)$ over the whole domain of possible price strategies, and for firm B vice verser. (cf. d'Aspremont et al. (1979), p. 1146)

the quartiles mark the threshold for the Nash equilibrium condition.[24] Additionally, the authors conduct the analysis with quadratic transport costs and show that Nash price equilibria $p_1^* = c(l - a - b)(l + \frac{1}{3}(a - b))$ and $p_2^* = c(l - a - b)(l + \frac{1}{3}(b - a))$ hold for all a and b on the line.[25] Furthermore, Nash profits $\pi_1(p_1^*, p_2^*)$ and $\pi_2(p_1^*, p_2^*)$ increase with decreasing a and b respectively. Thus, under quadratic transportation costs profit-maximizing sellers maximize their distance and locate at the ends of the city.

To sum up, d'Aspremont et al. (1979) provide evidence that a general PMD, as suggested by Hotelling (1929), is invalid which is attributable to the structure of the profit functions under the linear transportation cost scheme. Moreover, their note suggests a contrary principle of maximum differentiation for the location decision when applying quadratic transportation costs. Using quadratic transportation costs proves advantageous since then a Nash equilibrium in prices (for a simultaneous pricing game) exists for all locations on the city domain.

According to the negative result achieved by d'Aspremont et al. (1979) it would have not been surprising to accept the conclusion that the PMD in the Hotelling model should be finally discarded. However, the conflicting evidence spurred more research activities. Exemplarily, the papers of Economides (1984), Economides (1986), and Economides (1993) illustrate that the level of consumers' reservation price, the functional form of transportation costs and the number of competitors critically determine the equilibrium in prices and location for the Hotelling model.

In these works, the existence and the solutions for price and location equilibria are scrutinized for a two stage game where in the first stage firms simultaneously choose locations and in the second stage a simultaneous price competition takes place (for previously determined locations). To begin with the Hotelling model is restated in a generalized form. In particular, the utility function of consumer ω purchasing a unit of the differentiated product x is defined by $U_\omega(x, m) = m + V_\omega(\omega) - f(d(x, \omega)) - P_x$ with m standing for the budget (endowment with a Hicksian composite good), $V_\omega(\omega) = k$ the constant reservation price for all consumers, $f(d(x, \omega))$ a function for the disutility of traveling in space from ω to a firm's mill at x (for $f(d) = d$ and

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[24]Formally, for any $\epsilon > 0$: $\pi_1(p_1^*, p_2^*) = \frac{1}{2t}(p_1^*(p_2^*))^2 > l(p_2^* - c(l - a - b) - \epsilon)$. The same argument applies for firm 2. Then after a little algebra this yields the two conditions $l^2 + (\frac{a-b}{3})^2 \geq 2l(a+b)$ and $l^2 + (\frac{b-a}{3})^2 \geq 2l(a+b)$ (these correspond to equations (1) and (2) on p.1146). To complete the proof see that out of these conditions follows $(\frac{|a-b|}{3})^2 \geq 2l(a+b) - l^2 = 2l(a + b - \frac{l}{2})$, further evaluating $|p_1^* - p_2^*| < c(l - a - b)$ leads to $\frac{|a-b|}{3} < \frac{1}{2}(l - (a + b))$. Thus, a consistent solution with respect to the upper and lower bound requires $\frac{5}{2}l(\frac{l}{2} - (a + b)) + \frac{1}{4}(a + b)^2 \geq 0$ which is not fulfilled for all a and b on the domain, e.g. $a = b = \frac{1}{2}$.

[25]Traveling a distance x to a seller a consumer incurs transportation costs of cx^2. Thus, the utility of consuming at firm A and B (with a sufficiently high surplus s) is $u_A = s - p_1 - t(x - a)^2$ and $u_B = s - p_2 - t(1 - x - b)^2$. For the indifferent consumer between A and B set $u_A = u_B$ which yields the demand and profit functions. (cf. d'Aspremont et al. (1979), p.1148) Furthermore, proofs for the uniqueness of the price and location equilibrium for quadratic transportation costs are provided in Neven (1985).

$f(d(x,\omega)) = |x - \omega|$ this corresponds to linear transportation costs) and P_x the price of x.[26] (cf. Economides (1984), p. 347 and 349f) From the total utility the disutility function $g_x(\omega) = f(d(x,\omega)) + P_x$ can be separated. Thus, consumers' optimization problem is to minimize $g_x(\omega)$ for given k, that is depending on the constant reservation price a consumer will travel to the nearest seller. Firms' locations are taken both from the zero edge of the city with unit length and w.l.o.g. $x < y$ (for a duopoly), the indifferent consumer $\overline{z}$ between the two firms lies at $\overline{z}$ and represents also the most disadvantaged consumer since his value for g among consumers in the middle market is the highest.[27] Then, Nash prices in a duopoly are $P_x^* = \frac{1}{3}(2 + x + y)$ and $P_y^* = \frac{1}{3}(4 - x - y)$ with an equilibrium state only if: $x^2 + y^2 + 2xy - 8x + 28y - 20 > 0$ and $x^2 + y^2 + 2xy - 32x + 4y + 4 > 0$.[28] (cf. Economides (1984), proposition 2 on p. 352, and p. 353)

Now, in Economides (1984) the assumption that every buyer purchases one unit of the differentiated product is dropped.[29] As a consequence, sellers' demand and profit functions comprise of three parts segmented by different price bounds. (cf. Economides (1984), p. 354ff) This is rooted in the critical consumers who are indifferent between buying and not participating in the market and who are now not located at the market edges $z = 0$ and $z = 1$ anymore. Their behavior is characterized by the condition $k = g_x(z)$ and $k = g_y(z)$ respectively, that is the reservation price equals their total disutility of consumption. Formally, for these two conditions four solutions obtain, i.e. the locations for four indifferent consumers, two for each seller one on the right and the left side of the mill: $z_{1,3}(P_x) = x \mp (k - P_x)$ and $z_{2,4}(P_y) = y \mp (k - P_y)$. Then, the first part of firms' profits refers to the instance that undercutting one's rival is a viable strategy, the remaining firm (e.g. product x) serves the market as a monopolist with the undercutting price $P_x = P_y - (y - x)$ and demand by the amount of twice the distance to the indifferently reluctant consumer: $2(k - P_x)$. The second part describes the competitive scenario where the rivals establish a market boundary at $\overline{z}$ in their middle market (see above). Then, each seller takes demand determined by the distance from their market edge to $\overline{z}$ (e.g. for product x that is $\overline{z} - z_1$). The respective (upper) price bound is defined by the price to capture the reluctant indifferent consumer to the right of $\overline{z}$ (e.g. $\overline{z}(P_x) = z_3(P_x)$) The third part allows both firms to stay in the market as a local monopolist, clearly, then their market areas are not connected. Formally, demand corresponds to part one of the

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[26] Likewise $U_\omega(y,m) = m + V_\omega(\omega) - f(d(y,\omega)) - P_y$ for consuming one unit of the differentiated good sold at the second mill at y.

[27] To determine $\overline{z}$ set: $g_x(\overline{z}) = g_y(\overline{z})$

[28] This recaps the results of d'Aspremont et al. (1979). Demand is $D_x = \overline{z}$, $D_y = 1 - \overline{z}$. Then, $g_x(\overline{z}) = g_y(\overline{z})$ yields $\overline{z} = \frac{1}{2}(P_y - P_x + y + x)$. Profits are $\Pi_x = D_x P_x$, $\Pi_x = D_y P_y$ applying the first order condition and solving for P_x and P_y yields the Nash prices. The undercutting prices are obtained setting $\overline{z} = y$ and $\overline{z} = x$ or $P_x = P_y - (y - x)$. For an equillibrium $\Pi_x(P_x^*) > \Pi_x(P_y^* - (y - x))$ must hold and analogously for the second good y.

[29] Note that in Economides (1986) it is retained (cf. second paragraph on p. 67).

profit function and the upper price bound is, of course, the reservation price k. To sum up, the distinct feature of the total demand curve is a kink at the transition from part two to three and a discontinuity at the transition from part two to one due to the undercutting behavior. (cf. Economides (1984), Fig. 4, p. 355)

Subsequently, the Nash price equilibria and equilibrium profits are determined for all three parts and the consequences of the underlying demand structure are drawn for the existence of equilibria. For the case of local monopolies (either for one or two firms) and the competitive case unique price equilibria are derived, at the kink a continuum of price equilibria ('touching' Nash equilibria[30]) exists. (cf. Economides (1984), Theorem 1, p. 359f) The equilibrium configuration critically depends on the level of the reservation price k, therefore for given k according to their existence conditions all Nash price equilibria can be assigned to defined sets for the location of the mills x and y.[31] To derive the solutions of the location game for all three cases the derivatives of the corresponding profit functions $\Pi_x^*(x, y, P_x^*(x, y), P_y^*(x, y))$ and $\Pi_y^*(x, y, P_x^*(x, y), P_y^*(x, y))$ with respect to x and y respectively are evaluated.[32] The result is that in the case of competitive Nash price equilibria and the case of 'touching' Nash equilibria the two players move towards the edges of the city, if local monopolies are established no incentives to move prevail.

In sum the model of Economides (1984) confirms the intuition that for a large distance $y - x$ and comparatively low reservation prices, a Nash price equilibrium in two local monopolies obtains. As the distance decreases and the reservation price increases the price equilibrium is realized for a continuum of prices ('touching' Nash equilibrium at the kink of the demand for $P_x + P_y = 2k - (y - x)$). Subsequently, for closer distances and higher reservation prices a price equilibrium is established in competition and if firms approach even further undercutting strategies prohibit a Nash price equilibrium. Location preferences clearly suggest a tendency for sellers to separate from each other, in the case of local monopolies, however, firms do not move unless their respective market cannot fully be exploited. Put differently, Economides (1984) consolidates the results of the previous literature that is at odds

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[30]These result from the discontinuity of the derivative of the profit function. The economic interpretation of the Nash price continuum is that the value of the reservation price is such that the indifferent and most disadvantaged consumer in the middle market is at the same time the critical consumer for both firms to opt out of the market. He has equal utility for purchasing at one of both firms and for not consuming at all. (cf. Economides (1984), p. 357f)

[31]In equilibrium the Nash prices yield maximum profits and the price has to lie within the respective price bounds of the distinguished segment of the demand and profit function. Specifically, for the local monopolistic case with both players in the market the existence condition is $k < y - x$. (cf. Economides (1984), p. 356) For the competitive scenario the existence of the Nash equilibrium additionally requires that profits at the local peak in the price interval have to exceed profits at the left corner referring to the undercutting profits. In sum this corresponds to equations (6) and (7) summarized in (8) on p. 357. For the 'touching' Nash equillibria the existence conditions are given in equations (10) and (11) summarized in (13) on p. 358.

[32]Profit functions are abbreviated by Π_x^* and Π_y^*. Thus, the location equilibrium demands $\frac{\partial \Pi_x^*}{\partial x} = \frac{\partial \Pi_y^*}{\partial y} = 0$.

with the PMD (e.g. d'Aspremont et al. (1979), Prescott & Visscher (1977)) in a two stage price-location game for the linear city and provides evidence in favor of a Nash equilibrium in locations „*at a 'local monopolistic' configuration with the firms choosing to produce very different products.*" (Economides (1984), p. 366). Essentially, the fact that sellers are not able to exploit the whole market area towards the city edges is the explanation for this outcome. Moreover, based on d'Aspremont et al. (1979) he shows that Nash price equilibria are not principally inexistent if firms settle to close, rather the location threshold for a Nash price equilibrium to be discarded is a function of the level of reservation prices.

The purpose of the second paper (Economides (1986)) is to scrutinize the effect of transportation costs on sellers' profit-maximizing location decisions in a Hotelling duopoly.

This is achieved by extending consumers' utility function with a fixed but arbitrary exponent α (with $1 \leq \alpha \leq 2$) in the transportation cost term, i.e. set $f(d) = d^\alpha$ which leads to the utility function $U_z(x, m) = m + k - |x - z|^\alpha - P_x$ when consuming x. (cf. p. 68) Subsequently, firms' profit functions accounting for the Nash price equilibrium in the second stage are set up and the 'zero relocation locus' $\frac{\partial \Pi_x^*}{\partial x} = 0$ is determined. It is important to note that the location equilibrium is derived under the assumption of symmetric locations, i.e. $y = 1 - x$. (cf. Economides (1986), p. 69) Inserting equilibrium prices into the profit function and evaluating the profit derivative yields the derivative as a function of α, i.e. $\frac{\partial \Pi_x^*}{\partial x} \leq 0$ only if $x \geq \overline{x}(\alpha) = \frac{5}{4} - \frac{3}{4}\alpha$ which implies that for values of $\alpha < \frac{5}{3}$ firm x relocates to $\overline{x}$ as the profit-maximizing locus and that for $\alpha > \frac{5}{3}$ the profit-maximizing location is $x = 0$ (since then $\overline{x} < 0$).[33] Thus as stated in proposition 1, for a solution in the price game, the symmetric equilibrium in the location game is $x = \overline{x}(\alpha)$ and $\overline{y}(\alpha) = 1 - \overline{x}(\alpha)$. (cf. Economides (1986), p. 69) Now, what is the range of validity for the location equilibrium? Clearly, the boundary is determined by the relation of undercutting profits to the Nash profits. For this purpose the function $f(x, \alpha) = \Pi_x^{UC} - \Pi_x^*$ is used.[34] Then, for $f \leq 0$ the area of subgame perfect symmetric location equilibria is defined, and for $f > 0$ undercutting is profitable and no Nash price equilibrium exists and the location equilibrium is not defined. Subsequently, the boundary for equilibrium configurations is determined by

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[33]Recall that $x < y$, the indifference condition is $P_x - P_y = |z - y|^\alpha - |x - z|^\alpha$ where $\overline{z}$ denotes the solution for the location of the indifferent consumer. For $\Pi_x = P_x D_x = P_x \overline{z}$ this leads to $\frac{\partial \Pi_x}{\partial P_x} = \overline{z} + P_x \frac{\partial \overline{z}}{\partial P_x}$. Using $\frac{\partial P_x}{\partial \overline{z}}$ from the indifference condition yields the Nash price $P_x^* = \alpha \overline{z}((\overline{z} - x)^{\alpha-1} + (y - \overline{z})^{\alpha-1})$ and similarly $P_y^* = \alpha(1 - \overline{z})((\overline{z} - x)^{\alpha-1} + (y - \overline{z})^{\alpha-1})$. Consequently: $\Pi_x^* = \alpha \overline{z}^2 ((\overline{z} - x)^{\alpha-1} + (y - \overline{z})^{\alpha-1})$. For further details on the evaluation of $\frac{\partial \Pi_x^*}{\partial x}$ see Economides (1986) footnote 4 on p. 69.

[34]In the derivation of the profit functions $y = 1 - x$ and P_x^* and P_y^* are used. Further, note that for a symmetric location equilibrium $\overline{z} = \frac{1}{2}$. Then $P_x^* = \alpha(\frac{1}{2} - x)^{\alpha-1}$ and $\Pi_x^* = \frac{1}{2} P_x^*$, and similarly for Π_y^*. Undercutting profits are $\Pi_x^{UC} = P_x^{UC}$. The undercutting price fulfills the indifference condition $P_x^{UC} + (1 - x)^\alpha = P_y^* + (1 - y)^\alpha$, that is, firm y sets the Nash price and firm x undercuts and takes the whole market by shifting the indifferent consumer to the edge $z = 1$. Then, $P_x^{UC} = \alpha(\frac{1}{2} - x)^{\alpha-1} + x^\alpha - (1 - x)^{1-\alpha}$. (cf. Economides (1986), p. 70)

the solution for $f = 0$ and denoted with $\overline{\overline{x}}(\alpha)$. It follows that the intersection of $\overline{x}(\alpha)$ with $\overline{\overline{x}}(\alpha)$ yields a critical value for the exponent of the transportation costs ($\overline{\alpha}$) above which a symmetric equilibrium under a Nash price equilibrium is reached. Below $\overline{\alpha}$ no locational equilibria exist since then the optimal location $\overline{x}$ falls into the region where $f > 0$. (cf. Economides (1986), Fig. 1 on p. 69)

The crucial finding is that $\overline{\alpha} \approx 1.26$ and together with the definition of $\overline{x}$ this implies that firms are not inclined to maximally differentiate their products and take the extreme positions at the city edges. Particularly, for transportation costs with $\frac{5}{3} > \alpha \geq \overline{\alpha}$ they will take equilibrium positions on the line and charge Nash prices.[35] In sum, this result generalizes the findings in d'Aspremont et al. (1979) where a principle of maximum differentiation is suggested for $\alpha = 2$ and the indeterminate case for the Hotelling model if $\alpha = 1$. Essentially, it can be concluded that the equilibrium configuration, specifically the location equilibrium, is determined by the transportation cost scheme.

In the third paper (Economides (1993)) the two-stage location-price game is generalized from a duopoly to a setting with n firms competing on the linear city.

Firms' strategic variables are captured in the price and location vectors $\boldsymbol{p} = (p_1, ... p_n)$ and $\boldsymbol{x} = (x_1, ... x_n)$ with $\Pi_j(p_1^*, .., p_{j-1}^*, p_j, p_{j+1}^*, ..., p_n^* | \boldsymbol{x}) \leq \Pi_j(\boldsymbol{p} | \boldsymbol{x}) \; \forall j = 1, \ldots, n$ as the consistently defined Nash price equilibrium. (cf. p. 305f) In addition, marginal and fixed production costs (m, F) are introduced into the model. Also note that the linear transportation cost coefficient is denoted with λ. Then, the solution to the second stage price game reduces to the expression $\boldsymbol{p}^* = \boldsymbol{A}^{-1}\boldsymbol{y}$ where $\boldsymbol{A}$ is a nxn matrix (with well behaving properties) consisting of fraction numbers and the elements of $\boldsymbol{y}$ are a function of the locations $x_j(j = 1, \ldots, n)$.[36] As a result, equilibrium profits are for interior firms $\Pi_j = \frac{(p_j^*)^2}{\lambda}$, and for firms closest to the market edges $\Pi_j = \frac{(p_j^*)^2}{2\lambda}$. (cf. Economides (1993)), Proposition 1, p. 307f) These profit functions constitute the objective functions for the derivation of the optimal locations in the first stage of the game. (cf. Economides (1993)), p. 308) Since λ is a constant the signs of $\frac{\partial \Pi_j}{\partial x_j}$ and $\frac{\partial p_j^*}{\partial x_j}$ are equivalent and from the structure of $\boldsymbol{A}^{-1}$ it follows that $\frac{\partial p_j^*}{\partial x_j} = \boldsymbol{A}^{-1}\frac{dy_j}{dx_j}$. According to the structure of $\boldsymbol{y}$ the respective derivative reduces to expressions in λ, thus $\frac{\partial p_j^*}{\partial x_j} \; \forall j = 1, \ldots, n$ depends on real numbers (following from the elements $(a^{inv})_{i,j}$ from $\boldsymbol{A}^{-1}$) and λ. The relationship between the terms of the $(a^{inv})_{i,j}$ in the derivative expression can be further evaluated which leads to the important condi-

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[35] For instance, in Fig. 1 (cf. Economides (1986), p. 69) it is illustrated that at $\overline{\alpha}$ and slightly above, firms even locate closer than the quartile positions.

[36] This generalizes the solution for the price equilibrium in a duopoly (cf. (Economides (1984))). In particular, $\boldsymbol{p}^*$ represents the solution for the case of pure price competition which requires that consumers reservation prices are sufficiently high, i.e. $k \geq p_j + m + \lambda|w - x_j|$ and that undercutting is not profitable, i.e. $\overline{z}_j \geq x_{j-1}$. (cf. Economides (1993), p. 306) Consequently, setting up expressions for firms' market boundaries and profit functions, and applying the first order condition leads to the algebraic equilibrium equation. The properties and the existence of the inverse of $\boldsymbol{A}$ are shown in lemma 1 and corollary 1. (cf. Economides (1993), p. 307)

tion $\frac{\partial p_j^*}{\partial x_j} > 0$ (< 0) only if $\frac{n+1}{2} > j$ $(< j)$ in lemma 2. (cf. Economides (1993), p. 309 and the proof on p. 318f) Clearly, this implies that for every firm j located to the left of the centrally located firm(s) $(j < \frac{n+1}{2})$ an increase in its location increases its profits (and equilibrium price), likewise every firm located to right $(j > \frac{n+1}{2})$ can increase its profits (and equilibrium price) by decreasing its location. Thus, there is a dominant strategy for every not centrally located firm to move towards the central firm(s). Consequently, given equilibrium prices $\boldsymbol{p}^*$ there exists no subgame perfect equilibrium for the choice in locations if firms compete.[37]

Subsequently, more details on market equilibrium properties are given. Firstly, the intuition is confirmed that as firms move inwards the effects of the location change on the equilibrium prices and profits on the other firms decreases with distance and is thus strongest on the nearest neighbors. Ceteris paribus, prices and profits decrease for neighboring firms in the direction of the centrally spaced firm(s) and increase for more distant neighbors as one firm adapts its location. (cf. Economides (1993), Proposition 3, p. 310 and the proof on p. 319) Secondly, by a violation of the non-undercutting condition it is ruled out that a state where all firms in the market gain the same profits serves as an equilibrium since differences in the levels of equilibrium prices suggest that peripheral firms are undercut by their neighbors. (cf. Economides (1993), p. 308) Thirdly, the location scenario of an equidistant spacing is scrutinized where d denotes the distance of interior firms and c the distance from the edges, and $c < \frac{d}{2}$. (cf. Economides (1993), p. 310 and footnote 5) Nash prices are $p_j^* = \lambda(d + ce_j)$ and e_j is a variable determined by elements of $\boldsymbol{A}^{-1}$. The main result is that by the properties of e_j equilibrium prices reveal a strictly convex, symmetric, U-shaped structure over the line $[0,1]$. (cf. Economides (1993), Proposition 4, p. 310 and the proof on p. 319) Thus, in an equidistant setting the peripheral firms exploit the highest degree of monopoly power and charge the highest prices, by contrast the centrally located firms set the lowest prices. Due to firms' market sizes it is then the second (and $n - 1^{st}$) firm who make the highest profits, depending on the level of c the peripherals could be the second most profitable $(c \approx \frac{d}{2}, n \geq 5)$ or the least profitable firms $(c = 0)$. (cf. Economides (1993), p. 311) Finally, it is shown that the suggested pricing structure (in an exogenously imposed equidistant spacing structure) represents a competitive price equilibrium for $n \geq 4$ if reservation prices are sufficiently high, and also an equilibrium for $n = 3$ provided that c is bounded $(c < 0.435d)$. This can be interpreted as a generalization of the critique of d'Aspremont et al. (1979) on the existence of Nash price equilibria in the Hotelling duopoly for oligopolistic

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[37] Recall from the duopoly case in Economides (1984) that consumers' reservation price k determines the type of market interaction, for high k firms compete, for low k firms form local monopolies and not all consumers are served, and for intermediate k kink solutions obtain. This characteristic remains in the oligopolistic model with n players. The number of local monopolists is bounded by k and an equidistant spacing allows for the largest number of firms. There are no relocations incentives for local monopolists. (cf. Economides (1984), p. 312f) Additionally, as in the duopoly, at the kink multiple equillibria exist. (cf. Economides (1984), p. 314)

market structures ($n \geq 3$). (cf. Economides (1993), p. 312 and Theorem 2)

The study of Economides (1993) sends a key message in terms of firms' tendency to locate at the center or the periphery of the linear city: there is no perfect equilibrium for n firms in a simultaneous two-stage price-location game under linear transportation costs and provided that the whole market is served. The location of the central firm(s) represents an 'attractor' and rivals have a dominant strategy to locate away from the city edges. Furthermore, it is interesting that an equilibrium state for n firms with identical profits does not exist, thus, depending on the location configuration an order in firms' profitability obtains. A special case is made for an equidistant spacing pattern where it is shown that a price equilibrium implies a strictly convex, U-shaped structure over the line. For this equilibrium the condition to cover all consumers in the market (i.e. for sufficiently high reservation prices) restricts the peripheral firms to control a smaller market than the interior firms.

In a two stage sequential entry game in the Hotelling model with firms choosing their position in the first stage and selecting a profit-maximizing price in the second stage the application of quadratic transportation costs comes with the benefit of a well-defined set of Nash equilibrium prices for every location.[38] Thus, with quadratic transportation costs a perfect subgame equilibrium in locations can be examined which is exploited in the papers of Neven (1987), Economides et al. (2004) and Goetz (2005). The goal of these studies is to scrutinize location equilibrium configurations and determinants for market deterrence in the linear city.

The important feature of the underlying model is the assumption of fixed entry costs F and a sequential entry process while prices are chosen simultaneously in the second stage. Otherwise the classical assumptions of the Hotelling model apply: line of unit length, uniform consumer distribution, constant and zero marginal costs of production, and a constant consumers' reservation price. Moreover, analogous to the principle in Prescott & Visscher (1977), firms' subgame perfect location decisions are derived under the assumption of perfect foresight with respect to subsequent location decisions of other competitors.[39] Following the notation of Neven (1987), the demand function D_i for firm i is defined by the locations of the indifferent consumers in the middle markets to its left and right side, in sum demand equals a firm's total market area.[40] Profits are $\Pi_i = P_i D_i - F$ ($i = 1, ..., n$). In addition, as was previously shown for a duopoly in Neven (1985), the model implies that the simultaneous price game

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[38] Quadratic transportation costs are convex and guarantee concave profit functions, thus second order conditions are satisfied and a noncooperative price equilibrium is obtained. Here, once again, Economides (1984) proves to be a valueable source (see proposition 1 and p. 350ff).

[39] For an illustrative example of the calculus see Economides et al. (2004), section 4 on p. 11f.

[40] Assume n firms, then $D_i = max(0, minR - maxL) =: max(0, \overline{\alpha}_i - \underline{\alpha}_i)$ where L is the set of all possible market boundaries (α) with all neighbors to the left of i (including the city edge 0), and R is the corresponding set to the right, formally: $L = [0, \alpha_{i,k}, k = 1, ..., i-1]$ and $R = [1, \alpha_{i,k}, k = i+1, ..., n]$ (cf. Neven (1987), equation (3), p. 422f)

in the second stage yields a unique equilibrium also for n firms. (cf. Neven (1987), p. 432f) Thus, a perfect subgame in locations can be played.

In Neven (1987), the results for the case of an exogenously given number of firms yield the monopolist locating at the center ($x_1 = \frac{1}{2}$), and in a duopoly the two players locating at the opposite ends maximally differentiating their product. Further, it is mentioned (Neven (1987), p. 425) that the outcome of sequential entry in a duopoly is fully in accord with the result of a simultaneous location decision (cf. Neven (1985)). The equilibrium for three firms reveals an asymmetric pattern with the first firm located centrally (but not at $\frac{1}{2}$) and the subsequent competitors close to the edges. The asymmetry follows directly from the backward induced calculus, in particular, it proves to be more profitable for the second entrant to divert from a regular location pattern if the first firm took the center (and the third firm acted accordingly)[41], in turn, this implies that the optimal decision of the first firm is to locate asymmetrically with respect to the central position. For four firms a higher degree of symmetry prevails, the first two entrants balance the advantages from a central position with increasing market sizes (and locating farther apart). In sum, the equilibrium for $n = 4$ reveals a first-mover advantage with the first firm locating more centrally than the second and thus gaining the highest profits. Furthermore, it is concluded that the first-mover advantage and thus the asymmetry in the equilibrium location pattern decreases as the number of firms increases.

Next, let us turn to the results in Neven (1987) for the case of entry deterrence. In this case the number of firms and firms' location is endogenized and entry results from the level of fixed costs *and* the location choice of the incumbent firms. Clearly, the higher F, the lower n. Furthermore, the level of F determines firms' strategic behavior. As expected, for the monopoly case the first entrant locates at the center, in a duopoly with comparatively high fixed costs both firms maximally differentiate. However, if fixed costs are further reduced the two players apply a deterrence strategy and move inwards. The outcome then is a symmetric location pattern, and a continuum of unique possible equilibrium locations dependent on the level of fixed costs obtains with the extreme location pair given at 0.31 and 0.69. (cf. Neven (1987), table 2, p. 429) Similarly for three players, the market configuration starts with the unconstrained case for a defined interval of fixed costs.[42] As fixed costs further decrease deterrence strategies are devised and equilibrium locations result contingent on F. Specifically, the first firm locates close to the center and the second and third entrant choose an entry deterring position on each of its sides. For decreasing F the location pattern becomes more symmetric with the extreme case of firm 1 at $\frac{1}{2}$ and the other players locating symmetrically at a position slightly below

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[41] The second and third firm would locate at $\frac{1}{8}$ and $\frac{7}{8}$ respectively.

[42] Confirm that the locations in table 1 (Neven (1987), p. 425) and table 2 for $0.0245 < F < 0.0255$ (Neven (1987), p. 429) are the same.

the quartiles indicating an advantage for the first entrant. For four firms it is firstly observed that, in contrast to the previous cases, the unconstrained equilibrium can not be supported, and secondly, later entrants bear the cost of entry prevention. For a comparatively higher level of fixed cost (e.g. $F = 0.007$) the first entrant locates close to the center, and for low F (e.g. $F = 0.004$)) locations close to the city edges are more profitable for the early entrants. Additionally, the location pattern is symmetric for the boundary case such that a new firm is indifferent in which market slot to enter.

Economides et al. (2004) provide further details and consolidate Neven's findings. For a fixed number of firms the location equilibrium configurations for $n = 1, 2, 3$ are confirmed. Furthermore, the principle that the profits decrease with the consecutive order of entry, in particular, that the first mover locates centrally and gets the highest profits is supported. In contrast to Neven, however, in the case of $n = 4$ the results of Economides et al. (2004) suggest the second entrant locating not centrally but between the first firm and the left corner of the city (cf. Economides et al. (2004), table 3, p. 9 and Fig. 1, p. 7).[43]

In addition, as regards the entry deterring game a couple of differing points to Neven's study deserve attention. Firstly, Neven (1987) claims that in a duopoly both firms simultaneously move towards the center as fixed costs decrease leading to a symmetric pattern.[44] By contrast, Economides et al. (2004) show that initially only the first firm actively prohibits entry by moving inward. Clearly, then an asymmetric location equilibrium over the fixed cost range results. (cf. Economides et al. (2004), p. 13 and Fig. 2, p. 14) Moreover, this finding is also confirmed by Goetz (2005) (cf. Goetz (2005), p. 253 and Fig. 1, p. 252).[45] The second point pertains to the case where $n = 3$. Neven (1987) emphasizes that deterrence costs are mostly borne by the second and third entrants with an equilibrium for the boundary of a forth firm to enter at $(0.255, 0.5, 0.745)$.[46] On the contrary, the results of Economides et al. (2004) yield an evenly spaced out boundary equilibrium $(0.25, 0.5, 0.75)$ (cf. Econo-

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[43] Indeed the unconstrained case with the predefined number of four entrants in Economides et al. (2004) seems to resemble the outcome of the free entry game in Neven (1987) for $F = 0.007$ (cf. Neven (1987), paragraph (vi), p. 430 and table 2, p. 429).

[44] „[...] the duopolists will deter entry of a third firm. They will both move inside, to an extent such that the second firm will have to choose symmetric location to deter entry. This is a situation in which the burden of entry deterrence is shared equally between two firms." (Neven (1987), p. 429)

[45] Note also that in Goetz (2005) the profits of the first entrant increase as he takes on the task of entry deterrence and increases his location. This marks an important distinction of his model where equilibrium locations are calculated for changes in market size. Technically, his demand function is supplemented by the total population in the market denoted with N, formally: $D_i = max(0, N(minR - maxL))$. (cf. Goetz (2005), p. 251) Thus, a variation in entry costs is modeled by changes in N keeping fixed costs in the profit function constant, e.g. for increasing N entry costs fall. By contrast, in Economides et al. (2004) and Neven (1987) fixed costs are variable.

[46] „It is noticeable that [...] the burden of entry deterrence is mostly carried by the second and third firms which are being constrained." (Neven (1987), p. 430) The numbers are from table 2 on p. 429.

mides et al. (2004), table 6, p. 21). Again, Goetz (2005) provides similar results. (cf. Goetz (2005), paragraph x, p. 257) Furthermore, Economides et al. (2004) stress the fact that when comparing profit losses due to entry deterrence with the scenario of a fixed number of firms the first entrant bears the highest costs of the prohibitive action. Therefore, no clear sign of an advantage for the first firm in the deterrence game if $n = 3$ is recognizable. Thirdly, for the case of four active firms as for the triopoly Neven (1987) suggests that the first entrant unambiguously benefits from the entry deterring efforts of the later entrants.[47] The results of Economides et al. (2004) are in striking contradiction to this prediction of a first-mover advantage since they find that in certain ranges of fixed costs the order of profits does not correspond to the order of entry anymore (as in the unconstrained case). Particularly, regions are observed where profits of the second entrant are larger than of the first mover, and even profits of the third firm increase those of the first firm which leads them to conclude that late entry in a free entry game is profitable. (cf. Economides et al. (2004), p. 22 and Fig. 16, p. 23)

Goetz (2005) picks up the notion of advantages for late movers under free entry and focuses on the case of $n = 3$. He finds a third-mover advantage where profits of the third entrant exceed those of the second in a defined fixed cost range. Consider the following explanation. As the first firm takes the burden of entry deterrence it moves to the center. However, this implies that the advantage of the second firm against the third diminishes since, for a given location of the first firm, when taking his turn the second firm can always choose the 'better' side. Only when the first firm locates in a small interval around $\frac{1}{2}$ this strategic advantage vanishes which in turn gives a benefit to the third entrant.[48] Subsequently, the second firm bounces back when it starts to locate more centrally participating with the first firm in entry deterring actions which kicks off at the particular value of the market size for equilibrium locations of the simultaneous game at $(\frac{1}{8}, \frac{1}{2}, \frac{7}{8})$. (cf. Goetz (2005), paragraphs vi and vii, p. 255f) Eventually, the entry deterring behavior of the first and second firm leads to a discontinuity in the equilibrium locations for increasing market sizes (decreasing fixed costs) since it becomes profitable for the first firm to switch from the center to a more remote position. However, note that the profit function of the first firm remains continuous. (cf. Goetz (2005), Fig. 4 and Fig. 5, p. 256 and paragraph ix, p. 257) This finding is in contrast to the predictions given in Economides et al. (2004).

Now, what is to conclude from these studies of market entry dynamics with regards to the PMD? In sum, the papers argue that firms' location choice implies the strategic element of entry deterrence. It is illustrated that as market conditions become

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[47]„As observed in the case of three firms, the first entrant is able to use the entry deterring behavior of further entrants to its own advantage." (Neven (1987), p. 431)

[48]Note that Economides et al. (2004) already provided an analogous result but did not give a thorough explanation. (cf. Economides et al. (2004), p. 18f and the fixed cost range $[0.020, 0.022]$ in Fig. 10, p. 19)

more competitive, that is under decreasing levels of fixed costs (and increasing market size), firms move together and bear the cost of entry prevention. Consequently, fixed costs (and market size) and consecutive strategic decisions imply an attracting potential for firms' location even under quadratic transportation costs (which otherwise are an indicator of repellent forces for equilibrium locations). Generally, location configurations and thus the tendency to locate at the center are dependent on the level of entry costs revealing symmetrical and asymmetrical equilibria. In particular, examples are provided for boundary solutions with symmetric location patterns[49] which specifies previous findings of Hay (1976) and Prescott & Visscher (1977). Moreover, the sharing of the cost of entry prevention depends on the market structure, but predictions in the studies differ. The state of the field suggests that in defined parameter ranges late entry in free entry games can be beneficial for markets with three and four firms.

A way to circumvent the problem of the existence of Nash price equilibria and perfect subgame location equilibria in the Hotelling model is to apply a Stackelberg leader-follower structure in the price and location subgame. This approach is analyzed in Anderson (1987) for a duopoly where in the first stage one firm is the leader and the other the follower in choosing the location, and in the second stage a price leader-follower game is played. Subsequently, the goal of the paper is to determine the order of firms' actions, specifically, to endogenize the pricing decisions (since the assumption is that one firm will always be the first to locate in the market).
The setting is the linear city with linear transportation costs and the game is solved by backward induction. Thus, initially and w.l.o.g., the price reaction function for player A is derived, that is firm B is to be taken as the price leader. The price reaction covers the possibility to undercut B if he sets a price $p_B > \overline{p}_B$, react to an undercutting of B who charges $p_B < \tilde{p}_B$ by an adaptive price just to stay in the market, and set the profit-maximizing price if $p_B \in [\tilde{p}_B, \overline{p}_B]$. The set of A's pricing responses is dependent on the location pair (a, b). (cf. Anderson (1987), proposition 1, p. 373f)[50] Subsequently, firm B's leadership prices as a function of a and b are derived. (cf. Anderson (1987), proposition 2, p. 379ff) If the competitors are located fairly close, i.e. for $1 - a \leq \sqrt{b}$, B's best decision is to set the price $\hat{p}_B$ for which A is indifferent between undercutting and playing defensively. For greater distances B has three profit-maximizing pricing strategies which all lead to A optimizing the

[49] Neven (1987), table 2, p. 429: $(n = 2, F = 0.0255)$, $(n = 3, F = 0.009)$ and $(n = 4, F = 0.004)$. Economides et al. (2004), table 5−7, pp. 17, 21 and 25: $(n = 2, F = 0.025857)$, $(n = 3, F = 0.009)$ and $(n = 4, F = 0.00435)$. Goetz (2005), pp. 254 and 257: $(n = 2, F = 0.0258(N = 967.6))$ and $(n = 3, F = 0.00892(N = 2804.0))$.

[50] As firms get relatively close, the region of profit maximization M collapses and the price reaction is solely characterized by a discontinuous behavior. (cf. Anderson (1987), figures on p. 376f) Technically, the price intersections of the profit functions corresponding to the three different strategies are evaluated. The location regions for the applicability of the strategies follow from the order of the price intersections. (cf. ibid, paragraph (d) in proof of proposition 1 on p. 391f)

quadratic part of his profits over the price. For B these are the corner solutions $\tilde{p}_B$ and $\overline{p}_B$ as well as the price where the quadratic part of his profits is maximized.[51] Based on the solution for firms' optimal price setting behavior with B as the price leader and A as the price follower their location reaction functions are determined.[52] The distinct feature is that if both players locate very close at one of the city edges (either for $b \leq \frac{1}{2}(45 - 7\sqrt{41}) \approx 0.089$ and $b \geq \frac{1}{2}(7\sqrt{41} - 43) \approx 0.911$) then firm A's repellent move is defined by the reaction function $a(b) = 1 - \sqrt{b}$ and $a(b) = \sqrt{1 - b}$ respectively. However, for any other b, a continuum of favorable locations for A obtains, that is his location reaction function consists of a set of different combinations (a^*, b) yielding the same profits.[53] For firm B a defined location reaction function obtains dependent on which side of $\frac{1}{2}$ his rival locates. Considering firm A's profits along B's location reaction function yields the highest value for $a = \frac{1}{2}$, that is given B's location reaction firm A has the incentive to locate at the center. (cf. Anderson (1987), p. 397) Now, two scenarios are compared.

1. Firstly, firm A is the location leader, firm B the follower. The price leadership retains with B. Clearly, A then picks the center and B locates according to his reaction function for $a = \frac{1}{2}$ at the equilibrium location $b^* = 0.131$ and 0.869 respectively. Since A is the price follower he sets a lower price than B to expand his market area whereas B charges a higher price to maximize his profits. In sum, this yields the highest profits for A as the location leader, given the structure of the Stackelberg pricing game. (cf. Anderson (1987), proposition 5, p. 384f)

2. Secondly, firm B is the location leader and A the follower. The price leadership retains with B. Since B is the location and price leader he takes a remote position according to his location reaction function allowing him to charge a high price. To derive the highest possible profits for B it is assumed that $a = 0$ since the set of indifferent locations for A also allows for other less profitable outcomes. In sum, this yields the highest profits for B as the location lead-

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[51] For an illustration see figure 8 (Anderson (1987), p. 382). If both players are located close to the city edges, B wants to expand his market area with the the lowest possible price $\tilde{p}_B$ such that A remains accommodating (region 2b). If B locates more centrally and comes not too close, a high price strategy is the best reply where $\overline{p}_B$ represents the highest price such that A has no incentive to undercut. (region 2a) For distant locations of B and A locating centrally the profit-maximizing price is the best choice (region 2c).

[52] For firm A the algebraic conversions involve for every of the four cases $(\hat{p}_B, \tilde{p}_B, \overline{p}_B)$ as well as B's profit-maximizing price the evaluation of the reduced form price reactions, i.e., insert p_B into the corresponding price reaction of A. Subsequently, the resulting p_A is inserted into A's profit function and the corresponding market demand is calculated. Finally, profits are differentiated by a. (cf. Anderson (1987), proof of proposition 3, pp. 392-394) Analogously, prices and demand for firm B are derived for every of the four strategies and the partial derivative of his profit function by b is evaluated. (cf. Anderson (1987), proof of proposition 4, pp. 395-397)

[53] This is due to A's profit function evaluated at $\overline{p}_B$ which results as $\Pi^*(p_A(\overline{p}_B), \overline{p}_B) = 2t(1 - \sqrt{b})^2$ and is thus independent of a. (cf. Anderson (1987), p. 392f)

er, given the structure of the Stackelberg pricing game. (cf. Anderson (1987), proposition 6, p. 385f)

The final argument of the paper is concerned with the price leadership decision (and not with the location leadership decision). It is to prove that the first scenario yields an equilibrium, that is to show that price followership by the early entrant (firm A) is supported by the later entrant (firm B). Therefore consider that the difference between the location leaders in the two scenarios is their role in the Stackelberg game in prices. A comparison of respective profits yields the price follower in an advantageous position. This means that firm A prefers to be the price follower. But does firm B act accordingly? Indeed, this is the case since for firm A locating at $\frac{1}{2}$, that is for given locations in the first stage, firm B earns higher profits as a price leader than as a price follower.[54] (cf. Anderson (1987), proof of proposition 7, p. 387)

To conclude, in terms of the PMD the study of Anderson (1987) offers a new perspective where in the cause of a sequential price setting game an equilibrium is found with the first entrant locating at the city center and taking the role of the price follower. The location reaction of the second firm, however, suggests to locate distantly ($b^* = 0.131$ and 0.869) such that repelling forces in the location strategy dominate. Furthermore, the second entrant's dominant pricing strategy then is to take the position of the leader. Two aspects characterize the equilibrium outcome. Firstly, the sequential order in choosing price and location leads to a strategic advantage for the first firm exemplified by the taking of the central position and higher profits and lower prices. Secondly, the threat of being undercut leads the second player to take a secure market position with comparatively lower profits and a lower market size but charging higher prices than the incumbent firm.

More recently, Fleckinger & Lafay (2010) study a two stage game in the Hotellling model where in contrast to the previously presented papers the assumption of irrevocable product choices is relaxed. Instead, they introduce catalog competition, that is they examine equilibrium states in a duopoly where each firm decides for the product position (location) and price in one stage of the game. Consequently, the structure is such that in the first stage firm A (leader) chooses its strategic variables, and in the second stage firm B (follower) decides on his.

In their model the usual assumptions apply.[55] To recap, one consumer's problem located at x is to minimize his disutility $(p + C(x - a), q + C(b - x))$ where C shall be convex, monotonically increasing and differentiable. Thus, the indifferent consumer

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[54]Profits for B as a leader in the first scenario are $0.428t$, profits as a follower are derived using the follower profit function considering the optimal price-follower location, i.e. firm A's profits for $b = \frac{1}{2}$ which yields $0.172t$.

[55]These are: unit interval ($x \in [0, 1]$), even consumer distribution $f(x)$, zero production costs, sufficient reservation price. Note that locations a, and b are measured by their distance from zero.

between the two firms at position y shall be characterized by $p + C(a-y) = q + C(b-y)$. The set (p,q) denotes the product prices and (a,b) the locations where the pair (p,a) denotes the calls of the leader and (q,b) those of the follower. The consecutive findings essentially rest upon two propositions. Firstly, it is proved that the follower always gains (weakly) higher profits than the leader.[56] This confirms the intuition that the follower has two profit-maximizing choices, he could either undercut and kick the leader out of the market, or he could accommodate the leader. In the case of accommodation, it is secondly proved that the follower locates at the position of the indifferent consumer $b = y$ and that he charges a strictly higher price than the leader along consumers' disutility function, i.e., $q = p + C(b-a) > p$.[57] In what follows, these results are applied to the Hotelling model, i.e., set $C(x) = t|x|$. Then, accommodation yields profits of $\Pi_B^{Acc} = q(1-b) = (p + t(b-a))(1-b)$. Evaluating $\frac{\partial \Pi_B}{\partial b} = 0$ and using $q = p + t(b-a)$ yields the optimum pair $(b^* = \frac{1}{2}(1 + a - \frac{p}{t}), q^* = \frac{t}{2}(1 - a + \frac{p}{t}))$. Undercutting profits are $\Pi_B^{UC} = p$. Profits for firm A in the accommodation case are $\Pi_A^{Acc} = pb^*$ which shall be maximized subject to $\Pi_B^{Acc} \geq \Pi_B^{UC}$. Evaluating this constraint yields $a^* = 1 + \frac{p}{t} - 2\sqrt{\frac{p}{t}}$. Subsequently, $\frac{\partial \Pi_A^{Acc}(a^*)}{\partial p} = 0$ leads to $p^* = \frac{4}{9}t$. In sum, for $a \leq \frac{1}{2}$ the catalog solutions $(a^*, p^*, b^*, q^*) = (\frac{1}{9}, \frac{4}{9}t, \frac{1}{3}, \frac{2}{3}t)$ obtain. (cf. proposition 4, p.64) Walking through the same principles for quadratic transportation costs $C(x) = tx^2$ the equilibrium is found for the set $(a^*, p^*, b^*, q^*) = (0, \frac{1}{4}t, \frac{1}{2}, \frac{1}{2}t)$. (cf. Fleckinger & Lafay (2010), proof of proposition 5, p.68)

In conclusion, due to set up and the particular timing structure of choosing a catalog for the strategic variables price and location, Fleckinger & Lafay (2010) derive different results for the Hotelling model with linear transportation costs compared to Anderson (1987). In terms of the location it is revealing that firms share a comparatively shorter distance on one side of the market ($a = \frac{1}{9}, b = \frac{1}{3}$ for $a \leq \frac{1}{2}$) under catalog competition. It is then the leader's strategy to choose a remote location with a low price while the follower benefiting from the potential undercutting threat can decide on a more centralized location and charges a higher price. This emphasizes that players have to commit themselves differently under different circumstances which may turn out for their advantage or disadvantage. In Anderson (1987) the first entrant commits himself by his location and draws an advantage from the irre-

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[56] The indirect proof initially assumes that $\Pi_B < \Pi_A$ with corresponding equilibrium pairs (a^*, p^*) and (b^*, q^*). Since firm B is free to undercut A with $p^* - \epsilon$ this leads to a contradiction. Thus, $\Pi_B \geq \Pi_A$. (cf. Fleckinger & Lafay (2010), proof of Proposition 2, p.67)

[57] By symmetry assume $a \leq \frac{1}{2}$. If firm B accommodates, he does not undercut, thus $q > p$ for $b \geq a$. Since $q > p$ firm B exploits the consumers to its right market side. Thus, the goal is to minimize y. Applying $\frac{\partial}{\partial b}$ to the indifference condition yields: $\frac{\partial C(y-a)}{\partial b} = \frac{\partial C(b-y)}{\partial b} \rightarrow C'(y-a)y' = C'(b-y)(1-y')$ or $(C'(y-a) + C'(b-y))y' = C'(b-y)$. Two cases have to be differentiated. Firstly, $y \leq b$ which leads to $y' > 0$ and a decrease in b causes y to decrease. Then firm B seeks to minimize b. Secondly, $y \geq b$ implying $y' < 0$ and an increase in b leads to a decrease in y. Then firm B seeks to maximize b. The optimum is, of course, to locate at $b = y$. The profit-maximizing price follows directly from the indifference condition under consideration of the best location choice: $q = p + C(b-a)$. (cf. Fleckinger & Lafay (2010), proof of lemma 1, p.67)

vocability of this choice by becoming the price follower. By contrast, in Fleckinger & Lafay (2010) the commitment of the leader to set the strategic catalog leads to a disadvantage. It is interesting to see that the nature of the product and the market characteristics is the source for this asymmetry.[58] Market peculiarities that imply the requirement to chose the location (product position) and price simultaneously give the second entrant the credible power of an attrition strategy. As the follower he can wait for the leader to post his price before he chooses his position (and price). However, if the circumstances are binding only for the location and allow for flexibility in the pricing decision, then the first mover is far better off. As the presented models convincingly demonstrated, this instance has profound consequences for firms equilibrium locations and their respective distance.

Departing from the study of Economides (1984) the paper of Hinloopen & van Marrewijk (1999) investigates the impact of a variation of consumers' reservation price in a two stage sequential Hotelling game where location and price are each simultaneously chosen in one stage. Hinloopen & van Marrewijk (1999) re-examine previous results on sellers' tendency to disperse in the market and develop a general framework to analyze the effect of the reservation price on Nash equilibrium prices and equilibrium locations.

The common model structure of a Hotelling duopoly under linear transportation costs is used with locations of firm A and B taken from the respective edges of the interval $[0, l]$ and denoted as h_a and h_b. Throughout it is assumed that firms locate symmetrically, i.e. $h_a = h_b$. The reservation price is variable but shall be the same for all consumers in the market denoted as v. As a distinct model parameter α is introduced. (cf. Hinloopen & van Marrewijk (1999), p. 737) α measures the market size relative to the effective reservation price $\frac{v}{t}$ with t defined as the transportation cost for traveling one unit distance. Thus, total market length can be expressed as $l = \alpha \frac{v}{t}$. Keeping l constant, it is obvious that a high value of α corresponds to a low value of v and vice verser. Subsequently, the task is set to find equilibrium sets for the price-location pair for the range of intermediate reservation prices $\frac{8}{7} < \alpha < 2$ since the case of high reservation prices $\alpha < \frac{8}{7}$ is covered in Hotelling (1929)[59] and the case of low reservation prices $\alpha > 2$ is dealt with in Economides (1984)[60].

The subsequent analysis is separated into two parts that consider comparatively low

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[58] Fleckinger & Lafay (2010) provide some examples for flexible catalogs. See their discussion section on p. 65f.

[59] For the primary reference to check for this dependency see Hinloopen & van Marrewijk (1999), p. 738 and their reference section.

[60] Recall that in Economides (1984) the market equilibrium for the duopoly is determined by two local monopolists whose market areas do not intersect and who both completely serve their local markets implying that a fraction of consumers (between the boundary and the city edge) are not served. Thus for given l, the market size of the local monopoly is determined by the reservation price $\frac{v}{t} = \frac{l}{\alpha}$ (which essentially accounts for the indifferent consumer at the boundary). Since the city shall allow both monopolies to thrive this implies $2\frac{v}{t} < l$ or $\alpha > 2$.

reservation prices ($\frac{4}{3} < \alpha < 2$, cf. Hinloopen & van Marrewijk (1999), pp. 738-742) and high reservation prices ($\frac{8}{7} < \alpha < \frac{4}{3}$, cf. ibid., pp. 742-747).

- Part 1: Two cases have to be distinguished, firms locating below the quartiles $h_a = h_b < \frac{l}{4}$ and between the center and the quartiles. For $h_i < \frac{l}{4}(i = a, b)$ two local monopolies exist. Now, a firm could set a price such that the consumer at the city edge has a positive net utility of purchasing which is equivalent to his reservation price exceeding the total cost of buying (price plus transportation cost). Then demand comprises of h_i plus the area extending to the most distant consumer located at $x = \frac{1}{t}(v - p_i)$ who is indifferent between buying or not. Alternatively, a firm could charge a price such that the indifferent consumer sits at the city edge. Clearly, demand then extends the area twice of the firm's distance to the edge, i.e., $2h_i$. Based on this demand function the profit functions are set up, using $\frac{\partial \Pi_i}{p_i} = 0$ equilibrium prices p_i^* and equilibrium profits $\Pi_i^* = \Pi(p_i^*)$ are derived. From $\frac{\partial \Pi_i^*}{\partial h_i} > 0$ it follows that sellers are inclined to move towards the quartiles (if they are located at $h_i < \frac{l}{4}$).[61] For $\frac{l}{4} < h_i < \frac{l}{2}$ demand and profits correspond to Economides (1984), p. 355. However, since now a comparatively higher v is considered (that is for $\alpha < 2$) two supplementary remarks concerning the derivation of Nash price equilibria and respective price bounds are made.[62] (cf. Hinloopen & van Marrewijk (1999), p. 741) Nevertheless, these do not modify the prediction on sellers tendency to

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[61] The first demand-subcase yields $\Pi_i = p_i(h_i + \frac{v - p_i}{t})$, setting the first derivative zero gives $p_i^* = \frac{1}{2}(v + th_i)$. To be consistent recall that $p_i^* < v - th_i$ (also charging the Nash price gives the consumer at the city edge a positive net utility) which is equivalent to $h_i < \frac{v}{3t}$, and for $\frac{v}{3t} < \frac{l}{4}$ we get $\frac{4}{3} < \alpha$. To prove sellers' relocation tendency, see that $\frac{\partial \Pi_i(p_i^*)}{\partial h_i} = \frac{1}{2}(v + th_i) > 0$ and $h_i^* = -\frac{v}{t}$ leading to $\Pi_i^* = -\frac{(p_i^*)^2}{t}$. In the second demand-subcase $x = h_i$, thus $p_i^* = v - th_i$, and $\Pi^* = 2h_i(v - th_i)$, and $\frac{\partial \Pi^*}{\partial h_i} = 2v - 4th_i$. Clearly, the maximum is $h_i^* = \frac{1}{2}\frac{v}{t}$. The market area must suffice $2h_i < \frac{l}{2}$, and $\frac{\partial \Pi^*}{\partial h_i} > 0$ requires $h_i^* > \frac{l}{4}$ which is equal to $\alpha < 2$.

[62] The first remark specifies the range of α where the model of Economides (1984) can be applied without restrictions which is for α exceeding an approximate value of 1.884. (cf. Hinloopen & van Marrewijk (1999), equation (A.4), p. 750) This threshold indicates that by undercutting one's rival the complete market can be served, that is the indifferent consumers with their reservation price leveling total buying costs on both sides of the mill are served. For instance, for higher v (and lower α) the indifferent consumer to the left of firm A (locating h_a from zero) falls out of the city and would be located in the negative range. (cf. ibid, Fig. 3b, p. 740) The critical condition for all to remain in the market is given by $h_i < \frac{1}{t}(v - p_i^{UC})$. Thus, for $\alpha < 1.884$ the validity range for the price equilibria have to be adapted (see ibid., equations (6b) and (6c), p. 742). The second remark states that the 'touching' Nash price equilbrium of Economides (1984) on p. 357f does not necessarily exist. The existence is subject to a value of $\alpha > \frac{12}{7}$. As firms move away from the center (i.e. equilibrium locations decrease) Nash prices increase. A touching equilibrium will not exist if, starting from the competitive Nash price equilibrium p_a^c, a price (smaller than the Nash price of the 'touching' equilibrium p_a^t) is reached such that the indifferent consumer at the city edge is served (this price is found in the 'competitive equilibrium with full supply', cf. Hinloopen & van Marrewijk (1999), Fig. 4, p. 741). This happens to be the case for comparatively high values of v (horizontal line for v is shifted upwards). By contrast, the touching equilibrium is reached if the price for a competitive equilibrium in full supply lies below p_a^c which occurs for comparatively smaller values of v.

move outwards if they are located at $h_i > \frac{l}{4}$. Bringing these results together for $\frac{4}{3} < \alpha < 2$ the equilibrium location results with $h_i^* = \frac{l}{4}$.[63]

- Part 2: Again the two cases $h_i \leq \frac{l}{4}$ and $\frac{l}{4} < h_i < \frac{l}{2}$ have to be analyzed. If locations are below the quartiles, already a competitive situation arises. The economic interpretation is that as v increases (and α decreases) more consumers are inclined to purchase which leads both rivals - even though they are located comparatively far apart - to compete for the indifferent consumer at the market center at $\frac{1}{2}$.[64] It is clear that then $p_i^* = \frac{1}{2}(v + th_i)$ (the consumer at $x = 0$ enjoys a net utility surplus), and that the market area comprises $l \leq [h_a + \frac{v - p_a}{t}] + [h_b + \frac{v - p_b}{t}]$.[65] Further, profits are $\Pi_i^* = p_i^* \frac{l}{2}$, and from part 1 follows $\frac{\partial \Pi_i^*}{\partial h_i} > 0$ which implies both firms locating towards the quartiles (if they are located at $h_i < \frac{l}{4}$). Next consider locations above the quartiles and the case of market competition. Then, no net utility surplus obtains, thus $p_i^* = v - th_i$, and $\Pi_i^* = p_i^* (\frac{l}{2} + (h_i - h_j))$.[66] Subsequently for symmetrical locations, $\frac{\partial \Pi_i^*}{\partial h_i} = 0$ yields $h_i^* = \frac{v}{t} - \frac{l}{2} = l(\frac{1}{\alpha} - \frac{1}{2})$. Then, $h_i^* \geq \frac{l}{4}$ reduces to $\alpha \leq \frac{4}{3}$ which illustrates that under price competition firms will locate within the market quartiles if $\frac{4}{3} \geq \alpha \geq \frac{8}{7}$.[67] Clearly, for higher v (and lower α) both firms locate towards the center with the closest locations for $\alpha = \frac{8}{7}$ at $h_i = \frac{3}{8}l$ and $h_j = \frac{5}{8}l$ which equals a minimum distance of one quarter of the city length.

In conclusion, Hinloopen & van Marrewijk (1999) provide evidence that firms in a Hotelling duopoly have the tendency to agglomerate at the city center. However for the assumption of symmetrical locations, they emphasize that firms' optimal location decisions are crucially dependent on the level of consumers reservation price. In particular, they specify the corresponding bounds for which the Nash price-location equilibrium to solve the underlying two stage simultaneous game exists such that every consumer in the market is served. Accordingly, the closest firms could get is a quarter of the market length. Furthermore, in their model the economic intuition is

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[63] Then the equilibrium price is given by the corner solution $p_i^*(h_i^*) = v - th_i = v(1 - \frac{1}{4}\alpha)$. (cf. Hinloopen & van Marrewijk (1999), lemma 1, p. 742)

[64] Recall that if v is low in the limiting case $\alpha > 2$ firms only serve the customers in their local monopoly region and some consumers close to the edges are not served at all which is originally dealt with in Economides (1984). Also recall from part 1 in the first bullet above that for intermediate values $\frac{4}{3} < \alpha < 2$ no competition for $h_i < \frac{l}{4}$ occurs.

[65] Consequently, inserting profit-maximizing prices yields $2l - 2\frac{v}{t} \leq h_a + h_b$, and for $h_a, h_b \leq \frac{l}{4}$ this reduces to $h_a + h_b \leq \frac{l}{2}$. Consistency demands $2(l - \frac{v}{t}) \leq \frac{l}{2}$ or $\alpha \leq \frac{4}{3}$.

[66] Profits are described in a general form to show the location reaction function $h_i(h_j)$. Demand equals $\frac{l}{2}$ plus an additional term if firms would locate asymmetrically. (cf. Hinloopen & van Marrewijk (1999), p. 745)

[67] The lower bound is derived by ruling out the possibility of undercutting one's rival. The undercutting price demands to take the whole middle market, thus $p_i^{UC} = p_j^* - t(l - h_i - h_j) = v - tl + th_i$. Undercutting gives total demand of l and is discarded if $\Pi_i^{UC} \leq \Pi_i^*$ which reduces to $l(\frac{v}{2t} - l) \leq \frac{v}{t}(h_i - h_j) - h_i^2 + h_i h_j - \frac{3}{2}lh_i$, and for symmetrical locations to $h_i \leq -\frac{v}{3t} + \frac{2}{3}l$. Combining h_i^* with the undercutting threshold gives $\frac{v}{t} - \frac{l}{2} \leq \frac{2}{3}l - \frac{v}{3t}$ or $\frac{8}{7} \leq \alpha$.

confirmed, suggested by Hotelling and continuously revised by subsequent studies, that „*it is then profitable for a firm to move towards the market centre in order to strive for a larger market share, but never so much that undercutting will be profitable. But the closer firms locate (or, the more homogeneous the products are), the fiercer competition will be, and hence, the lower the price the firms can quote.*" (Hinloopen & van Marrewijk (1999), p. 747)

Recent work on the application of the Hotelling model analyzes the effects of entry regulation measures on price and location equilbria as well as on consumer surplus. Elizalde et al. (2015) motivate their work empirically by government action in the Spanish province of Navarra where between 2001 and 2007 restrictive and relaxing measures regulating the number of pharmacies have been enacted. For the authors this raised the question whether the regulatory decisions have been efficient in terms of social welfare. In particular, two ways of regulation, firstly, concessions on the number of licenses, and secondly, minimum distance rules between firms' locations have been investigated in a Hotelling duopoly with quadratic transportation costs and under a variation of the reservation price k.

For the first policy measure of conceding licenses a simultaneous choice of locations and a sequential entry game are studied (prices are assumed to be always chosen simultaneously after the location decision is made). The concession of licenses refers to a fixed number of firms constituting the market configuration, thus the results of Elizalde et al. (2015) have to be assessed in the context of the models of Economides (1984) and Hinloopen & van Marrewijk (1999) for the simultaneous location game, and Neven (1987), Economides et al. (2004) and Goetz (2005) in case of sequential entry. Now, the findings of Elizalde et al. (2015) for a duopoly under simultaneous entry reveal that the transportation cost scheme (linear or quadratic) does not make significant differences in terms of firms' equilibrium locations in the range of low reservation prices. (cf. Elizalde et al. (2015), p. 20f) Particularly, the results show that according to d'Aspremont et al. (1979) for sufficiently high reservation prices maximum differentiation obtains. As k decreases the demand effect dominantes the price effect and firms move towards the center. For further decreasing k firms remain at the quartiles. For very low k no pure location equilibrium obtains. In a duopoly under sequential entry the same pattern as under simultaneous entry is observed.

Concerning the second policy measure, the impact of a variation in the minimum distance d in the range $[\frac{1}{4}, \frac{1}{2}]$ at different levels of reservation prices k is evaluated in a sequential location subgame. For high k and over a comparatively high range of k an asymmetric location equilibrium is observed. For $d = \frac{1}{2}$ both firms locate at the city edges, as d decreases firm 1 moves towards the center while firm 2 remains at its remote position. At a value of $d = \frac{1}{3}$ the location pattern reverses and firm 2 turns to the center while firm 1 locates towards the city boundary. Due to the more

central position firm 1 gains higher demand and charges higher prices, thus taking away a higher profit than firm 2. Furthermore, the characteristic feature is an entry deterring behavior, that is firms' distance is kept at $2d$ and their distance to the city boundaries does not exceed d. (cf. Elizalde et al. (2015), p. 22f) For low k no first mover advantage pervails and firms choose the quartiles, as k further decreases the whole market can not be served. In terms of social welfare, consumer surplus is higher under the minimum distance rule provided that k is high (and that demand is inelastic), for low k no respective significant difference between the two policy measures is observed.

In a nutshell, the paper of Elizalde et al. (2015) contributes to previous work concerning equilbrium location patterns by emphasizing the impact of regulatory measures such as a minimum distance rules. It is revealing that under consideration of quadratic transportation costs a decrease in the minimum distance measure spurs the first entrant to locate towards the center leading to an asymmetric location equilibrium. Furthermore, a decrease in the reservation price increasingly levels the first mover advantage. In sum, these findings complement the results of the previously presented location models (e.g. Hinloopen & van Marrewijk (1999), Economides et al. (2004)).

To summarize this section, the general conclusion can be drawn that Hotelling's original proposal that sellers tend to agglomerate at the market center which is referred to as a principle of minimum differentiation (PMD) can not be unilaterally supported for pure Nash price equilibria. Notably, it has been shown that this conclusion depends on particular parameter assumptions. For instance, for quadratic transportation costs pure Nash price equilibria emerge and subsequently location equilibria for different oligopoly settings can be analyzed. Furthermore, the level of consumers' reservation price proved to be an important determinant for equilibrium location patterns in a duopoly with firms locating closer the higher the reservation price, i.e. the more inelastic demand in the market. Particularly, for oligopolistic market with more players ($n \geq 3$) no perfect equilibrium in a simultaneous two-stage price-location game exists which is due to firms' inclination to move away from the city edges. In addition, the literature suggests that the type of the underlying game impacts the equilibrium outcome, that is whether simultaneous and sequential entry under fixed cost is considered, or a Stackelberg setting is imposed on the strategic variable decision, or that the revocability of the product choice and the flexibility of the pricing strategy is assumed. Finally, evidence is provided for the impact of regulatory measures concerning firms' location choice on equilibrium results.

The limitations of this survey have already been mentioned in the introductory note. At this stage it should be subsequently remarked that under modifications of the Hotelling model that generalize the deterministic model structure and introduce

probabilistic measures further conclusions concerning firms' optimal location decision can be drawn. More specifically, in models with heterogeneous consumer preferences and multiple product characteristics the PMD can be restored. Pioneering work has been contributed by de Palma et al. (1985) who relax the original assumption of a fixed reservation price and conjecture that consumers have specific and varying assertions about the offered products about which firms are only informed on an aggregate level. Consequently, the utility function is modeled by a probabilistic measure. This implies that for sufficiently large heterogeneity in consumers' preferences, modeled by the parameter μ in the logit function, firms locate at the center in a location Nash equilibrium. (cf. de Palma et al. (1985), proposition 1 and 2, p. 774f) In a consecutive paper Rhee et al. (1992) extend the notion of heterogeneous consumer valuation to unobservable product characteristics and thus model uncertainty in the buying decision. They show that for sufficient heterogeneity minimum differentiation results. Furthermore, Irmen & Thisse (1998) introduce multiple dimensions in which firms differentiate and suggest that the PMD holds for all dimensions except for the particular dimension which is weighted most importantly. Additionally, in an interesting extension for the case of multi-dimensional product differentiation in an evolutionary model Hehenkamp & Wambach (2010) provide evidence for the restoration of the PMD when firms play a dynamic game and optimize their strategy between offering already established products and new products.

2.3 The Effect of Market Shapes

2.3.1 Models with Circles

First insights on location equilibria for markets with circular shapes are established by Eaton & Lipsey (1975).[68] The immediate implication of the circular shape is that the market is not bounded, thus, no firms at the market periphery exist and all firms face the equal optimization problem locating in a neighborhood with two nearest neighbors.

As a consequence, for Eaton and Lipsey's model 1 with ZCV different results in comparison with the linear city are derived. In particular, equilibrium configurations on

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[68] For a detailed description of their four model specifications refer to chapter 2 of this survey. Moreover note that in footnote 4 on p. 33 Eaton & Lipsey (1975) make reference to a previous discussion on location equilibrium patterns in circular markets that is concerned with the minimization of transportation costs and the existence of a socially efficient outcome. (cf. Grace (1970) and Samuelson (1970)) At least two points are of interest in this debate and cast a shadow on upcoming studies. Firstly, emphasis is given to the significance of relocation costs and its impact on regulatory issues and policy planning. Secondly, provided that transportation costs are increasing in distance traveled, it is shown that regardless of the functional form an equidistant spacing leads to an efficient location outcome in terms of minimal total transportation costs (for the proof see Samuelson (1970), p. 342).

the circle yield multiple equilibria. For instance, duopolists are not inclined to form a pair at $\frac{1}{2}$ since due to the unbounded spatial characteristic of the market for every seller location total demand is split in halves, that is each player shares a market boundary to his left and right and in sum sellers' distances are equally divided. As for $n = 3$ a set of possible equilibrium locations for the third firm obtains on the long side of the other two competitors' markets. (cf. Eaton & Lipsey (1975), Fig. 3, p. 32) Generally, locations are chosen according to the principle that no firm holds a market smaller than half of the market of any other firm (i.e. condition 1.i). In contrast to the linear city the socially optimal configuration with an equidistant spacing of firms locating at a distance of $\frac{1}{n}$ is part of the equilibrium solution set for the circle in model 1 since no peripheral firms exist whose dominant strategy is to move inwards and form a pair. (cf. Eaton & Lipsey (1975), p. 31f)

In model 2 (with a maximum loss conjecture) the socially optimal configuration yields the unique equilibrium for the circle since firms' dominant strategy is to maximize the short side of their market. (cf. Eaton & Lipsey (1975), p. 32f) The only deviation from the $\frac{1}{n}$-solution obtains for the case of a monopoly and a 'pure' duopoly (with no anticipation of a third player entering) where in contrast to the line multiple equilibria exist, that is firms' locations can not uniquely be determined. In addition, models 3 and 4 with variable consumer densities under ZCV and the maximum loss conjecture respectively do not produce different outcomes for the circle compared to the bounded line. (cf. Eaton & Lipsey (1975), pp. 36 and 39)

In a nutshell, the difference in market shapes between a bounded line and a circle in the models of Eaton & Lipsey (1975) (under a simultaneous location game and no price competition) implies that the central position at $\frac{1}{2}$ on the line loses its importance. Consequently, multiple equilibria for the circle obtain whereas in the linear city, for instance, the duopolists locate at the center and no equilibrium for three firms can be achieved. In addition, it becomes clear that the socially optimal configuration with $\frac{1}{n}$-spaced firms becomes more important in the treatment of circular market shapes and is, for instance, part of the solution set of model 1 under ZCV. Furthermore, the following papers represent particular examples for the case of equidistant location patterns in circular markets.

Pioneering work on spatial competition in a circular market is provided by Salop (1979). His goal is to apply the mechanics of monopolistic competition to the differentiated product market on a circle and study the equilibrium properties under free entry. Since two classes of products are considered and consumers either purchase one unit of the differentiated good on the circle (according to their positive net utility of consumption) or spend the remaining income on a homogeneous 'outside' good the model can be considered as an extension of a monopolistic competition model a

la Chamberlin.[69]

The set up follows a two stage order. In the first stage a set of potential entrants simultaneously chooses to enter the market, in the second stage they simultaneously compete in prices. The important assumption is that firms' locations are exogenously given by an equidistant spacing along the circle. As a consequence, this reduces firms to charge an uniform price p in equilibrium and lays emphasis on the equilibrium number of firms n that can be still supported by the market under a zero profit condition. Thus, technically the goal is to find the symmetric zero-profit Nash equilibrium. (cf. Salop (1979), p. 145)

The optimization problem of the L evenly distributed consumers per unit distance on the circle with unit circumference is to maximize their net utility $v - c|l_i - l^*| - p_i$ with v as their reservation price[70], p_i and l_i as the price and location of firm i, l^* as the most preferred brand specification (the consumer's location), and c as the constant marginal rate of transportation. Evidently, linear transportation costs are assumed. (cf. ibid.) Subsequently, the case of a monopoly region and a competitive region are distinguished.[71] Contingent upon v a monopolist captures consumers up to a maximal distance of $x^m = \frac{v-p}{c}$ on its right and left side, thus gains total demand of $q^m = 2Lx^m$. By contrast, if firms compete an indifferent consumer is located at $x^c = \frac{1}{2c}(p_i \pm p_{i\mp 1} + \frac{c}{n})$ and total demand is $q^c = 2Lx^c$. This translates into a total demand curve where the monopoly region exhibits a smaller slope than the competitive region and a kink marks the spot where the monopoly regions of two neighboring firms touch.[72] (cf. Salop (1979), Fig. 1, p. 143) Now, the market equilibrium requires the two standard conditions that, firstly, marginal revenue equals marginal cost $\frac{\partial \Pi}{\partial q} = \frac{\partial AC}{\partial q} = m$, and secondly, price equals average cost $p = AC = m + \frac{F}{q}$ (with F as the level of fixed costs). Further note that provided all consumers in the market are served equidistant spacing requires $q = \frac{L}{n}$. Then, the price and number of firms in equilibrium are determined as $p_m = m + \frac{c}{2n_m}$ and $n_m = \sqrt{\frac{cL}{2F}}$ for the monopoly region and $p_c = m + \frac{c}{n_c}$ and $n_c = \sqrt{\frac{cL}{F}}$ for the competitive region.[73] (cf. Salop (1979), p. 147) At the kink the tangent to the average cost curve is not defined, however, since the kink marks the interaction point of the monopoly regions of two firms the monopolistic demand function q^m evaluated towards the direction of one

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[69]For an introduction into Chamberlinian models see for instance chapter 7.2 in Tirole (2003).

[70]Consumption of the differentiated product only occurs if $v > 0$ which is equivalent to the utility from consuming the differentiated product exceeding the utility from consuming the outside good $u > \overline{s}$. (cf. Salop (1979), p. 142)

[71]The case of a 'supercompetitive' region refers to an undercutting strategy and proofs not to be profitable since the equilibrium price does sufficiently exceed marginal cost. (cf. Salop (1979), p. 148f)

[72]Limits and possibilities of equilibria at kinks in the Hotelling model (bounded $[0,1]$-line) are studied in Economides (1984) and Economides (1993).

[73]$\Pi = p(q)q$ yields $\frac{\partial \Pi}{\partial q} = p'q + p$, use $\frac{\partial p}{\partial q^m} = -\frac{c}{2L}$ and $\frac{\partial p}{\partial q^c} = -\frac{c}{L}$ to determine the prices, and the definition of average costs and $q = \frac{L}{n}$ to determine n.

nearest neighbor (i.e. $q^m = Lx^m = \frac{L}{n}$) can be used to express the equilibrium price $p_k = v - \frac{c}{n}$. Subsequently considering $p = AC$, the equilibrium number of firms at the kink follows with $v - m = n_k \frac{F}{L} + \frac{c}{n_k}$. (cf. Salop (1979), p. 148) These results establish the economics of monopolistic competition for product differentiation on the circle. Firms price above marginal cost with zero profits. The mark up declines as the number of firms rises, in terms of the exogenous variables the mark up is a positive function of transportation costs c and fixed costs F and a negative function of the market size L (number of consumers per unit distance). Corresponding relations obtain for the equilibrium number of firms. Furthermore, Salop emphasizes the counterintuitive equilibrium behavior at the kink solution where an increase in average costs leads to a decrease in equilibrium prices.[74] The economic argument is given that higher costs cause profits to become negative, as a result some firms exit the market and the remaining competitors enjoy higher demand and economies of scale leads them to charge lower prices in the subsequent equilibrium. (cf. Salop (1979), p. 149)

The final part of the paper is concerned with welfare implications. Firstly, it is shown that a local monopolist provides a positive net surplus which implies that the entire circular market should be served.[75] (cf. Salop (1979), p. 150f) Secondly, the number of firms that maximizes total market surplus is derived and found to be lower than the equilibrium number of firms.[76] (cf. Salop (1979), p. 151f)

The paper of Salop (1979) represents a classical study of monopolistic competition in a circular market providing solutions for equilibrium prices and the equilibrium number of firms in the case of local monopolies, market competition as well as the boundary case of touching local monopoly markets. To keep his analysis tractable, however, he does not endogenize firms' location choice in his model but rather imposes an equidistant spacing pattern.

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[74]Graphically the AC-curve moves to the right and the kink solution slides down the monopoly part of the demand function. (cf. Salop (1979), Fig. 8, p. 150)

[75]The proof proceeds in two steps. (1) The net surplus of a firm is given by the value of its production (quantified by the product price p) subtracting respective costs (m, F). Recall that the differentiated product is sold to all L consumers over a distance x up to the indifferent consumer (located at x^*) whose reservation price v equals transportation costs cx and price of the product p. The firm sells to this consumer at the lowest possible price, thus $x^* = \frac{v-m}{c}$. Generally, p and thus net surplus B can be stated as a function of the distance x. Total net surplus sums up the value of the sold products up to the critical distance where the indifferent consumer is located: $B = 2L \int_0^{x^*} (p(x) - m) dx - F = 2L \int_0^{x^*} (v - cx - m) dx - F = 2Lx^*(v - m - cx^* + \frac{1}{2}cx^*) = \frac{L}{c}(v-m)^2$. Then, $B \geq 0$ reduces to $v - m \geq \sqrt{\frac{cF}{L}}$. (2) Monopoly profits are $\Pi_m = (p(q_m) - m)q_m - F = (v - \frac{c}{2L}q_m - m)q_m - F$. Let q_m^{max} solve the first order condition, then $\Pi_m(q_m^{max}) \geq 0$ reduces to $v - m \geq \sqrt{\frac{2cF}{L}}$. Thus, $\Pi_m \geq 0$ always guarantees $B \geq 0$.

[76]In sum n firms serve $2n$ local markets with the indifferent consumer located at the market boundary $\frac{1}{2n}$. Recall that L is the number of consumers per unit distance. Thus, accordingly to the preceding footnote, total surplus for the whole circle is $W = 2n \int_0^{\frac{1}{2n}} (p(x) - m) L dx - nF = 2n \int_0^{\frac{1}{2n}} (v - cx - m) L dx - nF$. Solving the integral and evaluating $\frac{\partial W}{\partial n} = 0$ gives $n^* = \frac{1}{2}\sqrt{\frac{cL}{F}}$. Clearly, $n^* = \frac{1}{2}n_c$ and $n^* < n_m$.

The study of Economides (1989) serves to demonstrate the existence of equilibrium states for prices and location choices in circular markets and vindicates Salop's assumption of an equidistant location pattern by endogenizing firms' optimal location decisions. In particular, he develops a model that proceeds in three stages. Firstly, firms choose to enter, secondly, they simultaneously take their optimal location, and thirdly, given the resulting location setting they simultaneously choose their profit-maximizing prices. Consumers' utility function is taken to be $U_\omega = m - p_j + V_\omega(x_j) = m - p_j + k - (x_j - w)^2$ and comprises a budget endowment m, a constant reservation price k as well as the disutility covering the product price p_j and the transportation cost term.[77] Transportation costs are assumed to increase quadratically in distance traveled. Furthermore, consumers are assumed to be distributed uniformly with density μ over the circumference and firms incur total production costs of $C_j = F + c_j(q_j)$ with an increasing marginal cost function, i.e., $\frac{\partial c_j}{\partial q_j} \geq 0$. (cf. Economides (1989), p. 180f)

Now, each stage is separately analyzed following the principle of backward induction. The starting point for the treatment of the price game are the demand functions (exemplarily for firm j) for a local monopoly $D_j^M = 2\mu\sqrt{k - p_j}$ and for the case of price competition between nearest neighbors $D_j = \frac{p_{j+1} - p_j}{2(x_{j+1} - x_j)} + \frac{p_{j-1} - p_j}{2(x_j - x_{j-1})} + \frac{x_{j+1} - x_{j-1}}{2}$.[78] Based on these the total demand curve for a firm proves to be concave leading to a unique noncooperative equilibrium for the price subgame that depending on the level of the reservation price k yields a solution for an equilibrium of local monopolists (firms' markets do not overlap and not all consumers are served), a competitive equilibrium (firms' markets overlap), and kink equilibria (firms' markets touch).[79] (cf. Economides (1989), p. 180ff) The solution to the price game is denoted with $\boldsymbol{p}^*(\boldsymbol{x})$ where vector element j stands for the equilibrium price of firm j and each p_j^* is a function of the firms locations $\boldsymbol{x} = (x_1, ... x_n)$. Subsequently, to solve the location game in the second stage for each firm j the objective function $\hat{\Pi}_j(\boldsymbol{x}, \boldsymbol{p}^*(\boldsymbol{x}))$ has to be maximized. It is shown that for the case of a competitive price equilibrium (sufficiently high k) and constant marginal costs c the symmetric equidistant location

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[77] The model structure is analogous to previous work, e.g. Economides (1984), see chapter 2 of this survey.

[78] Monopoly demand accounts for the consumer who is - contingent upon his reservation price k - indifferent between buying or not, i.e. $p_j = V_\omega(x_j)$, with the position of this consumer at $x_j^* = \sqrt{k - p_j}$. Demand under price competition is the sum of the expressions for the location of the consumer who has equal utility between buying at nearest competitors j and $j - 1$ and j and $j + 1$ respectively, that is the sum of the market boundaries of firm j to its right and left side.

[79] Exemplarily, the case of price competition between j and $j + 1$ is considered. The price bounds between nearest neighbors lie on the monopoly curve. The graphical representation is that the monopoly demand curve engulfs the linear demand curves for all potential competition between neighboring firms. Thus, the total demand curve generally comprises sections of the concave monopoly demand curve and sections of decreasing linear demand curves. In sum a concave structure obtains. (cf. Economides (1989), Fig. 1, p. 182) In addition, the uniqueness of the equilibrium is linked to the second derivatives of the profit function. (cf. Economides (1989), Lemma 1, p. 190)

pattern where firms are separated by a distance d leads to equal equilibrium prices $p_j^* = p = c + d^2$ and further maximizes $\hat{\Pi}_j$. (cf. Economides (1989), p. 186f) Based on the expression of the objective function $\hat{\Pi}_j$ and equilibrium prices and locations equilibrium profits for each firm $\Pi^* = \frac{\mu}{n^3} - F$ as a function of the number of firms, fixed costs and the marked size obtain. This allows the number of firms to be endogenized by demanding $\Pi^*(n^*) \geq 0$ and $\Pi^*(n^* + 1) < 0$. Thus, the equilibrium number of firms is given by the integer of the root $\sqrt[3]{\frac{\mu}{F}}$. (cf. Economides (1989), p. 187) Finally, concerning welfare issues Salop's finding is vindicated that the equilibrium number of firms n^* under free entry exceeds the number of firms that maximizes total surplus (cf. Economides (1989), p. 188f).

In conclusion, Economides (1989) proves that in a three stage price-location entry game on the circle the setting of symmetrical equidistant locations with identical prices above marginal cost is an equilibrium with the number of firms proportional to the ratio of the market size with the level of fixed production costs. In comparison to the linear city it is striking that the use of quadratic transportation costs allows for an equidistant pattern. However, it is noted by the author that the model is restricted to the instance that „*the existence of other locational (varietal) structures as prefect equilibria, although unlikely, cannot be ruled out.*" (Economides (1989), p. 185)

Finally, in a more recent study Madden & Pezzino (2011) provide an example for a model of product differentiation on the circle with an additional firm located at the center of the circle and offering a homogeneous product. The focus of their work are the resulting welfare implications where a comparison with the results of the Salop model highlights the impact of the central firm.

The model proceeds in three stages. In stage 1, a firm from a set of potential candidates chooses to take the central position, in stage 2, N firms enter the market and take equidistant positions on the circumference, and in stage 3, all firms compete simultaneously in prices. (cf. Madden & Pezzino (2011), p. 7) Furthermore, the model assumes uniformly distributed consumers on the perimeter with unit length, demand is sufficiently inelastic so that every consumers purchases one unit of the good, the location pattern is exogenously imposed and symmetric, transportation costs on the perimeter are linear in distance and the coefficient is set to be $t = 1$. Moreover, transportation costs to purchase at the central firm are $\delta > 0$, entry costs for the perimeter firms are F and for the central firm G. Prices of the perimeter firms are denoted with $P_i, i = 1, ..., N$ and of the central firm with P_c. (cf. Madden & Pezzino (2011), p. 4)

To derive the market equilibrium (following backward induction) demand and profit functions are set up based on the position of the indifferent consumer on the perimeter. In addition, to account for the central firm the indifference condition

$P_i + z = P_c + \delta$ is considered for a consumer at the closest location z to the perimeter firm i. As a result, the Nash equilibrium prices are contingent upon the number of perimeter firms N. For small N perimeter firms and the central firm have positive Nash prices $(P_i^*, P_c^* > 0)$, for intermediate N the central firm charges the competitive price $(P_c^* = 0, \Pi_c = 0)$ and does not gain market share, however its presence creates competitive pressure on the prices of the perimeter firms, and finally, for large N the central firm is outperformed $(\Pi_c = 0)$ and the Salop price equilibrium obtains. (cf. Madden & Pezzino (2011), Lemma 2, p. 9) The profit functions for the equilibrium Nash prices are continuous in N and allow to solve for the zero profit condition $\Pi(c, N) = F$ ($c = 0$ if no central firm exists and $c = 1$ if a central firm has entered). In line with the solution of the equilibrium Nash prices the equilibrium number of perimeter firms for $c = 1$ is distinguished for three regimes according to the level of fixed costs, obviously for $c = 0$ the solution of the Salop model obtains. (cf. Madden & Pezzino (2011), p. 10) Using the equilibrium number of perimeter firms from the second stage, the decision to take the position of the central firm in the first stage is solved. Intuition is confirmed that for sufficiently high fixed costs G or for low levels of fixed costs for perimeter firms F entry of the central firm is deterred, and for sufficiently low G (and F intermediate or high) a central firm enters the market. The corresponding threshold for G is an increasing function in F and further determined by δ. (cf. Madden & Pezzino (2011), Theorem 2, p. 10 and Fig. 2, p. 11)

Finally, according to the welfare analyses social optima[80] consist of two exclusive states, that is the market should either be served only by perimeter firms ($N \neq 0, c = 0$) or only by a central firm ($N = 0, c = 1$). (cf. Madden & Pezzino (2011), p. 6) A comparison of the social optima with the market equilibrium yields the standard result of excessive product differentiation provided that G is sufficiently high and F is low. In addition for high F, the cases where the social optimum suggests $c = 1$ but the market equilibrium either reveals $N > 0$ and $c = 0$ (for high G), as well as $N > 0$ and $c = 1$ (for low G) are covered. The striking result emerges for a combination of low entry costs G and F. As F is low the social optimum suggests that only perimeter firms should serve the market and the standard result is reproduced (the equilibrium also covers the central firm). However, for intermediate F it turns out that the number of perimeter firms is too low compared to the social optimum which is in contrast to the standard result. (cf. Madden & Pezzino (2011), p. 13)

In sum, the study of Madden & Pezzino (2011) illustrates the example for an introduction of a measure of centrality into markets with a circular shape. Subsequently, the paper demonstrates the impact of such an extension on market equilibrium prices and the equilibrium number of firms. In addition, the effects on welfare and the emergence of potential market failures are shown. Insights on the determinants of location

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[80] In accordance with Salop (1979) social optima represent market configurations that minimize total costs due to entry and transportation. (cf. Madden & Pezzino (2011), p. 5)

patterns, however, are not provided since, as in Salop (1979), a symmetric location configuration is assumed.

To sum up this subsection, one important common property of spatial competition in bounded linear and circular market geometries is the notion of localized competition. For instance, according to Mulligan & Fik (1989) equilibrium prices are predominantly determined by the locations and marginal production costs of the nearest neighbors. Moreover, when assessing the influence of further neighboring firms, a dominant distance decay effect on equilibrium prices is inherent for spatial competition under linear transportation costs. However, differences between the bounded linear and the circular city lie in the determinants of equilibrium prices. Whereas on the circle equilibrium prices obtain as a function of the relative distance between firms, in the linear city the spatial boundary conditions lead to a dependence of the price on the length of the market as well as on firms' locations. (cf. Mulligan & Fik (1989))

As regards the location equilibrium, issues on the existence of a perfect subgame equilibrium for the linear city have already been addressed in section 2 of this survey. For circular market shapes the unique existence appears to be challenging to prove, however, as was demonstrated in Economides (1989) a symmetrical location configuration serves as the prominent candidate for a subgame perfect equilibrium. As is demonstrated in the seminal paper of Salop (1979), a consequence of the notion of localized competition is that firms retain monopoly power and set prices above marginal costs. As the number of firms increases the markup and profits decrease, in the limiting case of a zero profit condition firms still act as local monopolists. However, this prediction does not generally hold regardless of the market shape as will be presented - among other things - in the next subsection.

2.3.2 Models with Market Centers

2.3.2.1 Non-uniform consumer distributions

First findings on the impact of the consumer distribution on firms' location decision in the Hotelling model are provided in Eaton & Lipsey (1975). The optimal location of a firm is critically determined by the structure of the distribution which is exemplified by the relation of modes to the number of firms. As a result, firms maximize their market area according to the shape of the distribution, in a special case they form pairs.[81] Eaton & Lipsey (1975) did not, however, analyze how the consumer distribution affects price and location competition. This is, for instance, to be done

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[81] For details refer to Eaton & Lipsey (1975) pp. 33-39 and chapter 2 of this survey.

in the paper of Neven (1986) where in a simultaneous two-stage price-location game with two players under quadratic transportation costs on the unit line ($x \in [0,1]$) the market equilibrium contingent upon the density distribution $c(x)$ (and the cumulative density $C(x)$) is examined.[82] Specifically, $c(x)$ is assumed to be continuous, differentiable and symmetric with one mode at the center $x = \frac{1}{2}$. (cf. Neven (1986), p. 122) As usual, the indifferent consumer sits at $\alpha = \frac{p_2 - p_1}{2(x_2 - x_1)} + \frac{x_1 + x_2}{2}$, then demand and profits respectively for firm 1 follow with $\Pi_1 = p_1 C(\alpha)$, and for firm 2 $\Pi_2 = p_2 C(1 - \alpha)$.

To derive the solution for the game three propositions are made. Firstly, it is shown that under the given profit functions a Nash price equilibrium requires $c(x)$ to be concave. The intuition is that the concavity of $c(x)$ is linked to the existence of a single peak. A single peak of $c(x)$ in turn requires profits to be concave and single-peaked (e.g. for firm 1 $\frac{\partial^2 \Pi_1}{\partial p_1^2} > 0$ and $\frac{\partial^3 \Pi_1}{\partial p_1^3} < 0$ over the range $\alpha \in [0, \frac{1}{2}]$) which reduces to $c''(x) < 0$. (cf. Neven (1986), p. 123) Clearly, the set of possible concave distributions is bounded by the two extremes of a rectangular and a triangle distribution.[83] Secondly, the unique price equilibrium for symmetrical locations $x_1 = 1 - x_2$ is derived with $p_1^* = p_2^* = \frac{1 - 2x_1}{c(\frac{1}{2})}$.[84] Moreover, the uniqueness for the symmetrical case is proved by showing that the convex price reaction functions intersect in one defined point. (cf. Neven (1986), p. 123f) Thirdly, profits $\Pi_1(p_1^*)$ and $\Pi_2(p_2^*)$ are maximized with respect to $x_1 = 1 - x_2$ which reduces to the location equilibrium $x_1^* = \frac{1}{2} - \frac{3}{4} \frac{1}{c(\frac{1}{2})}$. (cf. Neven (1986), p. 124f) This proves that for the concave family of consumer distributions and provided that firms locate symmetrically the optimal locations are at the city edges if $1 \leq c(\frac{1}{2}) \leq \frac{3}{2}$, and optimal locations continuously move towards the center for increasing values of $c(\frac{1}{2})$ in the interval $\frac{3}{2} \leq c(\frac{1}{2}) \leq 2$. Thus, the closest position is at $x_1^* = \frac{1}{8}$ and $x_1^* = \frac{7}{8}$ under a triangular distribution.

In conclusion, the study of Neven (1986) shows that in a duopoly firms tend to locate where consumer are concentrated. This is exemplified by the comparison of the contracting forces due to quadratic transportation costs and the attracting forces due to an agglomeration of consumers at the city center. In particular, he provides a unique solution for a two-stage price location game under the restriction of symmetrical locations and concave distributions. The balance of gaining higher demand and being exposed to fiercer price competition is described by equilibrium locations as a function of the density peak with the extreme solutions of maximally differentiating (and confirming results of d'Aspremont et al. (1979)) or coming as close as 3/8 to the market center ($x_1^* = \frac{1}{8}$, $x_1^* = \frac{7}{8}$).

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[82]Demand is assumed to be perfectly inelastic.

[83]It follows that the maximum value of $c(x)$ is bounded by $[1, 2]$. The mass must equal one, thus, for a rectangular distribution $c(x) = 1$ for all x on the domain, and for a triangular distribution the peak is at $c(\frac{1}{2}) = 2$.

[84]$\frac{\partial \Pi_1}{\partial p_1} = C(\alpha) - p_1 \frac{c(\alpha)}{2(x_2 - x_1)} = 0$, thus $p_1 = 2(x_2 - x_1) \frac{C(\alpha)}{c(\alpha)}$, similarly, $\frac{\partial \Pi_2}{\partial p_2} = C(1 - \alpha) - p_2 \frac{c(1-\alpha)}{2(x_2 - x_1)} = 0$ and $p_2 = 2(x_2 - x_1) \frac{C(1-\alpha)}{c(1-\alpha)}$. In equilibrium, $p_1^* = p_2^*$ which equals $\alpha = \frac{1}{2}$, and then $C(\frac{1}{2}) = \frac{1}{2}$.

The study of Tabuchi & Thisse (1995) scrutinizes equilibrium states in a Hotelling duopoly under a triangular consumer density distribution (in comparison to a uniform density) in both a simultaneous, and a sequential location game. Their paper stresses the importance of asymmetric location equilibrium patterns.

As in Neven (1986) quadratic transportation costs are assumed and prices are chosen simultaneously after locations are taken. Moreover, an important assumption in their model is that firms are not restricted to locate on the interval $[0, 1]$ and can choose any position on the real line. For locations x_1, x_2 (taken from the city origin) and prices p_1, p_2, profits are $\Pi_1 = p_1 F(\hat{x})$ and $\Pi_2 = p_2(1 - F(\hat{x}))$ with, as expected, the indifferent consumer at $\hat{x} = \frac{p_2 - p_1}{2(x_2 - x_1)} + \frac{x_1 + x_2}{2}$ and $f(x)$ denoting the density distribution and $F(x)$ the cumulative density. Specifically, for $x \in [0, 1]$ the triangular density can be expressed by $f(x) = 2 - 2|2x - 1|$ and the uniform density is, of course, $f(x) = 1$.[85] (cf. Tabuchi & Thisse (1995), p. 215) The price game and first order conditions are analogue to Neven (1986), thus $\Pi_1^* = 2(x_2 - x_1)\frac{F^2(\hat{x})}{f(\hat{x})}$ and $\Pi_2^* = 2(x_2 - x_1)\frac{(1 - F(\hat{x}))^2}{f(\hat{x})}$. Note that $\hat{x}(p_1^*, p_2^*)$ has to be distinguished for the triangular density for the regions $\hat{x} < \frac{1}{2}$ (or $x_1 + x_2 < 1$) and $\hat{x} > \frac{1}{2}$ (or $x_1 + x_2 > 1$). (cf. Tabuchi & Thisse (1995), p. 217)

To solve the location game the first order conditions at p_1^* and p_2^* with respect to x_1 and x_2 are used.[86] For the simultaneous location game the solution for the uniform distribution is obtained by solving the two equations $\Pi_1^*(p_1^*, p_2^*)$ and $\Pi_2^*(p_1^*, p_2^*)$ in the two variables x_1 and x_2.[87] This yields a symmetric location equilibrium in the outside quartiles of the $[0, 1]$-city and gives the 'real' solution of the maximum differentiation result of d'Aspremont et al. (1979). (cf. Tabuchi & Thisse (1995), p. 218) The interesting result for the triangular distribution is that in a simultaneous location game symmetric equilibria do not exist.[88] The economic rationale is that due to the demand effect players can increase their profits by moving towards the center if they are located far apart, and owing to the price effect profits can be increased by moving outwards if they are located closely. An asymmetric location equilibrium exists and assigns one firm a location on $[0, 1]$ and the other a location outside $[0, 1]$.[89]

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[85]It is evident that the triangular distribution comprises two branches, for $x \in [0, \frac{1}{2}]$ $f(x) = 4x$, and for $x \in [\frac{1}{2}, 1]$ $f(x) = 4(1 - x)$.

[86]Exemplarily: $\frac{\partial \Pi_1^*}{\partial x_1} = \frac{\partial}{\partial x_1}(2(x_2 - x_1)\frac{F^2}{f}) = -2\frac{F^2}{f} + 2(x_2 - x_1)\frac{\partial}{\partial x_1}(\frac{F^2}{f}) = -2\frac{F^2}{f} + 2(x_2 - x_1)[2F\frac{\partial F}{\partial x_1}\frac{\partial \hat{x}}{\partial x_1}\frac{1}{f} - \frac{F^2}{f^2}\frac{\partial f}{\partial x_1}\frac{\partial \hat{x}}{\partial x_1}] = -2\frac{F^2}{f} + 2(x_2 - x_1)[2F\frac{\partial \hat{x}}{\partial x_1} - \frac{F^2}{f^2}\frac{\partial f}{\partial x_1}\frac{\partial \hat{x}}{\partial x_1}]$. (cf. Tabuchi & Thisse (1995), equation (8a), p. 217)

[87]Recall that $f = 1$ and $F = \hat{x}$. (cf. Tabuchi & Thisse (1995), equations (10a) and (10b), p. 218)

[88]In the proof small deviations from symmetric locations are considered and profits for these locations are compared to the profits under symmetric locations. In any case deviations from the symmetric pattern yield higher profits. This also becomes clear from the structure of the density distribution since at the center $x = \frac{1}{2}$ it shows a peak, thus $f'(x = \frac{1}{2})$ is discontinuous and therefore the location reaction functions are discontinuous. (cf. Tabuchi & Thisse (1995), p. 219f and Fig. 1)

[89]The exact Nash equilibrium locations are $x_1^* = 1 - \frac{5\sqrt{6}}{18} \approx 0.319$ and $x_2^* = 1 + \frac{\sqrt{6}}{9} \approx 1.272$, and by symmetry $x_1^* = -\frac{\sqrt{6}}{9} \approx -0.272$ and $x_2^* = \frac{5\sqrt{6}}{18} \approx 0.680$ respectively.

(cf. Tabuchi & Thisse (1995), Proposition 3, p. 220) In comparison with the uniform distribution equilibrium profits under the triangular distribution for both firms are lower. This indicates the impact of the price effect. Even though firms move comparatively closer towards the region where consumer density is concentrated, price competition drives profits down. (cf. Tabuchi & Thisse (1995), p. 221) The solution for the sequential location game also yields an asymmetric location equilibrium. As intuition suggests, the first entrant enjoys a first-mover advantage and irrespective of the distribution (uniform or triangular) chooses the central place for his location $x_1^* = \frac{1}{2}$ whereas the second entrant locates outside of $[0, 1]$.[90] Again a comparison of profits between the uniform and the triangular distribution suggests higher price competition in the latter case. (cf. Tabuchi & Thisse (1995), p. 222)

To conclude, the paper of Tabuchi & Thisse (1995) shows that irrespective of the timing of the location game a higher concentration of consumer density around the center leads firms to locate close by. Moreover in conjunction with the results of Neven (1986), it is illustrated that the specific form of the distribution critically impacts the market equilibrium. In particular, a triangular distribution implies an asymmetric location configuration and rules out the existence of symmetrical configurations. As will be shown by the next paper, this argument also applies to convex and log-concave density functions whereas as was shown in Neven (1986) concave and symmetric densities allow for symmetric location equilibria.

In the paper of Anderson et al. (1997) a rigorous examination of the impact of the consumer distribution on location equilibria in the standard simultaneous two-stage price-location game for a duopoly is conducted. They provide general expressions for the perfect market equilibrium dependent on the consumer distribution and conditions for the existence and uniqueness of the perfect subgame location equilibrium states. Thus, an important contribution of the study is to determine the properties for the density function $f(x)$ such that (symmetric or asymmetric) location equilibria emerge.[91]

The model of Anderson et al. (1997) is the standard model with quadratic transportation costs utilized in previous studies (e.g. d'Aspremont et al. (1979), Neven (1986), Tabuchi & Thisse (1995)). The critical assumption, however, is to impose the condition of log-concavity on the density $f(x)$ in order to easily set up the equilibrium of the price subgame. (cf. Anderson et al. (1997), Assumption 1, p. 106)

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[90] The equilibria for the uniform case are $x_1^* = \frac{1}{2}$ and $x_2^* = \frac{3}{2}$ (and by symmetry $x_2^* = -\frac{1}{2}$), and for the triangular case $x_1^* = \frac{1}{2}$ and $x_2^* = 1.443$ (and $x_2^* = -1.443$). (cf. Tabuchi & Thisse (1995), p. 222)

[91] Tabuchi & Thisse (1995) already anticipate that restrictions on the density functions to provide the existence and nonexistence of symmetrical location equilibria are lacking: „*a symmetric equilibrium might not exist even with a smooth consumer density if this density increases very sharply near the center. Clearly, more work is called for here to determine the conditions under which symmetric equilibrium never arises.*" (Tabuchi & Thisse (1995), p. 220)

Exploiting the first order conditions (f.o.c.) of the profit functions $\Pi_1 = p_1 F(\xi)$ and $\Pi_2 = p_2(1 - F(\xi))$ reveals that a unique solution for the location of the indifferent consumer given the profit-maximizing prices (that solve the f.o.c.) denoted with $\bar{\xi} := \xi(\bar{p}_1, \bar{p}_2)$ exists. This is a consequence of the log-concavity assumption. (cf. Anderson et al. (1997), equation (2.3), p. 107) It follows that the f.o.c. in the first stage location game reduce to an expression in terms of $f(\bar{\xi})$ and $F(\bar{\xi})$ and to derive the optimal location pair (x_1^*, x_2^*) a solution ξ^* for this expression has to be obtained. (cf. Anderson et al. (1997), equation (2.8), p. 108) Thus, the subgame perfect equilibrium $(x_1^*, x_2^*, p_1^*, p_2^*)$ is a result of the properties of f and F (that determine ξ^*) and is generally determined by the shape and the modes of f. (cf. Anderson et al. (1997), Proposition 1, p. 109) In the special case of a symmetric distribution with the median at $x = 0$ the location equilibrium is symmetric and prices are identical (cf. Anderson et al. (1997), corollary 1, p. 116), and the comparative static results are in line with the predictions of Neven (1986). A higher density at the center $f(M = 0)$ leads to higher price competition and lower prices, and to agglomeration and lower values for x_1^* and x_2^*. Furthermore, as the variance of f increases in the symmetric example equilibrium prices and locations rise, that is tighter distributions lead firms to cluster at the center. (cf. Anderson et al. (1997), p. 110f)

To prove the existence and uniqueness of the subgame perfect equilibrium (in addition to the log-concavity of f) two conditions are required. (cf. Anderson et al. (1997), Proposition 2 and the proof, p. 115) Firstly, for given f there must not be an incentive for one firm to change its location and jump over to its rival's side.[92] Secondly, the degree of concavity of f is bounded.[93] This implies that if f is, for instance, symmetric and log-concave this is not sufficient for an (symmetric) equilibrium to exist, rather f must not be 'too' concave.[94] In addition, given that f fulfills the restriction on the concavity for densities with a high degree of asymmetry a subgame perfect equilibrium does not exist since then the straddle condition is violated.[95] Thus, in sum the degree of concavity and asymmetry of the density f determines the existence of the location equilibrium and the perfect subgame equilibrium.

A further interesting point is raised by generalizing the results of Tabuchi & Thisse (1995) who demonstrated the nonexistence of symmetric location equilibria for a

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[92] According to Anderson et al. (1997) this is stated as the 'straddle condition'. (cf. ibid., p. 113f)

[93] More precisely, an auxiliary function $H(x)$ determined by f and F shall be strictly pseudoconcave. (cf. Anderson et al. (1997), Assumption 2 and equation (4.1), p. 114) Moreover, the example is provided that for a symmetric distribution f with median M the requirement of pseudoconcavity of $H(x)$ reduces to the condition that the measure for the normalized concavity of f is restricted by $-\frac{f''(M)}{(f(M))^3} < 8$. (cf. ibid., equation (4.2), p. 114)

[94] If $H(x)$ is strictly pseudoconcave then ξ^* is unique. Moreover, if $H(x)$ is not strictly pseudoconcave multiple equilibria obtain (see the examples of the logistic density on p. 116 and the Laplace density on p. 122).

[95] In general, the impact of the asymmetry of f has to be analyzed numerically on a case by case basis. An example is provided for the Weibull density. (cf. ibid., p. 117)

triangular density. Anderson et al. (1997) reduce the stability of the location equilibrium to the concavity properties of f.[96] (cf. Anderson et al. (1997), Proposition 3, p. 118) This implies that for symmetric, log-concave densities with moderate concavity $(-\frac{f''(x)}{(f(x))^3} < 8)$ a symmetric location equilibrium is unique and stable, for intermediate concavity $(8 < -\frac{f''(x)}{(f(x))^3} < 24)$ the symmetric location equilibrium is not stable and unique, and multiple equilibria obtain where potentially asymmetric location equilibria coexist; eventually, if concavity is high $(-\frac{f''(x)}{(f(x))^3} > 24)$ symmetric equilibria cease to exist, and asymmetric equilibria possibly obtain (if they fulfill the straddle condition).[97] The economic rationale behind this influence of the shape of f is that asymmetric locations reduce competition and allow to charge comparatively high prices in cases where consumer density is highly concentrated at the market center (i.e., for highly concave f), whereas symmetric locations intensify competition for the indifferent consumer at the point of concentration if f is symmetric.

In conclusion, Anderson et al. (1997) demonstrate the influence of the properties of the density distribution f on the market equilibrium and provide the critical conditions for its existence. Economically, the tendency of firms to agglomerate is explained by the concentration of demand which is attributable to the concavity properties of f. They demonstrate that under mild market conditions symmetric location equilibria exist, subsequently competition can be attenuated by choosing asymmetric locations; finally, under harsh market conditions equilibria do not exist.

Two recent studies illustrate the relationship of a modified uniform consumer distribution, and of a location cost distribution with firms' profit maximizing strategies in the Hotelling model. They serve as particular interesting examples since they do not directly test variations in the specifics of the density distribution on the market equilibrium but offer alternatives to model these effects. In particular, these studies demonstrate that nonlinear variations in market characteristics that subsequently influence the structure of market demand (and make certain market areas more attractive than others) have impacts on firms' profits and their profit-maximizing behavior.

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[96] In particular, they formulate an expression of the steady state point (i.e. the location equilibrium x_i^*) to which the location of a firm according to the best response dynamics of its reaction function converges. Take for instance player 1, in general his reaction function R_1 depends on x_2^* which is described by $\Omega_1 := R_1(R_2)$. For an arbitrary starting point of the iteration process $s_0 > x_1^*$ the best response is to decrease the location and move towards x_1^* provided that $s_0 > \Omega_1(s_0)$. (cf. Anderson et al. (1997), Fig. 1, p. 119) Thus, the condition for a steady state is $\Omega(x_1^*) = x_1^*$. Then the properties of $\Omega_1(x)$ at the equilibrium x_i^* are determined by the concavity of the density f. (cf. ibid., equation (4.5), p. 118). In the extreme of a very concave density the reaction function is discontinuous. (cf. ibid., Fig. 2, p. 120)

[97] They provide the example for $f_\alpha(x) = N(\alpha)(\alpha - \sqrt{1 + (\alpha^2 - 1)^2 x^2})$ with $x \in [-1, 1]$, $\alpha > 1$ and $N(\alpha)$ as a numerical constant such that $\int f_\alpha(x)dx = 1$. The triangular density converges to $f_\alpha(x)$ for $\alpha \to \infty$. (cf. Anderson et al. (1997), p. 120) Unique symmetric location equilibria exist for $\alpha <\approx 5.3$, symmetric and asymmetric equilibria coexist for $\approx 5.3 < \alpha <\approx 20$, and asymmetric equilibria solely obtain for $\alpha >\approx 20$. (cf. ibid., Fig. 3, p. 121)

Thomadsen et al. (2013) address the question of the effects of a decrease in the number of consumers and the market size on the equilibrium prices and profits if firms compete with a horizontally differentiated product. They make the, at first sight, surprising proposition that a decline in market size caused by a drain of consumers who are not particularly committed to the consumption of the differentiated product leads to an increase in equilibrium prices and profits. Thus, under specific circumstances but leaving the general market characteristics unchanged (number of competitors, cost structure, transportation cost scheme) a declining market leaves firms better off.

The framework of their analysis is the Hotelling duopoly with linear transportation costs[98], a uniform consumer distribution and constant marginal costs normalized to zero. Specifically, consumer j's utility for a defined location when purchasing at firm i is $u_{ij} = V - p_i - d_{ij}$ with V as the reservation price, p_i as the product price and d_{ij} the traveling distance. Two important assumptions underly firms' locational setting. Firstly, their locations are exogenous, secondly they locate symmetrically with relative distance $2D$ and a distance of $\frac{3}{5}V - \frac{2}{5}D$ from the respective ends of the city. In addition, it is assumed that the reservation price V is small enough such that not for every location configuration the whole market is served. (cf. Thomadsen et al. (2013), p. 1001)

A variation in the market size is examined in three different settings. In the first scenario the variation is such that consumers located around the city edges exit the market. That is the market is truncated symmetrically from both sides leaving the firms with a remnant hinterland of an amount denoted by K.[99] Comparing equilibrium prices and profits before and after the exit yields a well-defined range for K such that profits for both firms after the exit exceed profits before the exit, as well as a range for K such that an increase in profits results from a decrease in K, i.e. for a decrease of the peripheral market size.[100] (cf. Thomadsen et al. (2013), Theorem 1, p. 1002) The economic interpretation is evident, as firms need not attract consumers with comparatively low utility from the edges they are able to exploit their local

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[98] The robustness of the results is tested under quadratic transportation costs. (cf. Thomadsen et al. (2013), p. 1004)

[99] The distance from firms' location to the edges after the exit is K. (cf. Thomadsen et al. (2013), Fig. 1, p. 1002)

[100] Before the exit the hinterland is $d = \frac{3}{5}V - \frac{2}{5}D$, then equilibrium prices are derived from the utility function $p = V - d$ and demand and profits are $q = d + D$ and $\Pi_{bef} = pq$. Note that in equilibrium there is no competition in the region between the firms. After the exit assuming that all consumers in K are captured ($p < V - K$) the profit function is $\Pi_i = p_i(K + D + \frac{1}{2}(p_j - p_i))$, applying the first order condition yields $p = 2(K+D)$ and a transition obtains solving $2(K+D) < V - K$ for K. For K above the profit function is $\Pi_i = p_i(V - p_i + D + \frac{1}{2}(p_j - p_i))$. Then however, a higher price than the profit-maximizing price can be charged which is the kink solution $p = V - K$. Thus, in equilibrium profits after exit are $\Pi_{after} = (V - K)(K + D)$. Setting $\Pi_{after} > \Pi_{bef}$ and solving for K yields the lower bound on K for the level of profits, and $\frac{\partial \Pi_{after}}{\partial K} < 0$ the respective bound for changes in profits. Note that presumably they made a mistake for the undercutting case where setting undercutting profits < 0 reduces to $K > V - 2D$ and not $K > V - 4D$. (cf. Thomadsen et al. (2013), proof on p. 1006)

monopoly power by charging higher prices. In sum, the price effect dominates the demand effect and profits increase under consumer exit. Comparative statics confirm this intuition, equilibrium prices (at the kink) rise with decreasing K. Moreover, undercutting is ruled out, therefore firms' dominant strategy is to complement the rival's pricing behavior. The limit of this phenomenon is set by a lower bound on K, if this is breached a declining market size is not offset by a high price strategy anymore. The second scenario assumes an asymmetric truncation of the market size only on one side. The qualitative findings and the proceeding are analogous to the previous symmetric case. (cf. Thomadsen et al. (2013), Theorem 2, p. 1003) In the third case consumers exit from the market center at $\frac{1}{2}$ around a distance of G and by a fraction f. As before conditions for an increase in firms' profits compared to the pre-exit state are provided such that a price equilibrium is guaranteed. (cf. Thomadsen et al. (2013), Theorem 3, p. 1004)

In conclusion, Thomadsen et al. (2013) do not provide new insights on determinants of firms' location decision since locations are imposed exogenously. Nevertheless, their model represents an interesting case where it is demonstrated that the seemingly disadvantageous decline in market size does not lead to shrinking profits and a decline in prices as a desperate reaction to retain one's customer base. Rather, a counterintuitive change in the pricing strategy leads firms to a more profitable situation. Put differently, the model illustrates that given fixed absolute locations and thus no flexibility in one strategic variable, the optimization of the remaining strategic variable (product price) suffices to be profitable and absorb fundamental changes in the market environment that cause, for instance, relative locations to change.

Fairly recently, Hinloopen & Martin (2017) introduce a cost of location function for a Hotelling duopoly that imposes a cost for each firm to set up their mill that varies according to the chosen location on the unit line. This introduces an additional degree of freedom in firms' optimal decision in a classical simultaneous two-stage price-location game. Subsequently, conditions are provided and the properties of the cost of location function are examined such that a perfect subgame equilibrium can be established.

The model of Hinloopen & Martin (2017) is essentially the model of d'Aspremont et al. (1979) with the only notable exception that the profit function accounts for the term of the location cost function $c(y)$ where the argument y is defined as „*the distance from the firm's location to the nearest end of the line ($y = a$ for firm A, $y = b$ for firm B).*" (Hinloopen & Martin (2017), p. 120) Thus, $c(y)$ has to be defined for $y \in [0, \frac{1}{2}]$ and $c(y)$ is symmetric. Now, the inclusion of $c(y)$ and subsequent equilibrium conditions are analyzed for linear and quadratic transportation costs.

In the linear case d'Aspremont et al. (1979) gives the conditions for the subgame price equilibrium. (cf. d'Aspremont et al. (1979), equations (1) and (2), p. 1146, and Hinloopen & Martin (2017), equations (1) and (2), p. 120) Equilibrium prices p_A^*

and p_B^* remain unchanged, profit functions, however, are $\Pi_A = \frac{1}{2t}(p_A^*)^2 - c(a)$ and $\Pi_B = \frac{1}{2t}(p_B^*)^2 - c(b)$. As the price equilibrium is defined, conditions for the location equilibrium are set up which are (i) that undercutting is not profitable compared to equilibrium profits, (ii) the first order condition in a and b, (iii) the second order condition in a and b, and (iv) that equilibrium profits are positive. The fact that the profit function contains $c(a)$ and $c(b)$ respectively implies that the location cost function and its first and second derivatives are part in all equilibrium conditions. (cf. Hinloopen & Martin (2017), Proposition 2, p. 121) Evaluating the first order conditions for firm A and B yields the solution of symmetric locations ($a^* = b^*$), and since $c'(a^*), c'(b^*) > 0$ and $c''(a^*), c''(b^*) > 0$ the location cost function has to increase at an increasing rate towards the center at $\frac{1}{2}$. The increase of the location cost and therefore the particular solution for a^* and b^* is due to the specification of $c(y)$. The example for the case $c(y) = y^\beta$ is presented where for low values of $\beta \geq 1$ location costs increase rapidly and for $\beta = 1$ the equilibrium locations are at the city edges, and for high values of $\beta \leq 2.43$ locations costs increase moderately and for $\beta = 2.43$ firms locate at the quartiles. (cf. Hinloopen & Martin (2017), p. 123f and Fig. 3)

In the case of a quadratic transportation cost scheme again the results of d'Aspremont et al. (1979) for the price game are exploited by the authors. Since there are no restrictions on the location choice due to undercutting the conditions for the existence of a location equilibrium are (i) the first order conditions, (ii) the second order conditions, and (iii) that equilibrium profits are positive. (cf. Hinloopen & Martin (2017), Proposition 3, p. 125) As in the linear case the solution to the first order conditions is a symmetric location equilibrium. However, in contrast to the linear case, the first order conditions reveal $c'(a^*), c'(b^*) < 0$ and according to the second order condition $c''(a^*)$ and $c''(b^*)$ can be positive or negative. Thus, a location equilibrium under quadratic transportation costs requires the location cost function to decrease towards $\frac{1}{2}$ or to increase towards the city edges. Again the particular solution for a^* and b^* results from $c(y)$ with the general finding that for rapidly decreasing functions firms are inclined to locate towards the center and thus a principle of maximum differentiation would not generally hold.

To conclude, the paper of Hinloopen & Martin (2017) illustrates that possible perfect subgame location equilibria in a simultaneous two-stage price-location game are critically determined by the cost of location modeled by a cost distribution $c(y)$. The intuition is confirmed that, similar to respective variations of the consumer distribution (e.g. Neven (1986), Tabuchi & Thisse (1995)), the location cost function creates incentives to locate in particular regions of the market such that a subgame perfect equilibrium is guaranteed. A prominent example is provided with the case of quadratic transportation costs in which location costs are required to increase towards the city boundaries, and thus firms would not maximally differentiate but

rather tend to agglomerate at the center.

2.3.2.2 Intersecting roadways and networks

Intersecting roadways represent the case of two (or more) intersecting lines and spatial competition in this setting can be explained by applying the principles of the Hotelling model. In the standard formulation with uniformly distributed consumers one distinguishable feature of this market setting is that the intersection connects two markets and makes locations close by more attractive since a larger customer base can be potentially served. Consequently, intersecting roadways may be interpreted as a market setting in which a certain degree of centrality pervails that is exogenously imposed by the market geometry. As in the case of the linear city general interests arise concerning the conditions on the existence and the properties of price and location equilibria.

In Braid (1989) it is demonstrated that a perfect subgame equilibrium does not exist since firms always have an incentive to relocate towards the center.
The model assumes an arbitrary number of $n = 1, ..., N$ infinite roads radiating from a center at $x = 0$ where on each spoke an arbitrary number of firms i is located at positions $x_{n,i}$ charging prices $p_{n,i}$. Transportations costs increase linearly in travel distance by rate k. Furthermore, the assumption is made that the center is taken by firm 0 with $x_0 = 0$. Excluding undercutting strategies the solution to the second stage price game is straightforward. For an arbitrary firm i on spoke n market demand is derived by the location of its indifferent consumers on the centrally orientated and peripheral side ($z_{n,i-1}$ and $z_{n,i}$ respectively). The profit function, first order conditions and consecutive profits for profit-maximizing prices Π_i and Π_0 are derived. (cf. Braid (1989), p. 108f) The strategic advantage of a central position is exemplified by a comparison of the profit functions where a factor of $\frac{N}{2}$ indicates that on every spoke consumers are served by the central firm and in case of an equidistant spacing (and equal equilibrium prices) this clearly leads to higher profits. (cf. Braid (1989), equations (6) and (7), p. 109) To show the nonexistence of the location equilibrium it is considered that all firms but one hold their location and the concerned seller moves towards the center (w.l.o.g. firm 2 on spoke 1 which implies $\Delta x_{1,2} < 0$). Subsequently, the effects of the relocation on equilibrium prices of all firms are evaluated.[101] In particular, the effect of the price change for the moving

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[101]Specifically, the first order conditions of the price game are expressed in terms of variations of the prices and locations. This leads to a set of equations where on spoke $n = 1$ only the price changes of the neighboring firms $i = 1$ and $i = 3$ result as a function of $\Delta x_{1,2} < 0$ (see equations (12) to (14)). The solution for the zero-set equations of the price changes for $i > 3$ on spoke $n = 1$ and for $i > 0$ for all other spokes are given in equations (17) and (18). They are obtained by evaluating the recursive relationships and solving a quadratic equation for the coefficient of the Δp's (which is not explicitly carried out in the article). The proceeding is described on p. 110f

firm on its profits are provided for the case of two, three and four spokes. (cf. Braid (1989), p. 111) It is revealing that for the case of the unbounded linear city ($N = 2$) no advantage due to the inward move obtains. This is explained by equivalent price changes of the neighboring firms as a result of $\Delta x_{1,2}$ (the closer located neighbor decreases his price, the more distant neighbor increases his price by the same amount). In sum, the equilibrium price of the moving firm does not change and neither do the positions of the indifferent consumers, then clearly, $\Delta p_{1,2} = 0$ does not effect profits $\Delta \Pi_{1,2} = 0$. By contrast, for $N = 3$ and $N = 4$ the centrally located neighbor does not decrease his prices by the same amount as the peripheral neighbor increases his price ($|\Delta p_3| > |\Delta p_1|$). The asymmetry is due to the best reply of the central firm. At the center the price reaction does not directly respond to the stimulus of one particular spoke, rather firm 0's optimal behavior accounts for competition on all spokes which has a dampening effect on the price drop due to the inward move of only one firm.

In sum, the paper of Braid (1989) demonstrates that road intersections are a determinant for firms' optimal location decision. The prediction that firms agglomerate at the center is in line with previous studies on the subject of nonuniform consumer distributions (Anderson et al. (1997), Tabuchi & Thisse (1995), Neven (1986)). As a result of the underlying market geometry intersecting roadways can be interpreted as a limiting case of a highly concentrated consumer distribution that collapses at the intersecting point (provided that consumers are uniformly distributed on the lines). In this sense the nonexistence of a perfect subgame equilibrium fits in well with the findings of the previous literature.

The study of Braid (1993) extends the market setting with one intersection and bridges the gap between models with one-dimensional intersecting lines on the one hand and spatial competition models in two dimensions on the other hand. The goal is to determine the Nash equilibrium of a simultaneous pricing game and conditions for its existence given a network of roads spread over a two-dimensional plane. In addition, the paper also considers the notion of a varying consumer distribution by including demand concentrations at the grid points.

The market geometry consists of a square grid with side length R. Three types of consumers are served by firms that are assumed to be located at every grid point, thus firms' locations are an exogenous variable. The first type of consumers is evenly distributed over the plane with density D, the second type is evenly distributed on each road of the network with density G, and thirdly, consumers are distributed with density N at each node of the grid. Consumers' transportation cost increases by a linear rate k when traveling on the network (travel costs to approach the main roads are assumed to be negligibly small) and demand for every consumer of each type is

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and in the appendix p. 112.

assumed to be perfectly inelastic. (cf. Braid (1993), pp. 189 and 191) To determine the Nash price equilibrium the derivation of the demand function of a firm is required. For this purpose the location of the indifferent consumer is set up for each customer type. (cf. Braid (1993), p. 191f) Firstly, the N consumers concentrated at a grid point all purchase by assumption, consecutive demand is $q_{spot} = N$. Secondly, concerning the consumers distributed on the lines, for each firm in sum four indifference points, thus two linear market areas exist. Between each firm (grid point) the indifferent consumer locates at $\frac{R}{2}$ if equal prices are charged, under price competition the position is $d = \frac{R}{2} + \frac{p_i - p_j}{2k}$, thus in sum total demand on the roads for a firm is $q_{road} = 4dG$. Thirdly, the indifference condition for the road customers also applies to the customers on the plane (who take a costless trip to access the main road). Applying this argument in four directions leads to a market area of $4d^2$, thus, a total demand on the plane of $q_{plane} = 4d^2D$. In sum, this yields profits for each firm of $\Pi_0 = (p_0 - c)(q_{spot} + q_{road} + q_{plane})$ with c as the marginal cost of production. In a Nash equilibrium all firms charge equal prices which reduces the price differences in Π_0 to zero. According to the first order condition the Nash price (net of production cost) obtains as a function of the consumer densities N, G and D, the transportation cost coefficient k, and the distance between the grid points R: $p_E = c + kR \frac{\frac{1}{2}N + GR + \frac{1}{2}DR^2}{GR + DR^2}$. (cf. Braid (1993), p. 192) The condition for the existence of a price equilibrium rules out the case of undercutting and of a high price strategy such that only the customers at the gird point are exploited. (cf. Braid (1993), equations (5) and (6), p. 193)

Addressing the comparative statics the Nash price equilibrium reveals the following properties: (i) p_E increases in k and R which expresses the local monopoly power of each firm on its consumer base, (ii) p_E decreases in D (consumer density on the plane) which highlights that an increase in the global population of the market leads to higher competition between firms, and (iii) p_E increases in N which illustrates the significance of the consumer point concentrations at the road intersections, as these locations become significant incentives to attract and compete for consumers from the periphery vanish. In the limit an excessive increase in N causes the price equilibrium to break down and firms to devise a high price strategy or an undercutting strategy. It is interesting to follow the author's argument that the nonexistence of the equilibrium is a direct result of the impact of N and, importantly, existence can not be restored by changing to a quadratic transportation cost scheme. (cf. Braid (1993), p. 200)

In conclusion the paper of Braid (1993) emphasizes the role of the market geometry, and in particular consumer concentrations on the road network, in the determination of the price equilibrium. This marks a difference to the predictions of models for linear bounded markets (e.g. d'Aspremont et al. (1979)) and circular markets (e.g. Economides (1989)). Furthermore, the symmetric price equilibrium p_E is in line with findings of other articles in the field of intersecting roads and networks (cf. examples in Braid (1993), p. 194f). For the case of purely intersecting roadways ($N = D = 0$)

$p_E = c + kR$ reproduces the price equilibrium in Braid (1989) and hints at the more general results of Fik & Mulligan (1991).

The study of Braid (2013) departs from the concept of Braid (1989) and develops a model to examine the location patterns in a network of intersecting roads with finite distance of unit length where on each road one firm chooses its optimal location provided that the center is taken by an incumbent firm.[102] Put differently, this model refers to a set of linear cities where one firm is located at one end of the city and simultaneously competes (given its fixed location) with all other firms that optimize in the second stage over price and in the first stage over location.

The model follows the 'classical' approach in the fashion of d'Aspremont et al. (1979) with the assumptions of quadratic transportation costs, uniformly distributed consumers with density normalized to 1, and completely inelastic demand. All of these apply to each of the n unit road segments. Generally, there are $n + 1$ firms (and n roads) charging prices P_i but the main focus of the paper lies on the case $n = 4$. (cf. Braid (2013), p. 794)

The derivation of the Nash price equilibrium assumes that a particular firm (firm 1) locates at a distance a from the center where firm 0 has settled, while the remaining firms (indexed with subscript 2) locate at b. The positions of the indifferent consumers for firm 1 (z_1) and the remaining players (z_2) follow, and consequently the profit functions of all firms Π_0, Π_1 and Π_2. The first order conditions (f.o.c.) define the profit-maximizing prices, clearly, the f.o.c. for the central firm is a function of all n firms locations (a, b) and prices (P_1, P_2), while the f.o.c. of the peripheral firms capture only the interaction with firm 0. (cf. Braid (2013), p. 795) The set of these three equations can be solved for P_0, P_1, P_2, these are reinserted into the profit functions which in turn are the objective functions for the location game. Focusing on firm 1 and setting $\frac{\partial \Pi_1}{\partial a} = 0$ under the assumption of symmetric locations ($a = b$) the profit-maximizing locations obtain as a sole function of the number of spokes n. (cf. Braid (2013), equation (15), p. 796) Subsequently, setting $n = 4$ equilibrium locations $a = b = \frac{5}{9}$ are derived which demonstrates, for this particular case, the general finding that in a symmetric equilibrium with $n \geq 3$ firms locate closer to the center than the social optimum would suggest.[103] (cf. Braid (2013), Propositions 3 and 5, p. 797f) The crucial part of the analysis is that a perfect subgame equilibrium is not generally defined. For the case $n = 4$ the existence of the Nash price equilibrium requires the location of firm 1 to be distant from the center while the other three

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[102]Note that in contrast to Braid (1989) the roads have a finite distance and there is only competition with the central firm, and in contrast to Madden & Pezzino (2011) locations are endogenized and firms are allowed to move on a dimension connecting the circular periphery and the center.

[103]The social optimum is defined as the configuration that minimizes total transportation costs for all consumers in the market. The optimum is found in a symmetric setting with firms locating at a distance of $\frac{2}{3}$, then the maximum distance for a consumer to travel to a firm is $\frac{1}{3}$. (cf. Braid (2013), Proposition 1, p. 794)

players are located at their equilibrium location ($b = \frac{5}{9}$), i.e. formally $a > a^* \approx \frac{1}{4}$ and $\frac{2}{9} < a^* < \frac{1}{3}$ must hold. (cf. Braid (2013), Appendix, p. 806) Breaching this restriction implies that the central firm is undercut by firm 1. Moreover, the point is made that for $n > 4$ a Nash price equilibrium „*might fail to exist even if a* (sic!) *assumes its equilibrium value [...] In fact, this is almost certainly the case in the limit that n goes to infinity.*" (Braid (2013), p. 806f)

In addition, the comparative statics of the model suggest that as the number of firms (and spokes) increases firms move towards the center and equilibrium locations decrease. The argument is essentially the same as in Braid (1989) and draws upon the optimal price setting behavior of the central firm which is dependent upon the competition with all firms on all spokes and therefore price responses against one particular peripheral firm are alleviated (in comparison with the case of 'direct' competition). Also, equilibrium prices and firms' profits decrease as n increases. (cf. Braid (2013), Proposition 6, p. 798)

In sum, the model of Braid (2013) tackles a simultaneous two-stage price-location game in a general setting of intersecting roads where each of the roads represents a Hotelling city with unit length. A straight-forward solution to the game suggests symmetrical locations and prices for the peripheral firms, the comparative statics suggest sellers' tendency to agglomerate at the center for an increasing number of firms. Moreover, the spatial differentiation of oligopolistic markets in equilibrium does not represent a social optimal outcome. However, issues with the existence of the subgame perfect equilibrium remain.

A further example of an oligopolistic market model that makes use of intersecting roads is provided in the paper of Chen & Riordan (2007). Their study is related to the literature of nonlocalized monopolistic competition[104] and by imposing the network geometry in a market with n players they establish an equilibrium for spatial competition in a nonlocalized framework. (cf. Chen & Riordan (2007), p. 898) Since firms' market areas are connected over a central hub, their model implies that optimal pricing decisions are a result of direct competition between all sellers in contrast to localized market competition where equilibrium prices are derived from competition with nearest neighbors.

Their setting consists of a network of N spokes with length $\frac{1}{2}$ intersecting at a center. Firms' locations are assumed to be at the end of the spokes, in contrast to the previously presented studies no firm is located at the center. The total number of firms is n and only one firm shall be allowed to be located on a spoke. Generally, some spokes remain vacant, i.e. $n \leq N$. Locations are assumed to be given exogenously, thus, variations in firms' locations are not subject to the examination. Additional assumptions of the model are that consumers are distributed uniformly over the lines,

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[104]For an introduction see Tirole (2003) pp. 287f and 298ff.

the number of consumers on a spoke is normalized to one, consequently consumer density on a spoke is $\frac{2}{N}$.[105] Moreover, transportation costs are assumed to be a linear function in traveled distance (with $t = 1$). (cf. Chen & Riordan (2007), p. 901)

Two critical assumptions underly the spokes model. Firstly, consumers' reservation price v is finite. Secondly, optimizing their utility from consumption consumers only choose between two alternatives.[106] (cf. Chen & Riordan (2007), p. 903) Together with the market geometry, this implies that the demand function (exemplarily for a firm j) is a result of three different categories of consumers.[107] The first category refers to the standard case of a consumer who is indifferent between purchasing at firm i or firm j and whose location is a function of the price differential.[108] More-over, firm j competes in the same way with all firms in the market, by assumption a consumer prefers firm j with a probability of $\frac{1}{N-1}$ and firm j could maximally gain $N - 1$ of these indifferent consumers (cf. Chen & Riordan (2007), second equation on p. 902). The second category considers the indifferent consumer on the spoke of firm j who is according to his reservation price indifferent between buying at firm j or buying an outside good (the type of these consumers would purchase from a firm that has not yet located on a spoke). Again the outside good is preferred with a probability of $\frac{1}{N-1}$ (cf. ibid., third equation on p. 902). The third category accounts for consumers on vacant spokes that choose firm j as their preferred seller with prob-ability $\frac{1}{N-1}$ or buy an outside good. If the vacant spoke is occupied by a seller these consumers then fall into category 1 (cf. ibid., forth equation on p. 902). This illus-trates that the level of v is critical for firms' demand function, and subsequently for the level of equilibrium prices and profits.

Evaluating profit functions and first order conditions by cases yields the symmetric price equilibrium p^* as a monotonic function in v (over the range of interest). (cf. Chen & Riordan (2007), equation (3), p. 904 and for the derivation pp. 917-919) In the limiting case for high v (and high p^*) surplus from consumption is positive and more firms generate more intense competition which drives prices down.[109] As v de-creases the indifference condition for consumers on vacant spokes becomes binding, thus, firms charge their prices according to the corresponding marginal consumer. For decreasing v the willingness of the marginal consumer to buy declines and to

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[105] Density equals the number of consumers divided by the market length. There are $\frac{1}{N}$ consumers on one spoke of length $\frac{1}{2}$.

[106] These could be purchasing at two different sellers in the market, or buying at a seller and buying an outside good which is not buying at all. See explanations below.

[107] „*There are three relevant categories of consumers: consumers for whom brand j is preferred, and whose two preferred brands are both available; consumers for whom brand j is the first preferred brand, whose second preferred brand is not available; and consumers whose first brand is unavailable and for whom brand j is the second preferred brand.*" (Chen & Riordan (2007), p. 901)

[108] Locations are fixed, the distance is measured from the end of a spoke (towards the direction of the center) and utility from consuming at firm j is $u_j = v - p_j - x_j$, and at firm i: $u_i = v - p_i - ((\frac{1}{2} - x_j) + \frac{1}{2})$.

[109] There is no dependence on v here: $p^* = \frac{2N}{n-1} - 1$.

catch up firms decrease their price and stay on the kink.

As v further decreases the discrepancy between 'pure' competition with other firms for the indifferent consumer with two buying preferences for the differentiated good on the one hand, and the goal to capture the indifferent consumer on the vacant spokes on the other hand becomes eminent. The result is that the marginal consumers on the vacant spokes drive the price reaction and, importantly, the respective demand function reveals a higher price elasticity than demand from 'pure' competition. Now, if the number of firms increases, the number of vacant spokes decreases and competition intensifies. This leads to a relative increase of the 'pure' competition segment in the total demand function. Since demand from the vacant spokes is more price elastic, the price elasticity for the total demand decreases. As a consequence, equilibrium prices rise for an increasing number of competitors. (cf. Chen & Riordan (2007), Corollary 1, p. 905)

Concerning equilibrium profits the comparative statics further demonstrate the impact of n and v. As n increases it is clear that demand from consumers on the vacant spokes declines. This implies a reduction in profits. This effect is comparatively stronger for high levels of v since then a larger part of total demand is made up by the vacant spokes. In sum equilibrium profits decrease, despite rising prices (as argued above). Likewise, if v is relatively low, initially profits decrease (i.e. for increasing n at low levels), however, as n becomes sufficiently large the price effect takes hold since only small fractions on the vacant spokes are lost and rising prices lead profits to rise. (cf. Chen & Riordan (2007), p. 906f)

In conclusion, the model of Chen & Riordan (2007) illustrates the critical impact of market geometry on the determinants and the behavior of a price equilibrium for spatial competition in oligopolistic markets. Essentially, their paper stresses the importance and the differences in the outcomes of localized competition and non-localized competition which in their model results from the linkage over a central node in the spatial market. Contrary to the predictions of the models with circular shape (Salop (1979), Economides (1989), Madden & Pezzino (2011)) and of network models with a central firm (Braid (2013)), the example is stated that under specific parameter configurations an increase in the number of firms leads to an increase in equilibrium prices and ambiguous effects on equilibrium profits. As the authors emphasize, this is particularly noteworthy, since other determinants for the price setting behavior such as imperfectly informed consumers or mixed pricing strategies are explicitly excluded from the analysis. (cf. Chen & Riordan (2007), p. 900, footnote 8)

Finally, two studies shall be presented that incorporate network effects in a spatial price model and investigate the effect of market geometry on the price equilibrium under different exogenous pricing conjectures.[110]

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[110]In his introductory note Mulligan (1996) characterizes two strands in the literature of spatial

The paper of Fik & Mulligan (1991) departs from the classical assumptions of the Hotelling model: (i) uniformly distributed consumers on the line with unit density throughout the market, (ii) linear transportation cost scheme with coefficient t, (iii) perfectly inelastic demand, and additionally assumes (iv) a cost function with fixed costs F and marginal costs k. (cf. Fik & Mulligan (1991), p. 81) Evidently, the market area of an arbitrary interior firm i follows using the locations of the indifferent consumer to its left where competition with firm 1 shall take place (a_{i1}) and to its right with the nearest neighbor firm 2 (a_{i2}). In sum, market demand is $A_i = a_{i1} + a_{i2} = \frac{1}{2t}(p_{i1} + p_{i2} - 2p_i + t(D_{i1} + D_{i2}))$ where D_{ij} is the distance of firm i to any firm j. Essentially, network effects are captured by assuming that firm i can have n_i nearest neighbors which leads to $A_i = \sum_{j=1}^{n_i} a_{ij} = \frac{1}{2t}(\sum_{j=1}^{n_i} p_{ij} - n_i p_i + t \sum_{j=1}^{n_i} D_{ij})$. Then, firm i's profit-maximizing prices are determined by the first order condition with the underlying profit function $\Pi_i = (p_i - k_i)A_i - F_i$.[111] (cf. Fik & Mulligan (1991), p. 82) The term for the partial derivatives $\frac{\partial p_{ij}}{\partial p_i} = \phi_{ij}$ is the price conjectural parameter and accounts for the price change of neighbor j to a price variation of firm i, throughout it is assumed that price changes of i are equal towards all neighbors.[112] (cf. Fik & Mulligan (1991), p. 83) Given a particular conjectural scheme the first order condition connects firm i's optimal prices (p_i) with the prices of its nearest neighbors (p_{ij}), and with transportation costs (tD_{ij}) and marginal costs (k_i). The multiplicative and additive structure of the equation allows to separate the different factors and write the solution for the price equilibrium in algebraic form as $\boldsymbol{p^*} = \boldsymbol{C}^{-1}\boldsymbol{X}$ where $\boldsymbol{p^*}$ is the vector of equilibrium prices of all firms in the market, the nxn-matrix $\boldsymbol{C}$ incorporates the more or less complex topological structure[113], and the vector $\boldsymbol{X}$ includes

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competition models. (cf. p. 155f) One stream focuses on a game theoretical approach that separates location and pricing decisions and is extensively presented in this survey. The other is dedicated to a general form of the price reaction function and leaves open different forms about the type of price reaction (a description is provided in footnote 112 below). The studies of Fik & Mulligan (1991) and Mulligan (1996) are two examples that use the latter approach and apply the assumptions to network structures whereas for instance the work of Braid (2013) falls into the first of his categories. Also recall that in an earlier stage the study of Eaton & Lipsey (1975) considers two different conjectures (zero conjectural variation and the minmax-conjecture) under which firms optimize their location decisions. Moreover, in the article of Capozza & Order (1978) the common price conjectures are considered and the characteristics of price equilibria under spatial competition subject to a zero profit condition are scrutinized. Subsequently, they provide conditions under which an increase in the number of sellers leads to an increase in the equilibrium price.

[111] Precisely, the equation is $\frac{\partial \Pi_i}{\partial p_i} = \frac{1}{2t}[p_i(\sum_{j=1}^{n_i} \phi_{ij} - n_i) + \sum_{j=1}^{n_i} p_{ij} - n_i p_i + t \sum_{j=1}^{n_i} D_{ij} - k_i(\sum_{j=1}^{n_i} \phi_{ij} - n_i)]$. (cf. ibid., equations (10) and (11), p. 82)

[112] However, note that price conjectures among different firms, that is for different i, can vary. Generally, ϕ_{ij} can take three values. Under Loeschian competition ($\phi_{ij} = 1$) price movements are alike which implies the assumption that the market area of every firm is fixed. Under Hotelling-Smithies competition ($\phi_{ij} = 0$) nearest neighbors do not respond to price variations which implies that prices of competitors are fixed and market area and demand vary corresponding to price movements. Under Greenhut-Ohta competition ($\phi_{ij} = -1$) each firm assumes that the price at the market boundary is fixed. (e.g. Capozza & Order (1978), p. 898)

[113] Formally, the variable n_i measures the degree of connectivity. If firm i is located at a node and

distances and marginal costs. (cf. Fik & Mulligan (1991), p. 83)

Subsequently, differences in equilibrium price levels between the market setting of a linear city with equidistant locations and a modified linear city where the second firm competes with three nearest neighbors are examined. Firms' locations and the location pattern are exogenously imposed. (cf. Fik & Mulligan (1991), tables 1 and 2, and Fig. 1 and 2, pp. 83 and 86f) The corresponding results show that the existence of a node at the location of firm 2 in the network market leads to comparably lower equilibrium prices for all firms under different pricing conjectures. Moreover, it is demonstrated that the degree of connectivity in terms of n_i implies lower price levels, that is, firm 2 and firm 4 in the network market have the highest number of links and charge lower prices in equilibrium than the three peripheral firms (the effect is predominant under pricing conjectures that support competition).

In sum, this highlights the importance of market geometry for the determination of the price equilibrium and suggests that higher connected markets imply a higher degree of price competition for centrally located firms.

In Mulligan (1996) the model of Fik & Mulligan (1991) is used to generalize the analysis and incorporate firms' location to characterize the market equilibrium. The basic assumptions remain the same concerning linear transportation costs, perfectly inelastic demand, and the production cost function. The price reaction functions are derived from the first order conditions by setting up the demand and profit functions based on the position of the indifferent consumer. In particular, the case is made for interior firms (to investigate circular markets) and exterior firms sharing a market border with the end of the bounded market (to investigate linear bounded markets). (cf. Mulligan (1996), p. 158)

The general solution for a circular and a linear bounded market geometry with n firms follows from $\boldsymbol{Y^*} = \boldsymbol{C}^{-1}\boldsymbol{Z}$ where $\boldsymbol{Y^*}$ contains the locations and prices in equilibrium, the $2n$x$2n$-matrix the price and location coefficients accounting for the interactions in the market topology and $\boldsymbol{Z}$ covers the exogenous variables. (cf. Mulligan (1996), pp. 160 and 162) The information for this equation is deduced from the first order conditions in prices and from two assumption on firms' location behavior. Firstly, it is proposed that interior firms choose their location according to a principle of maximal differentiation.[114] Secondly, exterior firms' location behavior is character-

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competes with more than two neighbors ($n_i > 2$), in row i and column i of $\boldsymbol{C}$ the corresponding elements c_{ij} and c_{ji} represent the connection between firm i and neighbor j and are nonzero since the first order condition of firm i contains all p_j of the nearest competitors. Thus, in general the number of nonzero off-diagonal elements for a particular row (column) in $\boldsymbol{C}$ indicates the number of nearest neighbors. (cf. Fik & Mulligan (1991), example of firm 2 in equation (16) and Fig. 2, p. 83f)

[114] An arbitrary firm i locates at the position of the indifferent consumer between his nearest neighbors firm $i-1$ and $i+1$. (cf. Mulligan (1996), equation (7), p. 159) Mulligan argues that this location pattern is obtained for the case of elastic demand functions (see for instance Hay (1976)). Subsequently, he argues that in a limiting case of completely inelastic demand this assumption also holds.

ized by a principle of spatial aggression, that is, their location is assumed to lie in $0 \leq X_1 \leq X_{1,max}$ (for the left-sided firm 1) and $X_{n,min} \leq X_n \leq Z$ (for the right-sided firm n, and Z denoting the city length). The interval bounds $X_{1,max}$ and $X_{n,min}$ are determined by the condition to prohibit the undercutting of the respective nearest neighbor and the particular location choice is subsequently parameterized by λ (or μ).[115] From this the results for firms' equilibrium prices, locations and profits for the case of the circular city (cf. ibid., equations (13), (14) and (16), p. 161) and the linear bounded city (cf. ibid., equations (20), (21), (24), (25), (29) and (30), p. 162ff) follow. The comparative static analysis shows that profits and prices increase as the density of firms decreases while a variation in the spatial aggression parameter λ (or μ) leads to opposing effects on firms' equilibrium prices in the linear bounded market. As the two peripheral firms move towards the center their equilibrium prices increase whereas the prices of interior firms decrease, thus, the corresponding price ratio increases as λ (or μ) rises. Moreover, it is also interesting to see that the difference between the equilibrium prices in terms of the price ratio increases as seller density increases. (cf. Mulligan (1996), p. 167)

Finally, the model is extended to a network market consisting of four nodes and five links. (cf. Mulligan (1996), Fig. 1, p. 170) In the examination of the market equilibrium the proceeding assumes that a spatial leader anticipates his optimal location, subsequently, the competitors choose their position according to the principle of maximal differentiation and all firms charge their price contingent on the price conjecture. By backtesting the results the outcome is compared with the initial expectation to check on the existence of an equilibrium.[116] (cf. Mulligan (1996), p. 169) The calculations show that the market is characterized by the existence of multiple equilibria. (cf. ibid., table 4, p. 171) Generally, equilibrium prices and profits vary across firms and are a function of the market geometry (the length of the links). This highlights the interrelation of the geographical and economical properties of markets. Since profits across firms are a function of the geometrical properties, these in combination with the level of entry costs determine the number of firms and subsequently the location pattern in equilibrium. (cf. Mulligan (1996), p. 172)

To summarize this subsection, the literature shows that the market characteristics and the geometry of the Hotelling model can be extended by modifying the consumer distributions and by introducing intersecting roadways and road networks.

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[115]Consequently, the location parameter λ (or μ) is also bounded by a value $\lambda < \lambda_{max}$. The bounds are derived by setting up the problem for the case of the exterior firms behaving maximally aggressive and solving for their equilibrium price. Subsequently, this price is set equal to the equilibrium price of an interior firm, λ_{max} decreases as the number of firms n increases. (cf. Mulligan (1996), appendix, p. 175)

[116]This approach reflects the methodology of sequential entry with perfect foresight by Prescott & Visscher (1977).

Both approaches have in common that specific regions in market space become more attractive with profound implications on the equilibrium outcome for prices and locations. For instance, if the assumption of a uniform consumer distribution is dropped and more consumers are concentrated at the middle point of the Hotelling line (i.e. the region of the market center is valorized), the market share effect gains importance and firms have increased incentives to agglomerate at the center. However, it is shown that under nonuniform distributions the existence of symmetrical location configurations is not guaranteed and asymmetrical location equilibria are likely to emerge (e.g. Neven (1986), Tabuchi & Thisse (1995)). In general, the properties of the consumer distribution (the degree of symmetry and log-concavity) determine the particular equilibrium outcome (e.g. Anderson et al. (1997)). Additionally, it is suggested that alternative forces such as a nonlinear decline in the consumer distribution (e.g. Thomadsen et al. (2013)), and the introduction of a cost of location distribution (e.g. Hinloopen & Martin (2017)) impact firms' behavior.

For the case of intersecting roadways and networks the existence of market centers at the intersections implies more complex interaction patterns. Even though nonuniform consumer distributions impact profit-maximization, competition remains localized. By contrast, in the case of intersecting roads the nature of competition is fundamentally changed. This is attributable to the node in the market geometry allowing for spatial ramifications in price and location competition. Since linear submarkets are connected through a single point competition becomes nonlocalized and pricing and location decisions become relevant for a greater number of nearest rivals. This instance is demonstrated in different studies where a firm located at the intersection is subject to competition with all firms located on adjacent spokes (e.g. Braid (1989), Braid (2013)), and also for the case where the intersection remains unoccupied but price competition is transmitted over the node from one firm to all other competitors in the market (e.g. Chen & Riordan (2007)). Furthermore, according to the empirical papers which are presented in the next subsection, it is revealing that a central location in a local market (close to the intersection) implies an asymmetrical location pattern and endows the central firm with more market power. This in turn leads to different outcomes of the price game compared to a symmetrical location setting in the local market (e.g. Firgo et al. (2016)). In addition, two selected articles from the spatial network literature illustrate that the particular geometrical shape of a spatial network constitutes an important determinant for the price equilibrium in oligopolistic markets (e.g. Fik & Mulligan (1991), Mulligan (1996)). To generally conclude, it is forcefully shown in the literature that the extension of the geometry from a line to a setting with two intersecting lines changes the equilibrium outcome and introduces different roles for the market players.

2.4 Empirical Evidence for Price and Location Decisions

In this section some examples of the empirical literature concerning determinants of firms' price and location decisions as well as the implications of firms' tendency to locate at a central point in a local market shall be presented. In particular, recent studies dealing with retail gasoline markets are considered since these markets exhibit the driving forces for price and location decisions in a fairly pure form. Concerning sellers' locations, as has been shown in the theoretical studies, a market share effect leads firms to agglomerate at market centers and a price competition effect implies a tendency to locate far apart. Retail gasoline markets serve as a fruitful example to study the causes of price and location patterns due to the homogeneity of the purchased good which allows to control for alternative product differentiation characteristics easily as well as the intensity of price competition as a result of the transparency of mill prices.

In Netz & Taylor (2002) data on station-level characteristics for 4,000 gasoline stations in the metropolitan area of Los Angeles between 1992 and 1996 are used. Based on firms' geographical position the degree of spatial differentiation between two competitors is measured by their Euclidean distance where the local market of each seller is assumed to be of a circular shape with a predefined radius. Competition is proxied by the number of rivals in the circular local market. Moreover, gasoline stations are distinguished by their type of brand to capture effects due to perceived sellers' differences. To account for the effect of the spatial consumer distribution main roads are considered as areas with a high concentration of consumers and specifically a station's spatial characteristic of being as close to a main road as 0.25 miles is considered in the estimation. Further, the percentage of each station's rivals that are located near a major road is used as a proxy for the number of intersections in a local market. (cf. Netz & Taylor (2002), p. 167)

The empirical model is represented by a regression equation with the average distance of a station to its competitors in the local market as the dependent variable, and the competition variables (number of sellers, number of sellers squared[117]), a vector of station characteristics (e.g. brand, convenience store, car wash etc.) and other variables to control for demand and market characteristics (e.g. median household income, median value of housing) as the explanatory variables.[118] (cf. Netz &

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[117]The squared termi is used to account for a decreasing effect on firms' distance as the number of stations increases which is particularly the case if the number of sellers in a local market is comparatively high. (cf. Netz & Taylor (2002), p. 168)

[118]Location choices can be restricted by zoning laws, in addition there are entry costs for new sellers coming into the market. The proxies that account for these two effects are (i) the proportion of stations (in a local market) requiring prepayment, (ii) the proportion of housing (in a local market) that is rented rather than owner-occupied, (iii) the median value of housing, and (iv) the median household income. (cf. Netz & Taylor (2002), p. 166) In addition, zoning effects are

Taylor (2002), equation (1), p. 164) To address potential spatial relations between the observations and errors a spatial lag model and a spatial error model are applied. Estimations are carried out for two different data samples (entry stations and stations that are permanently present) and for different sizes of the local market (half a mile, a mile or two miles). The results show that (ceteris paribus) firms locate farer from one another as competition intensifies, that is, as the number of sellers increases, as the proportion of independent stations increases, and as the fraction of stations with the same brand as the central station in the local market increases. Moreover, the results illustrate that firms are inclined to locate closer to main roads and intersections which serves as an indication to confirm theoretical predictions that firms tend to locate where consumers are concentrated.

In the paper of Pennerstorfer (2009) the effect of a gasoline station's brand on the price level is examined and it is highlighted that two conflicting effects characterize competition for gasoline retailers in a local market. Firstly as in Netz & Taylor (2002), according to the competition effect price levels decrease as competition intensifies and as the number of unbranded stations increases. Secondly, due to perceived quality differences between branded and unbranded stations an increase in the number of unbranded sellers implies an increase in the local monopoly power of branded stations and causes an incentive to increase their price (composition effect). (cf. Pennerstorfer (2009), p. 138)

The statistical analysis is based on cross section data of 400 stations in Lower Austria from 2003 comprising price data and data on station level characteristics as well as statistical data of the municipalities from 2001. In the regression model the price level is estimated as a function of competition variables, station characteristics (incl. brand) and variables capturing local demand and market characteristics (e.g. size of the municipality, population density, speed limits). Competition is measured by the number of sellers in the local market. Comparable to Netz & Taylor (2002) each local market is defined by a circular shape but with different radii of 15.5 kilometer and 20 kilometer respectively owing to the rural characteristics of the spatial area. To examine the composition effect the fraction of unbranded stations among the number of competitors in a local market is considered. Dependent on whether the central firm is branded or unbranded two corresponding variables are included in the regressions. To control for spatial autocorrelation a spatial lag model is utilized, furthermore the spatial distribution of unbranded sellers is explicitly accounted for and spatial effects of other explanatory variables are included by applying a spatial weight matrix.[119]

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captured by using fixed effects on the the level of municipalities. (cf. ibid., p. 168)

[119]To measure the distance decay effect an element w_{ij} of the spatial weight matrix equals the reciprocal value of the Euclidean distance of firm i and j. Rows are standardized to one. (cf. Pennerstorfer (2009), p. 143)

The results indicate positive and highly significant effects of the brands as well as the spatially lagged price on the price level. Thus, unbranded stations charge on average lower prices and due to the positive spatial correlation lower prices of unbranded stations cause the prices of branded stations to decline. By contrast, the fraction of unbranded stations in a local market with a branded central firm (after spatial weights) is positively correlated with price levels implying that an increase in the fraction of unbranded direct competitors in the local market of a branded station leads to an increase of its average price. Further evidence for the existence of this composition effect is provided by results of numerical simulations. (cf. Pennerstorfer (2009), p. 148ff) In sum the paper of Pennerstorfer (2009) suggests that an increase in competition by unbranded stations does not necessarily lead to a decrease in price levels of branded stations since the total price effect is composed of a 'pure' price competition effect (regardless of the brand) and of a composition effect that is driven by perceived quality differences. Thus, evidence is found that a station's brand represents an important characteristic for product differentiation and has to be considered as a major determinant of gasoline price levels.

In the study of Pennerstorfer & Weiss (2013) the relationship of spatial clusters of stations with gasoline price levels is examined. Subject to the research interest is the intuition that - as a consequence of localized competition - nearest neighbors along a road which belong to the same brand and form a spatial cluster are exposed to less competition compared to a situation where each neighbor serves its product under a different brand. In particular the paper scrutinizes the case of a merger of gasstations in the Austrian retail market that affected the composition of brands and spatial clusters.

The data on station characteristics (including geographical location) covers the cross section of all 2.814 gasoline stations in Austria. Price data is available from 2000 to 2005 in quarterly observations in an unbalanced panel and covers the date of the merger of 98 stations by a major brand in 2003. Moreover, data on local market and demand characteristics is used (e.g. population density, land price, number of tourists). Spatial cluster of stations with the same brand are modeled by an index based on the concept of Thiessen polygons.[120] The index characterizes the local market of each station and is constructed by the total number of nearest competitors, the number of neighboring stations that form an adjacent cluster, and the cluster size (i.e. the total number of firms that form an adjacent cluster). (cf. Pennerstorfer & Weiss (2013), p. 665) Correspondingly, an increase in the index value refers to an

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[120]Thiessen polygons are a two-dimensional representation for spatial competition between two stations along a road and define the local market of a station in relation to its nearest neighbors. Conceptually, the position of the indifferent consumer based on stations' location is marked by a perpendicular line and the set of intersecting perpendiculars forms the polygon. (cf. Pennerstorfer & Weiss (2013), Fig. 1, p. 663)

increase in the number of stations forming a (brand) cluster, and therefore, to an increase in the cluster size which implies a potential increase in market power for these stations as a result of a lower degree of price competition. Now, due to the take-over of 98 stations of a major brand the structure of spatial competition and of spatial clusters changed which is measured in movements of the index. The price effects are estimated with a 'difference-in-difference' model. Accordingly, price levels are estimated (in a panel) as a function of station- and time-dependent variables, time- and station-fixed effects, and as a function of the spatial clustering index as well as two supplementary dummy variables to capture the merger effects.[121] The results reveal a significant and positive correlation of price levels with the spatial clustering index variable and on average an increase in prices by 0.14 cent is observed for stations which were directly affected by the take-over. (cf. Pennerstorfer & Weiss (2013), p. 668)

In sum the findings of Pennerstorfer & Weiss (2013) shed light on the interdependence of stations' location patterns with their price levels. In particular, it is demonstrated that a take-over of stations in the Austrian market caused a change in the spatial structure which in turn implied a decrease in price competition and in the price levels in the corresponding local markets.

The study of Firgo et al. (2015) highlights the empirical relationship of the degree of centrality that characterizes stations' locations and the level of gasoline prices. Theoretical predictions for their estimations are drawn from a model of intersecting roads (combining features of Chen & Riordan (2007) and Braid (2013)) under linear transportation costs and perfectly inelastic demand. Specifically, stations' locations are assumed to be exogenously given and each spoke is occupied by one player. Importantly, an asymmetric location pattern is imposed with one firm locating closer to the intersection than its rivals and the center is assumed to be a vacant spot. Then, from the expressions of market demand, the profit functions, and the first order conditions (in prices) of the centrally located station and the peripherals it follows that the price reaction of a remote station (to a price change of the central station) is stronger than the price reaction of the central firm (to a price change of a peripheral station). (cf. Firgo et al. (2015), Proposition 1, p. 82). This proposition is in line with the findings in Braid (1989) and Braid (2013) and extends the argument to an asymmetrical location pattern in a setting of intersecting roads. Additionally, conditions are provided such that the price of the central station exceeds the price of a remote station which highlights the countervailing forces of a price competition effect if the number of remote stations is high and they locate comparatively close, and of a market share effect if the central firm locates close to the intersection and

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[121] These are firstly a dummy that measures whether the price of the merged station varies, and secondly whether the price of a nearest neighbor in the radius of 1.5 miles varies. (cf. Pennerstorfer & Weiss (2013), p. 667)

holds a large market share. (cf. Firgo et al. (2015), Proposition 2, p. 83)

The empirical investigation uses data for gasoline prices for the metropolitan region of Vienna in an unbalanced panel for 22 points in time between 1999 and 2005, and station specific data for all stations surveyed in 2003 as well as local market and demand characteristics.[122] The degree of centrality is measured based on the number of times a station is a direct neighbor to another station.[123] (cf. Firgo et al. (2015), equation (6), p. 85) Subsequently, a diagonal centrality matrix C is constructed with diagonal elements c_{ii} expressing the degree of centrality for station i.

The regression equation specifies as the dependent variable the price and as the explanatory variables (i) the spatially lagged price, (ii) the spatially lagged price accounting for the degree of centrality, (iii) the degree of centrality (as a sole variable) and (iv) a set of explanatory variables relating to station and local market characteristics.[124] (cf. Firgo et al. (2015), equation (7), p. 85) In addition, the specification includes spatially lagged error terms using binary spatial weights. As estimation methods Maximum Likelihood estimation and the inclusion of instrumental variables are used. The results reveal a significant positive correlation of market prices with the spatially weighted prices of centrally located stations while the relationship with the spatially lagged price vector is not significantly different from zero. This confirms the prediction that changes in the price levels in local markets are critically determined by variations of the price decisions of the central player. Moreover, the regression coefficient for the centrality measure is not significantly different from zero which implies that the absolute price level is not explained by the degree of centrality. Thus, on average the price of a centrally located station does not exceed the market price suggesting that no evidence in favor or against the price competition effect or the market share effect for the central station is found. In addition, based on the regression coefficients simulations illustrate that an exogenous price shock imposed on a central station leads in total (after consideration of direct and indirect feedback effects) to higher price levels for the central station as well as to a higher average market price as the degree of centrality increases. (cf. Firgo et al. (2015), p. 88f)

In conclusion, the paper of Firgo et al. (2015) provides evidence for the interrelation of asymmetrical location patterns with the price level in retail gasoline markets. Specifically, controlling for spatial dependencies among stations their results imply a statistically significant relationship between the price of a centrally located sta-

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[122]Initially conducted tests (variance ratio tests, two-way fixed effects estimation on station and time fixed effects, rank reversal tests) reveal that the price variation in the cross sections does not follow random movements. (cf. Firgo et al. (2015), p. 84)

[123]A matrix G is constructed with all stations spawning the rows and columns. Then, an element g_{ij} equals one if station i and j are nearest neighbors and zero otherwise.

[124]The spatial weight matrix accounts for distance decay effects and contains reciprocal values of a distance measure between stations' locations. It is constructed based on a circular radius of 5 minutes driving time which implies that for stations farer away respective matrix elements are zero.

tion and the average local market price. The importance of the spatial dependence between market players and asymmetric location patterns is further illustrated in shock simulations where a higher degree of centrality implies that exogenous shocks have stronger impacts on prices of the centrally located firm as well as on the average local market price.

Recently, the paper of Firgo et al. (2016) corroborates previous findings of econometric analyses on the importance of central locations for the determination of price levels. They use data on station and local market characteristics for the Austrian gasoline market, and unbalanced price data available for 23 points in time between 1999 and 2005. Local markets are defined based on the average driving time to nearest competitors. Consistency requires that the shortest driving time to a station outside a local market always exceeds the shortest driving time to the closest rival within. Moreover, the market center is defined as the point which minimizes the sum of distances to all stations in the local market. (cf. Firgo et al. (2016), p. 80)

Two regression models are examined (cf. Firgo et al. (2016), equations (2) and (3), p. 81). Firstly, controlling for station and local market characteristics the relationship of the spatially lagged price with the market price for markets with three, four, five and six stations is estimated. Secondly, by including diagonal matrices to select central and remote stations the effect of the price of a central station on remote stations, the effect of a remote station's price on the central station as well as the coefficient of the price interaction between remote stations is estimated for the same variation in market size as in the first case.

The results show that for the first model - where stations are assumed to have equal (symmetric) market power in terms of their proximity to the center - the spatially lagged price is significantly and positively correlated with the market price. This confirms previous results (e.g. Pennerstorfer (2009), Firgo et al. (2015)) and indicates that on average stations increase prices if their nearest neighbors increase prices. Moreover, the regression results reveal that this interaction continuously declines as the market size increases from three to six players. For the second case evidence is provided that the asymmetric location pattern critically impacts the price interaction between stations. For markets with three stations the findings of the first regression model is confirmed. However, as the number of stations in the local market increases the coefficient of the central station on its rivals' prices increases (compared to the case with three stations) whereas the coefficient indicating the influence of the remote station on the central competitor as well as the coefficient measuring the interaction between remote stations continuously decrease.

In sum, Firgo et al. (2016) provide clear-cut evidence for the influence of a central location on the pricing behavior in local markets. Their results indicate that as the market size in terms of the number of competitors increases the importance of a

central market position increases. For growing market sizes remote stations are less influenced by the prices of nearest rivals and more likely to follow the price setting behavior of the centrally located station.

2.5 Models of Economic Agglomeration

Models of economic agglomeration use a more general approach to describe location patterns of firms compared to game theoretically based price-location models. These sort of models are rooted in the field of economic geography and address the fundamental questions of why economic activities of agents (households and firms) agglomerate in certain places in space, and consequently try to identify the determinants of agglomeration and dispersion forces. Typically cities represent central points in space and clearly the size and significance of cities widely differ. Therefore, a particular stream of the research activities is concerned with the functional and locational characteristics of cities to provide explanations for their existence and development as well as their hierarchical order.[125]

The goal in this subsection is to give a very brief overview on topics that are dealt with in agglomeration models. Intuition shall be developed for the research topics of regional economics that can have ramifications in very different fields of the economic literature such as growth and development theories or urban planning. For this reason this subsection firstly draws largely on the literature review of Fujita & Thisse (1996), and secondly presents the seminal paper of Eaton & Lipsey (1982) and a more recent article of Tabuchi & Thisse (2011) in greater detail.

In the preceding chapters of this survey it was made clear that firms' tendency to agglomerate or disperse in a linear and bounded market is the result of interdependent profit-maximizing decisions concerning price and location. In short, agglomerative and deglomerative forces emerge from spatial competition that is inherently strategic. According to Fujita & Thisse (1996) in the context of economic behavior and

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[125] The origins of economic geography models date back to the work of Johann Heinrich von Thuenen. His approach yields concentric circles around a central place (city). Depending on specific characteristics such as price, transportation cost and the production technology he argues that it is profitable to produce only one type of product in a certain spatial region with respect to the center. Furthermore, the geographer Walter Christaller developed the prototype of a model of central places. His model implies the existence of different central places distinguished by a degree of centrality which is defined by the amount and quality of services rendered to its population. Assuming a regular transportation network connecting a system of places, a basic outcome is that in order to distribute the areas between the centers with goods and services a triangular pattern of central locations emerges where the adjacent areas to each central place form a regular hexagonal spacing. Conditions for this pattern to emerge are, firstly, that firms need to sell at least to a minimum demand to remain profitable, and secondly, that consumers need to be close enough to a central place and be willing to incur transportation costs. Moreover, Christaller's hierarchy model implies that the goods and services only flow in one direction from a city with higher centrality to less central places. (cf. Mulligan (1984), p. 9f)

decision making in space two further general determinants have to be considered to explain agglomeration and dispersion forces which are (i) externalities, and (ii) increasing returns.

The standard approach for the first determinant is to introduce information externalities and explain agglomeration as a result of informational spillover effects among firms. (cf. Fujita & Thisse (1996), pp. 348-351) The intuition behind this idea is that firms share different types of information and mutually benefit from public good characteristics of information flows. Since close proximity fosters the use of communication channels and stimulates economic activity firms tend to settle at a central location. On the other side a higher concentration of firms in one area leads to an increase in the commuting distances of its employees that live in the surrounding neighborhoods, thus, in turn to higher wage rates and land rents in the agglomeration area. An equilibrium for the spatial distributions of firms and households is reached if these countervailing effects are balanced.

Formally, firms' profits are an increasing function in the aggregate benefit from exchanging information and a decreasing function in the amount of the two production factors land and labour (cf. Fujita & Thisse (1996), p. 349, equation 2.2). On the side households' income and total budget comprises of distance-dependent commuting costs (with a unit transportation cost coefficient) as well as the costs for the consumption of land and a composite good. The aggregate information benefit as well as land rates, wage rates and commuting costs are a function of the agents' locations. In equilibrium the land and labour markets are cleared and all households achieve an optimal level of consumption (utility) and all firms an optimal level of profits. Within this setting research interests focus on the impact of the functional form of the aggregate information benefit function (e.g. linear or exponential decay factor) on the characteristics of the equilibrium state and are associated with the prominent works of the Japanese regional economist Masahisa Fujita (for corresonding references see Fujita & Thisse (1996), p. 350f). Depending on the parameter ranges and the functional form of the benefit function either unique configurations (one city with a center) or multiple equilibria (polycentric cities) obtain. Moreover, in an extension of the model with costly intrafirm communication, solutions obtain that cover the case of a city with a central business district surrounded by residential areas in which firms' back units are located. This demonstrates that these sort of agglomeration models are capable of relating communication technologies and firms' intra-organizational structures with their spatial distribution and the structures of modern cities characterized by the dichotomy of a central business district and residential areas.

The second determinant of increasing returns refers to the notion of monopolistic competition and input differentiation. (cf. Fujita & Thisse (1996), p. 352ff) The prototype of this sort of model assumes a population of homogeneous consumers

distributed in space who choose among a homogeneous product and a variety of n differentiated goods. For a continuum of differentiated goods the utility is described by a constant elasticity of substitution (CES-type function). Likewise, firms' production function shall be of a CES-type. Formally, utility is $U = (z_0)^\alpha (\int_0^n [z(\omega)]^\rho d\omega)^{\frac{1-\alpha}{\rho}}$ and produced output $x = (z_0)^\alpha (\int_0^n [z(\omega)]^\rho d\omega)^{\frac{1-\alpha}{\rho}}$ where z denotes the consumption goods and the input factors respectively (with z_0 as the homogeneous good and the homogeneous input), ρ is a measure for the degree of substitution and differentiation respectively ($0 < \rho < 1$), and α is a scaling factor. Furthermore, it is assumed that labor represents the only input factor with a fixed labor requirement and a variable part in the production function (i.e. the marginal labor requirement a). Additionally, transportation costs shall be increasing in distance and strategic price setting of firms shall be ruled out, i.e. the standard assumption of monopolistic competition holds with a considerable number of differentiated products as substitutes and thus no firm having a significant impact on market price and total consumption. In sum, this model implies increasing returns to scale (through ρ), isoelastic demand curves, and an aggregate demand being independent of the spatial distribution of consumers.[126] In equilibrium the price for a firm is given by the marginal production cost of labor with equilibrium wages $W(x)$ varying in space times a mark-up which increases with the degree of product differentiation: $p^*(x) = \frac{aW(x)}{\rho}$. The basic intuition for an equilibrium state in which agents agglomerate and form a city is that a higher density of firms attracts more consumers to satisfy their needs for a greater variety of goods. Likewise, a higher density of consumers attracts more firms since they expect higher demand. Firms tend to agglomerate since they specialize in production to increase profits, and consumers tend to agglomerate since they prefer a greater variety of goods to increase utility. Repulsive forces are represented by a higher degree of competition in denser areas. Indeed the literature yields equilibrium states with bell-shaped distribution functions, however, to dissolve repulsive forces a certain degree of differentiation has to prevail. (cf. references in Fujita & Thisse (1996), p. 354)

A further stream of the regional economics literature, which is associated with the Nobel laureate Paul Krugman, applies the monopolistic competition model on a two sector economy and explains the emergence of core-periphery structures. (cf. references in Fujita & Thisse (1996), p. 355ff) As suggested above, two types of goods are in the economy: a homogeneous agricultural good produced by an immobile labor force and traded with zero transportation costs, and a continuum of differentiated industrial goods produced by a mobile work force with distance-dependent transportation costs and sold on a monopolistically competitive market. Agglomeration

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[126]Intuitively, since buying from a more distant seller proportionally decreases utility and no firm applying certain pricing strategies the elasticity of an individual demand at a certain location is the same throughout the whole spatial area.

forces emerge due to real income effects. As firms agglomerate price competition intensifies generating an increase in real incomes which leads to a migration of the mobile industrial workers, in turn causing more firms to locate at the central place etc. By contrast, the immobile workers and production of the agricultural good remain in the periphery. In sum this yields a core-periphery structure with the production of the differentiated industrial goods concentrated in one region. In the literature it is shown that the existence of an equilibrium depends on the degree of transportation costs, the degree of differentiation, and the share of the industrial sector to the economy. Moreover, it is emphasized that the equilibrium is not stable suggesting that the initial state of a regional economy plays a critical role in its development path. Extended versions of this model consider a totally mobile work force and generally positive transportation costs. In this case, a single city surrounded by agricultural regions represents an equilibrium state subject to the degree of differentiation and transportation costs as well as the total population of workers. Intuitively, the smaller the degree of differentiation, i.e. if products become closer substitutes, more incentives exist to locate at the periphery. Finally, consider that this model type is capable of explaining hierarchical structures in regional patterns (as in the earliest works on the subject by Walter Christaller). This is achieved by introducing different groups of differentiated industrial goods with different transportation rates. Then, higher ordered cities provide a greater amount of groups of differentiated goods. However, in contrast to Christaller the flow of goods also comprises reverse transactions from less central to more central cities.

An example for an agglomeration model that exemplifies the effect of a particular type of externality is provided in the seminal article of Eaton & Lipsey (1982). They motivate their work by developing a spatial model for firms' location decision that is based on their profit-maximizing behavior. The question that the paper tries to answer is „*why, in other words, do firms retailing different goods tend to cluster together?*“ (Eaton & Lipsey (1982), p. 58) This stands in contrast to previous models of central places, most prominently the Christaller model, which explains the hierachical pattern of agglomeration spots by geometrical arguments.

The model of Eaton and Lipsey assumes that households consume two goods (A and B) with a constant rate over time normalized to one. The market is the one-dimensional line with unit length and uniformly distributed consumers with density D. After each time period households assess their stock and decide to go on a shopping trip where they are allowed to buy either a bundle of good A consisting of $\frac{1}{\alpha}$ units, a bundle of good B with $\frac{1}{\beta}$ units, or both bundles. On their trip consumers minimize transportation costs which are increasing in traveled distance. Then, the probability to consume A in one period is α, and to consume B in one period is β. On the supply side firms are distinguished into respective groups A and B. They face fixed costs of K_A and K_B, marginal costs are assumed to be zero. Moreover,

prices are exogenously given (p_A, p_B), for the choice of the location each firm assumes zero conjectural variation, and it shall be allowed for more than one firm to locate on the exact same spot. Finally, for an equilibrium to prevail three conditions shall hold: firstly, locations are optimal (i.e. guarantee maximal profits), secondly, there is no exit (i.e. revenues must exceed fixed costs), and thirdly there is no entry. To characterize equilibrium states these three conditions are exploited.

Using the profit-maximizing condition, it follows that in equilibrium only central places of order two (i.e. a seller of type A and a seller of type B are located at the same place), or central places of order two and groups of sellers with the same type (A or B) exist, or there could be a configuration with groups of sellers of type A and B only if they are separated by a central place of order two. (cf. Eaton & Lipsey (1982), proposition 1 and 2, p. 62) In short, there will never exist an equilibrium where a firm of type A is a neighbor of a firm of type B, rather they must form a central place of order two. The intuition is that the two alternatives for firm A (to forming a pair with B) do not constitute an equilibrium since then profits for all players are not maximal.[127]

Using the exit condition the second main argument of the paper is that multiple equilibria obtain such that only one group of firms (A or B) and central places of order two emerge in the market, or such that only central places of order two prevail. Essentially, for one group of firms to exist the relation of fixed costs (e.g. K_A) to the expected revenue (i.e. $\alpha p_A D Y$)[128] must stay within defined boundaries. Formally, the necessary conditions for firms of group A and firms of group B can not be fulfilled simultaneously which rules out a state where they are located simultaneously. (cf. Eaton & Lipsey (1982), proposition 4, p. 64 and the verbal arguments on p. 65) The model of Eaton & Lipsey (1982) earns its significance by establishing the existence of central places based on firms' profit-maximizing decisions. It represents a generalization of traditional central place models and emphasizes that central places attract consumers who purchase two different goods and therefore steal demand from neighboring firms. In other words, there arise negative demand externalities for firms producing one type of product due to central places at which two types of products are served. Therefore, central places require larger market areas for neighboring firms to stay in the market. Moreover, the paper stresses that the equilibrium is not uniquely defined and that it depends on the transportation costs and the relative volumes of multipurpose and single-purpose shopping. (cf. Eaton & Lipsey (1982), p. 66f)

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[127]The first alternative is that the firm of type A (firm A_i) remains at its initial location a_i, then however, it could move towards firm B and increase profits by gaining more consumers who want product A and B. The second alternative is to locate at the spot of its nearest A-type neighbor firm A_{i-1} at a_{i-1}, this leads to optimal profits for the migrant firm A_i but only if the local market for consumers who solely purchase good A is big enough. This condition, however, leads the neighboring firm A_{i-1} to move to the left of A_i. (cf. Eaton & Lipsey (1982), p. 63)

[128]Y denotes the length of the required market area such that a firm bears fixed costs.

The article of Tabuchi & Thisse (2011) serves as an example for the second determinant for agglomeration and presents a model of monopolistic competition in space. Within an economy of different industrial sectors under monopolistic competition it describes the linkage between transportation costs and spatial equilibrium configurations. The intuition behind the model is to provide the theoretical background for the empirical observation that a substantial decrease in transportation costs over the past decades is correlated with an increase in the population of major cities in OECD countries.

The economcy consists of a homogenous (agricultural) good and a set of differentiated goods each produced in a specific industry ($i = 1, ..., I$). Each industry is subject to monopolistic competition and comprises a continuum of varieties ($v \in [0, n]$). The spatial setting is a circumference with length 1. The model can be considered as a general equilibrium model and deploys a two-stage procedure that unfolds by backward induction. In the first stage the distribution of workers is dealt with, workers are assigned to an industry and a city and the individual choices correspond to a maximum level of utility. In the second stage, the prices, demand functions and wage rates of the industries are derived.

The starting point of the analysis is a CES-type utility function. Total utility results from consuming industrial goods and the agricultural good, the intensity of competition is captured by the elasticity of substitution (σ^i) for each industry. (cf. Tabuchi & Thisse (2011), equation (2), p. 242) Maximizing total utility subject to the budget constraint (total consumption has its source from earned wages) yields the individual demand at a particular location for a variety of an industrial good that is produced at a particular location. Transport costs (τ) are assumed to be the same accross industries. (cf. Tabuchi & Thisse (2011), p. 243) The specification of the demand functions and the introduction of transportation costs allows to define a firm's profit assigned to a particular industry and located in a particular city where the solution for the population of workers in the city is subject to stage 1. (cf. ibid., equation (4), p. 243) Maximizing respecitve profit functions yields the equilibrium price for each industry which hinges upon the amount of transportation costs (i.e. the distance to travel from the production site in a city to the consuming worker) times a mark-up that depends on the competitiveness of the industry. For each industry wages are restricted by firms' operating profits. Based on the equilibrium price level the condition that all varieties can be produced in a city the equilibrium wage level is derived which differs by industry and location since operating profits are not equal across cities and industries. (cf. Tabuchi & Thisse (2011), equation (7), p. 243)

The distribution of the population over the whole economy yields multiple equilibria. Therefore, Tabuchi & Thisse (2011) focus on two equilibrium configurations to scrutinize the role of transportation costs in the model. In the first case all cities are of equal size and all industries are present in each city, but in every city different

varieties are produced. In the formal treatment of the first case an interval for the level of transportation costs is specified such that an equilibrium pattern of an even number of cities which are symmetrically and equidistantly located around the circle exists and is stable. The intuition for the condition to prove the existence is that an equilibrium requires all industries to be equally represented in each city, i.e. to set up an equilibrium industry share that is dependent on the elasticity of substitution and the coefficient in preference. (cf. Tabuchi & Thisse (2011), equation (12), p. 244) It follows that the problem reduces to the positiveness of an auxiliary function $g \geq 0$ which yields the threshold value for transportation costs. If transportation costs are sufficiently low (beneath the threshold) then the number of cities in equilibrium doubles compared to the opposite case. (cf. Tabuchi & Thisse (2011), Proposition 1, p. 245)

The second case deals with a situation where cities have different sizes and larger cities offer a larger set of industries. The corresponding section 4 in the paper deals with the simplified case of one differentiated industry ($i = 1$) and the case of multiple differentiated industries in the economy. It contains mathematical descriptions of the development of spatial patterns starting with an initial equilibrium state of a given number of cities and defines the conditions under which the evolution process evolves. The basic intuition for the outcome of the analysis for the single industry case is that the initial development state starts with a relatively high value of transportation costs, given a steady decrease as a first threshold is passed the symmetrical initial configuration becomes unstable and the size of the cities fluctuates. For further decreasing transportation costs bigger cities grow and smaller cities shrink. Eventually, the numer of cities is halved and the size of the remaining cities has doubled. (cf. Tabuchi & Thisse (2011), p. 247)

In conclusion, the article of Tabuchi & Thisse (2011) represents a prominent and demanding example for a recent model of economic agglomeration. It demonstrates that the emergence of agglomerations is the result of an outcome of economic interactions between workers and consumers respectively on the one side and firms on the other. The equilibrium states are not uniquely defined but pertain to the principles of profit- and utility-maximization. The main impact of the study is to provide explanations for the evolution of spatial configurations as the level of transportation costs decreases.

2.6 Conclusion

This survey tells the story of why different firms in a market would be likely to position their product and settle their premise or retail outlet in a similar segment, or why this would not be the case. Two arguments (and a final reference note) shall

close the examination:

1. The selected articles show that the location determinants can be assigned to groups representing the following different economic impact factors (a summary of the various model characteristics and predictions is provided in table 1):

   - consumers' reservation price and elasticity of demand

   - transportation costs incurred by consumers (t-costs)

   - number of firms n

   - characteristic and timing of the game

     - simultaneous and sequential entry

     - relocation costs and entry costs

     - revocability and commitment of the decision on the strategic variables

     - assumptions on the reaction of rivals' strategic behavior to one's actions (conjectural variations / strategic foresight)

   - characteristics of the consumer distribution

   - market geometry

To wrap up the state of the field, generally two countervailing effects impact firms' locations. In a duopoly, the market share effect suggests to maximize the hinterland and move towards the center whereas price competition increases the closer the two rivals get. Since all consumers purchase by assumption Hotelling (1929) introduced the proposition of an agglomerative behavior. Smithies (1941), and later Economides (1984) and Hinloopen & van Marrewijk (1999) focused their research questions on this assumption of perfectly inelastic demand and an infinite consumers' reservation price. If an increasing number of consumers opt out of the market since their net utility of consumption becomes negative, intuitively no additional consumers can be gained by relocating and price competition becomes the dominant force. Thus, Economides (1984) argues for a repulsive location behavior and eventually for the emergence of isolated local markets served by monopolists. Hinloopen & van Marrewijk (1999) demonstrate that Hotelling and Economides studied extreme cases of location patterns in terms of the value of the reservation price and shed light on the intermediary transition process partially vindicating Hotelling's suggestion to move towards the center.

A general proposition of minimized product and location differences can not

be obtained in a subgame perfect price and location equilibrium since for close distances an incentive to undercut exists. This is, of course, one of the main insights of d'Aspremont et al. (1979). Moreover, they show that this instance can be traced back to the underlying transportation cost regime. More specifically, Economides (1986) defined the exact parameters for a transportation cost function for which a subgame perfect market equilibrium exists. Within certain parameter ranges lower transportation costs imply that firms agglomerate in equilibrium which reflects the significance of the market share effect due to a higher flexibility of consumers choosing their utility-maximizing product. However, according to Economides (1993) who generalizes from a duopoly to oligopolistic markets the proposition holds that for $n \geq 3$ a subgame perfect equilibrium does not exist since the dominant strategy for remote firms relies upon the market share effect. This finding anticipates the importance of asymmetrical structures in spatial markets whose significance became clearer in more recent theoretical studies concerning nonuniform consumer distributions and intersecting roadways, and empirical studies on gasoline markets. Furthermore, the literature suggests that location patterns are critically determined by the strategic interaction between the players, and particularly, by the role of the timing in the game, that is the question of the sequence in which each player decides on his strategic variable(s). The contributions of Hay (1976) and Prescott & Visscher (1977) are among the first to study sequential entry games pinning down the strategic dependencies in the location choice by introducing prohibitively large relocation costs. Generally, their studies argue for equidistant location configurations which confirms the intuition that the size of local markets has to correspond to the level of entry costs. Thus, under sequential entry the market share effect loses its significance. Then, the dominant strategy is to identify market niches and to secure one's position against later entrants. Subsequent research interests focus on the question how location patterns vary contingent on the level of entry costs and which player takes the most advantages out of the sequential entry order. Exemplarily, the studies of Neven (1987), Economides et al. (2004) and Goetz (2005) show that symmetric and asymmetric location patterns result from sequential entry and that first-mover advantages exist, however, under certain parameter ranges also late entry may be profitable. Additionally, the studies of Anderson (1987) and Fleckinger & Lafay (2010) include sequential interactions for price and location decisions in a duopoly. They emphasize that particular product and market characteristics cause players to commit themselves differently which implies advantages for the first or the late mover in the game and subsequently location outcomes either reveal a dispersed pattern or the two players locating comparatively close.

As the assumption of a uniform consumer distribution is dropped and the market geometry is extended to a setting of intersecting roads the tendency is observed that firms are attracted to the region where consumers are concentrated. Generally however, the problem of characterizing the interaction patterns and finding an equilibrium for a price-location game becomes more complicated. For the case of nonuniform consumer distributions the studies of Neven (1986), Tabuchi & Thisse (1995) and Anderson et al. (1997) illustrate that the characteristics of the distribution determine the existence and the type of the location equilibrium. For intersecting roads the studies of Braid (1989) and Braid (2013) demonstrate that a subgame perfect equilibrium in a simultaneous price location game does not exist which is attributable to the inherent asymmetry in the competitive relationship of the players related to the notion of nonlocalized competition and the specifics of the market geometry.

Table 2.1: Model predictions on the location choice in spatial competition models a la Hotelling

Spatial competition model	number of firms	model assumptions	location tendency	location determinants and outcomes
Uniform consumer distribution:				
Hotelling (1929)	2	s2S-P-L game[129], linear t-costs	$(+)$[130]	perfectly inelastic demand, ZCV[131] in price and location
Smithies (1941)	2	location game solved for 3 types of competition, linear demand, linear t-costs	$(+,-)$	ratio of t-cost coefficient and intercept (demand curve)
Economides (1984)	2	s2S-P-L game, finite reservation price, linear t-costs	$(-)$	finite reservation price implies emergence of local monopolies
Hinloopen & van Marrewijk (1999)	2	s2S-P-L game, finite reservation price, linear t-costs, symmetric locations	$(+,-)$	$(+)$ for intermediate market sizes ($\frac{8}{7} \leq \alpha \leq \frac{4}{3}$) and increasing reservation prices
d'Aspremont et al. (1979)	2	s2S-P-L game, infinite reservation price, linear t-costs, price equilibrium not subgame perfect	$(-)$	quadratic t-costs yield subgame perfect price equilibrium

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[129]simultaneous two-stage price-location game
[130]$(+)$ indicates the tendency to agglomerate, $(-)$ to disperse in space.
[131]zero conjectural variation

Economides (1986)	2	s2S-P-L game, infinite reservation price, t-cost exponent α, symmetric locations	$(+,-)$	$(+)$ for decreasing α and $1.26 = \overline{\alpha} \leq \alpha < \frac{5}{3}$
Economides (1993)	$n \geq 3$	s2S-P-L game, infinite reservation price, linear t-costs	$(+)$	no subgame perfect location equilibrium, distance decay effects, exogenous equidistant spacing yields U-shaped Nash prices
Hay (1976)	$n \geq 2$	elastic demand, costly relocation, sequential entry	$(-)$	cost and demand parameters
Prescott & Visscher (1977)	$n \geq 2$	costly relocation, sequential entry, prefect foresight	$(-)$	n, endogenization of n, fixed costs, endogenization of mill price
Neven (1987), Economides et al. (2004), Goetz (2005)	$n \geq 2$	s2S-P-L game, quadratic t-costs, sequential entry, fixed costs	$(+,-)$	fixed costs and market size, entry deterrence, symmetrical and asymmetrical equilibria exist
Anderson (1987)	2	sequential game, one's price and location chosen at different stages, linear t-costs	$(-)$	location leader takes center and becomes price follower, first mover advantage, second mover locates remotely, irrevocable location choice
Fleckinger & Lafay (2010)	2	sequential game, one's price and location chosen at one stage, linear and quadratic t-costs	$(+)$	firms locate on same side of the market, second mover advantage (location closer to center), equal flexibility of product and price
Nonuniform cons. distribution:				
Neven (1986)	2	s2S-P-L game, quadratic t-costs, symmetrical locations, symmetrical distribution	$(+)$	peak of symmetrical distribution $c(\frac{1}{2})$, consumer concentration implies agglomeration
Tabuchi & Thisse (1995)	2	s2S-P-L game, quadratic t-costs, simultaneous and sequential location subgame, triangular distribution, unbounded line	$(+)$	asymmetrical equilibrium, first mover takes center, consumer concentration implies agglomeration
Anderson et al. (1997)	2	s2S-P-L game, quadratic t-costs, log-concave distribution	$(+)$	convexity properties determine symmetric or asymmetric equilibrium, consumer concentration implies agglomeration

Intersecting roads:

Braid (1989)	$n \geq 2$	s2S-P-L game, center occupied, linear t-costs	(+)	no subgame perfect location equilibrium, asymmetric price reaction between central and remote firms
Braid (2013)	$n \geq 2$	s2S-P-L game, center occupied, quadratic t-costs, symmetric locations	(+)	no subgame perfect location equilibrium, asymmetric price reaction between central and remote firms, increase in the number of firms leads to agglomeration

2. From the state of the field it can be concluded that a complex set of determinants has to be used to explain location configurations. The original setting of the linear city has been subject to various analyses that bring different impact factors in sharper focus. By contrast, theoretical models for the setting of intersecting roads allow for a wider range of research questions and still leave interesting research gaps open. In particular, a model that examines the strategic interaction in the price and location choice of an incumbent and an entrant firm in a setting of intersecting roads has not been developed yet. Clearly, findings for a two-stage game where prices and locations are chosen simultaneously are provided in the studies of Braid (1989) and Braid (2013) yielding the outcome that a subgame perfect equilibrium does not exist. Inspired by the treatment of the linear city and the approaches of Anderson (1987) (and Fleckinger & Lafay (2010)), however, it is interesting to apply a sequential entry game to the market type of intersecting roads and investigate firms' behavior and potential equilibrium outcomes. One of the considerable simplest settings would be to treat the case of a duopoly and take the location of the first entrant as exogenously given. This provides the case of a model for entry into a local monopoly market with spatial characteristics where location costs are prohibitively high (thus relocation does not occur). The strategic interaction in this game focuses on the choice of the entrant on his price and location, and the subsequent price reaction of the incumbent firm. The dichotomous options for a reaction to entry would be to accept the competitor, or to undercut him and deter entry. Consequently, conditions on the each of these have to be specified.

Admittedly, this survey took a narrow (but deep) focus to reflect the vast literature on the Hotelling model. Therefore, it is natural that some aspects that are related to model predictions on location decisions were not dealt with in greater detail. In order to account for these further aspects the following streams of the literature can be considered:

- Locations in mixed strategy equilibria

  An interesting example for the case of mixed strategies in the Hotelling model is provided in Gal-Or (1982). She demonstrates that a mixed strategy equilibrium exists in a Hotelling duopoly with the two firms picking prices randomly from a continuous price distribution. The existence condition requires that the drawing is from a defined price interval that decreases as firms move towards the center. Moreover, a general treatment of mixed strategy equilibria and conditions on their existence is provided in Dasgupta & Maskin (1986) who show that in the Hotelling model each subgame in prices has an equilibrium in mixed strategies.

  A further important contribution is the study of Osborne & Pitchik (1987). They characterize the set of mixed strategy price equilibria in the Hotelling model subject to different location combinations. Based on these results the location choices are examined and a unique subgame perfect location equilibrium is derived with the firms locating above the quartiles at 0.27. In addition, they find a subgame perfect equilibrium with mixed strategies in locations. Xefteris (2013) departs from Osborne & Pitchik (1987) but in contrast to their model he assumes an infinite reservation price. Subsequently, he proves that a subgame perfect equilibrium exists with both firms locating at the market center.

- Cournot competition in the Hotelling model

  Competition may take the form of price competition (Bertrand competition) or via competition in quantities (Cournot competition). An example for Cournot competition in the Hotelling model is given in the study of Anderson & Neven (1991). They show that for a linear demand function Cournot oligopolists ($n \geq 2$) who spatially discriminate their price locate at the center of the market. Pal (1998) confirms the finding that in a linear city model firms agglomerate at the market center and additionally finds that in a circular market an equidistant location pattern under Cournot competition obtains. Matsushima (2001) re-examines his results for the circular market and shows that an agglomerative location tendency is sustained in equilibrium with half of the firms locating at one point on the circle, and the other half locating at the opposite point. More recently, Matsumura et al. (2005) relate the contradictory predictions of

the previous studies for the circular market to differences in the transportation cost functions as well as to simultaneous and sequential location choices.

3 Does Entry Pay Off in a Linear City with a Center?[*]

3.1 Introduction

Every time a firm considers entering a market it is confronted with the issues of price setting and product positioning. Many examples illustrate that successful market entry is based on a balanced approach where a business strategy is required that differentiates the own product and defines the competitive space but also acknowledges competitors' strengths and avoids thriftless head-to-head battles. Take for instance the case of Capital One, which developed from a monoline credit card company to one of the largest U.S. bank holding companies in less than a decade. In 1994 there was little differentiation in the credit card industry which lead it to develop statistical models to offer custom-tailored products determining best combinations of product, price and credit limit. Key to success was not to aggressively enter the market exposing itself to direct price competition taking on the incumbents' uniform pricing strategies but to develop its own skills using analytics for product customization.

Under a current market state characterized by a given set of incumbent firms entry is by its nature a sequential phenomenon. Balancing the costs and profits the entrant firm determines his strategic variables, most importantly product and price, whereas the incumbents separately choose for an appropriate reaction by means of their strategic variables, which in a first reaction would be their price excluding the option to reposition products in the short run.[1] For the entrant to optimize his profits and make consecutive strategic decisions he has to address the following questions. What

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[1] The assumption that pricing decisions are more flexible than the choice of products can be disputed. An interesting paper providing results where products and prices are chosen simultaneously by one player is provided by Fleckinger & Lafay (2010). We stick to the classical approach to study entry and assume that price setting is less costly than to reposition a product. A nonexhaustive list of papers using this assumption are Goetz (2005), Lambertini (2002), Tabuchi & Thisse (1995), and Prescott & Visscher (1977).

is the most profitable choice to design a new product accounting for the position of the incumbent firms' competing products with regards to consumers' preferences? What is the best price considering the current incumbents' product prices and, more importantly, the anticipated future price decisions as a reaction to market entry? In sum, what is the combined optimal choice for a new product and its price such that entry profits are maximized and given the incumbents' price reaction to entry?

Based on and inspired by the paper of Anderson (1987) we pick up these issues and develop a model that examines market entry in a two-stage game with an entrant firm choosing his price and location in the first stage and one incumbent firm choosing his price in the second stage. Accordingly, the analysis is conducted within the framework of a spatially differentiated market with a linear transportation cost scheme where the only distinguishable characteristic between firms' products is given by firms' location.

Our contribution to previous studies is two-fold. Firstly, we extend the original linear spatial setting accentuating the central location and introduce an additional variable Z which represents a node in the center and thus a measure of centrality in the market. This enhances the strategic interaction between the players. Consequently, we are interested in the impact the variable Z has on the pricing behavior, the decision on the entry location, and the realized profits of the players. Secondly, in the light of the centrality bonus Z, we are interested if entry leads to a higher or lower degree of product differentiation in the market. In terms of the taxonomy of Fudenberg & Tirole (1984) we address the question whether a 'puppy dog' behavior of the entrant firm that entails choosing a differentiated product and applying a low price strategy proves to be a reasonable outcome of the entry game.

As usual, a short description of the structure of the paper is given at the beginning. In section 2 the assumptions of the model are depicted. In section 3 the strategic decision set of the incumbent firm is explained by sketching his reaction functions dependent on the entry price and the entry location. In section 4 the strategic decision of the entrant is scrutinized, based on the best replies of the incumbent, we derive profitable entry pricing strategies and pricing functions respectively, construct profit functions for each strategy and examine their dependency on the entry location. This drills down to a set of propositions on the entry decision in subsection 4.2.4. In section 5 we exemplify and interpret the results in three entry scenarios and compare our formulas with the model of Anderson (1987). Eventually, we summarize our results and provide conclusions.

3.2 The Model

The analysis departs from the classic duopoly model of spatial competition developed by Hotelling (1929). Consider two firms, firm I and firm E, on a line of unit length ($l = 1$), the linear city. Consumers are uniformly distributed on the line and each consumer shall purchase one unit of a homogenous product. The total number of consumers is normalized to $N = 1$. Further, firms' marginal costs c are assumed to be equal and set to $c = 0$. To extend the setting we assume that a spoke of length Z is attached to the market at a distance of $z = \frac{1}{2}$ taken from the respective ends of the linear city. The uniform distribution and unit demand assumption shall also apply for the Z consumers on the spoke.

Clearly, since competing firms sell a good of identical properties they differentiate themselves by two factors: price and location. Let p_I and d_I denote the price and location of firm I and p_E and d_E the respective quantities for firm E. To be consistent with the literature, locations d_I and d_E are defined as distances taken from respective extremes of the city as depicted in figure 3.1. Correspondingly, the market is divided into two hinterlands attached to the respective seller and an intermediate market consisting of the demand between the competing rivals $1 - d_I - d_E$ plus the demand given by Z concentrated in the market center at $z = \frac{1}{2}$. Consumers in the hinterlands will always purchase at their dedicated seller, i.e. demand is perfectly price inelastic. We now consider a two-stage sequential price-location game to study market entry.

Stage 1: the entrant firm E decides on his location d_E and price p_E

Stage 2: the incumbent firm I chooses his price p_I and reacts to entry

The following general assumptions precede the analysis:

- Firm E does not incur any entry costs ($f = 0$).

- Consumer's transportation costs increase linearly with a constant factor t when traveling one unit distance.

- The exogenous variable Z represents a measure for centrality in the market. The player seizing the market center shall be rewarded by a shift in the demand for his product. Consequently, all Z consumers from the additional spoke purchase at the seller that captures the indifferent consumer at $x = \frac{1}{2}$. This implies that no transportation costs incur when traveling on the spoke. Without loss of generality Z shall be restricted to $0 < Z \leq \frac{1}{2}$.

- Firm I's location d_I shall be exogenously given, e.g. resulting from a former market entry or a change in the regional market structure (length of the city). Due to the symmetry of the problem the range of d_I lies within $[0, \frac{1}{2}]$.

- In the asymmetric case, i.e., $d_I < \frac{1}{2}$ the entrant principally has two options to locate in the market. He could locate on the opposite end of the city with respect to the incumbent's location. This implies that for $d_E < \frac{1}{2}$ the center lies between the contenders (as depicted in figure 1). Alternatively, the entrant could choose his location such that the incumbent lies between the entrant's mill and the city center. To reduce complexity and study the effect central-ity has on the strategic decision of the incumbent firm we restrict firm E's hinterland to be on the opposite spoke of firm I's location and set $0 \leq d_E < \frac{1}{2}$.

- Consumers' behavior is solely characterized by maximizing their utility, likewise firms' behavior is determined by profit maximization. The endogenous variables of the model are firms prices p_E, p_I and the entry location d_E. These represent the strategic variables of the game.

We solve the game by backward induction.

Figure 3.1: Illustration of the Hotelling model with a centrality bonus Z
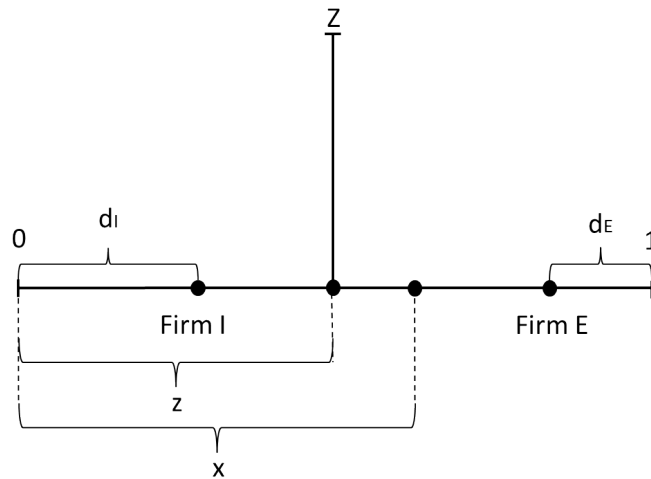
Comment: At given locations firm I and firm E earn a hinterland of size d_I and d_E. In the market center at $z = \frac{1}{2}$ a spoke of length Z is connected with the linear city. In the given setting firm I captures the market center and the indifferent consumer locates on the right hand side of the center at $x > z$.

3.2.1 Demand and Profit Functions

The starting point of the analysis is consumers' utility indifference condition from which the position x of an indifferent consumer is derived. Since we apply the sim-plifying assumption that consumers on the spoke bear no transportation costs when traveling to the center at $z = \frac{1}{2}$ the expression for x is the same as in the classi-

cal Hotelling setting with linear transportation costs. Under a sufficiently positive surplus $\bar{s}$ the familiar relation obtains:

$$u_I = u_E \tag{3.1}$$

$$\bar{s} - p_I - t(x - d_I) = \bar{s} - p_E - t(1 - d_E - x) \tag{3.2}$$

The indifferent consumer is attracted by comparatively lower prices and reacts proportionately to a change in firms' locations:

$$x = \frac{p_E - p_I}{2t} + \frac{1 - d_E + d_I}{2} \tag{3.3}$$

Based on $x(p_I, p_E, d_I, d_E)$ firms' demand and profits are contingent upon the relation of prices p_I, p_E, and locations d_I, d_E, e.g. for the incumbent I four different cases have to be distinguished:

$$q_I = \begin{cases} 1 + Z & p_I \leq p_E - t(1 - d_I - d_E) \\ x + Z & p_I < p_E + t(d_I - d_E), x > \frac{1}{2} \\ x & p_I > p_E + t(d_I - d_E), x < \frac{1}{2} \\ 0 & p_I > p_E + t(1 - d_I - d_E) \end{cases} \tag{3.4}$$

In the first case the incumbent undercuts the entrant and earns the whole market $1 + Z$. We refer to this as the deterrence strategy and denote the respective price as $p_I^{Det} = p_E - t(1 - d_I - d_E)$. In the second case the incumbent accommodates entry and earns the center Z by setting prices below the threshold of $p_E + t(d_I - d_E)$. In particular two options exist, firm I could either react modestly and choose to set a price such that his profit function is maximized, i.e. the first order condition $\frac{\partial \Pi_I(p_I, q_I = x(p_I, p_E, d_I, d_E) + Z)}{\partial p_I} = 0$ is fulfilled. Alternatively, he could react to an aggressive entry behavior and defend his claim for the center Z by setting a price that prohibits the entrant from taking the center. We refer to the first of these options as the accommodation-Z strategy, the second option as the deterrence-Z strategy, and denote the respective prices as $p_I^{AccZ} = \frac{1}{2}(p_E + t(1 - d_E + d_I + 2Z))$ and $p_I^{DetZ} = p_E + t(d_I - d_E)$. The third case refers to the situation where firm I accommodates entry but loses the center Z which occurs for prices above the boundary $p_E + t(d_I - d_E)$. The best price the incumbent could set in this situation is derived from the first order condition neglecting the centrality bonus Z. We refer to this as the accommodation strategy and denote the respective price as $p_I^{Acc} = \frac{1}{2}(p_E + t(1 - d_E + d_I))$. Finally, the incumbent may apply a defensive strategy avoiding being undercut and charge a price only to defend his hinterland d_I. We refer to this as the deference

strategy and denote the respective price as $p_I^{Def} = p_E + t(1 - d_I - d_E)$.
To sum up, the following profit function for firm I obtains:

$$\Pi_I = \begin{cases} p_I^{Det}(1 + Z) & p_I \leq p_I^{Det} \\ p_I^{AccZ}(x + Z) & p_I^{Det} < p_I < p_I^{DetZ}, x > \frac{1}{2} \\ p_I^{DetZ}(\frac{1}{2} + Z) & p_I = p_I^{DetZ} - \epsilon, \epsilon \to 0 \\ p_I^{Acc}x & p_I^{DetZ} < p_I \leq p_I^{Def}, x < \frac{1}{2} \\ 0 & p_I^{Def} < p_I \end{cases} \tag{3.5}$$

3.3 Incumbent's Reaction Functions

3.3.1 The Price Reaction Function

The objective in the next subsections is to to examine the structure of firm I's pricing behavior with respect to the entry price p_E and the entry location d_E. The analysis provides an understanding of firm I's decision in the second stage of the game and makes use of the predefined five strategies p_I^{Det}, p_I^{AccZ}, p_I^{DetZ}, p_I^{Acc} and p_I^{Def}.

Lemma 1: Based on a comparative analysis of the profit functions and provided that $d_E^\triangleleft < d_E \leq d_E^\triangleright$ the incumbent firm prefers to charge his prices p_I with respect to firm E's prices p_E according to:

(I) for $d_I \leq d_I^\triangleleft$

$$p_I = \begin{cases} p_I^{Det} = p_E - t(1 - d_I - d_E) & p_E > \overline{p}_E \\ p_I^{AccZ} = \frac{1}{2}(p_E + t - td_E + td_I + 2tZ) & \hat{p}_E < p_E < \overline{p}_E \\ p_I^{DetZ} = p_E + t(d_I - d_E) & p_E^\dagger < p_E < \hat{p}_E \\ p_I^{Acc} = \frac{1}{2}(p_E + t(1 - d_E + d_I)) & \tilde{p}_E < p_E < p_E^\dagger \\ p_I^{Def} = p_E + t(1 - d_I - d_E) & p_E < \tilde{p}_E \end{cases} \tag{3.6}$$

(II) for $d_I^\triangleleft < d_I < d_I^\triangleright$

$$p_I = \begin{cases} p_I^{Det} = p_E - t(1 - d_I - d_E) & p_E > \overline{p}_E \\ p_I^{AccZ} = \frac{1}{2}(p_E + t - td_E + td_I + 2tZ) & \hat{p}_E < p_E < \overline{p}_E \\ p_I^{DetZ} = p_E + t(d_I - d_E) & \check{p}_E < p_E < \hat{p}_E \\ p_I^{Def} = p_E + t(1 - d_I - d_E) & p_E < \check{p}_E \end{cases} \tag{3.7}$$

(III) for $d_I^\triangleright < d_I$

$$p_I = \begin{cases} p_I^{Det} = p_E - t(1 - d_I - d_E) & p_E > \overline{p}_E \\ p_I^{AccZ} = \frac{1}{2}\left(p_E + t - td_E + td_I + 2tZ\right) & \hat{p}_E < p_E < \overline{p}_E \\ p_I^{DetZ} = p_E + t(d_I - d_E) & p_E < \hat{p}_E \end{cases} \qquad (3.8)$$

with:
$\overline{p}_E = t(3 + d_E - d_I + 2Z - 4\sqrt{d_E(1 + Z)})$
$\hat{p}_E = t(1 + d_E - d_I + 2Z)$
$p_E^\dagger = t(1 + d_E - d_I + 4Z - 2\sqrt{2Z(1 + 2Z)})$
$\tilde{p}_E = t(3d_I + d_E - 1)$
$\check{p}_E = t\left(d_E + d_I\left(\frac{1 - 2d_I - 2Z}{1 - 2d_I + 2Z}\right)\right)$
$d_E^\triangleright = \frac{1}{4(1+Z)}$
$d_E^\triangleleft = 3\sqrt{Z(\frac{1}{2} + Z)} - \frac{1}{2}(1 + 6Z)$
$d_I^\triangleleft = \frac{1}{2} + Z - \sqrt{Z(\frac{1}{2} + Z)}$
$d_I^\triangleright = \frac{1}{4}(1 - 2d_E - 2Z + \sqrt{4d_E^2 + (1 - 2Z)^2 + 4d_E(1 + 6Z)})$

Lemma 2: The incumbent firm maximizes his profits for the accommodation-Z strategy Π_I^{AccZ} in the market region $x > \frac{1}{2}$ by charging p_I^{AccZ} only if $d_E < d_E^\triangleright$, and maximizes his profits for the accommodation strategy in the market region $x < \frac{1}{2}$ by charging p_I^{Acc} only if $d_I < d_I^\triangleleft$.

Proofs: See the appendix.

The first thing to note is that the incumbent's pricing strategies follow a distinct order dependent on the entrant's prices p_E. For high values of p_E the incumbent kicks the entrant out of the marked by shifting the market boundary to his opponent's mill setting p_I^{Det}. For a decreasing p_E maximizing profits over his prices and charging p_I^{AccZ} becomes profitable for firm I while still being in charge of the center Z ($x > \frac{1}{2}$). Next, the accommodation-Z strategy is dominated by the deterrence-Z strategy implying that the incumbent's best choice for a further decreasing p_E is to fence off his opponent to take the center and push the market boundary onto firm E's spoke by charging p_I^{DetZ}. Under specific locational settings and a low pricing strategy of the entrant firm the incumbent accepts the loss of the center Z ($x < \frac{1}{2}$) and chooses to set the profit maximizing price p_I^{Acc}. Finally, for an aggressive pricing behavior of firm E the incumbent defends his position in the market by applying the deference strategy and setting p_I^{Def}.

Lemma 1 demonstrates that firm I's pricing decision is a complex function of the location parameters d_I, d_E, and the centrality bonus Z. This relation is best highlighted by comparing our results with the findings in Anderson (1987) where a respective price reaction function is analyzed for the Hotelling case without the center Z. In Anderson (1987) the existence of the accommodation strategy is dependent on a sole relation between the locations. For market settings where the two players locate fairly close, i.e., $(1 - d_I)^2 < d_E$, the accommodation strategy is dominated by the deterrence and deference strategy, and thus, is not a feasible option for the incumbent firm. By contrast if the intermediate market becomes large enough, i.e., $(1 - d_I)^2 > d_E$, firm I optimizes his price p_I according to his accommodation profit function in a given price interval for p_E. For high p_E deterrence dominates accommodation and for low p_E deference dominates accommodation, but for an intermediate pricing behavior the accommodation case proves feasible.

The existence of the centrality bonus Z increases the strategy options for the incumbent and divides the market into two sections $x > \frac{1}{2}$ and $x < \frac{1}{2}$. In particular, Z causes a shift in firm I's accommodation profit function splitting it into a part for the area $x > \frac{1}{2}$ where Z increases accommodation profits (accommodation-Z strategy) and a part for the area $x < \frac{1}{2}$ where I does not occupy the center, i.e., $Z = 0$ (accommodation strategy). A comparison of firm I's profit functions shows that under consideration of all five strategy options no explicit relation between d_I and d_E to specify the viability for an accommodating pricing behavior of firm I exists (see proof 1). Instead the analysis demonstrates that a set of relations between d_E and Z for the part $x > \frac{1}{2}$, and between d_I and Z for the part $x < \frac{1}{2}$ obtains.[2] As regards d_E and Z intuition suggests that for an increase in Z the range for the two players to locate in the market becomes smaller such that the accommodation-Z strategy for firm I yields the comparatively highest profits which is highlighted by the reciprocal relation $d_E^{\triangleright} = \frac{1}{4(1+Z)}$. In line with this, a high value for Z corresponds with a smaller space in which the entrant firm could locate such that the incumbent would not promptly change from an undercutting behavior to the deterrence-Z strategy to defend the center. Put differently, if the entrant comes relatively close to the center $(d_E > d_E^{\triangleright})$ the accommodation-Z strategy is always dominated either by the deterrence strategy or the deterrence-Z strategy. Since the pricing functions p_I^{Det} and p_I^{DetZ} do not intersect the switchover point is described by a discontinuity given by the intersection of the respective profit functions at $p_E^{\times} = t(2 + 2Z - d_E(3 + 4Z) - d_I)$. (cf. figure 3.5 and 3.3) If the entrant remains below the critical distance $(d_E < d_E^{\triangleright})$ the incumbent plays the accommodation-Z strategy maximizing his profits by charging p_I^{AccZ} for prices p_E in $[\hat{p}_E, \bar{p}_E]$. Then, his pricing behavior is characterized by a discontinuity at $\bar{p}_E$. (cf. figure 3.4 and 3.2)

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[2]In fact the existence of the accommodation-Z and the accommodation strategy in terms of the thresholds $d_E^{\triangleright}$ and $d_I^{\triangleleft}$ is not dependent on respective locations d_I and d_E.

This finding is supplemented by the behavior of d_I contingent upon Z. Graphically, an increase in Z shifts deterrence profits, accommodation-Z profits and deterrence-Z profits upwards whereas the accommodation and deference profits are not affected. In particular, if Z is small and firm I loses the center the accommodation strategy reveals the comparatively highest profits only for locations below the threshold $d_I^{\triangleleft}$. Now, if Z increases the range for the incumbent to apply the accommodation strategy shrinks and if Z exceeds $\frac{(1-2d_I)^2}{8d_I-2}$ (which is equivalent to $d_I \geq d_I^{\triangleleft}$) the incumbent never accommodates entry when $x < \frac{1}{2}$ but rather plays the deterrence-Z strategy. Analogously, the profitability of the deference strategy is restricted to moderate values of Z or locations in the interval $[d_I^{\triangleleft}, d_I^{\triangleright}]$ respectively. This implies that given fixed values for d_I and d_E firm I has a dominant strategy to defend the center and charge p_I^{DetZ} for $Z \geq \frac{(d_E+d_I)(2d_I-1)}{2(d_E-d_I)}$ (equivalent to $d_I \geq d_I^{\triangleright}$). Put differently, given a fixed Z the incumbent prefers the deterrence-Z strategy to the deference strategy as well as the accommodation strategy for low p_E when having a location close enough to the center, i.e. when $d_I > d_I^{\triangleright}$.

Table 3.1: Model parameters for the extended Hotelling model

Z	Location parameters				Intermediate markets	
	$d_I^{\triangleleft}$	$d_E^{\triangleright}$	$d_E^{\triangleleft}$	$d_E^{\triangleright} - d_E^{\triangleleft}$	$1 - \frac{1}{2} - d_E^{\triangleright}$	$1 - d_I^{\triangleleft} - d_E^{\triangleright}$
0	0.5	0.25	−0.5	0.25	0.25	0.25
0.05	0.384	0.238	−0.153	0.238	0.262	0.387
0.1	0.355	0.227	−0.065	0.227	0.273	0.418
0.15	0.338	0.217	−0.013	0.217	0.283	0.445
0.2	0.326	0.208	0.022	0.186	0.292	0.466
0.25	0.317	0.2	0.049	0.151	0.3	0.483
0.3	0.310	0.192	0.070	0.123	0.308	0.489
0.35	0.305	0.185	0.086	0.099	0.315	0.510
0.4	0.3	0.179	0.1	0.079	0.321	0.521
0.45	0.296	0.172	0.112	0.061	0.328	0.531
0.5	0.293	0.167	0.121	0.045	0.333	0.540

An illustration of the relationships between d_I, d_E and Z is given in table 3.1 where the location parameters from lemma 1 and respective intermediate markets $(1 - d_I - d_E)$ are depicted. For equidistant and increasing values of Z the critical values for $d_I^{\triangleleft}$, $d_E^{\triangleright}$, and $d_E^{\triangleleft}$ are calculated. Recall that these are linked to different strategy options for firm I, namely $d_I^{\triangleleft}$ expresses the viability of the accommodation strategy (firm I loses Z, $x < \frac{1}{2}$), $d_E^{\triangleright}$ indicates the viability of the accommodation-Z strategy (firm I gains Z, $x > \frac{1}{2}$), and $d_E^{\triangleleft}$ marks a lower limit for the applicability of the deference strategy[3].

As expected an increase in Z leads to a decrease in the location parameters $d_I^{\triangleleft}$ and $d_E^{\triangleright}$ whereas $d_E^{\triangleleft}$ rises. This is also demonstrated by the size of the intermediate markets. Specifically, we can compare the intermediate market in the extended Hotelling

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[3]The negative values imply that for $Z < \frac{1}{6}$ the deference strategy is feasible for all $d_E < \frac{1}{2}$ (cf. proof 1).

setting for the case where the incumbent locates at the market center $(d_I = \frac{1}{2})$ with the intermediate market for the same fixed d_I in the classical Hotelling model according to the relation $d_E = (1 - d_I)^2$ (cf. Anderson (1987)). Clearly, the existence of the center as well as an increase in Z implies a higher required distance between the players for accommodation to still be a strategic option for the incumbent firm.

Figure 3.2: Illustration of the incumbent's profit functions over p_I for $\overline{p}_E \approx 1.86$
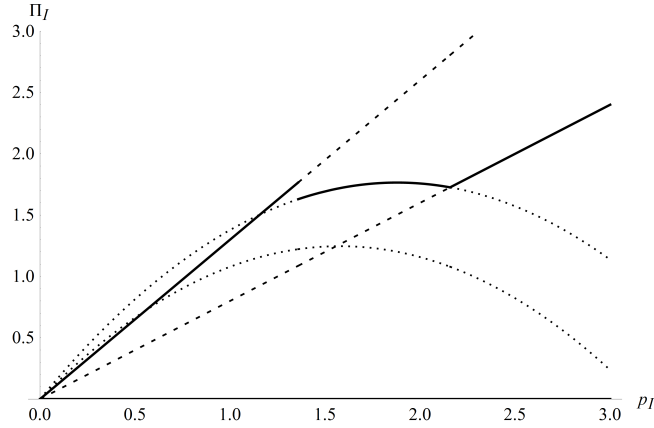
Comment: The solid lines depict the viable profit ranges. For small p_I the incumbent chooses the deterrence strategy and charges p_I^{Det}. For $p_I > \overline{p}_E - t(1 - d_I - d_E)$ he changes to the accommodation-Z strategy and rides the curve until the profit maximizing price $p_I^{AccZ} \approx 1.88$ is realized. The maximum is a feasible solution since firm E's location lies below the boundary $d_E < d_E^{\diamond} \approx 0.19$. For prices $p_I > \overline{p}_E + t(d_I - d_E)$ the deterrence-Z strategy is preferred. The parabola below represents the accommodation profits for $Z = 0$. Parameter values are $d_I = 0.4$, $d_E = 0.1$, $t = 1$, and $Z = 0.3$.

Figure 3.3: Illustration of firm I's profits over p_I for $\overline{p}_E \approx 1.00$
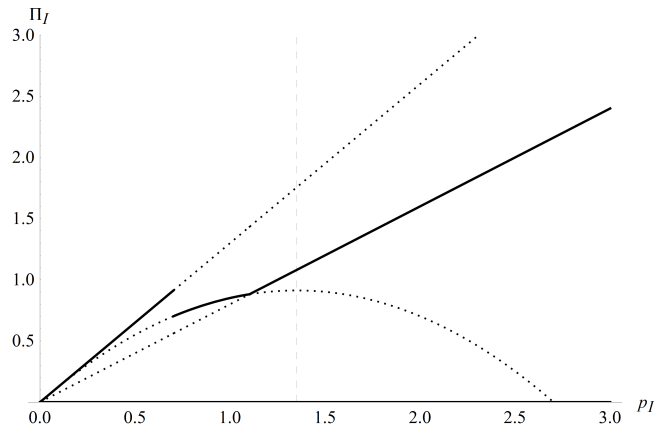
Comment: The entrant's location lies above the threshold $d_E > d_E^{\diamond} \approx 0.19$. As a result the profit maximizing price $p_I^{AccZ} \approx 1.35$ depicted by the vertical dashed line exceeds the boundary value $\overline{p}_E + t(d_I - d_E) \approx 1.10$ and the accommodation-Z strategy is not viable. Parameter values are $d_I = 0.4$, $d_E = 0.3$, $t = 1$, and $Z = 0.3$.

Figure 3.4: Illustration of the incumbent's profit and price functions over p_E

Incumbent's profit functions

Incumbent's price functions
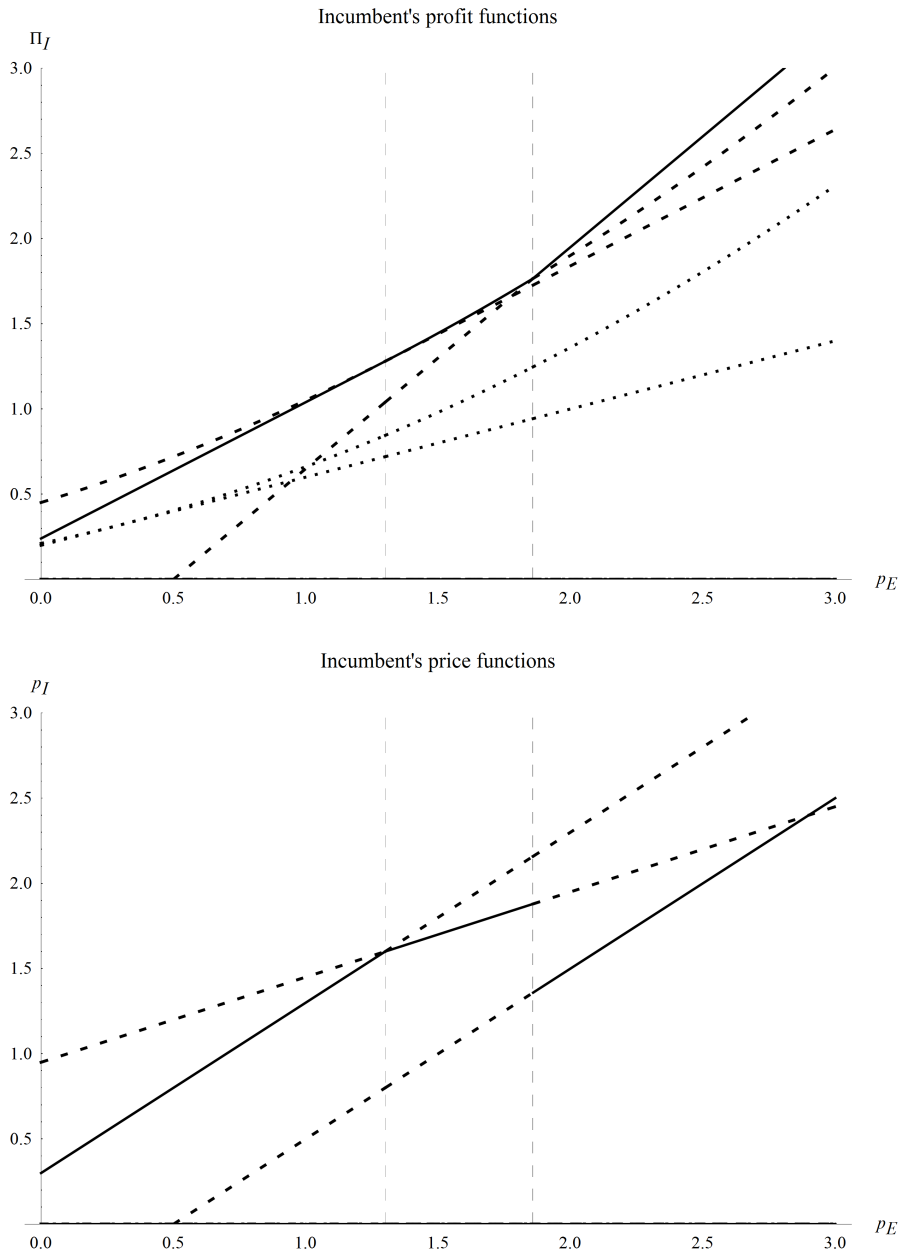
Comment: The solid lines depict the viable profit and price ranges. For high values of p_E the deterrence strategy yields the highest profits. At $\overline{p}_E \approx 1.86$ (first vertical dashed line) the incumbent changes to the accommodation-Z strategy, the change in prices is described by a discontinuity. At $\hat{p}_E \approx 1.3$ (second vertical dashed line) accommodation-Z profits intersect with deterrence-Z profits, the transition is described by a kink in the profit and pricing curves. Thus, for decreasing p_E charging p_I^{DetZ} proves to be the most profitable pricing strategy. Indeed the deterrence-Z strategy dominates the accommodation and deference strategy (small dashed profit curves) over the whole price range since $d_I > d_I^{\triangleright} \approx 0.34$ and $d_E > d_E^{\triangleleft} \approx 0.07$. This scenario represents part (III) of the reaction function in lemma 1. Parameter values are $d_I = 0.4$, $d_E = 0.1$, $t = 1$, and $Z = 0.3$.

Figure 3.5: Illustration of the incumbent's profits and price functions for the case $d_E > d_E^{\triangleright}$

Incumbent's profit functions

Incumbent's price functions
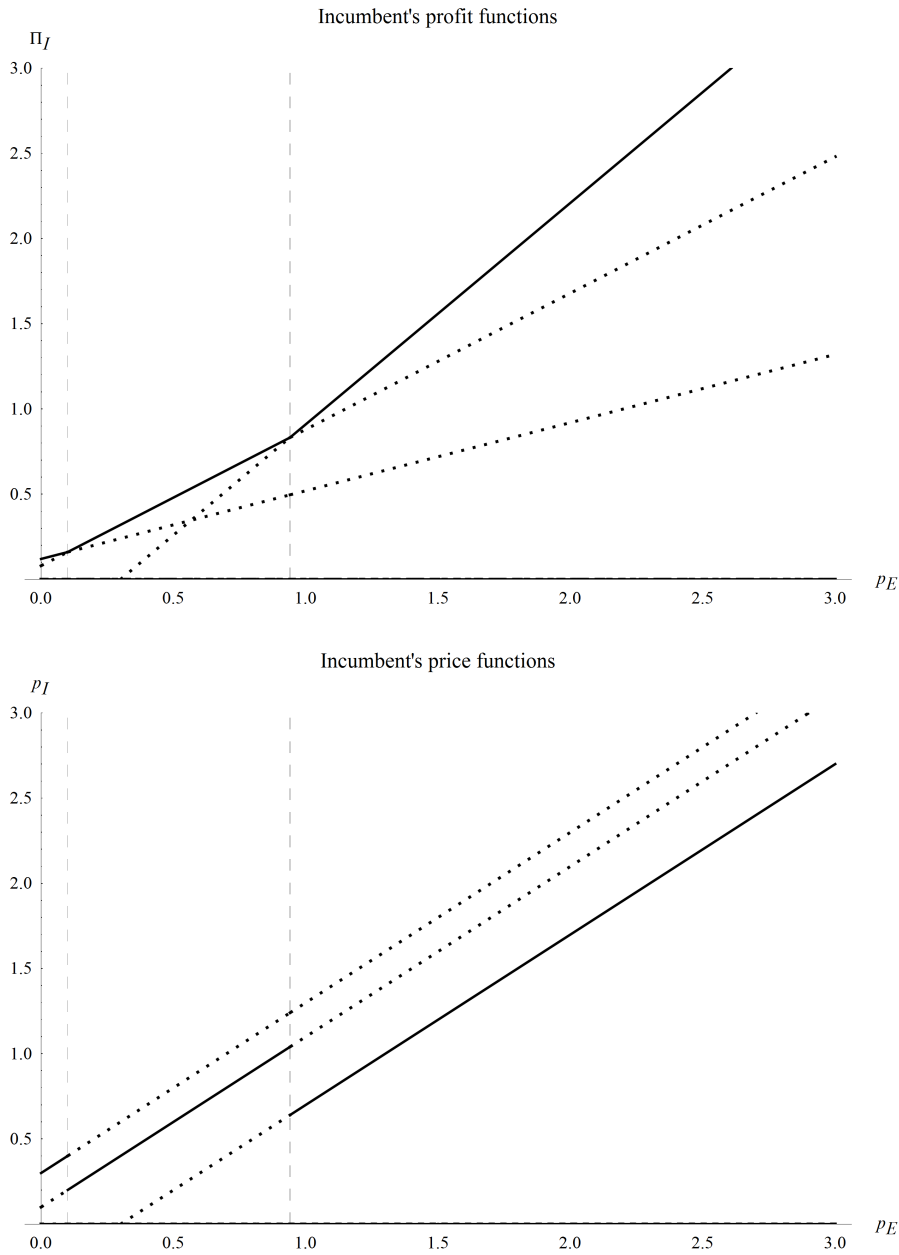
Comment: The entrant locates close to the center with $d_E > d_E^{\triangleright} \approx 0.19$. As a result the accommodation-Z strategy is not viable and the incumbent directly changes from the deterrence strategy to the deterrence-Z strategy. The intersection is given at $p_E^{\times} = t(2+2Z-d_E(3+4Z)-d_I) \approx 0.94$ (first vertical dashed line). Additionally, the incumbent switches from p_I^{DetZ} to the deference strategy provided that $p_E < \check{p}_E \approx 0.1$ (second vertical dashed line). The transitions in the profit functions are described by a kink whereas the price changes are characterized by two discontinuities. This scenario refers to part (II) of the reaction function in lemma 1 with the noticeable difference that charging p_I^{AccZ} is not a viable option for firm I. Parameter values are $d_I = 0.4$, $d_E = 0.3$, $t = 1$, and $Z = 0.3$.

3.3.2 The Location Reaction Function

We set up the incumbent's best price response as a function of the entrant's location choice for a defined price range. Analogous to the proceeding in lemma 1 a set of expressions of the form $p_I(d_E)$ obtains. Since we conduct a comparative analysis of firm I's profits, as expected, analogous results with respect to the price reaction function are derived. Additionally, for the low price range a particularity in the location reaction is found. In the following, the reaction function is dissected into a part corresponding to a remote location of the incumbent firm in lemma 3 and a part corresponding to a location close to the center of the city in lemma 4.

Lemma 3: Provided that $d_I \leq d_I^{\triangleleft}$ and for the price intervals $p_E^{\triangle} > p_E > p_E^{\triangledown\dagger}$ and $p_E^{\triangle} > p_E > p_E^{\triangleright}$ firm I's location reaction function comprises of:

(I) for $p_E^{\triangle} \geq p_E > p_E^{\triangleleft}$

$$p_I = \begin{cases} p_I^{AccZ} = \frac{1}{2}\left(p_E + t - td_E + td_I + 2tZ\right) & 0 < d_E < \overline{d}_E \\ p_I^{Det} = p_E - t(1 - d_I - d_E) & \overline{d}_E < d_E \end{cases} \qquad (3.9)$$

(II.a) for $Z > \zeta$ and $p_E^{\triangleleft} \geq p_E > p_E^{\triangleright}$, and
(II.b) for $Z < \zeta$ and $p_E^{\triangleleft} \geq p_E > p_E^{\triangledown\dagger}$

$$p_I = \begin{cases} p_I^{AccZ} = \frac{1}{2}\left(p_E + t - td_E + td_I + 2tZ\right) & 0 < d_E < \hat{d}_E \\ p_I^{DetZ} = p_E + t(d_I - d_E) & \hat{d}_E < d_E < d_E^{\times} \\ p_I^{Det} = p_E - t(1 - d_I - d_E) & d_E^{\times} < d_E \end{cases} \qquad (3.10)$$

(III.a) for $Z > \zeta$ and $p_E^{\triangleright} \geq p_E > p_E^{\triangledown\dagger}$

$$p_I = \begin{cases} p_I^{DetZ} = p_E + t(d_I - d_E) & 0 < d_E < d_E^{\times} \\ p_I^{Det} = p_E - t(1 - d_I - d_E) & d_E^{\times} < d_E \end{cases} \qquad (3.11)$$

(III.b) for $Z < \zeta$ and $p_E^{\triangledown\dagger} \geq p_E > p_E^{\triangleright}$

$$p_I = \begin{cases} p_I^{AccZ} = \frac{1}{2}\left(p_E + t - td_E + td_I + 2tZ\right) & 0 < d_E < \hat{d}_E \\ p_I^{DetZ} = p_E + t(d_I - d_E) & \hat{d}_E < d_E < d_E^{\dagger} \\ p_I^{Acc} = \frac{1}{2}(p_E + t(1 - d_E + d_I)) & d_E^{\dagger} < d_E < d_E^{\ddagger} \\ p_I^{Det} = p_E - t(1 - d_I - d_E) & d_E^{\ddagger} < d_E \end{cases} \qquad (3.12)$$

Lemma 4: Provided that $d_I \geq d_I^\triangleleft$ and for the price intervals $p_E^\triangle > p_E > p_E^{\nabla\ddagger}$ and $p_E^\triangle > p_E > p_E^\triangleright$ firm I's location reaction function comprises of:

(I) for $p_E^\triangle \geq p_E > p_E^\triangleleft$

$$p_I = \begin{cases} p_I^{AccZ} = \frac{1}{2}\left(p_E + t - td_E + td_I + 2tZ\right) & 0 < d_E < \bar{d}_E \\ p_I^{Det} = p_E - t(1 - d_I - d_E) & \bar{d}_E < d_E \end{cases} \tag{3.13}$$

$(II.a)$ for $Z > \zeta$ and $p_E^\triangleleft \geq p_E > p_E^\triangleright$, and
$(II.b)$ for $Z < \zeta$ and $p_E^\triangleleft \geq p_E > p_E^{\nabla\ddagger}$

$$p_I = \begin{cases} p_I^{AccZ} = \frac{1}{2}\left(p_E + t - td_E + td_I + 2tZ\right) & 0 < d_E < \hat{d}_E \\ p_I^{DetZ} = p_E + t(d_I - d_E) & \hat{d}_E < d_E < d_E^\times \\ p_I^{Det} = p_E - t(1 - d_I - d_E) & d_E^\times < d_E \end{cases} \tag{3.14}$$

$(III.a)$ for $Z > \zeta$ and $p_E^\triangleright \geq p_E > p_E^{\nabla\ddagger}$

$$p_I = \begin{cases} p_I^{DetZ} = p_E + t(d_I - d_E) & 0 < d_E < d_E^\times \\ p_I^{Det} = p_E - t(1 - d_I - d_E) & d_E^\times < d_E \end{cases} \tag{3.15}$$

$(III.b)$ for $Z < \zeta$ and $p_E^{\nabla\ddagger} \geq p_E > p_E^\triangleright$

$$p_I = \begin{cases} p_I^{AccZ} = \frac{1}{2}\left(p_E + t - td_E + td_I + 2tZ\right) & 0 < d_E < \hat{d}_E \\ p_I^{DetZ} = p_E + t(d_I - d_E) & \hat{d}_E < d_E < \check{d}_E^\dagger \\ p_I^{Def} = p_E + t(1 - d_I - d_E) & \check{d}_E^\dagger < d_E < \check{d}_E^\ddagger \\ p_I^{Det} = p_E - t(1 - d_I - d_E) & \check{d}_E^\ddagger < d_E \end{cases} \tag{3.16}$$

with:

$d_I^\triangleleft = \frac{1}{2} + Z - \sqrt{Z(\frac{1}{2} + Z)}$

$\bar{d}_E = 5 + d_I + \frac{1}{t}p_E + 6Z - 4\sqrt{\left(\frac{p_E}{t} + 1 + d_I + 2Z\right)(1 + Z)}$

$\hat{d}_E = \frac{1}{t}p_E + d_I - 1 - 2Z$

$d_E^\times = \frac{2(1+Z) - \frac{1}{t}p_E - d_I}{3 + 4Z}$

$d_E^\dagger = \frac{1}{t}p_E + (d_I - 1 - 4Z) + 2\sqrt{2}\sqrt{Z(1 + 2Z)}$

$d_E^\ddagger = 5 + d_I + \frac{1}{t}p_E + 4Z - 4\sqrt{\left(\frac{p_E}{t} + 1 + d_I + Z\right)(1 + Z)}$

$\check{d}_E^\dagger = \frac{1}{t}p_E - d_I\left(\frac{1 - 2d_I - 2Z}{1 - 2d_I + 2Z}\right)$

$\check{d}_E^\ddagger = 1 - d_I - \frac{p_E(1 - d_I + Z)}{t(1 + d_I + Z)}$

$p_E^\triangle = t(3 - d_I + 2Z)$

$p_E^\triangleleft = t\left(\frac{5 + 12Z + 8Z^2}{4(1+Z)} - d_I\right)$

$p_E^{\triangleright} = t(1 - d_I + 2Z)$

$p_E^{\triangledown\dagger} = t\dfrac{\left(5+18Z+16Z^2-4d_I(1+Z)-2\sqrt{2}\sqrt{Z(1+2Z)}(3+4Z)\right)}{4(1+Z)}$

$p_E^{\triangledown\ddagger} = t\dfrac{(1-2d_I)(1+d_I+Z)(1+2Z)}{2(1-2d_I+2Z)(1+Z)}$

$p_E^{\bowtie} = \frac{1}{2}t(1 - 2d_I)$

$\zeta \approx 0.015503$

Proofs: See the appendix.

Generally, the expressions in lemma 3 and 4 demonstrate that contingent on the price level of p_E different strategy combinations for the incumbent obtain to react to the entrant's choice for his location d_E. For the high price range (part (I)) the accommodation-Z strategy supports market entry, for the modest price range (parts (II.a) and (II.b)) the accommodation-Z strategy and the deterrence-Z strategy constitute the entry accommodating strategy set, and for the low price range (parts (III.a) and (III.b)) entry occurs either when the deterrence-Z strategy or a strategy triple $(p_I^{AccZ}, p_I^{DetZ}, p_I^{Acc})$ and $(p_I^{AccZ}, p_I^{DetZ}, p_I^{Def})$ is used. Furthermore, in the examined price ranges deterrence is always part of firm I's strategy set.

The location reaction function reveals the pattern that it is profitable for the incumbent to accept entry when the entry location lies below a threshold value and that entry will be deterred when the entrant locates too far from his edge of the city respectively. Particularly, for the parts (I), (II) and (III.a) the critical entry locations where the incumbent switches to an entry deterring behavior are given by the expressions $\overline{d}_E$ and $d_E^{\times}$. These represent firm I's indifference conditions between the deterrence strategy and the accommodation-Z strategy as well as with the deterrence-Z strategy. We see that $\overline{d}_E$ and $d_E^{\times}$ increase and $\hat{d}_E$ decreases with decreasing p_E which determines the range of the accommodation-Z strategy and firm I's profit maximizing behavior when being in charge of the center. Let us depart for instance at the high price interval (part (I): $p_E > p_E^{\triangleleft}$) where firm I optimizes his profits over p_I setting p_I^{AccZ} when the entry location lies below $\overline{d}_E$ and the transition in the pricing behavior is described by a discontinuity. (cf. figure 3.6) Now, in the modest price interval (part (II): $p_E > p_E^{\triangleright}, p_E^{\triangledown\dagger}, p_E^{\triangledown\ddagger}$) the accommodation-Z strategy is only profitable for locations until the threshold of $\hat{d}_E$ since charging p_I^{DetZ} and defending the claim for the center proves to be the best choice for locations exceeding $\hat{d}_E$. If the incumbent charged p_I^{AccZ} for locations $d_E > \hat{d}_E$ he would lose the center Z and thus would not realize accommodation-Z profits. Graphically, the profit function for the deterrence-Z strategy is a tangent to the parabola of the accommodation profit function at $\hat{d}_E$ and the price transition is defined by a kink. (cf. figure 3.7) Thus, for $\hat{d}_E < \overline{d}_E$ the deterrence-Z strategy fills up the space between the accommodation-Z strategy and deterrence, and the switchover point for entry deterrence is determined by $d_E^{\times}$.

Figure 3.6: Illustration of the incumbent's profit and price functions over d_E for part (I) in lemma 3 and 4
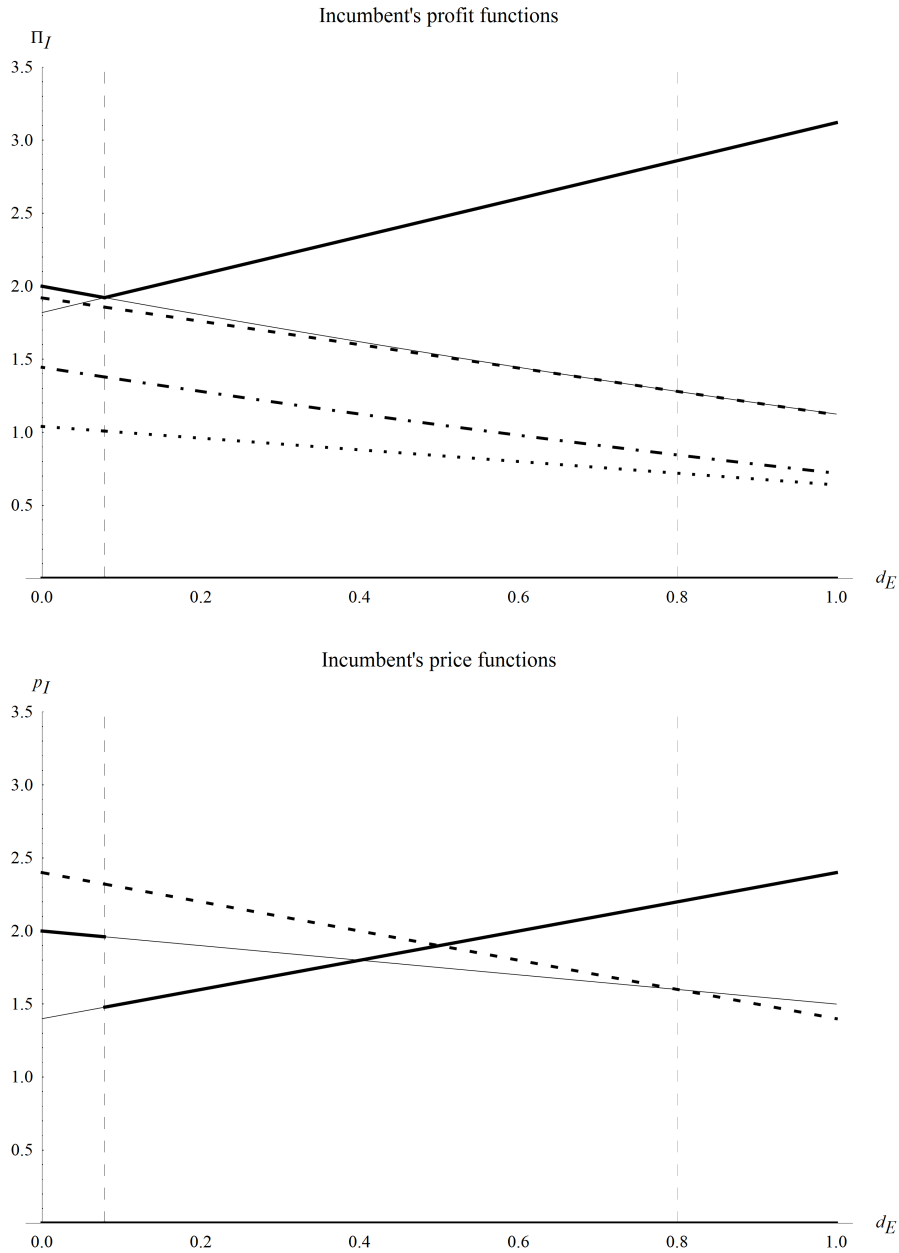
Comment: The marked solid lines depict the viable profit and price ranges. For low values of d_E the accommodation-Z strategy dominates until the profit function intersects with the deterrence profits at $\overline{d}_E \approx 0.079$ (marked by the first vertical dashed line). Thus, for $d_E > \overline{d}_E$ deterrence is the dominant strategy. The transition in the price functions is described by a discontinuity. Additionally, the tangential intersection of the accommodation profits with the profits for the deterrence-Z strategy (dashed profit function) is given at $\hat{d}_E = 0.8$ (marked by the second vertical dashed line). The profits for the accommodation strategy are depicted by the dash-dotted function, and the profits for the deference case by the dotted function. Parameter values are $d_I = 0.4$, $p_E = 2.0$, $t = 1$, and $Z = 0.3$.

Figure 3.7: Illustration of the incumbent's profit and price functions over d_E for part (II) in lemma 3 and 4

Incumbent's profit functions

Incumbent's price functions
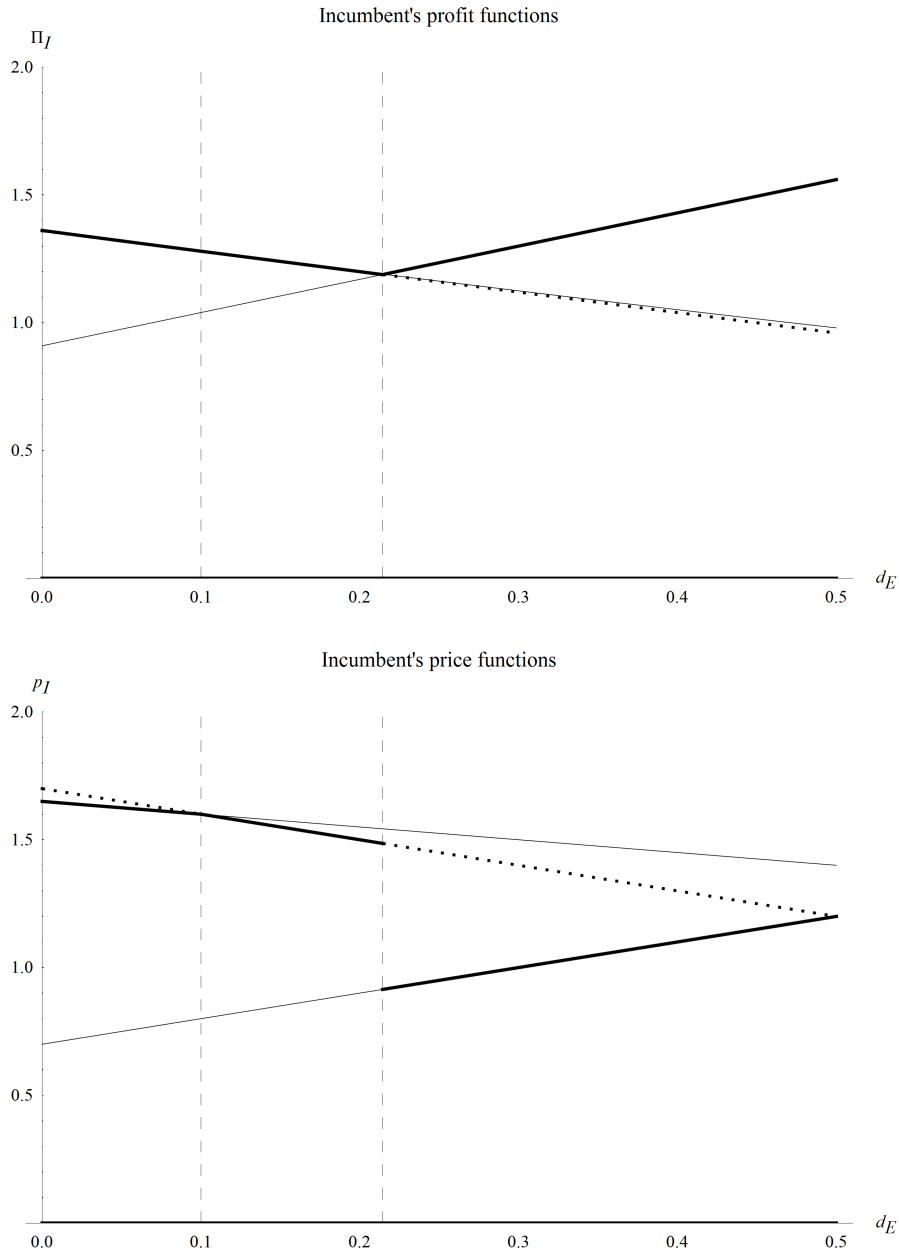
Comment: The marked solid lines depict the viable profit and price ranges. For entry locations $d_E < \hat{d}_E = 0.1$ (first vertical dashed line) the accommodation-Z strategy yields the highest profits and dominates the other strategies (due to the small resolution of the graphs this effect is not visualized). Since $\hat{d}_E < \overline{d}_E \approx 0.215$ the deterrence-Z strategy is preferred for locations between $\hat{d}_E$ and the intersection of the deterrence-Z profits with the deterrence profit function at $d_E^\times \approx 0.214$ (second vertical dashed line). For $d_E > d_E^\times$ deterrence is the dominant strategy. The transition in the pricing behavior from the accommodation-Z strategy to the deterrence-Z strategy is given by a kink, the change between the deterrence-Z strategy and the deterrence strategy is described by a discontinuity. Parameter values are $d_I = 0.4$, $p_E = 1.3$, $t = 1$, and $Z = 0.3$.

The centrality bonus Z has different effects on the location parameters. As argued for the price reaction function, profits for the accommodation and the deference strategy are not affected by a change in Z whereas profits for the deterrence, the deterrence-Z, and the accommodation-Z strategy are shifted upwards and slopes increase when Z rises. Consequently, as illustrated in table 3.2, an increase in Z causes the switchover points $\overline{d}_E$ and $d_E^\times$ to mount. Additionally, an increase in Z implies that the profitability of the accommodation-Z strategy shrinks and causes $\hat{d}_E$ that marks the kink solution to decrease. Thus, for these three location parameters an increase in Z shows the same impact on the location reaction as a decrease in p_E. In numbers, for $p_E = 1$, $Z = 0.2$ and $d_I = 0.4$ the accommodation-Z strategy is not feasible anymore and the strategy set solely consists of p_I^{DetZ} and p_I^{Det} which corresponds to a transition of part $(II.a)$ to part $(III.a)$ in the reaction functions in lemma 4, entry deterrence then occurs at the location $d_E^\times = 0.263$. This reveals an interesting result of the analysis: given the rise of $\overline{d}_E$ and $d_E^\times$ when Z increases and entry prices p_E decrease the incumbent is less inclined to accommodate entry when being in charge of the center $(x > \frac{1}{2})$ as well as to deter entry, rather as intuition might suggest the dominant strategy for moderate entry locations $d_E < d_E^\times$ is to defend the claim for the center and charge p_I^{DetZ}. To turn this argument around since, for instance, $\hat{d}_E > \overline{d}_E$ in the high entry price scenario $(p_E = 2)$ in table 3.2 the accommodation-Z strategy proves to be more profitable for all Z than the deterrence-Z strategy.

Table 3.2: Model parameters of the location reaction function for $d_I = 0.4$

Z	$\overline{d}_E$	$\hat{d}_E$	$d_E^\times$	$\overline{d}_E$	$\hat{d}_E$	$d_E^\times$
	$p_E = 2$			$p_E = 1$		
0	0.024	1.4	-0.133	0.203	0.4	0.200
0.05	0.032	1.3	-0.094	0.219	0.3	0.219
0.1	0.040	1.2	-0.059	0.235	0.2	0.235
0.15	0.049	1.1	-0.028	0.252	0.1	0.250
0.2	0.058	1.0	0.000	0.268	0.0	0.263
0.25	0.068	0.9	0.025	0.284	-0.1	0.275
0.3	0.079	0.8	0.048	0.301	-0.2	0.286
0.35	0.089	0.7	0.068	0.317	-0.3	0.295
0.4	0.101	0.6	0.087	0.334	-0.4	0.304
0.45	0.112	0.5	0.104	0.350	-0.5	0.313
0.5	0.124	0.4	0.120	0.367	-0.6	0.320

Secondly, the centrality bonus Z impacts the structure of the reaction function in the parts (II) and (III) in lemma 3 and 4 where a distinction by two cases determined by the numerical constant ζ has been made. Graphically, if Z shrinks the tangent Π_I^{DetZ} to the parabola Π_I^{AccZ} flattens (increase in $\hat{d}_E$), simultaneously the critical location for a switchover of the deterrence-Z strategy to the accommodation strategy and the deference strategy respectively decreases (decrease in $d_E^\times$) since the differ-

ence between the two parabolas Π_I^{AccZ} and Π_I^{Acc} constantly diminishes. Thus, there exists a threshold for Z given by ζ where the accommodation-Z strategy and the accommodation strategy (in (III.b)) as well as the accommodation-Z strategy and the deference strategy (in ($III.b$)) are part of the same location reaction function. Alternatively, this effect is found in the ambiguous relation of the price $p_E^{\triangleright}$ with $p_E^{\triangledown\dagger}$ and $p_E^{\triangledown\ddagger}$ respectively that mark the bounds for respective pricing strategies to exist. Furthermore, lemma 3 and 4 illustrate for the low price range ((III.b) and ($III.b$)) that firm I's pricing behavior hinges upon its location d_I. In particularly, for $Z < \zeta$ the accommodation strategy is part of the preferred strategy set for locations that lie in a remote part of the city, i.e. $d_I \leq d_I^{\triangleleft}$, and the deference strategy is included in the reaction function when I locates close to the center or $d_I \geq d_I^{\triangleleft}$. The expression for $d_I^{\triangleleft}$ is derived by a comparative profit comparison of the accommodation and deference strategy where the linear profit function for the deference case marks a tangential solution to the accommodation profit function denoted as $\tilde{d}_E$. Clearly, for locations $d_E > \tilde{d}_E$ charging p_I^{Acc} is not profitable since otherwise the incumbent would be undercut at his own mill. The existence of accommodation is only established when the intersection of deterrence-Z profits with accommodation profits ($d_E^{\dagger}$) falls below the tangential intersection $\tilde{d}_E$, for $d_E^{\dagger} > \tilde{d}_E$ accommodation is obsolete and deference exists. Note that the bound for the accommodation strategy $d_I^{\triangleleft}$ is dependent on Z and matches with the corresponding bound in the analysis of the price reaction in lemma 1.

3.4 The Entrant's Optimization Problem

This section provides an analysis of the entrant's decision on the strategic variables price p_E and location d_E. Based upon the price reaction function in subsection 2.2 our approach is to apply the calculus of a classical Stackelberg leader-follower game to derive expressions for p_E. (cp. chapter 3 in Anderson (1987)) These represent solutions for defined sets of d_I, d_E and Z which subsequently enable us to endogenize d_E and derive expressions for an optimal entry set (p_E^*, d_E^*). Additionally, by means of firm I's location reaction function in subsection 2.3. the incumbent's reaction to the suggested price-location combinations for entry is examined.

Lemma 1 demonstrates that the price reaction of the incumbent firm dissects into three different parts. These are parts where the incumbent takes a strong, a moderate and a weak market position in terms of his location d_I with regards to the center. Thus, we analyze the Stackelberg leader-follower game for each of these three parts.[4]

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[4] According to the price reaction function in subsection 2.2 and proof 1 we assume $d_I^{\triangleleft} < d_I^{\triangleright}$ and $d_E > d_E^{\triangleleft}$ respectively (this order strictly holds for all $Z \leq \frac{1}{6}$). For $d_I^{\triangleleft} > d_I^{\triangleright}$ which occurs for small d_I and d_E that is when the players locate far from one another the deference strategy (p_I^{Def}) is

Also, it is evident only to consider the cases where the incumbent does not opt for the deterrence strategy. Initially, we have to identify different combinations of locations d_I and d_E with the possible set of entry prices p_E.

3.4.1 Derivation of location segments

Generally, the entrant's optimization problem is formulated as[5]:

$$\max_{p_E, d_E} \Pi_E(p_E, d_E, p_I(p_E, d_E)) \tag{3.17}$$

Since we scrutinize a two-stage game firm E's profits and thus his optimization problem is contingent upon firm I's price. Now, let us analyze the case with the incumbent setting p_I^{AccZ} and seizing the center $(x > \frac{1}{2})$. Clearly, firm I then gains demand in the amount of $x + Z$ and firm E in the amount of $1 - x$. It follows:

$$\Pi_E(p_I^{AccZ}) =: \Pi_E^{Acc} = -\frac{1}{4t}p_E^2 + \frac{1}{4}\left(3 + d_E - d_I + 2Z\right)p_E \tag{3.18}$$

Applying the first order condition to Π_E^{Acc} yields firm E's best price as a function of the locations of the contenders d_E and d_I as:

$$p_E^{Acc} = \frac{1}{2}t\left(3 + d_E - d_I + 2Z\right) \tag{3.19}$$

The entrant chooses p_E^{Acc} only for a defined set of d_E and d_I. The highest possible price firm E could set is $\overline{p}_E$ where the incumbent is indifferent to the deterrence strategy or the accommodation-Z strategy. Subsequently, the corner solution $\overline{p}_E$ is preferred to p_E^{Acc} only if $\frac{\partial \Pi_E^{Acc}}{\partial p_E} > 0$, i.e. when profits can be increased by a slight price increase. Evaluating $\left.\frac{\partial \Pi_E^{Acc}}{\partial p_E}\right|_{\overline{p}_E} > 0$ reduces to:

$$d_I > 3 + d_E + 2Z - 8\sqrt{d_E(1 + Z)} \tag{3.20}$$

On the contrary, since for any price exceeding $\overline{p}_E$ entry is deterred, the existence of p_E^{Acc} demands $\overline{p}_E > p_E^{Acc}$ which reduces to $d_I < 3 + d_E + 2Z - 8\sqrt{d_E(1 + Z)}$.

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dominated by the accommodation strategy (p_I^{Acc}). The set $d_I \leq d_I^\triangleleft$ and $d_E < d_E^\triangleleft$ implies $\tilde{p}_E < 0$. This is not dealt with in the cause of the price reaction function. For the argument to omit the explicit treatment of this case refer to the profit comparison in subsection 3.2.3.

[5]By symmetry firm E's demand and profit functions are the same as firm I's only interchanging subscripts and the term x with $1 - x$.

The lowest possible price firm E charges under p_I^{AccZ} is given by $\hat{p}_E$ marking the point of indifference for firm I between the accommodation-Z strategy and the deterrence-Z strategy. Since profits increase linearly in p_E the corner solution $\hat{p}_E$ is the best price in this regard. Additionally, p_I^{AccZ} demands $1 - x(p_I^{AccZ}) < \frac{1}{2}$ which reduces to $p_E > \hat{p}_E$. It follows that p_E^{Acc} must fulfill this price restriction from which we derive:

$$d_I > d_E + 2Z - 1 \qquad (3.21)$$

Condition (3.21) is fully in accord with $p_E^{Acc} > t(1 + d_E - d_I + 2Z)$ derived from $p_I^{AccZ} < p_I^{DetZ}$. By contrast, under p_I^{AccZ} a price decrease is profitable for firm E for the location set $d_I < d_E + 2Z - 1$ which follows from $\left.\frac{\partial \Pi_E^{Acc}}{\partial p_E}\right|_{\hat{p}_E} < 0$.

Finally, we have to consider the imposed restrictions from lemma 1. For d_E we utilize the boundaries $d_E^{\triangleright}$ and $d_E^{\triangleleft}$ which both only depend on the value of Z, and for d_I the boundaries $d_I^{\triangleleft}$ and $d_I^{\triangleright}$ have to be accounted for. Note that $d_I^{\triangleright}$ is a function of d_E and Z whereas $d_I^{\triangleleft}$ is only determined by Z. Together with the expressions in (3.20) and (3.21) these four terms define sets of locations that are attached to firm I's price reactions according to his reaction function in lemma 1 and firm E's pricing decision based on Π_E^{Acc}.

Figure 3.8: Illustration of the location segments for $t = 1$
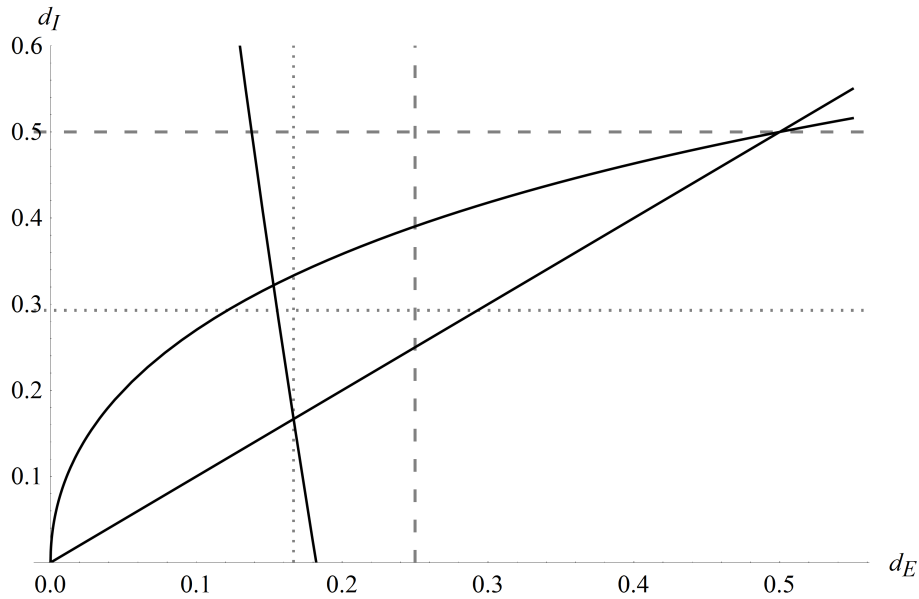
Comment: The downward sloping curve represents the condition for $\bar{p}_E$ in equation (3.20), the straight increasing line depicts the condition for $1 - x < \frac{1}{2}$ in equation (3.21), and the concave increasing curve refers to the term $d_I^{\triangleright}$. All three functions are drawn for $Z = \frac{1}{2}$. The lines that parallel the ordinate and abscissa depict the boundaries $d_E^{\triangleright}$ and $d_I^{\triangleleft}$, the dashed lines correspond to $Z = 0$, and the dotted lines to $Z = \frac{1}{2}$. The boundary $d_E^{\triangleleft}$ is found at the intersection of $d_I^{\triangleright}$ with $d_I^{\triangleleft}$ at a maximum value of $\frac{3}{\sqrt{2}} - 2 \approx 0.121$ for $Z = \frac{1}{2}$.

The findings of the previous section on the comparative statics are illustrated in figure 3.8. For any $d_E < d_E^{\triangleright}$ the incumbent can choose the accommodation-Z strategy, for any $d_I > d_I^{\triangleleft}$ the accommodation strategy is not applicable and both are decreasing with growing Z. This is supplemented with the results of the inequalities in (3.20) and (3.21). Accordingly, the entrant will optimize Π_E^{Acc} and charge p_E^{Acc} for sufficiently high values of d_I. If Z increases the condition in (3.21) increasingly restricts the location set such that the accommodating scenario with $x > \frac{1}{2}$ between firm I and firm E occurs. Particularly, the maximum is reached at $d_I = d_E$ when $Z = \frac{1}{2}$, and for $Z \leq \frac{1}{4}$ any set $d_I, d_E \in [0, \frac{1}{2}]$ implies that firm E maximizes his profits by a price increase. As regards the condition in (3.20) an increase in Z leads to a corresponding upward shift increasing the set of locations for the accommodating scenario. Note also that these two restrictions intersect at the boundary location $d_E^{\triangleright}$ and that $d_I^{\triangleright}$ collapses to the limit $\frac{1}{2}$ when Z converges to zero.

3.4.2 Analysis of the Stackelberg leader-follower game in prices

3.4.2.1 Strong market position of the incumbent

This case assumes that the incumbent locates within a distance of $d_I^{\triangleright}$ with respect to the center and refers to part (III) of the price reaction function in lemma 1. Graphically, this corresponds to the location set bounded by the concave function and the limit $d_I = \frac{1}{2}$ in figure 3.8. Now, consider firm E's pricing options in a descending order.

Let us begin with entry locations $d_E < d_E^{\triangleright}$ such that the accommodation-Z strategy is applicable for firm I. From the preceding subsection we can summarize that for prices exceeding $\overline{p}_E$ the entrant would be undercut by the incumbent. Subsequently, for the location set defined by condition (3.20) and flipping the inequality sign firm E's best response is to charge p_E^{Acc} and the incumbent would respond with charging p_I^{AccZ}. In addition, the location set in (3.21) implies that under p_I^{AccZ} a price decrease in p_E does not yield a profitable outcome for the entrant. Indeed, diverting from the accommodation scenario and charging $\hat{p}_E$ is not profitable for the entrant. Clearly, the entrant could never take the center as the price leader since firm I undercuts him at $x = \frac{1}{2}$ with p_I^{DetZ}. Then the entrant's profits would be $\frac{1}{2}\hat{p}_E$. Comparing $\Pi_E^{Acc}(p_E^{Acc})$ with $\frac{1}{2}\hat{p}_E$ reveals that profits are even only at $d_I = d_E + 2Z - 1$, for all other location sets $\Pi_E^{Acc}(p_E^{Acc})$ yields higher profits. Thus, in sum the increase in demand to $1 - x = \frac{1}{2}$ does not compensate for the price drop to $\hat{p}_E$.

For entry locations $d_E > d_E^{\triangleright}$ the incumbent's reaction to entry consists of charging p_I^{Det} and p_I^{DetZ}. Entry then only occurs under the deterrence-Z strategy with profits $\frac{1}{2}p_E$. The best price for the entrant in this case is given by $p_E^{\times}$ where the incumbent

is indifferent between kicking firm E out of the market and defending the center. For prices above $p_E^\times$ the entrant certainly is undercut, and prices below $p_E^\times$ are not profitable due to the linear increase in entry profits with growing price levels. Repeatedly, the entrant has no chance to capture Z as the first mover in the game.

In sum firm E's pricing behavior is described by the following options when firm I dominates the location setting with $d_I > d_I^\triangleright$:

- choose p_E^{Acc} if $d_I < 3 + d_E + 2Z - 8\sqrt{d_E(1+Z)}$ and $d_E < d_E^\triangleright$

- choose $\overline{p}_E$ if $d_I > 3 + d_E + 2Z - 8\sqrt{d_E(1+Z)}$ and $d_E < d_E^\triangleright$

- choose $p_E^\times$ if $d_E > d_E^\triangleright$

A variation in Z has the expected effects. The location set where p_E^{Acc} represents the best choice decreases whereas the set for $p_E^\times$ increases as Z grows since $d_E^\triangleright$ declines. In figure 3.8 the functions for $d_E^\triangleright$ and the condition in (3.20) are shifted towards the point of origin as well as the condition in (3.21) moves upwards for an increase in Z.

3.4.2.2 Moderate market position of the incumbent

This case assumes that firm I locates in a distance range of $d_I^\triangleleft < d_I < d_I^\triangleright$ which for $Z = \frac{1}{2}$ is graphically illustrated in figure 3.8 by the area below the concave function and the horizontally dotted line. Part (II) in the price reaction function in lemma 1 enhances the previous analysis by the fact that the entrant can possibly charge an aggressive price trying to undercut the incumbent at his own mill ($\check{p}_E > 0$). The incumbent would then react with the deference strategy charging p_I^{Def} to stay in the market.

For $d_E < d_E^\triangleright$ the entrant sets $\overline{p}_E$ for the location set given in (3.20) and charges p_E^{Acc} for the inverse. Also, the center is not accessible for firm E with the location of indifference between charging p_E^{Acc} and $\hat{p}_E$ at $d_I = d_E + 2Z - 1$. But does a further decrease to $\check{p}_E$ now prove to be profitable? Entry profits in this case are given by $(1 - d_I + Z)p_E$ and increase linearly in prices. Thus, the best price firm E can charge when undercutting the incumbent is $\check{p}_E$. It follows that we need to compare the entry profits for the deference case $(1 - d_I + Z)\check{p}_E$ and the accommodation scenario $\Pi_E^{Acc}(p_E^{Acc})$. This reveals that for $d_I, Z \in [0; \frac{1}{2}]$ charging p_E^{Acc} proves to be more profitable than charging $\check{p}_E$ (cf. proof 5a in the appendix). Put differently, the necessary decline in prices and subsequent gain in demand to attack the incumbent when entering the market does not offset profits earned in the scenario of a mutual accommodating behavior of the two contenders.

For $d_E > d_E^\triangleright$ the incumbent reacts to entry by setting p_I^{Det} or p_I^{DetZ} for high entry price levels. Then the entrant would choose $p_E^\times$ as his best price. For low entry price

levels firm I chooses p_I^{DetZ} or p_I^{Def} where firm E's best option is to charge $\check{p}_E$. Thus, a comparison between profits $\frac{1}{2}p_E^{\times}$ and $(1 - d_I + Z)\check{p}_E$ has to be made. It follows that for locations $d_E^{\triangleright} < d_E < d_E^{split}$ the entry price $p_E^{\times}$ is preferred, and for locations $d_E^{split} < d_E < \frac{1}{2}$ the entry price $\check{p}_E$ yields the highest profits. The switchover point is a complex function in d_I and Z given by $d_E^{split} = \frac{2 - 7d_I + 8d_I^2 - 4d_I^3 + 6Z - 4d_I Z + 4Z^2 + 4d_I Z^2}{5 - 12d_I + 4d_I^2 + 16Z - 16d_I Z + 12Z^2}$ (cf. proof $5b$ in the appendix). Thus, when an accommodating scenario between firm I and firm E does not exist, an aggressive pricing behavior of the entrant firm yields a profitable outcome only if he locates close enough to the center and the incumbent respectively.

In sum firm E's pricing behavior is described by the following options when firm I holds a moderate position in terms of his location with $d_I^{\triangleleft} < d_I < d_I^{\triangleright}$:

- choose p_E^{Acc} if $d_I < 3 + d_E + 2Z - 8\sqrt{d_E(1 + Z)}$ and $d_E < d_E^{\triangleright}$

- choose $\overline{p}_E$ if $d_I > 3 + d_E + 2Z - 8\sqrt{d_E(1 + Z)}$ and $d_E < d_E^{\triangleright}$

- choose $p_E^{\times}$ if $d_E^{\triangleright} < d_E < d_E^{split}$

- choose $\check{p}_E$ if $d_E > d_E^{split}$

In addition to the stated effects of the centrality bonus on firm E's pricing behavior in the previous subsection, a change in Z also impacts the pricing decision in terms of the location d_E^{split}. Figure 3.9 illustrates that with increasing Z the critical location to undercut the incumbent firm also increases.

Figure 3.9: Illustration of the location boundary d_E^{split} against Z for $t = 1$
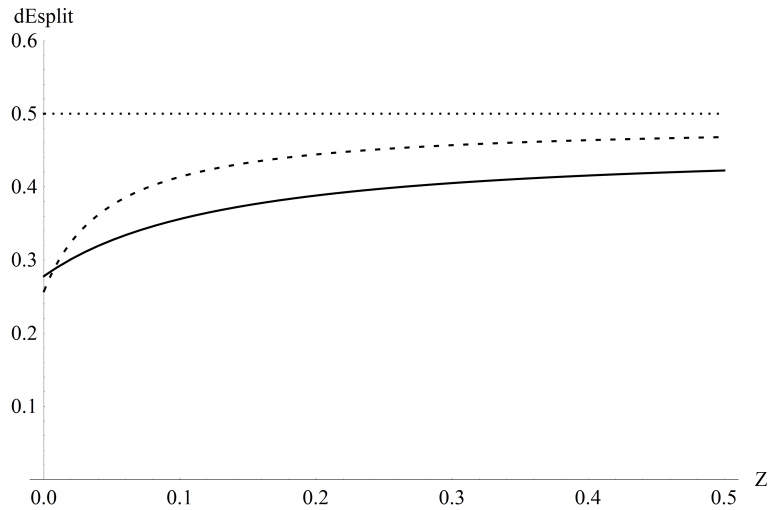
Comment: For $d_E > d_E^{split}$ the entrant prefers to charge $\check{p}_E$ to $p_E^{\times}$. The solid line depicts the location for $d_I = 0.35$, the small dashed line for $d_I = 0.45$, and the tiny dashed line for $d_I = 0.5$.

The higher Z the closer the entrant has to locate to the center such that a price attack against the incumbent pays off compared to setting $p_E^\times$. Moreover, the effect of a variation of d_I is depicted. As intuition suggests, with growing d_I the entrant has to locate closer to the center for the undercutting strategy to be profitable. An initially better market position of the incumbent forces the entrant to match up with his rival in terms of his own location.

3.4.2.3 Weak market position of the incumbent

This case refers to part (I) of the price reaction function in lemma 1 and assumes firm I's location to be restricted by $d_I < d_I^\triangleleft$ which is illustrated by the location segment below the horizontally dotted line in figure 3.8. Concerning the situation where the incumbent chooses the deterrence strategy, the accommodation-Z strategy, and the deterrence-Z strategy we follow the preceding treatments. Subsequently, we show that firm E can not charge a profit maximizing price when being in charge of the center and firm I setting p_I^{Acc}. Particularly, the best price the entrant can choose under this scenario is given by $p_E^Z := t(1 + d_E - d_I) - \epsilon, \epsilon \to 0$. In addition, an undercutting strategy with price $\tilde{p}_E$ is also not part of firm E's strategy set.
Recall that if the incumbent holds a comparatively weak market position locating at a distance below $d_I^\triangleleft$ from the center and given that $p_E \in [\tilde{p}_E, p_E^\dagger]$ his best strategy is to accommodate entry and optimize his profits charging p_I^{Acc} (with $Z = 0$). The entrant then seizes Z and gains a demand in the amount of $1 - x(p_I^{Acc}) + Z$. Respective entry profits reduce to:

$$\Pi_E(p_I^{Acc}) := \Pi_E^{AccZ} = -\frac{1}{4t}p_E^2 + \frac{1}{4}\left(3 + d_E - d_I + 4Z\right)p_E \qquad (3.22)$$

Applying the first order condition yields firm E's best price:

$$p_E^{AccZ} = \frac{1}{2}t(3 + d_E - d_I + 4Z) \qquad (3.23)$$

We see immediately that $p_E^{AccZ} > p_E^{Acc}$ for all d_E, d_I, Z. Clearly, $1 - x(p_I^{Acc}) > \frac{1}{2}$ must hold which reduces to $p_E < t(1 + d_E - d_I) = p_E^Z$. Setting $p_E^{AccZ} < p_E^Z$ yields the restriction $d_I < d_E - 1 - 4Z$ which results in an empty set for any $Z \geq 0$ if d_I and d_E are restricted to the interval $[0; \frac{1}{2}]$. Note also that $p_I^{Acc} > p_I^{DetZ}$ demands the same price restriction on p_E.
Why is the entrant unable to charge his profit maximizing price when being in charge of the center? Indeed, the explanation is found in the existence of the bonus Z that implies a higher price level between the contenders when both play according to an entry accommodating scenario. For instance, see that p_I^{AccZ} linearly increases in Z

from which it follows from Π_E^{Acc} that p_E^{Acc} grows linearly in Z as well. Now, if firm E takes the center and both players accommodate it is evident that firm I's profit maximizing price decreases ($p_I^{AccZ} > p_I^{Acc}$). Subsequently, this implies that the corresponding profit-optimizing price of firm E, i.e., p_E^{AccZ} also has to decrease since a higher demand certainly requires a lower price for firm E. By contrast, since firm E holds the center p_E^{AccZ} increases with growing Z. In particular, the difference between profits amounts to $\Pi_E^{AccZ} - \Pi_E^{Acc} = \frac{1}{2}p_E Z$, and prices differ by $p_E^{AccZ} - p_E^{Acc} = tZ$. The counter effects could only be dissolved by a corresponding variation in locations d_I and d_E, however, algebra shows no solution set in the feasible location range obtains. As a consequence, the solution for firm E when the incumbent charges p_I^{Acc} is given by p_E^Z since it represents the highest price such that the entrant stays in charge of the center. To reverse the argument, for every $d_I, d_E \in [0; \frac{1}{2}]$ the rank $p_E^Z < p_E^{AccZ}$ holds which also implies that $p_E^\dagger$ is no feasible price for the entrant. This is obvious since $p_E^\dagger > p_E^{AccZ}$ reduces to $d_I < d_E - 1 + 4Z - 4\sqrt{2Z(1+Z)}$ and $1 - 4Z + 4\sqrt{2Z(1+Z)} > \frac{1}{2}$ for all $Z \geq 0$. Also note that $p_E^Z < \hat{p}_E$ for all $Z > 0$. The lower price bound is given by $\tilde{p}_E$. As stated in the previous subsections the profit function for an undercutting strategy increases linearly in prices, thus, $\tilde{p}_E$ refers to the best price the entrant could set with respective profits of $(1 - d_I + Z)\tilde{p}_E$. The incumbent then reacts by setting p_I^{Def}. Also note that the rank $\tilde{p}_E < p_E^Z$ remains true for all $d_I \leq \frac{1}{2}$ and any real d_E.

A profit comparison for firm E's pricing strategies yields the following results. Firstly, charging $\tilde{p}_E$ is dominated by p_E^Z for all locations $d_I, d_E \in [0, \frac{1}{2}]$. Secondly, intersecting entry profits for the accommodating scenarios when firm E is in charge of Z and refrains from taking the center respectively, i.e., $\Pi_E^{AccZ}(p_E^Z) = \Pi_E^{Acc}(p_E^{Acc})$ shows that for moderate values of $Z > \frac{1}{4}\left(5 - 2\sqrt{6}\right)$ profits $\Pi_E^{AccZ}(p_E^Z)$ exceed $\Pi_E^{Acc}(p_E^{Acc})$ when the entrant locates closer to the center than a threshold value of $d_E > d_E^Z :=$ $1 + d_I + 6Z - 4\sqrt{Z + 2Z^2}$. (cf. proof of proposition 1 in the appendix) Correspondingly, the strategy to ride the profit function and leave the center to the incumbent is more profitable for distant locations, thus, when $d_E < d_E^Z$. Additionally, the pricing strategy to charge p_E^{Acc} dominates setting p_E^Z for all $Z < \frac{1}{4}\left(5 - 2\sqrt{6}\right)$. Thirdly, provided that the accommodation-Z strategy and p_I^{AccZ} is not feasible ($d_E > d_E^\triangleright$) charging $p_E^\times$ yields higher profits than p_E^Z for small entry locations $d_E < d_E^{Z\times} = \frac{1 + 2d_I Z}{2(2 + 3Z)}$ and setting p_E^Z leads to comparatively higher profits for locations above $d_E^{Z\times}$.

These results for the case p_I^{Acc} complete the previous derivation of location segments in subsection 3.4.1 for p_I^{AccZ}. (see figure 3.10) Rewriting the terms for d_E^Z and $d_E^{Z\times}$ yields the following two conditions that suggest for the entrant to pick the pricing strategy of charging p_E^Z compared to setting p_E^{Acc} and $p_E^\times$:

$$d_I < \frac{2 + 3Z}{Z}d_E - \frac{1}{2Z} \tag{3.24}$$

$$d_I < d_E + 4\sqrt{Z(1+2Z)} - 1 - 6Z \tag{3.25}$$

Note that for $Z \to 0$ the r.h.s. of (3.24) converges to $d_E^{\triangleright} = \frac{1}{4}$. Further, condition (3.25) shows the same behavior under a variation of Z for $Z \in [0, \frac{1}{2}]$ as condition (3.21), particularly, for $Z = \frac{1}{2}$ both reduce to $d_I = d_E$. Thus, for decreasing Z the location set for p_E^Z to be a profitable pricing strategy also decreases.

Figure 3.10: Illustration of the location segments for different pricing strategies for firm E when firm I applies the accommodation strategy (p_I^{Acc})
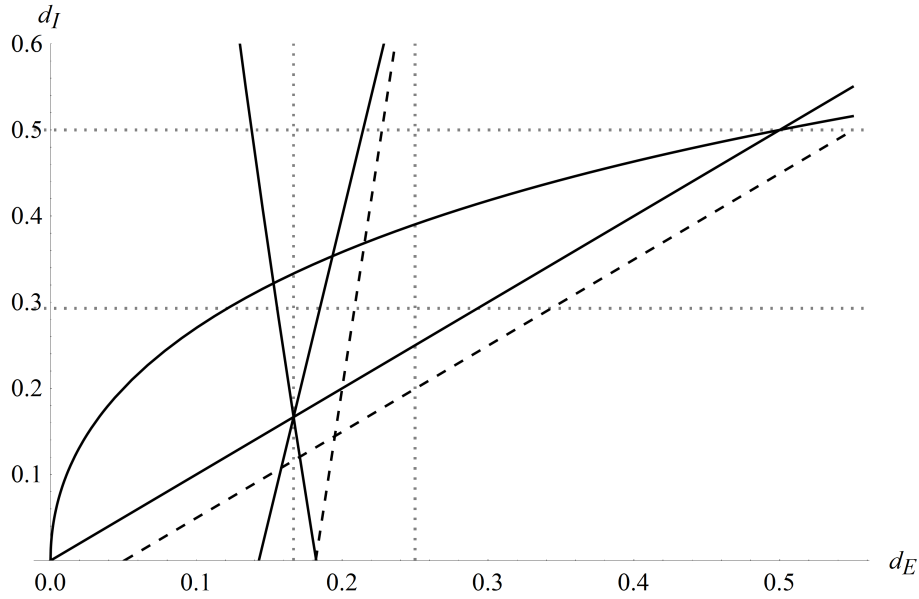
Comment: Graphs in figure 3.8 are enhanced by the restrictions given by d_E^Z and $d_E^{Z,\times}$. The solid lines depict the transitions in firm E's pricing behavior in terms of d_I and d_E for $Z = \frac{1}{2}$ and the dashed lines d_E^Z and $d_E^{Z,\times}$ for $Z = \frac{1}{4}$. Parameter values are $t = 1$.

In sum firm E's pricing behavior is described by the following options when firm I holds a weak position in terms of his location with $d_I < d_I^{\triangleleft}$:

- choose $\overline{p}_E$ if $d_I > 3 + d_E + 2Z - 8\sqrt{d_E(1+Z)}$ and $d_E < d_E^{\triangleright}$

- choose p_E^{Acc} if $d_I > d_E + 4\sqrt{Z(1+2Z)} - 1 - 6Z$ and $d_I < 3 + d_E + 2Z - 8\sqrt{d_E(1+Z)}$ and $d_E < d_E^{\triangleright}$

- choose p_E^Z if $d_I < d_E + 4\sqrt{Z(1+2Z)} - 1 - 6Z$ and $d_E < d_E^{\triangleright}$, and if $d_I < \frac{2+3Z}{Z}d_E - \frac{1}{2Z}$ and $d_E > d_E^{\triangleright}$

- choose $p_E^{\times}$ if $d_I > \frac{2+3Z}{Z}d_E - \frac{1}{2Z}$ and $d_E > d_E^{\triangleright}$

3.4.2.4 Entry decision

So far our approach covered an analysis of the strategy options of the entrant firm. According to the Stackelberg leader-follower game in prices we find that firm E's set comprises of $\overline{p}_E, p_E^{Acc}, p_E^{\times}, \check{p}_E$, and p_E^Z. The first two prices $\overline{p}_E$ and p_E^{Acc} correspond to the scenario with both players optimizing their profits and firm I gaining the center Z, the next two $p_E^{\times}$ and $\check{p}_E$ refer to firm E's best choice if the accommodation-Z strategy is not feasible and if the entrant sets an aggressive pricing behavior, finally p_E^Z represents firm E's best reply to the case where the incumbent accommodates entry but loses the center. Additionally, we derived conditions to establish the ranks $\overline{p}_E > p_E^{Acc} > \hat{p}_E$ and $p_E^{AccZ} > p_E^Z > \tilde{p}_E$. Moreover, we may state that the relations $p_E^{Acc} > \check{p}_E$ and $p_E^{Acc} > p_E^Z$ hold for all $d_I, d_E, Z \in [0; \frac{1}{2}]$.

The next step is to evaluate firm E's profits regarding the suggested five pricing strategies. Respective profits then are dependent on d_E, d_I, and Z and essentially determine firm E's payoffs as a function of his location. As intuition suggests the best choice for the entry set (p_E, d_E) depends on firm I's location. Thus, pursuing our previous approach we distinguish between the case where the incumbent has a strong market position and locates relatively close to the center $(d_I > d_I^{\triangleleft})$, and the case of remote locations $(d_I < d_I^{\triangleleft})$. Furthermore, it is evident that firm E's behavior hinges upon firm I's strategy set where emphasis lies on the scenario that the incumbent retains a strong market position and accommodates entry. Consequently, we have to differentiate the case where firm I's strategy set contains the pricing strategy p_I^{AccZ} or $d_E < d_E^{\triangleright}$ from the case where p_I^{AccZ} is excluded or $d_E > d_E^{\triangleright}$. We make the following propositions:

Proposition 1: For the location range $d_E < d_E^{\triangleright}$ and $d_I > d_I^{\triangleleft}$ the profit function $\Pi_E^{Acc}(p_E^{Acc})$ yields the highest values.

Proof: See the appendix.

Proposition 2: For the location range $d_E < d_E^{\triangleright}$ and provided that $d_I > d_I^{cMax}$ firm E's strategy is to charge p_E^{Acc} for locations $0 \leq d_E \leq d_E^{Ints}$ and to charge $\overline{p}_E$ for locations $d_E^{Ints} \leq d_E \leq d_E^{\triangleright}$. Then the local maximum and the entrant's profit-maximizing price-location pair is given by:

$p_E^*(d_I, Z) = \frac{2}{9}t \left(25 - 3d_I + 22Z - \Lambda(d_I, Z) - 6\sqrt{(1 + Z)(23 + 3d_I + 26Z - 2\Lambda(d_I, Z))} \right)$
$d_E^*(d_I, Z) = \frac{1}{9}(23 + 3d_I + 26Z - 2\Lambda(d_I, Z))$

with:
$\Lambda(d_I, Z) = 4\sqrt{(1 + Z)(7 + 3d_I + 10Z)}$
$d_I^{cMax} = \frac{4Z + 8Z^2 - 1}{4 + 4Z}$
$d_E^{Ints} = 29 + d_I + 30Z - 8\sqrt{13 + d_I + 27Z + d_I Z + 14Z^2}$

Proof: See the appendix.

Proposition 3: For the location range $d_E > d_E^{\triangleright}$ and $d_I > d_I^{\triangleleft}$, and provided that $d_I > d_I^{cMax}$ firm E's dominant strategy is to choose the profit-maximizing set (p_E^*, d_E^*) if the restriction $Z > \kappa \approx 0.0305$ is fulfilled.

Proof: See the appendix.

Proposition 4: As a result of firm I's location reaction function in lemma 3 the entrant firm discards the strategy to charge p_E^Z for any $d_E \in [0; \frac{1}{2}]$.

Proof: See the appendix.

The propositions address all five possible pricing strategies for firm E and their respective relations. Essentially, we determine conditions under which a local profit maximum given by the set (p_E^*, d_E^*) exists. (cf. proposition 2) The local equilibrium is established in an accommodating market entry scenario. This scenario is characterized by a modest behavior of the entrant firm in terms of his optimal entry location and entry price. Indeed, the characteristic feature of firm E's choice is described by a profit maximum that is reached at a particular location $d_E^{Max} = d_E^*$. As regards the pricing strategy the entrant aims at charging the highest possible price such that the incumbent refrains from an undercutting strategy given by $\bar{p}_E$. Since the profit-maximizing location lies within the boundary of $d_E^{\triangleright}$ the incumbent's best reaction is to charge p_I^{AccZ} and accommodate entry. Accordingly, the center of the city is taken by the incumbent firm. Furthermore, the condition for the profit maximum to exist $(d_E^{Max} < d_E^{\triangleright})$ is dependent on the location of the incumbent and limits d_I to exceed the threshold value of d_I^{cMax} which is a positive function in Z. In addition, a comparison with firm I's location reaction function reveals that under the entry price p_E^* the incumbent would react by accommodating entry and charging p_I^{AccZ} if $d_E < \bar{d}_E$ or undercutting firm E by setting p_I^{Det} if $d_E > \bar{d}_E$. Evidently, we find that d_E^* is approximated by $\bar{d}_E(p_E^*)$ but never exceeds $\bar{d}_E(p_E^*)$ for $0 \leq d_I \leq \frac{1}{2}$. An illustration of this market entry scenario is depicted in figure 3.11. It is interesting to see that initially – for entry locations $d_E < d_E^{Max}$ – firm E prefers the accommodation pricing strategy p_E^{Acc}. However, as d_E increases the accommodation price shows a corresponding increase and a maximum for $p_E^{Acc} \leq \bar{p}_E$ is reached at a critical location of $d_E^{Ints} \approx 0.14$ depicted by the first vertical dashed line. (cf. also equ. (3.20) in subsection 3.4.1) Certainly, higher profits are gained sticking to the accommodation price but are not feasible since otherwise firm E would be undercut. As a consequence, the entrant switches to $\bar{p}_E$ and increases his location until the profit maximum is reached at $d_E^{Max} \approx 0.17$ (second vertical dashed line). A further increase in d_E leads only to decreasing profits for firm E and an adaption in the pricing strategies of the two contenders. If d_E increases the threshold location

of $d_E^{\triangleright} \approx 0.18$ (third vertical dashed line) the accommodating entry scenario breaks down. For instance, if the incumbent locates close to the center ($d_I > d_I^{\triangleright}$) he reacts either by deterring market entry and charging p_I^{Det} or by undercutting the entrant at the center setting p_I^{DetZ} if firm E locates at positions $d_E > d_E^{\triangleright}$. (cf. lemma 1) Thus, the entrant's best strategy is then to play $p_E^{\times}$ with decreasing profits in d_E. Additionally, if $d_I^{\triangleleft} < d_I < d_I^{\triangleright}$ a further transition in the strategies of the two contenders occurs at $d_E^{split} \approx 0.43$ where an aggressive pricing behavior of the entrant becomes profitable and he drops prices from $p_E^{\times}$ to $\check{p}_E$.

Subsequently, provided that the incumbent holds a strong market position ($d_I > d_I^{\triangleleft}$) and comparing respective profits in the location ranges $d_E < d_E^{\triangleright}$ and $d_E > d_E^{\triangleright}$ the question arises which price and location pair yields the highest outcome. Is it most profitable for the entrant to behave modestly and optimize profits under the implicit constraints of firm I as the second mover or does an aggressive behavior against his rival and thus an undercutting strategy yield a higher outcome for firm E as the first mover? We admit not to provide a closed form solution but merely give an indication of the relation between the pricing strategies $\overline{p}_E$ and $\check{p}_E$. This could be a point of departure for future research on the subject. The scenario illustrated in figure 3.11 suggests that the local maximum (p_E^*, d_E^*) represents also the dominant entry strategy over the whole domain $d_E \in [0; \frac{1}{2}]$. In general the result depends on the parameter configuration and thus the variable set d_I and Z. In our nonexhaustive treatment we level the profits of the profit maximum in (p_E^*, d_E^*) with the highest possible profits that can be gained under an aggressive pricing of setting $\check{p}_E$ and locating infinitesimally close to the center. (cf. proposition 3) It turns out that the condition for $\check{p}_E$ to be a viable option restricts the centrality bonus to be comparatively small. This suggests that a significantly high value of Z reduces fierce competition in the market and thus has a stabilizing effect on the market setting. Put differently, under a high centrality bonus Z firm E's strategy to impose a self restricting behavior in terms of deliberately locating in a remote part of the city pays off compared to an aggressive entry behavior where the aim is to push the incumbent rival out of the market. It has to be noted that this finding is subject to a number of restrictions. However, under specific assumptions for the parameters d_I and Z we show that the incumbent reacts adaptively to an undercutting of firm E initially charging p_I^{Def}. As firm E increases his prices in accordance with his profit function $\check{\Pi}_E = (1 - d_I + Z)\check{p}_E$, the incumbent switches to the deterrence strategy setting p_I^{Det}. This also suggests that the entrant refrains from undercutting the incumbent for sufficiently close locations to the center and thus supports stable market conditions. (cf. the remark in the appendix and table 3.4 in section 4)

In proposition 4 firm E's entry decision for the case of remote locations $d_I < d_I^{\triangleleft}$ is considered. The important feature in this location range is that according to his profit structure the incumbent's reaction to entry requires him to accept the loss of

the center and accommodate entry by charging p_I^{Acc}. The prerequisite is that the entrant takes a comparatively strong market position exceeding a defined location $d_E^{\dagger}$ and that his prices fulfill $p_E^{\bigtriangledown\dagger} \geq p_E > p_E^{\triangleright}$ (cf. lemma 3 and the corresponding proof). Then the best strategy for firm E is to charge p_E^Z and gain profits Π_E^Z which are an increasing function in d_E. By a comparison of the threshold prices in the location reaction function, most notably $p_E^{\bigtriangledown\dagger}$, with the entry price at the switchover point for the pricing strategy p_E^Z we show that the pricing strategy p_E^Z yields no profitable result for firm E. This finding is a result of firm I's reaction to entry. If the entrant drops his prices to p_E^Z firm I does not react modestly and accommodates the loss of the center, depending on the parameter configuration of d_I and Z firm I rather replies aggressively and defends his claim for the center (p_I^{DetZ}) or deters entry (p_I^{Det}).

Finally, we sketch the comparative static behavior of firm E's feasible entry profits under a variation of Z and d_I. As expected, an increase in firm I's location leads to a downward shift for the profit functions of the pricing strategies $\overline{p}_E$, p_E^{Acc}, $p_E^{\times}$, and $\check{p}_E$ over the whole domain $d_E \in [0; \frac{1}{2}]$. Particularly, for a fixed Z such that $d_I > d_I^{cMax}$ it follows that the profit maximizing location d_E^{Max} shrinks as d_I grows. As already argued the impact of d_I on $\check{\Pi}_E$ is determined by the functional form of d_E^{split} and critically depends on the level of Z (cf. proof 5b and figure 3.13 in the appendix). Generally, we may state that $\check{\Pi}_E$ is shifted upward and the sensitivity of the profits to d_E increases as the value of firm I's location increases.

Regarding a variation in Z, the profit functions for the strategies $\overline{p}_E$ and p_E^{Acc} show an upward shift as Z increases. In addition, for a fixed $d_I > d_I^{cMax}$ see that the local maximum d_E^{Max} as well as the tangential intersection d_E^{Ints} increase with growing Z. Profits for the strategy $p_E^{\times}$ also increase as the centrality bonus Z increases and according to the interaction term between d_E and Z in the profit function the sensitivity with respect to Z correspondingly rises. Eventually, mirroring the behavior under a change in d_I the profit function for $\check{p}_E$ shows a complex reaction to Z. In particular, we observe that profits move up and the sensitivity of the profit function decreases with an increase in Z.

Figure 3.11: Illustration of firm E's profit function for different pricing strategies against his location d_E

Entrant's profit functions

Entrant's price functions
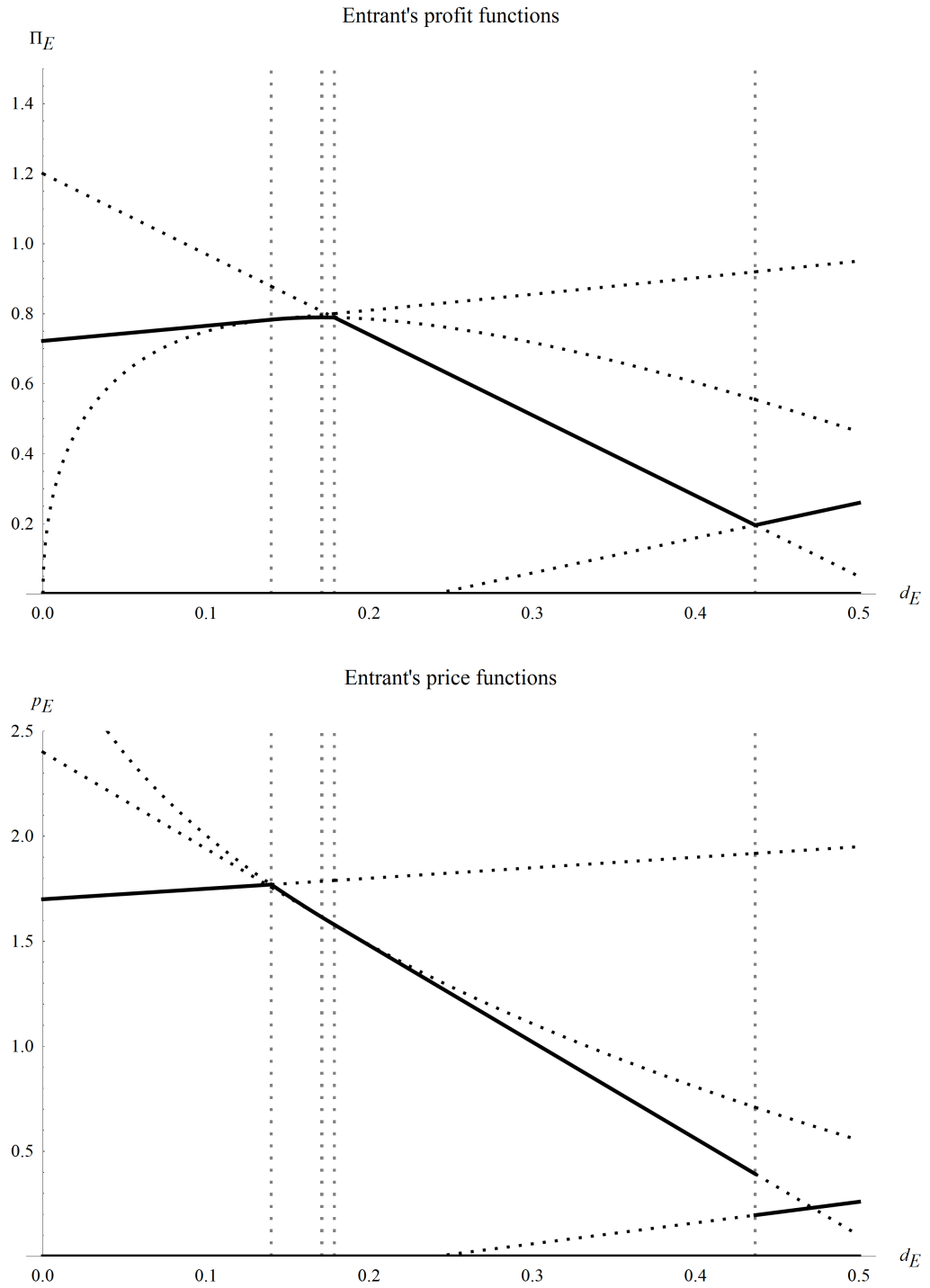
Comment: The solid lines depict the viable profit and price ranges. The first vertical dashed line indicates the transition of the pricing strategy p_E^{Acc} to $\bar{p}_E$ at the location d_E^{Ints}, the second marks the local profit maximum at d_E^{Max}. The next transition occurs from the pricing strategy $\bar{p}_E$ to $p_E^\times$ at $d_E^{\triangleright}$. Both transitions are described by a kink. Finally, the price drop from $p_E^\times$ to $\check{p}_E$ at d_E^{split} captures the last change in strategies. Parameter values are $d_I = 0.4$, $Z = 0.4$ and $t = 1$.

3.5 Interpretation and Comparison of Results

In this section we summarize and interpret the previous findings and give examples of the prices and profits for firm E and firm I in three selected scenarios depicted in tables 3.3, 3.4 and 3.5. Moreover, we provide a comparison of our results and implications with previous work on the subject by Anderson (1987).

Table 3.3: Summary of the entry accommodation scenario for $d_I = 0.4$ and $t = 1$

Z	d_E^*	p_E^*	Π_E^*	p_I^{AccZ}	Π_I^{AccZ}	d_I^{cMax}	x
0	0.143	1.228	0.465	1.242	0.772	-0.250	0.621
0.05	0.147	1.277	0.501	1.315	0.865	-0.186	0.607
0.1	0.150	1.325	0.538	1.387	0.963	-0.118	0.594
0.15	0.153	1.373	0.577	1.460	1.066	-0.048	0.580
0.2	0.157	1.422	0.617	1.532	1.174	0.025	0.566
0.25	0.160	1.470	0.658	1.605	1.288	0.100	0.552
0.3	0.164	1.518	0.700	1.677	1.406	0.177	0.539
0.35	0.167	1.566	0.744	1.750	1.530	0.256	0.525
0.4	0.171	1.615	0.790	1.822	1.660	0.336	0.511
0.45	n.a.	n.a.	n.a.	n.a.	n.a.	0.417	0.497
0.5	n.a.	n.a.	n.a.	n.a.	n.a.	0.500	0.483

The first scenario represents the entry accommodation scenario described in proposition 2. Accordingly, firm E chooses the profit-maximizing price-location pair (p_E^*, d_E^*) in the first stage of the game provided that $d_I > d_I^{cMax}$ or $d_E^{Max} < d_E^{\triangleright}$ respectively. Consequently, firm E discards the theoretical option to gain the centrality bonus Z and the incumbent firm reacts by accommodating entry and optimizing his resulting profit function in the second stage. This scenario therefore expresses the advantage of firm I to charge his prices as the second mover of the game and brings about a state of equilibrium. A change in price and location for the entrant does not increase his profits and induces the incumbent to adapt his accommodating strategy. Firstly, if firm E decreases his price, the resulting, comparatively low increase in market share does not compensate for the decline in profits. Furthermore, the incumbent is inclined to change his pricing strategy and takes the center Z due to the second-mover advantage. Secondly, an increase in firm E's location leads to lower net profits since $\bar{p}_E$ as the corresponding pricing function decreases in d_E. Additionally, at the respective price level the incumbent chooses to undercut the entrant for locations exceeding d_E^*. Thirdly, if firm E decreases his location, he only gains lower profits compared to (p_E^*, d_E^*), and a price increase at the maximum is not possible in order to avoid the incumbent to play the deterrence strategy.
It is important to note that the critical variable d_I^{cMax} determining the condition for the existence of the local maximum for firm E's corresponding profit function $\Pi_E^* = \overline{\Pi}_E(p_E^*, d_E^*)$ positively depends on Z. Certainly, this implies that not for all

combinations of d_I and Z on the domain $[0; \frac{1}{2}]$ the local profit maximum is feasible. For instance, in table 3.3 see that under an assumed value of $d_I = 0.4$ for Z exceeding the threshold of 0.4 the local profit maximum does not exist. This is owed to the reaction of firm I, if firm E locates at the corresponding location $d_E^* > d_E^{\triangleright}$ charging p_I^{AccZ} is not profitable anymore and therefore entry is not possible in the accommodation scenario. Additionally, the existence of firm E's profit maximum is reflected in the market boundary where for $d_I < d_I^{cMax}$ we obtain $x < \frac{1}{2}$ which of course is not reconcilable with an accommodating reaction of firm I.

Furthermore, we observe the impact of the centrality bonus on profits and prices of the contenders. By construction a higher value for Z corresponds to a larger market area. As expected, we confirm that firm I's profits and accommodation prices increase with increasing Z. As firm I is not induced to confront the entrant it follows that firm E also benefits from an increase of the market size in terms of Z even though he is not in the position to gain the center. In particular, see that both firm E's accommodation price p_E^* as well as the optimal location d_E^* increase as Z grows.

Table 3.4: Summary of the entry undercutting scenario for $d_I = 0.4$ and $t = 1$

Z	d_E^{split}	$\check{p}_E$	$\check{\Pi}_E^{min}$	$d_E^{\times}$	$\check{\Pi}_E^{th.}$	p_I^{Def}	Π_I^{Def}
0	0.267	0.667	0.400	0.311	0.427	1.000	0.400
0.05	0.339	0.473	0.307	0.384	0.336	0.700	0.280
0.1	0.375	0.375	0.263	0.419	0.293	0.550	0.220
0.15	0.396	0.316	0.237	0.440	0.270	0.460	0.184
0.2	0.410	0.277	0.221	0.454	0.256	0.400	0.160
0.25	0.420	0.248	0.211	0.463	0.248	0.375	0.143
0.3	0.427	0.227	0.204	0.470	0.243	0.325	0.130
0.35	0.432	0.210	0.199	0.475	0.240	0.300	0.120
0.4	0.436	0.196	0.196	0.479	0.239	0.280	0.112
0.45	0.440	0.185	0.195	0.482	0.239	0.264	0.105
0.5	0.443	0.176	0.194	0.485	0.240	0.250	0.100

The second scenario refers to the case of the entrant undercutting the incumbent at d_I in the first stage of the game and the incumbent defending his market position in the second stage. Thus, this scenario covers the instance of a possible first-mover advantage by considering that the entrant will take the center Z. In subsection 3.4.2.4 and proposition 3 we argue that only for small values of Z the undercutting strategy $\check{p}_E$ represents a viable option for the entrant. This is on the one hand linked to the complex behavior and dependency of the profit function $\check{\Pi}_E = \Pi_E(\check{p}_E(d_E, d_I, Z), d_I, Z)$ under a variation of d_I and Z and on the other hand explained by the rise of p_E^* and Π_E^* in the accommodation scenario under an increase in Z.

Now, table 3.4 provides further insights on the subject. Strikingly, the initial undercutting price $\check{p}_E$ taken at the switchover location d_E^{split} decreases as Z increases. This implies that a higher bonus that could be grasped by the entrant requires him to set

an increasingly lower undercutting price. This instance is confirmed by the behavior of the threshold d_E^{split} which increases in Z and thus expresses that the entrant needs to take an increasingly stronger market position for the pricing strategy $\check{p}_E$ to represent a viable option. As a consequence, corresponding profits $\check{\Pi}_E^{min}$ evaluated at $\check{p}_E$ and d_E^{split} decrease in Z and illustrate that a potential first-mover advantage by the entrant is diminishing. Clearly, since the incumbent has to react adequately to the price attack to hold his market position his behavior mirrors the entrant's with prices $p_I^{Def}(\check{p}_E(d_E^{split}))$ and corresponding profits Π_I^{Def} decreasing for growing values of Z.

The depicted profits $\check{\Pi}_E^{min}$ in table 3.4 represent the minimum values the entrant gains under the undercutting strategy $\check{p}_E$ since profits monotonically increase in d_E. By contrast in column 4 the numbers for $d_E^\times$ provide the location maxima and thus firm E's preferred location taken at the switchover point d_E^{split} and corresponding price $\check{p}_E(d_E^{split})$. According to firm I's location reaction function entry is deterred when firm E locates at $d_E > d_E^\times$ (cf. lemma 4). It is revealing that the situation is characterized by a recursive relation since the entrant is inclined to increase d_E leading to an increase in $\check{p}_E$ to realize higher profits whereas $d_E^\times$ decreases for growing p_E. To determine the theoretically highest profits for the entry undercutting strategy $\check{\Pi}_E^{th.}$ we evaluate the profit function at the lowest possible undercutting price $\check{p}_E(d_E^{split})$ and the theoretical maximum location $d_E^\times$. We confirm that $\check{\Pi}_E^{th.}$ decreases as Z grows. Additionally, we observe that the undercutting strategy is viable over all Z since $d_E^\times$ exceeds d_E^{split}, however, in the provided example ($d_I = 0.4$) a comparison of the theoretically highest undercutting profits with corresponding profits for the accommodation scenario in table 3.3 shows that the accommodation strategy (p_E^*, d_E^*) dominates.

Table 3.5: Summary of the entry undercutting-Z scenario for $d_I = 0.4$ and $t = 1$

Z	$p_E^\times(d_E^\triangleright)$	$\Pi_E^\times(d_E^\triangleright)$	$p_E^\times(d_E^{split})$	$\Pi_E^\times(d_E^{split})$	p_I^{DetZ}	Π_I^{DetZ}
0	0.850	0.425	0.800	0.400	1.000	0.500
0.05	0.938	0.469	0.614	0.307	1.100	0.605
0.1	1.027	0.514	0.525	0.263	1.200	0.720
0.15	1.117	0.559	0.474	0.237	1.300	0.845
0.2	1.208	0.604	0.442	0.221	1.400	0.980
0.25	1.300	0.650	0.422	0.211	1.500	1.125
0.3	1.392	0.696	0.408	0.204	1.600	1.280
0.35	1.485	0.743	0.399	0.199	1.700	1.445
0.4	1.579	0.789	0.393	0.196	1.800	1.620
0.45	1.672	0.836	0.389	0.195	1.900	1.805
0.5	1.767	0.883	0.387	0.194	2.000	2.000

The third scenario considers the possibility for firm E to undercut the incumbent firm at the location $x = \frac{1}{2}$ and seize the center Z. Referring to the profit-maximizing

price-location set (p_E^*, d_E^*), we argue that an undercutting for the center is not a profitable strategy option.

To begin with, in table 3.5 we introduce the undercutting price $p_E^\times$ and corresponding profits $\Pi_E^\times$ at the transition locations $d_E^\triangleright$ and d_E^{split}.[6] Note that the entry price and profit increase at $d_E^\triangleright$ and decrease at d_E^{split} as Z grows. This reflects on the one hand the relation of the strategy $p_E^\times$ with the strategy set (p_E^*, d_E^*), and on the other hand the relation of $p_E^\times$ and $\Pi_E^\times$ with the mill-undercutting price and profit. As already argued, with increasing Z entry profits under the pricing strategies $\overline{p}_E$ and p_E^{Acc} increase. Since $d_E^\triangleright$ marks the intersection of $\overline{p}_E$ and $\overline{\Pi}_E$ with $p_E^\times$ and $\Pi_E^\times$ the latter are shifted upwards. Additionally, due to the increase of $\check{\Pi}_E$ and $\check{p}_E$ under a decrease of Z also $\Pi_E^\times$ and $p_E^\times$ increase at the intersection d_E^{split}. Moreover, the price and profit of firm I for the deterrence-Z strategy are depicted provided that firm E chooses $p_E^\times(d_E^\triangleright)$. Note that in this particular case, i.e. at $d_E = d_E^\triangleright$, the defensive price p_I^{DetZ} is independent of d_I.

Turning to the subject of interest, why does the existence of Z not induce the entrant to undercut the incumbent at $x = \frac{1}{2}$? Why is the undercutting-Z strategy for firm E under specific circumstances not profitable and thus feasible? Our answer is that under consideration of firm I's response charging $p_E^\times$ and $\hat{p}_E$ is outperformed by the accommodating pricing strategies $\overline{p}_E$ and p_E^{Acc} (cf. proofs for propositions 1 and 2). Additionally, we can show that the incumbent does not accept the loss of the center in an accommodating scenario (cf. proof for proposition 4). Thus, regarding the distribution of the centrality bonus Z our model suggests a clear second-mover advantage on the side of the incumbent firm.

In particular, for distant entry locations $d_E < d_E^\triangleright$ the entrant effectively has two choices, he could concede Z to the incumbent and optimize his price setting behavior, alternatively, he could challenge the claim for Z charging $\hat{p}_E$. As was shown, the incumbent reacts to the first option by accommodating entry and in the second case undercuts the entrant at $x = \frac{1}{2}$ to retain the center (cf. proof of lemma 1). Note that setting $p_E^\times$ if $d_E < d_E^\triangleright$ is ruled out since it does not represent an appropriate undercutting price and proves not to be profitable (cf. proof of proposition 2). For close entry locations $d_E > d_E^\triangleright$ an accommodating reaction by the incumbent is not feasible, thus, his reaction would either be to defend the center (p_I^{DetZ}) or deter entry (p_I^{Det}). Concerning firm E, regardless of the strategy option $\check{p}_E$, the best entry price is therefore $p_E^\times$. The characteristic feature then is that increasingly lower undercutting-Z prices are required for increasingly closer entry locations which is highlighted in table 3.5 by a comparison of $p_E^\times$ evaluated at $d_E^\triangleright$ and d_E^{split}. Intuitively, when aiming for the bonus Z the entrant makes it also more difficult to be undercut by the incumbent as he locates closer to the center.

This leads to the following conclusion: under the pricing strategy $p_E^\times$ firm E has a

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[6] In the underlying demand function for firm E we set $Z = 0$.

clear preference to choose a distant location which is determined by the switchover point $d_E^{\triangleright}$. This further implies that if the local maximum (p_E^*, d_E^*) exists the strategy to choose this profit-maximizing price-location set dominates the strategy to opt for the pair $(p_E^{\times}, d_E^{\triangleright}) = (\bar{p}_E, d_E^{\triangleright})$, by contrast, if the local maximum does not exist the best entry set is given by the kink solution $(p_E^{\times}, d_E^{\triangleright}) = (\bar{p}_E, d_E^{\triangleright})$. To illustrate the point, for a comprehensive solution the non-applicable fields in table 3.3 are substituted with the values of table 3.5 for $Z = 0.45$ and $Z = 0.5$. Moreover, we confirm that profits Π_E^* exceed profits $\Pi_E^{\times}(d_E^{\triangleright})$ over the assumed parameter range.

Our modeling approach is in full accord with the model of Anderson (1987). In particular, the term for the profit-maximizing entry location d_E^* in proposition 2 of our paper represents a generalized form of the location reaction function $b^*(a)$ for $a \leq \frac{1}{2}$ accounting for the centrality bonus Z in his proposition 4 (cp. Anderson (1987), p. 383). Consequently, the results for firm B's entry location and corresponding prices and profits for both firms in proposition 5 and 6 in Anderson's article (ibid., pp. 385-387) match our results for the equivalent cases $d_I = \frac{1}{2}$ and $d_I = 0$ if $Z = 0$. This is, of course, not surprising since we also took account of firm E's profit functions under the most profitable pricing strategies following a Stackelberg leader-follower game in prices. In a nutshell, our main contribution and extension of Anderson's work is two-fold. Firstly, we evaluated more pricing options for the entrant which is a direct consequence of the introduction of the centrality bonus Z. Secondly, we determined the conditions for the existence and nonexistence – or at least the maximum bound in the case of the undercutting strategy $\check{p}_E$ – of different entry states taking into account not only firm I's price reaction to firm E's price setting but also the price reaction to the choice of the entry location. However, due to the emphasis that is given to the impact of Z we used a more simplified approach compared to Anderson's model reflected in the instance that firm I's location is not endogenized in our model.

3.6 Conclusion

In this paper we scrutinize a two-stage sequential market entry game between two players, an incumbent firm (firm I) and an entrant firm (firm E) in the geographical setting of a linear city a la Hotelling with linear transportation costs. The set up is extended by a center in the middle of the city ($x = \frac{1}{2}$) where additional demand in the amount of Z can be gained. We focus on the resulting strategic interaction between the players assuming that the incumbent already served the market in a pre-monopolisitic stage which implies that his location (d_I) is not disposable and, thus, does not represent a strategic variable in the game. This leads us to provide a solution set comprising the entry price and location (p_E, d_E) chosen in the first stage of the game and the price charged by firm I (p_I) in the second stage.

In the proceeding we firstly provide relevant parts of the reaction function to the entry price as well as to the entry location under consideration of five different pricing options for firm I. These follow from the introduction of the demand dependent centrality bonus Z and are: an undercutting strategy against firm E to deter entry, a deterrence strategy to defend the claim for Z, two entry accommodating strategies where Z is either taken by the incumbent or the entrant, and a strategy to defend the market position against an undercutting strategy of firm E. In a second step, based on the strategic set of firm I the best prices of firm E are derived. Subsequently, accounting for the corresponding demand these are inserted into firm E's profit function to analyze the dependency of different entry profits on the choice of the entry location.

Our analysis highlights the interrelation of the players' locations with the centrality bonus Z and generally shows that Z is crucial to determine the range for the pricing strategies to be applicable and to describe the transition points between different pricing strategies. Particularly, we are for instance able to confirm the intuitive assumption that firm I is less likely to apply the accommodation strategies for higher values of Z (exemplified by $d_E^{\triangleright}$ and $d_I^{\triangleleft}$). The main finding of the paper is that we provide a solution for an entry accommodation scenario where both firms optimize the profit functions over their strategic variables and the entry solution set given by (p_E^*, d_E^*). The center is retained by the incumbent firm setting a higher price and realizing higher profits than the entrant who chooses a distant location. An increase in Z increases prices and profits but reduces the domain since the existence of the equilibrium is bounded by firm I's location with the threshold location (d_I^{cMax}) decreasing as Z grows. This result is in full accord with the model of Anderson (1987) since for $Z = 0$ our solution matches firm E's location reaction function in his paper. In addition, our results suggest that firm E has no chance to capture Z. Firstly, we argue that an aggressive entry behavior aiming at undercutting the incumbent firm is not profitable compared to the accommodating equilibrium. Intuitively, a higher bonus Z requires even lower undercutting prices and leads to fiercer competition. Secondly, a more modest entry strategy to refrain from undercutting the rival and only capture the center Z is not feasible due to the second-mover advantage of firm I and the structure of firm E's profits. Thirdly, we exclude the case where firm E seizes Z in an accommodating scenario.

In the course of our paper we try to bridge a gap between the subject of sequential entry in spatial modeling (Anderson (1987), Fleckinger & Lafay (2010)) and spatial monopolistic competition in centralized markets (Braid (1989), Braid (2013), Chen & Riordan (2007)). To the best of our knowledge explicit expressions of reaction functions with respect to entry price and location taking account of a market center have not been used previously in analyses of sequential entry games under a linear transportation cost scheme. A distinct feature of our approach compared to, for in-

stance, Anderson (1987) is that after deriving firm E's best prices and corresponding profit functions the preferred entry locations are checked with respect to the price reaction of firm I. Thus, we examine the incumbent's price reaction to both entry price *and* location. This implies that our treatment therefore does not include a location subgame where the incumbent optimizes d_I with respect to firm E's entry decision in a prevenient stage. (cp. Anderson (1987), p. 384, Proposition 5)

The appealing feature of our approach, however, is that we drill down firm I's reaction to the most profitable pricing decision and subsequently assess the market structure as a result of firm E's best reply. Clearly, in our model entry is not principally deterred but occurs under particular circumstances, these being that the entrant deliberately restricts himself to a modest market position in a distant location to the center. This refers to a business strategy characterized by the term 'judo economics' introduced in the seminal paper of Gelman & Salop (1983) where the entrant improves his strategic position by credibly committing himself to a limited capacity. If he were to break his credible capacity-limitation commitment, the incumbent firm undercut the entry price and the entrant obtained no customers. In their model the strategic variable of the incumbent only consists of his price. (cf. Gelman & Salop (1983), p. 316) Our results are in line with this thesis in terms of the derived location thresholds in firm I's location reaction function, if these were exceeded, the incumbent deterred entry. We may also note that the credibility of firm E's decision is supported by the construction of the spatial setting since product heterogeneity is only attributed to the location in the city. Additionally, Fleckinger & Lafay (2010) show that in a sequential entry game under catalog competition, that is two firms deciding in one stage on both strategic variables, price and location, a second-mover advantage results as the leader's strategy is to choose a low-price and low-market share strategy to avoid being undercut by the follower. Thus, we conclude that a comparison with the literature shows that the implications of our model fit well into the scholarly discussion.

Eventually, we may summarize shortcomings of our model as well as potential future developments of it. For the price reaction function the case where the two contenders locate very far from one another is not explicitly dealt with in the analysis, i.e., for the range $d_E < d_E^{\triangleleft}$ and $d_I \leq d_I^{\triangleleft}$. However, with regards to the monotonicity of firm E's accommodation profits in d_E this does not affect the outcome of our analysis. For the location reaction function the case of low undercutting prices charged by the entrant is not explicitly analyzed, i.e., for the range $p_E < p_E^{\triangleright}$, $p_E < p_E^{\triangledown\dagger}$, and $p_E < p_E^{\triangledown\ddagger}$. The comparison of firm E's undercutting strategy and accommodation strategy is therefore subject to restrictions for d_I and Z (cf. remark in the appendix). Nevertheless, we are able to determine an upper bound solution that determines the range of where the profit-maximizing set (p_E^*, d_E^*) dominates the undercutting strategy (cf. proposition 3).

Future developments could further consolidate our model with the work of Anderson (1987) and Fleckinger & Lafay (2010). Potential enhancements comprise an endogenization of firm I's location d_I and subsequent analysis on the impact of Z with respect to firm I's location decision. Subsequently, referring to proposition 7 in Anderson's paper (p. 386f) new insights could be gained assessing the timing structure of the subgames in location and prices and to which extent Z determines first- and second-mover advantages. Furthermore, it would be interesting to study catalog competition as in Fleckinger & Lafay (2010) in the context of a spatially centralized market.

Appendix

Proof 1: Recapitulating the dependencies from equation (3.5) we define:

$\Pi_I(p_I^{Det}, 1 + Z) \equiv \Pi_I^{Det} = [p_E - t(1 - d_E - d_I)](1 + Z)$,

$\Pi_I(p_I^{DetZ}, \frac{1}{2} + Z) \equiv \Pi_I^{DetZ} = [p_E - t(d_E - d_I)](\frac{1}{2} + Z)$,

$\Pi_I(p_I^{Def}, d_I) \equiv \Pi_I^{Def} = p_E d_I + t d_I(1 - d_I - d_E)$,

$\Pi_I(p_I^{AccZ}, x(p_I^{AccZ}) + Z) \equiv \Pi_I^{AccZ} = \alpha(p_E)^2 + \beta p_E + \gamma$, and

$\Pi_I^{AccZ}(Z = 0) \equiv \Pi_I^{Acc}$

with: $\alpha = \frac{1}{8t}$, $\beta = \frac{1}{4}(1 - d_E + d_I + 2Z)$, and $\gamma = \frac{t}{8}(1 + d_E^2 + d_I^2) + \frac{t}{4}(d_I - d_E - d_E d_I) + \frac{t}{2}(Z^2 + Z + d_I Z - d_E Z)$.

See that all profits increase in p_E. Deterrence profits Π_I^{Det} show the highest linear increase by the amount of $1 + Z$, deterrence-Z profits Π_I^{DetZ} increase comparatively lower by the factor $\frac{1}{2} + Z$, and deference profits Π_I^{Def} increase by the smallest amount of d_I (recall that $d_I \leq \frac{1}{2}$). Profit functions for the two accommodation cases Π_I^{AccZ} and Π_I^{Acc} are strictly convex and monotonically increasing on $p_E \geq 0$ since the minimum $p_E^{min} = -t(1 - d_E + d_I + 2Z) < 0$ for all $d_I, d_E, Z \geq 0$. Clearly, $Z > 0$ implies that $p_E^{min}(Z) < p_E^{min}(Z = 0)$ or that $\Pi_I^{AccZ} > \Pi_I^{Acc}$ for all $p_E \geq 0$.

Next, we examine the profit intersections. For high values of p_E the deterrence and accommodation-Z strategy are preferable. Matching Π_I^{Det} with Π_I^{AccZ} yields the two solutions $\overline{p}_E = t(3 + d_E - d_I + 2Z - 4\sqrt{d_E(1 + Z)})$ and $\overline{\overline{p}}_E = t(3 + d_E - d_I + 2Z + 4\sqrt{d_E(1 + Z)})$. Further, we define $\overline{p}_E^{\diamond}$ as the intersection of the corresponding prices $p_I^{Det}(\overline{p}_E^{\diamond}) = p_I^{AccZ}(\overline{p}_E^{\diamond})$. See that the deterrence strategy is preferred to the accommodation-Z strategy for every $p_E \in [\overline{p}_E, \overline{\overline{p}}_E]$ since $\overline{p}_E < \overline{p}_E^{\diamond}$ for all $d_I, Z \geq 0$ and $d_E < 1 + Z$. This corresponds to the finding of Anderson (1987) in the original setting of the linear city (i.e. for $Z = 0$).

Matching Π_I^{AccZ} with Π_I^{DetZ} we obtain the tangential solution $\hat{p}_E = t(1 + d_E - d_I + 2Z)$. Likewise, $\hat{p}_E$ proves to be the solution for the intersection of the price functions p_I^{AccZ} and p_I^{DetZ} which is supported by $\left.\frac{\partial \Pi_I^{AccZ}}{\partial p_E}\right|_{\hat{p}_E} = \frac{1}{2} + Z$. This implies that the accommodation-Z strategy is preferred for every $p_E > \hat{p}_E$ and the deterrence-Z strategy is applied for $p_E < \hat{p}_E$. Intuitively, for $p_E < \hat{p}_E$ firm I loses the center when charging p_I^{AccZ} only realizing Π_I^{Acc} which induces him to play p_I^{DetZ} and gain higher profits Π_I^{DetZ}. This is consistent with the behavior of the market boundary where $x(p_I^{AccZ}) > \frac{1}{2}$ for all $p_E > \hat{p}_E$ and $d_E, d_I, Z \geq 0$.

Matching Π_I^{DetZ} with Π_I^{Acc} yields the two solutions $p_E^{\ddagger} = t(1 + d_E - d_I + 4Z - 2\sqrt{2Z(1 + 2Z)})$ and $p_E^{\ddagger} = t(1 + d_E - d_I + 4Z + 2\sqrt{2Z(1 + 2Z)})$. See that $p_E^{\ddagger} < \hat{p}_E < p_E^{\ddagger}$ holds for all $d_E, d_I, Z \geq 0$. This can also be concluded from the structure of Π_I^{AccZ} and Π_I^{Acc} where $\frac{\partial^2 \Pi_I^{AccZ}}{\partial p_E^2} = \frac{\partial^2 \Pi_I^{Acc}}{\partial p_E^2} = \frac{1}{4t}$. Further, intersect the respective price functions $p_I^{DetZ}(p_E^{\ddagger\diamond}) = p_I^{Acc}(p_E^{\ddagger\diamond})$ and see that $p_E^{\ddagger} < p_E^{\ddagger\diamond}$ for all $d_E, d_I, Z \geq 0$.

It follows that the deterrence-Z strategy dominates the accommodation strategy for $p_E^\dagger < p_E < \hat{p}_E$, and that the accommodation strategy is preferred to the deterrence-Z strategy for $p_E < p_E^\dagger$.

We turn to the deference case. Matching Π_I^{Acc} with Π_I^{Def} yields $\tilde{p}_E = t(3d_I + d_E - 1)$, and matching Π_I^{DetZ} with Π_I^{Def} we get $\check{p}_E = t\left(d_E + d_I\left(\frac{1-2d_I-2Z}{1-2d_I+2Z}\right)\right)$. Leveling the price functions p_I^{Acc} and p_I^{Def} the profit intersection $\tilde{p}_E$ obtains which can also seen by $\left.\frac{\partial \Pi_I^{Acc}}{\partial p_E}\right|_{\tilde{p}_E} = d_I$. It follows that even though $\Pi_I^{Def} < \Pi_I^{Acc}$ for $p_E < \tilde{p}_E$ firm I charges p_I^{Def} in order not to be undercut at his own mill. This finding specifies the analysis in Anderson (1987) where it is indicated that the deference strategy yields higher profits (see Fig.3 on p.376 and proof for proposition 1 lit.b on p.390). Additionally, due to the linear dependencies of Π_I^{DetZ} and Π_I^{Def} in p_E the deterrence-Z strategy is preferable for $p_E > \check{p}_E$, whereas the deference strategy dominates for $p_E < \check{p}_E$. Note that $p_I^{Def} > p_I^{DetZ} > p_I^{Det} \; \forall p_E$.

The final step of the proof is to derive the ranges of validity for the five pricing strategies with regards to Z. First note that the accommodation-Z strategy is only viable when $\overline{p}_E > \hat{p}_E$. This corresponds to a restriction for d_E contingent on Z, i.e., $d_E < d_E^\triangleright = \frac{1}{4(1+Z)}$. If $d_E > d_E^\triangleright$ the accommodation-Z strategy is always dominated either by the deterrence strategy or the deterrence-Z strategy and the switchover point is given by the intersection of Π_I^{Det} with Π_I^{DetZ} at $p_E^\times = t(2+2Z-d_E(3+4Z)-d_I)$. For $d_E > d_E^\triangleright$ the switchover point always lies below $\hat{p}_E$. The condition for the accommodation strategy to exist is given by $p_E^\dagger > \tilde{p}_E$ imposing a restriction on d_I contingent on Z, i.e., $d_I < d_I^\triangleleft = \frac{1}{2} + Z - \sqrt{Z(\frac{1}{2}+Z)}$. Also note that $p_E^\dagger > \tilde{p}_E$ restricts $p_E^\dagger$ to be positive. To check on the deference case we first see that the boundaries $d_E^\triangleright$ and $d_I^\triangleleft$ yield a positive value for $\tilde{p}_E$ for all $0 \le Z \le \frac{1}{2}$. Next we see that $\tilde{p}_E > 0$ only if $d_E > d_E^\triangleleft = 3\sqrt{Z(\frac{1}{2}+Z)} - \frac{1}{2}(1+6Z)$ is fulfilled for $d_I = d_I^\triangleleft$. Note that $\tilde{p}_E(d_I^\triangleleft) > 0$ for all $d_E \le \frac{1}{2}$ and $Z < \frac{1}{6}$. Since $d_E^\triangleright > d_E^\triangleleft$ for all $0 < Z \le \frac{1}{2}$ a nonnegative set for $d_I < d_I^\triangleleft$ obtains which proves part (I) of firm I's proposed price reaction. For parts (II) and (III) we find that $\check{p}_E < \tilde{p}_E$ for all $d_E, Z > 0$ when the deterrence-Z strategy dominates the accommodation strategy, i.e., $d_I > d_I^\triangleleft$. Again, Z determines the profitability of Π_I^{DetZ} in relation to Π_I^{Def} with $\check{p}_E$ strictly monotonically decreasing in Z. Setting $\check{p}_E \ge 0$ we get $d_I < d_I^\triangleright = \frac{1}{4}(1-2d_E-2Z+\sqrt{4d_E^2 + (1-2Z)^2 + 4d_E(1+6Z)})$, and $\check{p}_E < 0$ for $d_I^\triangleright < d_I \le \frac{1}{2}$ respectively (the negative root only yields $d_I^\triangleright < 0$ for $d_E, Z > 0$). Next we compare $d_I^\triangleleft < d_I^\triangleright$ and find the floor for d_E at the same $d_E^\triangleleft$ as for part (I). Since the interval $[d_E^\triangleleft; d_E^\triangleright]$ is nonempty the existence of parts (II) and (III) is established. Also note that $d_E^\triangleleft > 0$ only if $Z > \frac{1}{6}$ which implies that $d_I^\triangleleft < d_I^\triangleright$ holds for all $d_E \le \frac{1}{2}$ if $Z < \frac{1}{6}$. This completes the proof.

Proof 2: Consider the profit generating functions: $\Pi_I^{gAccZ}(p_I) = -\frac{p_I^2}{2t} + \frac{p_I}{2} - \frac{p_I d_E}{2} +$

$\frac{p_I d_I}{2} + \frac{p_I p_E}{2t} + p_I Z$ and $\Pi_I^{gAccZ}(Z = 0) = \Pi_I^{gAcc}(p_I)$. Further, recall $\Pi_I^{gDet}(p_I) = p_I(1 + Z)$ and $\Pi_I^{gDetZ}(p_I) = p_I(\frac{1}{2} + Z)$ from proof 1. Now, we will evaluate these profit functions at the switchover points $\overline{p}_E$, $\hat{p}_E$ and $p_E^\dagger$ to show under which restrictions the maxima in Π_I^{gAccZ} and Π_I^{gAcc} are covered.[7]

If firm E charges $\overline{p}_E$, the incumbent is indifferent between the deterrence strategy and the accommodation-Z strategy. Realizing Π_I^{gDet} and $q_I = 1 - d_E$ requires $p_I \leq p_I^{Det}$ and realizing Π_I^{gAccZ} corresponds to $p_I^{Det} < p_I < p_I^{DetZ}$. Thus, the incumbent applies the deterrence strategy for low p_I until $\overline{p}_E - t(1 - d_I - d_E)$ is reached. For higher p_I he switches to an accommodating behavior until the threshold of $\overline{p}_E + t(d_I - d_E)$ is reached. This of course restricts the market boundary to fullfill $1 - d_E \leq x \leq \frac{1}{2}$. Also we see that the intersection of Π_I^{gDet} with Π_I^{gAccZ} lies below p_I^{Det} for all $d_E, d_I, p_E > 0$. Inserting and expanding yields the following expressions: $p_I^{Det}(\overline{p}_E) = 2t(1 + d_E + Z - 2\sqrt{d_E(1 + Z)})$, $p_I^{AccZ}(\overline{p}_E) = 2t(1 + Z - \sqrt{d_E(1 + Z)})$, and $p_I^{DetZ}(\overline{p}_E) = 3t + 2t(Z - 2\sqrt{d_E(1 + Z)})$. The rank $p_I^{Det}(\overline{p}_E) < p_I^{AccZ}(\overline{p}_E) < p_I^{DetZ}(\overline{p}_E)$ only holds true for $d_E < \frac{1}{4(1+Z)}$ (and $d_E < 1 + Z$ respectively).

Analogously, taking $\hat{p}_E$ we get $p_I^{Det}(\hat{p}_E) = 2t(d_E + Z)$, $p_I^{AccZ}(\hat{p}_E) = t(1 + 2Z)$ and $p_I^{DetZ}(\hat{p}_E) = t(1 + 2Z)$. The existence of the maximum in Π_I^{gAccZ}, i.e., $p_I^{DetZ} > p_I^{AccZ}$ demands $p_E > \hat{p}_E$. Graphically, this corresponds to the intersection of the straight line Π_I^{gDetZ} with the slope $\frac{1}{2} + Z$ with the parabola Π_I^{gAccZ}. At $\hat{p}_E$ prices are identical and a kink in the price reaction function obtains which is also supported by the analysis of the price and profit intersections in proof 1. Note that at the kink p_I^{AccZ} and p_I^{DetZ} are independent of the locations d_E and d_I.

Finally, we compare Π_I^{gDetZ} with Π_I^{gAcc}. First we find the respective profit intersection at $p_I^c := p_E - 2tZ + t(d_I - d_E)$. See that $p_I^c \geq p_I^{Acc}$ only holds if $p_E \geq t(1 + 4Z - (d_I - d_E)) := p_E^c$. We can conclude that $\hat{p}_E < p_E^c$ for all $Z > 0$ which implies that $p_I^{Acc} > p_I^c$ for every $p_E < \hat{p}_E$. Now, inserting $p_E^\dagger$ yields $p_I^{DetZ}(p_E^\dagger) = t(1 + 4Z - 2\sqrt{2Z(1 + 2Z)})$, $p_I^{Acc}(p_E^\dagger) = t(1 + 2Z - \sqrt{2Z(1 + 2Z)})$ and $p_I^{Def}(p_E^\dagger) = 2t(1 - d_I + 2Z - \sqrt{2Z(1 + 2Z)})$. The relation $p_I^{DetZ}(p_E^\dagger) < p_I^{Acc}(p_E^\dagger)$ holds true for all $Z \geq 0$ and $p_I^{Acc}(p_E^\dagger) < p_I^{Def}(p_E^\dagger)$ holds true if $Z > 0$ and $d_I < d_I^\triangleleft$. This completes the proof.

Proof 3: The starting point are the profit functions Π_I^{Det}, Π_I^{DetZ}, Π_I^{Def}, Π_I^{AccZ}, and Π_I^{Acc} from proof 1. Now, their intersections and respective existence conditions are analyzed with respect to d_E.

Profits for the deterrence case Π_I^{Det} are linearly increasing in d_E with the slope $1 + Z$.

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[7]For clarity: Π_I^{gAccZ}, Π_I^{gAcc}, Π_I^{gDet}, and Π_I^{gDetZ} are general functions of the profits for arbitrary prices p_I, in proof 1 the rank of profits was derived with regards to firm I's five possible pricing strategies. Recall from equations (3.5) and (3.4) that Π_I^{gAccZ} gives the profits in p_I when firm I captures the indifferent consumer which equals a demand in the amount of the market boundary, i.e., $q_I = x$. Graphically, Π_I^{gAccZ} is depicted by a parabola, e.g., firm I's optimal choice is to charge $p_I^{AccZ}(\overline{p}_E)$ to realize the highest possible profits. Π_I^{gDet} incorporates the profits for any p_I when $q_I = x = 1 - d_E$. Graphically, Π_I^{gDet} is a line with the slope $1 + Z$.

All other profit functions decrease in d_E over the range $[0; \frac{1}{2}]$: Π_I^{DetZ} by the factor $\frac{1}{2} + Z$, Π_I^{Def} by the factor d_I, and the accommodation profits are convex in d_E with the minima at $\frac{1}{t}p_E + 1 + d_I + 2Z$ and $\frac{1}{t}p_E + 1 + d_I$ respectively. Both accommodation profits have the same covexity given by $\frac{\partial^2 \Pi_I^{AccZ}}{\partial d_E^2} = \frac{\partial^2 \Pi_I^{Acc}}{\partial d_E^2} = \frac{t}{4} > 0$. Thus, for $Z > 0$ the minimium of Π_I^{Acc} will always lie below the minimum of Π_I^{AccZ} and accommodation-Z profits will exceed accommodation profits over $d_E \in [0; \frac{1}{2}]$.

For high p_E deterrence dominates all other strategies. Setting $\Pi_I^{Det} = \Pi_I^{AccZ}$ yields the two solutions $d_E = 5 + d_I + \frac{1}{t}p_E + 6Z \pm 4\sqrt{(\frac{p_E}{t} + 1 + d_I + 2Z)(1 + Z)}$, the higher always lies above $\frac{1}{2}$ and thus the lower solution stated as $\overline{d}_E$ indicates the switchover point from the deterrence to the accommodation-Z strategy. Expression $\overline{d}_E > 0$ reduces to $p_E < t(3 - d_I + 2Z)$ for all $d_I, t, Z > 0$ named as $p_E^{\triangle}$ and marking the price limit for the accommodation-Z strategy to exist. Further, leveling respective pricing functions $p_I^{Det} = p_I^{AccZ}$ yields $\overline{d}_E^{\diamond} = 1 - \frac{1}{3}(d_I + \frac{1}{t}p_E - 2Z)$. We see that $\overline{d}_E < \overline{d}_E^{\diamond}$ for $p_E < p_E^{\triangle}$, thus at $\overline{d}_E$ firm I's price reaction is described by a discontinuity.

The intersections between the accommodation-Z strategy and the deterrence-Z strategy as well as between the accommodation strategy and the deference case each reveal a tangential solution. Firstly, setting $\Pi_I^{AccZ} = \Pi_I^{DetZ}$ and $p_I^{AccZ} = p_I^{DetZ}$ we obtain $\hat{d}_E = \frac{1}{t}p_E + d_I - 1 - 2Z$ or $\left. \frac{\partial \Pi_I^{AccZ}}{\partial d_E} \right|_{\hat{d}_E} = -t(\frac{1}{2} + Z)$. Secondly, matching $\Pi_I^{Acc} = \Pi_I^{Def}$ and $p_I^{Acc} = p_I^{Def}$ yields $\tilde{d}_E = 1 - 3d_I + \frac{1}{t}p_E$ or $\left. \frac{\partial \Pi_I^{Acc}}{\partial d_E} \right|_{\tilde{d}_E} = -td_I$. For $d_E > \hat{d}_E$ the accommodation-Z strategy is prefered to the deterrence-Z strategy, and for $d_E > \tilde{d}_E$ the accommodation strategy is prefered to the deference strategy. If the incumbent would show an accommodating behavior for locations below these boundaries he would be undercut by the entrant. Thus, his pricing behavior changes at $\hat{d}_E$ and $\tilde{d}_E$, and the corresponding price reactions are described by a kink. Additionally, intersecting accommodation profits with the deterrence-Z profits yields the two solutions $\frac{1}{t}p_E - (1 - d_I + 4Z) \pm 2\sqrt{2Z(1 + 2Z)}$ where only the higher term is applicable with respect to $d_E \in [0; \frac{1}{2}]$ which we denote as $d_E^{\dagger}$. The intersection of deterrence profits with accommodation profits yields $5 + d_I + \frac{1}{t}p_E + 4Z \pm 4\sqrt{(\frac{p_E}{t} + 1 + d_I + Z)(1 + Z)}$ where only the lower solution is feasible w.r.t. d_E denoted as $d_E^{\ddagger}$.

We turn to the intersections of the linear profit functions. Leveling $\Pi_I^{Det} = \Pi_I^{DetZ}$ yields $d_E^{\times} = \frac{2(1+Z) - \frac{1}{t}p_E - d_I}{3 + 4Z}$ with deterrence as the preferred strategy for $d_E > d_E^{\times}$ and deterrence-Z dominating for $d_E < d_E^{\times}$. Matching $\Pi_I^{DetZ} = \Pi_I^{Def}$ we obtain $\breve{d}_E^{\dagger} = \frac{1}{t}p_E - d_I \left(\frac{1 - 2d_I - 2Z}{1 - 2d_I + 2Z} \right)$ with the deterrence-Z startegy as the preferred option for $d_E < \breve{d}_E^{\dagger}$ and deference for the remaining part. Likewise, we define the intersection for $\Pi_I^{Det} = \Pi_I^{Def}$ as $\breve{d}_E^{\ddagger} = 1 - d_I - \frac{p_E(1 - d_I + Z)}{t(1 + d_I + Z)}$ with deterrence prefered for $d_E > \breve{d}_E^{\ddagger}$. Since p_I^{DetZ} and p_I^{Def} are parallel and p_I^{DetZ} intersects p_I^{Det} at $d_E = \frac{1}{2}$ $\forall p_E$ respective transitions in the pricing behavior of the three strategies are characterized by a discontinuity.

Next, we determine the ranges of validity for the derived intersections. As already argued part (I) refers to the condition $\overline{d}_E \geq 0$ or $p_E \leq p_E^{\triangle}$ which holds for all $d_I, t, Z > 0$, to rule out the deterrence-Z strategy $\overline{d}_E < \hat{d}_E$ must hold which reduces to $p_E > t\left(\frac{5+12Z+8Z^2}{4(1+Z)} - d_I\right) := p_E^{\triangleleft}$. Further, $\overline{d}_E = \frac{1}{2}$ yields $p_E = t\left(\frac{7}{2} - d_I + 2Z \pm 2\sqrt{2}\sqrt{(1+Z)}\right)$ with $p_E^{\triangle}$ and $p_E^{\triangleleft}$ lying within these boundary values for all $d_I, Z \geq 0$. Thus, the accommodation-Z strategy is profitable for $0 \leq d_E < \overline{d}_E$ and the deterrence strategy is prefered for $\overline{d}_E < d_E < \frac{1}{2}$. To check for consistency see that $p_E^{\triangle} > p_E^{\triangleleft}$ holds for $d_I, Z \geq 0$. In addition, $\overline{d}_E$ is monotonically decreasing in p_E $\forall p_E \in [0; p_E^{\triangle}]$ and $\hat{d}_E$ is monotonically increasing in p_E with slope $\frac{1}{t}$.

Part (II.a) and part (II.b) require $\overline{d}_E > \hat{d}_E$ or $p_E < p_E^{\triangleleft}$. The lower price bound for (II.a) is determined by $\hat{d}_E > 0$ or $p_E > t(1 - d_I + 2Z) := p_E^{\triangleright}$. A consistency check reveals that $p_E^{\triangleleft} > p_E^{\triangleright}$ holds for $d_I, Z \geq 0$. Consider also that $d_E^{\times} > \hat{d}_E$ is fullfilled for $p_E < p_E^{\triangleleft}$ and $d_I, Z \geq 0$. Further, $d_E^{\times} < \frac{1}{2}$ requires $p_E > \frac{1}{2}t(1 - 2d_I) := p_E^{\bowtie}$, and we see that $p_E^{\bowtie} < p_E^{\triangleright}$ holds for $d_I, Z \geq 0$. Thus, $p_E^{\bowtie}$ and $p_E^{\triangleleft}$ constitute the price bounds for part (II.a).

Part (III.a) accounts for $\hat{d}_E < 0$ or $p_E < p_E^{\triangleright}$ and covers solely the deterrence-Z strategy and the deterrence strategy with $d_E^{\times}$ as the point of indifference. The existence of the accommodation strategy (p_I^{Acc}, $x < \frac{1}{2}$) hinges upon the value of d_I and Z. Graphically, for a decreasing d_I the tangent in $\tilde{d}_E$ shifts downwards. This implies that the value of $\tilde{d}_E$ increases and charging p_I^{Acc} becomes profitable. Analytically, $d_E^{\dagger} < \tilde{d}_E$ states the condition for p_I^{Acc} to exist which reduces to the familiar relation $d_I < \frac{1}{2} + Z + \sqrt{Z(\frac{1}{2} + Z)} := d_I^{\triangleleft}$ (cf. lemma 1 and proof 1). Thus, for every $d_I > d_I^{\triangleleft}$ the accommodation strategy does not exist. If the accommodation strategy exists the profit functions Π_I^{DetZ} and Π_I^{Acc} intersect. Now, $d_E^{\times}$ is monotonically decreasing in p_E, thus, for $d_E^{\times} = d_E^{\dagger}$ profits for the deterrence-Z strategy, the deterrence strategy and the accommodation strategy are equal. Then $d_E^{\times} < d_E^{\dagger}$ reduces to $p_E > \frac{t\left(5+18Z+16Z^2-4d_I(1+Z)-2\sqrt{2}\sqrt{Z(1+2Z)(3+4Z)^2}\right)}{4(1+Z)} := p_E^{\triangledown\dagger}$. In addition, the deterrence strategy in part (III.a) is prefered for every $d_E > d_E^{\times}$ which requires $p_E > p_E^{\bowtie}$. To establish the reaction function in (III.a) we find $p_E^{\triangledown\dagger} > p_E^{\bowtie}$ $\forall d_I, Z \geq 0$.

To derive the expressions in part (II.b) and (III.b) we have to compare $p_E^{\triangleright}$ with $p_E^{\triangledown\dagger}$. There is no definite order between these price boundaries instead it depends on the value of Z. Particularly, we find that $p_E^{\triangleright} > p_E^{\triangledown\dagger}$ holds if Z exceeds a numerical value of $\zeta \approx 0.015503$ (this refers to the numerical solution of the third root of a third degree polynomial in Z)[8]. Consequently, for $Z < \zeta$ $p_E^{\triangledown\dagger} > p_E^{\triangleright}$ holds. It follows that under these conditions the accommodation-Z strategy is prefered for $0 \leq d_E < \hat{d}_E$ since $p_E > p_E^{\triangleright}$. Further, the accommodation strategy is feasible since $p_E < p_E^{\triangledown\dagger}$ or $d_E^{\times} > d_E^{\dagger}$, the ranges for p_I^{Acc} are determined by respective profit intersections, i.e., $d_E^{\dagger} < d_E < d_E^{\ddagger}$. A consistency check shows $p_E^{\triangleleft} > p_E^{\triangledown\dagger}$ for all

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[8]in Mathematica: $Root\left[-1 + 62\#1 + 160\#1^2 + 96\#1^3 \&, 3\right]$

$d_I, Z, t > 0$, thus $p_E^{\nabla\dagger}$ constitutes the price bound for parts (II.b), (III.a) and (III.b). Finally, for part (III.b) for deterrence to exist see that $d_E^{\ddagger} = \frac{1}{2}$ yields a price set with the bounds $\frac{1}{2}t\left(7 - 2d_I + 8Z \pm 4\sqrt{2}\sqrt{1 + 3Z + 2Z^2}\right)$. This set contains $p_E^{\triangleright}$ for all $d_I, Z \geq 0$ which completes the proof.

Proof 4: We use the profit functions and intersections from the preceeding proof. Part (I) refers to lemma 3 since the price relation $p_E^{\triangle} > p_E^{\triangleleft}$ holds for all $d_I, Z \geq 0$. For parts (II) and (III) we follow the same line of argumentation but examine the case $d_I > d_I^{\triangleleft}$.

If the accommodation strategy does not exist, Π_I^{DetZ} and Π_I^{Det} intersect with Π_I^{Def}. Then the condition $d_E^{\times} = \breve{d}_E^{\dagger}$ marks the transition. Setting $d_E^{\times} < \breve{d}_E^{\dagger}$ yields $p_E > \frac{(1-2d_I)t(1+d_I+Z)(1+2Z)}{2(1-2d_I+2Z)(1+Z)} := p_E^{\nabla\ddagger}$, and as expected $\breve{d}_E^{\dagger}(p_E^{\nabla\ddagger}) = \breve{d}_E^{\ddagger}(p_E^{\nabla\ddagger})$. Additionally, $d_E^{\times} < \frac{1}{2}$ or $p_E > p_E^{\bowtie}$ must hold. A consistency check reveals that $p_E^{\nabla\ddagger} > p_E^{\bowtie}$ holds true for $Z > 0$ and $0 < d_I < \frac{1}{2}$.

Next, we have to compare $p_E^{\triangleright} > p_E^{\nabla\ddagger}$. Solving for equality yields the two solutions $d_I^{(1,2)} = \frac{5+14Z+8Z^2\pm\sqrt{1-60Z-284Z^2-416Z^3-192Z^4}}{2(6+8Z)}$. This implies that for $d_I^{(1)} < d_I < d_I^{(2)}$ we obtain $p_E^{\triangleright} < p_E^{\nabla\ddagger}$. Since $p_E^{\nabla\ddagger}$ has a singularity at $d_I = \frac{1}{2} + Z$ for $Z > 0$ we see that the intersections only occur at the lower branch of the splitted function. Further, the existence of the intersections $d_I^{(1,2)}$ is restricted to the positive discriminant of the root, i.e., to a value $Z = \zeta \approx 0.015503$ (which corresponds to the same root as in proof of lemma 3). Thus, for $Z > \zeta$ the inequality $p_E^{\triangleright} > p_E^{\nabla\ddagger}$ holds for all $d_I \in [0; \frac{1}{2}]$. We still have to check $p_E^{\triangleleft} > p_E^{\nabla\ddagger}$ which holds for all $d_I, Z \geq 0$. Finally, for deterrence to exist $\breve{d}_E^{\ddagger} < \frac{1}{2}$ yields $p_E > t\frac{(1-2d_I)(1+d_I+Z)}{2(1-d_I+Z)}$, and $p_E^{\triangleright} > t\frac{(1-2d_I)(1+d_I+Z)}{2(1-d_I+Z)}$ holds for $d_I, Z \geq 0$ which completes the proof.

Proof 5a: Firstly, we set $\breve{\Pi}_E := (1 - d_I + Z)\breve{p}_E$ and see that $\breve{\Pi}_E$ linearly increases in d_E with slope $\frac{t\left(1-3d_I+2d_I^2+3Z-4d_I Z+2Z^2\right)}{1-2d_I+2Z}$; the slope is positive for all Z and $d_I < 1+Z$. Secondly, consider that the minimum for $\Pi_E^{Acc}(p_E^{Acc})$ given at $d_E = d_I - 3 - 2Z$ is negative for $d_I, Z \in [0, \frac{1}{2}]$ which implies that respective profits monotonically increase in d_E over the positive range. Next, we match the profit functions $\breve{\Pi}_E = \Pi_E^{Acc}(p_E^{Acc})$. This yields a solution function with linear and quadratic terms in d_E, and linear, quadratic terms and terms to the power of three in d_I and Z as well as mixed terms between d_E, d_I and Z. We find the following two solutions in d_E:

$$d_E^{(1,2)} = \frac{1}{1 - 2d_I + 2Z}\left(5 + 14d_I^2 + 16Z + 12Z^2 - d_I(17 + 26Z) \pm 4\Gamma\right) \qquad (3.26)$$

with:
$\Gamma = \sqrt{\alpha d_I^4 - \beta d_I^3 + \gamma d_I^2 - \delta d_I + \xi}$

$\alpha = 8$

$\beta = 20(1 + 2Z)$

$\gamma = 2\left(9 + 36Z + 32Z^2\right)$

$\delta = \left(7 + 40Z + 72Z^2 + 40Z^3\right)$

$\xi = (1 + Z)(1 + 2Z)^3$

Graphically, this solution set corresponds to the intersections of the linear function $\check{\Pi}_E$ and the parabola $\Pi_E^{Acc}(p_E^{Acc})$ (see figure 3.12). Restricting the lower boundary solution to $d_E^{(1)} < \frac{1}{2}$ shows that this inequality is not fulfilled for any $d_I < \frac{1}{2}$ if $Z > 0$. Additionally, see that $d_E^{\triangleright} < d_E^{(1)}$ and $d_E^{(1)} < d_E^{(2)}$ for all $0 < Z < \frac{1}{2}$ and $d_I < \frac{1}{2}(1 + 3Z) - \frac{1}{2}\sqrt{2Z + 5Z^2}$. At $d_I = \frac{1}{2}(1 + 3Z) - \frac{1}{2}\sqrt{2Z + 5Z^2}$ the intersections $d_E^{(1,2)}$ collapse ($\Gamma = 0$) and the profit functions intersect in a tangential point, if d_I further increases Π_E^{Acc} dominates $\check{\Pi}_E$ for all $d_E \geq 0$. This completes the proof.

Figure 3.12: Illustration of the relation between $\check{\Pi}_E$ and $\Pi_E^{Acc}(p_E^{Acc})$ against d_E
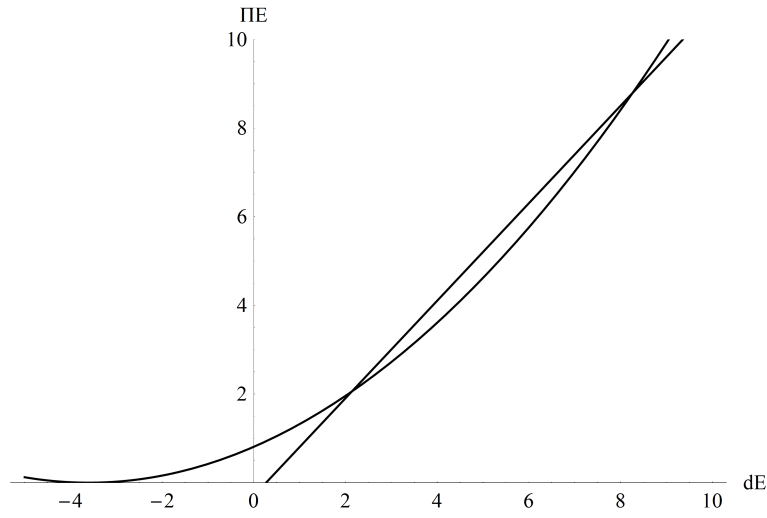
Comment: Parameter values are $t = 1$, $d_I = 0.4$, and $Z = \frac{1}{2}$.

Proof 5b: Let us denote $\Pi_E^{\times} := \frac{1}{2}p_E^{\times}$ and take over $\check{\Pi}_E$ from proof 5a. By taking the respective first derivatives it is straightforward to see that $\Pi_E^{\times}$ linearly decreases and $\check{\Pi}_E$ linearly increases in d_E for $d_I \in [0, \frac{1}{2}]$ and $Z \geq 0$. Thus, we find the profit intersection at:

$$d_E^{split} = \frac{2 - 7d_I + 8d_I^2 - 4d_I^3 + 6Z - 4d_I Z + 4Z^2 + 4d_I Z^2}{5 - 12d_I + 4d_I^2 + 16Z - 16d_I Z + 12Z^2} \tag{3.27}$$

d_E^{split} shows a complex dependency on d_I and Z since it comprises of two polynomials in the nominator and denominator. Let us first consider the impact of d_I. For $Z = 0$

d_E^{split} reduces to $\frac{2-7d_I+8d_I^2-4d_I^3}{5-12d_I+4d_I^2}$ where the nominator and the denominator equal zero at $d_I = \frac{1}{2}$ and the denominator yields a second null at $\frac{5}{2}$. Thus, $d_E^{split}(Z=0)$ is not defined at $d_I = \frac{1}{2}$ and over the range $0 \leq d_I < \frac{1}{2}$ the function is monotonically decreasing in d_I. For $Z > 0$ the dependency of d_E^{split} on d_I is solely determined by the null of the denominator given at $\frac{1}{2}(1+2Z)$. For increasing Z the singularity increases and the convexity of d_E^{split} in d_I on the interval $[0, \frac{1}{2}]$ decreases. As a result d_E^{split} is shifted upwards for reasonably large locations d_I (see figure 3.13). As regards the dependency of d_E^{split} on Z we consider the singularity at $\frac{1}{2}(-1+2d_I)$, for Z exceeding this boundary d_E^{split} is monotonically increasing in Z. The singularity shifts towards the origin for increasing d_I and d_E^{split} converges to $\frac{1}{2}$ for $d_I \to \frac{1}{2}$. (see figure 3.9) Further, we see that $d_E^{split} < \frac{1}{2}$ for $Z > 0$ and $\frac{1}{2} - \sqrt{\frac{1}{2}+Z+Z^2} < d_I < \frac{1}{2}$ and that $d_E^{split} > 0$ for all $d_I, Z \in [0, \frac{1}{2}]$. Clearly, for $d_E^{split} < 0$ we get $\check{\Pi}_E > \Pi_E^{\times}$, and if $d_E^{split} > \frac{1}{2}$ the relation $\check{\Pi}_E < \Pi_E^{\times}$ obtains. A consistency check shows that $d_E^{\triangleright} < d_E^{split}$ for all $Z, d_I \in [0, \frac{1}{2}]$ and that $\frac{1}{2} - \sqrt{\frac{1}{2}+Z+Z^2} < d_I^{\triangleleft}$ for all $Z \geq 0$ which completes the proof. We conclude firstly that undercutting the incumbent setting $\check{p}_E$ will never strictly dominate the strategy of charging $p_E^{\times}$, and secondly that only for locations of $d_I > \frac{1}{2} - \sqrt{\frac{1}{2}+Z+Z^2}$ the strategy to charge $\check{p}_E$ will yield higher profits than setting $p_E^{\times}$.

Figure 3.13: Illustration of the relation between d_E^{split} and d_I for different values of Z
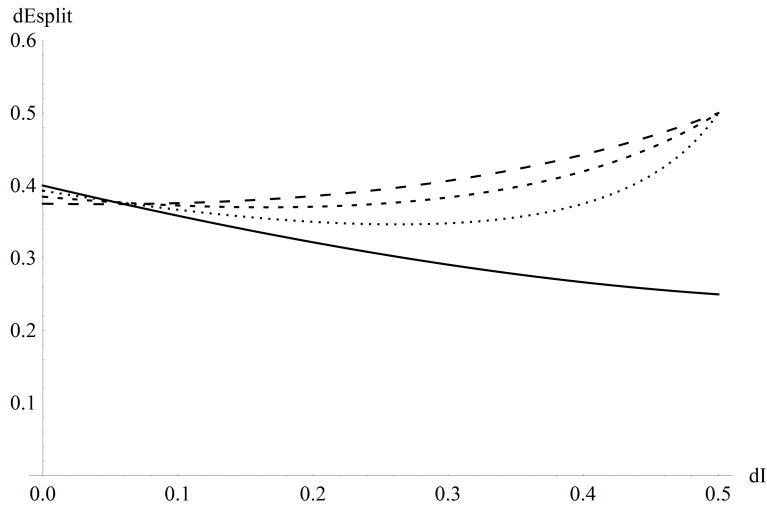
Comment: An increase in Z leads to an increase in d_E^{split} as d_I grows depicted for $Z = 0$ (solid curve), $Z = 0.1$ (tiny dashed curve), $Z = 0.25$ (small dashed curve), and $Z = 0.5$ (medium dashed curve). Parameter values are $t = 1$.

Proof of Proposition 1: Based on the previous analyses in proof $5a$ and $5b$ we firstly define firm E's profit functions and summarize their characteristics. Secondly, we conduct a comparative profit analysis. The relevant profit functions for firm E's

profit optimization problem are:

$\Pi_E(\overline{p}_E, 1 - x(p_I^{AccZ})) \equiv \overline{\overline{\Pi}}_E = t((3 + d_E - d_I + 2Z)\sqrt{d_E(1 + Z)} - 4d_E(1 + Z))$,

$\Pi_E(p_E^{Acc}, 1 - x(p_I^{AccZ})) = \Pi_E^{Acc}(p_E^{Acc}) = \frac{1}{16}t(3 + d_E - d_I + 2Z)^2$,

$\Pi_E(p_E^Z, 1 - x(p_I^{Acc}) + Z) \equiv \Pi_E^Z = \frac{1}{2}t(1 + d_E - d_I)(1 + 2Z)$,

$\Pi_E(p_E^\times, 1 - x(p_I^{DetZ})) = \Pi_E^\times = \frac{1}{2}t(2 - 3d_E - d_I + 2Z - 4d_EZ)$, and

$\Pi_E(\check{p}_E, 1 - x(p_I^{Def}) + Z) = \check{\Pi}_E$.

Π_E^Z and $\check{\Pi}_E$ increase linearly in d_E with steeper slopes as Z grows. Further, $\Pi_E^\times$ decreases linearly in d_E also with increasing slopes in Z. Profits $\Pi_E^{Acc}(p_E^{Acc})$ increase monotonically in d_E over the positive range for all $d_I, Z \in [0, \frac{1}{2}]$. Finally, $\overline{\overline{\Pi}}_E$ shows a local maximum at $d_E^{Max} = \frac{1}{9}\left(23 + 3d_I + 26Z - 8\sqrt{7 + 3d_I + 17Z + 3d_IZ + 10Z^2}\right)$ and intersects $\Pi_E^{Acc}(p_E^{Acc})$ at $d_E = 29 + d_I + 30Z \pm 8\sqrt{13 + d_I + 27Z + d_IZ + 14Z^2}$ where we denote the lower intersection with d_E^{Ints}. See that the first derivatives of $\overline{\overline{\Pi}}_E$ and $\Pi_E^{Acc}(p_E^{Acc})$ in d_E are identical at the profit intersections, particularly at d_E^{Ints}, from which we conclude that $\overline{\overline{\Pi}}_E$ is dominated by $\Pi_E^{Acc}(p_E^{Acc})$ over the domain.

As regards the comparison of $\Pi_E^{Acc}(p_E^{Acc})$ with Π_E^Z two intersections at $d_E = 1 + d_I + 6Z \pm 4\sqrt{Z + 2Z^2}$ obtain. A nonnegative set requires $1 + d_I + 6Z - 4\sqrt{Z + 2Z^2} < \frac{1}{2}$ which holds for all $Z > 0$ and $d_I < \frac{1}{2}(-1 - 12Z) + 4\sqrt{Z + 2Z^2}$. Since d_I is bounded Z is restricted to $0 < \frac{1}{2}(-1 - 12Z) + 4\sqrt{Z + 2Z^2} < \frac{1}{2}$. The upper bound is fullfilled for all $0 \leq Z < \frac{1}{2}$, the lower bound reduces to $\frac{1}{4}\left(5 - 2\sqrt{6}\right) < Z < \frac{1}{4}\left(5 + 2\sqrt{6}\right)$. Thus, for $Z < \frac{1}{4}\left(5 - 2\sqrt{6}\right)$ the rank $\Pi_E^{Acc}(p_E^{Acc}) > \Pi_E^Z$ holds for all $d_E, d_I \in [0, \frac{1}{2}]$, and for Z above this boundary it holds if $d_E < 1 + d_I + 6Z - 4\sqrt{Z + 2Z^2} := d_E^Z$. Additionally, a further match reveals that $d_E^Z > d_E^\triangleright$ holds for a threshold value of $d_I > \frac{1}{4 + 4Z} + 4\sqrt{Z + 2Z^2} - 1 - 6Z := d_I^{critZ}$ for all $Z \geq 0$. Referring to firm I's reaction functions see then immediately that $d_I^\triangleleft > d_I^{critZ}$ for all $Z \geq 0$. In sum this implies that for locations $d_I < d_I^\triangleleft$ and $d_E < d_E^\triangleright$ the rank $\Pi_E^{Acc}(p_E^{Acc}) > \Pi_E^Z$ is always true. For distant locations of the incumbent the strategy of charging p_E^{Acc} does not dominate p_E^Z anymore.

Next we consider the relation between $\Pi_E^{Acc}(p_E^{Acc})$ and $\Pi_E^\times$. We demonstrate that $p_E^\times$ is only an alternative for firm E for the location range $d_E > d_E^\triangleright$, i.e. when p_I^{AccZ} is not a profitable strategy option, but rather p_I^{Det} and p_I^{DetZ} respectively are preferred. This follows from the rank $\hat{p}_E < p_E^\times < \overline{p}_E$ which holds true for all $d_I, Z \in [0, \frac{1}{2}]$ and as expected for $d_E < d_E^\triangleright$. For $d_E < d_E^\triangleright$ firm E's strategy to capture the center is given by $\hat{p}_E$ which is dominated by p_E^{Acc} for all $0 \leq d_E, Z \leq \frac{1}{2}$. Clearly, firm E could charge $p_E^\times$ when $d_E < d_E^\triangleright$, profits then would be $\Pi_E^{Acc}(p_E^\times)$ since the incumbent reacts with p_I^{AccZ}. Thus, match $\Pi_E^{Acc}(p_E^{Acc}) = \Pi_E^{Acc}(p_E^\times)$ and the tangential solution $d_E = \frac{1 - d_I + 2Z}{7 + 8Z}$ obtains which is also evident since p_E^{Acc} solves the first order condition. Also note that $p_E^\times$ and p_E^{Acc} intersect at this d_E. Alternatively, see that profits $\Pi_E^{Acc}(p_E^\times)$ and $\Pi_E^\times$ intersect at $d_E^\triangleright$ which is fully in accord with the suggested order of entry prices. We conclude that the strategy p_E^{Acc} dominates $p_E^\times$ for all $d_I, Z \in [0, \frac{1}{2}]$

and $d_E < d_E^\triangleright$. Put differently, the match $\Pi_E^{Acc}(p_E^{Acc}) = \Pi_E^\times$ is not of economic interest in the range $d_E < d_E^\triangleright$ since firm I would never choose p_I^{DetZ} when $p_E \in [\hat{p}_E; \overline{p}_E]$.
To conclude the proof recall from proof $5a$ that $\Pi_E^{Acc}(p_E^{Acc}) > \check{\Pi}_E$ holds over the domain $d_E, d_I, Z \in [0, \frac{1}{2}]$.

Proof of Proposition 2: Recall $\frac{\partial \Pi_E^{Acc}}{\partial d_E} > 0$ and from the preceding proof that $\frac{\partial \overline{\Pi}_E}{\partial d_E} = 0$ yields d_E^{Max} and that $\Pi_E^{Acc}(p_E^{Acc})$ and $\overline{\Pi}_E$ intersect at d_E^{Ints}. Note that the intersection lies below the maximum on the given domain, i.e., $d_E^{Ints} < d_E^{Max}$ for all $d_I, Z \in [0, \frac{1}{2}]$.
Next, we compare the maximum with the boundary for p_I^{AccZ} to be applicable and set $d_E^{Max} < d_E^\triangleright$ which holds for all $Z \geq 0$ and $d_I > \frac{4Z + 8Z^2 - 1}{4 + 4Z} := d_I^{cMax}$. This implies that for every $Z < \frac{1}{4}\left(\sqrt{3} - 1\right)$ the maximum lies within the range of $d_E^\triangleright$ and for a higher Z there exists a set of $d_I < d_I^{cMax}$ such that d_E^{Max} is out. More specifically by applying $d_I^{cMax} < d_I^\triangleleft$ we find that[9] for $Z < Z' \approx 0.379$ for every $d_I > d_I^\triangleleft$ also $d_I > d_I^{cMax}$. By contrast for $Z > Z'$ there exist some d_I such that $d_I^\triangleleft < d_I < d_I^{cMax}$. So far we have established conditions such that firm E maximizes his profits $\overline{\Pi}_E$ over d_E, however, from the preceding proof we know that $\Pi_E^{Acc}(p_E^{Acc})$ yields higher profits. It is straightforward to show that this is only true for locations $d_E < d_E^{Ints}$. Intuitively, the profit maximizing accommodation price of firm E p_E^{Acc} can not become arbitrarily large. The limit is set by the price such that the incumbent is indifferent between deterring the entrant or playing p_I^{AccZ}. This price is of course $\overline{p}_E$. Now for certain location pairs (d_I, d_E) the rank $p_E^{Acc} > \overline{p}_E$ obtains. (cf. equ. (3.20) in subsection 3.1) Evaluating this particular expression with respect to d_E yields the term for d_E^{Ints} which essentially proofs our proposition. In short $\overline{\Pi}_E = \Pi_E^{Acc}(p_E^{Acc})$ and $p_E^{Acc} = \overline{p}_E$ at d_E^{Ints}. Thus, for $d_E < d_E^{Ints}$ the entrant charges p_E^{Acc} and for $d_E > d_E^{Ints}$ he sets $\overline{p}_E$. Since $d_E^{Ints} < d_E^{Max}$ the profit maximizing location is of course $d_E^{Max} = d_E^*$. The transition in profits and prices is described by a kink. Clearly, inserting d_{Max} into $\overline{p}_E$ and evaluating the expression yields p_E^*.
For p_E^* and d_E^* to be the optimal solution in the range $d_E < d_E^\triangleright$ we have to show that the pricing strategy $\overline{p}_E$ also dominates the two strategies $p_E^\times$ and $\hat{p}_E$ that lead the entrant to decrease his prices. Thus, we firstly match $\overline{\Pi}_E = \Pi_E^{Acc}(p_E^\times)$ and find the intersections at $d_E^\triangleright$ and $s^{(1,2)} = \frac{14 - 3d_I + 22Z - 4d_I Z + 8Z^2 \pm 4\sqrt{10 - 3d_I + 28Z - 7d_I Z + 26Z^2 - 4d_I Z^2 + 8Z^3}}{(3 + 4Z)^2}$.
See that at $d_E^\triangleright$ the profit functions share a tangential intersection since the first derivatives in d_E are equal and that for $d_E \in [s^{(1)}, s^{(2)}]$ $\overline{\Pi}_E > \Pi_E^{Acc}(p_E^\times)$ since $\overline{p}_E > p_E^\times$. (see figure 3.14) Furthermore, we find that $s^{(1)} \leq d_E^{Ints}$ holds for all $d_I \in [0, \frac{1}{2}]$ and $Z \geq 0$ which proves that $\overline{\Pi}_E > \Pi_E^{Acc}(p_E^\times)$ holds for all $s^{(1)} < d_E < d_E^\triangleright$ and thus rules out the strategy to charge $p_E^\times$. Secondly, we set $\overline{\Pi}_E = \hat{\Pi}_E$ which yields the three solutions $d_E^\triangleright$ and $s^{(1,2)} = 7 + d_I + 6Z \pm 4\sqrt{3 + d_I + 5Z + d_I Z + 2Z^2}$ with $\overline{\Pi}_E > \hat{\Pi}_E$

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[9] in Mathematica: $Root\left[-9 - 4\#1 + 52\#1^2 + 56\#1^3 \&, 3\right]$

for $\mathbf{s}^{(1)} < d_E < d_E^{\triangleright}$. Also note that $\mathbf{s}^{(2)} > \frac{1}{2}$ for all $d_I, Z \geq 0$. Moreover $\mathbf{s}^{(1)} < d_E^{Max}$ holds for $Z \geq 0$ and $2\sqrt{Z + Z^2} - 2 < d_I < 2(15 + 16Z) - 10\sqrt{8 + 17Z + 9Z^2}$ where the boundaries comprise the domain $d_I \in [0; \frac{1}{2}]$ for all $0 \leq Z \leq \frac{1}{2}$. A further drill down shows that $\mathbf{s}^{(1)} < d_E^{Ints}$ for all $Z \geq 0$ and $d_I^{cMax} < d_I$, thus, $\hat{p}_E$ is not a profitable pricing strategy compared to $\bar{p}_E$ if $d_E < d_E^{\triangleright}$ and $d_I^{cMax} < d_I$.

To conduct a final consistency check we use part (I) of the location reaction function in lemma 3 and 4. See that the rank $p_E^* < p_E^{\triangle}$ holds for all $d_I, Z \in [0; \frac{1}{2}]$ and that $p_E^* > p_E^{\triangleleft}$ remains true for the location set $d_I > d_I^{cMax}$ for all $Z \geq 0$. Thus, if firm E chooses his profit maximizing set (p_E^*, d_E^*) firm I reacts with p_I^{AccZ} or p_I^{Det} according to his location reaction function. We use the boundary $\bar{d}_E$ and insert p_E^*. Comparing $\bar{d}_E(p_E^*)$ with d_E^* reveals that the local maximum d_E^* does not exceed the boundary for all $d_I, Z \in [0; \frac{1}{2}]$. By contrast, we find that the relation $d_E^* > \bar{d}_E(p_E^*)$ holds for $Z \geq 0$ and $d_I > 31 + 30Z$ which shows that under the entry set (p_E^*, d_E^*) deterrence does not occur. This completes the proof.

Figure 3.14: Illustration of the relation between the profit functions $\overline{\Pi}_E$, $\Pi_E^{Acc}(p_E^{Acc})$, $\Pi_E^{\times}$, and $\Pi_E^{Acc}(p_E^{\times})$
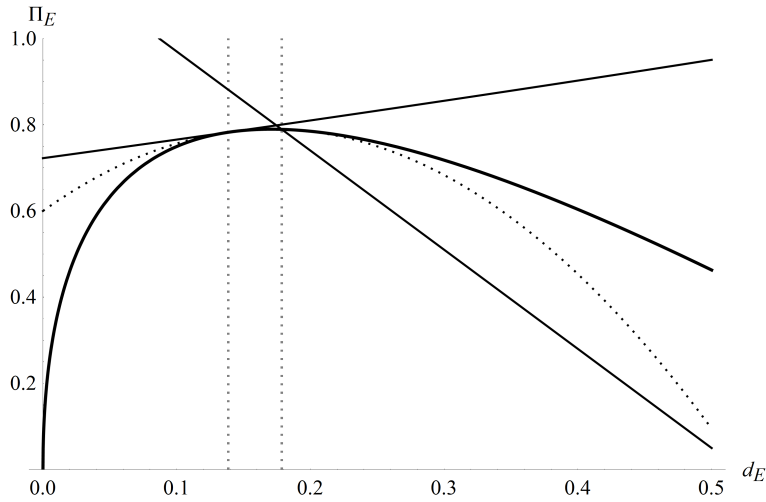
Comment: $\overline{\Pi}_E$, $\Pi_E^{Acc}(p_E^{Acc})$, and $\Pi_E^{\times}$ are depicted as solid functions, $\Pi_E^{Acc}(p_E^{\times})$ is depicted as the dotted curve. The vertical dashed lines indicate the intersections of $\overline{\Pi}_E$ with $\Pi_E^{Acc}(p_E^{\times})$. For locations $d_E > \mathbf{s}^{(1)} \approx 0.139$ charging $\bar{p}_E$ yields higher profits than $p_E^{\times}$, at $d_E = d_E^{\triangleright} \approx 0.179$ respective profit functions share a tangential intersection. Parameter values are $d_I = 0.4$, $Z = 0.4$ and $t = 1$.

Proof of Proposition 3: For $d_E > d_E^{\triangleright}$ charging p_I^{AccZ} is not feasible, thus $\bar{p}_E$ and p_E^{Acc} are not part of firm E's strategy set. Therefore we are left to evaluate the relation between Π_E^Z, $\Pi_E^{\times}$ and $\check{\Pi}_E$ and derive solutions sets for market entry on the domain $d_E > d_E^{\triangleright}$. In addition, we compare these with the profit-maximizing set (p_E^*, d_E^*). Firstly, matching the linear functions $\Pi_E^{\times}$ and Π_E^Z yields $\Pi_E^{\times} > \Pi_E^Z$ for $d_E < \frac{1 + 2d_I Z}{2(2 + 3Z)} :=$ $d_E^{Z\times}$ with $0 \leq d_E^{Z\times} < \frac{1}{2}$ for all $d_I, Z \in [0, \frac{1}{2}]$. Secondly, recall from proof 5b that

$\Pi_E^\times > \check{\Pi}_E$ holds for $d_E < d_E^{split}$ and $d_I, Z \in [0, \frac{1}{2}]$, and thirdly see that $\Pi_E^Z > \check{\Pi}_E$ holds for all $Z > 0, d_I < \frac{1}{2}$.

Referring to lemma 1 and subsection 3.2.2 recall that for $d_I^\triangleleft < d_I < d_I^\triangleright$ and $d_E > d_E^\triangleright$ charging $p_E^\times$ is the preferred strategy for locations $d_E < d_E^{split}$ and $\check{p}_E$ for $d_E > d_E^{split}$. The transition in prices is described by a discontinuity since for all $Z \geq 0$ and $d_I < \frac{1}{2}$ the profit intersection d_E^{split} lies below the location d_E where $\check{p}_E$ and $p_E^\times$ intersect. For close locations $d_I > d_I^\triangleright$ firm E's undercutting price is $\check{p}_E < 0$, thus the pricing strategy $p_E^\times$ dominates. Next we have to consider that $\frac{\partial \Pi_E^\times}{\partial d_E} < 0$ and $\frac{\partial \check{\Pi}_E}{\partial d_E} > 0$. (cf. proof of proposition 1) This implies that firm E has no incentive to increase his location d_E when $d_E < d_E^{split}$ or when $p_E^\times$ is the only pricing option respectively. In particular, the best location firm E can choose under $p_E^\times$ is $d_E = d_E^\triangleright$ since this is where profits $\Pi_E^\times$ and $\overline{\Pi}_E$ as well as prices $p_E^\times$ and $\overline{p}_E$ intersect. Accounting for the results from proof of proposition 2 we can conclude that provided $d_I > d_I^{cMax}$ the set (p_E^*, d_E^*) yields higher profits than the best price-location set under the pricing strategy $p_E^\times$. Further, we find that the transition from $\overline{p}_E$ to $p_E^\times$ and $\overline{\Pi}_E$ to $\Pi_E^\times$ respectively is described by a kink at the location $d_E^\triangleright$.

Provided that $\check{p}_E > 0$, the behavior of $\check{\Pi}_E$ suggests that firm E increases his location when $d_E > d_E^{split}$. Recall from proof 5b that $d_E^\triangleright < d_E^{split}$ for all $Z, d_I \in [0, \frac{1}{2}]$. Thus, we compare the profits for the profit-maximizing set (p_E^*, d_E^*) with profits under the pricing strategy $\check{p}_E$ at $d_E = \frac{1}{2} - \epsilon, \epsilon \to 0$. We omit the fact that firm I deters entry at a location below $d_E = \frac{1}{2}$, however, since $\check{\Pi}_E$ and $\check{p}_E$ monotonically increase in d_E we can conclude that the comparison indicates an upper boundary for the profitability of the undercutting strategy. Clearly, for locations $d_E < \frac{1}{2}$ and particularly for locations where the undercutting strategy $\check{p}_E$ is viable undercutting profits do not exceed $\overline{\Pi}_E(d_E^{Max})$ under the following conditions.

Evaluating $\overline{\Pi}_E(d_E^{Max}) = \check{\Pi}_E(d_E = \frac{1}{2})$ yields a complex numerical solution set in d_I and Z. For the domain $d_I, Z \in [0; \frac{1}{2}]$ we find that the profits for the set (p_E^*, d_E^*) strictly increase profits $\check{\Pi}_E(d_E = \frac{1}{2})$ if $Z > \kappa \approx 0.0305$.[10] For $Z < \kappa$ playing $\check{p}_E$ implies $\overline{\Pi}_E(d_E^{Max}) < \check{\Pi}_E(d_E = \frac{1}{2})$ if d_I lies in a defined interval $[\check{d}_I^1; \check{d}_I^2]$ where respective boundaries are again numerical functions in Z. This completes the argument and demonstrates that the level of Z determines whether (p_E^*, d_E^*) constitutes a local or a global maximum in comparison with the undercutting strategy $\check{p}_E$.

Proof of Proposition 4: We examine the location ranges $d_E > d_E^{Z\times}$ and $d_E > d_E^Z$, and show that the reaction of firm I does not allow for the entry pricing strategy p_E^Z

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[10]In Mathematica evaluating $\overline{\Pi}_E(d_E^{Max}) > \check{\Pi}_E(d_E = \frac{1}{2})$ leads to the following expression for Z

$$\text{Root}[-2049517 - 157017785\#1 + 5288937559\#1^2 + 59510555641\#1^3 + 262944127002\#1^4$$
$$+648457185140\#1^5 + 997636619952\#1^6 + 997008491936\#1^7 + 647640924928\#1^8$$
$$+263301336320\#1^9 + 60596977664\#1^{10} + 5985009664\#1^{11}\&, 7]$$

to be profitable. Initially, recall that $\frac{\partial \Pi_E^Z}{\partial d_E} > 0$.

We begin by setting $d_E^{Z\times} > d_E^{\triangleright}$ which reduces to $d_I > \frac{1}{4+4Z}$ for $Z > 0$. Also see that $\frac{1}{4+4Z} < d_I^{\triangleleft}$ holds true for all $Z \geq 0$. Thus, there is a nonempty set of locations d_I such that strategy $p_E^{\times}$ is preferred for $d_E^{\triangleright} < d_E < d_E^{Z\times}$ and strategy p_E^Z for $d_E > d_E^{Z\times}$. It follows that when opting for the pricing strategy p_E^Z then the lowest possible price is given by $p_E^Z(d_E^{Z\times})$. It is straightforward to compare this price with the thresholds of the location reaction function in lemma 3 to assess firm I's reaction. Matching $p_E^Z(d_E^{Z\times}) < p_E^{\triangleleft}$ reveals that part (I) is not applicable since the rank holds for all $d_I, Z \in [0; \frac{1}{2}]$. Evaluating $p_E^Z(d_E^{Z\times}) < p_E^{\triangleright}$ reveals an ambigous result depending on the interaction of the parameters d_I and Z. We turn to the next price threshold and find that the rank $p_E^Z(d_E^{Z\times}) > p_E^{\triangledown\dagger}$ remains true for all $d_I, Z > 0$. This implies that firm I will never apply part (III.b) of his location reaction function and that firm I never reacts with charging p_I^{Acc} but for instance with p_I^{DetZ} to the entry strategy p_E^Z. Consequently, if firm E drops his price from $p_E^{\times}$ to p_E^Z at $d_E^{Z\times}$ his profits will not be $\Pi_E^Z(p_E^Z)$ but e.g. $\Pi_E^{\times}(p_E^Z)$ since $\frac{\Pi_E^{\times}}{\partial d_E} < 0$ and thus clearly not profitable. This is due to the fact that the incumbent will not adapt his pricing strategy and accommodate entry, he rather applies his defensive reaction strategies. In particular, firm I reacts with p_I^{DetZ} to the price drop from $p_E^{\times}$ to p_E^Z. To prove this argument see that the relation $d_E^{\times}(p_E^Z(d_E^{Z\times})) > d_E^{Z\times}$ holds for all $d_I, Z \in [0; \frac{1}{2}]$ and furthermore that $\hat{d}_E(p_E^Z(d_E^{Z\times})) < d_E^{Z\times}$ holds on the same domain. It suffices to compare the critical locations $d_E^{\times}$ and $\hat{d}_E$ with $d_E^{Z\times}$ since $p_E^Z(d_E^{Z\times}) < p_E^{\triangleleft}$ and part (I) is ruled out.[11]

In the second step of the proof we focus on the threshold d_E^Z or a comparison of the strategies p_E^{Acc} and p_E^Z. Analogously to the preceding above, we determine the price level at the discontinuity $p_E^Z(d_E^Z)$ and assign it to a corresponding price range of the location reaction function in lemma 3. Noteably, $p_E^Z(d_E^Z) > p_E^{\triangledown\dagger}$ holds again for all $d_I, Z > 0$ which rules out an accommodating reaction of firm I under a loss of Z. The further assignments of $p_E^Z(d_E^Z)$ to the parts (I), (II.a), (II.b) and (III.a) hinge upon interactions of d_I and Z. Especially, $p_E^{\triangleleft} > p_E^Z(d_E^Z)$ reduces to $d_I < 4\sqrt{Z + 2Z^2} - \frac{3+20Z+16Z^2}{4(1+Z)}$. It remains to compare the locations $\hat{d}_E$, $d_E^{\times}$, and $\overline{d}_E$ for the entry price $p_E^Z(d_E^Z)$ with d_E^Z. Firstly, a consistent result is obtained due to the rank $\hat{d}_E(p_E^Z(d_E^Z)) < d_E^Z$ which holds for all $d_I, Z > 0$ since firm I would never react with p_I^{AccZ} for $d_E > d_E^Z$. Secondly, the rank $d_E^{\times}(p_E^Z(d_E^Z)) < d_E^Z$ holds for all $d_I > 4\sqrt{Z + 2Z^2} - \frac{3+26Z+24Z^2}{4(1+Z)}$ and thus depends on the value of Z. Particularly, for any $d_I \in [0; \frac{1}{2}]$ firm I reacts with p_I^{Det} if $Z < Z''' \approx 0.077$ and with p_I^{DetZ} if $Z > Z'''$.[12] Thirdly, $\overline{d}_E(p_E^Z(d_E^Z)) < d_E^Z$ remains true for any $d_I < 4\sqrt{Z + 2Z^2} - \frac{1}{4}(3 + 23Z)$. We compare this limit with the boundary derived from $p_E^{\triangleleft} > p_E^Z(d_E^Z)$ and find that

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[11] Note that the demonstrated interdependence between the contenders holds for all $0 < d_I < \frac{1}{2}$ and is not restricted to the location range $d_E > d_E^{\triangleright}$ (or $d_I > \frac{1}{4+4Z}$) since the reaction of firm I is determined by the price level of the entry price p_E.

[12] In Mathematica reducing $4\sqrt{Z + 2Z^2} - \frac{3+26Z+24Z^2}{4(1+Z)} < 0$ yields $0 \leq Z < \text{Root}\left[9 - 100\#1 - 204\#1^2 - 32\#1^3 + 64\#1^4 \&, 3\right]$

$4\sqrt{Z+2Z^2}-\frac{1}{4}(3+23Z)<4\sqrt{Z+2Z^2}-\frac{3+20Z+16Z^2}{4(1+Z)}$ is true for $Z>0$. This implies that for any $p_E^Z(d_E^Z)>p_E^\triangleleft$ entry is deterred or $\overline{d}_E(p_E^Z(d_E^Z))<d_E^Z$.

Remark: The aim is to identify to which price range of the location reaction function in lemma 4 the undercutting price $\check{p}_E$ is assigned. At the location d_E^{split} the pricing behavior of the entrant firm shifts from $p_E^\times$ to $\check{p}_E$ and therefore for $d_E>d_E^{split}$ the undercutting strategy dominates. (cf. figure 3.15) Thus, we use $\check{p}_E$ at d_E^{split} for the allignment and note that the strategic behavior of firm E does not change under an increase in d_E since $\frac{\partial \check{\Pi}_E}{\partial d_E}>0$.

The rank $p_E^\triangleleft>\check{p}_E(d_E^{split})$ holds for all $Z>0$ and $d_I\in[0;\frac{1}{2}]$. Secondly, we set $\check{p}_E(d_E^{split})>p_E^\triangleright$. This rank holds true if Z and d_I remain in certain numerically defined intervals, i.e. for $Z<Z''\approx0.0128$ and the boundaries for d_I are a decreasing numerical function in Z with the maximum interval at $Z=0$ ($\approx[0.363;\frac{1}{2}]$) and the minimum at $Z=Z''$.[13] Consequently, for $Z<Z''$ and d_I outside the defined interval as well as $Z>Z''$ and any $0\le d_I\le\frac{1}{2}$ the relation $p_E^\triangleright>\check{p}_E(d_E^{split})$ obtains. Thirdly, we set $p_E^{\triangledown\ddagger}>\check{p}_E(d_E^{split})$ and find that this rank holds for all $d_I,Z\in[0;\frac{1}{2}]$. Considering the threshold value ζ from lemma 4 it follows that for any $Z<Z''$ and given that d_I lies in the predefined numerical interval part $(III.b)$ from the location reaction function is applicable. Now, due to the symmetry of $\check{d}_E^\dagger$ and $\check{p}_E$ – the two expressions describe the same intersection between firm I's profits for the deterrence-Z strategy and the deferrence strategy – we find the identity $\check{d}_E^\dagger(\check{p}_E(d_E^{split}))=d_E^{split}$. This implies that the incumbent initially reacts with p_I^{Def} to the undercutting of firm E at d_E^{split}. Recall that $\check{p}_E(d_E^{split})$ marks the lowest feasible price for the undercutting strategy and that $\check{p}_E$ increases in d_E. Thus, the entrant increases d_E when choosing the pricing strategy $\check{p}_E$ since corresponding profits move up. In turn, it follows that p_E will exceed $p_E^{\triangledown\ddagger}$ at a certain location d_E. Clearly, firm I's reaction then switches from part $(III.b)$ to $(II.b)$ and from p_I^{Def} to p_I^{Det} since $p_E>p_E^{\triangledown\ddagger}$ is equivalent to $d_E^\times<\check{d}_E^\dagger$. (cf. proof 4)

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[13]In Mathematica:

$$0<Z<\text{Root}\left[-57+4114\#1+26856\#1^2+63792\#1^3+66592\#1^4+25728\#1^5\&,3\right]$$

$$\text{Root}\left[-3-20Z-40Z^2-24Z^3+(15+48Z+36Z^2)\#1+(-20-32Z)\#1^2+4\#1^3\&,1\right]<\text{dI}<$$
$$\text{Root}\left[-3-20Z-40Z^2-24Z^3+(15+48Z+36Z^2)\#1+(-20-32Z)\#1^2+4\#1^3\&,2\right]$$

Figure 3.15: Illustration of firm E's profit function for different pricing strategies against his location d_E

Entrant's profit functions

Entrant's price functions
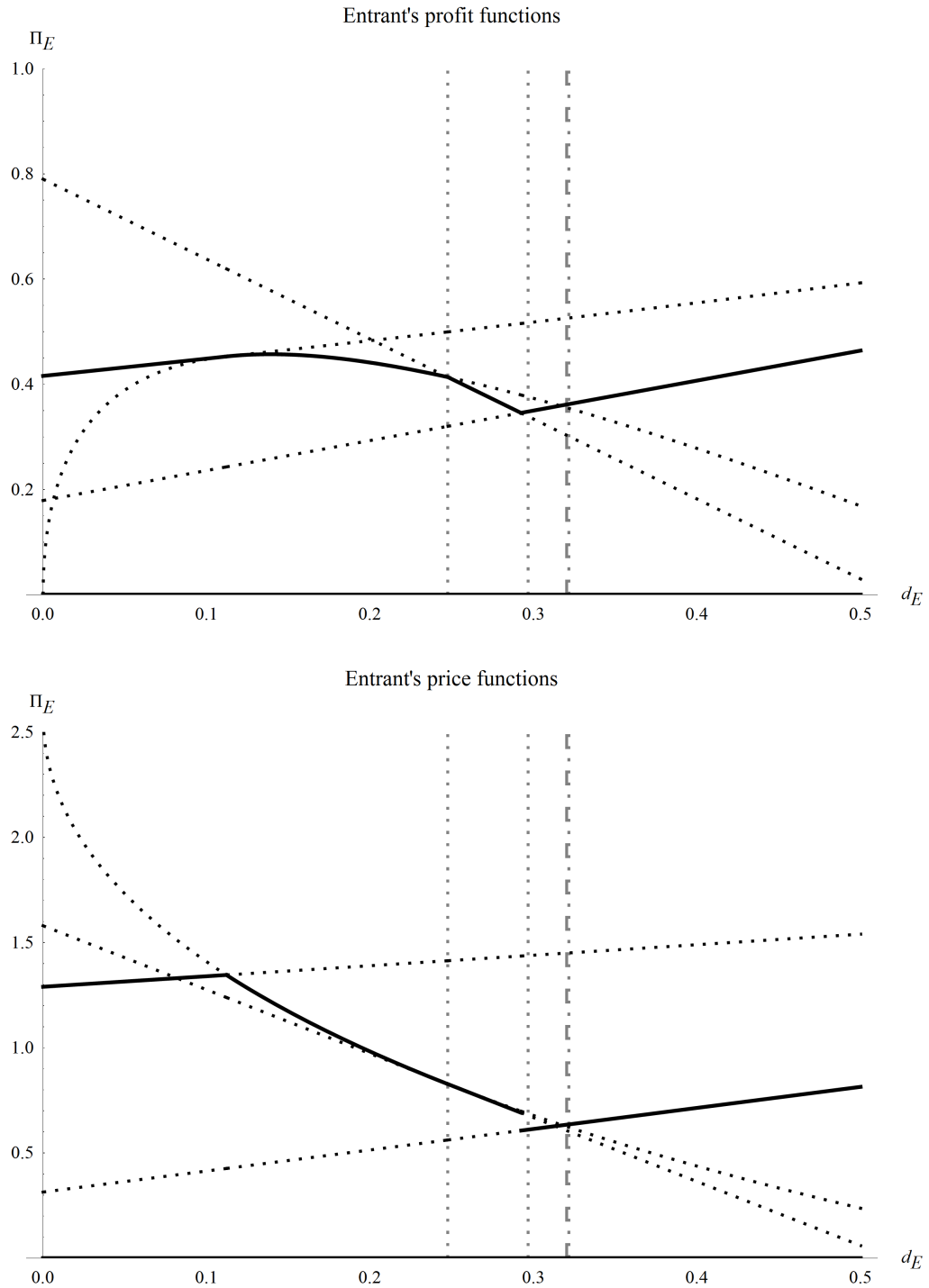
Comment: The solid lines depict the viable price and profit ranges. Profits for the undercutting strategy amount to 0.464 at $d_E = \frac{1}{2}$ and exceed profits for the set (p_E^*, d_E^*) which are 0.457. However, firm E is only able to play the undercutting strategy in the location interval $[\check{d}_E^\dagger, \check{d}_E^\dagger] \approx [0.292; 0321]$ (second and third vertical tiny dashed lines). The location $d_E^\times \approx 0.320$ is the threshold for deterrence if $p_E > p_E^{\bigtriangledown\ddagger}$ (medium dashed line) and $d_E^\triangleright$ is approximately 0.248 (first vertical tiny dashed line). Parameter values are $d_I = 0.44$, $Z = 0.01$ and $t = 1$.

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4 Price Dispersion, Search Costs and Spatial Competition[*]

4.1 Introduction

In contrast to predictions of simple textbook models, causal observations suggest that homogeneous products are sold at different prices by rival firms even in markets with intense competition. In fact, empirical studies reveal that prices, different firms charge for the same product, differ significantly and persistently and deviations from the 'law of one price' are the norm, rather than the exception. Since the existence of price differences for identical products is an indication of the efficiency of markets, programs and policies to improve access to information may result in lower prices for consumers and enhance consumer welfare.[1] The effects of competition-enhancing policies, however, might not be as straightforward as those implied by standard models. Rather, increased competition can potentially affect the price distribution asymmetrically and may have different impacts on the welfare of different types of consumers.[2] Thus, a thorough examination of the price distribution may shed light on structural relationships of a market economy and in turn provide a useful basis for advising policy makers.

Intuition suggests there are two straightforward explanations for the behavior of prices and the existence of price differences. Firstly, price levels and price dispersion are related to product and seller heterogeneity. Even though products might appear homogenous in terms of physical characteristics, as in the case of gasoline, they are being sold at different stores. In turn, retail outlets differ in convenience

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[1] The Austrian economics ministry for instance recently passed an act on the conduct rules of gasoline station operators effectively regulating gasoline pricing. It lays down that individual gasoline stations are only permitted to raise gasoline prices once a day. More evidence on the issue of government intervention and price discrimination in retail gasoline markets is available in Borenstein (1991) and Borenstein & Bushnell (2005).

[2] Cp. Lach & Moraga-Gonzalez (2009) and Morgan et al. (2006).

and amenities. Thus consumers may be willing to pay a premium for products that they perceive to be of higher quality, e.g. peculiar brands are perceived to be in the premium segment or shops having a reputation for extraordinary services to be rendered to customers. Additionally, products may also differ in a spatial context. In particular, buying decisions and customer satisfaction may be related to sellers' locations allowing, for instance, consumers to access a retail outlet conveniently or in a very short amount of time since outlets may be distributed across different market areas.[3]

Secondly, prices and price differences are determined by the factor information and explained by the economics of information.[4] Accordingly, variance in prices are associated with costs incurred by consumers and firms while processing market information. Sellers have incentives to charge different prices since consumers may differ, for instance, in their willingness and capabilities to collect information about sellers' location, prices and pricing strategies. In turn, demand is fragmented into consumer groups that differ according to their knowledge of the price distribution. As a result, markets are characterized by arbitrage opportunities and market equilibrium outcomes are cleary impacted by the existence of consumer groups that differ in their preferences to balance the costs and benefits of price search activities.

These two distinct approaches in mind, this paper examines the diesel price distribution in the Austrian retail market with respect to the influence competition and differing levels of search costs have on the mean price and the price variance. The Austrian gasoline market is particularly useful in testing hypothesis on the comparative static behavior of the price distribution. Firstly, it is characterized by a high level of concentration where the four major chains controll almost 60% of the market.[5] Secondly, gasoline can be considered an almost perfectly homogenous good with respect to its physical and chemical properties. From the perspective of the consumer, the key issue of differentiation in this market is the location of the individual station. Thus, competition intensity is directly related to the geographical proximity of sellers. We use two measures that account for sellers' distance relations in local markets. Additionally, from firms' perspective a critical issue in optimizing pricing strategies relates to the differences in the knowledge of stations' pricing behavior among consumers. The distribution of information reflects consumers' likelihood to police excessive market pricing. Intuitively, a station sets low prices in areas where consumers are susceptible to spotting gasoline prices and thus likely to switch be-

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[3]Products that differ via their quality characteristics are considered to be vertically differentiated since consumers agree over the preference ordering. In contrast, distinct seller locations refer to the notion of horizontal product differentiation implying that optimal consumer choices strongly depend on consumers' preferences that generally differ with respect to the observed characteristic, cp. Tirole (2003), p. 96ff.

[4]In the words of George Stigler: „*it would be metaphysical, and fruitless, to assert that all dispersion is due to heterogeneity*" (Stigler (1961), p. 214).

[5]Cf. Report of the Austrian Ministry of Economics, BMWA (2005).

tween sellers. On the other hand, a profitable strategy for a station in an area where consumers' ignorance of price levels and price changes is prevalent is to charge high prices.

In conclusion, the following research questions will be addressed: How does spatial competition between different gasoline stations affect the price distribution of diesel? How does the fraction of informed consumers and of uninformed consumers impact the price distribution and what are possible implications for the relationship for the level of search costs and the price distribution?

By using data on stations' characteristics and local market characteristics, our strategy is to initially control for product heterogeneity and estimate a model that examines the relationship between competition, search cost variables and proxies for sellers' location as well as station specific characteristics with price levels. In a second step we will test hypotheses on the relation between the fraction of informed consumers, search costs and competition with price dispersion measured by the price variance. Besides the usual OLS techniques we will also test for spatial autocorrelation and to avoid misspecification apply a spatial error model (SEM) to estimate market prices. In addition, further robustness checks concerning the use of alternative spatial weights matrices and search cost proxies will be carried out.

The remainder of this paper is as follows: subsection 4.2 presents a selected overview of the theoretical literature on the existence and behavior of the equilibrium price distribution with respect to the spatial distribution of firms and the existence of search costs among consumers. Special attention is drawn to the relationship between the fraction of informed consumers, the level of search costs and the number of sellers with the average price and price dispersion. Subsection 4.3 gives details on data and methodology and outlines the strategy for the empirical analysis. Subsection 4.4 presents the results and eventually subsection 4.5 closes with a concluding discussion on the findings.

4.2 Survey of Price Models

4.2.1 Spatial Competition Models

Standard models of spatial competition assume a uniform distribution of consumers in a one dimensional market setting for a homogenous product. (e.g. Hotelling (1929), Salop (1979)) Each firm serves a customer base that is within a certain local market range. Implications of these sort of models are that competition is a localized phenomenon since sellers compete in prices for potential customers with comparable transportation costs and settled within a common local market of interest. In particular, by reducing his price a seller i could attract consumers that are located farer

away and who would otherwise be indifferent between purchasing at firm i's outlet or at the sites of his close competitors. In equilibrium prices obtain as the sum of sellers' (equal) marginal cost and a markup that depends on the degree of spatial differentiation. Put differently, due to his location a seller holds a certain degree of monopoly power over its local market customer base which is reflected in the negative dependency of prices with geographical distance.

Now, an increase in competition intensity in terms of the number of sellers (due to reductions in fixed costs or increases in the number of consumers) is associated with a decrease in the ability to employ market power upon the original customer base. If a new firm enters the market it locates in the previously segmented market space effectively shrinking the local monopolies of neighbouring competitors by establishing its own local niche market. Accordingly, increased competition aggravates firms' possibilities to (spatially) differentiate themselves, leads to a denser net of outlets and thus erodes the basis to extract a surplus from nearby consumers since they reconsider their purchase decision due to this increase in seller variety. What follows is that under entry competition the price elasticity of demand increases, the price markup decreases and thus the market price declines.[6]

Further, unit transportation cost t can be interpreted as a measure of consumers' search cost incurred to compare prices of neighbouring sellers and get additional price quotes. Consequently, an increase in t decreases consumers' incentives to visit stores competing in prices. In turn, sellers are able to appropriate more surplus from their captives and as a result prices are expected to rise as the unit transportation cost increases.

While spatial competition models are straightforward in explaining the relation between market prices, transportation costs and competition intensity, the issue of market information and price dispersion is not explicitly addressed.[7] Rather, price dispersion is expected to arise due to asymmetries in the spatial distribution of firms. Comparable to the behavior of price levels, the competitive pressure of market entry

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[6]Salop (1979) shows that in equilibrium firms realize zero profits with prices above marginal cost and a markup that inversely depends on the number of competitors by assuming maximal product differentiation. In turn, the number of firms is endogenously determined by the amount of fixed costs and market size (total number of consumers), cf. p. 147ff.

Additionally, Perloff & Salop (1985) abstract from spatial competition models and examine the properties of a more general consumer model of product differentiation. They show that for a symmetric (and differentiable) probability distribution of brand preferences equilibrium prices increase with increases in the intensity of consumers' preferences. Further, they provide two conditions under which the competitiveness of markets is established. Accordingly, increased entry competition (number of firms converging to infinity) leads to a decrease in the equilibrium price only if either consumers' brand preferences are bounded or if price elasticity of demand is sufficiently increasing (cp. p.111 and the example on p. 113).

[7]Noteable exceptions are given in the work of Sheppard et al. (1992) and Haining et al. (1996). They scrutinize the existence and properties of spatial price equilibria under special distributions of consumer choice sets. However, they do not focus on the comparative static behavior of the resulting equilibrium price distribution under entry competition or a variation in consumers' search costs and the fraction of informed consumers respectively.

then leads price differences across the market to decrease. Thus, price dispersion is expected to decrease under an increase in seller density.[8]

4.2.2 Consumer Search Models

4.2.2.1 Price Distribution, Search Costs and the Fraction of Informed Consumers

For price dispersion to be an equilibrium outcome all firms and consumers in a market must not have incentives to reconsider buying decisions and change pricing strategies.[9] In other words, charging a range of prices would be a rational response of sellers to consumers' optimal search behavior. Thus, generally the existence and behavior of an equilibrium price distribution is a function of consumers' search costs. Two extreme cases highlight the intuition behind this argument. If for instance all consumers in a market got a price quote from every firm at no cost, the firm with the cheapest price would serve the whole market. Not surprisingly, fierce price competition is the result of this specific distribution of information. What follows is that for identical sellers with identical cost functions, price dispersion would not occur since sellers' best response is the Bertrand outcome with every firm charging the perfect competitive price at marginal cost (Bertrand (1883)).

In contrast, the existence of imperfect information among consumers does not naturally imply that sellers set distinct prices in equilibrium. Rather, Diamond (1971) was the first to emphasize the paradox that consumers behave rationally under sequential search with strictly positive and identical search costs when they do not search at all. Thus, a natural outcome in a setting where consumers lack price information is not price dispersion but monopoly pricing by sellers. The argument is as follows. Consumers purchase a unit of a good only at prices equal or below their reservation price r. In contrast they would search if prices exceeded r and incur a cost s for every new price quote. The reservation level varies with the unit search cost and price distribution, thus, for a given distribution $F(p)$ and identical unit search cost reserveration prices are identical among consumers. Clearly, sellers' rational response in this case is to set the identical optimal price r. In turn, if there exist no price dif-

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[8]Based on the findings of Perloff & Salop (1985), Barron et al. (2004) for instance argue that in asymmetric demand cases an increase in the number of sellers tends to increase the price elasticity across different seller types, and thus reduces respective markups and prices. Under a given common marginal cost this implies that the increase in the number of sellers leads to a reduction in the variance of prices or reduced price dispersion (p. 1045).

[9]In a survey of then existing equilibrium models, Rothschild proposed the following conditions: „*A satisfactory model of adjustment to equilibrium will have at least three parts: a discussion of the rules which market participants follow when the market is out of equilibrium; a description of how a market system in which individuals follow these rules operates; and, of course, a convergence theorem.*“ (Rothschild (1973), p. 1285f.)

ferences among sellers, it does not pay off for any customer to search. Consequently, sellers optimal response is to charge the monopoly price. The conclusion from the Diamond model is that, the existence of an equilibrium price distribution requires some form of heterogeneity among buyers and/or sellers and that the resulting price distribution reflects this underlying buyer and seller characteristics.[10]

In his well-known 'model of sales' Varian (1980), for instance, shows that under a special dichotomous distribution of search costs among consumers prices are dispersed in equilibrium. On the supply side he assumes that firms sell a homogenous product with an identical production cost structure unable to discriminate in pricing. On the demand side consumers are devided into two groups. Shoppers who are perfectly informed about sellers' locations and prices and therefore purchase the good at the cheapest firm without incurring any cost of search. By contrast, the regular or uninformed buyers initially observe only the price from the seller that they chose at random. If the actually charged price is below their reservation level, they purchase. Any further seller visit and additional price quote causes them nonnegative visiting or search costs. Now, the appealing feature of Varian's model is that firms' optimal pricing strategy has to reconcile conflicting goals to realize profits in the two distinct consumer segments. Intuitively, a seller could focus on the informed consumers and make profits by undercutting all his rivals; or he could aim at appropriating surplus from the uninformed consumers employing a high price strategy. Clearly, these strategies can not be applied simultaneously, rather firms' optimize their price setting in a dynamic and probabilistic context. The upshot is that in equilibrium firms price in mixed strategies randomizing their prices between a lower bound and an upper bound determined by their average costs and the consumers' reservation price respectively. As a result, extreme prices are more frequently charged whereas the frequency of intermediate prices diminishes. Thus, in equilibrium there is price dispersion.

In an effort to close the gap between the equilibrium outcomes of marginal cost pricing and monopoly pricing, Stahl (1989) developed a model of equilibrium price

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[10]Since early versions of models based on sequential consumer search were not capable of showing that price dispersion arises in equilibrium a special type of search models, so called clearinghouse models were developed. These incorporate a third party —an information clearinghouse —that sells a list of prices charged by different firms in a homogenous product market to a subset of consumers who subsequently purchase at the seller with the lowest listed price. Additionally, firms are also charged fees by the clearinghouse to be listed. Baye & Morgan (2001) show that when consumers' and firms' decision to access the clearinghouse, as well as respective subscription and advertisement fees are endogenized, the owner of the clearinghouse maximizes his profits in a dispersed price equilibrium in which all consumers have access to the market price list. Furthermore, the simplest clearinghouse assumption states that consumers may potentially become informed of all current prices on a market at once. Thus, a clearinghouse serves the role of interconnecting dispersed local markets and establishing competition between locally separated sellers. Consequently, predictions of clearinghouse models have been tested on online markets and studies showed that price dispersion on the internet is pervasive and significant (cf. Baye et al. (2004), and Baylis & Perloff (2002)).

dispersion for two distinct consumer groups: shoppers with zero search costs and uninformed regular buyers. Importantly, he extends Varian's work by assuming that the regular consumers do not remain uninformed. Rather, their search behavior is characterized by optimal sequential search with nonnegative search costs for every additional price quote.[11] The asymptotic behavior of the resulting equilibrium price distribution reveals interesting findings.[12] Firstly, as the fraction of informed consumers goes to zero, the lower bound converges to the upper bound, eventually converging to the monopoly price. Secondly, as the fraction of informed consumers approaches one, the price mark-up vanishes leading to perfectly competitive pricing. Thirdly, as search costs converge to zero, the reservation price continously declines, the upper bound converges to the lower bound with the end result of marginal cost pricing. Further, comparative static analyses support previous findings and show that, ceteris paribus, an increase in the fraction of informed consumers or a decrease in search costs causes the reservation price and the lower bound to decline respectively.

In conclusion, price dispersion in equilibrium obtains as a function of the fraction of informed consumers and the level of search costs. In the limit of only perfectly informed consumers in the market or search costs converging to zero, the price distribution degenerates to the competitive price revealing the Bertrand outcome. By contrast, as the fraction of the informed goes to zero, firms charge the monopoly price and the Diamond result obtains. These findings imply a negative correlation between the expected price and the fraction of informed consumers and a positive correlation between the expected price and the level of search costs.[13] Intuitively, under constant search intensity of the regular buyers, a higher proportion of shoppers raises firms' incentives to charge lower prices.[14] Likewise, a decrease in search costs leads to more intense search of the uninformed consumers and creates more competitive pressure on market prices.[15]

A general treatment of the comparative static behavior of prices and price dispersion with respect to the proportion of perfectly informed consumers is found in a recent study by Waldeck (2008). He investigates the properties of the first two moments of the equilibrium price distribution (expected price and variance) for two different

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[11]Important features of his model are that, (1) the first price quote for the uninformed is free, and (2) consumers' reservation price ρ is endogenized. Thus, the equilibrium price distribution $F(p, \mu, N, \rho(\mu, N, c))$ is dependent on the fraction of informed consumers μ, the level of search costs c and the number of firms N.

[12]Cf. proposition 2 and 3, p. 705.

[13]This is a special outcome for the case of full consumer participation. If search is truly costly, i.e. the uninformed consumers incur a search cost for *every* price quote, some may drop out of the market. Under this partial consumer participation equilibrium there is no net effect of the fraction of informed on expected price and expected price increases as the level of search costs decreases. For details see Janssen et al. (2005).

[14]The reservation price ρ is a decreasing function in the fraction of informed consumers μ. Cf. Stahl (1989), p. 704, equation 8 and 9.

[15]ρ is also an increasing function in the level of search costs c. Cf. ibid.

search modes that refer to the previous work of Varian (1980) and Stahl (1989): fixed sample search and sequential search.[16] His results are in line with previous findings. For both search specifications average prices paid by the uninformed and informed consumers decrease as the fraction of informed consumers rises. Further, in both cases price dispersion is an inverse U-shaped function of the fraction of informed consumers and for the sequential search mode an increase in search costs implies higher price dispersion.

4.2.2.2 Price Distribution and the Number of Sellers

What do search models tell us about the comparative static behavior of the price distribution under entry competition? In contrast to predictions of spatial competition models, findings by Varian (1980) and Stahl (1989) suggest that an increase in the number of sellers leads to an increase in the expected price. The intuition behind this odd result is given by the tension in firms' pricing strategies accruing from inherent differences in consumers' search activities. In particular, a larger number of competitors implies that the probability of gaining a profit from the uninformed consumer segment falls less rapidly than the probability of realizing profits by focusing on the informed consumers. Firms face increased competition intensity in two ways: firstly, an increase in the number of firms enlarges the choice set for the well informed consumers, and secondly, increased competition implies a decrease in the number of captive consumers per firm, thus, reducing the average purchase per uninformed consumer. In sum expected profits due to the chance of being the lowest priced seller and catching the informed segment are outbalanced by expected profits obtained from imposing high prices on a reduced uninformed consumer segment. In sum a high-pricing strategy proves to be more attractive under entry competition and average prices which are essentially the prices uninformed consumers pay rise. Morgan et al. (2006) elaborate on the findings of the comparative competition analysis in the Varian model. Interestingly, they provide theoretical and empirical evidence that entry competition implies an ambigous effect on the market price distribution and highlight that information imposes beneficial externalities on their holders. In particular, they show in a variant[17] of the Varian model that not only the average price rises with the number of firms but that simultaneously the expected value of the minimum price (of the equilibrium distribution) declines. Referring to the com-

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[16]In the step of endogenizing consumers' reservation price a connection is established between fixed sample search and sequential search. Under certain parameter values (small fraction of informed and large number of firms) sequential search 'converges' to the fixed sample type. The reservation price then exceeds a certain threshold and is further assumed to be exogenously given. Cf. Waldeck (2008), lemma 15 and table 3, p. 353.

[17]They assume the ratio of uninformed and informed consumers as well as the number of competing firms to be exogenously fixed whereas in the original Varian model the number of firms is determined by a zero profit condition (cf. Varian (1980), equation (5) on p. 656).

petition effect in the lower price segment, firms react by reducing competitive prices. The upshot is that under entry competition informed consumers pay on average lower prices and, as argued before, uninformed consumers are charged on average higher prices. As a result, price dispersion increases in the number of sellers.[18]

Similar findings on the distinct effects of competition on the equilibrium price distribution through the distribution of price information among consumers are obtained in the work of Lach & Moraga-Gonzalez (2009). Assume that a fraction μ_s of consumers is informed about s price quotes in the market. In total there are N firms, thus the ratio of the perfectly informed consumers corresponds to μ_N and the ratio of the fully uninformed consumers to μ_1.[19] Generally, every fraction of consumers $\mu_s(N)$ depends on the actual number of sellers in the market since a typical consumer in an informational segment is exposed to a particular number of price quotes while, for instance, in the case of gasoline pricing driving to work.[20] Further, the total number of consumers is denoted as L and marginal costs c for every firm are identical. Now, in the fashion of Varian, every firm sets a profit maximizing price p in a mixed pricing strategy according to the cumulative distribution $F(p)$.[21] Thus, given the random pricing strategies of all other competitors, expected profits for an arbitrary firm i from all types of consumers are given by:

$$\Pi_i(p, F) = L(p - c) \left[\sum_{s=1}^{N} \mu_s \frac{s}{N} (1 - F(p))^{s-1} \right] \tag{4.1}$$

Intuitively, a firm has the chance to make a profit in every information segment

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[18] In contrast to these findings, the evidence on the comparative behavior of price dispersion in the Varian model is not clear-cut. Baye et al. (2004), for instance, examine the theoretical relationship between the number of firms and the level of price dispersion, measured by the difference between the lowest and the second lowest price. According to their simulations price dispersion in the Varian model is a nonmonotonic function in the number of sellers. Nonmonotonicity arises due to strategic price effects that dominate when the number of sellers is small, and contrarily, order statistic effects determine the course of price dispersion for large numbers of sellers. In sum, they find that as the number of sellers rises, price dispersion initially increases and then smoothly declines (cf. Baye et al. (2004), p. 486f). Furthermore, Janssen & Moraga-Gonzalez (2004) scrutinize comparative statics in a closed form of the Varian model and find no analytical characterization of the behavior of price dispersion with respect to the number of firms (cf. p. 1096ff).

[19] The uninformed consumers know at least the price at the one seller they would purchase the good (this equals the definition of uninformed consumers in the Varian model). Additionally, the segment $s = 0$ and μ_0 would refer to the uninformed consumers that do not find it beneficial anymore to stay in the market. Consequently, from this segment no profits can be obtained. Further, every consumer group can be completely and unambiguously characterized: $\sum_{s=0}^{N} \mu_s = 1$.

[20] Making the fractions of consumers μ_s endogenous implies twofold. Firstly, enhanced competition may only affect the price distribution via the distribution of information among consumers, and secondly, it enables an analysis of the change in search behavior or a change in these different kinds of consumer fractions s respectively under entry competition. Details for special cases of this dependence are discussed below.

[21] They focus only on symmetric equilibria, cf. Janssen & Moraga-Gonzalez (2004), p. 1093 and Lach & Moraga-Gonzalez (2009), p. 5.

s. Correspondingly, the quantity sold is dependent on (i) the fraction of informed consumers in the respecitve segment μ_s, (ii) the likelihood s/N that the consumers observe the price quote from firm i, and (iii) the probability $(1 - F(p))^{s-1}$ that firm i sells the good to the segment s at price p. In equilibrium, firms maximize profits by randomizing prices. Thus, they are indifferent between charging any price in the support of F, especially the upper bound marked by consumers' reservation level $\bar{p} = v$ since profits for either pricing strategy are equal.[22] Formally, $\Pi_i(p, F) = \Pi_i(\bar{p}, F)$ has to be satisfied which yields the equilibrium condition:

$$(p - c)\left[\sum_{s=1}^{N} \mu_s s(1 - F(p))^{s-1}\right] = (v - c)\mu_1 \qquad (4.2)$$

Equation (4.2) determines the price distribution $F(p)$ in an equilibrium with firms setting profit maximizing prices in mixed strategies and consumers searching according to their information set depicted in the vector $\boldsymbol{\mu}(N) = (\mu_1(N), ..., \mu_N(N))$ representing the overall distribution of price information in the market. Generally, equation (4.2) can not be solved with respect to the equilibrium distribution $F(p)$. Special cases are investigated, however, in the study of Janssen & Moraga-Gonzalez (2004). In particular, they distinguish between three different search modes: (i) low search intensity ($\mu_1 < 1$) where consumers randomize between searching for one price quote or dropping out of the market, (ii) moderate search intensity and every uninformed consumer searching once ($\mu_1 = 1$), and (iii) high search intensity with uninformed consumers randomizing between searching for one price and searching for two prices $\boldsymbol{\mu} = (\mu_1 < 1, \mu_2 < 1)$. Now, comparable to the findings of Morgan et al. (2006), the equilibrium distribution $F(p)$ for exogenously given μ_1 in moderate search intensity is characterized by increased frequencies to charge low and high prices, thus increased price dispersion, as the number of sellers rises.[23] Interestingly, as the propensity to search μ_1 is endogenized[24] no equilibrium obtains and consumers change their search behavior. In particular, as the number of sellers grows average prices paid by the uninformed consumers rise continuously reducing their incentives

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[22] The lower bound is obtained by setting $F = 0$ and solving for p in the equilibrium condition (4.2).

[23] Technically, the distribution $F(p, N)$ is not stochastically first-order dominated by $F(p, N + 1)$. This result of Varian (1980), Janssen & Moraga-Gonzalez (2004) and Morgan et al. (2006) is in contrast to the findings of Rosenthal (1980). He provides evidence for stochastic dominance in the cdf of the price, formally $F(p, N) > F(p, N + 1)$, implying that in a mixed strategy equilibrium charging higher prices for any price level is preferred under the entry of an additional competitor. Thus, average prices for uninformed and informed consumers rise. His findings are due to the fact that the average number of informed consumers or the common market per firm decreases as the number of sellers rises. On the contrary, Varian assumes that the number of captive consumers per seller falls with N.

[24] The search process occurs under the condition that the net utility from purchasing the product is positive and that there are no incentives to change the search mode (cf. Janssen & Moraga-Gonzalez (2004), conditions 3.1. and 3.2. on p. 1097).

to stay in the market.[25] Turning to the low search intensity scenario, results show that the drop out in demand counteracts with the tendency to increase prices in the upper price segment. In turn, the rise in the average price comes to a halt and firms react to adjusted search preferences by equally strengthening their strategies of setting extreme prices merely inducing more price dispersion as the number of sellers increases.

Finally, findings for the high search intensity mode provide further evidence for the adaption of consumers' search behavior to the market structure promoting the impact enhanced competition has on the price distribution. Specifically, given the number of sellers is sufficiently small consumers search more intensely under increased entry competition. In contrast, provided that the number of sellers is large incentives to search are reduced as an additional competitor enters the market.[26] As expected, firms' profit maximizing pricing strategy under entry is determined by charging low prices to attract informed and high prices to extract surplus from uninformed consumers with the latter effect dominating. As a result, expected prices increase monotonically in N. Now, the non-monotonic search behavior of the captive customers interacts with firms' pricing strategies in two ways. In markets with a large status quo number of sellers both effects concur and unambigously lead to an increase in the average price. In markets with a low status quo number of firms, however, increased propensities to search counteract with firms' incentive to raise prices. Thus, the authors conclude that in line with the non-monotonic search behavior the expected price decreases with the number of sellers to begin with but subsequently increases as the number of firms gets sufficiently large. Further, they find numerical evidence for an increase in price dispersion as the expected price declines. This also leads them to conclude that price dispersion may have a non-monotonic relationship with respect to the number of sellers in the high search intensity mode.[27]

The intuition behind these results is that in local markets with a small number of competitors an increase in the number of firms induces higher price competition in the lower price segment. The reason is twofold; firstly, firms compete as expected for the informed consumers, and secondly, the uninformed consumers show increased search activity and consequently the probability for these to purchase at a low listed price increases. In sum, incentives for firms to focus on stealing rivals' business by offering the potentially lowest market price increase whereas at the same time extracting surplus from uninformed consumers remains viable. This tendency reverses as the profitability of focusing on the low price segment declines with a growing

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[25]The equilibrium fraction of uninformed consumers who find it still beneficial to remain in the market is essentially determined by the level of marginal search costs incurred for every new quote (cf. Janssen & Moraga-Gonzalez (2004), p. 1100).

[26]Simulations show that the turning point is given for a number of firms of 6 or 7 (cf. Janssen & Moraga-Gonzalez (2004), p. 1107).

[27]Cf. Janssen & Moraga-Gonzalez (2004), footnote 18 on p. 1108.

number of rivals in the market accompanied by a decrease in search intensity of the uninformed consumers. In turn, firms shift to the strategy of appropriating surplus from their captives, pressures on the low priced segment are released and the average market price increases in N.

4.2.3 Summary of Models' Predictions, Review of Empirical Evidence and Postulation of Hypotheses

The previous discussion revealed that the existence and behavior of the equilibrium price distribution is critically related to the distribution of (price) information among consumers and the degree of competition in the market. Table 4.1 summarizes suggested correlations between the fraction of informed consumers, the number of sellers and average prices and price dispersion, respectively.

Table 4.1: Summary of correlations in different price models

Dispersion model	Predicted correlation between:			
	Fraction of informed consumers and		Number of sellers and	
	average price	price dispersion	average price	price dispersion
Spatial competition models with asymmetries across firms (Hotelling (1929), Salop (1979))	negative		negative	negative
Consumer search models with heterogenous search costs & fixed sample search (Varian (1980))	negative	inverse U	positive	positive*
with heterogenous search costs & sequential search (Stahl (1989))	negative	inverse U	positive	negative
with fixed-sample-size search: • low search intensity ($0 < \mu_1 < 1$) • moderate search intensity ($\mu_1 = 1$) • high search intensity ($0 < \mu_1, \mu_2 < 1$) (Janssen & Moraga-Gonzalez (2004))			none positive nonlinear	positive ? nonlinear

(*) consumer search and number of firms exogenously fixed (cp. Morgan et al. (2006))

Firstly, firms' market position is characterized by their location which implies a straightforward relation between prices, competition intensity and consumers' propensity to search. In short, market power and thus the ability to raise profits is reflected

in geographical proximity to (potential) customers and rivals. Increased competition via a higher number of sellers leads to lower average prices and price dispersion. In the search theoretic context a decrease in unit transportation costs can be interpreted as an increase in the fraction of informed consumers and implies a decrease in the average price.

Secondly, the interrelation between search and pricing activities is explained in search models. Intuitively, given a certain purchasing behavior and a division of market demand into distinct consumer groups, firms could either offer very low prices and sell large quantities or charge high prices and sell a small amount of goods. Their pricing strategies and thus the average market price and price variance, i.e. the resulting market price distribution is determined by these conflicting profit-seeking goals. In this setting variations in consumers' information sets have a direct impact on the price distribution. An increase in the proportion of shoppers and a decrease in the level of search costs respectively causes average prices to decline. Further, in the limit the price distribution degenerates, thus price dispersion is expected to show an inverse U-shaped relation with the fraction of informed consumers.

Table 4.2: Survey of selected empirical studies investigating price dispersion

Empirical study in industry	Correlation between competition and price dispersion
Gasoline industry	
Marvel (1976)	negative
Png & Reitman (1994)	positive
Barron et al. (2004)	negative
Clemenz & Gugler (2006)	negative
Lewis (2008)	ambigous
Lach & Moraga-Gonzalez (2009)	positive
Airline industry	
Borenstein & Rose (1994)	positive
Gerardi & Shapiro (2009)	negative
Grocery products	
Walsh & Whelan (1999)	positive
Electronics products	
Baye et al. (2004)	negative

A comparison on models' predictions on the relation between competition intensity and the price distribution reveals ambiguous results. Clearly, the reason is that a variation in the number of sellers impacts average prices and price dispersion through

changes in consumers' search and purchasing behavior.[28] Indeed, as depicted in table 4.2 the existing empirical literature across different industries also provides mixed evidence on the correlation between the number of sellers and price dispersion.

Marvel (1976), for instance, reports that an increase in the number of competitors reduces the range in the price of gasoline. Barron et al. (2004) study the structural determinants of price dispersion in the retail gasoline industry in four geographic locations, and find empirical support that an increase in station density decreases both price levels and price dispersion. For the Austrian retail gasoline market Clemenz & Gugler (2006) find a negative correlation between seller density and price dispersion. More recently, Lewis (2008) also observes a negative relationship between the number of sellers and price dispersion of gasoline though his results reveal that correlations vary significantly for different types of sellers and different measures of dispersion. Furthermore, Png & Reitman (1994) find that prices of gasoline stations are more dispersed in markets with greater number of competitors. Likewise, Lach & Moraga-Gonzalez (2009) display empirical evidence that the distribution of gasoline prices spreads out as the number of stations increases implying an asymmetric affect of competition on the welfare distribution of consumers. In the airline industry, Borenstein & Rose (1994) similarly find that dispersion among airfares increases on routes with more competition or lower flight density whereas Gerardi & Shapiro (2009) recently show that higher competition has a negative effect on price dispersion. Finally, Walsh & Whelan (1999) report that brand price dispersion in the Irish grocery market increases with competition and Baye et al. (2004) provide evidence that price dispersion of consumer electronics products on Internet price comparison sites decreases with the number of sellers.

Our empirical work scrutinizes determinants of the price distribution of the Austrian retail gasoline market. Concluding the previous discussion this paper addresses two interesting issues. First, predictions on the relationship between the proportion of informed consumers and the level of search costs with the mean and the variance of the price distribution will be tested, respectively. Second, the relationship between entry competition and the mean and variance of prices will be examined. Referring to the predictions of the discussed models in subsection 4.2.1, subsection 4.2.2 and table 4.1 we propose the following hypotheses:

Hypothesis 1.1: The mean price is a decreasing function of the fraction of informed consumers in the market.

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[28]Lach & Moraga-Gonzalez (2009), for instance, argue that the effect of firm entry on the share of consumers who perceive a selected range of seller's prices is principally undetermined. Thus, different percentiles of the price distribution may be affected differently by increased competitive pressure in the market. As a result, the net effect of an increase in the number of sellers on the dispersion of prices can either be positive or negative, which leads them to conclude that „the direction and magnitude [of such effect] remains an empirical matter" (p. 19).

Hypothesis 1.2: The mean price is an increasing function of the level of search costs.

Hypothesis 1.3: The price variance is a non-monotonic function of the fraction of informed consumers in the market.

Hypothesis 1.4: The price variance is an increasing function of the level of search costs.

Hypothesis 2.1: The comparative static behavior of the price distribution is a complex function of entry competition. Thus, the mean price may be a positive or negative function of the number of sellers. Likewise, the price variance may be a positive or negative function of the number of sellers.

Hypothesis 2.2: If the mean price is negatively (positively) correlated with the number of sellers, then the mean price is expected to be positively (negatively) correlated with a distance measure between a station and its next neighbour.

Hypothesis 2.3: If the price variance is negatively (positively) correlated with the number of sellers, then the price variance is expected to be positively (negatively) correlated with a distance measure between a station and its next neighbour.

4.3 Empirical Analysis

4.3.1 Description of Data and Variable Specification

In our analysis we use data from various sources. Gasoline price data are collected by the Austrian chamber of labor and are available quarterly at irregular intervals from the period October 1999 to March 2005 (23 time periods). For the sample of gasoline stations used, the price data are unbalanced and consist of prices of diesel ranging from a maximum of 1,386 observations per time period (September 2003) to a minimum of 598 observations (March 2003) with 25,150 nonzero price observations over time. In total, price information was available for 61% of the station sample or for 1,718 out of all 2,822 Austrian gasoline stations for at least one of the given 23 time periods. The price data is merged with data for the geographical location of stations and other station specific as well as regional data. Demographical and regional data are obtained from the Austrian statistical office as part of the population census in 2001 and are available on a municipality level. Information about stations' characteristics covering their geographical coordinates is collected by Catalist in 2003 on an individual level.

Descriptive statistics are reported in table 4.3 where the set of covariates is arranged

into four main categories.

1. **Station Competition Proxies**

   We specify the degree by which stations spatially differentiate themselves and characterize stations' competitive environments with three main variables: the number of competitors in a circular radius of 3 kilometer (*Density*), the road travelling distance of a station to its nearest competitor (*Distance*) and a dummy variable with nonzero entries if a station does not have a rival seller within its 3km-periphery and thus is to be considered a local monopolist (*Monop*).

   The existing empirical literature suggests different measures of spatial differentiation.[29] The market geography of gasoline retailing in Austria is characterized by the dichotomy of local monopoly structures in remote areas and high seller density zones in metropolitan areas or along principal roadways. Accordingly, more than three quarters or 78.25% of stations have their nearest competitor within a distance of $3km$. In contrast, for a local market radius comparable to the median of the distance distribution ($0.96km$) 838 stations are considered to be a monopolist having no competitor in the respective geographical range. Thus, intuition might suggest that the choice of a circular competition zone should not be too narrow to capture the (complex) spatial patterns of competitive interactions between stations. However, by definition the local market radius may also establish a commensurable match between the demand and search cost proxies, given on an aggregate municipaly level, and the stations' (cirucular) competitive environment.[30] Consequently and to be consistent with the previous literature (Barron et al. (2004), Lewis (2008)) we define a local market radius of $3km$.

   Further, the number of firms in a particular region will generally be proportional to the average distance between firms as long as firms are equally distributed over the geographic area. If, however, individual shops are not distributed equally, the number of sellers in a local market is an inaccurate measure for the degree of spatial differentiation (cp. Pinkse et al. (2002)). Taking account of asymmetric location patterns, we additionally use the road distance between nearest competitors measured in km to proxy for competition intensity.[31]

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[29] A group of studies uses a circular approach with a local market radius of 1.5 miles to operationalize seller density in urban areas (cp. Barron et al. (2004) and Lewis (2008)) while other studies calculate the number of sellers in local municipals (cp. van Meerbeck (2003) and Clemenz & Gugler (2006)).

[30] According to the descriptives in table 4.3, the median of the municipal area amounts to $35.23km^2$ or an approximated circular radius of $3.35km$.

[31] Netz & Taylor (2002) use the Euclidean distance between retail outlets to account for spatial differentiation. Nonetheless this procedure ignores the fact that stations are connected via a network of roads. Consequently, the Euclidean distance might capture the relevant dimension of distance only very poorly. Based on the information on stations' geographic coordinates by using GIS-software we link the location of individual stations to the road network and calculate distances between neighbouring retail outlets.

2. **Search Cost Proxies**

On the demand side, consumers' search costs are proxied by variables corresponding to two different consumer groups that accordingly differ in their knowledge on local market pricing patterns. Particularly, we have information on the number of people commuting out of and commuting into a municipality and the number of overnight stays in a municipality.

Commuters are considered to purchase gasoline in high frequencies for their daily working trips. Consequently, they are characterized by a relatively high price elasticity of demand that relates to a low level of search costs and a profound knowledge of local sellers' pricing strategies respectively. On the contrary, the number of overnight stays proxies for the number of people that are not familiar with pricing patterns in the local gasoline market. Hence, this consumer group supposedly shows a lower price elasiticity and incurs higher costs for their search for the cheapest gasoline retailer.

Accordingly, in the further analysis the consumer group with a low level of search costs is represented by the share of commuters commuting out of a municipality relative to the number of employed people in that respective municipality ($Coms$). Further, the group of consumers characterized by a high level of search costs refers to the share of overnight stays relative to the number of inhabitants again on the municipal level ($Nights$).[32]

3. **Location and Regional Characteristics**

The third set of price determinants characterizes stations' location features. Accordingly, population density, given as the number of inhabitants in a municipality per square kilometer, allows to specifiy whether a station is located in a metropolitan or rural area ($PopDens$). Further, refering to respective speed limits, two dummy variables indicate if a station is located on a highway or a freeway distinguished by respective speed limits ($100kmh$, $130kmh$). Similarily, the dummy *Access* denotes if a station is considered to be highly accessible by cardrivers and finally nine state dummies display in which of the Austrian states a station is located.

4. **Station Characteristics**

Except for stations' plot size, individual station characteristics are all represented by dummy variables and specify different features that may affect sellers' pricing behavior. These comprise stations' opening times ($Open24h$), car services ($Wash$), service policies ($Service$, $Leisure$), product features ($Microwave$) as well as its payment facilities ($Creditcard$). Further, a set of dummies indicate a station's brand affiliation.

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[32]Additional variables listed in table 4.3 will be used for robustness checks.

Finally, based on information on stations' geographical coordinates and distance relations, spatial interdependencies between individual gas sellers are incorporated in a spatial weights matrix W. It states that, if one station (i) is to be considered a neighbour to another station (j) the respective element in the weights matrix (w_{ij}) will be nonzero. Further, each matrix element is weighted according to the inverse distance of the corresponding neighbourhood relationship (d_{ij}). Given this weighting scheme each element of the matrix is recalculated subject to the row standardization of W.[33] Formally:

$$(W)_{ij} = \tilde{w}_{ij} = \frac{w_{ij}}{\sum_j w_{ij}} \tag{4.3}$$

$$w_{ij} = \begin{cases} \frac{1}{d_{ij}} & \text{i } and \text{ j } neighbours \\ 0 & other \end{cases}$$

$$\sum_j \tilde{w}_{ij} = 1$$

In the construction of W two neighbourhood criteria are applied. Firstly, the nearest neighbour of every single station is included and secondly every seller in a 3-kilometer-periphery around each station is to be considered a neighbour.[34] Technically, this ensures that all row sums of the weighting matrix are nonzero and implies that every station is spatially related to another station. In addition, W takes into account that stations in more densly populated and presumably in more competitive areas are subject to more complex spatial interdependencies compared to local monopolies.

Table 4.3: Descriptive statistics (Number of observations = $25, 150$)

Variable	Description	Mean	Median	S.D.
pd	Diesel price (euro cents)	76.49	75.9	6.36
Station competition proxies:				
Density	Number of stations in 3km radius	6.672	3.0	8.68
Distance	Distance to nearest station	1.816	0.898	2.514
Monop	No neighbour in 3km radius	0.187	0.0	0.390
Search cost proxies:				
Coms	ComsOut/Employed	0.325	0.311	0.141
ComsTot	ComsSum/Inhabitants	0.361	0.331	0.140

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[33] Thus, closer neighbours are related to higher values of corresponding weights with every row of W summing up to 1. It follows that stations with single neighbours (special case: local monopolists) correspond to matrix elements equal to 1.

[34] To test for robustness two alternative weighting matrices will be used. For details see table 4.6 in the appendix.

Nights	*Overnight/Inhabitants*	2.082	0.143	6.481
NightsC	*Overnight/ComsSum*	8.000	0.368	34.543
ComsSum	*ComsIn+ComsOut*	10,860.0	3,034.0	16,078.0
ComsOut	Number of commuters out of a municipality	3,591.0	1,176.0	5,067.0
ComsIn	Number of commuters into a municipality	7,270.0	1,746.0	12,098.0
Overnight	Number of overnight stays in a municipality	79,610.0	1,062.0	206,197.0
Location and regional characteristics:				
PopDens	*Inhabitants/Area*	1,292.0	281.1	2,943.0
Inhabitants	Number of inhabitants in a municipality	36,470.0	7,368.0	57,162.0
Employed	Number of employed in a municipality	17,890.0	3,462.0	27,891.0
Area	Municipal area (km^2)	51.50	35.23	48.98
100*kmh*	On Freeway (80-100km/h)	0.018	0.0	0.132
130*kmh*	On Highway (100-130km/h)	0.023	0.0	0.151
Access	Highly accessible	0.6	1.0	0.49
Bgld	Station in state: Burgenland	0.035	0.0	0.184
Ktn	Station in state: Carinthia	0.095	0.0	0.294
Noe	Station in state: Lower Austria	0.161	0.0	0.368
Ooe	Station in state: Upper Austria	0.144	0.0	0.352
Slb	Station in state: Salzburg	0.092	0.0	0.289
Stk	Station in state: Styria	0.183	0.0	0.387
Tir	Station in state: Tyrol	0.129	0.0	0.335
Vbg	Station in state: Vorarlberg	0.031	0.0	0.175
Vie	Station in state: Vienna	0.126	0.0	0.332
Station characteristics:				
Open24h	Station is open 24h	0.145	0.0	0.353
Wash	Station has a carwash	0.704	1.0	0.456
Service	Station offers full-service	0.227	0.0	0.418
Leisure	Station offers leisure facilities	0.509	1.0	0.5
Microwave	Station sells microwave products	0.265	0.0	0.441
Creditcards	Station offers credit card payment	0.921	1.0	0.271
Agip	Station brand: Agip	0.074	0.0	0.262
Aral	Station brand: Aral	0.002	0.0	0.044
Avanti	Station brand: Avanti	0.049	0.0	0.216
Avia	Station brand: Avia	0.019	0.0	0.137
BP	Station brand: BP	0.221	0.0	0.415
Esso	Station brand: Esso	0.076	0.0	0.266

Jet	Station brand: Jet	0.045	0.0	0.208
OMV	Station brand: OMV	0.181	0.0	0.385
Shell	Station brand: Shell	0.132	0.0	0.338
Stroh	Station brand: Stroh	0.022	0.0	0.145
Unbranded	Station Unbranded	0.178	0.0	0.384
Plotsize	Station Plotsize	$1,785.0$	$1,500.0$	$1,718.0$

4.3.2 Model Specification

4.3.2.1 The Price Equation

The empirical analysis of the price distribution takes advantage of the panel struc-
ture of the dataset and will proceed in two steps.[35] We first estimate a model on the
relationship between gasoline prices and local market characteristics as well as indi-
vidual seller characteristics. The aim is to analyze the determinants of gasoline prices
and control for price differences resulting from station heterogeneity.[36] According to
the four sets of covariates described in section 4.3.1 the structure of the empirical
model for gasoline prices is given by

$$
\begin{aligned}
log(pd_{it}) = \; & \alpha_0 + \alpha_1 Competition_i + \alpha_2 Search_{it} + \alpha_3 Location_i + \alpha_4 Station_i \\
& + \sum_{t=1}^{T} \chi_t Time_t + u_{it}
\end{aligned}
\tag{4.4}
$$

with:

$$
\begin{aligned}
Competition_i &= \{log(Density_i), log(Distance_i)\} \\
Search_{it} &= \{Coms_i, log(Nights_{it})\} \\
Location_i &= \{log(PopDens_i), 100kmh_i, 130kmh_i, Access_i, State_i\} \\
Station_i &= \{Open24h_i, Wash_i, Service_i, Leisure_i, Microwave_i, Creditcards_i, log(Plotsize_i), Brand_i\}
\end{aligned}
$$

where the dependent variable is denoted pd_{it}, the self service, regular price of diesel
at station i at a point of time t, measured in euro cents per liter; $Competition_i$
represents proxies for the competitive environment of each station, essentially (the
logarithms of) seller density and the distance to the closest rival; $Search_{it}$ subsumes
the search cost proxies share of commuters and share of overnight stays; $Location_i$
specifies characteristics of a station's location, including state fixed effects; $Station_i$
captures station specific price determinants, including brand fixed effects; finally time

---

[35] The data is structured in a repeated cross section and sorted according to the different time
periods.

[36] Since all of the explanatory variables except for *Overnight* and *Nights* respectively do not change
over time, fixed effects for individual gasoline stations will not be included in the regressions.

fixed effects are included in the estimation and u_{it} denotes an error term.

Price equation (4.4) is estimated using two different statistical approaches. Previous studies for the Austrian market (Clemenz & Gugler (2006); Pennerstorfer (2009)) have investigated determinants of gasoline price levels. Their findings highlight the impact of spatial competition in gasoline retail pricing. Since our analysis focuses on determinants of the unexplained price variance, additionally to standard OLS, we use a GMM approach[37] accounting for spatial effects in the price residuals

$$u_{it} = \lambda \sum_{j=1}^{n_t T} \tilde{w}_{ij} u_{jt} + \nu_{it} \tag{4.5}$$

where the weighting matrix $W = (\tilde{w}_{ij})$ equals a single block diagonal matrix of dimension $n_t T \times n_t T$ consisting of the spatial weight matrices W_t for each period $t = 1, ...23$ and with n_t as the number of corresponding cross sectional observations and T as the number of periods.

The intuition behind the Spatial Error Model (SEM) in (4.5) is that certain effects remain outside the model of the price equation and enter the price residuals. In turn, these unmodeled price effects could show a spatial pattern, i.e. they spill over across neighbouring gasoline stations. Correspondingly, the residuals comprise of a part that is explained by the spatial structure imposed by W and the innovations ν_{it}; the existing spatial autocorrelation is then captured by the autoregressive parameter λ. On the contrary, ignoring spatial autocorrelation in the residuals implies biased standard errors and is thus associated with difficulties in the interpretation of coefficients (cp. LeSage (1997)).

4.3.2.2 Analysis of Price Dispersion

The second step of the analysis determines the relationship between price dispersion, competition, search costs and other location and station characteristics by

$$
\begin{aligned}
log(\epsilon_{it}^2) = \ & \beta_0 + \beta_1 Competition_i + \beta_2 Search_{it} + \beta_3 Location_i + \beta_4 Station_i \\
& + \sum_{t=1}^{T} \chi_t Time_t + \eta_{it}
\end{aligned}
\tag{4.6}
$$

---

[37] Technically, spatial autoregressive models or spatial error models respectively can be implemented via defining a maximum likelihood (ML) function and subsequently solving for the autoregressive parameter λ and variance. Generally, however the weights matrix W will be characterized by an asymmetric structure and for large datasets the computation of the corresponding eigenvalues is not feasible anymore. We implement a GMM estimator for a spatial simultaneous autoregressive error model in R using the *spdep* package. For technical details of the estimation procedure see Kelejian & Prucha (1999); an introduction into spatial econometrics focusing on the ML approach is provided in Anselin (1988) and LeSage & Pace (2009).

with:

$$Competition_i = \{log(Density_i), log(Distance_i)\}$$
$$Search_{it} = \{Coms_i, Coms_i^2, Nights_{it}, Nights_{it}^2\}$$
$$Location_i = \{log(PopDens_i), 100kmh_i, 130kmh_i, State_i\}$$
$$Station_i = \{Brand_i\}$$

where as the dependent variable $\epsilon_{it} = \{u_{it} \ or \ \nu_{it}\}$ is interpreted as a measure of unexplained price variation, free of store-, time-, and spatial effects of a station i during period t relative to the statewide average gasoline price. As with the price regression the main covariates of interest for explaining price differences are (the logarithms of) *Density* and *Distance*, the search cost proxies *Coms* and *Nights* as well as their quadratic forms $Coms^2$ and $Nights^2$. Analogous to the approach in modeling market price levels, other location and station specific parameters act as controls to isolate the competition effects and the (indirect) effects of search costs. Again, time fixed effects are included and η_{it} is an error term.

Ning & Haining (2003) highlight in their case study of gasoline pricing in the Sheffield metropolitan area the spatial structure of price residuals as a feature of localized interaction between stations. Similarly, Lewis (2008) constructs a localized dispersion measure that captures stations' average price deviation from its neighbouring competitors and finds significant competition effects. We will use the residuals from price equation (4.4) as a measure of (local) price dispersion in the market. These quantify the unknown deviation from the average gasoline price for each seller in the market after controlling for significant price determinants relating to competition intensity, local demand structures, stations' location and additional individual seller characteristics. Further spatial interdependencies or potential spatial autocorrelation in the OLS residuals respectively will be accounted for by applying the Spatial Error Model specified in equation (4.5). Accordingly, the dependent variable $\epsilon_{it} = \{u_{it} \ or \ \nu_{it}\}$ in the price dispersion regressions consists of two sets of price residuals: u_{it} denoting the residuals calculated in the OLS specification and ν_{it} representing the residuals as an outcome of the Spatial Error Model.

4.4 Results

This section provides results for the effects of competition, search costs, location and station characteristics on price levels and price dispersion. Estimates of the price model parameters in equation (4.4) are presented in table table 4.4 and estimated coefficients of the disperson model in (4.6) are reported in table 4.5 respectively.[38]

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[38]All of the reported t-statistics are robust to heteroscedasticity. We apply a White-$HC0$-estimator to fit the OLS regression models. For details see Zeileis (2004).

4.4.1 Price Determinants

Although small in magnitude, the results for the competition proxies $log(Density)$ and $log(Distance)$ in the price equation reveal consistent findings that are in line with theoretical predictions from spatial competition models (Salop (1979)) and with previous empirical studies (Clemenz & Gugler (2006); Pennerstorfer (2009)). Generally, the findings suggest that stations' price levels are a result of their local geographic monopoly structure whose size depends on rivals' market power and the prices charged by neighbouring competitors. In a nutshell the price regressions show that enhanced competition has on average a diminishing effect on market price levels, other factors held constant. Correspondingly, in both the OLS and the GMM specifications the coefficient on $log(Density)$ is negative and significant at the 1% level or better. Further, the coefficient on $log(Distance)$ is positive and significant ($p < 0.02$) across OLS estimations. Particularly, the elasticity of $Density$ amounts to -0.0023 (-0.0021) in the standard specification of the OLS (GMM) model indicating that as the number of competitors in a radius of $3km$ increases by 1% gasoline prices on average decrease by 0.0023% (0.0021%).[39] Similarly, increasing a station's distance to its closest competitor by 1% leads to an average increase in price levels by 0.0006% (0.0003%). We also check for robustness in an alternative specification by substituting the $Distance$ variable with the monopoly dummy $Monop$ and find positive and highly significant effects for the latter for both the OLS and GMM estimations.[40] Accordingly, stations who have no neighbour in their $3km$ market periphery charge on average higher prices by approximately 1.1% (0.65%) other factors equal.

Results for the search cost proxies $Coms$ and $log(Nights)$ support predictions of spatial competition between stations. They provide evidence that differences between consumer groups concerning their knowledge and information on market prices affect sellers' pricing behavior. Other variables held constant, an increase in the share of informed consumers causes the average market price level to decrease whereas the price level tends to increase as the share of uninformed consumers rises. In context with the unit transportation cost t this implies a positive correlation between consumers' search cost levels and the mean price. Correspondingly, in all specifications (OLS and GMM) coefficients on $Coms$ are negative and highly significant and coefficients on $log(Nights)$ are positive and significant at the 1% level or better. In particular, this implies for the standard OLS (GMM) specification that the percentage change in gasoline prices is given by -0.0327 (-0.0166) when the share of people commuting out of a municipality relative to the employed in that region increases by one percentage point. Likewise, the elasticity of $Nights$ amounts to 0.0016 implying

---

[39]See columns 1 and 2 in table 4.4.
[40]See columns 5 and 6 in table 4.4.

price increases of approximately 0.002% as the proportion of overnight stays relative to the number of inhabitants in a municipality rises by one percent.

In an additional specification (columns 3 and 4), we also include a spatially lagged variable for $Coms$, denoted $W * Coms$, accounting for the spatial influence of the search cost proxy for neighbouring stations.[41] Respective coefficient estimates show significant negative signs and thus are in line with estimation results of the $Coms$ variable. In particular, they indicate that an increase of the spatially averaged commuter share of neighbouring stations by one percentage point leads to a decrease in prices of -0.0072% and -0.0068% in OLS and GMM specifications. Eventually, column 5 and 6 also report results for an interaction term between the monopoly dummy $Monop$ and $Coms$. Respective coefficient estimates in the OLS and GMM models are highly significant and negative. This implies that stations with no neighbours in their market periphery of $3km$ show a substantial stronger price reaction to an increase in the number of informed consumers than stations with competitors in their local market. Specifically, the difference amounts to -0.0224% (-0.0142%) as the share of commuters increases by one percentage point for the OLS (GMM) estimation.[42]

Estimated coefficient results for the location parameters reveal that stations located on major roadways and stations that are highly accessible set on average higher prices. In addition, regression results provide statistical evidence that stations located in more densly populated areas are more likely to charge lower prices. Particularly, results of the OLS estimations reveal that stations located on major highways routes (speedlimits: up to 130km/h) charge significantly higher prices by about 3.9%; respective estimates in the GMM specifications show mark-ups of more than 4%. If a station is located on a major roadway (speedlimits up to 100km/h) significant price increases of more than 1% are observed across OLS and GMM specifications and finally accessibility mark-ups amount to more than 0.1% in all model estimations. The impact of population density $log(PopDens)$ on price levels is small but highly significant: the OLS estimates indicate that an increase of density by one percent leads to price decreases of around 0.002%; corresponding estimates for the elasticities in the GMM specifications yield smaller values between -0.0006 and -0.0009 below a significance level of 1%.[43]

---

[41]Since the local market radius does not differentiate if competitors are within the same municipality, these effects arise due to the irregular shape of administrative units as well as the fact that stations may be located close to a municipal border. We also included a spatially lagged term of the second search cost proxy $W * Nights$ but found no significant effects in any of the specifications.

[42]Again the interaction effect between the search cost proxy for the uninformed consumers and the monopoly dummy $Nights * Monop$ was not significant.

[43]Clemenz & Gugler (2006) show that population density predominantely explains location and thus density patterns of gasoline stations in the Austrian market. Leaving out the population density proxy $log(PopDens)$ across different price specifications (OLS and GMM) does not change signs and significance levels of all coefficients for one notable exception (correspond-

After controlling for stations' spatial differentiation, estimate results for station specific characteristics show further dimensions in which product differentiation occurs. All variables except for the dummies controlling for carwash and leisure facilities ($Wash, Leisure$) have significant positive signs. In particular, stations that have permanent opening times ($Open24h$: 0.32% to 0.38%) or offer full-service ($Service$: 0.22% to 0.56%) tend to charge higher prices. Further, sellers who offer potential for financing purchases using company credit cards ($Creditcards$: 0.31% to 0.49%) as well as a broad variety of other goods in a convenience store ($Microwave$: 0.20% to 0.22%) also impose higher mark-ups.

The spatial autoregressive coefficient λ is estimated roughly as 0.612, and in all three GMM specifications it is highly significant. The test result for the Moran's I statistic of the OLS model in equation (4.4) amounts to 0.771 (p < 2.2e-16) indicating spatial autocorrelation in the residuals. We conduct a Lagrange Multiplier (LM) Test to generally determine which alternative specification to use and to confirm the application of the Spatial Error Model suggested in equation (4.5). In line with the Moran's I test statistic, both standard LM-Error and LM-Lag test statistics reveal highly significant results and reject the null hypothesis (LM-Error: 14682.41 (p < 2.2e-16); LM-Lag: 14248.40 (p < 2.2e-16)). Finally, the robust forms of the test statistics show also both highly significant results but with a substantial larger value for the test statistic of the Error Model (RLM-Error: 457.93 (p < 2.2e-16); RLM-Lag: 23.92 (p < 1.005e-06)).

Anselin (2005) provides a simple decision rule concerning the selection process in spatial regression modeling (p. 199).[44] In the classical textbook case one of the (robust) test statistics significantly rejects the null hypothesis whereas the other alternative does not reject or only rejects the null on much smaller orders of statistical magnitude. Since both robust statistics (RLM-Error and RLM-Lag) in our case are highly significant we proceed with the specification with the higher robust test value. To check for robustness and avoid possible misspecifications relating to the imposed spatial structure, we rerun the GMM price regressions and LM specification tests with alternative weighting matrices.[45] Results on signs and significance levels of the

---

ing results are given in table 4.8 in the appendix): due to multicolinearity the coefficient on $log(Density)$ is partly affected. In the OLS specifications its value approximately doubles and its significance level rises substantially. In contrast, in the GMM specification, respective coefficient values remain stable and only significance levels increase. Since our main conclusions in terms of coefficients signs and significance levels are not affected, we report the results including the population density covariate in the regression tables.

[44] For technical details on LM tests regarding spatial model specifications see Anselin et al. (1996). In addition, LeSage & Pace (2009) provide a concise overview of the comparison between spatial and non-spatial model applications (p. 155ff). Further, a more conceptual overview of spatial modeling in applied econometrics is given in the well known seminal paper of Anselin (2002).

[45] Accordingly, two additional matrices are constructed: $W2$ is based on the same neighbourhood criterion as W but with weights relating to the squared inverse distances; $W10$ captures the ten nearest neighbours of every station (if applicable) with weights depending on the inverse distance measure. See table 4.6 in the appendix.

regression coefficients as well as conclusions regarding the model selection remain unchanged.[46] Additionally, further robustness checks are carried out to test the results of OLS and GMM price models with regards to their functional form. Results indicate that structural relationships in terms of coefficient signs and significance levels are independent of log-log and lin-lin model specification forms.[47]

Table 4.4: Price function regressions (log-log)

Independent :	Dependent : $log(pd)$					
	Weights Matrix : W					
	OLS	GMM	OLS	GMM	OLS	GMM
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	−0.3772***	−0.3907***	−0.3764***	−0.3882***	−0.3798***	−0.3906***
	(0.0035)	(0.0033)	(0.0035)	(0.0034)	(0.0037)	(0.0036)
$log(Density)$	−0.0023***	−0.0021***	−0.0026***	−0.0024***	−0.0024***	−0.0023***
	(0.0005)	(0.0004)	(0.0005)	(0.0004)	(0.0005)	(0.0005)
$log(Distance)$	0.0006**	0.0003	0.0006**	0.0002		
	(0.0002)	(0.0002)	(0.0002)	(0.0002)		
Monop					0.0110***	0.0065***
					(0.0024)	(0.0016)
Coms	−0.0327***	−0.0166***	−0.0281***	−0.0173***	−0.0219***	−0.0128***
	(0.0024)	(0.0021)	(0.0030)	(0.0021)	(0.0032)	(0.0023)
$(W)(Coms)$			−0.0072**	−0.0068***	−0.0092***	−0.0077***
			(0.0029)	(0.0021)	(0.0029)	(0.0022)
$(Monop)(Coms)$					−0.0224***	−0.0142***
					(0.0051)	(0.0033)
$log(Nights)$	0.0016***	0.0016***	0.0015***	0.0016***	0.0014***	0.0015***
	(0.0005)	(0.0004)	(0.0005)	(0.0004)	(0.0005)	(0.0004)
$log(PopDens)$	−0.0022***	−0.0009***	−0.0022***	−0.0008***	−0.0020***	−0.0006***
	(0.0003)	(0.0002)	(0.0003)	(0.0002)	(0.0002)	(0.0002)
$100kmh$	0.0118***	0.0144***	0.0117***	0.0141***	0.0121***	0.0142***
	(0.0020)	(0.0010)	(0.0020)	(0.0010)	(0.0020)	(0.0010)
$130kmh$	0.0387***	0.0425***	0.0384***	0.0424***	0.0387***	0.0425***
	(0.0021)	(0.0011)	(0.0021)	(0.0011)	(0.0021)	(0.0011)
Access	0.0018***	0.0012***	0.0018***	0.0012***	0.0019***	0.0012***

[46] Again the standard as well as the robust LM-Error and LM-Lag test statistics are highly significant with higher robust test statistics for the SEM. Refer to table 4.7 and for corresponding GMM regression results to table 4.9 in the appendix.

[47] For the log-log form see table 4.4 and the lin-lin form table 4.10 in the appendix.

	(0.0006)	(0.0003)	(0.0006)	(0.0003)	(0.0006)	(0.0003)
Open24h	0.0032***	0.0038***	0.0032***	0.0038***	0.0032***	0.0038***
	(0.0008)	(0.0004)	(0.0008)	(0.0004)	(0.0008)	(0.0004)
Wash	−0.0025***	−0.0025***	−0.0026***	−0.0026***	−0.0026***	−0.0026***
	(0.0007)	(0.0004)	(0.0007)	(0.0004)	(0.0007)	(0.0004)
Service	0.0023***	0.0056***	0.0024***	0.0056***	0.0022**	0.0055***
	(0.0009)	(0.0005)	(0.0009)	(0.0005)	(0.0009)	(0.0005)
Leisure	−0.0033***	−0.0018***	−0.0033***	−0.0018***	−0.0033***	−0.0019***
	(0.0006)	(0.0004)	(0.0006)	(0.0004)	(0.0006)	(0.0004)
Microwave	0.0022***	0.0020***	0.0022***	0.0020***	0.0021***	0.0020***
	(0.0007)	(0.0004)	(0.0007)	(0.0004)	(0.0007)	(0.0004)
Creditcards	0.0032***	0.0050***	0.0031**	0.0049***	0.0031**	0.0049***
	(0.0012)	(0.0007)	(0.0012)	(0.0007)	(0.0012)	(0.0007)
$log(Plotsize)$	0.0035***	0.0021***	0.0036***	0.0022***	0.0036***	0.0021***
	(0.0008)	(0.0005)	(0.0008)	(0.0005)	(0.0008)	(0.0005)
lambda		0.6122***		0.6127***		0.6124***
		(0.0055)		(0.0055)		(0.0055)
$Median(\nu_{it})$		0.0025		0.0024		0.0023
$S.E.(\nu_{it})$		0.0229		0.0229		0.0229
$Median(u_{it})$	0.0021		0.0021		0.0021	
$S.E.(u_{it})$	0.0364		0.0364		0.0364	
$Adj.R^2$	0.804		0.804		0.804	
Obs	25, 150	25, 150	25, 150	25, 150	25, 150	25, 150

*** denotes significance at the 1% level, ** at the 5% level, * at the 10% level

Omitted brand category = Unbranded, omitted state category = Vienna, omitted time category = 199910

(Coefficients for the time-, state- and brand-fixed effects have been omitted)

White heteroscedasticity correction ($H0$) is applied to OLS standard errors.

(Standard errors in parentheses)

4.4.2 Determinants of Price Dispersion

Coefficient estimates of the elasticities for the competition proxies *Density* and *Distance* reveal consistent findings regarding changes in the competitive environment on stations' price differences. Correspondingly, the pricing behavior of stations exposed to a higher degree of (spatial) competition intensity is likely to be characterized by a higher variance in prices. In short, higher spatial competition is associated with higher price dispersion. Particularly, we find a highly significant positive relationship between the logarithm of the number of stations in a $3km$ periphery,

$log(Density)$, as well as statistcal evidence for a negative relationship between the logarithm of the distance of a station to its closest competitor, $log(Distance)$, with the logarithm of the squared price residuals from equation (4.4) and (4.5) respectively. Corresponding estimates, for instance, show that an 1% increase in the number of competitors in a $3km$ local market radius leads to an increase in price dispersion of roughly 0.1% holding other factors equal.[48] Likewise, dispersion declines by about 0.04% to 0.045% (0.021% to 0.027%) as the distance measure (in km) increases by 1% when using the OLS (GMM) residuals in the estimations.[49]

As regards predictions of search models, regression results across the two different sorts of residuals (u_{it} and ν_{it}) suggest a significant nonlinear relationship between the share of informed consumers proxied by the $Coms$ variable and the level of price dispersion. According to the coefficient estimates of the linear and quadratic term in table 4.5 the course of price dispersion is characterized by an inverse U-shaped function of $Coms$.[50] In particular, the maximum of price dispersion is reached for a commuters-employed ratio in the interval of 0.4 to 0.5 implying that price variance increases with the share of commuters (informed consumers) in municipalities with a low proportion of commuting people whereas similarly dispersion decreases with increases in the $Coms$ proxy in municipalities with high shares of commuters. Correspondingly, for a share of commuters to the number of employed people of 0.2 estimates in column 2 of table 4.5 for instance suggest an increase of gasoline price variance of 0.75% as the respective ratio increases by one percentage point. Likewise, as the commuters-employed ratio amounts to 0.5 price dispersion decreases by 0.12% for a corresponding one-percentage-point increase in the share of informed consumers. Note that for the turning point of 0.4662 (cf. column 2) about 20% of the observations for $Coms$ lie in the decreasing part of the inversed U and for a value of 0.4333 (column 4) for more than 25% of the sample this is true.

Coefficient estimates of the other search cost variable $Nights$ show that price dispersion increases as the share of uninformed consumers in a municipality rises. Inclusion of the squared term $Nights^2$ does not reveal significant results and supports the hypothesis of a positive linear relationship. Specifically, all else equal dispersion increases across different specifications by roughly 0.01% when the share of overnight stays in a municipality to the number of inhabitants in the respective administrative unit increases by one percentage point.

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[48] Coefficients are significant at the 0.05% level or better.

[49] Estimates in the model with the OLS residuals u_{it} are significant at the 1% level or better; p-values for estimations with the GMM residuals ν_{it} amount to $p < 0.07$ (table 4.5 column 3) and $p < 0.16$ (column 4).

[50] Across different specifications respective linear coefficients are statistically different from zero at the 2% significance level or better; p-values for the quadratic terms amount to $p < 0.069$ (table 4.5 column 2) and $p < 0.036$ (column 4).

Table 4.5: Results of price dispersion regressions for different residuals

Independent :	Dependent : $log(u_{it}^2)$ (OLSResiduals)		Dependent : $log(\nu_{it}^2)$ (GMMResiduals)	
	(1)	(2)	(3)	(4)
Intercept	-8.0711^{***}	-8.3536^{***}	-7.9964^{***}	-8.2982^{***}
	(0.1556)	(0.2095)	(0.1547)	(0.2099)
$log(Density)$	0.1012^{***}	0.1153^{***}	0.1210^{***}	0.1363^{***}
	(0.0289)	(0.0297)	(0.0289)	(0.0297)
$log(Distance)$	-0.0450^{***}	-0.0400^{***}	-0.0266^{*}	-0.0208
	(0.0140)	(0.0143)	(0.0145)	(0.0148)
Coms	0.3312^{**}	1.3289^{**}	0.2771^{**}	1.4223^{**}
	(0.1364)	(0.5644)	(0.1380)	(0.5658)
$Coms^2$		-1.4252^{*}		-1.6412^{**}
		(0.7847)		(0.7842)
Nights	0.0065^{***}	0.0104^{***}	0.0072^{***}	0.0093^{**}
	(0.0021)	(0.0039)	(0.0020)	(0.0038)
$Nights^2$		$-3.74e^{-05}$		$-1.75e^{-05}$
		$(2.92e^{-05})$		$(2.39e^{-05})$
$log(PopDens)$	-0.0912^{***}	-0.0863^{***}	-0.0995^{***}	-0.0946^{***}
	(0.0162)	(0.0163)	(0.0158)	(0.0160)
$100kmh$	0.2944^{***}	0.2931^{***}	0.2254^{**}	0.2237^{**}
	(0.0996)	(0.0995)	(0.1133)	(0.1132)
$130kmh$	0.6047^{***}	0.6053^{***}	0.4574^{***}	0.4565^{***}
	(0.0869)	(0.0868)	(0.0895)	(0.0894)
$Median(\eta_{it})$	0.4670	0.4655	0.4445	0.4421
$S.E.(\eta_{it})$	2.174	2.174	2.158	2.158
$Adj.R^2$	0.101	0.101	0.107	0.108
Obs	$25,150$	$25,150$	$25,150$	$25,150$

*** denotes significance at the 1% level, ** at the 5% level

* at the 10% level

Omitted brand category = Unbranded

Omitted state category = Vienna

Omitted time category = 199910

(Respective FE-Coefficients have been omitted)

White heteroscedasticity correction ($H0$) is applied.

(Standard errors in parentheses)

With respect to the location parameters estimates provide significant evidence for a negative relationship between the logarithm of population density $log(PopDens)$ and the price dispersion measures $log(u_{it}^2)$ and $log(\nu_{it}^2)$. Particularly, variance shrinks by about 0.09% (0.1%) in the specifications using the OLS (GMM) residuals as the number of inhabitants per square kilometer rises by 1% holding other factors constant. Further, corresponding estimation results for the dummies indicating a station's location on a major road or highway show significant positive signs. Using the OLS

residuals, stations on freeways with a speed limit of 100km/h are on average charc-
terized by a higher price dispersion of about 30% and stations on interstates with
speed limits of 130km/h have a higher price variance of about 60% both compared
to stations that are not located on main roads and other factors held constant. Cor-
responding estimates for the specification with the GMM residuals yield a mark-up
of 22% for the $100kmh$ and 46% for the $130kmh$ covariate.

As with the price function additional checks on specification form and robustness are
carried out. Particularly, we conduct a LM Test and Moran's I Test on the residuals
of the standard OLS dispersion model given in equation (4.6) with $log(u_{it})$ as the de-
pendent variable. Corresponding results reveal highly significant values for Moran's
I and for the standard LM-Error and LM-Lag test statistics. As regards the robust
test statistics, the RLM-Lag consistently rejects the null hypothesis whereas the
RLM-Error shows only weak significant or insignificant results.[51] As a consequence,
we apply a spatial lag model to the OLS specification.[52] Corresponding results con-
firm previous findings and reveal highly significant positive elasticities for $Density$,
highly significant negative elasticities for $Distance$, highly significant evidence for a
quadratic dependence of price dispersion on $Coms$ as well as a significant positive
relationship between $Nights$ and the level of price dispersion.[53]

Finally, we also test the robustness of our results and run regressions with an alterna-
tive set of search cost proxies leaving the remaining regression model in equation (4.6)
unchanged. Accordingly, the proxy for the ratio of informed consumers in the market
($Coms$) is substituted with the share of the total number of commuters (number of
people commuting out and into the municipality) relative to the number of inhabi-
tants in the respective administrative unit, denoted $ComsTot$. Further, the measure
for the share of uninformed gasoline consumers ($Nights$) is replaced with the proxy
$NightsC$, denoting the number of overnight stays devided by the total number of
commuters in a municipality.[54] In addition to the residual derivatives $log(u_{it}^2)$ and

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[51]Again test results are provided for different weights matrices, cf. table 4.11 in the appendix.

[52]The spatial lag model takes the form

$$log(u_{it}^2) = \rho \sum_{j=1}^{n_t T} w_{ij} log(u_{jt}^2) + \boldsymbol{\beta X} + \sum_{t=1}^{T} \chi_t Time_t + \kappa_{it} \qquad (4.7)$$

with the same set of covariates $\mathbf{X} = \{Competition_i, Search_{it}, Location_i, Station_i\}$ as in equation
(4.6) and a spatial weights matrix $(W)_{ij} = w_{ij}$ (cp. equation (4.3) and table 4.6, regression
results are reported for weights matrices W, $W2$ and $W10$ in non row standardized form).
Technically, the SAR model is implemented as a Generalized Spatial Two Stage Least Square
model (GS2SLS) in R via the *spdep* package. The estimator fits the regression model by using
spatially lagged X variables as instruments for the spatially lagged dependent variable with ρ
as the coefficient of spatial dependence, for details see Kelejian & Prucha (1998).

[53]Accordingly, coefficients of $log(Density)$ and $log(Distance)$ across various specifications are signif-
icant at the 1% level or better (all standard errors are subject to a White-$H0$ heteroscedasticity
correction). In addition, using the weights matrix W the turning point for $Coms$ is given at
0.4673 with 80.75% of the observations in the decreasing part of the parabola. For details see
table 4.12 in the appendix.

[54]For descriptives of the variables see also table 4.3.

$log(\nu_{it}^2)$, we also introduce an alternative dispersion measure ω_{it} accounting for observed and unobserved price differences due to seller heterogeneity.[55] In line with previous results estimated elasticities for *Density* and *Distance* across alternative specifications are significant at the 1% level or better with expected signs. Likewise, the linear and quadratic coefficients of *ComsTot* and *ComsTot²* are significant and support the hypothesis of an inverse U-shaped form of price dispersion with respect to the share of informed consumers in the market. Further, coefficient estimates on *NightsC* indicate a significant positive linear relationship between the share of uninformed consumers and the level of price dispersion.[56]

4.5 Conclusion

This research paper examines determinants of the empirical diesel price distribution in the Austrian retail gasoline market. Particularly, we specified a model to test the relationship between the mean price and price variance with a set of spatial competition proxies and a set of search cost proxies referring to two distinct consumer groups. Our work is motivated by the fact that predictions by search models highlight the importance of information differentials among consumers for the existence and comparative static behavior of the equilibrium price distribution. In addition, model predictions and the empirical evidence on the impact of entry competition, in terms of the number of sellers, on the average price and price dispersion is not straightforward. Consequently, our contribution focuses on the question how the fraction of informed and uninformed consumers in the gasoline market relate to the mean price and price variance. Further, we provide evidence for the statistical correlation of the mean and variance with the number of competitors in the local market periphery of

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[55] With reference to Lach (2002) (p. 436f) and Lewis (2008) (p. 658f) we estimate a two stage panel regression on gasoline prices using seller-fixed effects and time-fixed effects to control for any price differences resulting from observed and unobserved heterogeneities in time. Consequently, the remaining residuals are used in the dispersion analysis and are denoted ω_{it}. Formally:

$$p_{it} = \alpha + \sum_{i=1}^{I} \zeta_i Station_i + \sum_{t=1}^{T} \chi_t Time_t + \omega_{it}$$

[56] For details see regression table 4.13 in the appendix. The turning point for *ComsTot* in regressions using the OLS (GMM) residual derivative $log(u_{it}^2)$ $(log(\nu_{it}^2))$ as the dependent variable amounts to 0.8297 (0.7573) with 1.82% (2.11%) of the observations lying in the decreasing part of the inverse U (columns 2 and 4). Corresponding results for estimations on the dispersion measure $log(\omega_{it}^2)$ yield a maximum for *ComsTot* at a share of 0.6327 with a comparably higher percentage of observations (5.24%) in the upper interval (column 6). In contrast, the significant coefficient on the squared variable *NightsC* in the regressions on $log(\omega_{it}^2)$ does not support predictions on a decreasing effect of the number of uninformed consumers on price dispersion since the sample of observations in the respective interval (0.07%) is so small it can practically be ignored. Eventually, the alternative specifications in table 4.13 are also tested with a spatial lag model structurally specified according to equation 4.7. Respective results are given in table 4.14 in the appendix and support all previous findings.

a station as well as with the distance between nearest rivals.

In the analysis a two-step procedure is applied. Firstly, the price function is estimated to identify the impact of price determinants, most notably, the competition and search cost variables, and to effectively control for further observed station and product characteristics. In turn, the price residuals are interpreted as a measure for unexplained price differences free of time, store and spatial effects. Thus, in the second step the squared price residuals are regressed on a set of covariates to investigate the impact of consumer fractions with differing information sets and competition intensity on price dispersion. Besides the usual OLS techniques, spatial econometric tools are used in the analysis to control for spatial spillover effects in the price residuals. Particularly, we apply a Spatial Error Model (SEM) in the estimation of price levels and pertinent tests (LM and Moran's I) to suggest a proper model specification. In addition, to check for robustness we conduct the SEM estimations with different kinds of weighting matrices.

Econometric results are in line with theoretical predictions and confirm our proposed hypotheses.[57] As implied by spatial competition models and search models the fraction of informed consumers, represented by the share of out-commuters to the employed in a municipality (*Coms*), is found to be negatively correlated with the mean price (Hypothesis 1.1). Likewise, findings reveal a positive correlation between the quotient of overnight stays with the number of inhabitants in a municipality (*Nights*) and the mean price. This indirectly supports predictions of the positive impact of the level of search costs on average prices (Hypothesis 1.2). Arguably, visitors in a particular region or municipality lack knowledge of sellers who charge especially high or low prices. As a consequence, they are considered to be uninformed consumers and associated with high costs of search to compare sellers prices.

Our main result refers to predictions from the search models of Varian (1980), Stahl (1989) and Waldeck (2008). Accordingly, price dispersion is expected to show a non-monotonic relationship with the fraction of informed consumers in the market. Indeed, regression results confirm that price dispersion, as measured by the squared price residuals, is characterized by an inverse U-shaped form with respect to *Coms* (Hypothesis 1.3).[58] Commuters arguably are attentive towards price changes in local markets and could easily identify stations offering low prices on their daily commuting paths. Thus, consistent with theoretical considerations, price dispersion initially increases as the fraction of commuters rises and starts to decline when the fraction of these informed consumers exceeds about 43%.

As regards the relation of the fraction of uninformed consumers with price dispersion,

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[57]See Subsection 4.2.3.

[58]Brown & Goolsbee (2002) provide similar results in their study that examines the impact the internet has on the relation between consumer search behavior and term-life insurance pricing. To our knowledge the relationship between the fraction of informed consumers and price dispersion in the gasoline retailing industry has not been examined yet.

a positive correlation between *Nights* and the price variance obtains. Varian (1980) initially argues that in equilibrium an increase in the share of uninformed consumers causes the average price to rise and the expected minimum price, i.e. the price paid by the informed consumers, to decline.[59] Accordingly, price disperison would increase with an increasing proportion of uninformed consumers in the market. By contrast, Morgan & Sefton (2001) revise this finding and prove that an increase in the fraction of uninformed consumers in Varian's model unambigously leads to an increase in the expected minimum price. They argue that a higher proportion of uninformed consumers induces more firms to enter the market which in turn causes two conflicting effects. An increase in the number of sellers raises incentives to focus on the high price segment, as a consequence average prices increase (cf. Hypothesis 1.2). In contrast an increase in the number of firms enlarges the choice set of the informed consumers putting pressure on minimum prices. In sum, they show that the first effect dominates the latter. However, the behavior of price dispersion has eluded their proof. Now, our results suggest an increase in the mean price and price variance as the fraction of uninformed consumers increases, other factors constant. Referring to the findings of Morgan & Sefton (2001) this suggests that the increase in the upper price segment is stronger than in the lower price segment. Further, provided that the fraction of uninformed consumers is associated with a high level of search costs, our results indirectly support predictions of Stahl (1989) and Waldeck (2008).[60]

Intuition suggests that entry of an additional firm in a fixed local market area impedes spatial differentiation among stations and thus reduces the average distance between nearest neighbours. As a consequence, we use two measures to capture the potential effect of increased competition: the number of competitors in a circular range of $3km$ around each station and the distance to its closest competitor. According to predictions of spatial competition models an increase in competition intensity is associated with a decrease in the mean market price and price dispersion whereas consumer search models forcefully argue that higher competition leads to an increase in average market prices (predictions on the behavior of price dispersion remain ambigous). Thus, as regards our variable set we would expect consistent results in the case of both competition proxies having opposite signs (Hypothesis 2.2 and Hypothesis 2.3).

Concerning the price levels, our findings support hypothesis of spatial competition between stations. Firstly, a significant negative relationship between the number of sellers and the mean price obtains. Secondly, the distance measure is found to be

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[59] „ [...] p_{min} will *decrease* with M - the uninformed consumers confer a *beneficial* externality on the informed consumers." p. 657.

[60] Cp. Hypothesis 1.4. Stahl shows that as search costs decrease the equilibrium price distribution degenerates to the competitive price, i.e. the average price and price dispersion decrease. Waldeck proposes that price dispersion (variance of the equilibrium distribution) increases as search costs increase in a setting of sequential search and the reservation price endogenized (cf. Proposition 19, p. 355).

significantly positively correlated with the mean. Further, these results are in line with previous work on spatial competition analysis in the Austrian gasoline market. Most notably, Clemenz & Gugler (2006) provide evidence that an increase in station density reduces average prices. As competition proxies they use the number of stations per square kilometer and the Herfindahl index as a measure of market concentration.

With respect to the relationship between price dispersion and entry competition results suggest a significant positive correlation of price variance with the number of sellers and a significant negative correlation of the variance with the distance measure. In sum, the comparative static behavior of the price distribution reveals that as competition increases the mean price decreases and dispersion increases. In context with predictions from search models this shift can be interpreted by a higher propensity of sellers to focus on the lower price segment whereas at the same time incentives to extract surplus from uninformed consumers remain intact. Generally, search models emphasize that sellers price in mixed strategies and tend to extreme pricing patterns. Thus, an increase in price dispersion is reflected in a shift of the probability mass towards the upper and lower bounds of the distribution. Now, results indicate that a higher number of sellers and closer proximity between stations leads to a lower mean price and higher dispersion. Consequently, these findings suggest that increased competition implies a downward shift in the probability mass and highlights that pricing activities in the lower segment become more frequent. Referring to the discussion of search models in section 4.2 this outcome of the competition analysis is partly in line with predictions of Janssen & Moraga-Gonzalez (2004) for the high search intensity mode.[61] Further, theoretical evidence for a negative relation between the number of sellers and the average price and a positive relation between the number of sellers and price variance is given in the model of Carlson & McAfee (1983).

In conclusion, our empirical analysis highlights that both competition and the dissemination of market information among consumers have a critical impact on the price distribution. It is shown that an increase in the fraction of (un)informed consumers leads to a decrease (increase) in the mean market price. Further, the course of price dispersion is determined by the fraction of informed consumers and accordingly follows an inverse U-shaped form whereas an increase in the fraction of uninformed consumers implies higher price dispersion. In turn, the effect of increased competition intensity is twofold. First, in line with previous empirical research the behavior of price levels supports predictions of spatial competition models. Specifically, an increase in the number of sellers and a decrease in the distance between nearest rivals implies a decrease in the mean price. Second, price dispersion significantly increases as competition intensifies. Together with the behavior of the mean price this finding

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[61] The empirical evidence supports the case of a small initial number of firms in the market.

emphasizes the interrelation between competition, consumers' search behavior and firms' pricing strategies.

Referring to the triangulation of consumer search, competition and strategic pricing future directions for empirical research might address predictions of extended versions of the model by Stahl (1989). In particular, it would be interesting to examine the relationship between search intensity, for instance in terms of purchase frequency, and the degree of market competition. Intuitively, a higher number of firms would be associated with a positive or negative change in search behavior. This would give further insights in the mechanisms and effects increased competition has on the equilibrium price distribution. Further, theoretical evidence is found that for the case of consumers opting out of the market (if search costs become considerably high and net utility becomes negative) relations between the number of firms, search costs and the fraction of shoppers with the expected price deviate from predictions of classical models (cp. Janssen et al. (2005)). Consequently, it would be interesting to test these alternative model predictions in an appropriate setting.

Appendix

Table 4.6: Summary of different spatial weights matrices used in spatial price regressions

	W	W2	W10
Neighbourhood criterion:	Nearest neighbour plus all stations in a circular range of $3km$	Nearest neighbour plus all stations in a circular range of $3km$	10 nearest neighbours
Number of stations:	$25,150$	$25,150$	$25,150$
Number of nonzero links:	$113,864$	$113,864$	$251,489$*
Symmetric:	No	No	No
Row standardized:	Yes	Yes	Yes
Weights: w_{ij}	$w_{ij} = \frac{1}{d_{ij}}$	$w_{ij} = \frac{1}{(d_{ij})^2}$	$w_{ij} = \frac{1}{d_{ij}}$
$Min(w_{ij})$:	0.0001129	0.000003	0.001588
$Max(w_{ij})$:	1	1	0.967300
$Mean(w_{ij})$:	0.2209	0.2209	0.1
$Median(w_{ij})$:	0.0889	0.0597	0.0779
Percentage of nonzero weights:	0.018	0.018	0.040

(*) Due to missing price information two stations had a maximum of eight and seven stations had a maximum of nine neighbours.

Table 4.7: Summary of test statistics of the standard OLS price specification for different spatial weights matrices

	W		W2		W10	
	Statistic	p-value	Statistic	p-value	Statistic	p-value
LM-Err	14,682.41	$< 2.20\text{e-}16$	13.215.28	$< 2.20\text{e-}16$	36,556.21	$< 2.20\text{e-}16$
LM-Lag	14,248.40	$< 2.20\text{e-}16$	12.873.88	$< 2.20\text{e-}16$	35,385.22	$< 2.20\text{e-}16$
RLM-Err	457.93	$< 2.20\text{e-}16$	366.83	$< 2.20\text{e-}16$	1,231.70	$< 2.20\text{e-}16$
RLM-Lag	23.92	1.01e-06	25.43	4.59e-07	60.70	6.66e-15
Moran's I	0.78	$< 2.20\text{e-}16$	0.78	$< 2.20\text{e-}16$	0.72	$< 2.20\text{e-}16$

Figure 4.1: Moran's I plot of standard OLS specification for weights matrix W
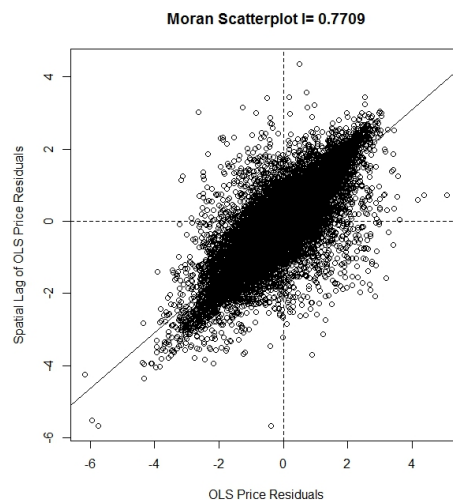

Moran Scatterplot I= 0.7709

Table 4.8: Price function regressions (log-log) without $log(PopDens)$ variable

Independent:	Dependent: $log(pd)$					
	Weights Matrix: W					
	OLS	GMM	OLS	GMM	OLS	GMM
	Coefficient (t-value)	Coefficient (z-value)	Coefficient (t-value)	Coefficient (z-value)	Coefficient (t-value)	Coefficient (z-value)
Intercept	-0.3888 (123.19)	-0.3972 (143.44)	-0.3875 (121.77)	-0.3943 (135.65)	-0.3899 (115.77)	-0.3956 (128.77)
$log(Density)$	-0.0046 (11.70)	-0.0026 (7.20)	-0.0048 (12.04)	-0.0029 (7.77)	-0.0045 (9.97)	-0.0026 (5.81)
$log(Distance)$	0.0005 (2.22)	0.0003 (1.71)	0.0005 (1.95)	0.0002 (1.42)		
Monop					0.0129 (5.52)	0.0073 (4.70)
Coms	-0.0313 (13.27)	-0.0141 (7.19)	-0.0257 (8.67)	-0.0150 (7.57)	-0.0183 (5.61)	-0.0105 (4.83)
(W)(Coms)			-0.0088 (3.05)	-0.0071 (3.27)	-0.0113 (3.87)	-0.0080 (3.63)
(Monop)(Coms)					-0.0284 (5.60)	-0.0158 (4.83)
$log(Nights)$	0.0021 (4.64)	0.0019 (4.39)	0.0021 (4.52)	0.0018 (4.24)	0.0019 (4.11)	0.0017 (3.88)
$100kmh$	0.0124 (6.35)	0.0144 (13.99)	0.0124 (6.31)	0.0141 (13.83)	0.0127 (6.44)	0.0143 (13.86)
$130kmh$	0.0391 (18.47)	0.0426 (38.02)	0.0388 (18.34)	0.0424 (37.94)	0.0391 (18.45)	0.0426 (38.10)
Access	0.0020 (3.60)	0.0012 (3.56)	0.0020 (3.56)	0.0012 (3.56)	0.0020 (3.67)	0.0013 (3.70)
Open24h	0.0036 (4.83)	0.0040 (9.55)	0.0035 (4.69)	0.0039 (9.42)	0.0035 (4.68)	0.0039 (9.29)
Wash	-0.0026 (3.95)	-0.0025 (6.77)	-0.0027 (4.06)	-0.0025 (6.87)	-0.0027 (4.05)	-0.0026 (6.96)
Service	0.0021 (2.42)	0.0056 (11.92)	0.0021 (2.48)	0.0056 (11.93)	0.0020 (2.27)	0.0055 (11.70)
Leisure	-0.0036 (5.68)	-0.0018 (4.67)	-0.0035 (5.65)	-0.0018 (4.72)	-0.0035 (5.65)	-0.0019 (4.96)
Microwave	0.0019 (2.99)	0.0020 (5.25)	0.0019 (3.02)	0.0019 (5.20)	0.0019 (2.91)	0.0019 (5.17)
Creditcards	0.0030 (2.46)	0.0050 (6.98)	0.0029 (2.40)	0.0050 (6.98)	0.0029 (2.38)	0.0049 (6.82)
$log(Plotsize)$	0.0034 (4.26)	0.0021 (4.34)	0.0035 (4.39)	0.0022 (4.48)	0.0034 (4.33)	0.0021 (4.32)
lambda		0.613 (112.05)		0.614 (112.20)		0.613 (112.10)
$Adj.R^2$	0.803		0.804		0.804	
Obs	25,150	25,150	25,150	25,150	25,150	25,150
$Median(u_{it})$	0.0021		0.0020		0.0021	
$S.E.(u_{it})$	0.0365		0.0365		0.0365	
$Median(\nu_{it})$		0.0024		0.0023		0.0023
$S.E.(\nu_{it})$		0.0230		0.0229		0.0229

Omitted brand category = Unbranded

Omitted state category = Vienna

Omitted time category = 199910

(Coefficients for the time-, state- and brand-fixed effects have been omitted)

White heteroscedasticity correction ($H0$) is applied to OLS standard errors.

(Absolute values of t-statistic and z-statistic in parentheses)

Table 4.9: GMM Price function regressions (log-log) with weights matrices $W2$ and $W10$

Independent:	Dependent: $log(pd)$					
	Weights Matrix: $W2$			Weights Matrix: $W10$		
	GMM	GMM	GMM	GMM	GMM	GMM
	Coefficient (z-value)	Coefficient (z-value)	Coefficient (z-value)	Coefficient (z-value)	Coefficient (z-value)	Coefficient (z-value)
Intercept	-0.3900 (119.53)	-0.3872 (115.28)	-0.3840 (109.75)	-0.3883 (94.78)	-0.3881 (93.97)	-0.3918 (91.98)
$log(Density)$	-0.0022 (5.65)	-0.0025 (6.38)	-0.0023 (4.89)	-0.0021 (6.06)	-0.0021 (6.04)	-0.0018 (4.41)
$log(Distance)$	0.0003 (1.53)	0.0002 (1.19)		0.0005 (3.13)	0.0005 (3.06)	
Monop			0.0068 (4.28)			0.0090 (6.11)
Coms	-0.0174 (8.32)	-0.0181 (8.63)	-0.0134 (5.69)	-0.0142 (7.78)	-0.0140 (7.47)	-0.0084 (3.97)
$(W)(Coms)$		-0.0079 (3.64)	-0.0087 (3.95)		-0.0009 (0.51)	-0.0017 (0.88)
$(Monop)(Coms)$			-0.0146 (4.35)			-0.0177 (5.76)
$log(Nights)$	0.0016 (3.72)	0.0016 (3.58)	0.0015 (3.40)	0.0015 (3.92)	0.0015 (3.90)	0.0014 (3.52)
$log(PopDens)$	-0.0009 (3.63)	-0.0008 (3.44)	-0.0006 (2.63)	-0.0008 (3.79)	-0.0008 (3.75)	-0.0006 (2.56)
$100kmh$	0.0143 (13.89)	0.0141 (13.71)	0.0143 (13.75)	0.0117 (11.76)	0.0117 (11.74)	0.0119 (11.95)
$130kmh$	0.0424 (37.53)	0.0422 (37.41)	0.0424 (37.57)	0.0422 (39.49)	0.0422 (39.45)	0.0425 (39.76)
Access	0.0011 (3.07)	0.0011 (3.07)	0.0012 (3.21)	0.0009 (2.77)	0.0009 (2.77)	0.0010 (2.95)
Open24h	0.0037 (8.86)	0.0037 (8.72)	0.0037 (8.68)	0.0041 (10.26)	0.0041 (10.25)	0.0041 (10.25)
Wash	-0.0025 (6.70)	-0.0026 (6.81)	-0.0026 (6.90)	-0.0022 (6.15)	-0.0022 (6.17)	-0.0022 (6.20)
Service	0.0056 (11.79)	0.0056 (11.78)	0.0055 (11.58)	0.0048 (10.80)	0.0048 (10.81)	0.0047 (10.54)
Leisure	-0.0018 (4.74)	-0.0018 (4.78)	-0.0019 (4.99)	-0.0015 (4.15)	-0.0015 (4.14)	-0.0015 (4.32)
Microwave	0.0020 (5.41)	0.0020 (5.34)	0.0020 (5.29)	0.0018 (4.97)	0.0018 (4.98)	0.0017 (4.86)
Creditcards	0.0049 (6.73)	0.0048 (6.72)	0.0048 (6.61)	0.0052 (7.63)	0.0052 (7.61)	0.0052 (7.63)
$log(Plotsize)$	0.0023 (4.57)	0.0023 (4.72)	0.0023 (4.58)	0.0022 (4.80)	0.0022 (4.81)	0.0021 (4.60)
lambda	0.596 (110.50)	0.596 (110.63)	0.596 (110.52)	0.808 (80.99)	0.808 (80.95)	0.808 (80.91)
Obs	25,150	25,150	25,150	25,150	25,150	25,150
$Median(\nu_{it})$	0.0023	0.0023	0.0022	0.0021	0.0021	0.0018
$S.E.(\nu_{it})$	0.0231	0.0231	0.0231	0.0207	0.0207	0.0207

Omitted brand category = Unbranded

Omitted state category = Vienna

Omitted time category = 199910

(Coefficients for the time-, state- and brand-fixed effects have been omitted)

(Absolute values of z-statistic in parentheses)

Table 4.10: Price function regressions (lin-lin)

Independent:	Dependent: pd Weights Matrix: W					
	OLS	GMM	OLS	GMM	OLS	GMM
	Coefficient (t-value)	Coefficient (z-value)	Coefficient (t-value)	Coefficient (z-value)	Coefficient (t-value)	Coefficient (z-value)
Intercept	0.6746 (283.61)	0.6684 (318.40)	0.6753 (280.16)	0.6705 (305.85)	0.6765 (284.25)	0.6717 (307.35)
Density	-0.0003 (8.29)	-0.0002 (5.03)	-0.0003 (8.67)	-0.0002 (5.63)	-0.0003 (9.00)	-0.0002 (6.12)
Distance	0.0010 (11.16)	0.0006 (9.87)	0.0010 (10.99)	0.0006 (9.78)		
Monopy					0.0134 (8.21)	0.0072 (6.70)
Coms	-0.0221 (13.29)	-0.0106 (7.44)	-0.0185 (8.54)	-0.0110 (7.73)	-0.0134 (5.64)	-0.0079 (5.04)
(W)(Coms)			-0.0056 (2.61)	-0.0054 (3.34)	-0.0078 (3.60)	-0.0064 (3.95)
(Monop)(Coms)					-0.0223 (5.90)	-0.0120 (4.86)
Nights	0.0001 (3.48)	0.0001 (3.66)	0.0001 (3.54)	0.0001 (3.74)	0.0001 (3.31)	0.0001 (3.56)
PopDens	-5.0e-07 (6.07)	-3.0e-07 (3.85)	-4.8e-07 (5.82)	-3.0e-07 (3.87)	-4.6e-07 (5.52)	-2.7e-07 (3.53)
100kmh	0.0104 (7.03)	0.0112 (14.60)	0.0104 (6.99)	0.0111 (14.45)	0.0107 (7.19)	0.0112 (14.68)
130kmh	0.0311 (19.18)	0.0328 (39.19)	0.0309 (19.08)	0.0327 (39.14)	0.0314 (19.33)	0.0331 (39.61)
Access	0.0013 (3.08)	0.0008 (3.13)	0.0012 (3.04)	0.0008 (3.09)	0.0013 (3.17)	0.0009 (3.36)
Open24h	0.0025 (4.39)	0.0029 (9.27)	0.0024 (4.30)	0.0029 (9.15)	0.0025 (4.47)	0.0029 (9.21)
Wash	-0.0019 (3.95)	-0.0019 (6.89)	-0.0020 (4.05)	-0.0020 (6.99)	-0.0020 (4.04)	-0.0020 (7.07)
Service	0.0015 (2.36)	0.0043 (12.07)	0.0015 (2.42)	0.0042 (12.07)	0.0016 (2.47)	0.0043 (12.20)
Leisure	-0.0028 (5.98)	-0.0014 (5.12)	-0.0028 (5.97)	-0.0015 (5.18)	-0.0027 (5.87)	-0.0015 (5.17)
Microwave	0.0012 (2.51)	0.0014 (5.14)	0.0012 (2.54)	0.0014 (5.07)	0.0011 (2.37)	0.0014 (4.93)
Creditcards	0.0020 (2.22)	0.0036 (6.67)	0.0019 (2.15)	0.0036 (6.64)	0.0020 (2.22)	0.0036 (6.72)
log(Plotsize)	0.0027 (4.62)	0.0020 (5.38)	0.0028 (4.69)	0.0020 (5.52)	0.0023 (4.00)	0.0017 (4.55)
lambda		0.608 (110.20)		0.608 (110.35)		0.608 (110.14)
Adj.R^2	0.820		0.820		0.820	
Obs	25,150	25,150	25,150	25,150	25,150	25,150
Median(u_{it})	0.0012		0.0012		0.0012	
S.E.(u_{it})	0.0270		0.0270		0.0270	
Median(ν_{it})		0.0014		0.0014		0.0013
S.E.(ν_{it})		0.0171		0.0171		0.0171

Omitted brand category = Unbranded

Omitted state category = Vienna

Omitted time category = 199910

(Coefficients for the time-, state- and brand-fixed effects have been omitted)

White heteroscedasticity correction ($H0$) is applied to OLS standard errors.

(Absolute values of t-statistic and z-statistic in parentheses)

Table 4.11: Summary of test statistics of the standard OLS price dispersion model (dependent: $log(u_{it}^2)$) for different weights matrices

	W		W2		W10	
	Statistic	p-value	Statistic	p-value	Statistic	p-value
LM-Err	4,999.55	< 2.20e-16	4,580.66	< 2.20e-16	11,115.33	< 2.20e-16
LM-Lag	5,036.85	< 2.20e-16	4,610.88	< 2.20e-16	11,186.95	< 2.20e-16
RLM-Err	4.44	0.04	2.87	0.09	1.94	0.16
RLM-Lag	41.75	< 2.20e-16	33.08	< 2.20e-16	73.57	< 2.20e-16
Moran's I	0.45	< 2.20e-16	0.46	< 2.20e-16	0.40	< 2.20e-16

Table 4.12: Results of price dispersion regressions for different weights matrices estimated by S2SLS

Independent:	Dependent: $log(u_{it}^2)$					
	(Weights Matrix: W)		(Weights Matrix: $W2$)		(Weights Matrix: $W10$)	
	Coefficient (z-value)	Coefficient (z-value)	Coefficient (z-value)	Coefficient (z-value)	Coefficient (z-value)	Coefficient (z-value)
Intercept	-8.0506	-8.3234	-8.0714	-8.3536	-8.0632	-8.3406
	(51.67)	(39.69)	(51.79)	(39.87)	(51.65)	(39.67)
$log(Density)$	0.0943	0.1080	0.1014	0.1153	0.1004	0.1143
	(3.25)	(3.63)	(3.48)	(3.86)	(3.47)	(3.84)
$log(Distance)$	-0.0560	-0.0509	-0.0448	-0.0340	-0.0510	-0.0452
	(3.82)	(3.40)	(3.03)	(2.66)	(3.07)	(2.67)
Coms	0.3218	1.2830	0.3314	1.3288	0.3295	1.3045
	(2.36)	(2.27)	(2.43)	(2.35)	(2.42)	(2.31)
$(Coms)^2$		-1.3728		-1.4251		-1.3925
		(1.75)		(1.82)		(1.78)
Nights	0.0065	0.0103	0.0065	0.0104	0.0065	0.0104
	(3.05)	(2.65)	(3.04)	(2.67)	(3.04)	(2.65)
$(Nights)^2$		-3.64e-05		-3.74e-05		-3.69e-05
		(1.25)		(1.28)		(1.26)
$log(PopDens)$	-0.0899	-0.0852	-0.0912	-0.0863	-0.0904	-0.0857
	(5.56)	(5.22)	(5.64)	(5.28)	(5.59)	(5.25)
$100kmh$	0.2941	0.2928	0.2944	0.2931	0.2953	0.2939
	(2.95)	(2.94)	(2.96)	(2.95)	(2.97)	(2.95)
$130kmh$	0.6077	0.6082	0.6046	0.6053	0.6076	0.6078
	(6.99)	(7.00)	(6.96)	(6.97)	(6.98)	(6.98)
rho	0.0002	0.0002	-9.32e-08	-1.21e-08	0.0354	0.0301
	(2.39)	(2.34)	(0.06)	(0.01)	(0.63)	(0.54)
Obs	25,150	25,150	25,150	25,150	25,150	25,150
$Median(\kappa_{it})$	0.4653	0.4640	0.4670	0.4655	0.4653	0.4647
$S.E.(\kappa_{it})$	2.173	2.174	2.174	2.174	2.173	2.173

Weight matrices not row standardized

Omitted brand category = Unbranded

Omitted state category = Vienna

Omitted time category = 199910

(Coefficients for the time-, state- and brand-fixed effects have been omitted)

White heteroscedasticity correction ($H0$) is applied to standard errors.

(Absolute values of z-statistic in parentheses)

Table 4.13: OLS price dispersion regressions with alternative search cost proxies

Independent:	Dependent: $log(u_{it}^2)$ (OLS Residuals)		Dependent: $log(\nu_{it}^2)$ (GMM Residuals)		Dependent: $log(\omega_{it}^2)$ (2WFE Residuals)	
	Coefficient (t-value)	Coefficient (t-value)	Coefficient (t-value)	Coefficient (t-value)	Coefficient (t-value)	Coefficient (t-value)
Intercept	-7.9140	-8.0716	-7.8535	-7.9753	-9.9591	-10.2480
	(57.05)	(51.16)	(57.39)	(51.21)	(36.52)	(33.68)
Density	0.0878	0.0934	0.1071	0.1120	0.1047	0.1161
	(3.22)	(3.41)	(3.92)	(4.08)	(3.14)	(6.12)
Distance	-0.0404	-0.0376	-0.0239	-0.0213	-0.0427	-0.0368
	(2.86)	(2.64)	(1.64)	(1.45)	(2.74)	(2.38)
ComsTot	0.3475	1.0041	0.2286	0.7980	0.2246	1.5442
	(3.19)	(2.74)	(2.08)	(2.21)	(2.06)	(3.63)
$(ComsTot)^2$		-0.6051		-0.5269		-1.2204
		(1.89)		(1.66)		(3.35)
NightsC	0.0009	0.0020	0.0011	0.0017	0.0010	0.0026
	(2.70)	(2.58)	(3.51)	(2.10)	(2.72)	(2.89)
$(NightsC)^2$		-1.39e-06		-6.99e-07		-1.85e-06
		(1.55)		(0.91)		(1.82)
PopDens	-0.1068	-0.1081	-0.1103	-0.1123	-0.0803	-0.0845
	(6.43)	(6.46)	(6.82)	(6.90)	(4.09)	(4.29)
100kmh	0.2992	0.2962	0.2301	0.2269	0.0851	0.0779
	(3.00)	(2.98)	(2.03)	(2.00)	(0.80)	(0.73)
130kmh	0.6108	0.6073	0.4633	0.4584	0.0025	0.0080
	(7.03)	(6.99)	(5.19)	(5.13)	(0.02)	(0.08)
$Adj.R^2$	0.101	0.101	0.107	0.107	0.055	0.055
Obs	25,150	25,150	25,150	25,150	25,150	25,150
$Median(\eta_{it})$	0.465	0.466	0.446	0.443	0.458	0.459
$S.E.(\eta_{it})$	2.174	2.173	2.158	2.158	2.675	2.675

Omitted brand category = Unbranded

Omitted state category = Vienna

Omitted time category = 199910

(Coefficients for the time-, state- and brand-fixed effects have been omitted)

White heteroscedasticity correction ($H0$) is applied to standard errors.

(Absolute values of t-statistic in parentheses)

Table 4.14: S2SLS price dispersion regressions with alternative search cost proxies

Independent:	Dependent: $log(u_{it}^2)$ (OLS Residuals)		Dependent: $log(\nu_{it}^2)$ (GMM Residuals)		Dependent: $log(\omega_{it}^2)$ (2WFE Residuals)	
	Coefficient (z-value)	Coefficient (z-value)	Coefficient (z-value)	Coefficient (z-value)	Coefficient (z-value)	Coefficient (z-value)
Intercept	-7.8978 (56.88)	-8.0486 (50.94)	-7.8297 (57.16)	-7.9413 (50.91)	-9.9403 (36.50)	-10.2230 (33.65)
Density	0.0812 (2.96)	0.0868 (3.15)	0.0973 (3.56)	0.1019 (3.72)	0.0972 (2.91)	0.1087 (3.22)
Distance	-0.0509 (3.44)	-0.0479 (3.21)	-0.0392 (2.59)	-0.0366 (2.40)	-0.0544 (3.29)	-0.0479 (2.92)
ComsTot	0.3257 (2.98)	0.9448 (2.58)	0.1968 (1.79)	0.7107 (1.96)	0.2005 (1.83)	1.4819 (3.49)
$(ComsTot)^2$		-0.5697 (1.78)		-0.4749 (1.49)		-1.1833 (3.25)
NightsC	0.0010 (2.74)	0.0020 (2.58)	0.0011 (3.57)	0.0017 (2.10)	0.0010 (2.76)	0.0026 (2.89)
$(NightsC)^2$		-1.37e-06 (1.54)		-6.75e-07 (0.88)		-1.83e-06 (1.81)
PopDens	-0.1047 (6.30)	-0.1060 (6.32)	-0.1073 (6.64)	-0.1091 (6.70)	-0.0780 (3.97)	-0.0822 (4.17)
$100kmh$	0.2989 (3.00)	0.2962 (2.98)	0.2298 (2.03)	0.2269 (2.00)	0.0850 (0.80)	0.0780 (0.73)
$130kmh$	0.6137 (7.07)	0.6105 (7.03)	0.4675 (5.23)	0.4632 (5.18)	0.0057 (0.05)	0.0048 (0.04)
rho	0.0002 (2.21)	0.0002 (2.14)	0.0002 (2.49)	0.0002 (2.44)	0.0002 (2.73)	0.0001 (2.52)
Obs	25,150	25,150	25,150	25,150	25,150	25,150
$Median(\kappa_{it})$	0.462	0.465	0.446	0.444	0.456	0.458
$S.E.(\kappa_{it})$	2.173	2.173	2.157	2.157	2.675	2.674

Weights matrix W (not row-standardized) is applied in S2SLS regressions.
Omitted brand category = Unbranded
Omitted state category = Vienna
Omitted time category = 199910
(Coefficients for the time-, state- and brand-fixed effects have been omitted)
White heteroscedasticity correction ($H0$) is applied to standard errors.
(Absolute values of z-statistic in parentheses)

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