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Abstract

Piketty's main theoretical prediction is that a small elite will own all wealth if capitalism is left to its own devices. We formulate and calibrate a Post-Keynesian model with an endogenous distribution of wealth between workers and capitalists. The model permits Piketty's corner solution of all wealth held by capitalists; however, it also shows that interior solutions with a stable, non-zero wealth share of workers, a stable wealth-to-income ratio, and a stable and positive gap between the profit and the growth rate determined by the Cambridge equation. Furthermore, simulations show that the model conforms to Piketty's empirical findings in a transitional phase, in which the wealth share of capitalists rises to over 60%, the wealth-to-income ratio increases, and income inequality rises. Finally, we show that the introduction of a wealth tax as suggested by Piketty could neutralize the rise in wealth concentration.

1. Introduction

Thomas Piketty's best-selling book 'Capital in the 21st Century' (Piketty 2014) triggered a renewed interest in empirical research regarding the accumulation and distribution of wealth, and a lively debate about their causes and consequences. Wealth determines income, power and opportunities, and lies at the very heart of economic inequalities. Understanding the dynamics of wealth accumulation and distribution is thus crucial to tackle these inequalities.

In a nutshell, Piketty's (2014) theoretical argument is that, since the profit rate is usually higher than the growth rate in an economy (an empirical regularity which he finds for most countries and time periods), wealth increases over time faster than income. This entails a more unequal distribution of income, because the share of profits increases and wealth ownership and capital income are more concentrated than labour income. A rising income inequality finally feeds back into a more unequal distribution of wealth, so that wealth will be ever-increasingly concentrated in the hands of a small elite.

Empirically, Piketty (2014) provides extensive data on the historical evolution of wealth-to-income ratios, wealth, and the personal income distribution. He shows that the wealth-to-income ratio has risen, and that wealth and income have become more unequally distributed in high-income countries since about the 1980s. Regarding the profit rate and the growth rate, he argues that they have been largely stable over the long run, but that the former is empirically higher than the latter.

The reception of the book in Post-Keynesian economics has been mixed. On the one hand, Post-Keynesian economists recognize the empirical contributions of the book: the collection of historical data and the carving out of observable patterns therein (Rowthorn 2014, Rehm and Schnetzer 2015, King 2017). On the other hand, Piketty's simple, neoclassical theoretical framework by which he explains the dynamics of wealth and income inequality has attracted the criticism of Post-Keynesian economists, in whose theoretical frameworks distribution has long played a major role (e.g. Galbraith 2014, Palley 2014, López-Bernardo et al. 2016a). Based

on the Cambridge equation (Pasinetti 1962), they point out that the wealth distribution in the long run can be stable, a statement that is clearly in contradiction to Piketty's conclusion.

Not only does Post-Keynesian theory show that the distribution of wealth can be stable in the long run, but it is also capable of explaining the short-run dynamic of wealth accumulation and distribution that Piketty (2014) presents abundant empirical evidence for. In the 'transitional phase', i.e. when the wealth share of capitalists is below its long-run equilibrium value, a rising wealth-to-income ratio and increasingly unequal distributions of wealth and income are all perfectly compatible with Post-Keynesian theory. Due to its focus on the long run, these short-run dynamics have not been investigated by Piketty's Post-Keynesian critics so far. The paper intends to close this gap.

To do so, we build a Post-Keynesian model in the tradition of Bhaduri and Marglin (1990) by incorporating an endogenous wealth distribution. We extend the model by blended incomes of workers and capitalists, differential rates of return, and capital gains. We show that a stable wealth share is a likely outcome in the long run, and both the euthanasia and the triumph of the rentier are special cases, and thus reiterate the critique of Piketty's hypothesis of an ever-increasing wealth concentration. Furthermore, we use the model to explain a 'transitional dynamic' that resembles the empirical evidence presented by Piketty and his projections to the future. A rising wealth-to-income ratio, rising wealth and income inequality and a profit rate that is higher than the growth rate of the capital stock (and thus income) are all consistent with our extended Post-Keynesian model. Finally, we show that a wealth tax, as suggested by Piketty, reduces both wealth and income inequality.

The paper is structured as follows. The literature review in section 2 discusses both Piketty and his Post-Keynesian critics, as well as the Post-Keynesian models of distribution. Section 3 describes the model and its extensions in detail. Section 4 presents a numerical simulation of both short-run dynamics and the long-run equilibrium. Section 5 discusses the effects of a wealth tax. Section 6 concludes.

2. Piketty and the Post-Keynesians

Piketty (2014)'s theoretical framework, formally elaborated in Piketty and Zucman (2014), consists of two so-called 'fundamental laws' and a 'fundamental contradiction of capitalism'. Piketty's first law states that the share of income from capital in total national income is equal to the rate of profit times the wealth-income ratio, or $\alpha = r \times \beta$, an accounting identity (Piketty 2014: 52). The second law is the Harrod-Domar equation (Piketty and Zucman 2014: 1274) that the ratio of wealth to income equals the ratio of the saving rate to the growth rate. In Piketty's denotation, $\beta = s/g$ (Piketty 2014: 166). Piketty's fundamental contradiction argues that wealth (which he treats synonymously to capital) increases faster than income if the profit rate is higher than the growth rate of GDP, i.e. if the oft-cited formula $r > g$ holds (Piketty 2014: 571). Piketty's analysis is based on a neoclassical production function, it assumes that the elasticity of substitution between labour and capital is higher than one

(Piketty and Zucman 2014: 1271), and the rate of profit is technologically determined (Piketty and Zucman 2014: 1270).

Empirically, Piketty's work is ground-breaking in the breadth and depth of his coverage of both wealth and income time series. In particular, he documents an increasingly unequal distribution of wealth (Piketty 2014: 332f), rising wealth-to-income ratios (ibid.: 164f), and an increasing concentration of the personal income distribution (ibid.: 290f) in Europe and the U.S., roughly since the 1980s. He also documents a slowing growth rate (ibid.: 93f) and a declining rate of return (ibid.: 199f) over that time horizon, even though he sees both as roughly stable over the long run. Crucially for his argument, he finds his 'fundamental contradiction' of capitalism to be empirically valid for most economies and historical periods, i.e. the profit rate to be higher than the growth rate (ibid.: 350f). Piketty sees a number of positive feedback mechanisms, which lead to an ever-increasing inequality in the distribution of income and wealth. Barring political intervention and notwithstanding his detailed discussion of institutional and systemic factors in the wealth distribution, following this train of thought to its logical conclusion means that Piketty's main prediction is that all wealth will be concentrated in the hand of a very small elite.

The Post-Keynesian and structuralist reception of Piketty's work has been largely sceptical. King's (2016) comprehensive review summarizes the arguments (of neoclassical, Austrian and Post-Keynesian economists) in nine over-arching points of critique. These range from differences in opinion regarding the relevance of wealth inequality and the suggested policy conclusions, but also empirical disagreements and theoretical critiques, the latter including the importance of institutions and of low-income countries. Relevant for this paper are the two critiques that 1) "Piketty uses the wrong (neoclassical) theory" (King 2016: 7) and 2) that the "predicted increase in wealth is implausible" (King 2016: 3).

Regarding the first point, many Post-Keynesian and structuralist economists criticized Piketty for using a neoclassical framework. Piketty (2014: 230f) misrepresents the Cambridge capital controversy of the 1950s and 1960s (Galbraith, 2014; Palley, 2014; López-Bernardo et al. 2016a), which showed that neoclassical economics is logically inconsistent in its valuation of capital since the value of capital is simultaneously determined by and determines the rate of profit (Harcourt, 1972; Felipe and McCombie, 2013). Connected to this is the critique of Piketty's conflation of wealth and capital. In particular, Post-Keynesians and Marxists objected to treating housing wealth as productive capital, measuring wealth at highly volatile market prices untethered from 'fundamental' values (Duménil and Levy 2014, Rowthorn, 2014).³

The bulk of Post-Keynesian the criticism was directed at the second point (López-Bernardo et al., 2016; Michl, 2014; Rowthorn, 2014; Taylor, 2014; van Treek, 2015). As discussed, from Piketty's fundamental laws and in particular the inequality $r > g$ follows a long-run dynamics of wealth accumulation that leads to an ever-increasing share of wealth concentrated in the hands of capitalists. Treating the variables as independent, van Treek (2015) numerically

³ We briefly address this 'valuation problem' in the model section, and take the larger point regarding productive (rather than total) capital on board in the formulation of our model.

illustrates that the profit rate exceeding the growth rate is not a sufficient condition for the wealth-to-income ratio and inequality continue to rise indefinitely. Differential saving rates of households with different positions in the income distribution are a necessary precondition for Piketty's conclusion to hold, and the wealth distribution is highly sensitive to changes in these differential saving rates.

Based on Pasinetti's Cambridge equation $s_r r = g$ (i.e., the equilibrium growth rate equals the saving rate of capitalists times the profit rate), Taylor (2014) argues that the two corner solutions euthanasia and the triumph of the rentier, as well as an interior solution, are all possible results. He focuses on the interaction between the wealth share of capitalists, capacity utilization and the profit share, and points out that the feedback mechanisms between these variables can stabilize or destabilize the system, depending on the structure of the economy. He shows that a rising wealth share entails the possibility of a rising profit share and chronic underutilisation of production capacities and stagnation in the long run, if the profit share responds strongly positively to an increase in the wealth share.

López-Bernardo et al. (2016a) also focus on the long run and the Cambridge equation. They argue that the profit rate must necessarily always be greater than the growth rate, provided that capitalists' savings rate is less than unity (that is, they consume a certain share of their income). Piketty's empirical finding that r is greater than g is thus fully consistent with a stable income distribution and, by extension, a stable wealth distribution. Only if the profit rate exceeds the ratio of the growth rate and the saving rate of capitalists will there be redistribution from wages to profits, and thus a rising profit share and ultimately, a rising concentration of wealth. Furthermore, although individual capitalists can increase their rate of accumulation by saving more, capitalists as a whole cannot, because an increase in the saving rate would inevitably entail a fall in the profit rate. López-Bernardo et al. (2016a) argue that Piketty overlooked this 'fallacy of composition', even though he explicitly discusses (but dismisses by referring to technology) its relevance (Piketty 2014: 215f). However, they point out that Piketty's cumulative feedback mechanism between accumulation and wealth concentration is indeed valid when it comes to personal income distribution, in particular among capitalists, because they have no access to wage income in a simple formulation.⁴

Post-Keynesian models are well suited for the analysis of both short- and long-run dynamics of wealth accumulation and distribution. They have long been used to investigate the distribution between labour and capital in the wage-/profit-led debate (Bhaduri and Marglin 1990, Stockhammer and Ederer 2008, Stockhammer et al. 2009, Lavoie and Stockhammer 2013, Barbosa and Taylor 2006, Kiefer and Rada 2015) and the distribution of personal income (Lavoie 2009, Carvalho and Rezai 2016, Palley 2017a). It is therefore indeed vexing that Piketty's writing shows no reception of the extensive Post-Keynesian body of literature on the theory of distributions.

⁴ Meade (1964) points out this disadvantage. Pasinetti (1974) argues that the thrickest group among capitalists will eventually dominate the others, as noted by Taylor (2014). We return to this point in the following section.

Two seminal Post-Keynesian models of the distribution of wealth are Palley (2012, 2017b) and Taylor et al. (2015). In Taylor et al.'s (2015) model, workers receive blended wage and profit income. The model incorporates capital gains due to its empirical relevance in the US. Taylor et al. (2015) simulate a wealth concentration ratio of about 60% - the top 1% owns roughly 60% of total wealth. Wealth is split between "the rich" and the "middle class-workers", as the "bottom 60%" empirically do not own any wealth. Palley's (2017b) model also includes an endogenous wealth distribution. The paper permits doubly blended income sources – both workers and capitalists receive both work and profit income in a Post-Keynesian framework. The parameters influencing the personal income distribution (and growth) are the functional income distribution, the distribution of the wage bill between workers and capitalists, and the distribution of wealth between workers and capitalists. The latter, in turn, depends on the differential propensities to save.

Post-Keynesian models have paid less attention to the transitory dynamics for the phase during which the wealth share (and, indeed, the functional and personal income distribution) differ from their (long-run) equilibria. The differential equations often used by the profit-led strand of the literature (and their concomitant VAR analyses) lend themselves to investigating these transition phases. Indeed, Taylor (2014) derives a differential equation for the wealth share. However, the paper then focuses on stable and instable equilibria, while pointing to possible feedback mechanisms between the distribution of wealth with the profit share and capacity utilization.

This paper extends the model of Ederer and Rehm (2017), which follows the Post-Keynesian tradition along the lines of Palley (2017b) and Taylor et al. (2015) with an endogenous wealth distribution in a two-class economy. The question we are asking is, whether Piketty's empirical evidence regarding a rising wealth share for decades can be integrated into a Post-Keynesian model, which permits for interior solutions rather than the corner solution of the entire wealth concentrated in the hands of capital. The main aim is to investigate the transition phase between short- and long-run dynamics of wealth accumulation and distribution, and thereby contribute to closing this gap in the Post-Keynesian literature of growth models.

3. A Model of the Wealth Distribution

The model is a standard two-class, Post-Keynesian formulation in the tradition of Bhaduri and Marglin (1990), drawing on Palley (2017b) and Taylor et al. (2015) and closely following Ederer and Rehm (2017)⁵. We introduce four innovations: (1) Wealth is accumulated through saving; (2) blended wage and capital income goes to both workers and capitalists; (3) beyond differential savings rates, workers and capitalists also have differential returns on their assets; (4) firms save and accumulate wealth, which is passed on to owners of shares via capital gains.

⁵ The model is briefly restated here for the convenience of the reader, since we discuss the solution and the relevance of our analytical assumptions relating to Piketty's theoretical and empirical definitions in detail.

Section 3a introduces a basic version of the model, and section 3b includes extensions (2) to (4).

a. Basic model

In the basic formulation of the model, income Y is divided between total profits R and the wage bill W according to the (exogenous) functional income distribution π (the profit share).

$$R = \pi Y$$

$$W = (1 - \pi)Y$$

All wages accrue to workers in the simple version of the model. Income of capitalists (subscript r) thus amounts to profits R on their share of wealth z . Workers (denoted by subscript w) also receive a part of profits R proportional to their share of wealth ownership $(1 - z)$, which together with wages make up total income of workers Y_w .

$$Y_r = zR$$

$$Y_w = W + (1 - z)R$$

We follow Post-Keynesian convention by assuming a positive differential between savings rates of capitalists s_r and workers s_w . Per definition, consumption propensities of workers and capitalists multiply with their respective incomes for total consumption C .

$$C = (1 - s_w)Y_w + (1 - s_r)Y_r$$

The investment equation is formulated according to the standard Post-Keynesian functional form in the Bhaduri-Marglin tradition, i.e. growth of the capital stock K depends on capacity utilization u and the profit share π . This formulation allows for both wage-led and profit-led demand growth regimes, depending on the values of the parameters β_1 and β_2 and the saving rates.

$$I = (\beta_0 + \beta_1 u + \beta_2 \pi)K$$

The aggregate goods market is in equilibrium, output equals demand. Since we abstract from all sectors other than households and firms, total demand consists of consumption and investment.⁶

$$Y = C + I$$

In order to de-trend income, profits, and investment, we follow convention by normalizing them to the capital stock. This yields stable solutions for capacity utilization u , the profit rate r and the growth rate of the capital stock g .

$$u = \frac{Y}{K}$$

⁶ The abstraction from government and an external sector is in line with Piketty (2014), who in much of his work omits them on empirical grounds following careful examination of their importance and valuation problems.

$$r = \frac{R}{K} = \pi u$$

$$g = \frac{I}{K}$$

The only asset in the model is productive wealth V , which is equal to the capital stock K . The ownership of (productive) capital entitles to the receipt of the corresponding share in profits. Both workers and capitalists accumulate wealth through savings. The level of savings is the difference between income and consumption.

In focusing on productive capital, we take an important strand of criticism of Piketty's neoclassical basis, discussed in section 2, on board, even though we appreciate Piketty's argument that wealth categories are to a certain degree fungible. Departing from total wealth as measured in microeconomic surveys introduces a potential wedge between Piketty's and our empirical results, since the critiques were not solely based on theoretical arguments regarding unproductive capital, but also revolved around a rising empirical importance of housing wealth as a share of total private wealth (Duménil and Levy, 2014). Our model is conceptually based on capital (rather than wealth), since it is adapted from a framework which focuses on productive investment and growth. Because productive wealth is distributed more unequally (i.e., housing is distributed more equally), our simulations are on the conservative side as we are more likely to find Piketty's extreme inequality.

In order to trace the behaviour of the model compared to Piketty's empirical evidence, we include the wealth-to-income ratio x (which is the inverse of capacity utilization u) and the ratio of capitalists' to workers' income, as a measure for the personal income distribution θ in the model.

$$x = \frac{K}{Y} = \frac{1}{u}$$

$$\theta = \frac{Y_r}{Y_w}$$

Table 1: Stocks and flows in the Post-Keynesian model with endogenous wealth distribution

	Households		Firms		Total
	Workers	Capitalists	Current	Capital	
Consumption	$-C_w$	$-C_r$	$+C$		0
Investment			$+I$	$-I$	0
Wages	$+W$		$-W$		0
Profits	$+R_w$	$+R_r$	$-R$		0
Wealth	$-\Delta V_w$	$-\Delta V_r$		$+\Delta K$	0
Total	0	0	0	0	0

Source: own elaboration.

Table 1 shows the stock-flow matrix of the model. A plus sign denotes a source of funds, a minus sign is a use of funds, and rows and columns sum to zero. Firms pay wage income to workers (row 3), and distribute profits (row 4), which both workers and capitalists receive. Workers and capitalists consume their income (row 1), and firms invest (row 2). Both workers and capitalists save and thus accumulate wealth in the form of productive capital (row 5). Households' savings finance firms' investment (column 4).

b. Short-run equilibrium

The short-run solution of the model assumes wealth shares to be constant, since wealth accumulates over long time periods. Capacity utilization, the profit rate and the growth rate adjust simultaneously, so that the equilibrium in the aggregate goods market (i.e. the IS-condition) is fulfilled for any wealth share of capitalists z . The short-run solutions for these three variables thus depend on the distribution of wealth:

$$u^* = \frac{\beta_0 + \beta_2\pi}{s_w + (s_r - s_w)\pi z - \beta_1}$$

$$r^* = \frac{\beta_0\pi + \beta_2\pi^2}{s_w + (s_r - s_w)\pi z - \beta_1}$$

$$g^* = \frac{(\beta_0 + \beta_2\pi)[s_w + (s_r - s_w)\pi z]}{s_w + (s_r - s_w)\pi z - \beta_1}$$

In particular, a higher wealth share of capitalists lowers capacity utilization, the profit rate and the growth rate of the capital stock. The reason is that a higher wealth share of capitalists transfers profit income to capitalists, which depresses total consumption due to capitalists' higher saving rate.

Contrary to a rise in the profit share, which can either raise or reduce the growth rate (depending on whether growth is wage-led or profit-led), a rise in the wealth share unambiguously depresses growth, since consumption decreases and there is no counteracting effect on investment. The latter is determined by the profitability of firms, not by the distribution of profits among workers and capitalists. Note that the profit rate is more sensible to the wealth share than the growth rate, because its effect on the latter works indirectly via capacity utilization. This is the reason why the profit rate decreases faster than the growth rate when the wealth share rises, and the two variables will eventually fulfil the Cambridge equation (see below).

Since the wealth-to-income ratio x is the inverse of capacity utilisation, it increases the more wealth is concentrated in the hands of capitalists. Likewise, the personal income distribution as measured by the ratio of income of capitalists to the income of workers, θ , is more unequal

when the wealth share of capitalists is high, because a higher share of profits accrues to capitalists in that case.

$$x^* = \frac{s_w + (s_r - s_w)\pi z - \beta_1}{\beta_0 + \beta_2\pi}$$

$$\theta^* = \frac{\pi z}{1 - \pi z}$$

c. Long-term equilibrium

Over time, both capitalists and workers accumulate wealth until the wealth share adjusts to its long-term equilibrium. Pasinetti's (1962) Cambridge equation $s_r r = g$ implies that the wealth share is stable when capitalists save exactly the amount that corresponds to their share in the increase in total capital (Palley, 2012, 2017b; Taylor, 2014). Consequently, Palley (2017b) points out that the Cambridge equation should be interpreted as an 'ownership equilibrium condition', since capitalists must save just enough to maintain their ownership share.

Since both the profit rate and the growth rate depend on the wealth share of capitalists z , we get a long-run equilibrium value for the wealth distribution:

$$z^{**} = \frac{s_r \pi - s_w}{(s_r - s_w)\pi}$$

As long as difference between the saving rates of workers and capitalists is sufficiently high, the equilibrium value for z is positive. The (long-run) distribution of wealth only depends on the differential saving rates and the profit share. Capitalists' long-run equilibrium wealth share z^{**} is higher: (1) the higher the profit share, (2) the higher the saving rate of capitalists, and (3) the lower the saving rate of workers. It does however *not* depend on the growth rate (i.e. on the parameters of the investment equation), which contradicts Piketty's argument. In the long-run equilibrium, in our model the wealth share is constant, so each class has to save the exact amount that keeps their wealth share constant, independent of how slowly or how fast the economy grows.

There are two corner solutions for the wealth share: (1) If workers do not save at all, all wealth will (naturally) be concentrated in the hand of capitalists ($z = 1$). In this case, the share of total income going to profits is the only determinant of the distribution of income. (2) If capitalists do not save, it is self-evident that eventually all wealth will belong to workers ($z = 0$). Furthermore, if the two saving rates are equal ($s_w = s_r$), the model has no meaningful solution for z . All wealth would be concentrated in the hand of workers, if capitalists have no access to wage income.⁷

The long-run solution for capacity utilisation, the profit rate and the growth rate are:

⁷ Meade (1964) points this out, see also Taylor (2014).

$$u^{**} = \frac{\beta_0 + \beta_2\pi}{s_r\pi - \beta_1}$$

$$r^{**} = \frac{(\beta_0 + \beta_2\pi)\pi}{s_r\pi - \beta_1}$$

$$g^{**} = \frac{s_r(\beta_0 + \beta_2\pi)\pi}{s_r\pi - \beta_1}$$

Note that they are now independent from workers' saving rate. The Kaleckian principle, that 'capitalists earn what they spend' (Kalecki, 1971) applies here in a broader sense, i.e. that a higher saving rate of capitalists would diminish capacity utilisation, the profit rate and the growth rate in equilibrium.

The long-run solutions for the wealth-to-income ratio and personal income distribution are

$$x^{**} = \frac{s_r\pi - \beta_1}{\beta_0 + \beta_2\pi}$$

$$\theta^{**} = \frac{s_r\pi - s_w}{s_r(1 - \pi)}$$

The personal income distribution also does not depend on the growth rate, but only on the profit share and the two saving rates. The wealth-to-income ratio on the other hand is higher, the lower the growth parameter β_0 . This is in line with Piketty's argument that slow economic growth would raise x . Nevertheless, it has no impact on the distribution of wealth.

For the wealth-to-income ratio, Piketty's second fundamental 'law' (which they refer to as an accounting identity, Piketty and Zucman (2014: 1274) and call the "Harrod-Domar-Solow formula" (Piketty and Zucman 2014: 1257)) is fulfilled:

$$x^{**} = \frac{s}{g}$$

where $s = s_w[(1 - \pi) + \pi(1 - z)] + s_r\pi z$ is the aggregate saving rate of the economy.⁸ In contrast to Piketty, however, as noted above, in our model the aggregate saving rate and the growth rate adjust endogenously through the change in the wealth share until they reach the long-run equilibrium.

In the Post-Keynesian model, the wealth-to-income ratio and the personal income distribution are thus determined simultaneously with all the other (short-term) variables for any wealth share z . This is in contrast to Piketty's model where the wealth-to-income ratio is the result of the developments of the (constant) profit rate and the (constant) growth rate over time.

It should be mentioned that one of Piketty's main theoretical repercussion of a rising wealth-to-income ratio is that the profit share falls. In the Post-Keynesian model, in contrast, we follow convention and treat the profit share as exogenous. In contrast to Piketty' model, it is the profit rate that varies according to changes in capacity utilisation. For Post-Keynesians, the

⁸ The mathematical proof is available upon request.

profit share is determined by the power of workers and capitalists, which has significantly shifted in the direction of the latter since the 1980s. The deregulation of trade and capital flows as well as of financial markets and institutional changes has brought labour in the defensive; the result was a secular fall of the profit share.

Nevertheless, the profit share can also be endogenously determined within the model. Taylor (2014) briefly discusses the stabilising and destabilising mechanisms in such a model. The outcome however is ambiguous. While the profit share in such a model increases along with a rising wealth share, its long-run stability depends on the parameters of the model. For a wide range of values, a stable long-run equilibrium wealth share is very likely, so that the results of the abovementioned analysis remain valid. However, there is also the possibility of an ‘explosive trajectory’ where at its end all wealth is concentrated in the hand of capitalists. This would obviously validate Piketty (2014)’s predictions. To examine the dynamics of such a model is however beyond the scope of this paper.

d. Transitional dynamics

Another way to derive the Cambridge equation is to take the derivatives of the wealth share z with respect to time and rewrite the differential equation.

$$\dot{z} = \frac{\dot{V}_r V - V_r \dot{V}}{V^2} = \left(\frac{\dot{V}_r}{V_r} - \frac{\dot{V}}{V} \right) \frac{V_r}{V} = (s_r r - g)z$$

It immediately follows that the wealth share is only stable if the Cambridge equation is fulfilled.⁹ Furthermore, it is obvious that the wealth share z rises if $s_r r > g$. If this inequality holds, the (percentage) increase in capitalists’ wealth is higher than the (percentage) increase in total wealth. Piketty’s famous ‘fundamental contradiction of capitalism’ $r > g$ would thus be a special case of this inequality when $s_r = 1$, i.e. when capitalists accumulate all their income.¹⁰ The only (albeit very important) difference is, that in the Post-Keynesian model the profit rate and the growth rate are not exogenous and constant, as in Piketty’s model, but are both endogenously determined and adjust simultaneously until the long-run equilibrium is reached.

From the short-run solutions for r^* and g^* we see that both the profit rate and the growth rate decrease unambiguously when the wealth share rises. A higher wealth share reduces aggregate demand and thus also the profit rate and the growth rate. As long as $z < z^{**}$, both rates are higher than their long-run equilibria, i.e. $r^* > r^{**}$ and $g^* > g^{**}$. For low wealth shares, the increase in capitalists’ wealth is higher than the one in total wealth, and the inequality $s_r r > g$ is fulfilled. Consequently, capitalists’ wealth share rises.

⁹ Another (trivial) solution of the differential equation would be $z = 0$.

¹⁰ Piketty (2014, p. 26, emphasis added) writes that „...when the rate of return *significantly* exceeds the growth rate of the economy...“, which can be interpreted along the lines of the abovementioned inequality (López-Bernardo et al., 2016).

Piketty's 'fundamental contradiction' of capitalism, that the wealth concentration increases if and because $r > g$, is therefore valid in the Post-Keynesian model when z is below its long-term equilibrium. If r is "significantly greater" than g , the wealth share of capitalists will rise. Nevertheless, in contrast to Piketty's view, this can only be a temporary situation. When the wealth share reaches its equilibrium value, the Cambridge equation is fulfilled and the wealth share remains constant.

The profit rate and the growth rate are not the only variables that depend on the wealth share, as discussed in section 3.b. As long as z goes up, capacity utilisation decreases. Furthermore, the wealth-to-income ratio, which is the inverse of the former, rises. The personal income distribution becomes more unequal, since a higher wealth share shifts profits from workers to capitalists.

In the 'transitional phase', i.e. as long as the wealth share is below its long-run equilibrium, the Post-Keynesian model thus predicts a development of the variables in line with Piketty's (2014) empirical data. Both wealth and income distributions become more unequal, the wealth-to-income ratio increases, and economic growth weakens. However, the mechanisms behind these developments are in stark contrast to those Piketty had in mind. Not only will the wealth share eventually reach its stable long-run equilibrium, but all variables are determined within the model and adjust simultaneously.

Finally, let us briefly look at the effects of a slowdown in growth. In Piketty's view (2014: 233), a lower growth rate raises the wealth-to-income ratio, which entails a more unequal income distribution. In the Post-Keynesian model, a fall in the growth rate (which is represented by a lower value for β_0), reduces both the long-run profit rate and the long-run growth rate, so that the Cambridge equation is unaffected. It has thus no effect on the distribution of wealth and income. However, it reduces capacity utilisation and raises the wealth-to-income ratio. In short, while a higher wealth share entails a reduction in the growth rate (due to changes in the saving rates or the profit share), the reverse is not true.

e. Model extensions

The simple version of the model above is analytically tractable, but it arguably does not yet capture the spirit of Piketty's many-faceted analysis. This section therefore discusses three possible extensions to the simple model, which nonetheless retains the Post-Keynesian properties that were derived in the previous section. We introduce (1) blended income for capitalists, (2) differential rates of returns across workers and capitalists, and (3) capital gains.

First, in the basic version of the model, all wages accrued to workers. However, Piketty emphasizes that wages play an important (if diminishing) role even at the very top of the income distribution. Empirically, the share of wages received by capitalists lies between 5 and 10 per cent for the US and most EU-countries (Taylor et al. 2015, Ederer and Rehm 2017). In the vein of Palley (2017b), the model can thus be extended by blending not just workers', but also capitalists' income sources, i.e. distributing wages between workers and capitalists. As

Ederer and Rehm (2017) show, the personal income distribution is then more skewed towards capitalists than in the simple case since workers now receive less (wage) income.

Second, Piketty points to differential rates of return – the higher the wealth owned, the higher the returns on this wealth (Piketty 2014: 447f). The reasons for differential returns across the wealth distribution might lie, among others, in more professional wealth management at higher wealth levels, the ability to take higher risk, or a higher likelihood of insider knowledge. Empirical analysis (Ederer and Rehm 2017) finds that the composition of wealth varies between workers and capitalists in particular, with the former holding a larger share of their wealth in low-yield asset classes (in particular bank deposits). The implication for the distribution of profits is that capitalists receive higher capital income and thus benefit more from the compound interest effect. We thus distinguish between two asset types within productive wealth: deposits, which we assume for simplicity to be non-interest bearing, and profit-generating assets, which yield profit income.

Third, capital gains are an important form through which retained earnings of firms are distributed to the owners. Even though Piketty does not emphasize them in his theoretical considerations, his empirical results show that they are highly relevant. These capitals gains solely depend on the saving rate of firms and can be integrated into a Post-Keynesian model (López-Bernardo et al., 2016b; Taylor et al., 2017). Since capitalists typically hold a larger share of their wealth in profit-generating assets, a higher saving rate of firms skews the distribution of wealth and income towards them. Furthermore, a higher saving rate of firms can be expected to reduce demand, capacity utilisation and growth, since firms by definition have a saving rate equal to one and therefore a higher saving rate than capitalists and workers.

With these extensions, the analytical solution of the model becomes more complicated than in the basic version (see Appendix B). Capitalists' wealth share now depends not only on the saving rates of workers and capitalists and on the profit share, but furthermore on the distribution of wages between workers and capitalists, on their respective shares of wealth held as profit-generating assets, and on the saving rate of firms. The higher capitalists' share of the wage bill, the higher is their wealth share in the long run. The higher the share of capitalists' wealth held in the form of profit-generating assets, the higher is their wealth share (the same holds for workers). The higher the saving rate of firms, the higher is the wealth share of capitalists. This extended model nevertheless exhibits the same short- and long-run dynamic as its basic version.

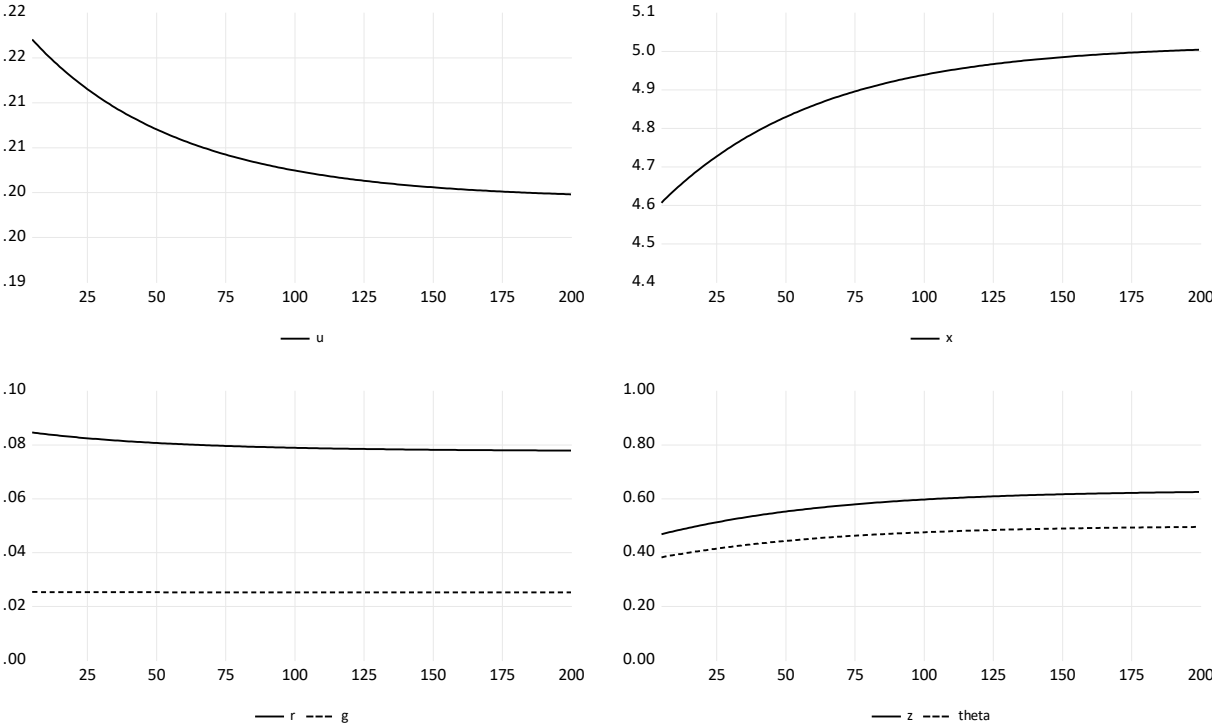
4. Simulating the dynamics

As section 3.d showed, even though the Post-Keynesian wealth model does not corroborate Piketty's theory of a corner solution for the wealth concentration, his empirical analysis is consistent with a transitional phase during which the wealth share of capitalists is below its long-term equilibrium. We therefore focus on the transitional dynamics next in order to illustrate the behaviour of the extended model in this phase.

The model is calibrated using parameters from the empirical literature (Ederer and Rehm 2017). As discussed in section 3e, the relevant parameters are the saving rates of workers and capitalists, the share of wealth held as profit-generating assets by workers and by capitalists, the profit share, the distribution parameter for wages (between workers and capitalists) and the saving rate of firms. For details on the parameter values and sources, see Table A1 in the Appendix.

Figure 1 shows a benchmark simulation using the average parameter values of all ten European countries¹¹ for which data is available. It depicts the dynamics of the model for capacity utilization u in the top left panel and for the wealth-income ratio x (which, as is clearly visible in the graphic, is the inverse of capacity utilization) at the top right. On the bottom, it shows the profit rate r and the growth rate g on the left, and wealth concentration z and income concentration θ (as above, measured as the ratio of capitalists wealth and income to worker’s wealth and income, respectively).

Figure 1: Short- and long-run dynamics of the Post-Keynesian wealth model for Europe



Source: own elaboration.

Note: This graph shows the dynamic in capacity utilisation u , the wealth-to-income ratio x , the profit rate r , the growth rate g , capitalists’ wealth share z , and the ratio of capitalists’ income to workers’ income (personal income distribution) θ . For parameter starting values, see Appendix A.

¹¹ These countries comprise Austria, Belgium, Cyprus, Spain, Finland, France, Greece, Malta, Portugal, and Slovakia.

There are three main findings: First, the profit rate is always higher than the growth rate. However, the differential decreases over time – if slightly – as the wealth share rises until it reaches an equilibrium. Second, the model clearly approaches an equilibrium in the long run; the wealth-to-income ratio, wealth and income inequality do not rise indefinitely. Third, however, during a transitional phase over the next 50 to 100 years, the model points to a steep rise in these variables, and a concomitant fall in capacity utilization.

The first result confirms Piketty's 'first law' of capitalism, $r > g$. This, however, holds by definition in a Post-Keynesian model due to Pasinetti's 'Cambridge equation', $s_r r = g$ (except for the unrealistic case in which capitalists do not consume, so that their saving rate is equal to one). The direction of change, however, contrasts with Piketty, who suggests that the differential between the two rates, if anything, will widen.

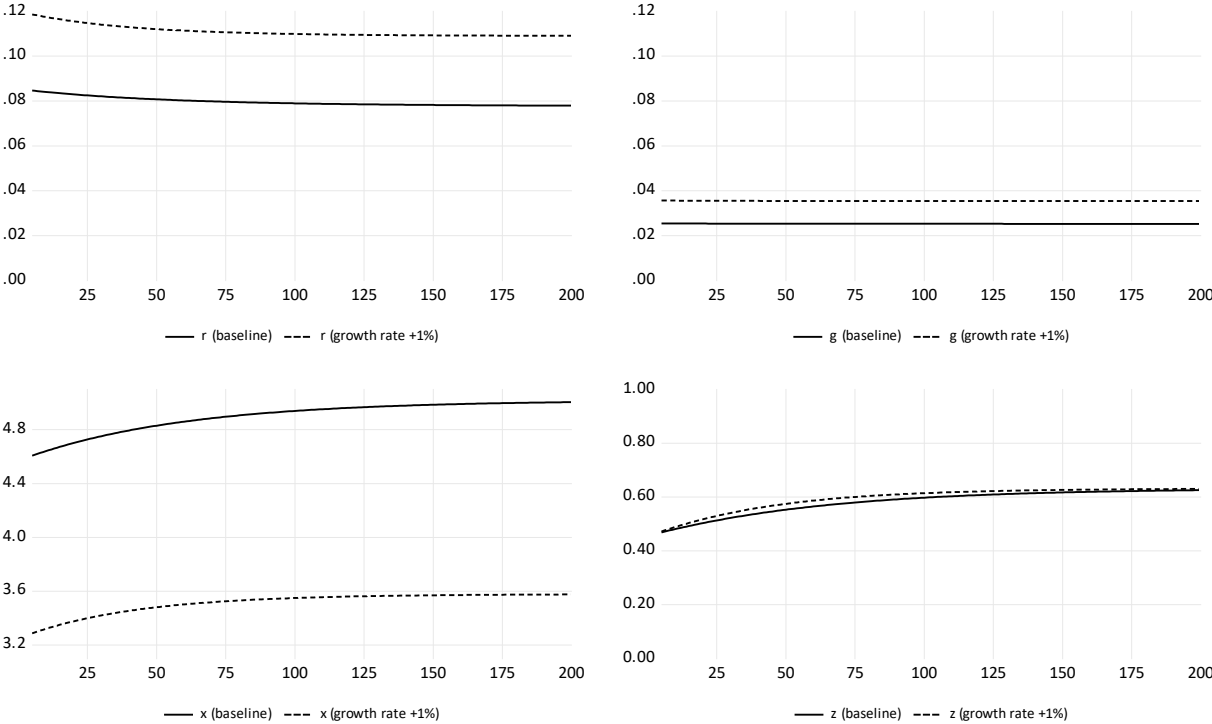
The second finding is in clear contradiction to Piketty (2014: 361), who says that "The fact that the return on capital is distinctly and persistently greater than the growth rate is a powerful force for a more unequal distribution." Leaving aside historical contingencies and caveats, which Piketty (2014: 361f) is careful to discuss and include, Piketty's bare-bones model predicts an ever-increasing wealth share, and thus eventually a corner solution for the concentration of wealth, in which a tiny elite owns all wealth. In contrast, our calibrated simulation confirms the theoretical finding from our analytical model, namely that the wealth share held by capitalists usually stabilizes at an interior solution. In the Post-Keynesian world, both workers and capitalists own a stable share of wealth.

Our third finding bridges the gap between a Post-Keynesian analysis and Piketty's empirical findings, or, put differently, it provides an analytical underpinning to Piketty's work. During the transition phase from the current state towards the long-run equilibrium, all variables exhibit the development described by Piketty: The share of wealth owned by capitalists, the Post-Keynesian equivalent to Piketty's 'elite' defined by percentiles of the wealth distribution, rises unequivocally and by significant levels from under 50% to more than 60%. Similarly, the wealth-to-income ratio increases to around 5. In addition, personal income inequality increases, i.e. the ratio of capitalists' income to workers' income rises from just below 40% to roughly 50%. As with wealth, Piketty measures these as income share of percentiles of the income distribution. To conclude, these are the main messages of Piketty's (2014) book: the wealth-to-income ratio, the wealth concentration, and personal income inequality will all increase, if capitalism is left to its own devices.

Figure 2 shows the effect of a rise in 'animal spirits', i.e. the exogenous component of the investment function β_0 . Piketty does not focus on the growth regime in too much detail, but here, too, our findings from the calibrated model are in line with his broad predictions – and, as expected, with Post-Keynesian models: If autonomous investment increases, the profit rate and the growth rate rise, as does capacity utilization, so the wealth-to-income ratio falls. However, the simulations also make clear that a change in autonomous investment has only transient effects on the distribution of wealth. In the long-run, capitalists' wealth share

approaches the same value as in the baseline simulation. The equilibrium value for the wealth share does not depend on the growth rate since the profit rate also increases endogenously, so that the Cambridge equation is fulfilled at the same value for z as before.

Figure 2: Effects of an increase in the growth rate on short- and long-run dynamics of the Post-Keynesian wealth model in Europe



Source: own elaboration.

Note: This graph shows the dynamic in the profit rate r , the growth rate g , the wealth-to-income ratio x , and capitalists' wealth share z following an increase in autonomous investment, compared to a baseline scenario. For parameter starting values, see Appendix A.

5. The effects of a wealth tax

Piketty's (2014: 532) solution for his predicted increasing wealth concentration and rising wealth-to-income ratio is a global wealth tax. Although Post-Keynesian criticize the proposal for being utopian (Palley 2014), it is nevertheless interesting to investigate the effects of a wealth tax in the Post-Keynesian model.

Piketty (2014: 571) (tentatively) suggests a tax of 0.1% on wealth up to 200.000 Euro, 0.5% up to 1 million Euro, 1% up to 5 million Euro, 2% up to 1 billion Euro, and 5% (to 10%) on wealth greater than a billion Euro. For simplicity, we use the average for the lower two brackets (0.3%) as workers' average tax rate and the average of the higher three tax brackets (4.3%) as capitalists' tax rate. Furthermore, we formulate the tax a flat rate t on the total wealth of both workers and capitalists and we assume that tax earnings are spent as government

consumption to maintain the stock-flow consistency of the model.¹² Income of capitalists and workers becomes

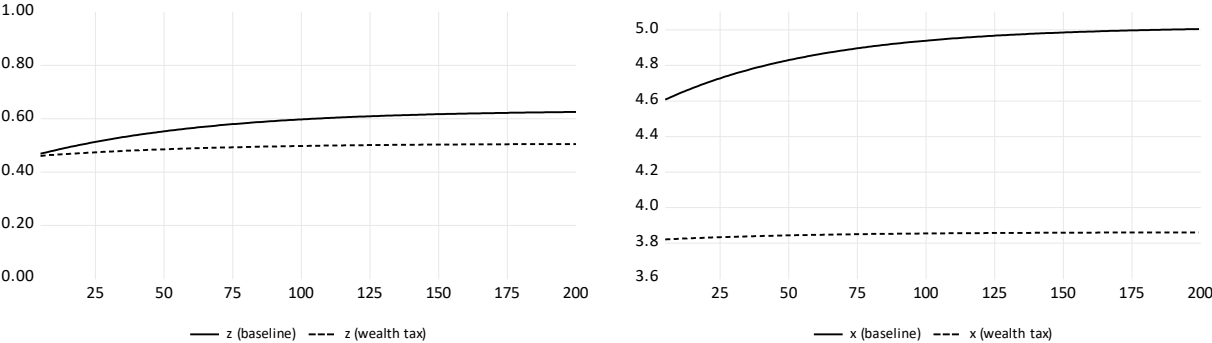
$$Y_r = zR - t_r V_r$$

$$Y_w = W + (1 - z)R - t_w V_w$$

For the dynamic equations, see Appendix B.

Figure 3 shows that the wealth tax redistributes income from capitalists (with a higher saving rate) to workers. It therefore raises capacity utilisation and simultaneously reduces the wealth-income ratio, while the growth rate and the profit rate also rise. Most importantly however, from Piketty’s point of view, the wealth share of capitalists would fall noticeably, from roughly 0.62% in the baseline scenario to about 50%. That is, a wealth tax as suggested by Piketty would neutralize the rise in wealth inequality predicted by the model. This reduction in the wealth share is permanent, as long as the tax is imposed periodically. Conversely, one-off levies only have a transitory effect as the wealth concentration returns to its long-run equilibrium.

Figure 3: Effects of a wealth tax on short- and long-run dynamics of the Post-Keynesian wealth model in Europe



Source: own elaboration.

Note: This graph shows the dynamic in the wealth-to-income ratio x , and capitalists’ wealth share z for a wealth tax of 0.3% on workers’ and 4.3% of capitalists’ wealth, compared to a baseline scenario. For parameter starting values, see Appendix A.

Other taxes that reduce the income of capitalists and thus dampen their ability to accumulate wealth, such as an inheritance tax or income taxes for capitalists, are fungible to a wealth tax; they will lead to the same effects as the ones described in Figure 4. In fact, in the Post-

¹² Another way to balance the budget would be to spend all tax revenue on monetary transfers. Transfers directly increase the income of households and have a distributive effect themselves given that workers and capitalists benefit differently from them. Public consumption on the other hand has no direct distributive effect in the model, which allows us to isolate the sole effect of Piketty’s tax rates.

Keynesian model, these different taxes can easily be calibrated to yield identical results. If wealth is passed on to the next generation every 25 years, an inheritance tax of 60% has on average the same incidence as a general (yearly) wealth tax of 2,4% (which is our average tax rate). Similarly, introducing a tax on capital income of approximately 30% leads to a similar result, given that the profit rate in the simulation is about 8%.¹³

6. Conclusion

This paper developed a Post-Keynesian model with an endogenous distribution of wealth between workers and capitalists, and extended it by including blended wage and capital income of both workers and capitalists, differential returns on assets between workers and capitalists, and capital gains which reflect the increase in firm value due to retained earnings. We looked into the short- and long-run dynamic of the model, discussed Piketty's theoretical arguments against its background, and evaluated his proposal of a wealth tax.

Piketty's main prediction is that a small elite will own all wealth if capitalism is left to its own devices. Our model permits this corner solution of all (or zero) wealth held by capitalists, but usually economies will show a stable long-run wealth distribution in which workers have a positive wealth share. In such an equilibrium, the wealth-to-income ratio is stable, and there is a (stable and positive) gap between the profit rate and the growth rate, which is given by the Cambridge equation. The specific level of the equilibrium wealth distribution between workers and capitalists depends on their saving rates, the profit share, the share of wage income that accrues to capitalists, the differential returns on wealth for the two household groups, and the saving rate of firms.

We therefore reject this theoretical conclusion of Piketty. However, we show that the model has a transitional phase, i.e. when the wealth share of capitalists is below its long-term equilibrium, in which the model behaves according to Piketty's (2014) empirical findings for high-income countries since the 1980s. In this situation, the wealth share of capitalists increases endogenously. Furthermore, the wealth-to-income ratio rises, the differential between the profit rate and the growth rate gradually decreases (but is always higher than the long-term gap), and income inequality rises. Consistent with Keynesian logic, a rising wealth share reduces aggregate demand and thus capacity utilization and growth. The paper thus provides theoretical foundations to Piketty's abundant empirical findings.

Concretely, our simulations show that wealth inequality in Europe – as measured by capitalists' wealth share – would rise from well under 50% to more than 60%. This level differs substantially across individual countries, as Ederer and Rehm (2017) show. Finally, we evaluate the effects of a wealth tax, which Piketty suggested for addressing the increasing concentration of wealth. The model shows that the introduction of a permanent wealth tax at levels suggested by Piketty (or, equivalently, a suitable inheritance tax or capital income tax)

¹³ However, it should be noted that this back-of-the-envelope calculation does not take into account that an income tax does not capture capital gains.

can indeed neutralize the rise in wealth inequality predicted by the model by reducing the equilibrium value for the wealth share owned by capitalists in Europe – and thus the wealth concentration – to 50%. It also reduces the wealth-to-income ratio and dampens income inequality.

There are a number of interesting avenues for future work. First, expanding the analysis to other countries, such as the U.S., is an obvious next step. Second, endogenizing the profit share and working through the stability aspects of such a model might provide valuable insight into potential ‘Piketty dynamics’ in a Keynesian framework. Third, delving into the policy research might yield more detailed information on the relative merits of a wealth tax versus an inheritance tax or capital income taxes.

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Appendix A: Parameter values

Table A.1: Parameter values for the model calibration, average over countries

Parameter	Value	Source
s_w	0.07	Ederer/Rehm 2018, Data: EHBS
s_r	0.24	Ederer/Rehm 2018, Data: EHBS
π	0.39	Ederer/Rehm 2018, Data: HFCS
α	0.06	Ederer/Rehm 2018, Data: HFCS
γ_w	0.49	Ederer/Rehm 2018, Data: HFCS
γ_r	0.91	Ederer/Rehm 2018, Data: HFCS

Note: Rows refer to: (1) s_w saving rate of workers, (2) s_r saving rate of capitalists, (3) π profit share, (4) α share of capitalists in the wage bill, (5) γ_w share of workers' wealth held in profit-generating assets, (6) γ_r share of capitalists' wealth held in profit-generating assets. In order to reproduce the empirical results of Ederer and Rehm (2017), we set the parameter for the saving rate of firms equal to zero. Since the empirical value of this parameter is usually higher, the long-run values of the simulations are on the conservative side.

Appendix B: Extended model

Table B.1: Transaction flow matrix in the extended model

	Households		Firms		Banks	Total
	Workers	Capitalists	Current	Capital	Capital	
Consumption	$-C_w$	$-C_r$	$+C$			0
Investment			$+I$	$-I$		0
Wages	$+W_w$	$+W_r$	$-W$			0
Profits	$+R_w$	$+R_r$	$-R$	$+R_f$		0
Equity	$-\Delta E_w$	$-\Delta E_r$		$+\Delta E$		0
Deposits	$-\Delta D_w$	$-\Delta D_r$			$+\Delta D$	0
Loans				$+\Delta L$	$-\Delta L$	0
Total	0	0	0	0	0	0

Source: own elaboration.

Table B.2: Balance sheet matrix in the extended model

	Households				Total
	Workers	Capitalists	Firms	Banks	
Capital			$+K$		$+K$
Equity	$+E_w$	$+E_r$	$-E$		0
Deposits	$+D_w$	$+D_r$		$-D$	0
Loans			$-L$	$+L$	0
Wealth	$-V_w$	$-V_r$	$-V_f$		$-V$
Total	0	0	0	0	0

Source: own elaboration.

Retained profits:

$$R_f = \eta R$$

Disposable income:

$$Y_w = (1 - \alpha)W + \frac{\gamma_w(1 - z)}{\gamma_w(1 - z) + \gamma_r z} (1 - \eta)R$$

$$Y_r = \alpha W + \frac{\gamma_r z}{\gamma_w(1 - z) + \gamma_r z} (1 - \eta)R$$

Wealth dynamic:

$$\dot{z} = \frac{\dot{V}_r}{V} - gz = \left\{ s_r \left[(1 - \pi)\alpha + \frac{\gamma_r z}{\gamma_w(1 - z) + \gamma_r z} (1 - \eta)\pi \right] + \frac{\gamma_r z}{\gamma_w(1 - z) + \gamma_r z} \eta\pi \right\} u - gz$$

Appendix C: Basic model with a (progressive) wealth tax

Table C.1: Stocks and flows in the model with a wealth tax

	Households		Firms		State	Total
	Workers	Capitalists	Current	Capital	Current	
Consumption	$-C_w$	$-C_r$	$+C$		$+C_g$	0
Investment			$+I$	$-I$		0
Wages	$+W$		$-W$			0
Profits	$+R_w$	$+R_r$	$-R$			0
Taxes	$-T_w$	$-T_r$			$+T$	0
Wealth	$-\Delta V_w$	$-\Delta V_r$		$+\Delta K$	0	0
Total	0	0	0	0	0	0

Source: own elaboration.

Wealth dynamic:

$$\dot{z} = [s_r(r - t_r) - g]z$$

$$z^* = \frac{(\beta_0 + \beta_2\pi)(s_r\pi - s_w) + \beta_1(s_rt_r - s_w t_w) - s_w s_r(t_r - t_w\pi)}{(\beta_0 + \beta_2\pi)(s_r - s_w)\pi + \beta_1(s_rt_r - s_w t_w) - s_w s_r\pi(t_r - t_w)}$$