Risk models with premiums adjusted to claims number

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Abstract

Classical compound Poisson risk models always consider the premium rate to be constant. By adjusting the premium rate to the history of claims, one can emulate a Bonus-Malus system within the ruin theory context. One way to implement such adjustment is by considering the Poisson parameter to be a continuous random variable and use credibility theory arguments to adjust the premium rate aposteriori. Depending on the possibility of this random variable to have a mass at zero or not, respectively referred to as latent versus historical claims, one obtains different relations between the ruin probability with constant versus adjusted premium rates. A combination of these two kinds of claims leads to a relation between the two ruin probabilities, when the aposteriori estimator of the number of claims is carefully chosen. Examples for specific claim sizes are presented throughout the paper.

Keywords. Ruin Probability, Mixed Poisson Process, Bonus-Malus, Bayesian Estimation, Lukacs' Theorem.

1 Introduction

One of the main assumptions of the classical collective risk models is that premiums are arriving at a constant rate c and thus the surplus of the company evolves over time as

$$U(t) = u + ct - \sum_{j=0}^{N(t)} Y_j, \quad t \geq 0, \tag{1}$$

where u is the initial capital, Y_j are the claim sizes (iidrv) arriving according to a Poisson process $N(t)$ with intensity λ. Inspired by the merit rating feature of a Bonus-Malus system,

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premium rates adjusted according to claim histories have been first introduced in a risk theory context by Buhlmann [4] and further by Dubey [8] as

$$U(t) = u + c\int_0^t \hat{\lambda}(s)\mathrm{d}s - \sum_{j=0}^{N(t)} Y_j, \quad t \geq 0. \tag{2}$$

Hence, instead of a constant premium rate as shown in the classical collective risk process, Dubey [8] adopted an adjustment of premium levels by randomising the expected number of claims overtime. More precisely, the intensity of the classical Poisson process is a random variable Λ, that is estimated based on the history of claims. Note that when $\mathbb{P}(\Lambda > 0) = 1$, the compound Poisson process is referred to as mixed Poisson process. Moreover, when Λ follows a Gamma distribution, the credibility estimator constitutes the basis for pricing Bonus-Malus systems in the Swiss liability car insurance.

Furthermore, Dubey [8] was able to show that for $\mathbb{P}(\Lambda = 0) = 0$, in other words $\mathbb{P}(\Lambda > 0) = 1$, and for a Bayesian estimator $\hat{\lambda}(t)$,

$$\psi(u) = \psi_0(u), \tag{3}$$

where ψ describes the probability of ruin (ruin being the first time the surplus process crosses zero) in a mixed Poisson model (2), whereas ψ_0 is the ruin probability in the classical Poisson model (1), with intensity one.

In the same setup of premium rates varying according to already known information about frequency of claims, we conduct a risk analysis in situations where the possibility of no-claims is allowed. We refer to these claims with a non-zero possibility of never happening as 'latent risks,' as opposed to 'historical risks', with zero probability of no claims. 'Latent claims' refer to claims that are not known to the insurer when signing a policy but bear the potential of causing claims many years later. One existing example is asbestos, which have led to a burst of related diseases and thus an increase in claims to be paid. In recent years, these emerging risks are always included in the models. We are presenting here one way to account for that, since the ever increasing awareness of legal rights and standards of healthcare raises the possibilities of the unaccounted for claims occurring in the future.

We show that, even if $\mathbb{P}(\Lambda = 0) = p > 0$, one can establish a relationship between the ruin probability in a model with adjusted premiums, versus the one with constant rates (see Theorem 6)

$$\psi(u) = \psi_0(u) - p\psi_0\left(u + c\ln\frac{1}{p}\right). \tag{4}$$

Thus, Dubey [8] found that the ruin probability for the adjusted model does not differ from a classical case when only historical risks are considered, whereas if all the claims are latent, the relationship changes as in (4). However, the most realistic scenario is a combination of both classical and latent claims. In order to combine the above two cases we need a well chosen estimator for the intensity of this number of claims process. The estimator we need is defined as

$$\hat{\lambda}(t) = \mathbb{E}[\Lambda^{(1)} + \Lambda^{(2)} | N(t)],$$

where $\mathbb{P}(\Lambda^{(1)} = 0) = 0$ and $\mathbb{P}(\Lambda^{(2)} = 0) = p > 0$. In this case, the relation between the ruin probability of the adjusted premium model ψ versus the classical one ψ_0 is more elaborated, and can be derived only when $\Lambda^{(1)} \sim \Gamma(\alpha, \lambda_0)$ and $\Lambda^{(2)}|_{\Lambda^{(2)}>0} \sim \Gamma(\beta, \lambda_0)$ for some $\alpha, \beta, \lambda_0 > 0$,

$$\psi(u) = \frac{1-p}{B(\alpha, \beta)} \int_{(0,1)} \psi_\theta(u) \theta^{\alpha-1}(1-\theta)^{\beta-1}\, d\theta + p \cdot \psi_1(u).$$

where $B(\alpha, \beta)$ is a Beta function and $\psi_\theta(u)$ is the conditional probability of ruin defined by

$$\psi_\theta(u) = P\left(\tau < \infty \middle| U(0) = u, \frac{\Lambda^{(1)}}{\Lambda^{(1)} + \Lambda^{(2)}} = \theta\right).$$

Apart from [8], the adjustment on premium rates in a risk model has actually been extensively studied in literature. Several criteria were employed for premiums to be adjusted accordingly. The majority of papers studied a premium function which varies with the current risk reserve, i.e., premium denoted as a function of $U(t)$. An introduction of earlier studies on this topic is addressed in Chapter VIII [3], where the premium rate is a discrete function responding to the level of the risk process. With respect to the randomisation of the Poisson paratemeter, or we would now call it 'parameter risk'[5], the exploration of which in literature dates back to as early as 1948 ([11], [2]), where either the randomness at the beginning of the process or that varying over time were considered.

While their work considered parameter risks only, as already mentioned before, [4] assumed Gamma distribution for the Poisson parameter and additionally introduced a model where premiums are adjusted stochastically based on the claim experience to date. Furthermore, [8] followed [4] employing the Bayesian estimation to describe premium adjustment but for a general distribution of the Poisson parameter. Apart from this, [8] way of estimating the claim intensity can be understood as a moving average of claim frequencies overtime. This approach was recently extended from a Poisson process to any operational time by [6]. Another relevant recent development on this topic comes from [9] who obtained the ruin probability of a surplus process where the premium rate is a function of the occurence of claims and the arrival of claims is represented by a Cox process.

This paper uses Dubey's[8] setup, incorporating the novelty of latent claims. It presents the probability of ruin in novel premium adjusted models in terms of the classical one. The paper is organised as follows. Section 2 provides a detailed proof of obtaining the underlying ruin probability for a model with only latent risks considered. The case of latent combined with historical claims is found in Section 3, where main lemmas and results are presented. Alternative representations using Pollaczek-Khinchin formula and an example for exponential claims are derived. Finally conclusions is Section 4. Examples and illustrations for various claim scenarios are presented throughout the paper.

To avoid confusion, a ruin probability is defined as follows throughout the paper,

$$\psi(u) = \mathbb{P}\left(\inf_{t\geq 0} U(t) < 0 | U(0) = u\right) = \mathbb{P}(\tau < \infty | U(0) = u), \tag{5}$$

where $\tau = \inf\{t > 0 : U(t) < 0\}$ denotes the time of ruin and $u \geq 0$.

2 Ruin Probability with only Latent Risks

Consider the model (2) with latent claims only.

Theorem 2.1. *The probability of ruin of the adjusted surplus process* (2) *with Poisson intensity* Λ*,* $\mathbb{P}(\Lambda = 0) = p > 0$*, is given by*

$$\psi(u) = \psi_0(u) - p\psi_0\left(u + c\ln\frac{1}{p}\right), \tag{6}$$

where ψ_0 *denotes the ruin probability in a classical risk model with jump intensity 1.*

Proof. The main difference from Dubey here is that Λ has a mass at $\{0\}$ which led us to consider $\{\Lambda = 0\}$ and $\{\Lambda > 0\}$ separately. Due to the fact that no claims are expected to be attributed to the risk process on the set $\{\Lambda = 0\}$, then recall (5) we can write,

$$\mathbb{P}(\tau < \infty | U(0) = u) = \mathbb{P}(\tau < \infty, \Lambda > 0 | U(0) = u) = \mathbb{P}(\tau < \infty | U(0) = u, \Lambda > 0)\mathbb{P}(\Lambda > 0). \tag{7}$$

Therefore, only the study under the measure $\mathbb{P}(\cdot|\Lambda > 0)$ is essential to be taken into consideration here.

Let $T_i, i = 1, \ldots, n$ represent the arrival time of the i^{th} claim with the convention that $T_0 = 0$, and $P_i, i = 1, \ldots, n$ denote the premium collected inbetween the $(i-1)^{th}$ and the i^{th} claim. Similar to Dubey [8], we first analyse the conditional distribution of the sequence of premiums $\{P_n, n \geq 1\}$. From his definition, we have $P_1 = \ln\{V(0)/V(T_1)\}$ and

$$P_{n+1} = \ln\{V^{(n)}(T_n)/V^{(n)}(T_{n+1})\},\ n \geq 1,$$

where $V(x) = \mathbb{E}(e^{-\Lambda x})$. On the one hand, adopting very similar steps in [8] yields for $n \geq 1$,

$$\mathbb{P}(P_{n+1} \geq x | T_n = y, \Lambda > 0) = e^{-x},\ n \geq 1.$$

On the other hand, regarding the distribution of P_1, it could be first noticed that $V(x)$ is a continuous and decreasing function and $V(x) \in (p, 1]$ since,

$$V(x) = \int e^{-\lambda x} dS(\lambda) = \int_{(0,+\infty)} e^{-\lambda x} dS(\lambda) + \mathbb{P}(\Lambda = 0),$$

and

$$\lim_{x\to 0} V(x) = 1; \quad \lim_{x\to\infty} V(x) = \mathbb{P}(\Lambda = 0) = p. \tag{8}$$

Hence, for $x \leq -\ln p$,

$$\begin{aligned} & \mathbb{P}(P_1 \geq x|\Lambda > 0) = \mathbb{P}(V(T_1) \leq e^{-x}|\Lambda > 0) \\ = \;& \mathbb{P}(T_1 \geq V^{-1}(e^{-x})|\Lambda > 0) = \int_{(0,+\infty)} e^{-\lambda V^{-1}(e^{-x})} \cdot \mathbb{P}(\Lambda \in d\lambda|\Lambda > 0) \\ = \;& \frac{1}{1-p}(V(V^{-1}(e^{-x})) - p) = \frac{e^{-x} - p}{1-p}. \end{aligned}$$

That is to say, under the measure $\mathbb{P}(\cdot|\Lambda > 0)$, the sequence $\{P_n, n \geq 2\}$ again follows an exponential distribution with parameter 1 and P_1 conforms to a truncated exponential distribution.

Furthermore, we study the conditional indpendence structure for the sequence of premiums under the measure $\mathbb{P}(\cdot|\Lambda > 0)$. As presented in [8], recall that $\{\Lambda > 0\} = \{T_1 < \infty\} = \{T_k < \infty\}$ for every $k \geq 1$, and $P_{n+1} = \ln\{V^{(n)}(T_n)/V^{(n)}(T_{n+1})\}$ on $\{T_n < \infty\}$. Therefore, when their joint Laplace Transform is considered, we have for $n \geq 1$,

$$\begin{aligned} & \mathbb{E}\left[e^{-\sum_{i=1}^{n+1} s_i P_i}\Big|\Lambda > 0\right] = \mathbb{E}\left[\mathbb{E}\left(e^{-\sum_{i=1}^{n+1} s_i P_i}\Big|T_1, T_2, \cdots, T_n, \Lambda > 0\right)\right] \\ = \;& \mathbb{E}\left\{\mathbb{E}\left[e^{-\sum_{i=1}^{n} s_i P_i} \cdot \mathbb{E}\left(e^{-s_{n+1} P_{n+1}}\big|T_1, T_2, \cdots, T_n, \Lambda > 0\right)\Big|T_1, T_2, \cdots, T_n, \Lambda > 0\right]\right\} \\ = \;& \mathbb{E}\left\{\mathbb{E}\left[e^{-\sum_{i=1}^{n} s_i P_i} \cdot \mathbb{E}\left(e^{-s_{n+1} P_{n+1}}\big|T_n, \Lambda > 0\right)\Big|T_1, T_2, \cdots, T_n, \Lambda > 0\right]\right\} \\ = \;& \frac{1}{1+s_{n+1}}\mathbb{E}\left[\mathbb{E}\left(e^{-\sum_{i=1}^{n} s_i P_i}\Big|T_1, T_2, \cdots, T_n, \Lambda > 0\right)\right] \\ = \;& \frac{1}{1+s_{n+1}}\mathbb{E}\left[e^{-\sum_{i=1}^{n} s_i P_i}\Big|\Lambda > 0\right], \end{aligned}$$

thus proving the indpendence structure of $\{P_n, n \geq 1\}$ through induction.

As a result, following the argument of renewal theory, the conditional probability of ruin is given by,

$$\mathbb{P}(\tau < \infty|U(0) = u, \Lambda > 0) = \int_0^{ln(1/p)} \frac{1}{1-p} e^{-t} \left(\int_0^{u+ct} \psi_0(u + ct - y) dF(y) + \overline{F}(u + ct)\right) dt,$$

where $\psi_0(u)$ is

$$\psi_0(u) = \int_0^\infty e^{-t}\left(\psi_0 * F(u+ct) + \overline{F}(u+ct)\right) dt.$$

Eventually, following the identity above we are able to write

$$\begin{aligned}
\psi(u) &= \int_0^\infty e^{-t}\left(\psi_0 * F(u+ct) + \overline{F}(u+ct)\right)\, dt - \int_{ln(1/p)}^\infty e^{-t}\left(\psi_0 * F(u+ct) + \overline{F}(u+ct)\right)\, dt \\
&= \psi_0(u) - \int_0^\infty e^{-(x+\ln\frac{1}{p})}\left(\psi_0 * F(u+c(x+\ln\frac{1}{p})) + \overline{F}(u+c(x+\ln\frac{1}{p})\right)\, dx \\
&= \psi_0(u) - p\psi_0(u+c\ln(1/p)).
\end{aligned}$$

This completes the proof. □

Now we apply this formulae to calculate ruin probabilities for specific claim distributions including Dubey's [8] examples.

Example 2.1. *Y_i follows an exponential distribution with $\mathbb{E}(Y_j) = 1$. A classical risk model (with jump intensity 1) gives an explicit ruin function, $\psi_0(u) = \frac{1}{c}exp\left(-\frac{c-1}{c}u\right)$. Substituting this into (6) yieds,*

$$\psi(u) = \frac{1}{c}e^{-\frac{c-1}{c}u} - \frac{p}{c}e^{-\frac{c-1}{c}\left(u+c\ln\frac{1}{p}\right)} = (1-p^c)\psi_0(u).$$

Example 2.2. *$Y_j = 1$. An approximation has been shown in classical models, $\psi_0(u) \sim \frac{c-1}{1+cr-c}e^{-ru}$, where r is the positive solution to $e^x = 1+cx$. Applying (6) with the above identity also verifies our result.*

$$\psi(u) \sim \frac{c-1}{1+cr-c}e^{-ru} - p\frac{c-1}{1+cr-c}e^{-r\left(u+c\ln\frac{1}{p}\right)} \sim (1-p^{cr+1})\psi_0(u),$$

when $u \to \infty$ with r the same as above.

Example 2.3. *$Y_j \sim Gamma(\frac{m}{n}, \alpha)$ with density function $f_Y(x) = \frac{\alpha^{\frac{m}{n}}}{\Gamma(\frac{m}{n})}x^{-\frac{m}{n}}e^{-\alpha x}, x \geq 0$. It is worth emphasising that $\frac{m}{n}$ taking integer values also covers the case of Erlang distributed claims. Employing recent results from [12], the required ruin probability is demonstrated in the following equations.*

$$\begin{aligned}
\psi(u) &= 1 - p - e^{-\alpha u}u^{\frac{1}{n}-1}\sum_{k=0}^{m+n-1} m_k E_{\frac{1}{n},\frac{1}{n}}\left(s_k u^{\frac{1}{n}}\right) \\
&\quad + pe^{-\alpha\left(u+c\ln\frac{1}{p}\right)}\left(u+c\ln\frac{1}{p}\right)^{\frac{1}{n}-1}\sum_{k=0}^{m+n-1} m_k E_{\frac{1}{n},\frac{1}{n}}\left(s_k\left(u+c\ln\frac{1}{p}\right)^{\frac{1}{n}}\right),
\end{aligned}$$

where $E_{\frac{1}{n},\frac{1}{n}}\left(s_k u^{\frac{1}{n}}\right) = \sum_{i=0}^\infty \frac{\left(s_k u^{\frac{1}{n}}\right)^i}{\Gamma\left(\frac{k+1}{n}\right)}$.

All these examples calculate ruin probabilities for a risk model when only the latent risks are present through connection with classical results. We will derive a similar approach to obtain ruin probabilities while both historical and latent risks are taken into account in the next section.

3 Ruin Probability with Both Historical and Latent Risks

Inspired by Dubey's results, we are proposing a third estimator for Λ by combining the two scenarios indicating a consideration for both historical and latent risks.

$$\hat{\lambda}(t) = \mathbb{E}[\Lambda^{(1)} + \Lambda^{(2)}|N(t)],$$

where $\mathbb{P}(\Lambda^{(1)} = 0) = 0$ and $\mathbb{P}(\Lambda^{(2)} = 0) = p > 0$.

Before continuing further analysis, the practical significance of our model will be addressed first. As we have seen in previous sections, historical and latent risks are taken into account by Dubey [8] separately. Latent risks normally do not have clear information at present. However, once they broke out in a negative way, it would possibly be too late for insurance companies to control the losses. They are often refleted by a sudden burst of the number of claims rather than a single large-size claim. Therefore, it makes sense to consider these risks at the modelling stage and adjust the premiums accordingly which is exactly what we are emphasising here. Now the remaining problem is to figure out the ruin probability in such a model.

3.1 Preliminary Technical Results

Correspondingly, denoting the respective claim sizes by $Y^{(1)}$ and $Y^{(2)}$ and claim counts by $N^{(1)}(t)$ and $N^{(2)}(t)$, we write our adjusted risk surplus process in the following way,

$$\begin{aligned} dU(t) &= c\hat{\lambda}(t)dt - dS^{(1)}(t) - dS^{(2)}(t) \\ &= c\hat{\lambda}(t)dt - dS(t). \end{aligned} \tag{9}$$

where $S^{(i)}(t) = \sum_{j=1}^{N^{(i)}(t)} Y_j^{(i)}$, $i = 1, 2$; $S(t) = S^{(1)}(t) + S^{(2)}(t) = \sum_{k=1}^{N(t)} Y_k$; $N(t) = N^{(1)}(t) + N^{(2)}(t)$. We propose the following results for the underlying risk surplus process.

Lemma 3.1. *Conditioning on* $\{\Lambda^{(1)} + \Lambda^{(2)} = \lambda\}$, $N(\cdot)$ *is a Poisson process with intensity* λ. *Conditoning on* $\left\{\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}} = \theta\right\}$, *the claim sizes are i.i.d with a common distribution function* $H_\theta(y) \stackrel{def}{=} \theta F(y) + (1-\theta)G(y)$. *But the counting process* $\{N(t), t \geq 0\}$ *depends on the sequence of claim sizes* $\{Y_k, k \geq 1\}$.

Lemma 3.2. *If* $(\Lambda^{(1)}+\Lambda^{(2)})$ *is independent from* $\left(\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}\right)$ *under* $\mathbb{P}(\cdot|\Lambda^{(2)}>0)$*, then for a given* $\theta\in(0,1]$*, under* $\mathbb{P}\left(\cdot\middle|\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}=\theta\right)$*, Model (9) could be reduced to Dubey's[8] model for Case 1.*

Both proofs could be seen from Appendix. To rephrase these lemmas, we claim that if $(\Lambda^{(1)}+\Lambda^{(2)})$ and $\left(\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}\right)$ are indpendent under $\{\Lambda^{(2)}>0\}$, then for any fixed $\theta\in(0,1]$, under $\mathbb{P}\left(\cdot\middle|\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}=\theta\right)$, the risk surplus process of this extended model is shown as follows.

$$dU_\theta(t)=c\hat{\lambda}(t)dt-d\sum_{k=1}^{N(t)}Y_k^\theta,$$

with a mixed Poisson process $N(\cdot)$ described by a randomized intensity and i.i.d claim sizes having identical distribution function $H_\theta(y)$. This risk surplus process is then exactly the same as Dubey's [8] model. More specifically, it is applicable for Case 1, because it is true that the underlying mixed Poisson process has a positive intensity.

Therefore, it is intriguing now to look into more details about the condition. It has been found that the independent property is satisfied if and only if the two variables have Gamma distributions with the same scale parameter. (See Lukacs's proportion-sum independence theorem in [10].) More precisely, we propose the following lemma whose proof is found in Appendix.

Lemma 3.3. *If* $\Lambda^{(1)}\sim\Gamma(\alpha,\lambda_0)$ *and* $\Lambda^{(2)}|_{\Lambda^{(2)}>0}\sim\Gamma(\beta,\lambda_0)$ *for some* $\alpha,\beta,\lambda_0>0$*, then we have,* $\left(\Lambda^{(1)}+\Lambda^{(2)}\right)|_{\Lambda^{(2)}>0}\sim\Gamma(\alpha+\beta,\lambda_0)$*,* $\left.\left(\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}\right)\right|_{\Lambda^{(2)}>0}\sim Beta(\alpha,\beta)$*, and they are independent.*

In other words, we found particular distribution functions for $\Lambda^{(1)}$ and $\Lambda^{(2)}|_{\Lambda^{(2)}>0}$ in order to ensure the desired condition satisfied. Additionally, a specific distribution for $\left.\left(\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}\right)\right|_{\Lambda^{(2)}>0}$ could also be determined which is a Beta in this case.

3.2 Calculate the Ruin Probability

In the following part, we explain two possible methods to calculate the ruin probability. To simplify notations, we denote $\xi=\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}$ in the sequel.

First of all, according to previous discussions, we know that when both $\Lambda^{(1)}$ and $\Lambda^{(2)}|_{\lambda^{(2)}>0}$ are Gamma distributed with the same scale parameter λ_0, and have the shape parameter α and β respectively, $\xi|_{\xi\neq 1}$ is $Beta(\alpha,\beta)$ distributed with $\{\xi\neq 1\}=\{\lambda^{(2)}>0\}$ (Lemma 3.3). In addition, for a fixed $\theta\in(0,1]$, conditioning on $\{\xi=\theta\}$, the surplus process can be reduced to the one in [8] where the conditional ruin probability (7) coincides with that of the classical risk process with parameter $(c, 1, H_\theta(\cdot))$ (Lemma 3.1, 3.2). Hence, the ruin probability for the underlying risk surplus process depends mostly on ξ which could be calculated using,

$$\begin{aligned}\psi(u) &= \mathbb{E}(\psi_\xi(u)) \\ &= \frac{1-p}{B(\alpha,\beta)} \int_{(0,1)} \psi_\theta(u)\theta^{\alpha-1}(1-\theta)^{\beta-1}\, d\theta + p \cdot \psi_1(u).\end{aligned}$$

where $\psi_\theta(u)$ is the conditional probability of ruin defined by

$$\psi_\theta(u) = P\left(\tau < \infty \middle| U(0) = u, \frac{\Lambda^{(1)}}{\Lambda^{(1)} + \Lambda^{(2)}} = \theta\right). \tag{10}$$

Firstly, notice here that $\psi_1(u)$ denotes the ruin probability when $\xi = 1$, i.e., $\Lambda^{(2)} = 0$. This clearly reduces the model to Dubey's [8] Case 1 which means $\psi_1(u) = \psi_0(u)$ (Recall $\psi_0(u)$ from Theorem 2.1). Secondly, since $\psi_\theta(u)$ is still dependent on the claim size distribution $H_\theta(\cdot)$, only when specific distribution functions are taken into account could we obtain an explicit formula for the ruin probability.
However, even for a mixture of two exponential distributions where we could have applied the result in [7], due to computational complexity, it does not appear to be trivial to obtain an explicit formula for the probability of ruin.

A direct calculation does not seem plausible thus encouraging us to seek for other approaches. We found that when the Pollaczek-Khinchin formula is employed, for $\theta \in (0, 1]$,

$$1 - \psi_\theta(u) = \begin{cases} (1 - \frac{\mu_\theta}{c}) \sum_{n\geq 0} \left(\frac{\mu_\theta}{c}\right)^n H_{e,\theta}^{*n}(u) & \text{if} \quad \mu_\theta < c, \\ 0 & \text{if} \quad \mu_\theta \geq c. \end{cases} \tag{11}$$

where $\mu_\theta = \theta\mu_F + (1-\theta)\mu_G$, $H_{e,\theta}(dy) = \frac{1}{\mu_\theta}(1 - H_\theta(y))\, dy = \mu_\theta^{-1}\overline{H}_\theta(y)\, dy$ is the integrated tail distribution of H_θ, and $H_{e,\theta}^{*n}(u)$ is the n-th convolution of $H_{e,\theta}$.

Theorem 3.1. *If* $\max\{\mu_F, \mu_G\} < c$*, then we obtain, for* $u > 0$*,*

$$\begin{aligned}& P\left(\tau < \infty | U(0) = u, \xi \neq 1\right) \\ = \; & 1 - (1-\eta) \sum_{l\geq 0, m\geq 0} \eta^l \rho^m \binom{m+l}{l} \frac{B(l+1+\alpha, m+\beta)}{B(\alpha,\beta)} F_e^{*l} * G_e^{*m}(u) \\ & -(1-\rho) \sum_{l\geq 0, m\geq 0} \eta^l \rho^m \binom{m+l}{l} \frac{B(l+\alpha, m+1+\beta)}{B(\alpha,\beta)} F_e^{*l} * G_e^{*m}(u)\end{aligned} \tag{12}$$

where $\eta = \mu_F/c$, $\rho = \mu_G/c$, *and* $F_e(y) = \frac{1}{\mu_F}\int_0^y (1 - F(x))\, dx$, $G_e(y) = \frac{1}{\mu_G}\int_0^y (1 - G(x))\, dx$.

This proof is to be seen in Appendix. If we further introduce $F^\gamma(t,u)$ and $G^\gamma(t,u)$ as follows, for $t \in (0,1)$ and $\gamma > 0$,

$$F^\gamma(t,u) = \sum_{l \geq 0} \binom{-\gamma}{l} (-t\eta)^l \, (F_e)^{*l}(u) \quad \text{and} \quad G^\gamma(t,u) = \sum_{l \geq 0} \binom{-\gamma}{l} (-t\rho)^l \, (G_e)^{*l}(u), \tag{13}$$

Together with the notations introduced above, the ruin probability could be rewritten in the following way which is proved in Appendix,

Corollary 3.1. *If* $\max\{\mu_F, \mu_G\} < c$*, then for* $u > 0$*,*

$$\begin{aligned} \psi(u)|_{\xi \neq 1} &= 1 - \alpha(1-\eta) \int_0^1 (1-t)^{\alpha+\beta-1} \int_0^u F^{\alpha+1}(t, u-y) G^\beta(t, dy)\, dt \\ &\quad - \beta(1-\rho) \int_0^1 (1-t)^{\alpha+\beta-1} \int_0^u F^\alpha(t, u-y) G^{\beta+1}(t, dy)\, dt. \end{aligned} \tag{14}$$

Our results could be further interpreted by the following example which simply considers two different exponential distributions for F and G respectively.

Example 3.1. *If* $F \sim \exp(\zeta_1)$*,* $G \sim \exp(\zeta_2)$ *and* α, β *are integers, the probability of ruin could be shown by the following formulae.*

$$\begin{aligned} \psi(u)|_{\xi \neq 1} &= 1 - \alpha(1-\eta) \left[\frac{1}{\alpha+\beta} + e^{-\zeta_1 u} \sum_{j=1}^{\beta} \binom{\beta}{j} (\rho\zeta_2)^j \sum_{i=1}^{\alpha+1} \binom{\alpha+1}{i} (\eta\zeta_1)^i \frac{u^{i+j-1}}{\Gamma(i+j)} \right. \\ &\quad \left. \times \int_0^1 (1-t)^{\alpha+\beta-1} t^{i+j} e^{\zeta_1 t\eta u} M_{X(i,j)}(-[(\zeta_1\eta - \zeta_2\rho)t - \zeta_1 + \zeta_2]u)\, dt \right] \\ &\quad - \beta(1-\rho) \left[\frac{1}{\alpha+\beta} + e^{-\zeta_1 u} \sum_{j=1}^{\beta+1} \binom{\beta+1}{j} (\rho\zeta_2)^j \sum_{i=1}^{\alpha} \binom{\alpha}{i} (\eta\zeta_1)^i \frac{u^{i+j-1}}{\Gamma(i+j)} \right. \\ &\quad \left. \times \int_0^1 (1-t)^{\alpha+\beta-1} t^{i+j} e^{\zeta_1 t\eta u} M_{X(i,j)}(-[(\zeta_1\eta - \zeta_2\rho)t - \zeta_1 + \zeta_2]u)\, dt \right], \end{aligned} \tag{15}$$

where ${}_1F_1(\cdot)$ *is a hypergeometric function with order 1,1 whose definition is given as follows.*

$${}_1F_1(a;b;z) = \sum_{k=0}^{\infty} \frac{(a)_k}{(b)_k} \frac{z^k}{k!},$$

where $(c)_k = c(c+1)\ldots(c+k-1)$ *with* $(c)_0 = 1$*.*

Detailed proof is available in Appendix.

Remark 3.1. *From the other perspective, we could take the Laplace Transform of (14) and the following result is obtained.*

$$\begin{aligned}\hat{\psi}_\xi(s)|_{\xi\neq 1} &= \frac{1}{s} - \alpha(1-\eta)\int_0^1 (1-t)^{\alpha+\beta-1}\left(1-\frac{t\eta\zeta_1}{\zeta_1+s}\right)^{-(\alpha+1)}\left(1-\frac{t\rho\zeta_2}{\zeta_2+s}\right)^{-\beta} dt \\ &\quad -\beta(1-\rho)\int_0^1 (1-t)^{\alpha+\beta-1}\left(1-\frac{t\eta\zeta_1}{\zeta_1+s}\right)^{-\alpha}\left(1-\frac{t\rho\zeta_2}{\zeta_2+s}\right)^{-(\beta+1)} dt.\end{aligned}$$

4 Conclusions

In modern era, the use of mobile phones may or may not cause significant losses to insurance companies, however it is worthwhile accounting for its risk. These kind of emerging risks are referred to as latent claims. In this paper we incorporate these latent risks together with historical ones in a risk model, by means of adjusting the premium rates. Considering first a model where only latent risks are considered and then a combination of both latent and classical claims we derive relationships between the probability of thin in the classical case, versus the case where the premiums are adjusted to the history of claims. The differences are amenable and thus this theory should encourage insurance companies to use adjusted premium rates in an attempt to reward their good customers, as in a classical Bonus-Malus system.

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APPENDIX

Proof of Lemma 3.1

Proof. It is true that under the condition $\{\Lambda^{(1)}+\Lambda^{(2)}=\lambda, \frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}=\theta\}$, $S(\cdot)$ is a compound Poisson process with parameter $(\lambda, H_\theta(y))$, and for any $n\in\mathbb{N}$ and $t_k, x_k\geq 0$, we have,

$$\mathbb{P}\left(\tau_k>t_k, Y_k\leq y_k, k=1,\ldots,n\middle|\Lambda^{(1)}+\Lambda^{(2)}=\lambda, \frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}=\theta\right)=\prod_{k=1}^{n}e^{-\lambda t_k}H_\theta(y_k), \quad (16)$$

where τ_k denotes the interarrival time between the $(k-1)^{th}$ and the k^{th} claim. Then,

$$\mathbb{P}\left(Y_k\leq y_k, k=1,\ldots,n\middle|\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}=\theta\right)=\prod_{k=1}^{n}H_\theta(y_k);$$

$$\mathbb{P}\left(\tau_k>t_k, k=1,\ldots,n\middle|\Lambda^{(1)}+\Lambda^{(2)}=\lambda\right)=\prod_{k=1}^{n}e^{-\lambda t_k}.$$

The conclusion of the first assertion is straight forward. □

Proof of Lemma 3.2

Proof. If $(\Lambda^{(1)}+\Lambda^{(2)})$ and $\left(\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}\right)$ are conditionally independent under $\{\Lambda^{(2)}>0\}$. Given any $\theta\in(0,1)$, the conditional independence implies

$$\mathbb{P}\left((\Lambda^{(1)}+\Lambda^{(2)})\in d\lambda\middle|\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}=\theta\right)=\mathbb{P}\left((\Lambda^{(1)}+\Lambda^{(2)})\in d\lambda\middle|\Lambda^{(2)}>0\right),$$

since $\left\{\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}=1\right\}=\{\Lambda^{(2)}=0\}$ and $\left\{\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}\in B\right\}=\left\{\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}\in B\right\}\cap\{\Lambda^{(2)}>0\}$, $\forall B\in\mathcal{B}(0,1)$. Therefore, it follows from identity (16) that $\forall A\in\mathcal{B}(\mathbb{R}^+)$,

$$\begin{aligned}
&\mathbb{P}\left(\tau_k>t_k, Y_k\leq y_k, k=1,\ldots,n, \Lambda^{(1)}+\Lambda^{(2)}\in A\middle|\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}=\theta\right)\\
=&\int_{\lambda\in A}\prod_{k=1}^{n}e^{-\lambda t_k}H_\theta(y_k)\mathbb{P}\left(\Lambda^{(1)}+\Lambda^{(2)}\in d\lambda\middle|\frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}=\theta\right)\\
=&\prod_{k=1}^{n}H_\theta(y_k)\left[\int_{\lambda\in A}\prod_{k=1}^{n}e^{-\lambda t_k}\mathbb{P}\left(\Lambda^{(1)}+\Lambda^{(2)}\in d\lambda\middle|\Lambda^{(2)}>0\right)\right]\\
=&\prod_{k=1}^{n}H_\theta(y_k)\times\mathbb{P}(\tau_k>t_k, k=1,\ldots,n, \Lambda^{(1)}+\Lambda^{(2)}\in A|\Lambda^{(2)}>0).
\end{aligned}$$

The identity above implies that, for every $\theta \in (0,1)$, under measure $\mathbb{P}\left(\cdot \middle| \frac{\Lambda^{(1)}}{\Lambda^{(1)}+\Lambda^{(2)}}=\theta\right)$, the claim sizes $\{Y_k, k\geq 1\}$ are i.i.d with a common distribution function $H_\theta(\cdot)$, the counting process $N(\cdot)$ is a mixed Poisson process with intensity $(\Lambda^{(1)}+\Lambda^{(2)})|_{\Lambda^{(2)}>0}$. More importantly, they are mutually independent so is the case conditioning on $\{\Lambda^{(2)}=0\}$ with $N(\cdot)$ as a mixed Poisson process with intensity $\Lambda^{(1)}$. □

Proof of Lemma 3.3

Proof. Basically, denoting $\gamma_1=\Lambda^{(1)}, \gamma_2=\Lambda^{(2)}|_{\Lambda^{(2)}>0}$, we have

$$\mathbb{P}\left((\gamma_1+\gamma_2)\in du, \frac{\gamma_1}{\gamma_1+\gamma_2}\in dv\right) = f_{\gamma_1}(uv)f_{\gamma_2}(u(1-v))u\,du\,dv$$

$$= \frac{\lambda_0^\alpha\lambda_0^\beta}{\Gamma(\alpha)\Gamma(\beta)}(uv)^{\alpha-1}(u(1-v))^{\beta-1}e^{-\lambda_0 u}u\,du\,dv$$

$$= \left(\frac{\lambda_0^{\alpha+\beta}}{\Gamma(\alpha+\beta)}u^{\alpha+\beta-1}e^{-\lambda_0 u}\,du\right)\cdot\left(\frac{1}{B(\alpha,\beta)}v^{\alpha-1}(1-v)^{\beta-1}\,dv\right),$$

where $f_{\gamma_1}(\cdot), f_{\gamma_2}(\cdot)$ are the density functions for γ_1 and γ_2 respectively, and $B(\alpha,\beta)=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$. And the lemma is proved. □

Proof of Theorem 3.1

Proof. If we let $\mu_F/c=\eta, \mu_G/c=\rho$, then for any fixed $\theta\in(0,1)$ such that $\theta\mu_F+(1-\theta)\mu_G<c$,

$$\mu_\theta H_{e,\theta}(y)=\int_0^y(1-\theta F(x)-(1-\theta)G(x))\,dx=\theta\mu_F F_e(y)+(1-\theta)\mu_G G_e(y),$$

where $F_e(y)=\frac{1}{\mu_F}\int_0^y(1-F(x))\,dx$, $G_e(y)=\frac{1}{\mu_G}\int_0^y(1-G(x))\,dx$. Hence,

$$\begin{aligned}
1-\psi_\theta(u) &= (\theta(1-\eta)+(1-\theta)(1-\rho))\sum_{n\geq 0}\left(\frac{1}{c}\right)^n(\theta\mu_F F_e(\cdot)+(1-\theta)\mu_G G_e(\cdot))^{*n}(u)\\
&= (\theta(1-\eta)+(1-\theta)(1-\rho))\sum_{n\geq 0}\sum_{0\leq l\leq n}\binom{n}{l}\theta^l(1-\theta)^{n-l}\left(\frac{\mu_F^l\mu_G^{n-l}}{c^n}\right)F_e^{*l}*G_e^{*(n-l)}(u)\\
&= (1-\eta)\sum_{l\geq 0,m\geq 0}\binom{m+l}{l}\eta^l\rho^m\theta^{l+1}(1-\theta)^m\left(F_e^{*l}*G_e^{*m}\right)(u)\\
&\quad+(1-\rho)\sum_{l\geq 0,m\geq 0}\binom{m+l}{l}\eta^l\rho^m\theta^l(1-\theta)^{m+1}\left(F_e^{*l}*G_e^{*m}\right)(u).
\end{aligned}$$

Then an integration over ξ on $\{\xi\neq 1\}$ using the probability density function of $Beta(\alpha,\beta)$ on both sides will lead to the desired result as shown in the theorem. □

Proof of Corollary 3.1

Proof. It can be seen that for $l, m \geq 0$, we have

$$\begin{aligned}
& \binom{m+l}{l}\frac{B(l+1+\alpha, m+\beta)}{B(\alpha,\beta)} = \frac{\Gamma(l+1+\alpha)}{\Gamma(l+1)\ \Gamma(\alpha)}\ \frac{\Gamma(m+\beta)}{\Gamma(m+1)\ \Gamma(\beta)}\ \frac{(m+l)!\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+m+l+1)} \\
= \ & \alpha\frac{(\alpha+l)(\alpha+l-1)\cdots(\alpha+1)}{l(l-1)\cdots 1}\cdot\frac{(\beta+m-1)(\beta+m-2)\cdots(\beta+1)\beta}{m(m-1)\cdots 1}\cdot B(\alpha+\beta, m+l+1) \\
= \ & \alpha(-1)^{l+m}\binom{-\alpha-1}{l}\binom{-\beta}{m}\int_0^1 t^{m+l}(1-t)^{\alpha+\beta-1}\,dt,
\end{aligned}$$

by adopting the property of a negative binomial distribution function where it allows for real-valued α, β. We further introduced notations from (13) through which we could write,

$$\begin{aligned}
& \sum_{l\geq 0, m\geq 0}\eta^l\rho^m\binom{m+l}{l}\frac{B(l+1+\alpha, m+\beta)}{B(\alpha,\beta)}F_e^{*l} * G_e^{*m}(u) \\
= \ & \alpha\int_0^1 (1-t)^{\alpha+\beta-1}F^{\alpha+1}(t, u-y)G^{\beta}(t, dy)\,dt.
\end{aligned}$$

Clearly, $F^\gamma(t,u)\ (G^\gamma(t,u))$ increases on $[0,1)\times\mathbb{R}^+$ with respect to (t,u), $F_\gamma(t,0)=1, F^\gamma(t,\infty)=(1-t\eta)^{-\gamma}$, and $G^\gamma(t,0)=1, G_\gamma(t,\infty)=(1-t\rho)^{-\gamma}$. Actually, taking the Laplace transform of $F^\gamma(t_0,\cdot)$ yields,

$$\int_{[0,\infty)} e^{-su}F^\gamma(t_0, du) = \sum_{l\geq 0}\binom{-\gamma}{l}(-t_0\eta)^l(\hat{F}_e(s))^l = \left(1-t_0\eta\hat{F}_e(s)\right)^{-\gamma},$$

which demonstrates that $F^\gamma(t,u)$ is proportional to a cumulative distribution function of a γ-convolution of compound geometry distribution.
Similarly, we have

$$\binom{m+l}{l}\frac{B(l+\alpha, m+1+\beta)}{B(\alpha,\beta)} = \beta(-1)^{l+m}\binom{-\alpha}{l}\binom{-\beta-1}{m}\int_0^1 t^{m+l}(1-t)^{\alpha+\beta-1}\,dt.$$

These directly lead to the equations shown in Corollary 3.1. □

5.1 Proof of Example 3.1

Proof. In fact, $\hat{F}_e(s)=\frac{\zeta_1}{\zeta_1+s}$, $\eta=(\zeta_1 c)^{-1}$, for $t_0\in(0,1)$,

$$\left(1-\frac{t_0\eta\zeta_1}{\zeta_1+s}\right)^{-1} = \int_0^\infty e^{-sy}\left(\delta_0(dy)+t_0\eta\zeta_1 e^{-\zeta_1(1-t_0\eta)y}\right)\,dy,$$

then, for any $\gamma \in \mathbb{N}$, we have

$$F^{\gamma}(t_0, dy) = \delta_0(\, dy) + \left(\sum_{l=1}^{\gamma} \binom{\gamma}{l} (t_0 \eta \zeta_1)^l \frac{y^{l-1}}{\Gamma(l)} e^{-\zeta_1 (1 - t_0 \eta) y} \right) dy, \tag{17}$$

where δ_0 denotes the Dirac measure centered at 0. Similarly, we have $\hat{G}_e(s) = \frac{\zeta_2}{\zeta_2 + s}$, $\rho = (\zeta_2 c)^{-1}$ and

$$\int_{[0,\infty)} e^{-sy} G^{\gamma}(t_0, dy) = \sum_{l \geq 0} \binom{-\gamma}{l} (-t_0 \rho)^l (\hat{G}_e(s))^l = \left(1 - t_0 \rho \hat{G}_e(s)\right)^{-\gamma}.$$

Hence, for any $\gamma \in \mathbb{N}$,

$$G^{\gamma}(t_0, dy) = \delta_0(\, dy) + \left(\sum_{l=1}^{\gamma} \binom{\gamma}{l} (t_0 \rho \zeta_2)^l \frac{u^{l-1}}{\Gamma(l)} e^{-\zeta_2 (1 - t_0 \rho) y} \right) dy.$$

Before continuing (14), first the following convolution is calculated,

$$\begin{aligned}
& \int_0^u F^{\alpha+1}(t, u-y) G^{\beta}(t, y) dy \\
= \; & 1 + e^{-\zeta_1 u + \zeta_1 t \eta u} \sum_{j=1}^{\beta} \binom{\beta}{j} \frac{(t\rho\zeta_2)^j}{\Gamma(j)} \sum_{i=1}^{\alpha+1} \binom{\alpha+1}{i} \frac{(t\eta\zeta_1)^i}{\Gamma(i)} \int_0^u e^{-[(\zeta_1 \eta - \zeta_2 \rho)t - \zeta_1 + \zeta_2] y} (u-y)^{i-1} y^{j-1} dy \qquad (18) \\
= \; & 1 + e^{-\zeta_1 u + \zeta_1 t \eta u} \sum_{j=1}^{\beta} \binom{\beta}{j} (t\rho\zeta_2)^j \sum_{i=1}^{\alpha+1} \binom{\alpha+1}{i} (t\eta\zeta_1)^i \frac{u^{i+j-1}}{\Gamma(i+j)} {}_1F_1(i, i+j, -[(\zeta_1 \eta - \zeta_2 \rho)t - \zeta_1 + \zeta_2] u),
\end{aligned}$$

where 1 results from an integration of the product of two Dirac measures, $1 = \int_0^u \delta_0^2(dy)$, and ${}_1F_1(\cdot)$ is a hypergeometric function with order 1,1 whose definition is given as follows.

$${}_1F_1(a; b; z) = \sum_{k=0}^{\infty} \frac{(a)_k}{(b)_k} \frac{z^k}{k!},$$

where $(c)_k = c(c+1)\dots(c+k-1)$ with $(c)_0 = 1$. In fact, it relates to a moment generating function of a Beta distributed random variable X with parameters i, j, i.e., $X \sim Beta(i, j)$.

$$M_X(-[(\zeta_1 \eta - \zeta_2 \rho)t - \zeta_1 + \zeta_2] u) = {}_1F_1(i, i+j, -[(\zeta_1 \eta - \zeta_2 \rho)t - \zeta_1 + \zeta_2] u),$$

which could be seen from the nature of the integral in (18). Thus, (14) could be written as,

$$\begin{aligned}\psi(u)|_{\xi\neq 1} &= 1-\alpha(1-\eta)\left[\frac{1}{\alpha+\beta}+e^{-\zeta_1 u}\sum_{j=1}^{\beta}\binom{\beta}{j}(\rho\zeta_2)^j\sum_{i=1}^{\alpha+1}\binom{\alpha+1}{i}(\eta\zeta_1)^i\frac{u^{i+j-1}}{\Gamma(i+j)}\right.\\ &\quad\left.\times\int_0^1(1-t)^{\alpha+\beta-1}t^{i+j}e^{\zeta_1 t\eta u}M_{X(i,j)}(-[(\zeta_1\eta-\zeta_2\rho)t-\zeta_1+\zeta_2]u)\,dt\right]\\ &\quad-\beta(1-\rho)\left[\frac{1}{\alpha+\beta}+e^{-\zeta_1 u}\sum_{j=1}^{\beta+1}\binom{\beta+1}{j}(\rho\zeta_2)^j\sum_{i=1}^{\alpha}\binom{\alpha}{i}(\eta\zeta_1)^i\frac{u^{i+j-1}}{\Gamma(i+j)}\right.\\ &\quad\left.\times\int_0^1(1-t)^{\alpha+\beta-1}t^{i+j}e^{\zeta_1 t\eta u}M_{X(i,j)}(-[(\zeta_1\eta-\zeta_2\rho)t-\zeta_1+\zeta_2]u)\,dt\right]. \qquad (19)\end{aligned}$$

□

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