View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by INRIA a CCSD electronic archive server

HAL
archives-ouvertes.fr

An H_∞ observer for descriptor nonlinear systems with nonlinear output equations

Gridsada Phanomchoeng, Ali Zemouche, Woongsun Jeon, Rajesh Rajamani, Frédéric Mazenc

▶ To cite this version:

Gridsada Phanomchoeng, Ali Zemouche, Woongsun Jeon, Rajesh Rajamani, Frédéric Mazenc. An H_∞ observer for descriptor nonlinear systems with nonlinear output equations. American Control Conference, ACC 2018, Jun 2018, Milwaukee, WI, United States. hal-01849018

HAL Id: hal-01849018

https://hal.inria.fr/hal-01849018

Submitted on 25 Jul 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

$\mathcal{H}_\infty$ Observer for Descriptor Nonlinear Systems with Nonlinear Output Equations

G. Phanomchoeng[1,2], A. Zemouche[3,4], W. Jeon[5], R. Rajamani[5], and F. Mazenc[3]

***Abstract*— This paper deals with the problem of $\mathcal{H}_\infty$ observer design for a class of descriptor nonlinear systems with nonlinear output measurements. The established result is broad and can be applied to many estimation problems in nonlinear systems, such as unknown input estimation, fault diagnosis and state estimation. Based on the introduction of new assumptions involving a rank condition that specifies a constraint between the numbers of allowed unknown inputs, nonlinear functions, states and the number of measurements, a new Linear Matrix Inequality condition (LMI) for observer design is proposed. The obtained LMI is more general and less conservative, from a feasibility point of view, than those existing in the literature.**

Keywords—Descriptor systems; LMIs; $\mathcal{H}_\infty$ analysis; nonlinear systems.

I. INTRODUCTION

State observer design for nonlinear systems has attracted the attention of many researchers in the field of automatic control. The topic is a subject of constant evolution [1], [2], [3], [4], [5], [6], [7], [8] and is motivated by several applications such as disturbance input estimation, fault diagnosis, control system design with unknown partial dynamics or synchronization and encryption in chaotic communication systems. One of the well-established problems that uses the theory of descriptor systems is the simultaneous state and unknown input estimation problem [9], [10], [11], [12]. Observer design for the class of descriptor systems with nonlinearities in both the dynamics of the system and the output measurements is an important unsolved problem.

While the problem of state observer design for nonlinear systems has been widely investigated in the recent literature and several research results have been established, the study becomes complicated when we face systems in descriptor/singular structures [13]. Indeed, very few results have been investigated or established in the case of descriptor systems, especially when the output measurements contain nonlinear terms.

Motivated by the lack of results in the field of observer design for descriptor nonlinear systems in the presence of nonlinearities in the measured outputs, we propose in this paper a new LMI design technique aimed at meeting a $\mathcal{H}_\infty$ performance criterion. The presence of nonlinear terms in the output measurements renders the design of the observer gain parameters to be particularly complicated. To overcome this obstacle, we propose a new LMI technique, where the method requires a certain assumption on system parameters concerning the numbers of outputs, unknown inputs, states and numbers of nonlinear functions in the measurement equations. Moreover, the proposed LMI technique is less conservative compared to the existing results in the literature. Indeed, the nonlinearities are handled in a less conservative way. The Young's inequality is used in a convenient manner that leads to dilated LMIs [14]. The contributions of this paper can be summarized as follows:

- The technique is able to estimate unknown inputs with only a minimum number of measurements. Indeed, the rank condition requires a necessary number of outputs.
- The contribution can be viewed as an extension of the work in [14] to a more general class of nonlinear systems: descriptor structure of the system and presence of nonlinearities in the outputs.

The rest of this paper is organized as follows. In Section II we introduce the problem formulation and the motivations of the proposed work. Section III is devoted to the proposed LMI design method based on a rank condition based assumption. Then the application to unknown input estimation is presented in Section IV. Finally, a conclusion and some perspectives are presented in Section V.

Notation: The following notation will be used throughout this paper.

- $\|\cdot\|$ is the usual Euclidean norm;
- $(*)$ is used for the blocks induced by symmetry;
- A^T represents the transposed matrix of A;
- $\mathbb{I}_r$ represents the identity matrix of dimension r;
- For a square matrix S, $S > 0$ ($S < 0$) means that this matrix is positive definite (negative definite);
- The notation $\|x\|_{\mathcal{L}_p^r} = \left(\int_0^{+\infty} \|x(t)\|^p \, dt\right)^{\frac{1}{p}}$ is the $\mathcal{L}_p^r$

[1] Department of Mechanical Engineering, Faculty of Engineering, Chulalongkorn University, Bangkok 10330, Thailand.
[2] Smart Wireless Communication Ecosystem Research Group, Chulalongkorn University, Bangkok 10330, Thailand.
[3] EPI Inria DISCO, Laboratoire des Signaux et Syst`emes, CNRSCentraleSup´elec, 91192 Gif-sur-Yvette, France.
[4] University of Lorraine, 186, rue de Lorraine, CRAN UMR CNRS 7039, 54400 Cosnes et Romain, France.
[5] Laboratory for Innovations in Sensing, Estimation, and Control, Department of Mechanical Engineering, University of Minnesota, Minneapolis, USA.

norm of the vector $x \in \mathbb{R}^r$. The set $\mathcal{L}_p^r$ is define by

$$\mathcal{L}_p^r = \left\{x \in \mathbb{R}^r : \|x\|_{\mathcal{L}_p^r} < +\infty\right\}$$

and then $\left(\mathcal{L}_p^r, \|\cdot\|_{\mathcal{L}_p^r}\right)$ is called the *Lebesgue space*;

- $e_s(i) = \Big(\underbrace{0, \dots, 0, \overbrace{1}^{i\,th}, 0, \dots, 0}_{s\ components}\Big)^T \in \mathbb{R}^s, s \geq 1$ is a vector of the canonical basis of $\mathbb{R}^s$;

II. Problem Formulation and Motivations

A. System description

The class of system to be investigated in this paper is defined by the following equations:

$$\begin{cases} \overset{\cdot}{\overbrace{E_\xi \xi(t)}} &= A_\xi \xi(t) + B_f f(\xi(t), u(t)) \\ & \quad + G_g g\big(y(t), u(t)\big) + D_\xi \omega(t) \\ y(t) &= C_\xi \xi(t) + B_h h\big(\xi(t), u(t)\big) + D_y \omega(t) \end{cases} \tag{1}$$

where $\xi \in \mathbb{R}^{n_\xi}$; $u \in \mathbb{R}^{n_u}$, and $y \in \mathbb{R}^{n_y}$ are respectively the state vector of the system, the known control input vector, and the output measurements vector. The vector $\omega \in \mathbb{R}^{n_\omega}$ represents the unknown disturbances affecting the system dynamics and the measured variables. The matrices $E_\xi \in \mathbb{R}^{n_d \times n_\xi}$, $A_\xi \in \mathbb{R}^{n_d \times n_\xi}$, $B_f \in \mathbb{R}^{n_d \times n_f}$, $G_g \in \mathbb{R}^{n_d \times n_g}$, $D_\xi \in \mathbb{R}^{n_d \times n_\omega}$, $C_\xi \in \mathbb{R}^{n_y \times n_\xi}$, $B_h \in \mathbb{R}^{n_y \times n_h}$, and $D_y \in \mathbb{R}^{n_y \times n_\omega}$ are known and constant. The functions $f : \mathbb{R}^{n_\xi} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}^{n_f}$ and $h : \mathbb{R}^{n_\xi} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}^{n_h}$ are globally Lipschitz[1] with respect to the variable ξ, uniformly on u, with Lipschitz constants γ_f and γ_h, respectively. Assume also that the pair (E_ξ, C_ξ) satisfies the following condition:

$$rank\left(\begin{bmatrix} E_\xi \\ C_\xi \end{bmatrix}\right) = n_\xi. \tag{2}$$

The aim consists in finding an observer to estimate asymptotically or exponentially the state ξ. Contrary to regular systems[2] for which a simple Luenberger observer with one constant gain can be used to investigate the $\mathcal{H}_\infty$ criterion, the problem of state observer for the descriptor system (1) turns out to be difficult in some cases. In this paper, we will first consider a well-known two-stage structure of the observer. The results we will propose can then be extended to more complicated structures [5], [16]. Although the structure of the state observer is simple, the synthesis of the observer gain parameters has complicated obstacles that we need to overcome in order to have tractable sufficient LMI conditions.

B. The two-stage observer structure

The two-stage state observer we consider in this note is described by the following equations:

$$\begin{cases} \dot{z}(t) &= A_z z(t) + A_y y_{\eta_1} + B_z f(\hat{\xi}(t), u(t)) \\ & \quad + G_z g\big(y(t), u(t)\big) \\ \hat{\xi}(t) &= z(t) + Q_z y_{\eta_2}(t) \\ y_{\eta_i}(t) &= y(t) - \eta_i(t), \quad i = 1,2 \end{cases} \tag{3}$$

where $\hat{\xi}(t)$ is the estimate of $\xi(t)$. The auxiliary variables $\eta_1(t), \eta_2(t)$ and the matrices A_z, A_y, B_z, G_z, and Q_z are the observer parameters to be determined so that the estimation error $\tilde{\xi}(t) = \hat{\xi}(t) - \xi(t)$ converges towards zero in sense of a pre-defined performance criterion, namely $\mathcal{H}_\infty$ or $\mathcal{W}^{1,2}$ criteria.. By using (1) and (3) $\tilde{\xi}(t)$ is expressed as:

$$\begin{aligned} \tilde{\xi}(t) = z(t) + \left(Q_z C_\xi - \mathbb{I}_{n_\xi}\right) \xi(t) + \\ Q_z B_h \left[h\big(\xi(t), u(t)\big) - \eta_2(t)\right] + Q_z D_y \omega(t). \end{aligned} \tag{4}$$

In the case $y(t) = C_\xi \xi(t)$, the problem becomes easy and the results are well known in the literature, because with $\eta_2(.) \equiv 0$, equation (4) reduces to

$$\tilde{\xi}(t) = z(t) + \left(Q_z C_\xi - \mathbb{I}_{n_\xi}\right) \xi(t). \tag{5}$$

Hence, from the condition (2), there exist two matrices P_z and Q_z such that

$$P_z E_\xi + Q_z C_\xi = \mathbb{I}_{n_\xi}. \tag{6}$$

Therefore, we can write the dynamics of the estimation error as:

$$\dot{\tilde{\xi}}(t) = \dot{z}(t) - P_z \overset{\cdot}{\overbrace{E_\xi \xi(t)}} \tag{7}$$

and then we can exploit the singular equation in (1) and the dynamics of $z(t)$ in (3) to lead to an appropriate and standard estimation error dynamics after some matrix manipulations [5], [10], [13].

However, the presence of $h((\xi(t), u(t))$ and $v(t)$ in the output measurements renders the problem more complicated. Indeed, the computation of the parameters A_z, A_y, B_z, G_z, Q_z, and the choice of the performance criterion depend on additional assumptions on the matrices B_h, D_v on the disturbance $v(t)$, and on the nonlinear function $h(\xi, u)$. In particular, the presence of nonlinearities in the output measurements renders the problem difficult and original from a LMI point of view. Depending on the distribution of the nonlinearity $h(.,.)$ in $y(t)$ (this is related to the structure of the matrix B_h) and its time derivative, a different LMI technique can be provided in each case.

C. $\mathcal{H}_\infty$ criterion

This subsection is devoted to some definitions related to the $\mathcal{H}_\infty$ performance criterion. The aim consists in finding $\eta_1(t)$, $\eta_2(t)$ and the matrices A_z, A_y, B_z, G_z, and Q_z so that the estimation error $\tilde{\xi}(t)$ satisfies the criterion:

$$\left\|\tilde{\xi}(t)\right\|_{\mathcal{L}_2^{n_\xi}} \leq \sqrt{\mu_\infty \|\omega\|^2_{\mathcal{L}_2^{n_\omega}} + \nu_\infty \left\|\tilde{\xi}_0\right\|^2} \tag{8}$$

where $\mu_\infty > 0$ is the disturbance attenuation level and $\nu_\infty > 0$ is to be determined. In fact, $\sqrt{\mu_\infty}$ is the gain from ω to $\tilde{\xi}$.

[1] If f and h are only locally Lipschitz, we may consider their saturated versions f^s and h^s, respectively, on an invariant compact set $\mathcal{X}$ in which f^s and h^s satisfy the global Lipschitz property [15].

[2] This means that the matrix E_ξ is invertible, i.e. $rank(E_\xi) = n_\xi$.

Usually we use Lyapunov functions to get checkable conditions guaranteeing (8). In the LMI framework, we take a quadratic Lyapunov function $V(\tilde{\xi})$, such that

$$\vartheta(t) \triangleq \frac{dV}{dt}(\tilde{\xi}) + \|\tilde{\xi}\|^2 - \mu_\infty \|\omega\|^2 \leq 0. \tag{9}$$

The objective is to develop some LMI conditions under which the inequality (9) holds.

III. Observer Design Methodology

In this section we will introduce a new LMI method to deal with the problem of state observer design for the described descriptor nonlinear systems.

To avoid the nonlinear term $h(\xi(t), u(t)) - \eta_2(t)$ in (4), an obvious solution consists in choosing Q_z so that $Q_z B_h = 0$. However, this solution requires the following additional rank condition:

$$rank\begin{pmatrix} E_\xi & 0 \\ C_\xi & B_h \end{pmatrix} = n_\xi + n_h. \tag{10}$$

which implies implicitly

$$n_d + n_y \geq n_\xi + n_h.$$

This means that the nonlinear function h is considered as unknown input, which requires additional measurements to satisfy (10). However, for practical reasons, it is often difficult to measure additional variables because of their expensive cost or unavailability at any cost.

To avoid the above constraint, we will introduce a new assumption on the nonlinear function h, which will not affect the number of measurements required to achieve the procedure.

Assumption 3.1: There exist two matrices A_κ, B_κ and a nonlinear function κ: $\mathbb{R}^{n_\xi} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_h}$ so that the time derivative of the nonlinear output function $h(\xi(t), u(t))$ satisfies the following conditions:

$$\frac{dh}{dt}(\xi(t), u(t)) = A_\kappa \xi(t) + B_\kappa\, \kappa(\xi(t), u(t)), \tag{11}$$

$$\|\kappa(\xi, u) - \kappa(\hat{\xi}, u)\| \leq \gamma_\kappa \|\xi - \hat{\xi}\|. \tag{12}$$

A. A new two-stage observer structure

Let us consider the two-stages observer (3) by specifying the variables η_i, $i = 1,2$. That is, we use the following observer structure:

$$\begin{cases} \dot{z}(t) &= A_z z(t) + A_y y_{\eta_1}(t) + B_z f(\hat{\xi}(t), u(t)) \\ & \quad + G_z g(y(t), u(t)) \\ \hat{\xi}(t) &= z(t) + Q_z y_{\eta_2}(t) \\ y_{\eta_i}(t) &= y(t) - \eta_i(t), \ i = 1,2 \\ \eta_1(t) &= B_h h(\hat{\xi}(t), u(t)) \\ \eta_2(t) &= B_h \int_0^t \left[A_\kappa \hat{\xi}(s) + B_\kappa \kappa\left(\hat{\xi}(s), u(s)\right)\right] ds. \end{cases} \tag{13}$$

From the rank condition (2), there exist two matrices P_z and Q_z so that

$$[P_z \quad Q_z] = \left(\begin{bmatrix} E_\xi \\ C_\xi \end{bmatrix}^T \begin{bmatrix} E_\xi \\ C_\xi \end{bmatrix}\right)^{-1} \begin{bmatrix} E_\xi \\ C_\xi \end{bmatrix}^T \tag{14}$$

which leads to (6). Then, the dynamics equation (4) becomes

$$\zeta(t) = z(t) + P_z E_\xi \xi(t) \tag{15}$$

$$+ Q_z B_h\left[h(\xi(t), u(t)) - \eta_2(t)\right].$$

where

$$\zeta(t) = \tilde{\xi}(t) - Q_z D_y \omega(t). \tag{16}$$

The new variable $\zeta(t)$ is introduced in order to avoid the derivative of the disturbance $\omega(t)$. For some details on this issue, we refer the reader to [18].

From Assumption 3.1, after developing the calculations, we get the following dynamics of $\zeta(t)$:

$$\begin{aligned} \dot{\zeta}(t) = A_z \tilde{\xi}(t) &+ (A_z - P_z A_\xi + \mathbb{L} C_\xi)\xi \\ &+ (B_z - P_z B_f) f(\xi, u) \\ &+ (G_z - P_z G_g) g(y, u) \\ &+ (\mathbb{E}_\omega - \mathbb{L}\mathbb{D}_\omega)\omega + B_z \delta f_t \\ &- A_y B_h \delta h_t - Q_z B_h B_\kappa \delta\kappa_t \end{aligned} \tag{17a}$$

$$\delta f_t = f(\hat{\xi}, u) - f(\xi, u) \tag{17b}$$

$$\delta h_t = h(\hat{\xi}, u) - h(\xi, u) \tag{17c}$$

$$\delta\kappa_t = \kappa(\hat{\xi}, u) - \kappa(\xi, u) \tag{17d}$$

$$\mathbb{A} = P_z A_\xi - Q_z B_h A_\kappa, \tag{17e}$$

$$\mathbb{L} = A_y - A_z Q_z \tag{17f}$$

$$\mathbb{E}_\omega = -P_z D_\xi, \mathbb{D}_\omega = -D_y. \tag{17g}$$

Since f, h, and κ are globally Lipschitz, then from [2] there exist functions

$$\phi_{ij} : \mathbb{R}^{n_f^i} \times \mathbb{R}^{n_f^i} \to \mathbb{R}$$

$$\psi_{ij} : \mathbb{R}^{n_h^i} \times \mathbb{R}^{n_h^i} \to \mathbb{R}$$

$$\varphi_{ij} : \mathbb{R}^{n_i} \times \mathbb{R}^{n_i} \to \mathbb{R}$$

and constants a_{ij}, b_{ij}, c_{ij}, d_{ij}, e_{ij}, and f_{ij} such that

$$\delta f_t = \sum_{i,j=1}^{i,j=n_f, n_f^i} \phi_{ij}(t)\mathcal{H}_{ij} H_i \tilde{\xi}(t), \tag{18}$$

$$\delta h_t = \sum_{i,j=1}^{i,j=n_h, n_h^i} \psi_{ij}(t)\mathcal{F}_{ij} F_i \tilde{\xi}(t), \tag{19}$$

$$\delta\kappa_t = \sum_{i,j=1}^{i,j=m, n_i} \varphi_{ij}(t)\mathcal{T}_{ij} T_i \tilde{\xi}(t), \tag{20}$$

with

$$a_{ij} \leq \phi_{ij}(t) \leq b_{ij}, \tag{21}$$

$$c_{ij} \leq \psi_{ij}(t) \leq d_{ij}, \tag{22}$$

$$e_{ij} \leq \varphi_{ij}(t) \leq f_{ij}. \tag{23}$$

and

$$\begin{aligned} \mathcal{H}_{ij} = e_{n_f}(i) e_{n_f^i}^T(j), \ \mathcal{F}_{ij} = e_{n_h}(i) e_{n_h^i}^T(j), \\ \mathcal{T}_{ij} = e_{n_\kappa}(i) e_{n_\kappa^i}^T(j). \end{aligned} \tag{24}$$

The dynamics of the estimation error is now rewritten under the detailed form:

$$\begin{aligned}\dot{\zeta}(t) = &\left((\mathbb{A} - \mathbb{L}C_\xi) + \sum_{i,j=1}^{i,j=m,n_i} \left[\phi_{ij} B_z \mathcal{H}_{ij} H_i\right] \right)\tilde{\xi}(t) \\ &- \left(\sum_{i,j=1}^{i,j=m,n_i} \left[\psi_{ij} \mathbb{L} B_h \mathcal{F}_{ij} F_i\right] \right)\tilde{\xi}(t) \\ &- \left(\sum_{i,j=1}^{i,j=m,n_i} \left[\varphi_{ij} \mathbb{L} B_h B_\kappa \mathcal{T}_{ij} T_i\right] \right)\tilde{\xi}(t) \\ &+ (\mathbb{E}_\omega - \mathbb{L}\mathbb{D}_\omega)\omega. \end{aligned} \quad (25)$$

B. New LMI design conditions

By calculating the derivative of the Lyapunov function

$$V(\zeta) = \zeta^T(t)\mathbb{P}\zeta(t), \quad (26)$$

along the trajectory of (25), we get $\vartheta(t) \leq 0$ if the following inequality holds:

$$\begin{aligned} &\overbrace{\begin{bmatrix} \mathbb{A}_{\mathbb{L}}^T\mathbb{P} + \mathbb{P}\mathbb{A}_{\mathbb{L}} + \mathbb{I}_n & -\Omega_1 + \mathbb{P}(\mathbb{E}_\omega - \mathbb{L}\mathbb{D}_\omega) \\ -\Omega_1^T + (\mathbb{E}_\omega - \mathbb{L}\mathbb{D}_\omega)^T\mathbb{P} & -\Omega_2 - \mu_\infty \mathbb{I}_q \end{bmatrix}}^{LINEAR} \\ &+ \sum_{i,j=1}^{i,j=n_f,n_f^i} \phi_{ij} \left(\overbrace{\begin{bmatrix} \mathbb{P}B_z\mathcal{H}_{ij} \\ \bar{D}_y\mathbb{P}B_z\mathcal{H}_{ij} \end{bmatrix}}^{\mathbb{X}_{ij}^T} \overbrace{[H_i \quad 0]}^{\mathbb{Y}_i} + \mathbb{Y}_i^T\mathbb{X}_{ij} \right) \\ &+ \sum_{i,j=1}^{i,j=n_h,n_h^i} \psi_{ij} \left(\overbrace{\begin{bmatrix} \mathbb{P}\mathbb{L}B_h\mathcal{F}_{ij} \\ \bar{D}_y\mathbb{P}\mathbb{L}B_h\mathcal{F}_{ij} \end{bmatrix}}^{\bar{\mathbb{X}}_{ij}^T} \overbrace{[-F_i \quad 0]}^{\bar{\mathbb{Y}}_i} + \bar{\mathbb{Y}}_i^T\bar{\mathbb{X}}_{ij} \right) \\ &+ \sum_{i,j=1}^{i,j=n_\kappa,n_\kappa^i} \varphi_{ij} \left(\overbrace{\begin{bmatrix} \mathbb{P}\bar{B}_\kappa\mathcal{T}_{ij} \\ \bar{D}_y\mathbb{P}\bar{B}_\kappa\mathcal{T}_{ij} \end{bmatrix}}^{\bar{\bar{\mathbb{X}}}_{ij}^T} \overbrace{[-T_i \quad 0]}^{\bar{\bar{\mathbb{Y}}}_i} + \bar{\bar{\mathbb{Y}}}_i^T\bar{\bar{\mathbb{X}}}_{ij} \right) \leq 0 \end{aligned} \quad (27)$$

where $\bar{B}_\kappa = Q_z B_h B_\kappa$, $\bar{D}_y = Q_z D_y$, $\mathbb{A}_{\mathbb{L}} = \mathbb{A} - \mathbb{L}C_\xi$ and

$$\Omega_1 = (\mathbb{E}_\omega - \mathbb{L}\mathbb{D}_\omega)^T\mathbb{P}Q_z D_y + \left(Q_z D_y\right)^T\mathbb{P}(\mathbb{E}_\omega - \mathbb{L}\mathbb{D}_\omega),$$

$$\Omega_2 = A_z^T Q_z D_y.$$

Using the Young's relation [14], we deduce that for all symmetric positive definite matrices $\mathbb{S}_{ij}$, $\mathbb{M}_{ij}$ and $\mathbb{N}_{ij}$ we have

$$\mathbb{X}_{ij}^T\mathbb{Y}_i + \mathbb{Y}_i^T\mathbb{X}_{ij} \leq \frac{1}{2}\left(\mathbb{X}_{ij} + \mathbb{S}_{ij}\mathbb{Y}_i\right)^T\mathbb{S}_{ij}^{-1}\overbrace{\left(\mathbb{X}_{ij} + \mathbb{S}_{ij}\mathbb{Y}_i\right)}^{\Delta_{ij}}, \quad (28)$$

$$\bar{\mathbb{X}}_{ij}^T\bar{\mathbb{Y}}_i + \bar{\mathbb{Y}}_i^T\bar{\mathbb{X}}_{ij} \leq \frac{1}{2}\left(\bar{\mathbb{X}}_{ij} + \mathbb{M}_{ij}\bar{\mathbb{Y}}_i\right)^T\mathbb{M}_{ij}^{-1}\overbrace{\left(\bar{\mathbb{X}}_{ij} + \mathbb{M}_{ij}\bar{\mathbb{Y}}_i\right)}^{\bar{\Delta}_{ij}}. \quad (29)$$

$$\bar{\bar{\mathbb{X}}}_{ij}^T\bar{\bar{\mathbb{Y}}}_i + \bar{\bar{\mathbb{Y}}}_i^T\bar{\bar{\mathbb{X}}}_{ij} \leq \frac{1}{2}\left(\bar{\bar{\mathbb{X}}}_{ij} + \mathbb{N}_{ij}\bar{\bar{\mathbb{Y}}}_i\right)^T\mathbb{N}_{ij}^{-1}\overbrace{\left(\bar{\bar{\mathbb{X}}}_{ij} + \mathbb{N}_{ij}\bar{\bar{\mathbb{Y}}}_i\right)}^{\bar{\bar{\Delta}}_{ij}}. \quad (30)$$

Finally, by following the methodology in [14], we get the following theorem.

Theorem 1: Assume that there exist symmetric positive definite matrices $\mathbb{P} \in \mathbb{R}^{n_\xi \times n_\xi}$, $\mathbb{S}_{ij} \in \mathbb{R}^{n_f^i \times n_f^i}$, $\mathbb{M}_{ij} \in \mathbb{R}^{n_h^i \times n_h^i}$, $\mathbb{N}_{ij} \in \mathbb{R}^{n_\kappa^i \times n_\kappa^i}$ and a matrix $\mathcal{R} \in \mathbb{R}^{n_y \times n_\xi}$ such that the following convex optimization problem is solvable:

$$\min(\mu_\infty) \; subject \; to \; (32) - (53) \quad (31)$$

$$\begin{bmatrix} Block & \overbrace{[\Sigma_1^f \quad \dots \quad \Sigma_m^f]}^{\Sigma_f} & \overbrace{[\Sigma_1^h \quad \dots \quad \Sigma_q^h]}^{\Sigma_h} \\ (*) & -\Lambda^f\mathbb{S} & 0 \\ (*) & (*) & -\Lambda^h\mathbb{M} \end{bmatrix} \leq 0 \quad (32)$$

$$Block = \begin{bmatrix} \mathbb{A}_{11} & (\mathbb{P}\mathbb{E}_\omega - \mathcal{R}^T\mathbb{D}_\omega - \Omega_1) \\ (*) & -(\Omega_2 + \Omega_2^T) - \mu_\infty \mathbb{I}_q \end{bmatrix} \quad (33)$$

$$\mathbb{A}_{11} = \mathbb{A}^T\mathbb{P} + \mathbb{P}\mathbb{A} - C_\xi^T\mathcal{R} - \mathcal{R}^TC_\xi + \mathbb{I}_{n_\xi} \quad (34)$$

$$\Sigma_i^f = \left[\mathcal{N}_{i,1}^f(\mathbb{P}, \mathbb{S}_{i1}) \quad \dots \quad \mathcal{N}_{i,n_f^i}^f(\mathbb{P}, \mathbb{S}_{in_f^i})\right] \quad (35)$$

$$\Sigma_i^h = \left[\mathcal{N}_{i,1}^h(\mathcal{R}, \mathbb{M}_{i1}) \quad \dots \quad \mathcal{N}_{i,n_h^i}^h(\mathcal{R}, \mathbb{M}_{in_h^i})\right] \quad (36)$$

$$\Sigma_i^\kappa = \left[\mathcal{N}_{i,1}^\kappa(\mathbb{P}, \mathbb{N}_{i1}) \quad \dots \quad \mathcal{N}_{i,n_\kappa^i}^\kappa(\mathbb{P}, \mathbb{N}_{in_\kappa^i})\right] \quad (37)$$

$$\mathcal{N}_{i,j}^f(\mathbb{P}, \mathbb{S}_{ij}) = \begin{bmatrix} \mathbb{P}B_z\mathcal{H}_{ij} \\ \bar{D}_y\mathbb{P}B_z\mathcal{H}_{ij} \end{bmatrix} + \begin{bmatrix} H_i^T\mathbb{S}_{ij} \\ 0 \end{bmatrix} \quad (38)$$

$$\mathcal{N}_{i,j}^h(\mathcal{R}, \mathbb{M}_{ij}) = \begin{bmatrix} \mathcal{R}^TB_h\mathcal{F}_{ij} \\ \bar{D}_y\mathcal{R}^TB_h\mathcal{F}_{ij} \end{bmatrix} + \begin{bmatrix} F_i^T\mathbb{M}_{ij} \\ 0 \end{bmatrix} \quad (39)$$

$$\mathcal{N}_{i,j}^\kappa(\mathbb{P}, \mathbb{N}_{ij}) = \begin{bmatrix} \mathbb{P}\bar{B}_\kappa\mathcal{T}_{ij} \\ \bar{D}_y\mathbb{P}\bar{B}_\kappa\mathcal{T}_{ij} \end{bmatrix} + \begin{bmatrix} T_i^T\mathbb{N}_{ij} \\ 0 \end{bmatrix} \quad (40)$$

$$\Lambda^f = block - diag(\Lambda_1^f, \dots, \Lambda_{n_f}^f) \quad (41)$$

$$\Lambda_i^f = block - diag(\frac{2}{b_{i1}}\mathbb{I}_{n_f^i}, \dots, \frac{2}{b_{in_f^i}}\mathbb{I}_{n_f^i}) \quad (42)$$

$$\mathbb{S} = block - diag(\mathbb{S}_1, \dots, \mathbb{S}_{n_f}) \quad (43)$$

$$\mathbb{S}_i = block - diag(\overbrace{\mathbb{S}_{i1}, \dots, \mathbb{S}_{in_f^i}}^{n_f^i \; times}) \quad (44)$$

$$\Lambda^h = block - diag(\Lambda_1^h, \dots, \Lambda_{n_h}^h) \quad (45)$$

$$\Lambda_i^h = block - diag(\frac{2}{d_{i1}}\mathbb{I}_{n_h^i}, \dots, \frac{2}{d_{in_h^i}}\mathbb{I}_{n_h^i}) \quad (46)$$

$$\mathbb{M} = block - diag(\mathbb{M}_1, \dots, \mathbb{M}_{n_h}) \quad (47)$$

$$\mathbb{M}_i = block - diag(\overbrace{\mathbb{M}_{i1}, \dots, \mathbb{M}_{in_h^i}}^{n_h^i \; times}) \quad (48)$$

$$\Lambda^\kappa = block - diag(\Lambda_1^\kappa, \dots, \Lambda_{n_f}^\kappa) \quad (49)$$

$$\Lambda_i^\kappa = block - diag(\frac{2}{f_{i1}}\mathbb{I}_{n_\kappa^i}, \dots, \frac{2}{f_{in_\kappa^i}}\mathbb{I}_{n_\kappa^i}) \quad (50)$$

$$\mathbb{N} = block - diag(\mathbb{N}_1, \dots, \mathbb{N}_{n_\kappa}) \quad (51)$$

$$\mathbb{N}_i = block - diag(\overbrace{\mathbb{N}_{i1}, \dots, \mathbb{N}_{in_\kappa^i}}^{n_\kappa^i \; times}) \quad (52)$$

$$A_z = P_zA_\xi - \mathbb{L}C_\xi \quad (53a)$$

$$A_y = \mathbb{L} + A_zQ_z \quad (53b)$$

$$B_z = P_zB_f \quad (53c)$$

$$G_z = P_zG_g \quad (53d)$$

Then, the $\mathcal{H}_\infty$ criterion (8) is satisfied with $\nu_\infty = \lambda_{max}(\mathbb{P})$. The observer gain parameter $\mathbb{L}$ is then computed by

$$\mathbb{L} = \mathbb{P}^{-1}\mathcal{R}^T. \quad (54)$$

IV. Application to Unknown Input Estimation

This section is devoted to unknown input estimation for nonlinear systems with nonlinear outputs. The aim is to use the results of the previous section to estimate, in the $\mathcal{H}_\infty$ sense, unknown inputs occurring in the system. The class of systems concerned by this study is described by the following nonlinear equations:

$$\begin{cases} \dot{x} = A_xx + A_\mu\mu + B_\sigma\sigma(x,\mu,u) \\ \qquad + G_\varrho\varrho(y,u) + D_x\omega \\ y = C_xx + C_\mu\mu + B_\varsigma\varsigma(x,\mu,u) + \bar{D}_y\omega \end{cases} \quad (55)$$

where $x(t) \in \mathbb{R}^{n_x}$ is the system state and $\mu(t) \in \mathbb{R}^{n_\mu}$ is the unknown input vector. The nonlinearities $\sigma(.)$ and $\varsigma(.)$

satisfy the global Lipschitz assumption as in the previous section.

We will propose some schemes to estimate simultaneously the system state $x(t)$ and the unknown input $\mu(t)$. In each estimation scheme, we rewrite system (55) under the descriptor form (1) under specific assumptions on the parameters of (55), namely the matrices A_μ, C_μ, B_ς, C_x, and the nonlinear function $\varsigma(x,u)$. There are several ways to organize the presentation of all the possible scenarios due to different assumptions required in each scenario. In this initial conference paper, we will only investigate the specific case of:

$$rank(C_\mu) = n_\mu \tag{56}$$

For this specific case, LMI conditions ensuring the simultaneous asymptotic estimation of the states and the unknown inputs will be provided.

System (55) can be rewritten below

$$E_\xi = [\mathbb{I}_{n_x} \quad 0_{n_x \times n_\mu}], \qquad A_\xi = [A_x \quad A_\mu], \tag{57a}$$

$$C_\xi = [C_x \quad C_\mu], \qquad \xi = \begin{bmatrix} x \\ \mu \end{bmatrix}, \tag{57b}$$

$$\begin{gathered} B_f = B_\sigma, \qquad G_g = G_\varrho, \qquad B_h = B_\varsigma, \\ D_\xi = D_x, \qquad D_y = \bar{D}_y, \end{gathered} \tag{57c}$$

$$\begin{gathered} f(\xi,u) = \sigma(x,\mu,u), \\ h(\xi,u) = \varsigma(x,\mu,u), \\ g(y,u) = \varrho(y,u). \end{gathered} \tag{57d}$$

Since the matrix C_μ is full column rank then it follows that the condition (2) holds. Indeed,

$$\begin{aligned} rank\left(\begin{bmatrix} E_\xi \\ C_\xi \end{bmatrix}\right) &= rank\left(\begin{bmatrix} \mathbb{I}_{n_x} & 0_{n_x \times n_\mu} \\ C_x & C_\mu \end{bmatrix}\right) \\ &= n_x + n_\mu = n_\xi. \end{aligned} \tag{58}$$

Hence the LMI design techniques given in Section III can be applied under additional assumptions.

Under Assumption 3.1, the observer based filter (13) provides a simultaneous estimation of the state $x(t)$ and the unknown input $\mu(t)$ in the $\mathcal{H}_\infty$ sense defined in (8) provided that the filter parameters are obtained by solving the convex optimization problem (31) in Theorem 1 according to the definitions (57).

V. Conclusion

In this note we have proposed a new LMI design technique that satisfies a $\mathcal{H}_\infty$ performance criterion for a class of descriptor nonlinear systems. The presence of nonlinearities in the output measurements renders the synthesis problem particularly complicated. To overcome this obstacle, we have proposed a different LMI method, where the method requires a certain assumption on system parameters involving the numbers of measurements, unknown inputs and nonlinear functions in the output equations. We have shown that the introduction of these new assumptions allows handling the problem of observer synthesis. New and less conservative LMI conditions for observer design are proposed.

As future work, we aim to apply this technique to complicated real-word models, such as the tripped rollover problem and radar vehicle tracking problems in automotive applications.

References

[1] Y. Wang, R. Rajamani, and D. Bevly, "Observer design for differentiable lipschitz nonlinear systems with time-varying parameters," in *53th IEEE Conference on Decision and Control*, Los Angeles, CA, USA, December, 2014.

[2] A. Zemouche and M. Boutayeb, "On LMI conditions to design observers for Lipschitz nonlinear systems," *Automatica*, vol. 49, no. 2, pp. 585–591, 2013.

[3] M. Oueder, M. Farza, R. B. Abdennour, and M. M'Saad, "A high gain observer with updated gain for a class of MIMO non-triangular systems," *Systems & Control Letters*, vol. 61, no. 2, pp. 298–308, 2012.

[4] V. Andrieu, L. Praly, and A. Astolfi, "High gain observers with updated gain and homogeneous correction terms," *Automatica*, vol. 45, no. 2, pp. 422–428, 2009.

[5] M. Darouach, "Observers for Lipschitz nonlinear descriptor systems: Application to unknown inputs systems," *IEEE Transactions on Automatic Control*, vol. 54, no. 8, pp. 1966–1972, 2009.

[6] J. Tsinias, "Time-varying observers for a class of nonlinear systems," *Systems & Control Letters*, vol. 57, no. 12, pp. 1037–1047, 2008.

[7] A. Alessandri, "Design of observers for Lipschitz nonlinear systems using LMI," in NOLCOS, *IFAC Symposium on Nonlinear Control Systems*, Stuttgart, Germany, 2004.

[8] J. P. Gauthier, H. Hammouri, and S. Othman, "A simple observer for nonlinear systems, applications to bioreactors," *IEEE Transactions on Automatic Control*, vol. 37, no. 6, pp. 875–880, 1992.

[9] M. Alma and M. Darouach, "Adaptive observers design for a class of linear descriptor systems," *Automatica*, vol. 50, no. 2, pp. 578 – 583, 2014.

[10] Q. P. Ha and H. Trinh, "State and input simultaneous estimation for a class of nonlinear systems," *Automatica*, vol. 40, pp. 1779–1785, 2004.

[11] A. Zemouche and M. Boutayeb, "Sobolev norms-based state estimation and input recovery for a class of nonlinear systems. Design and experimental results," *IEEE Transactions on Signal Processing*, vol. 57, no. 3, pp. 1021–1029, 2009.

[12] A. Zemouche and M. Boutayeb, "Nonlinear observer-based $\mathcal{H}_\infty$ synchronization and unknown input recovery," *IEEE Transactions on Circuits and Systems I*, vol. 56, no. 8, pp. 1720–1731, 2009.

[13] M. Darouach, M. Zasadzinski, and M. Hayar, "Reduced-order observer design for descriptor systems with unknown inputs," *IEEE transactions on automatic control*, vol. 41, no. 7, pp. 1068–1072, 1996.

[14] A. Zemouche, R. Rajamani, B. Boulkroune, H. Rafaralahy, and M. Zasadzinski, "$\mathcal{H}_\infty$ circle criterion observer design for Lipschitz nonlinear systems with enhanced LMI conditions," in *IEEE American Control Conference*, Boston, MA, USA., July 2016.

[15] H. Shim, Y. Son, and J. Seo, "Semi-global observer for multi-output nonlinear systems," *Systems and Control Letters*, vol. 43, 2001.

[16] M. Darouach and L. Boutat-Baddas, "Observers for a class of nonlinear singular systems," *IEEE Transactions on Automatic Control*, vol. 53, no. 11, pp. 2627–2633, 2008.

[17] L. Hassan, A. Zemouche, and M. Boutayeb, "$\mathcal{H}_\infty$ observer design for a class of nonlinear time-delay systems in descriptor form," *International Journal of Control*, vol. 84, no. 10, pp. 1653–1663, 2011.

[18] L. Hassan, A. Zemouche, and M. Boutayeb, "Robust unknown input observers for nonlinear time-delay systems," *SIAM Journal on Control and Optimization*, vol. 51, no. 4, pp. 2735–2752, 2013.

[19] M. Hou and P. C. Muller, "Design of observers for linear systems with unknown inputs," *IEEE Transactions on Automatic Control*, vol. 37, no. 6, pp. 871–875, 1992.

[20] R. Rajamani and Y.M. Cho. "Existence and design of observers for nonlinear systems: relation to distance to unobservability." *International Journal of Control* 69, no. 5 (1998): 717-731.