



Hydrodynamics of elastic micro-filaments : model comparison and applications

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Hydrodynamics of elastic micro-filaments : model comparison and applications

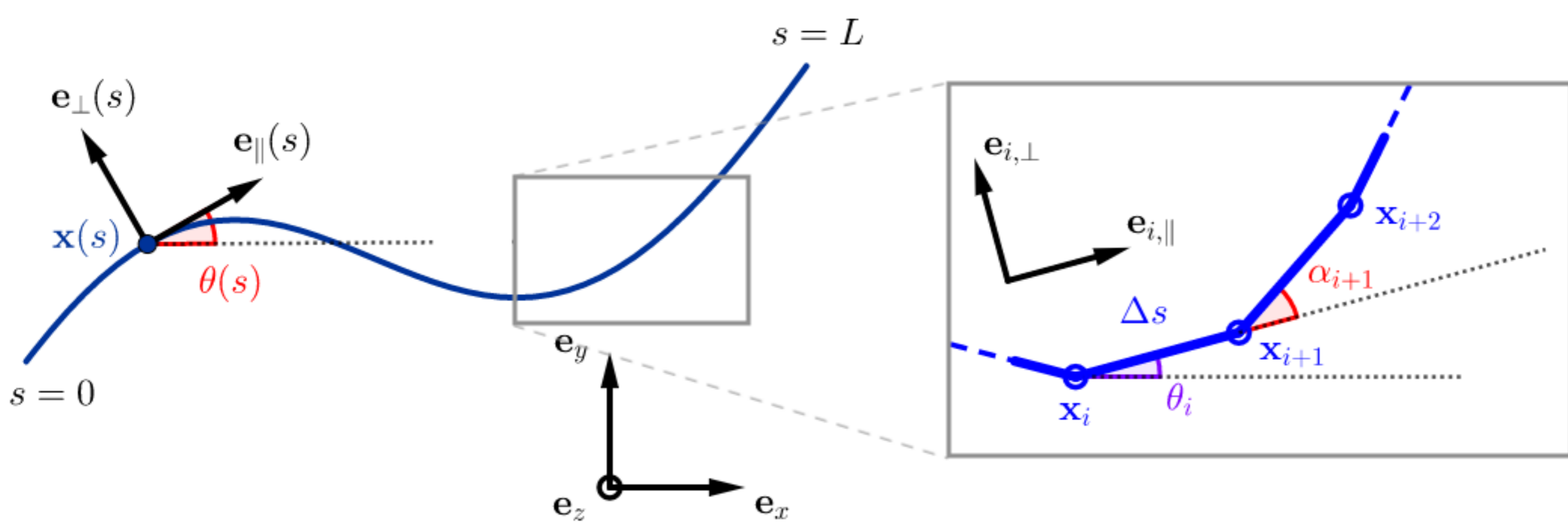
Clément Moreau, Laetitia Girdali, and Hermes Gadêlha

Motivations

- ▶ Efficiently model and simulate the dynamics of **inextensible micro-filaments** in 2D
- ▶ Deal with the coupling between elastic and hydrodynamic interactions
- ▶ Propose a model that is
 - ▷ **simple** to implement
 - ▷ numerically **robust**
 - ▷ **adaptable** for a range of applications

Modeling

- ▶ We consider an **inextensible flexible filament** immersed in a fluid at **low Reynolds number** → inertia is negligible, viscous effects dominate: **Stokes regime**



Parametrisation for the continuous and coarse-grained models

- ▶ **Hydrodynamics**: segment force density (Resistive Force Theory [2])

$$f_i(s) = \eta_{\parallel}(\mathbf{v}_i(s) \cdot \mathbf{e}_{i,\parallel})\mathbf{e}_{i,\parallel} + \eta_{\perp}(\mathbf{v}_i(s) \cdot \mathbf{e}_{i,\perp})\mathbf{e}_{i,\perp}$$

- ▶ **Elasticity**: linear model

$$\mathbf{M}_i^{\text{el}} = \kappa(\theta_{i+1} - \theta_i)\mathbf{e}_z$$

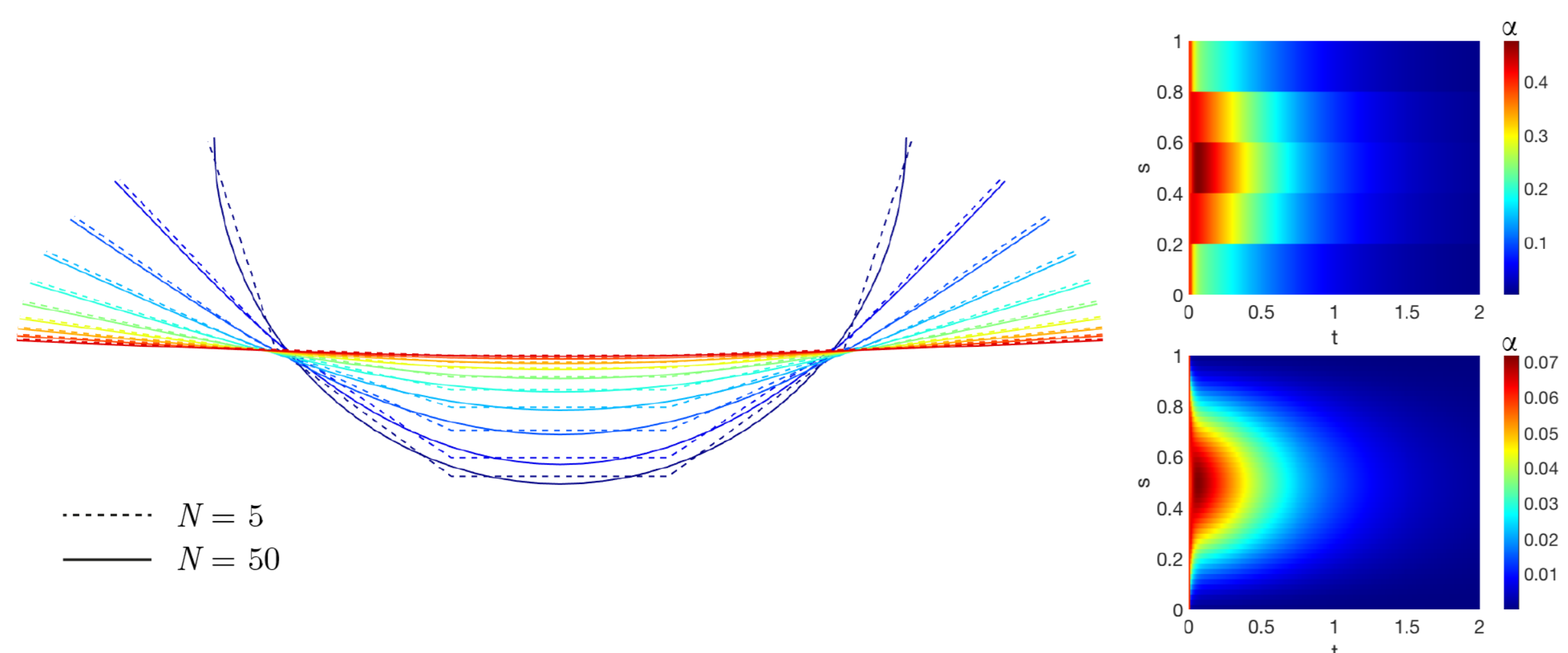
- ▶ 2nd Newton law gives the balance of forces and torques :

▷ Continuous model → **PDE system**

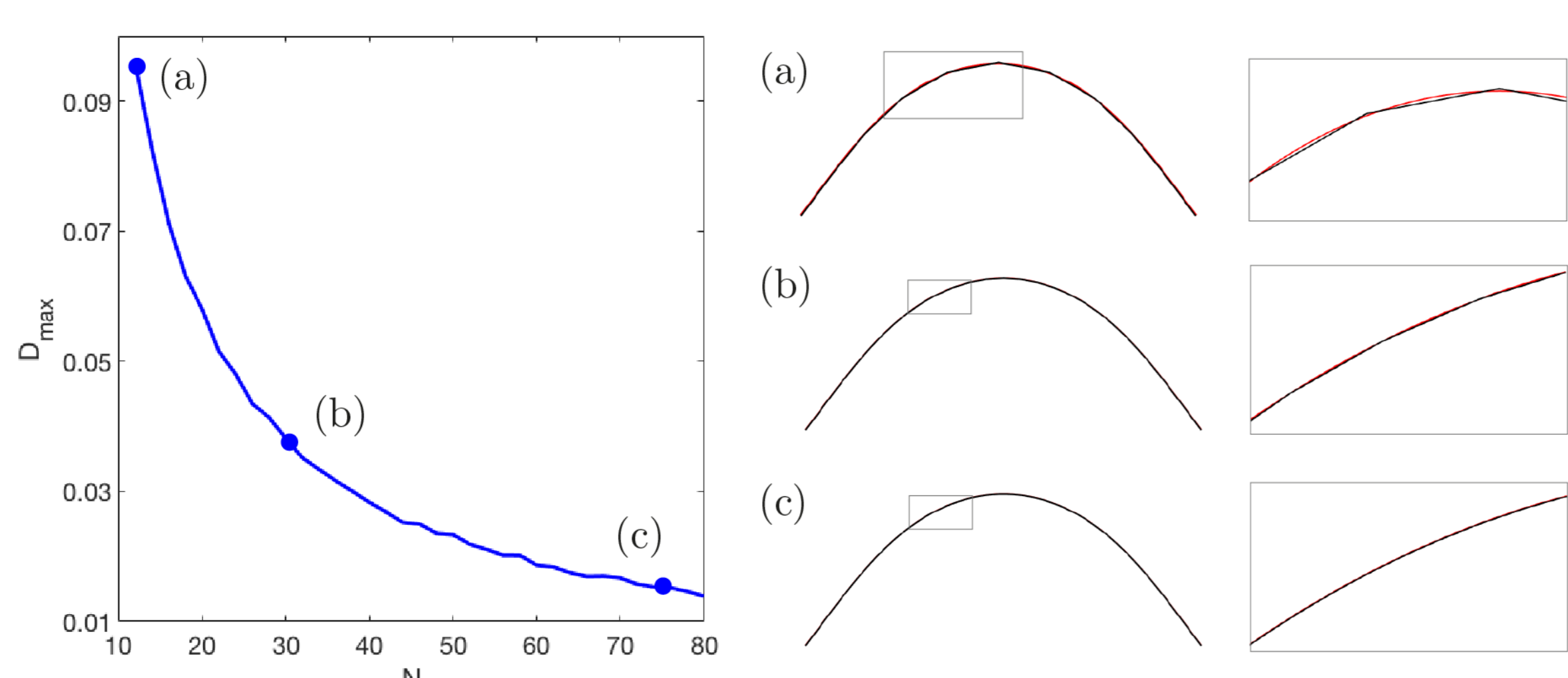
▷ Discrete model → integration on the segments → **ODE system**

Numerical Comparison

- ▶ Similar behavior on the “relaxation test”



- ▶ Fast convergence of the discrete model (error criterion $D_{\text{max}} = \max_{s,t} |\mathbf{x}_c - \mathbf{x}_{\text{cg}}|$)

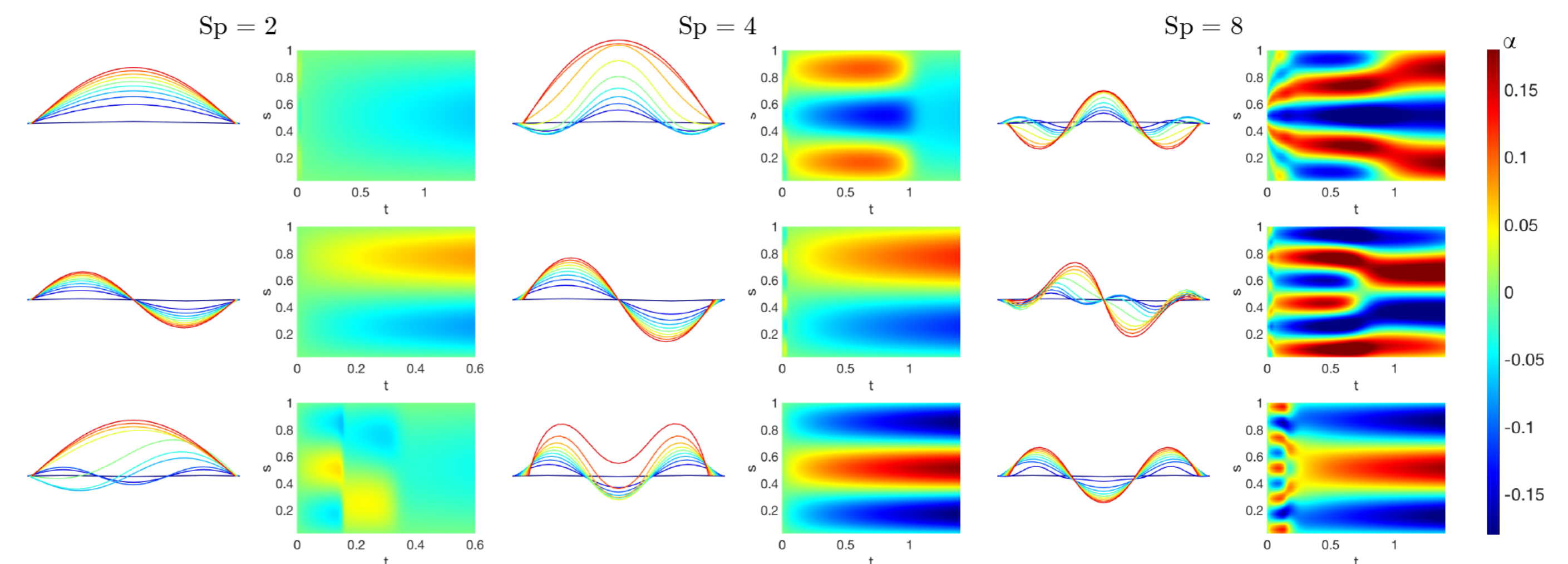


Comparison between the classical and coarse-grained systems for increasing number of segments.

- ▶ Discrete approach up to **100 times faster** than PDE approach
- ▶ **More robust** for rigid filaments and sharp curvatures

Applications : Buckling

- ▶ Filament submitted to **compression**
- ▶ Triggering different modes by changing initial condition



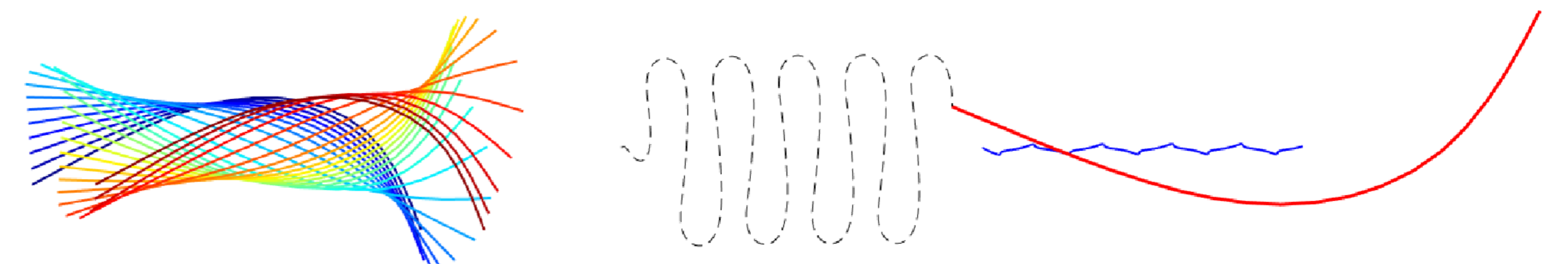
Visualization of the buckling phenomenon for three different rigidities and three different initial conditions.

- ▶ Study made easier by the discrete approach, under investigation

Applications : Magnetic Swimmers

- ▶ **Magnetised filament** – magnetisations μ_i
- ▶ External magnetic field \mathbf{H} creating a torque

$$\mathbf{T}_i^m = \mu_i(\mathbf{e}_{i,\parallel} \times \mathbf{H})$$

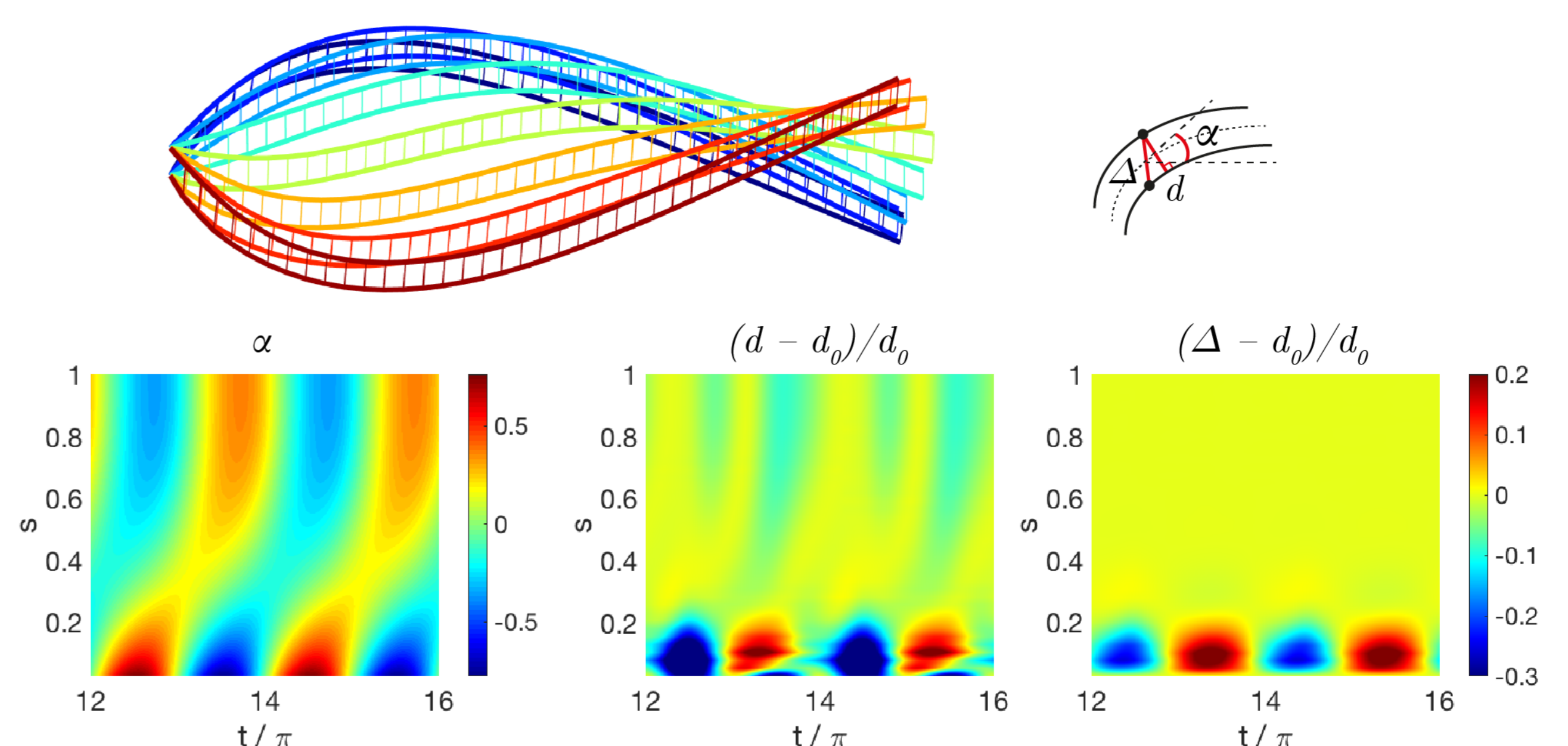


Example of magnetic drive with a sinusoidal orthogonal magnetic field.

- ▶ **Artificial self-propulsion** at micro-scale [3]
- ▶ Controllability and **optimal control**

Applications : Filament Bundle

- ▶ Model a **biological flagellum** through two filaments linked together
- ▶ Highlight non-trivial coupling phenomena



Coupling between two filaments obtained with the coarse-grained approach.

- ▶ Applications include sperm cell motility studies

References and Contact Information

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